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Cellular automata for one-lane traffic flow modeling

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Abstract

A cellular automata model is proposed to simulate microscopic traffic flow. In this type of models, based on methods from statistical physics, vehicles follow a reduced set of rules that can be applied in parallel. This allows simulation of large traffic networks with a reasonable computational effort. A cellular automata model is applied to a single-lane highway with a ring topology. Simulation results show the ability of this modeling paradigm to capture the most important features of the traffic flow phenomena.

Keywords: Traffic flow models; Microsimulation; Cellular automata

1. Introduction

In recent years, most industrialized societies have begun to see the limits of the growth of urban traffic. The traffic demand in metropolitan areas has largely exceeded the vehicular capacity producing a decrease in overall safety and an increase in economic losses and environmental pollution due to the longer trip times in these congested systems. Typical solutions, such as the expansion of

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roadway systems or road improvement, do not work well anymore for several social, physical and economical reasons (Varaiya, 1993; Shladover et al., 1991; Rillings, 1997). Solutions are now oriented to a better management of existing systems, using advanced technologies for that purpose. An accurate forecast of the impacts of these technologies is therefore critical before their final deployment.

Computer simulations as a means for evaluating control and management strategies in traffic systems have gained considerable importance because of the possibility of taking into account the dynamical aspects of traffic (see, for example, Eskafi et al., 1995; Antoniotti et al., 1998; SIAS-Transport-Planners, 2002; Transport-Simulation-Systems, 2002). Micro-simulation of traffic has become more popular as it can reproduce a large variety of phenomena observed in traffic. There is a recent trend on taking advantage of models developed originally in other research areas such as physics, mathematics, and computer science and applying them to traffic flow micro-simulation. Examples can be found in Los-Alamos-National-Laboratory (2001), Center for Parallel Computing (1995), US Department of Transportation (1992).

This paper describes one such model, the cellular automata model (CA) that originated in statistical physics in the study of particle behavior. [1] Several applications of CA to traffic flow can be found in Blue et al. (1996), Nagel (1996), Schadschneider and Schreckenberg (1993), Villar and de Souza (1994). Cellular automata are dynamic models developed to describe transportation phenomena. Space is discrete and consists of a regular grid of cells, each one of which can be in one of a finite number of possible states. The number and array of cells in the grid depends on the specific transportation phenomena being modeled. All cell states are updated synchronously in discrete time steps. Updating obeys a finite set of local interaction rules, that can have probabilistic influence. The new state of a cell is determined by the actual state of the cell itself and its neighbor cells. This local interaction allows to capture micro-level dynamics and propagates it to macro-level behavior. From the perspective of traffic flow, it is possible to relate the cell states with significant quantities such as travel time, vehicle speed, throughput, etc. Rules of interaction can be designed for complex highway topologies, such as multilane highways (Nagel, 1996).

The discrete nature of space, time and state of CA allows very fast calculations in comparison with other models that are based on continuous models. Still, CA are able to appropriately capture traffic behavior, as was demonstrated in Nagel (1996) with standard traffic flow in Germany. This velocity of calculation can be a critical advantage when simulating large highway networks with thousand of vehicles circulating on them.

This paper presents a CA probabilistic model for traffic simulation. The model extends the work of Nagel and Schreckenberg (1992), that first introduced CA for traffic simulation and applied them in Los-Alamos-National-Laboratory (2001). Rules of interaction in Nagel and Schreckenberg (1992) are modified to better capture driver reactions to traffic that are intended to preserve safety on the highway. As a result, a safety distance parameter is included in the model. This parameter is related to other safety analysis, as those presented in Alvarez and Horowitz (1999), Godbole and Lygeros (1994), Carbaugh et al. (1997) for manual and automated highway systems. By appropriately tuning this parameter, different traffic situations of manual, automated and mixed traffic can be considered. Although the CA model can be applied to multiple lane

[1] For this reason CA are often referred to as *particle hopping models.*

highways, in this paper a one-lane highway with a ring topology is used. The goal is to show the ability of the CA model paradigm to capture the basic phenomena of traffic flow. This closed boundary approach has been used by other authors, such as Li and Shrivastava (2002), to analyze traffic flow stability.

Simulation results presented confirm that this CA model can reproduce most common regimes in traffic: free flow, transition flow and congested flow (Lárraga and del Río, 2002). The relations derived from the density/velocity and density/flow curves are in agreement with the empirical fundamental diagrams that describe these relations in traffic analysis. The influence of the variation of speed on the flow is also found to be a factor of great importance in traffic synchronization.

The paper is organized as follows. A description of the proposed CA model is presented in Section 2. Section 3 contains simulation results in the form of fundamental diagrams. Special emphasis on the newly introduced safety parameter is given. Finally, Section 4 contains concluding remarks and a summary of findings.

2. Definition of the model

The model presented here is a probabilistic cellular automaton. It consists of N vehicles moving in one direction on a one-dimensional lattice of L cells arranged in a ring topology. The number of vehicles is fixed. Each cell is either empty, or is occupied by just one vehicle traveling with velocity v, that takes values ranging from 0 to $v_{\max}$. This speed limit can be different depending on the kind of vehicle under consideration: trucks, cars, etc. For simplicity, in this paper only one type of vehicles is considered and therefore the same maximum velocity will be used for all vehicles.

The integer velocity, that corresponds to one of the vehicle states in this CA, is related with the number of cells that a vehicle advances in one time step. [2] The other state, position, is related with the cell that each vehicle is occupying.

The typical length of a cell (Δx) is around 7.5 m. It is interpreted as the length of a vehicle plus the distance between vehicles in a jam, but it can be suitably adjusted according to the problem under consideration. The time step (Δt) is taken to be 1 s. Therefore, transitions are from $t \rightarrow t + 1$. This time step is on the order of humans reaction time. It can also be easily modified. With these values of Δx and Δt, $v = 1$ corresponds to a vehicle moving from one cell to the downstream neighbor cell, and translates into 27 km/h. The maximum velocity is set to $v_{\max} = 5$, which is equivalent to 135 km/h.

Due to the discrete nature of space and time in CA models, it is convenient to normalize units of distance, velocity and time with respect to the length of each cell Δx and the time step Δt. Therefore, units in position x denote the number of cell in the lattice; in velocity v, number of cells per unit time, and in time t, number of time steps. [3]

Let v_i and x_i denote the current velocity and position of vehicle i, and v_p and x_p be the velocity and position of the vehicle ahead (preceding vehicle) at a given time; $d_i := x_p - x_i - 1$ denotes the distance (number of empty cells) in front of the vehicle in position x_i. [4]

[2] Provided that there are no vehicles that obstruct its forward movement.

[3] For example, $v < d$ is used instead of $v < d/\Delta t$, because $\Delta t = 1$.

[4] Bumper to bumper headway.

State transitions are defined with the following set of rules, which are applied simultaneously to all vehicles:

S1: *Acceleration*. If $v_i < v_{\max}$, the velocity of vehicle i is increase by one

$$v_i \to \min(v_i + 1, v_{\max})$$

S2: *Randomization*. If $v_i > 0$, the velocity of vehicle i is decreased randomly by one, with probability R

$$v_i \to \max(v_i - 1, 0) \quad \text{with probability } R.$$

S3: *Deceleration*. [5] If $\text{round}(d_i + (1 - \alpha) \cdot v_p) < v_i$, the velocity of vehicle i is reduced to $\text{round}(d_i + (1 - \alpha) \cdot v_p)$

$$v_i \to \min(v_i, \text{round}(d_i + (1 - \alpha) \cdot v_p)) \quad \alpha \in [0, 1].$$

where the function 'round' truncates its argument to the closest integer.

S4: *Vehicle movement*. Each vehicle is moved forward according to its new velocity determined in steps 1–3

$$x_i \to x_i + v_i$$

Rules S1, S2 and S3 are designed to update velocity of vehicles; rule S4 updates position. According to this, state updating is divided into two stages, first velocity, second position. The rationale behind rules S1, S2 and S3 is as follows.

S1. This rule postulates that all the drivers strive to reach the maximum velocity whenever possible. This is in agreement with other velocity policies, as is the case with the greedy policy in Broucke and Varaiya (1996).
S2. This rule represents unseen factors that cause drivers to reduce their speed for no apparent reason. These situations include, for example, incidents along the highway that distract drivers. This random braking can contribute to creation of traffic jams.
S3. This rule is designed to avoid vehicles collision. The term $\text{round}(d_i + (1 - \alpha) \cdot v_p)$ represents the speed that driver estimates it should have in order to avoid collision with the preceding vehicle during the upcoming time step. By varying the parameter α, referred to here as the safe distance parameter, this estimation can be tuned. The physical interpretation of α will be discussed latter.

Rule S3 is the main modification to the model in Nagel and Schreckenberg (1992), that will be referred to as the NaSch model. First, it should be noticed that the deceleration rule of the NaSch model is recovered when the value $\alpha = 1$ is used. Therefore, rule S3 can be seen as a

[5] It may be needed to apply this rule several times, as will be explained later on.

generalization of NaSch- model deceleration rule. The NaSch-deceleration rule allows for deceleration values that are not physically feasible. For example, if a vehicle is traveling at maximum speed $v = 5$ and runs into a stopped vehicle, the NaSch-deceleration rule commands a reduction from $v = 5$ to $v = 0$ in 1 s. This represents a deceleration of 37.5 [m/s^2], clearly beyond physical capabilities of ground vehicles. On the other hand, there are scenarios where vehicles are not required to keep separation from the vehicles in front. This is the case, for example, in platooning schemes (Varaiya, 1993; Swaroop et al., 1994; Swaroop and Hedrick, 1996) that accept values of α closer to zero. [6] The parameter α, denoted here as the safety parameter, is mainly related with the degree of drivers aggressiveness (the closer α to zero the more aggressive the behavior), although it can be also related with the degree of highway automation (with high levels of automation allowing small inter-vehicle spaces at high speeds). For normal highways, with low levels of automation, the values of α are closer to one and can be tuned to represent feasible deceleration values.

It should be noticed that the quantity $((1-\alpha)\cdot v_p)$ in rule S3 can be seen as a safety distance that is added to d_i. This distance allows for vehicle deceleration to take more than one time step. Values of α are selected in such a way that $(d_i + (1-\alpha)\cdot v_p)$ takes discrete values. If this is not the case, the value is rounded to the closest integer. The value of α can be set on a spatial dependent manner, trying to represent, for example, places with changes of topology or curvature.

There is a price to pay with this modification that limits deceleration values. It could be the case that a deceleration in a vehicle implies also decelerations of vehicles behind, that now also have limited deceleration capability. The most critical scenario would be represented by a chain of vehicles, each one of them traveling at maximum speed, with the first vehicle running into a stopped vehicle. To set all velocities of vehicles in the chain consistently, each one of them using the maximum allowable deceleration, S3 has to be iterated $(v_{\max} - 1)$ times, assuming that velocity decreases by one from one vehicle to another in the chain.

The other relevant modification to the NaSch model is the change in the application of the deceleration and randomization rules. In the NaSch model, randomization is applied after deceleration, while in the model here proposed randomization is applied first and deceleration later. The main reason for this change is that when the deceleration values are limited, rule S3 has to be iterated to produce consistent velocity values. If randomization would occur afterwards, an additional deceleration could induce inconsistency in the velocities. In the model proposed here, the randomization rules is designed primarily to model erratic behavior from drivers. However, it must be noted that if rule S2 commands deceleration, this does not prevent deceleration rule S3 to require additional braking. In this manner overreaction while braking can still be modeled.

This CA model is a minimal model in the sense that all the four steps are necessary to reproduce the basic features of real traffic. However, additional rules may be incorporated to capture more complex situations (Knospe et al., 2000). The parameters of the model are the following: the number of cells L, the number of vehicles N, the limit speed $v_{\max}$, the random braking parameter R and the parameter of safe distance α, that in this paper takes the values $\{0, 0.25, 0.5, 0.75, 1\}$.

[6] There are, however, additional requirements to preserve safety, such as coordinated braking (Alvarez and Horowitz, 1999; Choi and Swaroop, 2001).

3. Simulation results

To simulate the CA model of the previous section, a closed system with $L = 10^4$ cells, representing a one-lane loop, is used. N vehicles are randomly distributed on the lane around the loop with an initial speed taking a discrete random value between 0 and $v_{\max}$. Since the system is closed, the average density, $\rho = N/L$, remains constant in time. Different values of N, R and α were simulated.

Each run was simulated for $T = 6*L$ time steps. To analyze results, the first half of the simulation was discarded to allow the system to reach its steady state. For each simulation, a value for parameter α was established to set the desired degree of safe distance among vehicles. For example, the value $\alpha = 0$ corresponds to minimal safety requirements allowing vehicles to occupy neighboring cells at high speeds.

A convenient way to demonstrate that the model reproduces traffic flow behavior is to plot the fundamental diagrams, that is, the relation between flow and density. Fig. 1 shows the fundamental diagram obtained for different values of α and a fixed value of $R = 0.4$. The form of the curves in Fig. 1 resembles those found in empirical studies of traffic flow analysis (Payne, 1971; Payne, 1979). Each plot in Fig. 1 is composed by the results of many simulations using the same value of R and values of number of vehicles N ranging from 0 to L. The value of N/L for each simulation is in the horizontal axis, while the average flow for that run of N is in the vertical axis.

Fig. 1 also illustrates the impact of the driving strategies coded in α. The maximum flow changes with the inverse of α, i.e. smaller values of α imply larger flows. This is in accordance with, for example, the use of platooning strategies (Varaiya, 1993) that require less safety distance and exploit the knowledge of the velocity of precedent vehicles (Swaroop and Hedrick, 1996). It is

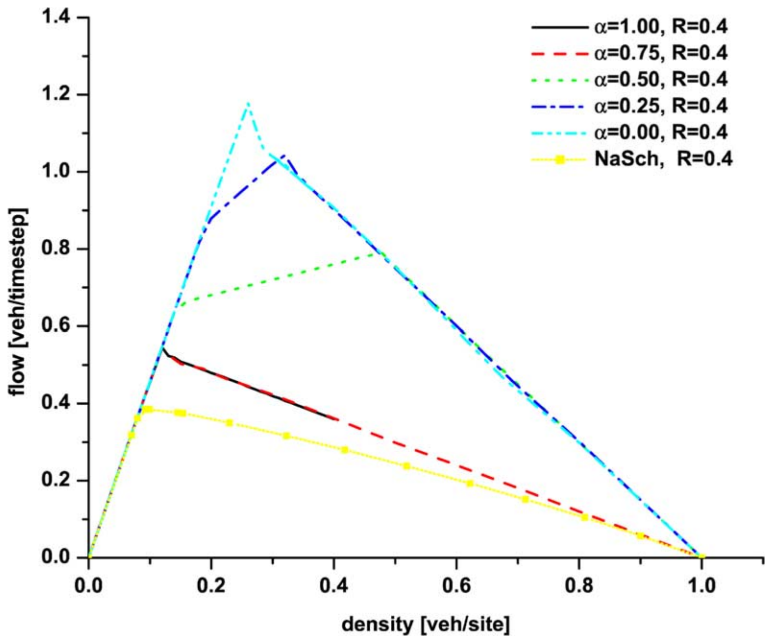

Fig. 1. Fundamental diagram for different values of safety factor, and noise $R = 0.4$.

interesting to note that values of $\alpha = 0.25$ and $\alpha = 0.5$ lead to forms of the diagram which are significantly different from the other cases, indicating a mixed condition where the maximum flow is attained with a non-maximal velocity. This is due to the presence of platoons of vehicles traveling at high speed in a synchronized fashion. The value of the density for maximum flow, ρ_m, decreases with smaller values of α, when $\alpha \leqslant 0.5$. It is also important to note that for low densities and all values of α, the slope of the fundamental diagrams is similar, indicating that vehicles travel at near maximum speed, while the slope in the congested region is only equal for values of $\alpha \leqslant 0.5$, indicating a similar velocity behavior for this situation. For comparison purposes, in Fig. 1 the fundamental diagram obtained from NaSch model with $R = 0.4$ is also presented, note that its maximum flow is lower than the all the results for the model presented here.

Fig. 2 shows the velocity diagram corresponding to the same simulation conditions as in Fig. 1. The first thing to note is that due to the presence of rule S2 the velocity does not reach the value of v_{max}. The role of the safe distance parameter α is also clear: the density interval for free-flow increase as α decreases.

The effect of the parameter R, that represents the probability of traffic disturbances, is shown in Fig. 3, where the induced velocity reduction due to large values of R can be appreciated. The greater its value, the smaller the flow, for the same density values. The location of ρ_m is pushed to the left for greater values of R. Fig. 4 corresponding to $R = 0.2$ shows that for $\alpha > 0.5$ the fundamental diagram is the same, as in Fig. 1. This indicates that when inter-vehicle distance is large, a probability of disturbances does not alter traffic flow.

It is possible to analyze higher moments of the velocity distribution, in particular speed variance, with the intention of showing the presence of synchronized traffic, that in this paper is

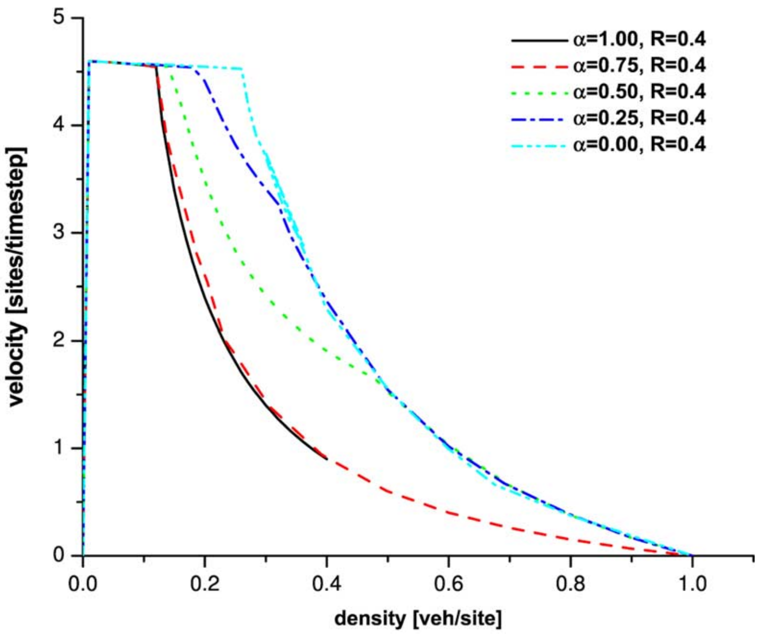

Fig. 2. Relationship between mean velocity and density for $R = 0.4$ and different values of α.

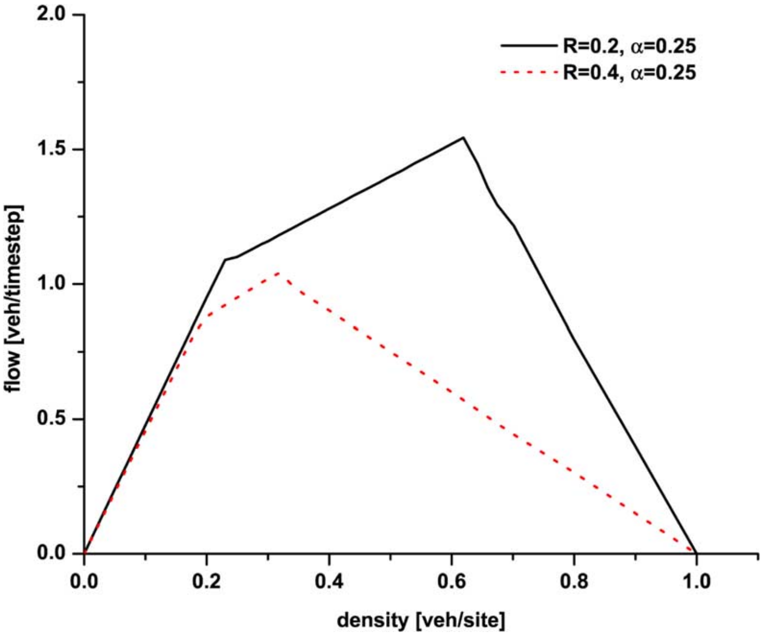

Fig. 3. Fundamental diagram for safety factor $\alpha = 0.25$ and different values of R.

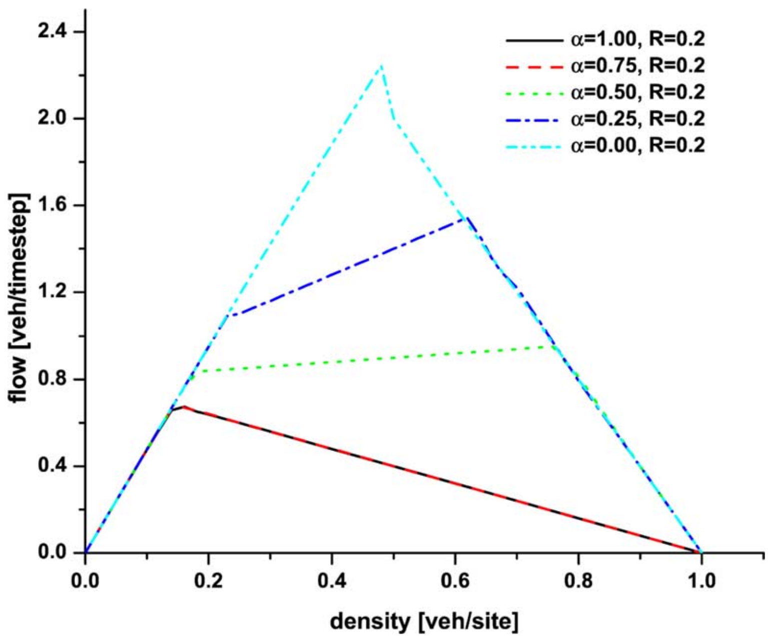

Fig. 4. Fundamental diagram for different values of the safety parameter α and $R = 0.2$.

explained by the presence of platoons of vehicles. For that purpose, the velocity of vehicles in last $L/3$ cells was analyzed during the last half of each simulation in order to detect platoons traveling

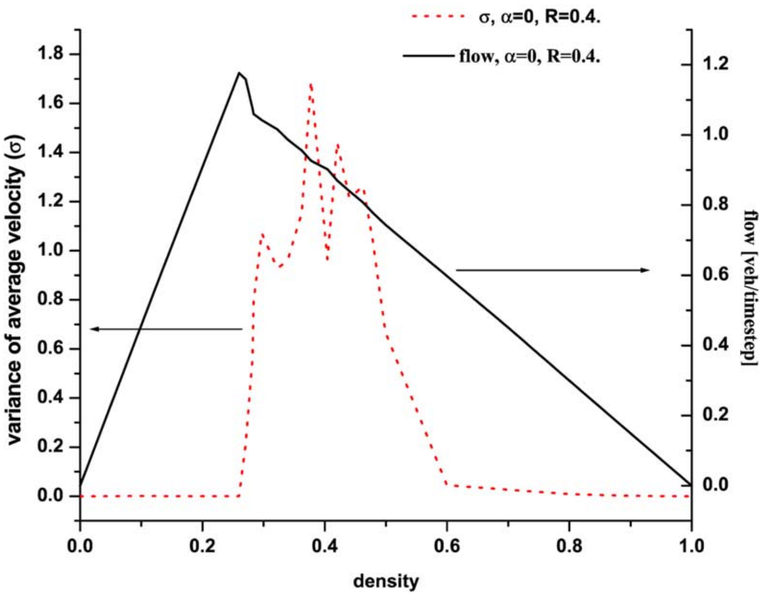

Fig. 5. Speed variance versus density for $R = 0.4$, $\alpha = 0$.

over it. Let $\bar{v}(t)$ be the average velocity of vehicles in the last $L/3$ cells at time and $\bar{v}$ the average velocity of vehicles in the same cells from time $T/2$ to time T. The speed variance is then calculated as

$$\sigma = \sqrt{\frac{1}{T - T/2} \sum_{t=T/2+1}^{T} (\bar{v}(t) - \bar{v})^2}$$

As an example, consider Fig. 5 that shows flow and the corresponding velocity variance for $R = 0.4$ and $\alpha = 0$. It is clear from this figure that variance is negligible for small density values, indicating that vehicles are traveling at almost maximum speed forming no platoons. At densities around 0.25, however, there is a sudden formation of platoons. The larger velocity variance indicates that platoons with different velocities co-exist in the highway. [7] For values of density around 0.5, variance of velocity sharply decreases, indicating a tendency for platoons to consolidate in larger platoons with vehicles traveling at the same low speed. For higher densities, above 0.6, variance is again almost zero, indicating that there are mostly very large platoons with low speed traveling on the highway. Values of $\alpha < 0.25$ and $\alpha > 5$ produce similar behavior in terms of velocity variance, although for the later magnitude of velocity variance is smaller.

Another result of velocity variance for values of $0.25 \leqslant \alpha \leqslant 0.5$ is shown in Fig. 6, that illustrates flow and velocity variance for $R = 0.4$ and $\alpha = 0.25$. In this case velocity variance shows bimodal behavior. There is small local maximum for densities around 0.25, that indicates the presence of synchronized traffic by the formation of small platoon traveling at high speed. The global

[7] In some cases, there are stopped vehicles or platoons.

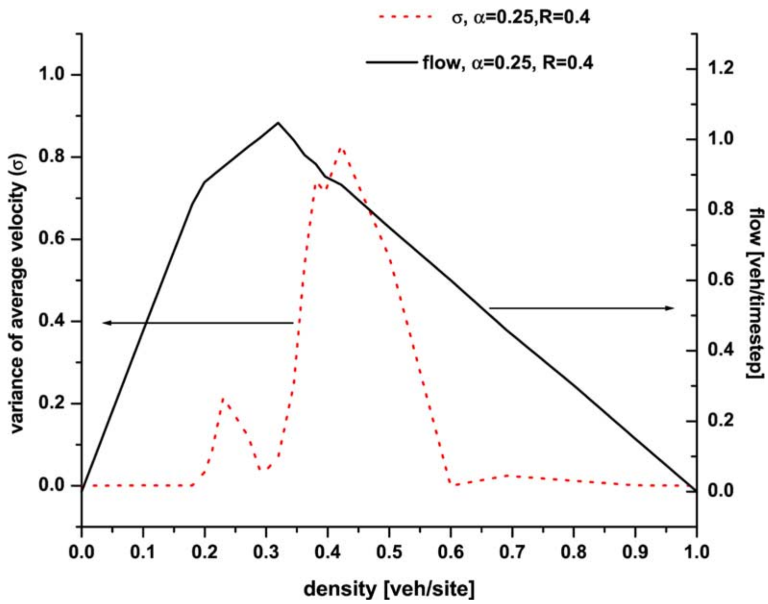

Fig. 6. Speed variance versus density for $R = 0.4$, $\alpha = 0.25$.

maximum for velocity variance is located for densities around 0.45 and is preceded and followed by sudden formation of large platoons traveling at low speed. Velocity variance for very low densities shows no platoons and for very high densities indicates again very large platoons with low speed, as in the results in Fig. 5.

By comparing the maximum flow and the maximum speed variance for $\alpha = 0$ and $\alpha = 0.25$ (Figs. 5 and 6, respectively), it is noted that the maximum flow for $\alpha = 0$ is 12% higher than for $\alpha = 0.25$. However, the maximum speed variance at $\alpha = 0.25$ is 50% smaller than for $\alpha = 0$. This is a consequence of the safety distance factor. When this factor is higher, there exists a smaller propagation of fluctuations that prevent variations of speeds by the formation of platoons or clusters of vehicles.

It should be noted that for all values of α the density where the speed variance is maximum is larger than the maximum flow density, ρ_m. However, high speed variance implies that vehicles in the system have widely varying speeds and that a vehicle would experience frequent speed changes per trip. This could also increase the probability of traffic accidents.

4. Conclusion

In this paper a cellular automata model is applied to simulate traffic flow. The model is based on a discrete time, space and state description that allows calculations in parallel. This model modifies a previous CA model, the Nagel–Shreckenberg model (Nagel and Schreckenberg, 1992) in significant ways. A safe distance parameter, α, that allows to simulate different driving strategies is included and the order of rules application is changed. The addition of this parameter proves to be useful to describe manual, automated and mixed traffic.

Although in this paper the model is simulated in a simple loop one-lane topology, it is possible to apply it to complex highway topologies and to include different vehicles types and behaviors, provided that these can be described with a finite set of rules.

Simulation results illustrate that this model captures the essential features of traffic flow encoded in the fundamental diagram, while preserving simplicity that allows rapid simulation that can prove useful for application to large scale traffic networks.

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