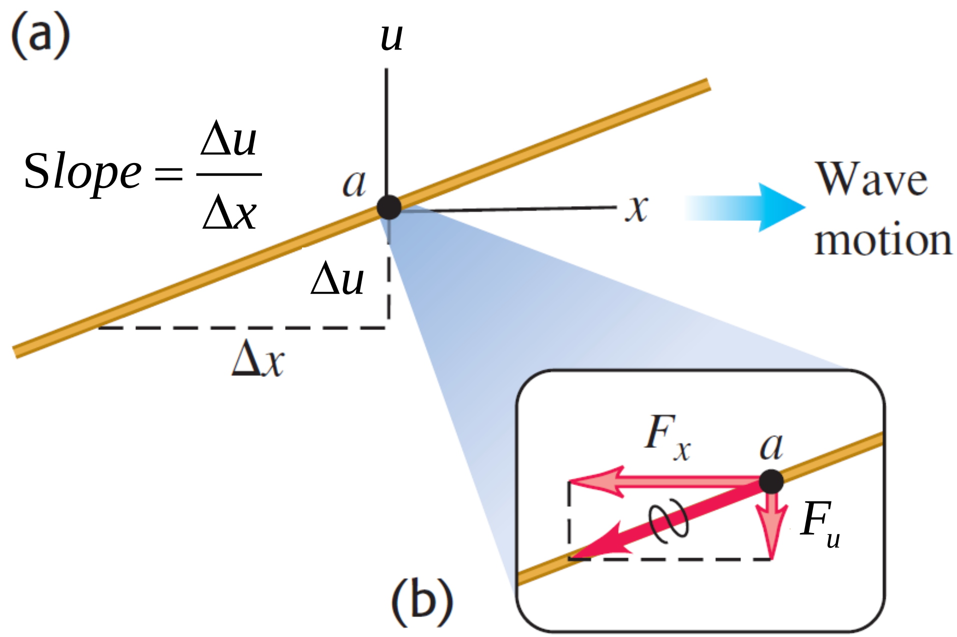
波的传播的能量（推导方式1）

波传播时，介质中每一点施加力，对相邻质点做功。



F_u / F :弦的斜率

$$F_u(x,t) = -F \frac{\partial u(x,t)}{\partial x}$$

a沿u方向运动，力F_u做的功功率P为F_u乘以横向速度：

$$P(x,t) = F_u(x,t) v_u(x,t) = -F \frac{\partial u(x,t)}{\partial x} \frac{\partial u(x,t)}{\partial t}$$

波的传播的能量

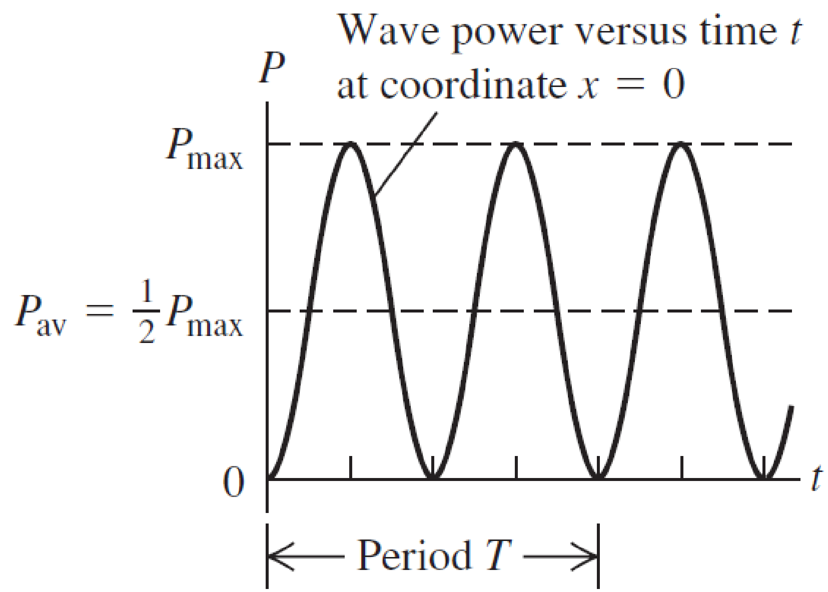
$$P(x,t) = F_u(x,t)v_u(x,t) = -F\frac{\partial u(x,t)}{\partial x}\frac{\partial u(x,t)}{\partial t}$$

$$u(x,t) = A\cos(kx - \omega t)$$

$$\frac{\partial u(x,t)}{\partial x} = -kA\sin(kx - \omega t)$$

$$\frac{\partial u(x,t)}{\partial t} = \omega A\sin(kx - \omega t)$$

$$P(x,t) = Fk\omega A^2\sin^2(kx - \omega t)$$



代入:

$$\omega = vk \text{ 及 } v^2 = F/\mu$$

$$P(x,t) = \sqrt{\mu F}\,\omega^2A^2\sin^2(kx - \omega t)$$

$$P_{av} = \frac{1}{2}\sqrt{\mu F}\,\omega^2A^2$$

$$P_{max} = \sqrt{\mu F}\,\omega^2A^2$$

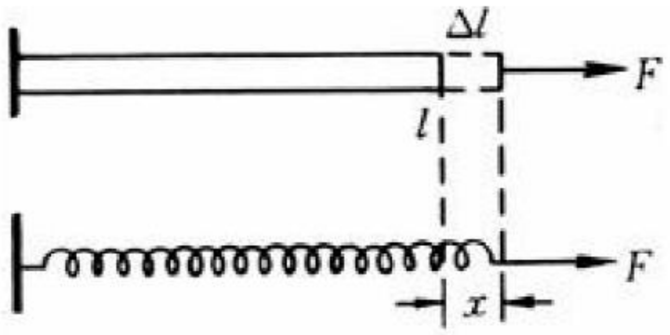
平均功率

$$P_{av} = \frac{1}{2}\mu v\omega^2A^2$$

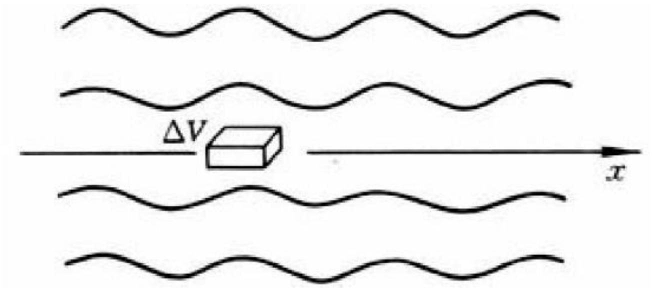
波的传播的能量 (推导方式2）

$u(x,t)=A\cos(wt-kx)$

如何求势能？类比弹簧



介质元的振动动能和弹性势能



弹簧		介质棒
$F=kx$	$\leftrightarrow$	$F=ES\dfrac{1}{l}\Delta l$
x	$\leftrightarrow$	Δl
k	$\leftrightarrow$	$k_c=\dfrac{ES}{l}$
$E_p=\dfrac{1}{2}kx^2$	$\leftrightarrow$	$E_p=\dfrac{1}{2}k_c(\Delta l)^2=\dfrac{1}{2}E(\dfrac{\Delta l}{l})^2V$

介质元$:\Delta m=\rho\Delta V$

振动动能ΔE_k和弹性势能ΔE_p

位移函数（波函数)$u(x,t)$,则

$$\Delta E_k=\frac{1}{2}\rho(\frac{\partial u}{\partial t})^2\Delta V$$

对波场代换$:V\to\Delta V,\Delta l/l\to\partial u/\partial x$

波的能量

势能：

纵波：$\Delta E_p = \frac{1}{2}E(\frac{\partial u}{\partial x})^2 \Delta V$

动能：$\Delta E_k = \frac{1}{2}\rho(\frac{\partial u}{\partial t})^2 \Delta V$

横波：$\Delta E_p = \frac{1}{2}G(\frac{\partial u}{\partial x})^2 \Delta V$

$$u(x,t) = A\cos(wt - kx)$$

$$\frac{\partial u(x,t)}{\partial t} = -wA\sin(wt - kx)$$

$$\frac{\partial u(x,t)}{\partial x} = kA\sin(wt - kx)$$

$$\Delta E_p = \frac{1}{2}Ek^2A^2\Delta V\sin^2(wt - kx)$$

$$\Delta E_k = \frac{1}{2}\rho w^2A^2\Delta V\sin^2(wt - kx)$$

机械能：$\Delta E = \Delta E_k + \Delta E_p = \frac{1}{2}(\rho w^2 + Ek^2)A^2\Delta V\sin^2(wt - kx)$

波的能量

机械能：$\Delta E = \Delta E_k + \Delta E_p = \frac{1}{2}(\rho w^2 + Ek^2)A^2\Delta V\sin^2(wt-kx)$

由波相速度公式: v=$\omega/k=\sqrt{E/\rho}$，得到$\rho w^2 = Ek^2$(动能等于势能）

因此ΔV包含机械能：

$\Delta E = \rho w^2 A^2 \Delta V \sin^2(wt-kx)$

$$\Delta E_p = \frac{1}{2}Ek^2A^2\Delta V\sin^2(wt-kx)$$

$$\Delta E_k = \frac{1}{2}\rho w^2A^2\Delta V\sin^2(wt-kx)$$

波场动能和势能同时达到最大或最小（和振动不同，
振动时平衡位置动能最大，势能最小。最大位移时相反。）

波场平衡位置时动能最大，介质间单位元相距也是最远，势能最大。

平均能量密度与平均能流密度

平均能量密度：单位体积内蕴含的能量

$$w(x,t)=\lim_{\Delta V\to 0}\frac{\Delta E}{\Delta V}=\rho w^2 A^2\sin^2(wt-kx)$$

时间平均的平均能量密度

$$\bar{w}=\lim_{\Delta V\to 0}\frac{\Delta E}{\Delta V}=\frac{1}{T}\int_0^T w(x,t)dt$$

$$=\frac{1}{2}\rho w^2 A^2$$

声音状态	声强/(W·m^{-2})
刚能听到的声音	$1\times10^{-9}\sim10^{-12}$
钟表的滴答声	1×10^{-7}
平和的谈话声	1×10^{-5}
中等强度的演讲声	1×10^{-3}
叫喊声	1×10^{-1}
流行乐队演唱声	1×10
震耳欲聋声	1×10^{3}

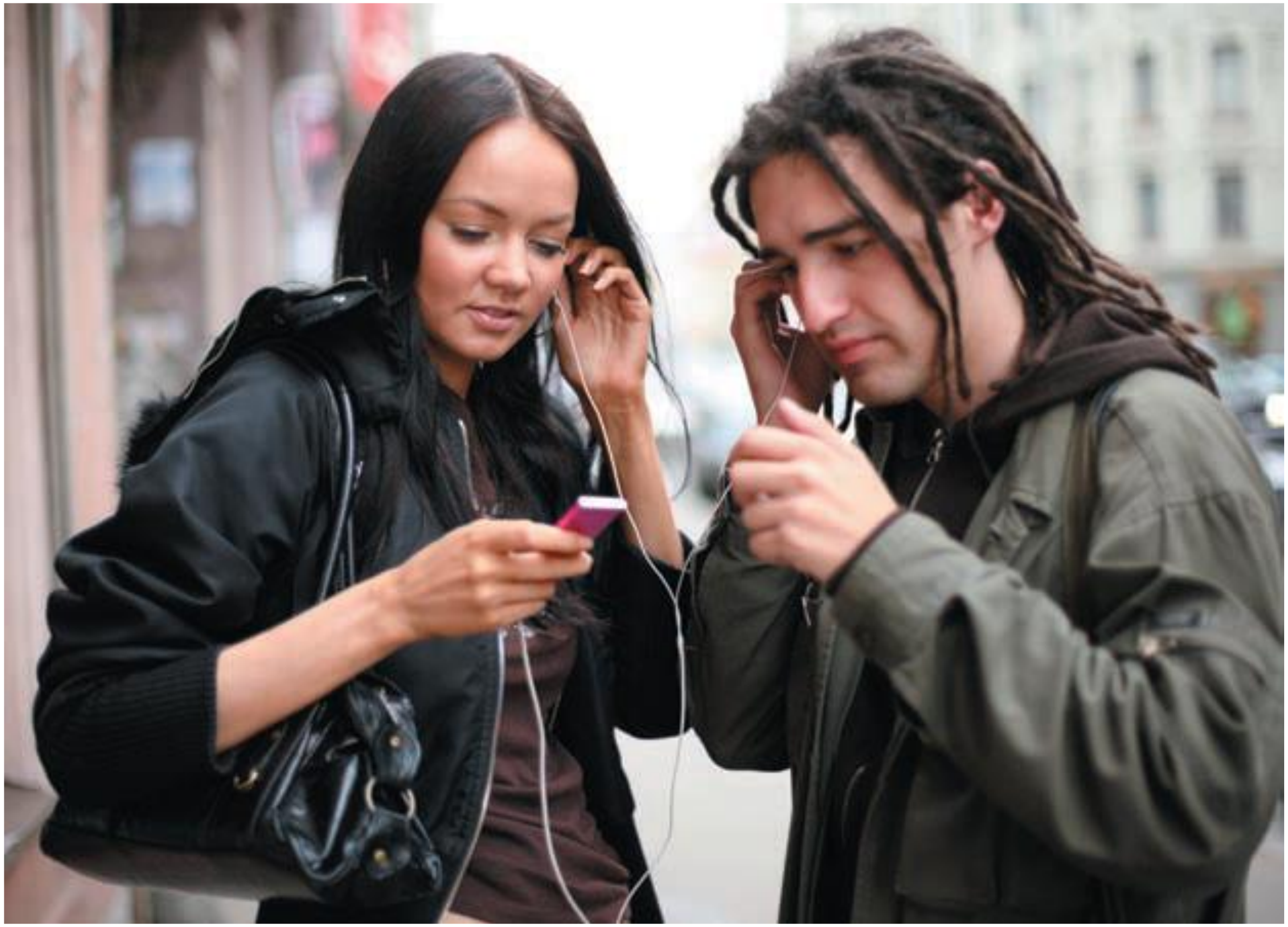
平均能流密度

$$I=\bar{w}v=\frac{1}{2}\rho w^2 A^2 v(W/m^2)$$

$$P_{av}=\frac{1}{2}\mu\omega^2 A^2 v$$

$$P_{av}/\Delta s=I$$

声波



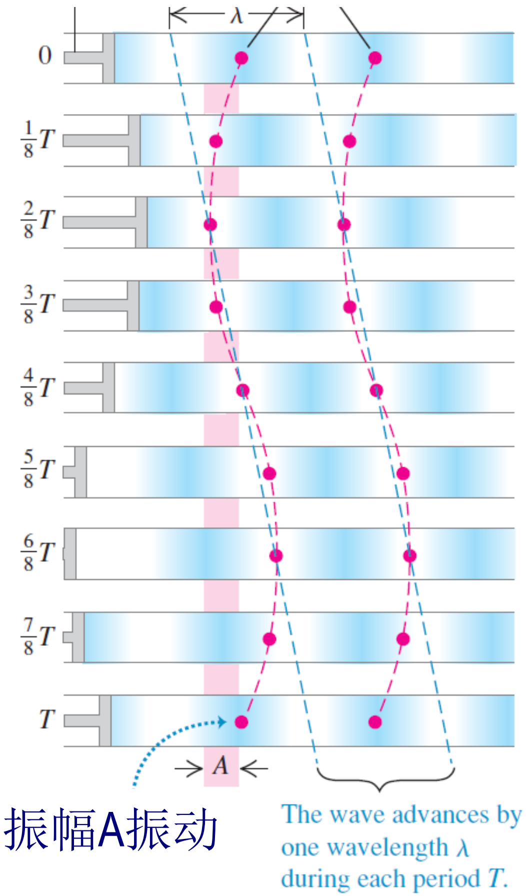
人耳接收频率：20Hz - 20000Hz

声波

纵波 按 1/8 T为间隔

两个粒子在介质中，相隔一个波长λ

声波在介质中
从左往右传播



粒子以振幅A振动

$p(x,t)$: x处时刻t和正常大气压强p_a之差

绝对压强：$p(x,t)+p_a$

波可以表示为：

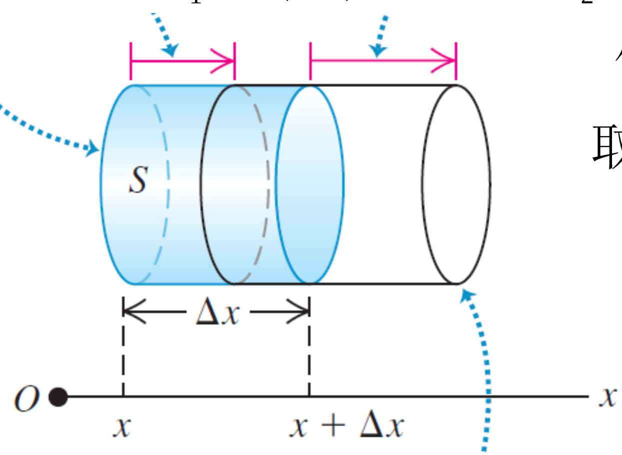
$$u(x,t) = A\cos(wt - kx)$$

声波：压强的变化

$$u(x,t) = A\cos(wt - kx)$$

未扰动的圆柱状气体，其截面面积S，长度Δx，体积$S\Delta x$

声波圆柱左端偏移$u_1 = u(x,t)$，右端偏移$u_2 = u(x+\Delta x, t)$（红线）



体积变化$\Delta V = S(u_2 - u_1) = S[u(x+\Delta x, t) - u(x, t)]$

取极限$\Delta x \to 0$

$$\frac{dV}{V} = \lim_{\Delta x \to 0} \frac{S[u(x+\Delta x, t) - u(x,t)]}{s\Delta x} = \frac{\partial u(x,t)}{\partial x}$$

$$p(x,t) = -K\frac{\partial u(x,t)}{\partial x}$$

$$p(x,t) = KkA\sin(kx - wt)$$

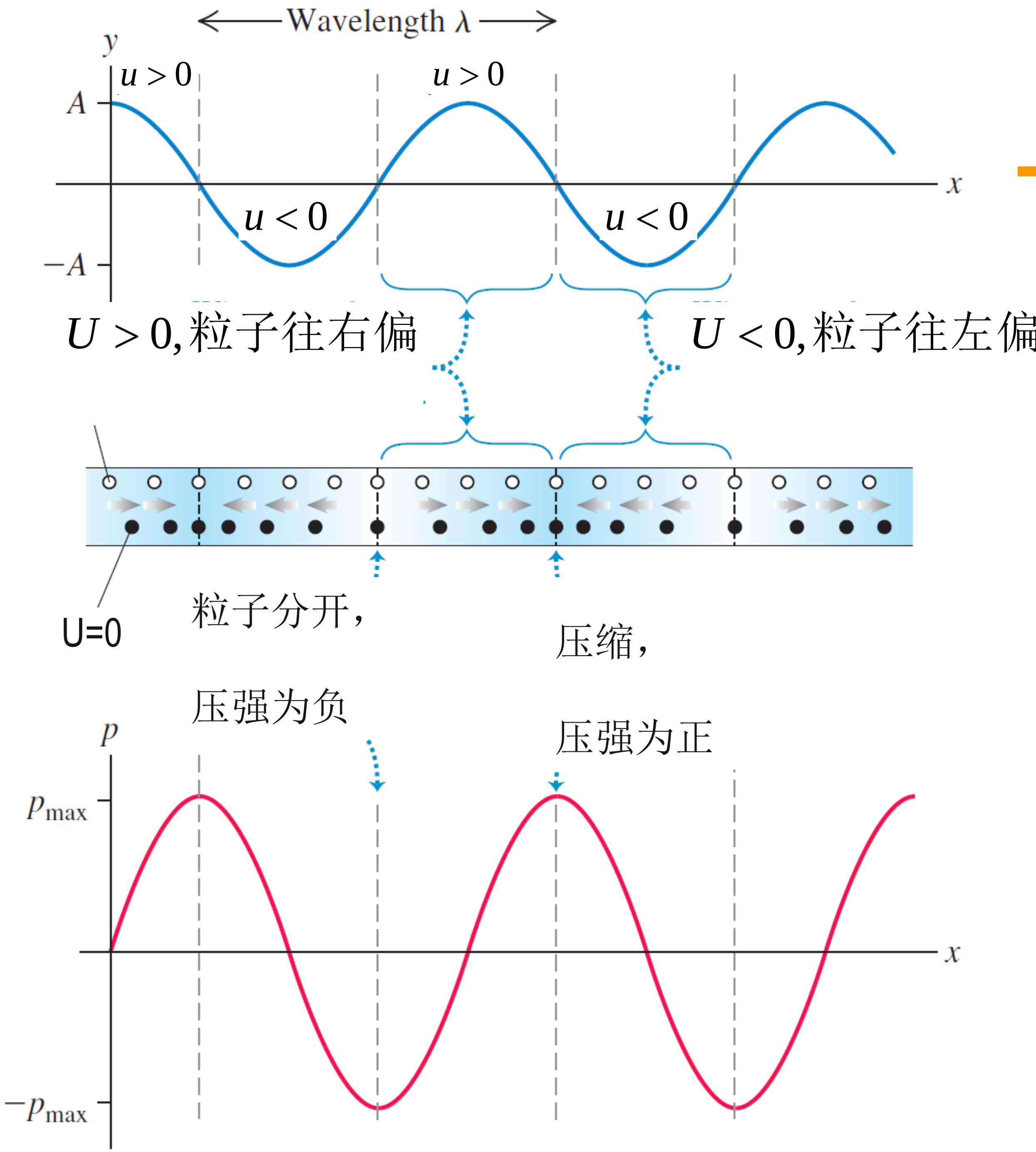
扰动的圆柱状流体体积变化$S(u_2 - u_1)$

声波：压强的变化

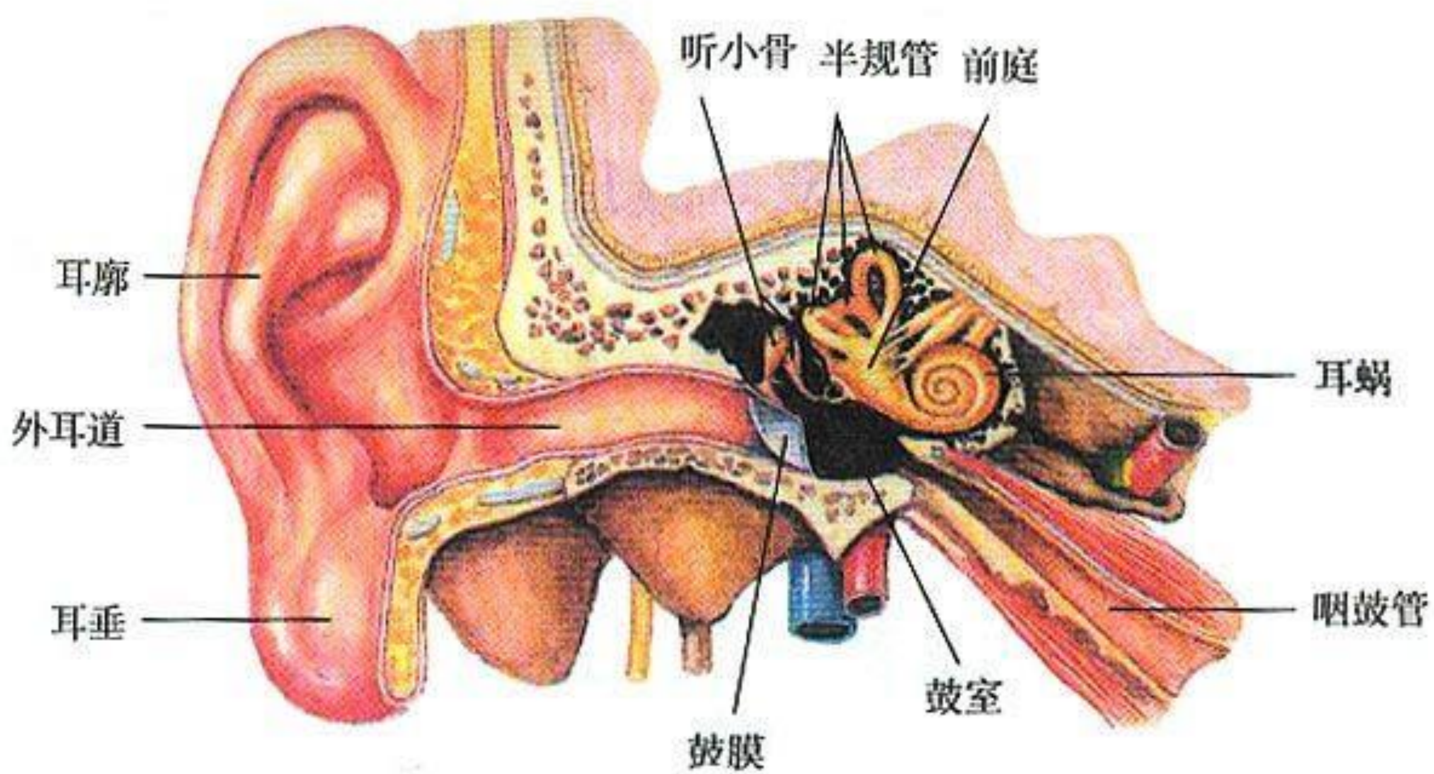
(a) 显示x处偏移u（t=0）时刻

(b) 显示粒子在t=0时刻的偏移

(c) 显示t=0时刻的压强变化



人耳的结构



声压和声压级 声强级

$$I=\frac{1}{2}\rho_0 v w^2 A^2=\frac{1}{2}\frac{A_p^2}{\rho_0 v}$$

A_p :空气压强波振幅

如$20\mu Pa$(刚听到声音）

200Pa(震耳欲聋)

L_p :声压级：实际声压值p与基准声压值p_b比值的对数乘以20

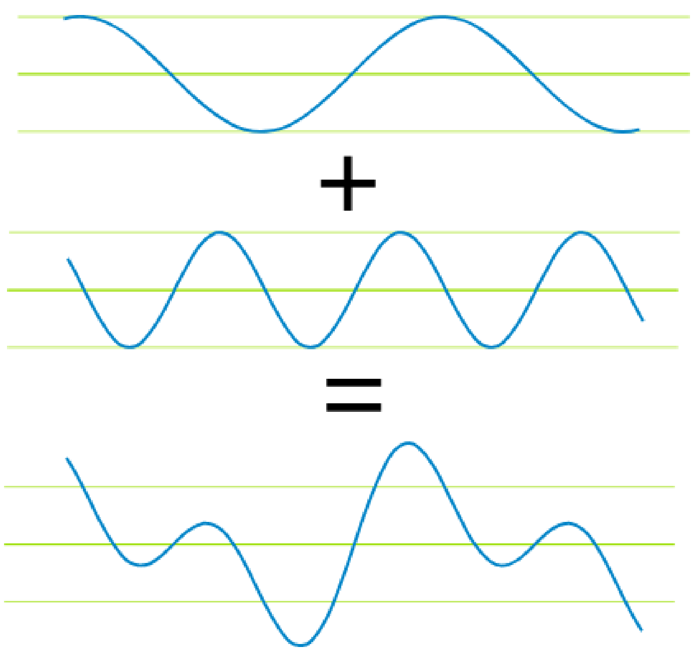
$\mathrm{L}_p=20\log(p/p_b)$,单位分贝（dB）。中国基准声压空气中为$20\mu Pa$.

$0dB$:声压$20\mu Pa$

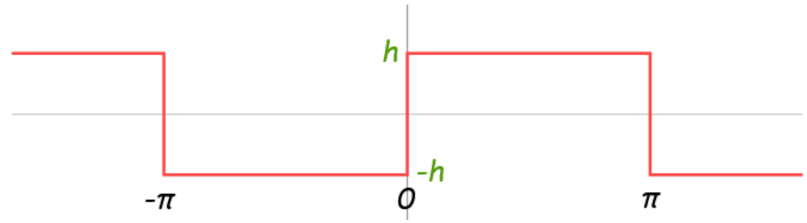
$100dB:2Pa$

$140dB:200Pa$

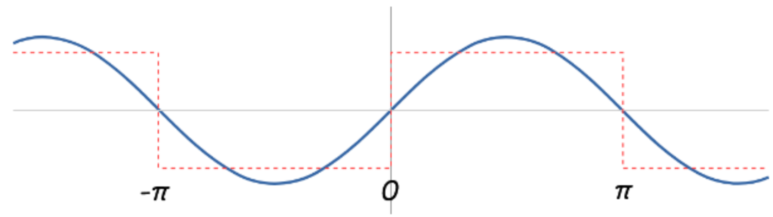
傅里叶级数



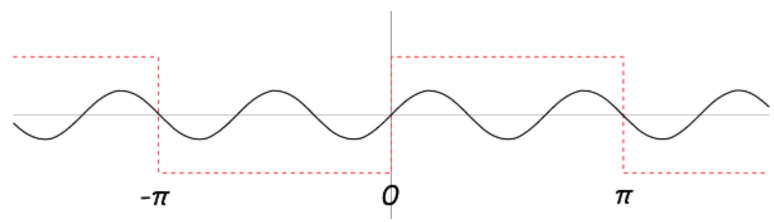
sin(x)+sin(2x)



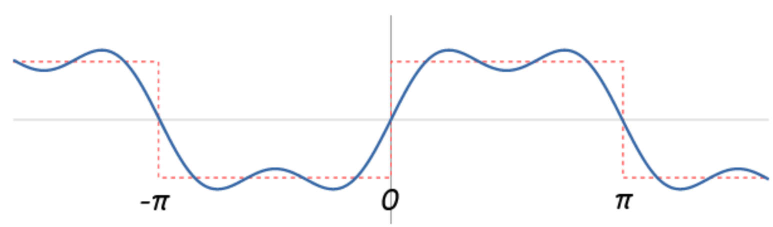
Start with **sin(x)**:



Then take **sin(3x)/3**:

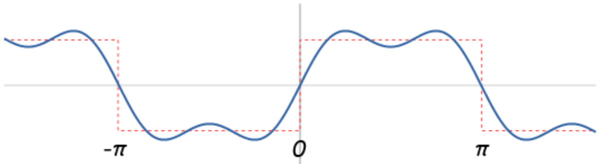


And add it to make **sin(x)+sin(3x)/3**:



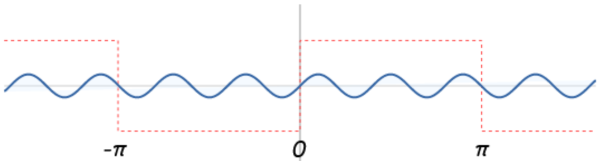
Can you see how it starts to look a little like a square wave?

傅里叶级数

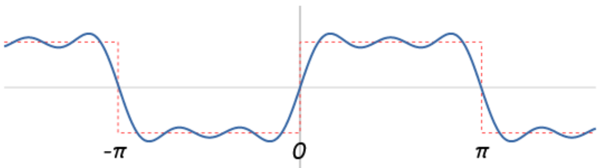


Can you see how it starts to look a little like a square wave?

Now take **sin(5x)/5**:

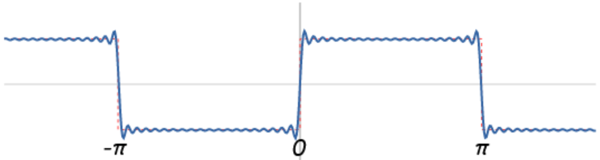


Add it also, to make **sin(x)+sin(3x)/3+sin(5x)/5**:

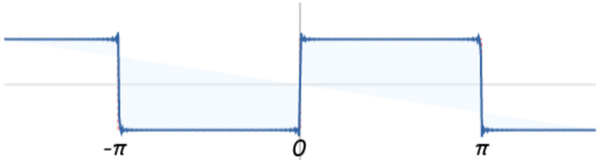


Getting better! Let's add a lot more sine waves.

Using 20 sine waves we get **sin(x)+sin(3x)/3+sin(5x)/5 + ... + sin(39x)/39**



Using 100 sine waves we get **sin(x)+sin(3x)/3+sin(5x)/5 + ... + sin(199x)/**

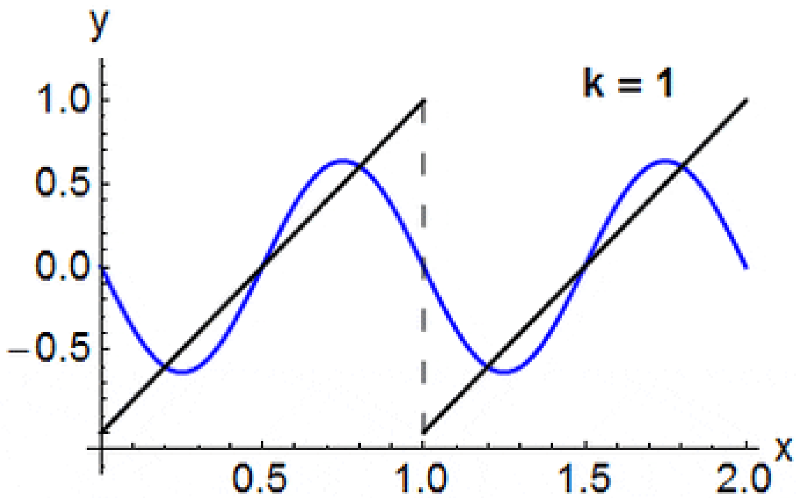


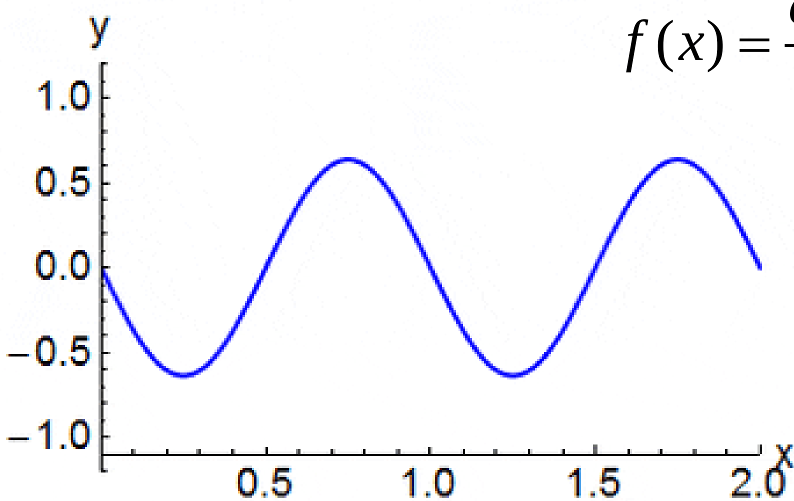
方波：

a square wave = sin(x) + sin(3x)/3 + sin(5x)/5 + ... (infinitely)

傅里叶级数

锯齿波





$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx+\sum_{n=1}^{\infty}b_n\sin nx$$

傅里叶级数

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx+\sum_{n=1}^{\infty}b_n\sin nx$$

一个函数表示为cos和sin的函数的组合

Square Wave	sin(x) + sin(3x)/3 + sin(5x)/5 + ...	sin((2n-1)*x)/(2n-1)
Sawtooth	sin(x) + sin(2x)/2 + sin(3x)/3 + ...	sin(n*x)/n
Pulse	sin(x) + sin(2x) + sin(3x) + ...	sin(n*x)*0.1
Triangle	sin(x) − sin(3x)/9 + sin(5x)/25 − ...	sin((2n-1)*x)*(-1)^n/(2n-1)^2

$f(x)$是我们想获得的函数

a_n, b_n是我们需要计算出的系数

$$a_n=\frac{1}{\pi}\int_0^{2\pi}f(x)\cos nx dx, n=0,1,2...$$

$$b_n=\frac{1}{\pi}\int_0^{2\pi}f(x)\sin nx dx, n=0,1,2...$$

傅里叶级数

写成指数形式：

$$f(x)=\sum_{n=-\infty}^{\infty} c_n e^{inx}$$

其中：

$$c_n=\frac{1}{2}(a_n-ib_n), c_{-n}=\frac{1}{2}(a_n+ib_n), n>0$$

$$c_0=\frac{1}{2}a_0$$

或者写成

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n\cos\frac{n\pi x}{L}+\sum_{n=1}^{\infty} b_n\sin\frac{n\pi x}{L}$$

$$a_n=\frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}dx, n=0,1,2\ldots$$

$$b_n=\frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}dx, n=1,2\ldots$$

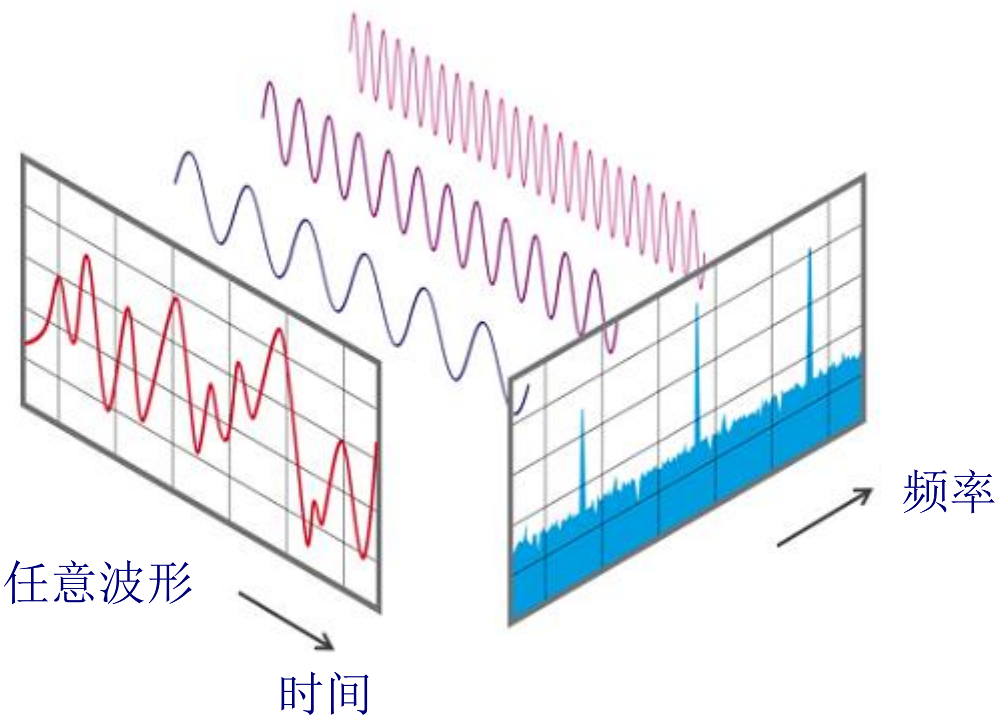
傅里叶变换

傅里叶变换

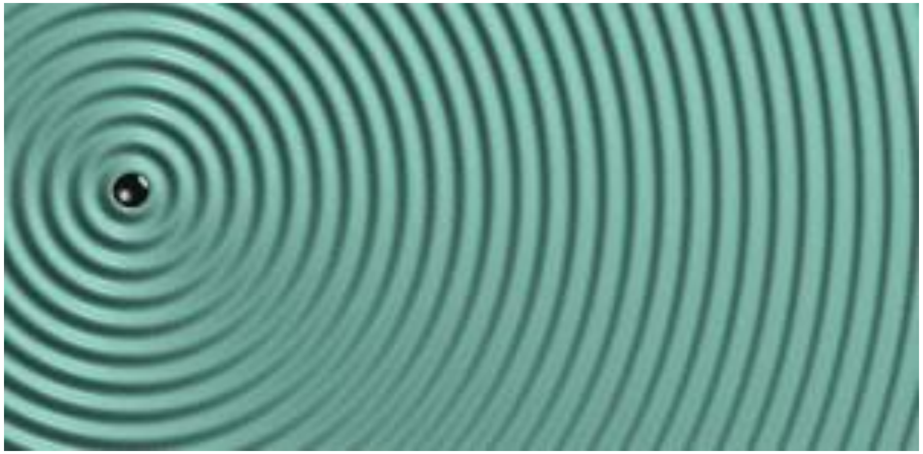
$$g(w)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)e^{iwt}dt$$

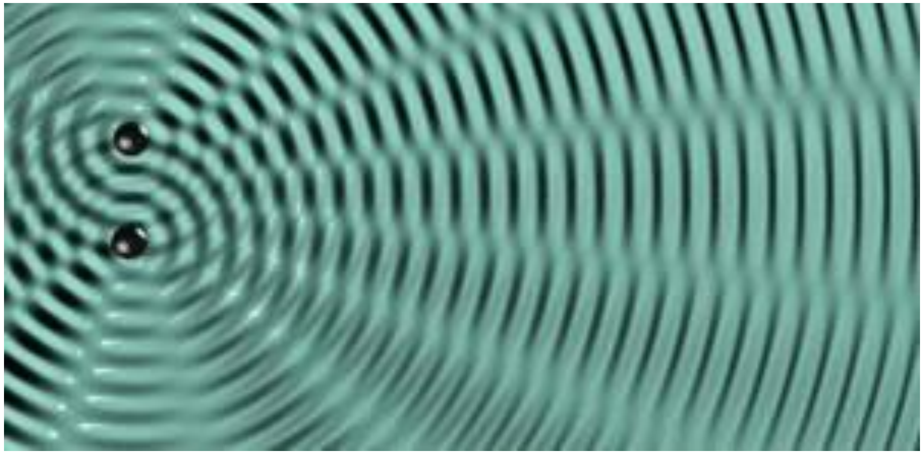
反傅里叶变换

$$f(t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} g(w)e^{-iwt}dw$$

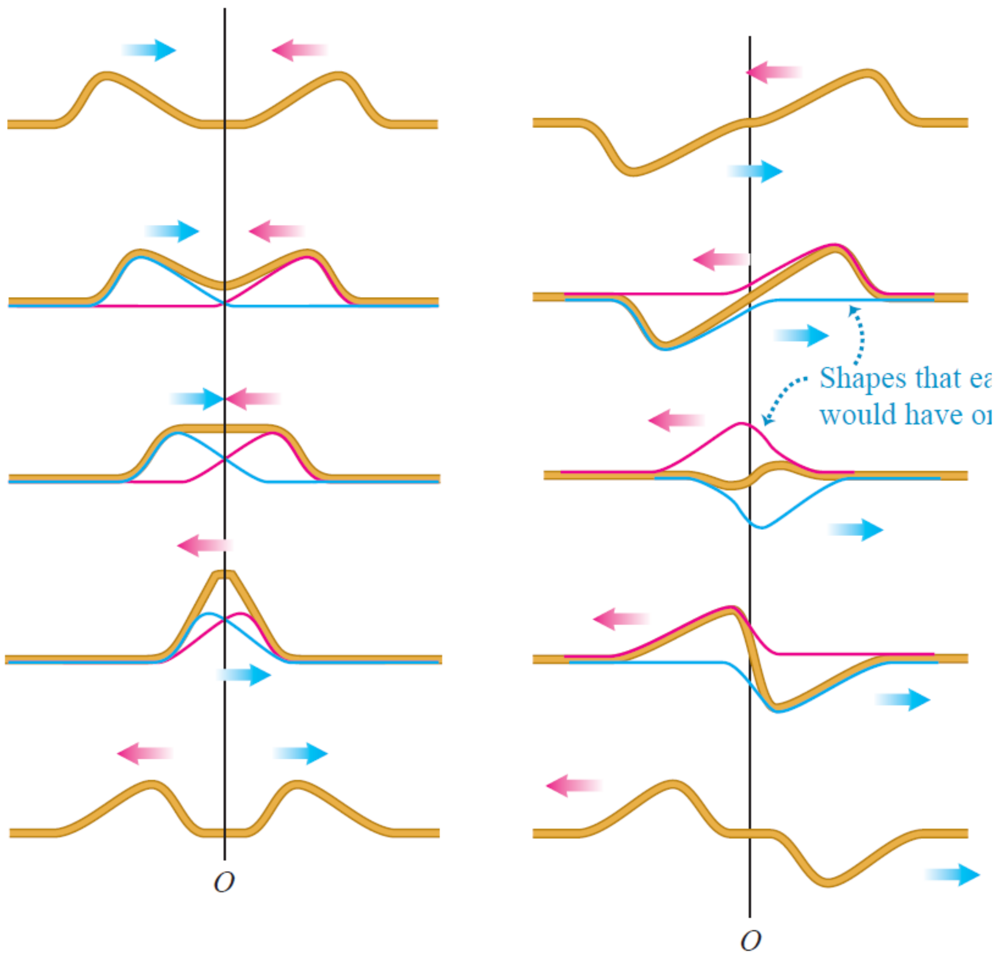


波的叠加





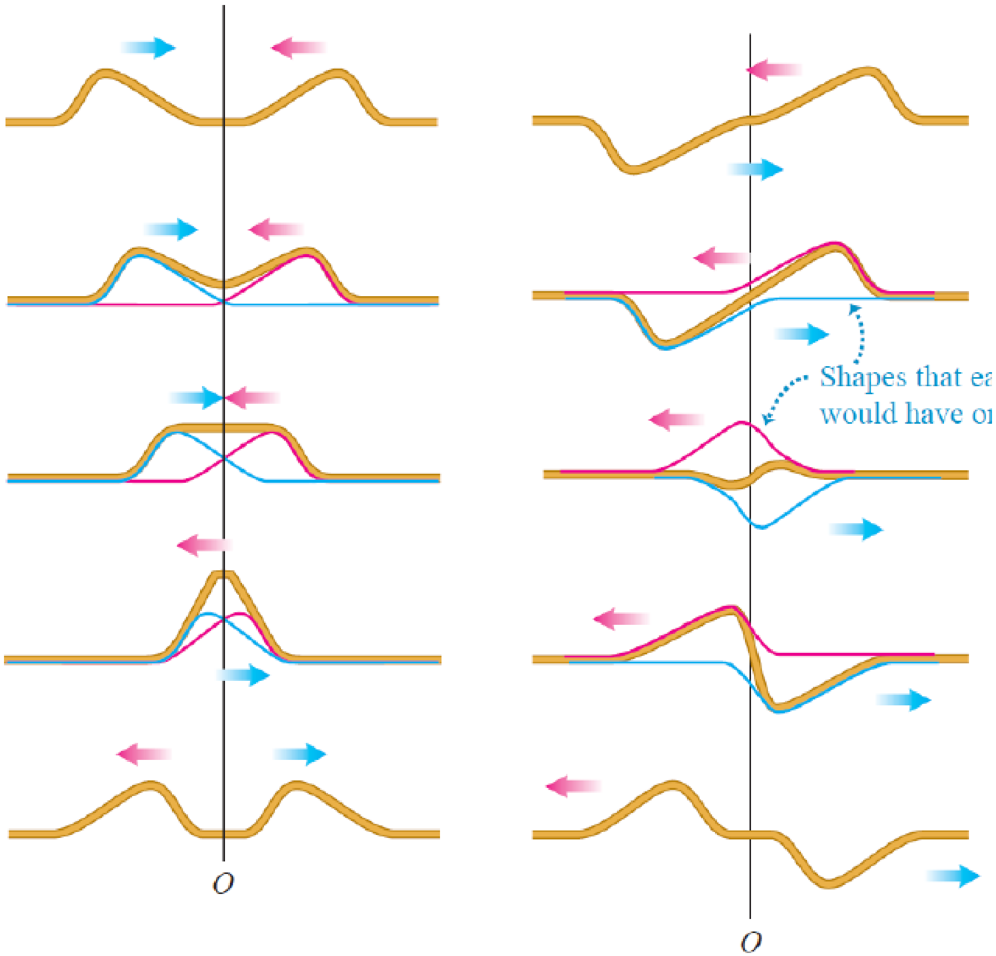
波的叠加



线性叠加：

$$u(x,t) = u_1(x,t) + u_2(x,t)$$

波的叠加



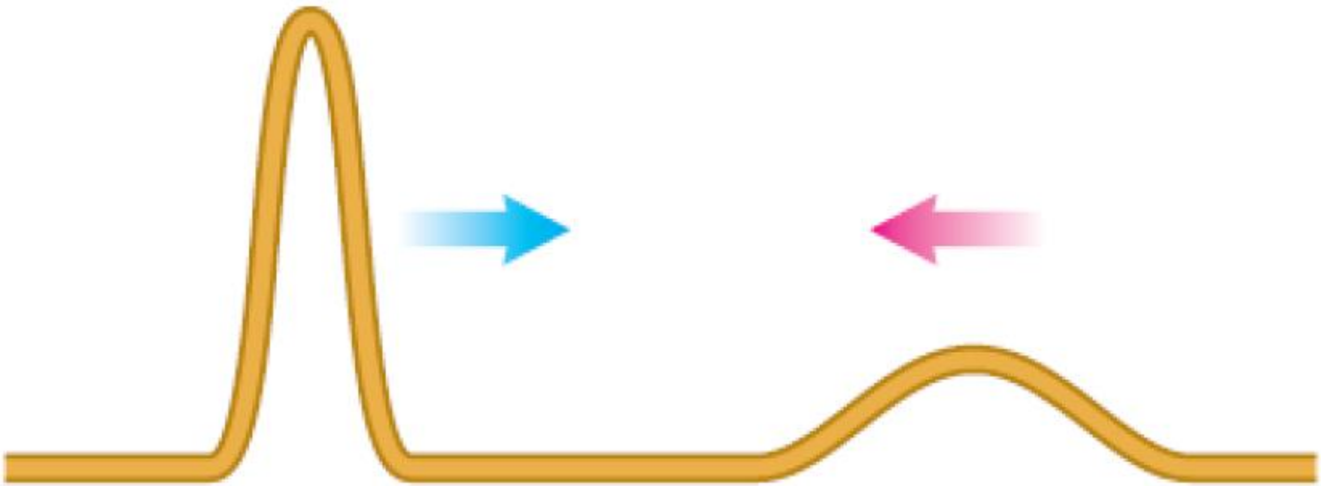
线性叠加：

$$\vec{u}(x,t)=\vec{u}_1(x,t)+\vec{u}_2(x,t)$$

任意一个时刻在任意一点，两列波的叠加引起的偏移，等于单一的波在该时刻该点引起的位移的叠加，为波函数的叠加。

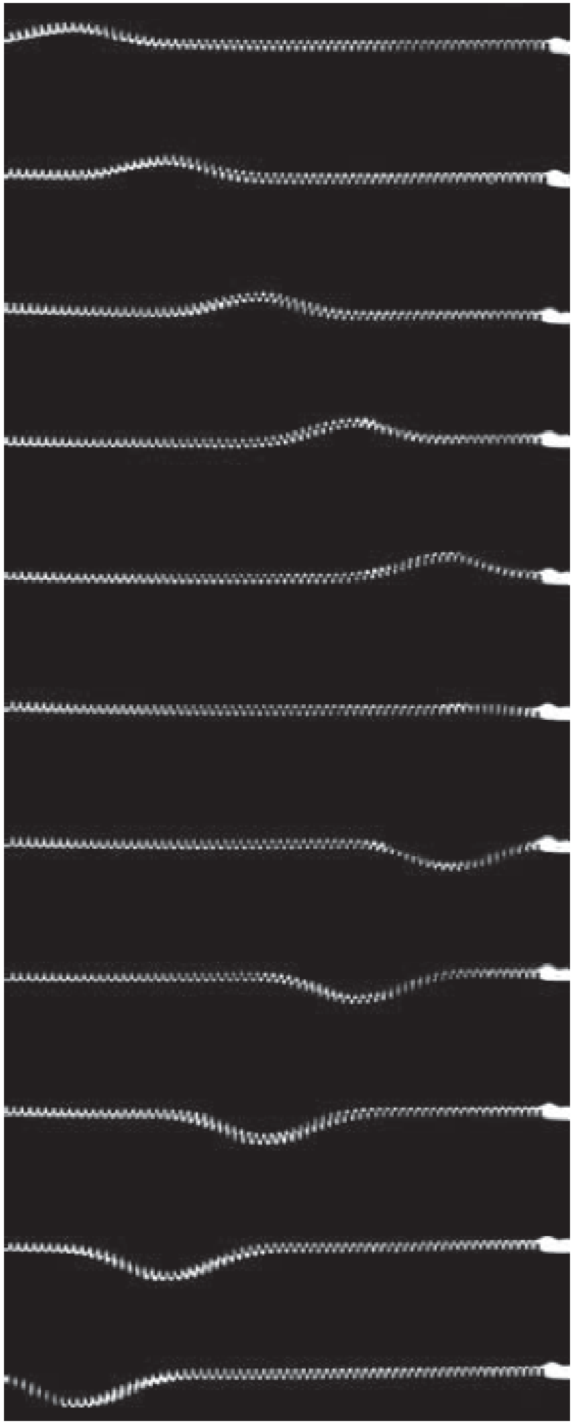
仅在特殊的情况下，才有非线性的叠加（如强场光学非线性）。

练习



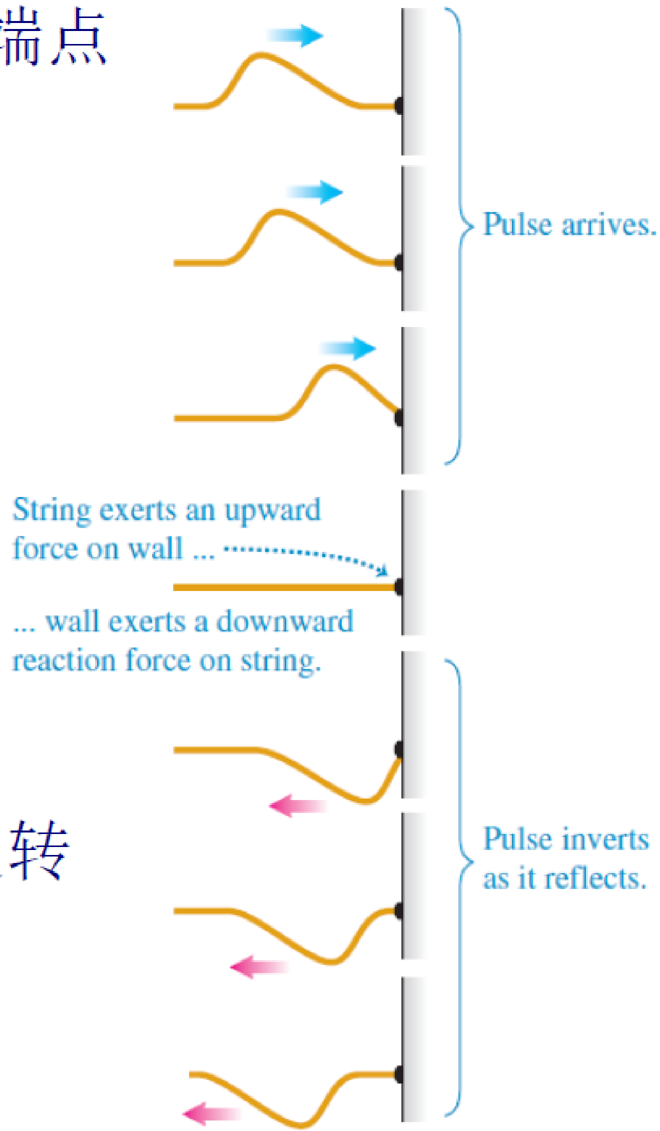
试画出两列波相遇时及相遇后的波形

波的反射



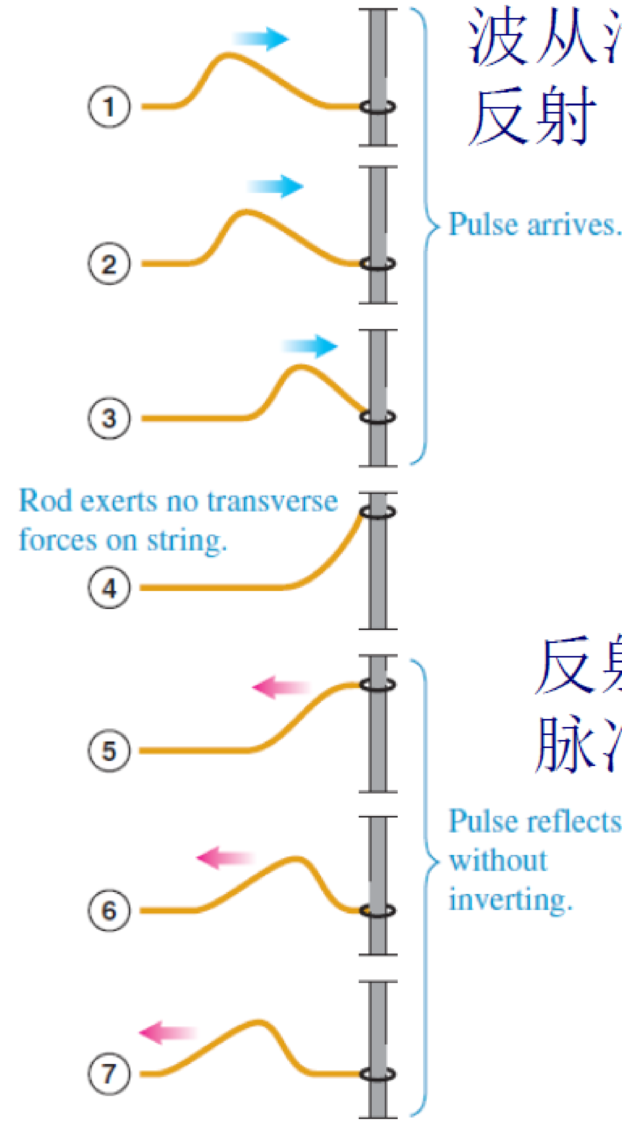
波的反射边界条件

波从固定端点反射



反射时
脉冲发生反转

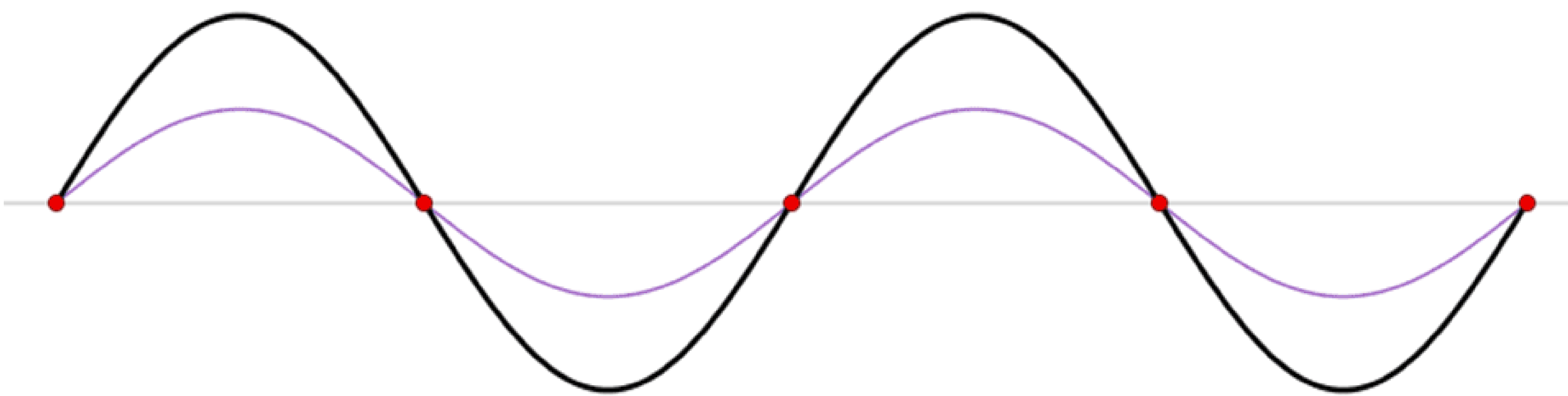
波从活动端点反射



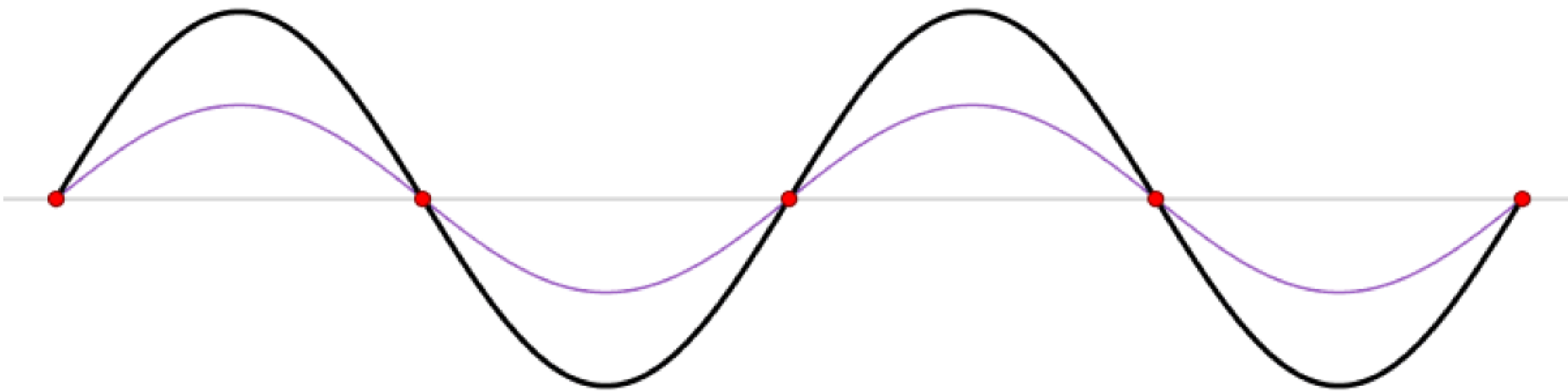
反射时
脉冲不反转

驻波的形成

两列波，频率相同，振动方向相同，传播方向相反，形成驻波。



驻波的形成



$$u_1(x,t) = A_0\cos(wt - kx)$$

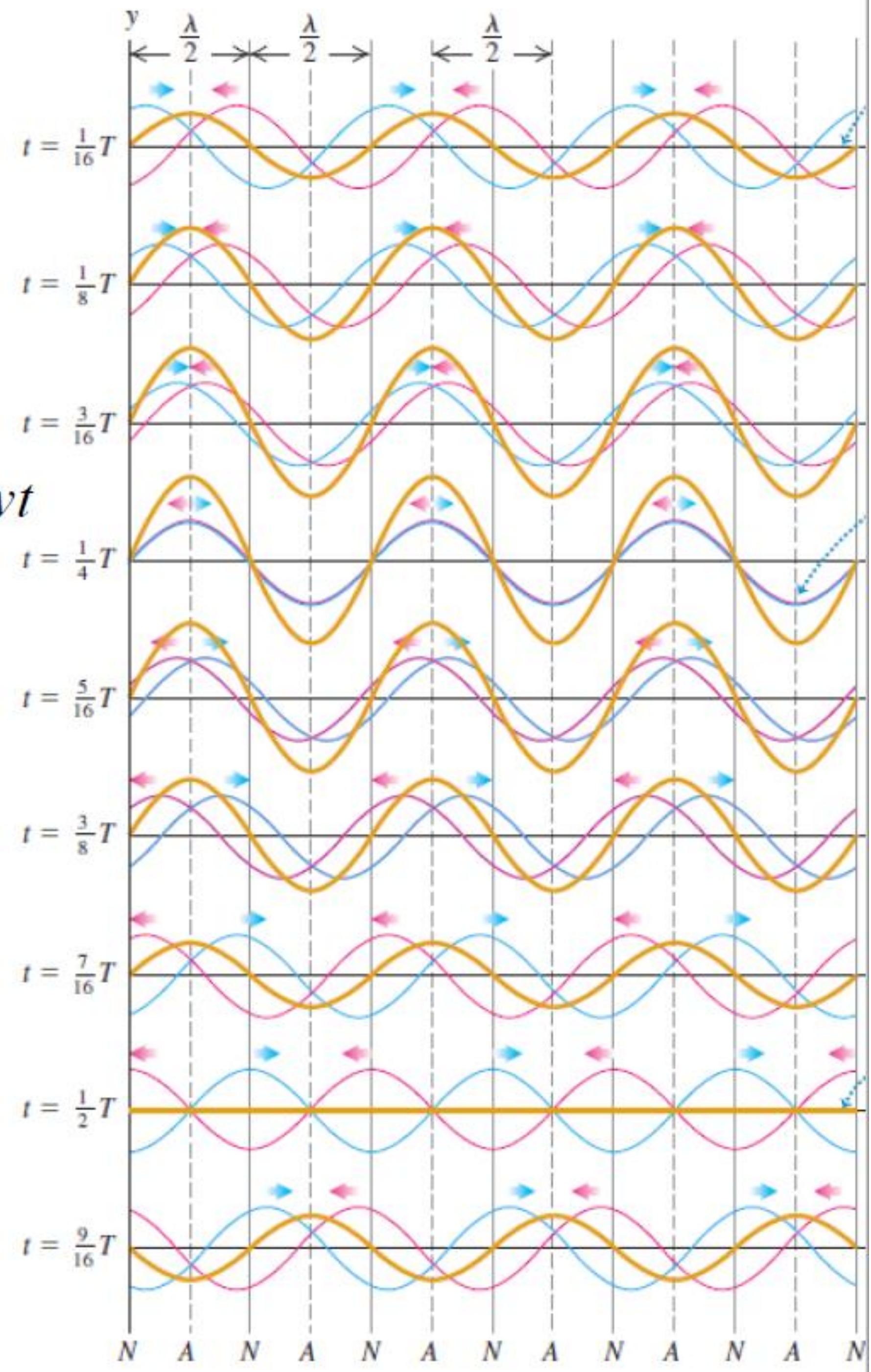
$$u_2(x,t) = A_0\cos(wt + kx)$$

设两波分别向右和向左传播，并假定振幅相同，原点处初始相位均为0.

$$u(x,t) = u_1(x,t) + u_2(x,t) = 2A_0\cos kx \cdot \cos wt$$

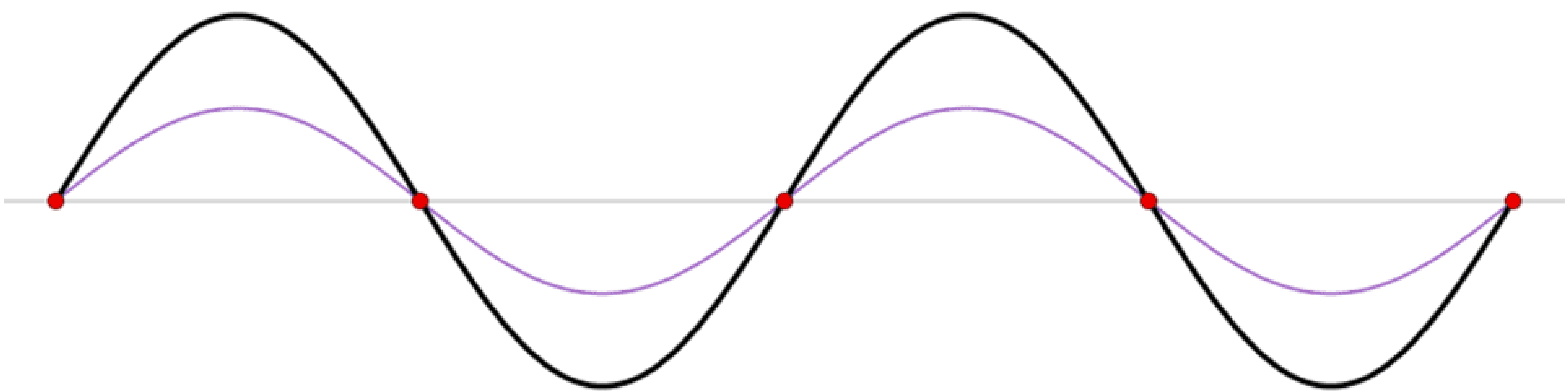
驻波

$$u(x,t) = u_1(x,t) + u_2(x,t) = 2A_0 \cos kx \cdot \cos wt$$



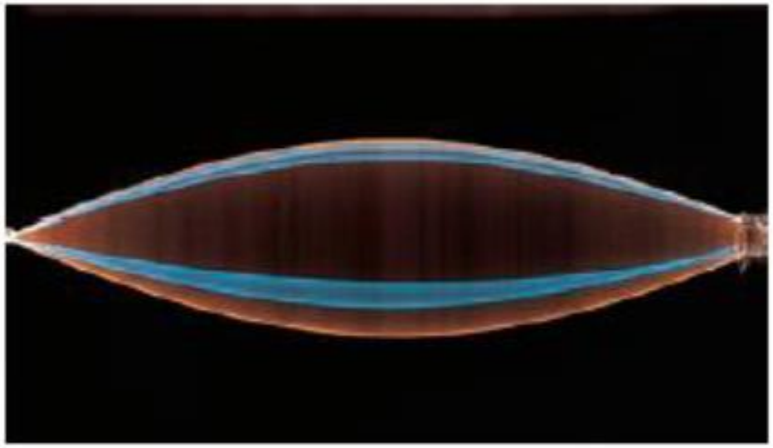
驻波的形成

两列波，频率相同，振动方向相同，传播方向相反，形成驻波。

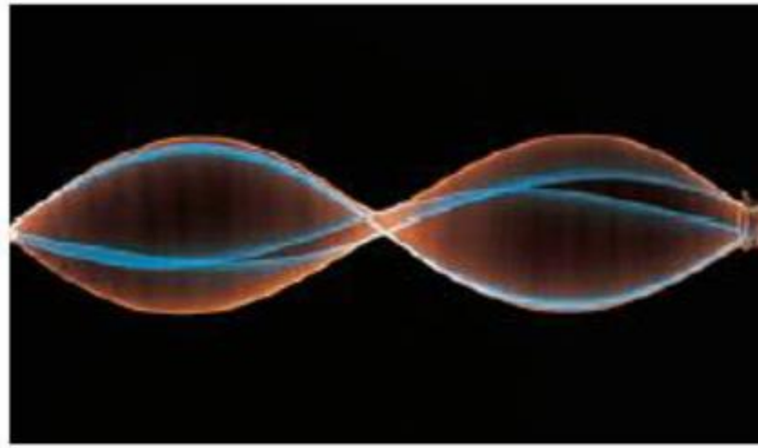


驻波

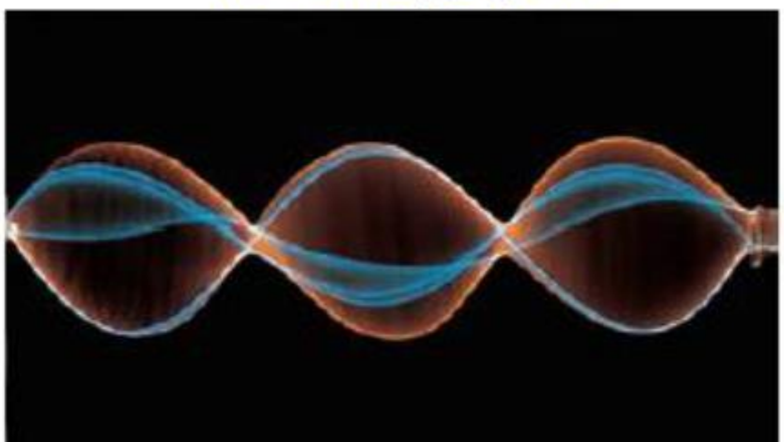
弦为波长一半



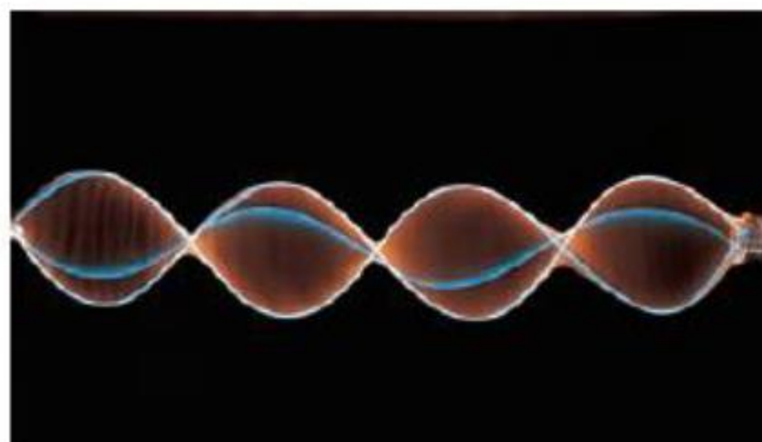
弦等于波长



弦为1.5倍波长

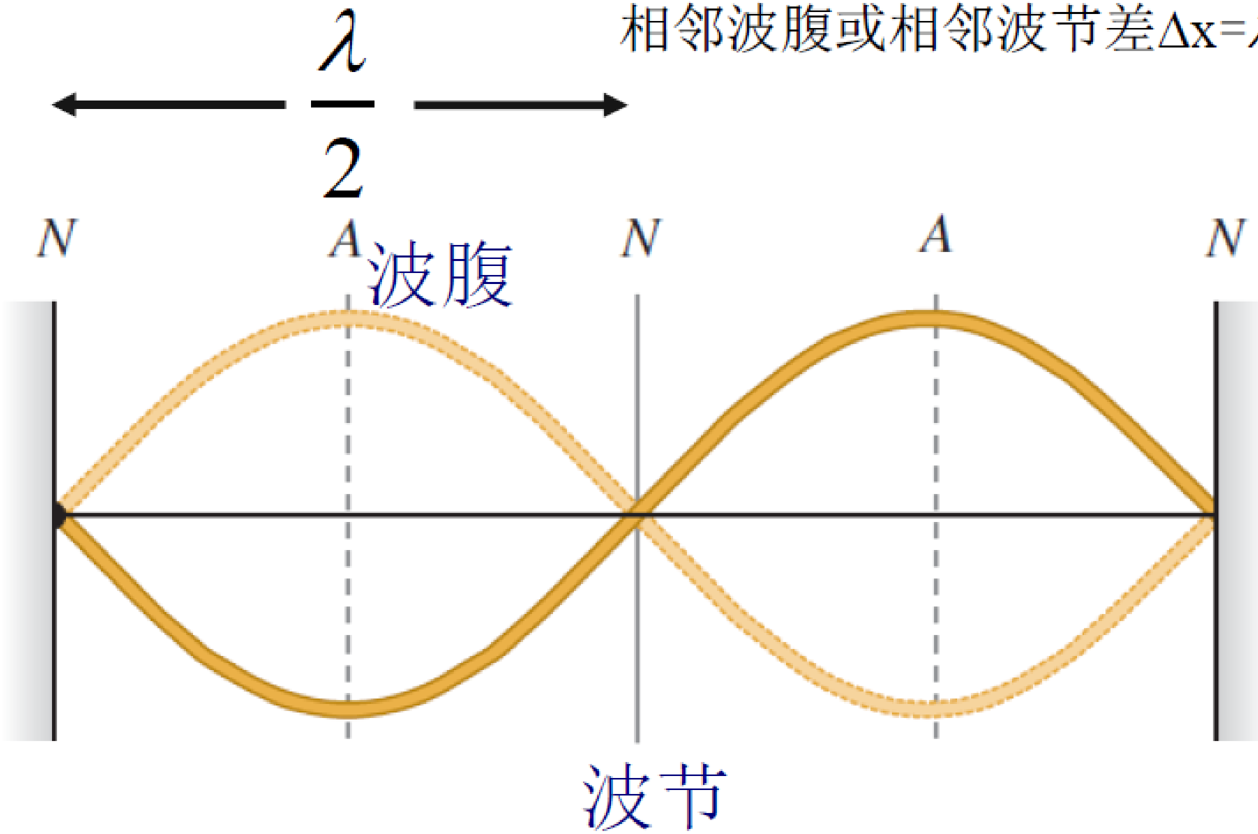


弦为2倍波长



驻波

相邻波腹或相邻波节差Δx=λ/2



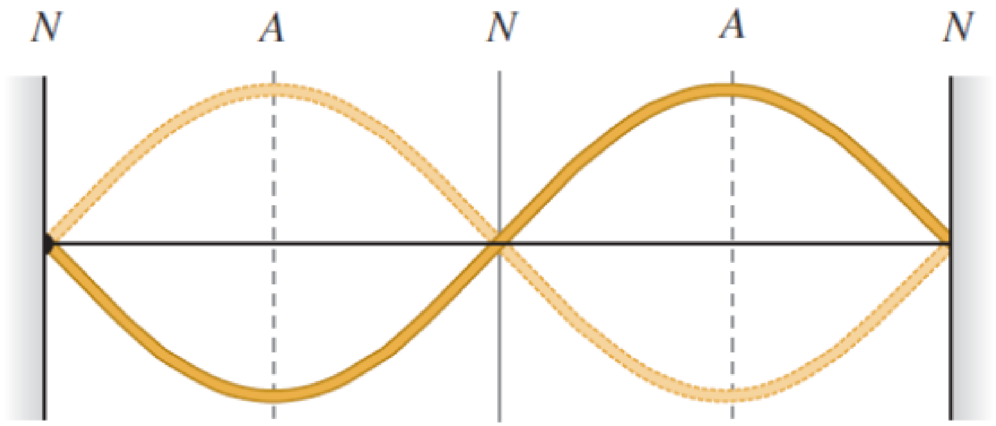
$$k = \frac{2\pi}{\lambda}$$

$$u(x,t) = u_1(x,t) + u_2(x,t) = 2A_0 \cos kx \cdot \cos wt$$

$x_n = n\dfrac{\lambda}{2}, n = 0, \pm 1, \pm 2, \ldots$ 波节 $x_n = (n+\dfrac{1}{2})\dfrac{\lambda}{2}, n = 0, \pm 1, \pm 2, \ldots$ 波腹

驻波

驻波场平均能流密度为0，
各点平均能量密度不为0.



驻波波腹处只有动能（两侧位移函数对称，无形变。

波节处两侧位移函数反对称，存在形变势能。

驻波

$$\lambda_n = \frac{2L}{n} \qquad n = 1, 2, 3, \ldots$$

$$f_n = \frac{v}{\lambda_n} = \frac{n}{2L} v \qquad n = 1, 2, 3, \ldots$$