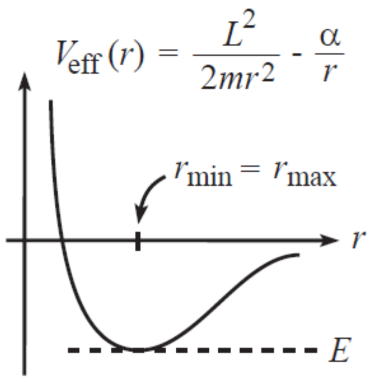
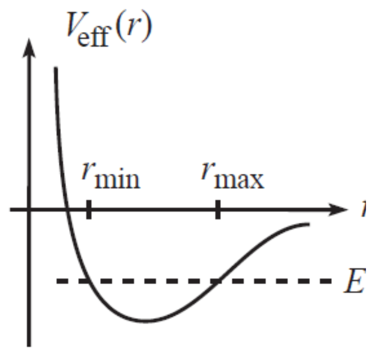
平方反比力场中的轨道

机械能守恒，角动量守恒。轨道形状和能量相关

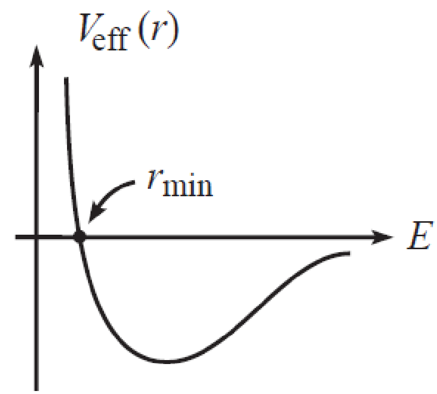
定义有效势



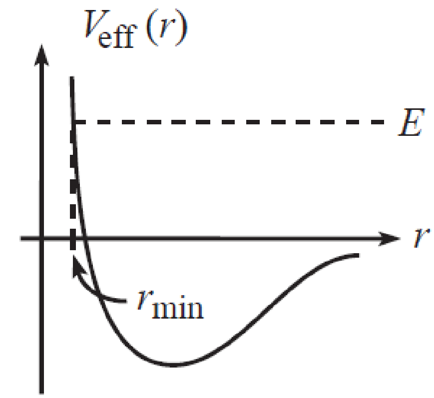
圆形轨道



椭圆轨道



抛物线轨道

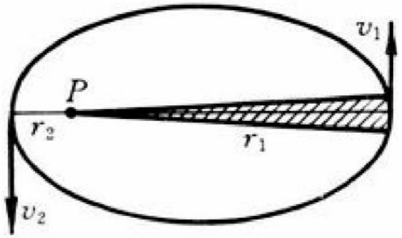


双曲线轨道

太阳在焦点位置

地球绕日运动：椭圆轨道

$1/r^2$



近日点和远日点附近，

机械能守恒

$$\frac{1}{2}mv_1^2 - G\frac{Mm}{r_1} = \frac{1}{2}mv_2^2 - G\frac{Mm}{r_2}$$

角动量守恒

$$\mathrm{r}_1 mv_1 = r_2 mv_2$$

最后得：$\mathrm{r}_2 = a - c$

地球同步卫星

1. 卫星轨道平面和赤道平面重合
角动量守恒，角速度方向与地球自转轴一致

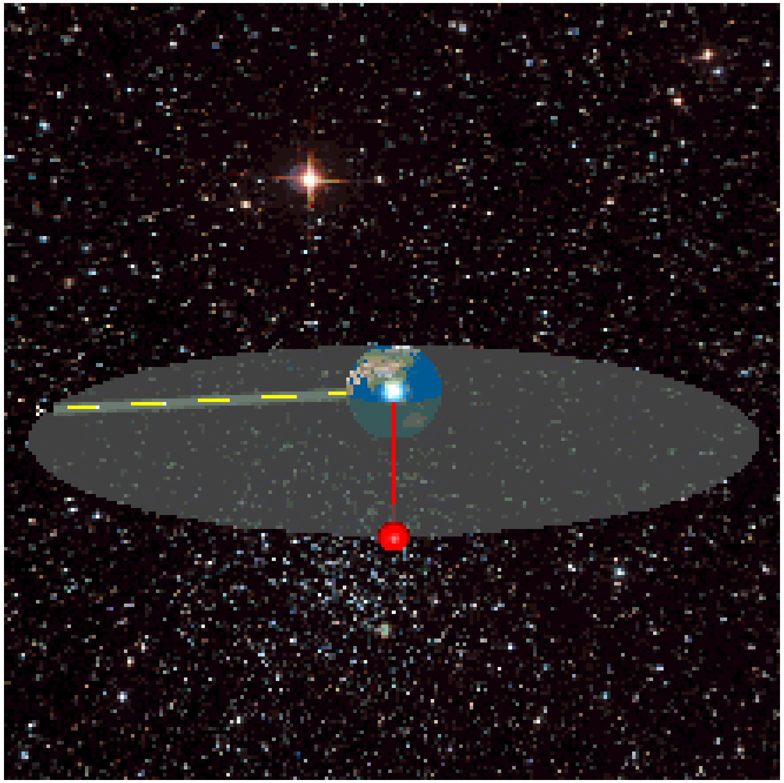
2. 卫星角速度等于地球自转速度

$$w = 2\pi / T, T = 24 \times 60 \times 60s$$

3. 卫星高度不高不低，引力等于向心力

$$G\frac{Mm}{(R+h)^2} = m\omega^2 (R+h)$$

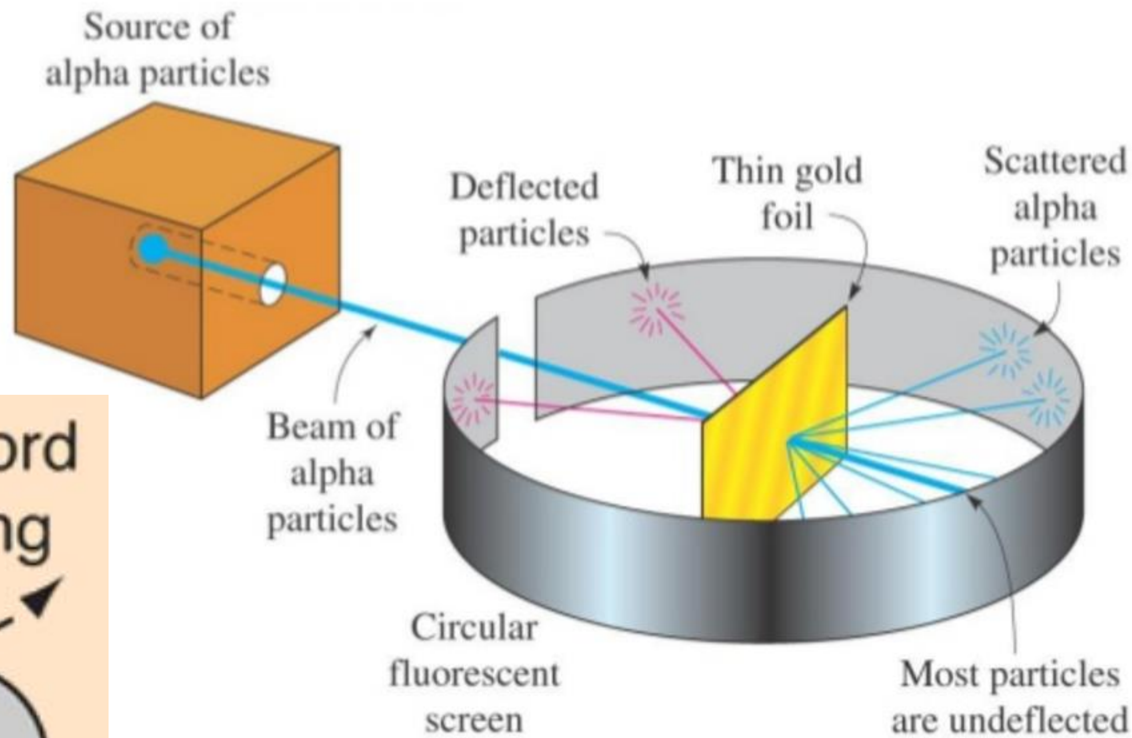
解出$h \approx 42000km - 6300km \approx 35630km$

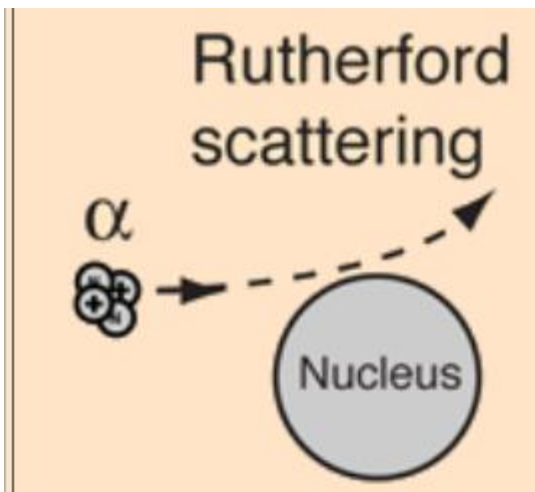


地球同步轨道

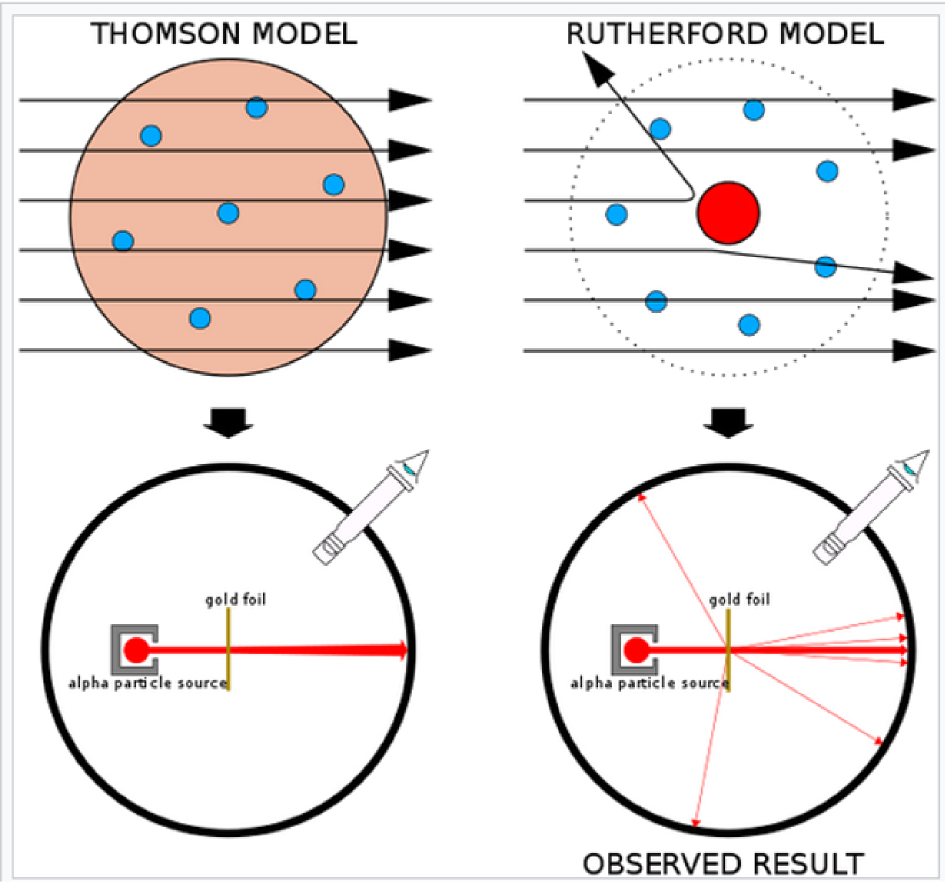
散射

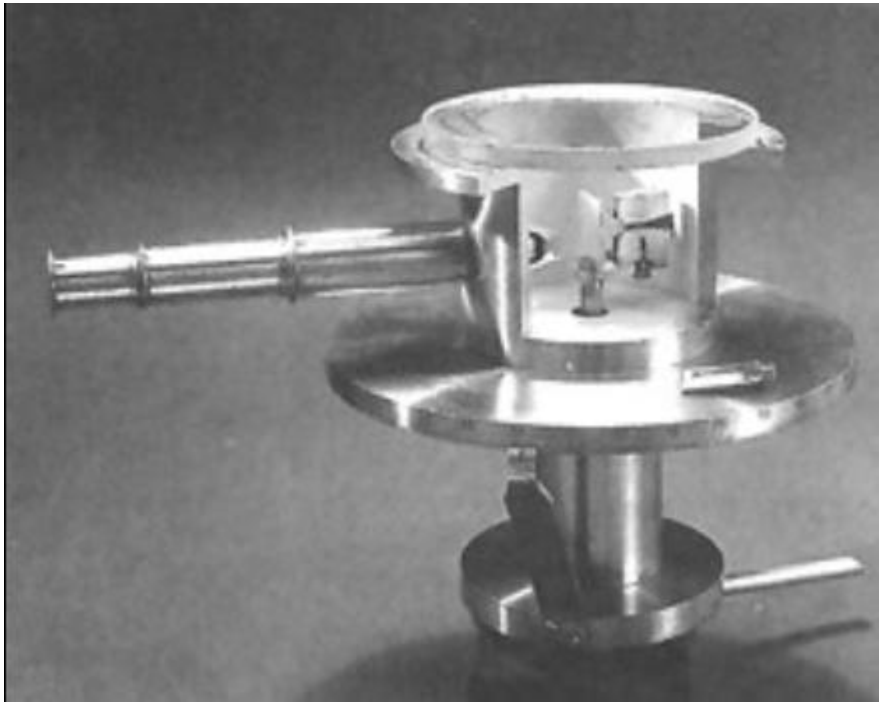
Rutherford散射

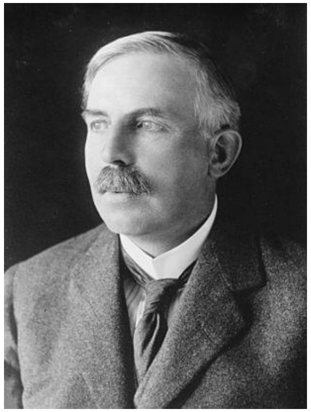




Rutherford散射







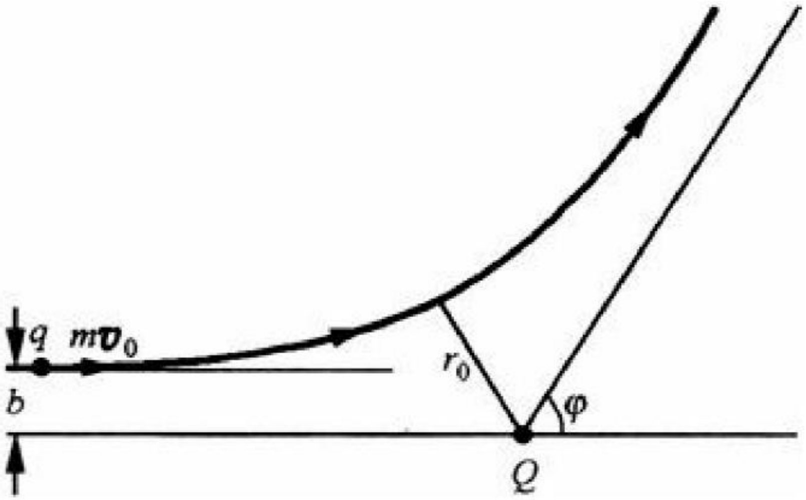
粒子散射

库仑力： 平方反比
库伦势： kQq/r

角动量守恒

机械能守恒

E>0, 双曲线



瞄准距离b

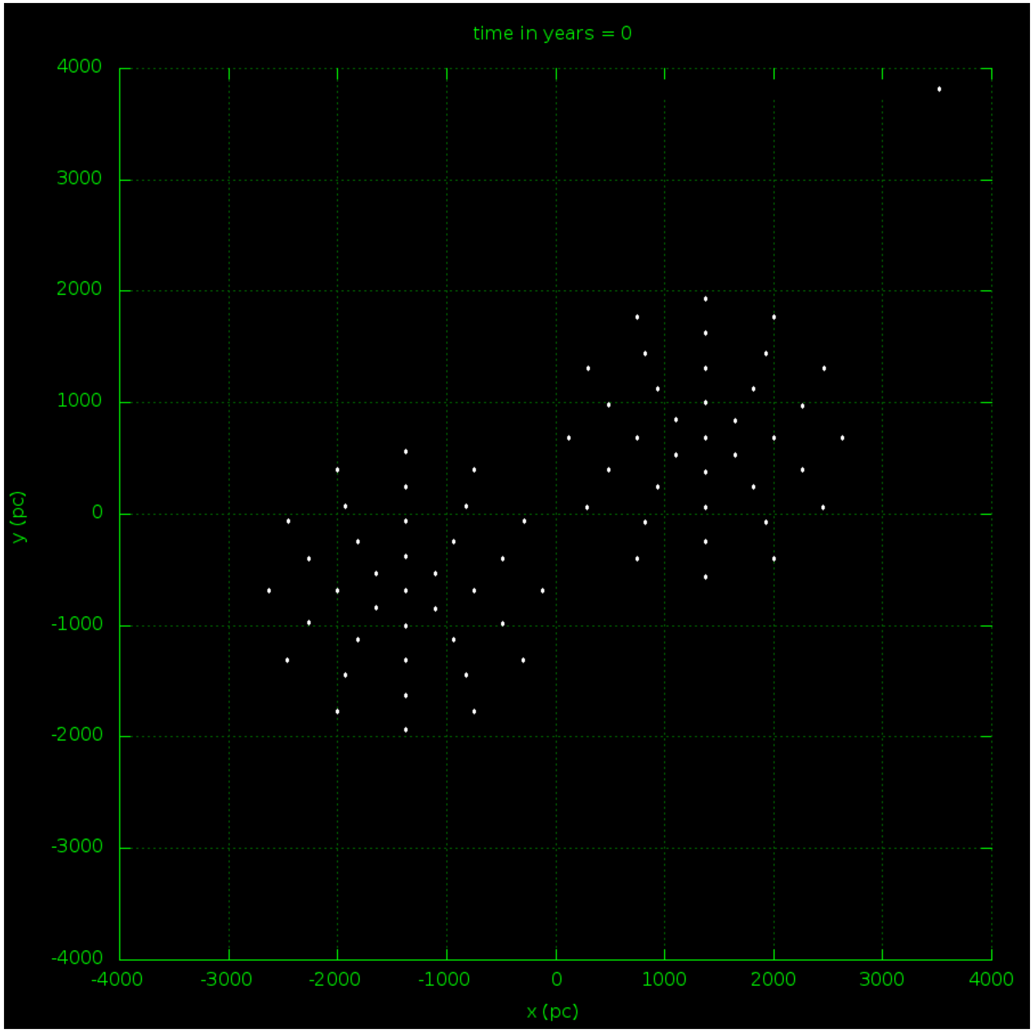
$$\cot\frac{\varphi}{2}=\frac{1}{k}\frac{mv_0^2}{qQ}\cdot b,$$

散射角φ

$$r_0=k\frac{qQ}{mv_0^2}+\sqrt{\left(k\frac{qQ}{mv_0^2}\right)^2+b^2},\quad k=\frac{1}{4\pi\varepsilon_0}.$$

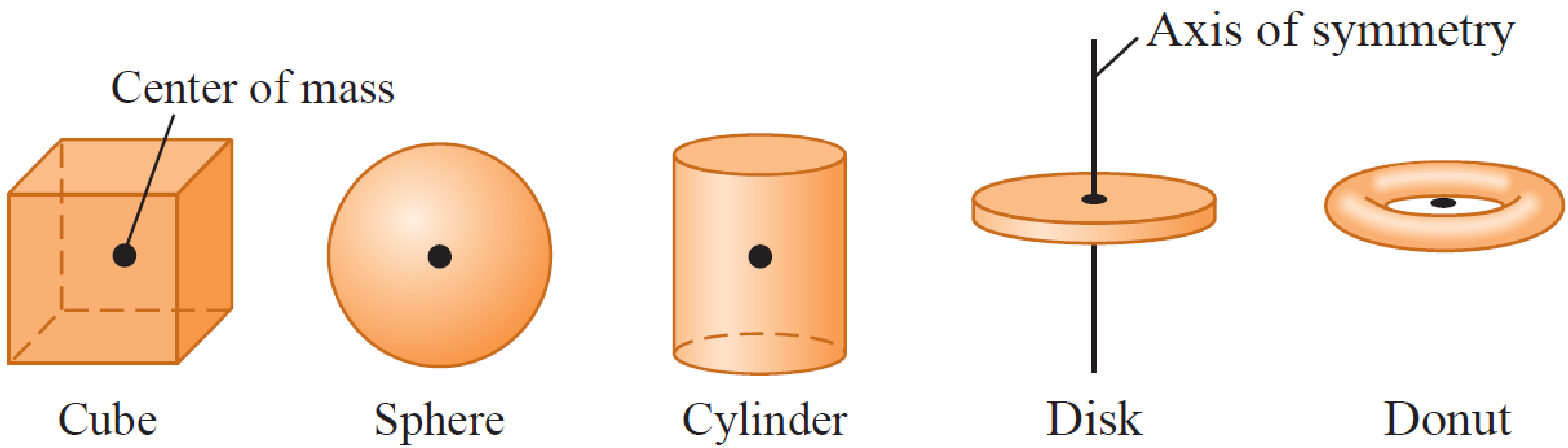
最近距离r

质心力学定理



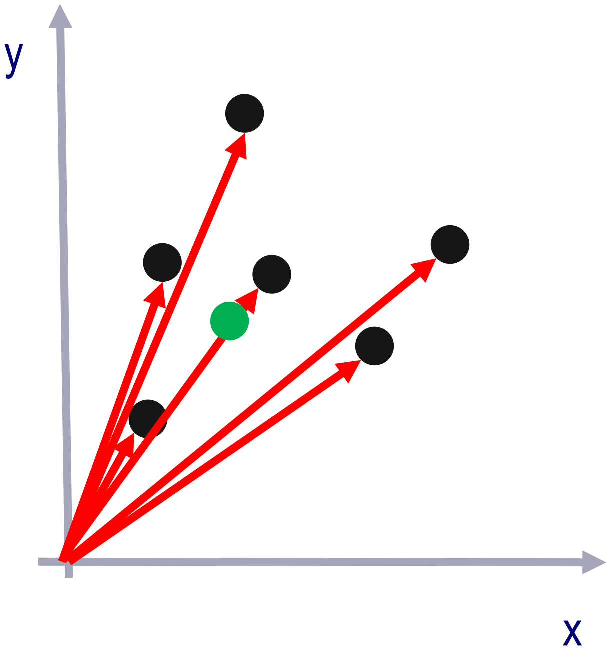
time in years = 0
4000
3000
2000
1000
0
-1000
-2000
-3000
-4000
y (pc)
-4000
-3000
-2000
-1000
0
1000
2000
3000
4000
x (pc)

质心



Axis of symmetry
Center of mass
Cube
Sphere
Cylinder
Disk
Donut

质心



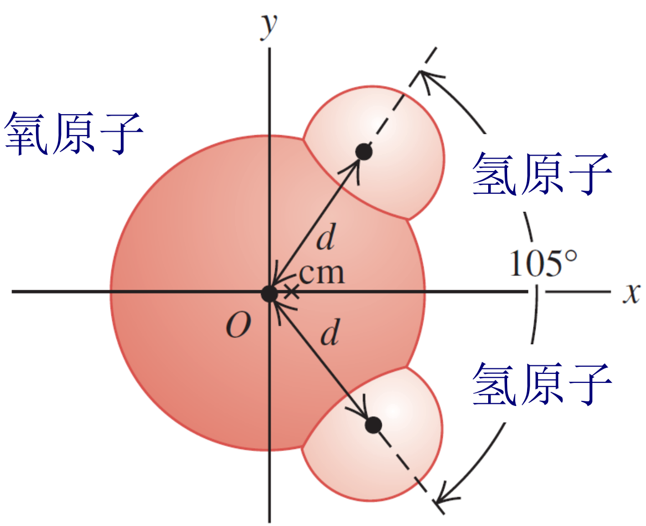
$$\vec{r}_C = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum \mathrm{m}_i \vec{r}_i}{M}$$

$$x_c = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

$$y_c = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum_i m_i y_i}{\sum_i m_i}$$

质心

求水分子的质心



$$x_c = \frac{\left[\begin{array}{c}(1.0\ \mathrm{u})(d\cos 52.5°) + (1.0\ \mathrm{u})(d\cos 52.5°) \\ + (16.0\ \mathrm{u})(0)\end{array}\right]}{1.0\ \mathrm{u} + 1.0\ \mathrm{u} + 16.0\ \mathrm{u}} = 0.068d$$

$$y_c = \frac{\left[\begin{array}{c}(1.0\ \mathrm{u})(d\sin 52.5°) + (1.0\ \mathrm{u})(-d\sin 52.5°) \\ + (16.0\ \mathrm{u})(0)\end{array}\right]}{1.0\ \mathrm{u} + 1.0\ \mathrm{u} + 16.0\ \mathrm{u}} = 0$$

H-O 原子间距： $d = 9.75 \times 10^{-11}$ m

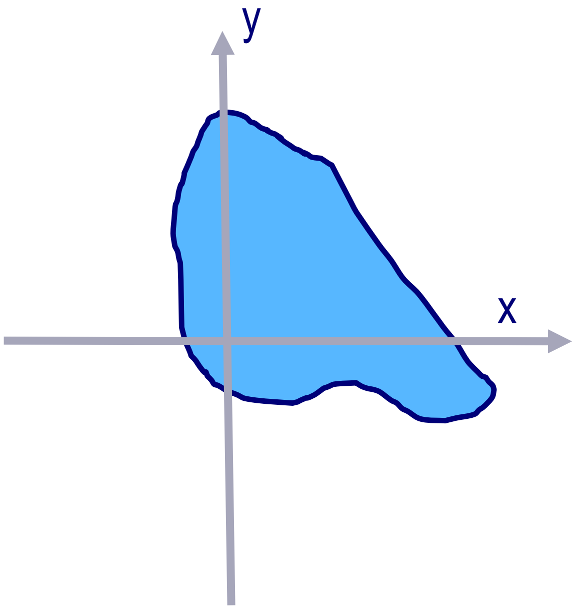
H 原子质量： 1.0 u

O 原子质量： 16.0 u

$$x_{cm} = (0.068)(9.57 \times 10^{-11}\ \mathrm{m}) = 6.5 \times 10^{-12}\ \mathrm{m}$$

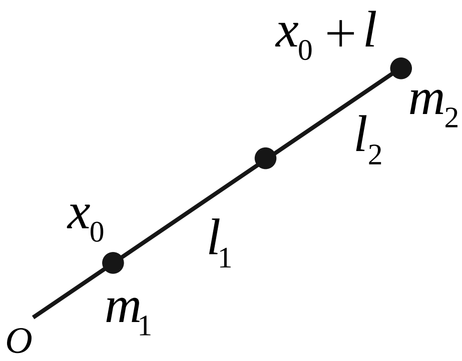
质心

连续分布：



$$\vec{r}_C = \int \frac{\vec{r}dm}{M} = \frac{\int_V \rho(\vec{r})\vec{r}d\tau}{\int_V \rho(\vec{r})d\tau}$$

两体质心与杠杆关系



m_1坐标： x_0

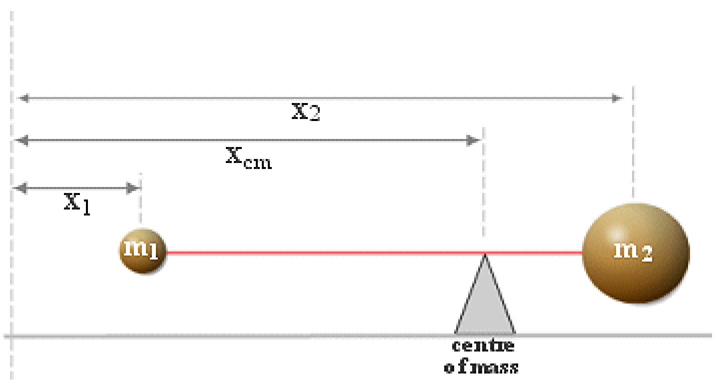
m_2坐标： x_0+l

两体质心坐标：

$$X_C=\frac{m_1x_0+m_2(x_0+l)}{m_1+m_2}=x_0+\frac{m_2l}{m_1+m_2}$$

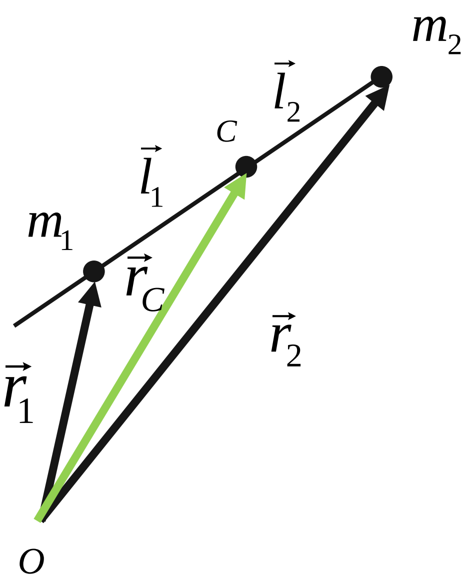
质心距两质点连线之间，与m_1和m_2距离分别为：

$$l_1=\frac{m_2}{m_1+m_2}l,l_2=\frac{m_1}{m_1+m_2}l$$



杠杆关系：$m_1l_1=m_2l_2$

两体质心与杠杆关系



$$\vec{r}_1 = \vec{r}_C + \vec{l}_1$$

$$\vec{r}_2 = \vec{r}_C + \vec{l}_2$$

所以

$$\vec{r}_C = \frac{m_1(\vec{r}_C + \vec{l}_1) + m_2(\vec{r}_C + \vec{l}_2)}{m_1 + m_2} = \vec{r}_C + \frac{m_1\vec{l}_1 + m_2\vec{l}_2}{m_1 + m_2}$$

$$\frac{m_1\vec{l}_1 + m_2\vec{l}_2}{m_1 + m_2} = 0$$

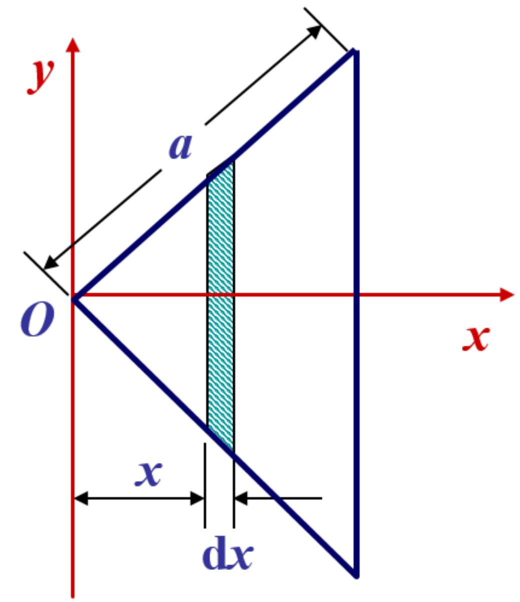
$$m_1\vec{l}_1 + m_2\vec{l}_2 = 0$$

所以参考点的选择并不会改变质心实际的位置

质心

例2-1

例题2-1求腰长为a等腰$\boxed{\text{直角}}$三角形均匀薄板的质心位置。



解:建立图示坐标,

由于面积元的高度为2y，所以其面积为2ydx = 2xdx。设薄板每单位面积的质量为 σ，则此面积元的质量

$$\mathrm{d}m = 2x\sigma\,\mathrm{d}x$$

$$x_c = \frac{\int x\mathrm{d}m}{m} = \frac{\int_0^{a/\sqrt{2}} 2\sigma\, x^2\mathrm{d}x}{\sigma\,\frac{1}{2}a^2} = \frac{\sqrt{2}}{3}a$$

质心和重心

质心位置： $$\mathbf{r}_C = \frac{\sum_i m_i \mathbf{r}_i}{\sum m_i} = \frac{\int_V \rho(\mathbf{r})\mathbf{r}dV}{\int_V \rho(\mathbf{r})dV}$$

重心位置： $$\mathbf{r}_G = \frac{\sum_i m_i g_i \mathbf{r}_i}{\sum m_i g_i} = \frac{\int_V \rho(\mathbf{r})g(\mathbf{r})\mathbf{r}dV}{\int_V \rho(\mathbf{r})g(\mathbf{r})dV}$$

质心和重心

$$mg = G\frac{Mm}{r^2}$$

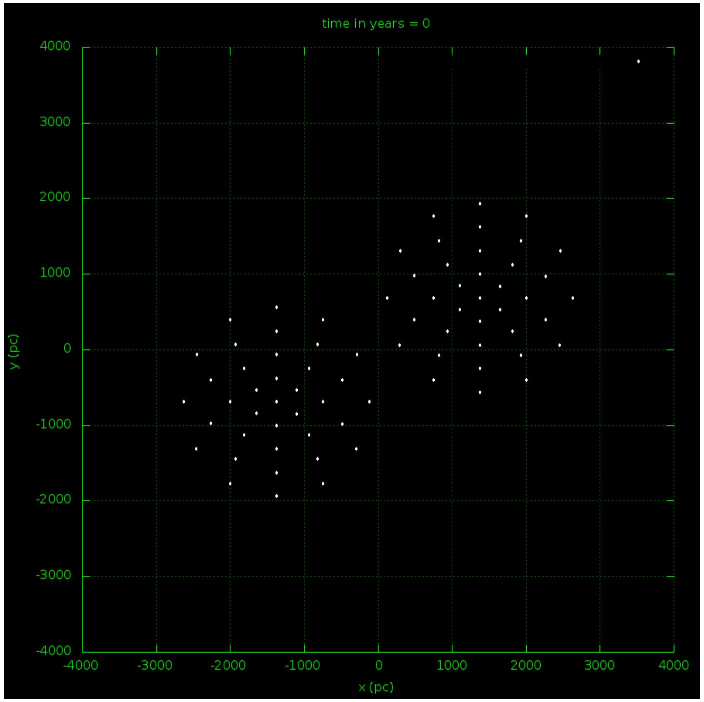
$$g = G\frac{M}{r^2}$$

$$\frac{dg}{dr} = \frac{-2GM}{r^3}$$

$$r_0 = 6000\ \text{km}, h = 500\ \text{m}$$

质心比重心高约2cm

质心运动定理



质心：

$$\vec{r}_C = \frac{\sum m_i \vec{r}_i}{M}$$

质心速度：

$$\vec{v}_C = \frac{d\vec{r}_C}{dt} = \frac{\sum m_i \frac{d\vec{r}_i}{dt}}{M} = \frac{\sum m_i \vec{v}_i}{M}$$

质心加速度：

$$\vec{a}_C = \frac{d\vec{v}_C}{dt} == \frac{\sum m_i \frac{d\vec{v}_i}{dt}}{M} = \frac{\sum m_i \vec{a}_i}{M}$$

质心动量、质心动能与质心角动量

质心动量是否等于质点组动量?

质心动能是否等于质点组动能?

质心角动量是否等于质点组角动量?

质心动量等于质点组总动量

质心动量等于质点组总动量

质心速度：

$$\vec{v}_C = \frac{d\vec{r}_C}{dt} = \frac{\sum m_i \frac{d\vec{r}_i}{dt}}{M} = \frac{\sum m_i \vec{v}_i}{M}$$

M是质点组总质量M=$\sum m_i$

质心动量等于质点组总动量 $MV_C = \sum m_i v_i$

质心动量变化定量

动量变化定理：

$$\frac{d\vec{p}}{dt}=\vec{F}$$

质心动量变化

$$\frac{d\vec{p}_C}{dt}=\frac{d\sum m_i v_i}{dt}=\frac{\sum \vec{p}_i}{dt}=\sum(\vec{f}_i+\vec{F}_i)$$

质心动量变化等于每个质点的受力之和

$$m_1\vec{a}_1=\boxed{\vec{F}_1}+\vec{f}_{12}+\vec{f}_{13}+\vec{f}_{14}+\cdots\vec{f}_{1n}$$

$$m_2\vec{a}_2=\boxed{\vec{F}_2}+\vec{f}_{21}+\vec{f}_{23}+\vec{f}_{24}+\cdots\vec{f}_{2n}$$

$$m_\mathrm{n}\vec{a}_n=\boxed{\vec{F}_n}+\vec{f}_{n1}+\vec{f}_{n2}+\vec{f}_{n3}+\cdots\vec{f}_{nn-1}$$

成对的内力大小相同，
方向相反（牛顿第三定律）

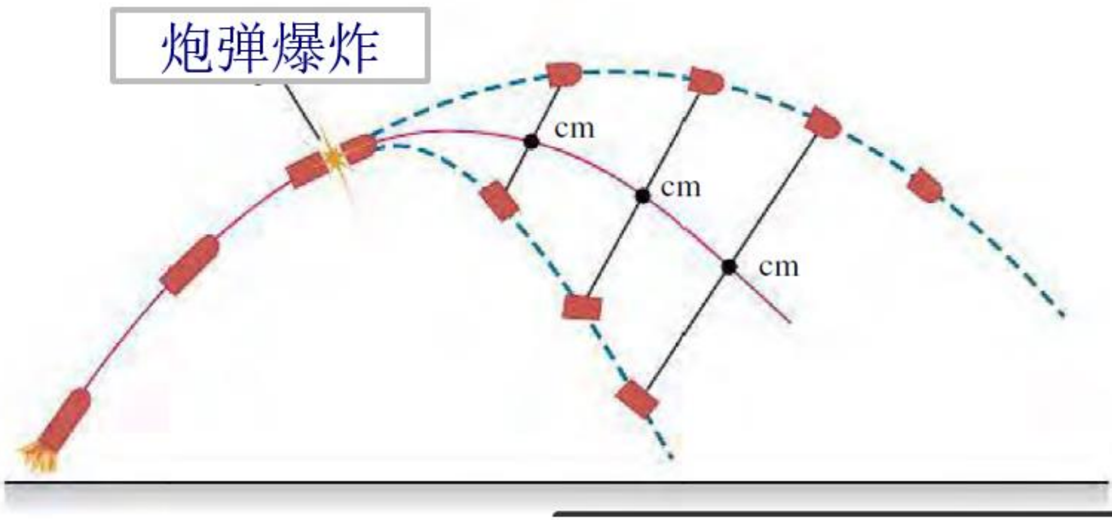
$$\vec{f}_{nm}+\vec{f}_{mn}=0$$

$$\frac{d\vec{p}_C}{dt}=\sum\vec{F}_i$$

仅剩合外力 $\vec{F}_1+\vec{F}_2+\vec{F}_3+\cdots\vec{\mathrm{F}}_n=\sum\vec{F}_i$

质心动量变化等于质点组所受合外力之和

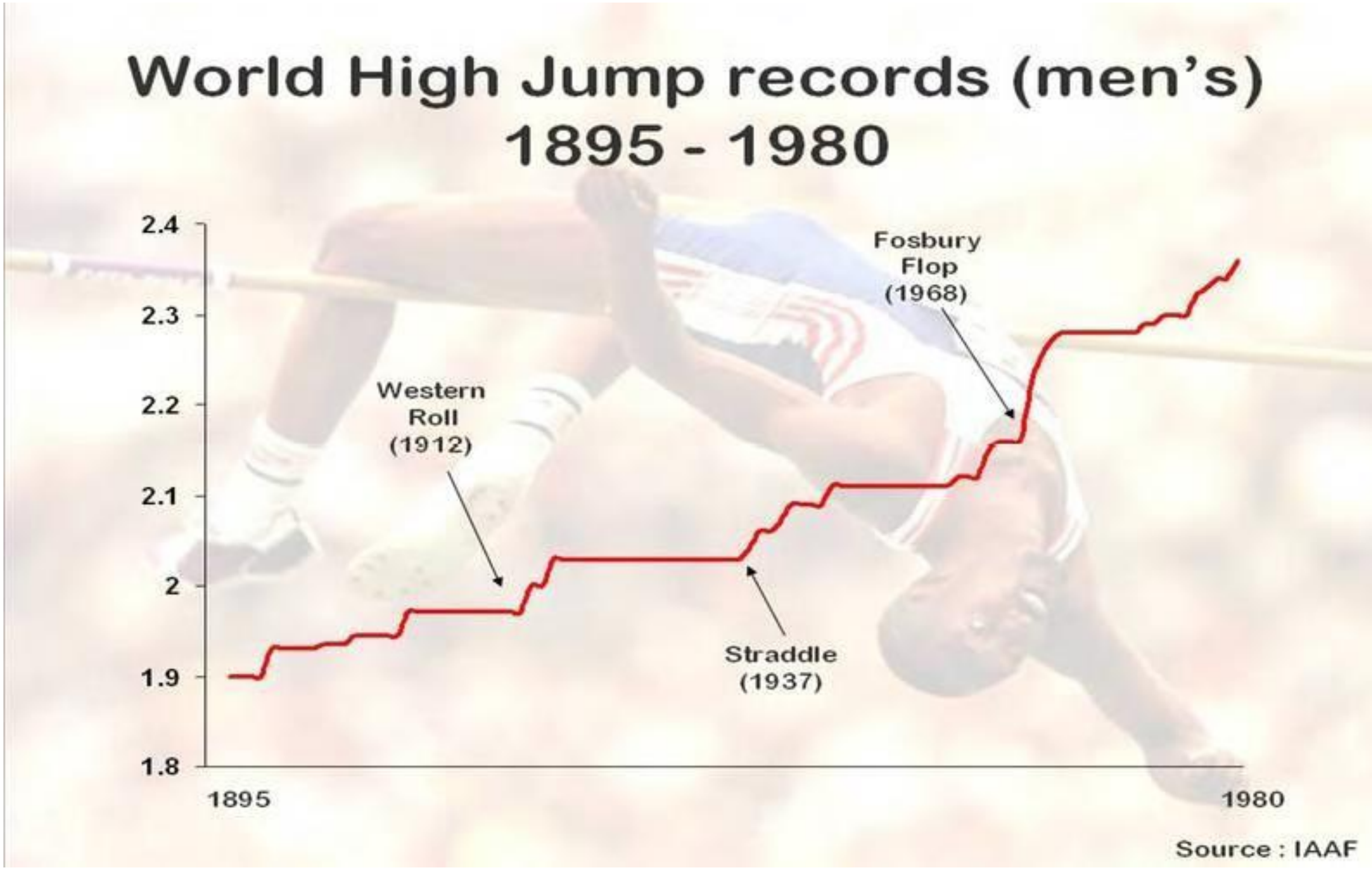
质心运动的例子



跳高

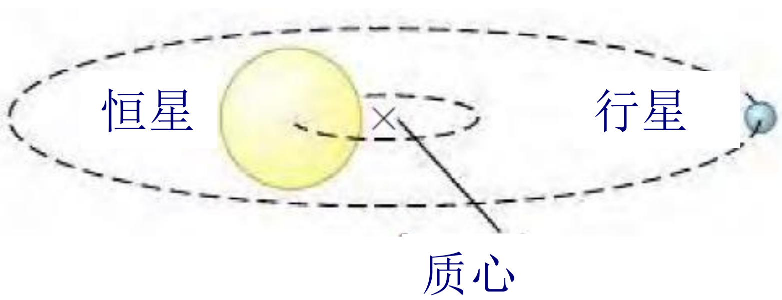
Bundesarchiv, Bild 183-S0305-0030
Foto: Mittelstädt, Rainer | 5. März 1977

跳高世界纪录



World High Jump records (men's)
1895 - 1980
2.4
2.3
2.2
2.1
2
1.9
1.8
1895
1980
Fosbury
Flop
(1968)
Western
Roll
(1912)
Straddle
(1937)
Source : IAAF

发现地外行星



恒星在周期性晃动,进而影响光谱变化