1. **Introduction**

Two dimensional ultrasound (US) has several disadvantages since 2D US images represent thin planes of the patients and cannot provide a complete volume of tissues and organs. These disadvantages include: the user must mentally integrate many 2D images to form a 3D impression of the anatomy, making the choice of the visualization plane difficult; finding the same location of the visualization plane is difficult when the probe is controlled manually; not all visualization planes are available because of anatomy and probe constrains1,2 [Fenster, Chen]. In the past ten years, 3D US imaging has attracted more and more researches because it can correct some of the 2D disadvantages3 [Zhu].

Several 3D US techniques have been reported in the past few years, but two main classifications can be found; techniques derived from 3D US probes and techniques that construct 3D data volumes with data obtained from 2D images4 [Estepar]. 3D US probes are more expensive compared with conventional 2D US probes, the 3D probes are also bulkier having mayor constrains for fine control of the location of the 2D planes, and the reconstructed volume geometry and size is limited by the probe5 [Honggang]. These drawbacks have lead researches to develop new approaches to convert 2D US transducers into systems capable of creating 3D images6 [Fenster 2]. Freehand 3D reconstruction technique has taken special interest; this technique offers special advantages because the operator can select optimal views and accommodate complex patient surfaces, it is not limited to the transducer geometry and size, and is low cost and flexible7 [Fenster 3]. This technique consists of manual manipulation of the probe in the usual manner over the anatomy to be view while acquiring images in arbitrary positions and orientations2 [Chen]. After image acquisition, the irregularly spaced B-scans are used to reconstruct a 3D regular volume by interpolation or approximation algorithms, the basic concept is the mapping of pixel and voxel estimation8 [Wen].

Volume reconstruction algorithms can be coarsely classified in two main approaches, Voxel Based Methods (VBM) and Pixel Based Methods (PBM). VBMs are methods that traverse the regular volume and use the position of voxels to find the appropriate pixels within the US images, these methods are able to keep the original image texture, but can produce reconstruction artifacts when the distance between planes are too big, and finding the corresponding pixel to each voxel can have a high computational cost2 [Chen]. PBMs take the position of each pixel within the B-scans and find the appropriate voxel positions within the volume; these methods usually consists of two steps: the bin-filling step (BFS), where the input pixels are traversed and the pixel value is assigned to one or several voxels; and the hole-filling step (HFS), where the voxels are traversed and empty voxels (voxels that have not been filled in the BFS) are filled9 [Solberg]. PBMs are often more accurate and allow several pixels to contribute the values of each voxel; the main disadvantages of these algorithms is that actual pixel positions are not used directly, but first resampled by inserting the pixels into the nearest voxel lowering the accuracy of the methods and also boundaries can be found between the areas filled in the BFS and areas filled in the HFS10 [Miller]. Using a 3D kernel around the pixels for determining the impact that each pixel will have on the nearby voxels may improve the accuracy of the reconstruction and reduce the number of voxels to be filled in the HFS. In this approach, usually a weight value is associated to each voxel and is used to calculate the final voxel values when several pixels contribute to a voxel9 [Solberg]. Barry et al.11 [Barry] used and spherical kernel around the pixels with an inverse distance weighting. A trilinear interpolation with a 2x2x2 kernel with linear weighting is used by Gobbi et al.12 [Gobbi]. Ohbuchi et al.13 [Ohbuchi] and Meairs et al.14 [Meairs] used Gaussian kernel, since it shape resembles the point-spread function of the ultrasound echography, where each pixel in the input slices is convolved with the truncated Gaussian kernel, with the only differences that Meairs et al.14 use an alpha blending method for increased computation speed.

The interpolation nature of reconstruction algorithms tends to reduce speckle, which may be desirable depending on the application, but it also can greatly blur image details, especially tissue boundaries15 [Huang 2]. Distance weighted algorithms, like the ones previously mentioned, are able to reduce reconstruction error, suppress speckle and preserve some coarse image details. However, not much attention has been paid to increase the preservation of important fine anatomical features, such as tissue boundaries, while reducing the speckle in the reconstruction algorithms [Huang]. In this work we propose a new Pixel Based Method that uses an adaptive Gaussian kernel in the bin-filling step, which variance is chosen as a function of the 2D image pixel gradient magnitude, in order to preserve borders while reducing speckle during the reconstruction.

1. **Previous Works**

Some VBMs have been proposed in order to reduce speckle noise and preserve tissue edges in reconstruction algorithms, but such methods preserve the mentioned disadvantages of VBMs and require huge time cost because of the shortest-distance-finding process8 [Wen]. Huang et al.16 [Huang otro] proposed an improved method for the voxel based distance weighted algorithm (DW) named squared distance weighted (SDW) interpolation that reduces the blurring in the reconstruction, using the square of the inverse distance as weight function, however the blurring of the image depends of a positive constant α (large values of α are good for speckle reduction and low values are good for edge preservation); an adaptive algorithm (Adaptive Square Distance Weighted, ASDW) for the SDW method was proposed, where the value of α is an homogeneity index dependent of the mean and variance of the region15 [Huang 2006], later Huang et al. change the squared distance of the ASDW for a Gaussian weighted distance (AGDW)17 [Huang 2009], but this method does not completely resolves the blurring problem. Doud et al.18 [Doud] proposed an edge-preserving distance weight method (EPDW) based on the ASDW, where all the images are first filtered with a speckle reducing anisotropic diffusion filter (SRAD) and the edges of each preprocessed image are detected using eight newton filters, later the edge information is used to perform the adaptive weighting, however the iterative nature of adaptive smoothing like anisotropic diffusion does not allow to predict the convergence process at all, which is rarely fast and makes it difficult to estimate the number of required iterations19 [Gomez].

Based on the strategy that different areas of the images should be smoothed differently, adaptive Gaussian filters makes the variance adapt to local characteristics of an image in order to reduce noise and preserve edges20 [Deng]. This filters compare favorably with anisotropic diffusion and they have good results at a low computational cost without supplying any variables19 [Gomez]. Taking these into account, Estepar et al.21 [Estepar] proposed a PBM with a conventional BFS (pixel nearest neighborhood) and an adaptive HFS that is based on a basis function interpolation similar to the one proposed by Meairs et al.14 [Meairs], where a spherical Gaussian kernel is placed over each empty voxel after the BFS and the voxel value is computed as the weighted sum of all filled voxels; the kernel variance is a function of the variance of the intensity of the nearby pixels, that carries information about the extent of speckle formation; this method performs much better than well-known methods, as long as the percentage of empty holes leaved by the BFS is low. However the blurring and number of empty voxels left by the BFS is not resolved.

1. **Proposed Method: Adaptive Gaussian pixel based reconstruction**

In this work we propose a new reconstruction method based on the work by Ohbuchi et al.13 and Meairs et al.14, where they used a Gaussian Kernel around the pixels in order to fill every voxel in the 3D regular volume that intersects the kernel. Based on the idea of adaptive Gaussian filters, where the variance of the kernels adapts to local characteristics of the image20 [Deng], and the fact that anisotropic diffusion filters are the best iterative methods for reducing speckle and preserve edges, where the amount of diffusion in each pixel is controlled by gradient of the image22 [Abd], this method controls the variance and size of the kernel using the gradient magnitude of each pixel in the 2D US images.

The first step of the PBMs is to found the spatial coordinates pixels of the acquired images. This is done by finding a transformation matrix that transforms de image coordinates into the regular volume coordinates , usually a position and orientation measurement (POM) device along with a calibration process is used to find this transformation9 [Solberg]. Equation (ecuacion) transformation of pixels to spatial coordinates.

where are the column and row indices of the pixel; and are scale factors with units of mm/pixels; is the transformation matrix that transforms the coordinates in the image coordinate system to the POM device receiver coordinate system , this transformation is usually found by a calibration process, Prager et al.23 [Prager] and Hsu et al.24 have done a complete review of different calibration methods; and transforms from to the transmitter or reference coordinate system , this is done automatically by the POM device.

After finding the pixel position in the volume coordinates, a spherical Gaussian kernel is placed over the pixel and every voxel that intersects with the kernel is filled with the pixel value weighted by the Gaussian kernel. In order to reduce speckle and preserve boundaries, we would want to encourage the smoothing within a region in preference to smoothing across boundaries, this could be achieved by adapting the variance and kernel size in the interior of each region with respect to the location of the region boundary. It has been demonstrated that the simplest estimation of edge position, the gradient of the image (), gives excellent results when detecting edges in anisotropic filtering25 [Perona]. Based on this, we use the gradient magnitude of the pixel to choose the variance and the size of the Gaussian kernel to control the amount of blurring of the BFS, where bigger gradient magnitudes will lead to smaller variances and sizes and vice versa. In order to reduce computational cost, instead of constructing the exact Gaussian kernel for each pixel, a bank of precomputed kernels is used to speed up the interpolation process21 [Estepar]. A number of kernels is constructed, where , and is the maximum defined variance and is the defined step in variance between kernels. The radius size of each kernel is chosen to be 3 times the variance in order to propagate the valid information, since a greater pixel distance than 3 sigma have negligible weights26 [Kiarash]. The kernels are constructed according to equation (ecuacion):

where is the distance from a point inside the kernel to the center of the kernel.

To choose which Gaussian kernel corresponds to each pixel first the local gradient for each pixel () in the image is computed and normalize into values between 0 and 1, then it is used to choose which kernel corresponds to the pixel gradient value (ecuacion).

The Gaussian weighted propagation of the pixel value to the voxels ) inside the reconstruction kernel is done by an incremental volume reconstruction proposed by Dai et al. where the voxel value is computed as (ecuacion)

where is the corresponding Gaussian weight for that voxel.

After all pixels are propagated into the volume using the adaptive reconstruction Gaussian kernel, the empty voxels are filled by a Gaussian HFS21 [Estepar]. In the HFS, the algorithm loops over the target volume and fill the gaps; for every empty voxel, non-empty neighboring voxels in a certain radius around the empty one are averaged into the resulting value, a growing radius starting at 1 voxel is used, if non-empty voxels are found the region will grow until non empty pixels are found or a maximum radius is reached10 [Miller]. In order to avoid the visible boundaries between the voxels assigned directly by the 2D ultrasound images during the BFS and the interpolated voxels computed during the HFS18 [Doud], a weighted distance can be used in the HFS in order to avoid far away pixels to contribute on the voxel and have smoother transitions2 [Chen], here we use a Gaussian weighted distance between the center voxel and the non-empty voxel which variance is dependent of the intensity of the voxels inside the region21 [Estepar].

1. **Experiments and Results**

To evaluate the reconstruction quality and the ability to preserve borders of the proposed methods, a set of ultrasound sweeps of different objects where taken using the freehand technique. An Aloka 1000 ultrasound machine with a 7.5 Mhz probe was used with an Epiphan DIV2USB 3.0 frame grabber to acquire digital ultrasound images. A Polaris Spectra was used as an optical POM device with a passive sensor attached to the US probe; the calibration process was made using the cross-wire phantom method reported by Barry et al.11 [Barry], because one point calibration methods are optimal due to their ease of construction and small reconstruction error, and the cross-wire phantom is a remarkably accurate one-point method23 [Prager].

Along with the proposed method, a conventional PBM (Pixel Nearest Neighborhood) with the HFS proposed by Estepar et al.21 [Estepar] and a fixed Gaussian Kernel Reconstruction method14 [Meairs] were implemented for comparison. Several reconstructions of an Ultrasound resolution Phantom (CIRS Model 044), which includes three sets of cylinders with known dimensions, were made with the three implemented methods. To compare the performance of each method to suppress speckle and preserve boundaries we use similar experiments to the ones reported by Huang et al.17 in [Huang]. The signal-to-noise ratio () of 3D homogeneous regions and the local contrast of target regions (cylinders) were studied to compare the performance of the implemented methods in reducing speckle and edge preservation respectively. Four homogenous regions, that only contained speckle, were selected at the same location in all reconstructed volumes and their was computed; larger SNR values indicate better performance in speckle reduction. The signal to noise ratio is computed as (ecuacion)

where and are the mean and the standard deviation of the homogeneous region respectively.

The local contrast measure in a local neighborhood can be express as (ecuacion)

where denotes a group of voxel intensities at the location and its voxel neighborhood. In order to use the local contrast measurement to represent the performance of contrast enhancement in a 3D inhomogeneous region, Huang et al.17 [Huang] used the averaged local contrast described by (ecuacion)

where is a sub-volume of a reconstructed volume and is the number of voxels within the sub-volume. This average was computed for all reconstructed volumes in four regions with mainly including one of the cylinders of the resolution phantom. Larger indicate that the region preserves sharper edges, more texture objects and details of the raw B-scan.

Along with the SNR and measurements, Estepar et al.21 and Huang et al.17 used a testing approach introduced by Rohling et al.27 [Rholing] that evaluates the ability of the reconstruction methods to interpolate and fill in the gaps on a plane. This method consists of artificially remove data from the images. A good reconstruction method will interpolate the removed data points with values near t the original data. A percentage of the pixels in a B-scan are randomly removed creating gaps of various sizes. The average of the difference between the interpolated and the original data over all missing data points is calculated as (ecuacion)

where is the original removed pixel value, is the reconstructed voxel value in the position of the pixel and is the number of removed pixels. The tests are performed removing different percentages of data: 25%, 50%, 75%, 100%, 300%, and 500%, where 100% refers to removing a complete B-scan, and 300% and 500% refers to removing a complete B-scans along with their two and four adjacent B-scans respectively.