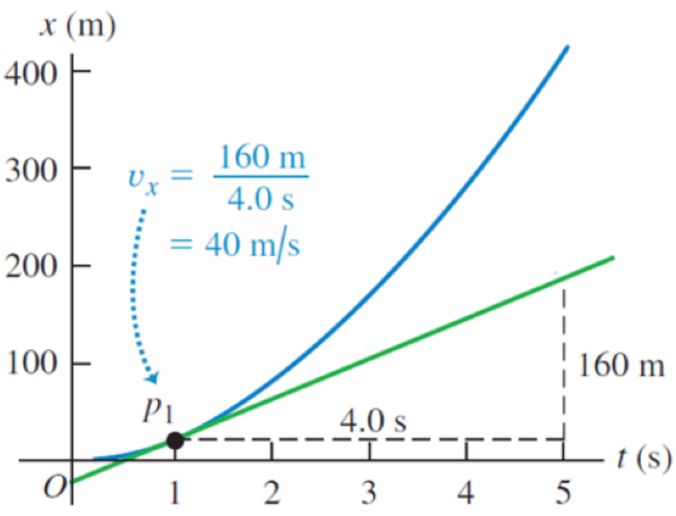
小结

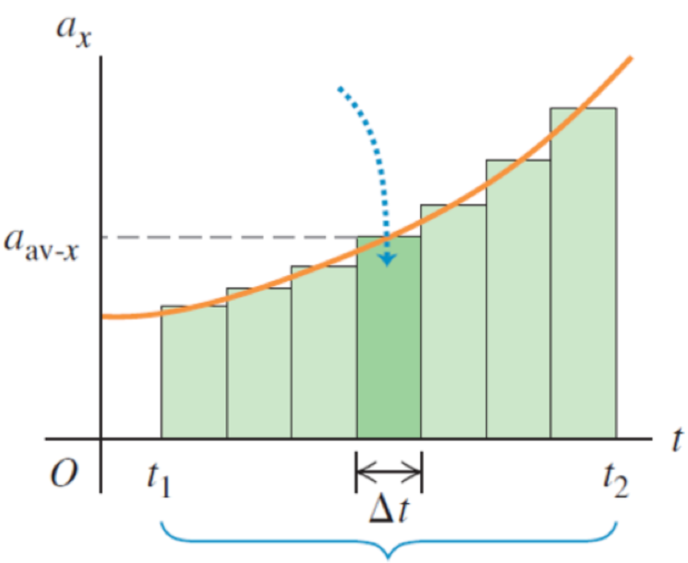
$$v_x = \frac{dx}{dt}$$

$$v_x = v_0 + \int_0^t a_x dt$$

$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$$

$$x = x_0 + \int_0^t v_x dt$$





惯性导航系统

$$v_{2x} - v_{1x} = \int_{v_{1x}}^{v_{2x}} dv_x = \int_{t_1}^{t_2} a_x\,dt$$

$$x_2 - x_1 = \int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v_x\,dt$$

$$v_x = v_{0x} + \int_0^t a_x\,dt$$

$$x = x_0 + \int_0^t v_x\,dt$$

记录飞机的加速度，并根据起飞时初始的位置和速度，利用加速度的记录算出飞机在空中的位置和速度。

标量与矢量

标量(Scalar): 只有大小，没有方向

温度(T)，质量(M)，体积(V)

矢量(Vector): 既有大小，也有方向

力（$\vec{f}$），位置矢量($\vec{r}$)，速度($\vec{v}$)， 风速

$\vec{A}$ $\vec{B}$

$\vec{A} = \vec{B}$

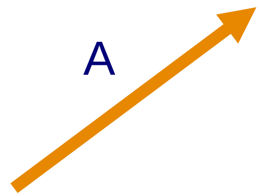
$\vec{A}$ $\vec{B}$

$\vec{A} = -\vec{B}$

矢量

矢量长度

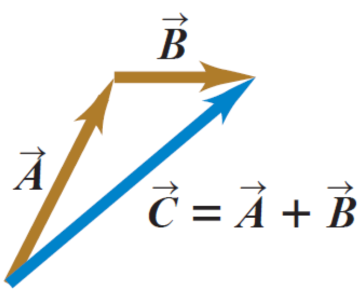
$$A = \left|\vec{A}\right|$$

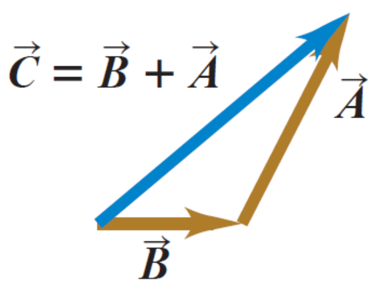


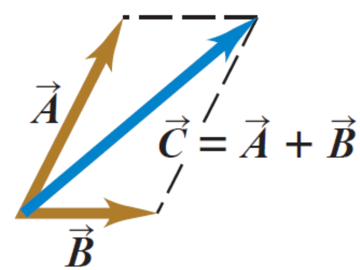
矢量加法

$$\vec{C} = \vec{A} + \vec{B}$$

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

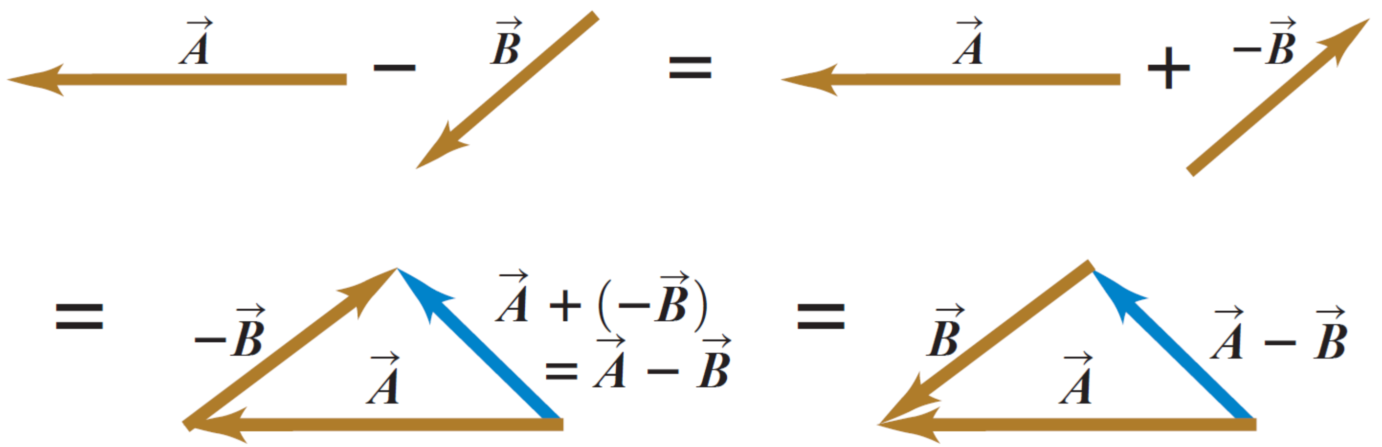




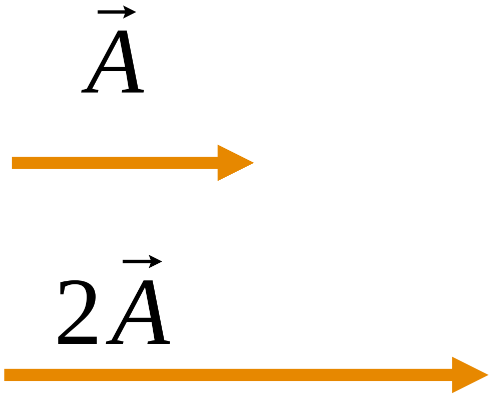


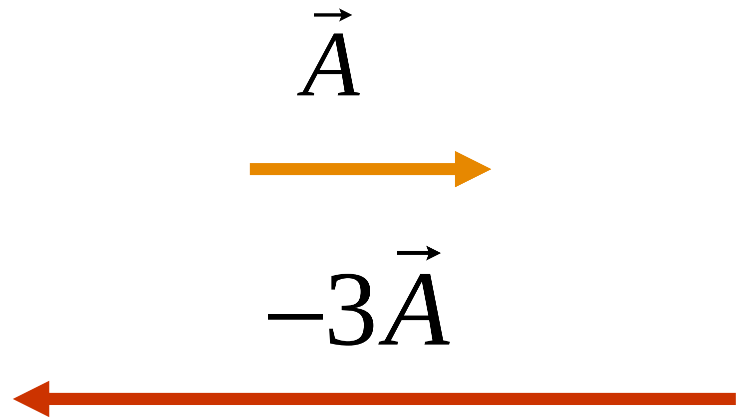
矢量减法

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$



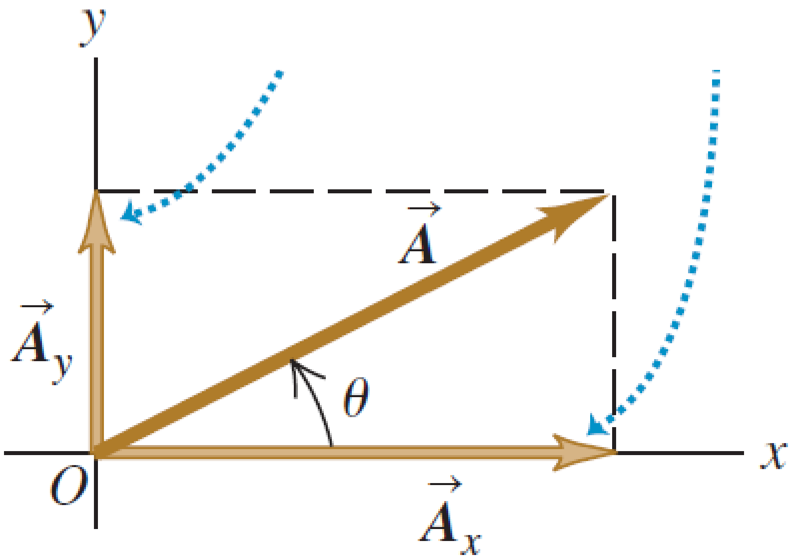
矢量与标量相乘



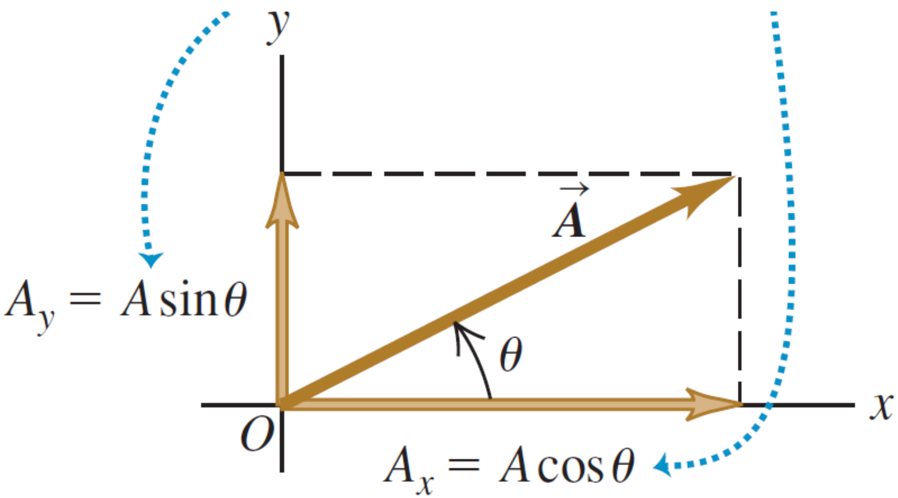


矢量分量

矢量$\vec{A}$的分量



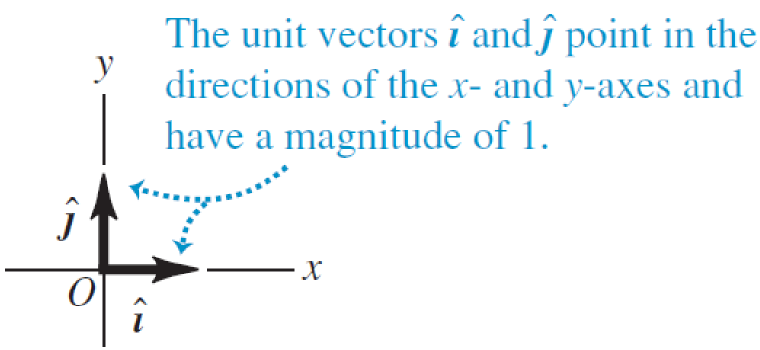
矢量$\vec{A}$的分量



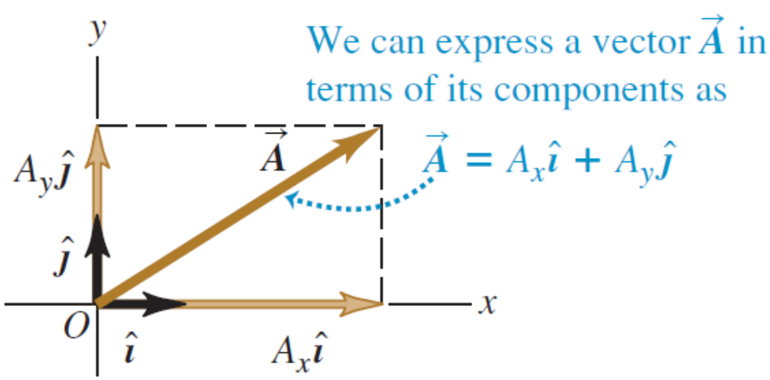
$$\vec{A} = \vec{A}_x + \vec{A}_y$$

单位矢量

(a)



(b)



$$\vec{A}_x = A_x\hat{\boldsymbol{\imath}}$$

$$\vec{A}_y = A_y\hat{\boldsymbol{\jmath}}$$

$$\vec{A} = A_x\hat{\boldsymbol{\imath}} + A_y\hat{\boldsymbol{\jmath}}$$

单位矢量

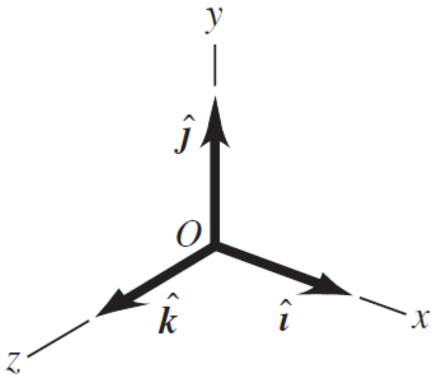
矢量相加

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$$

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath}$$

$$\begin{aligned}\vec{R} &= \vec{A} + \vec{B} \\ &= (A_x\hat{\imath} + A_y\hat{\jmath}) + (B_x\hat{\imath} + B_y\hat{\jmath}) \\ &= (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} \\ &= R_x\hat{\imath} + R_y\hat{\jmath}\end{aligned}$$

三维单位矢量

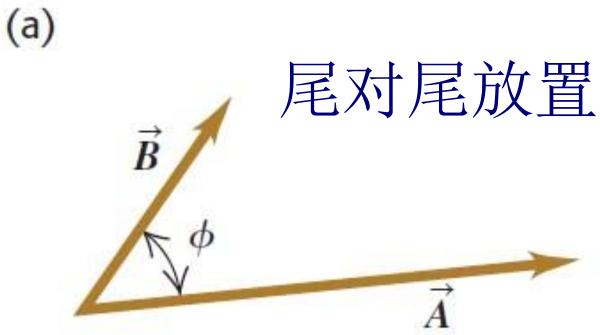


$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

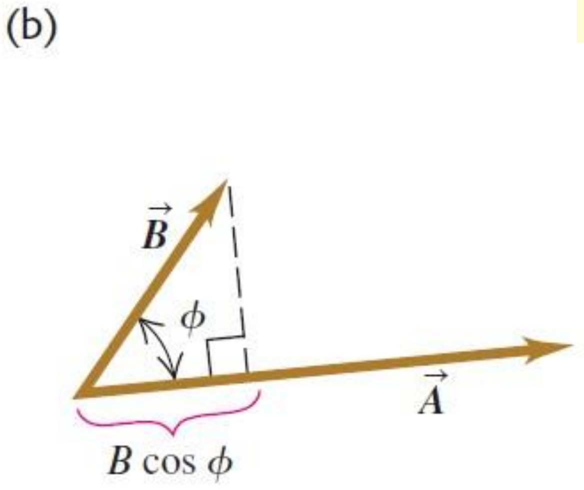
$$\begin{aligned}\vec{R} &= (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} + (A_z + B_z)\hat{k} \\ &= R_x\hat{\imath} + R_y\hat{\jmath} + R_z\hat{k}\end{aligned}$$

矢量点乘

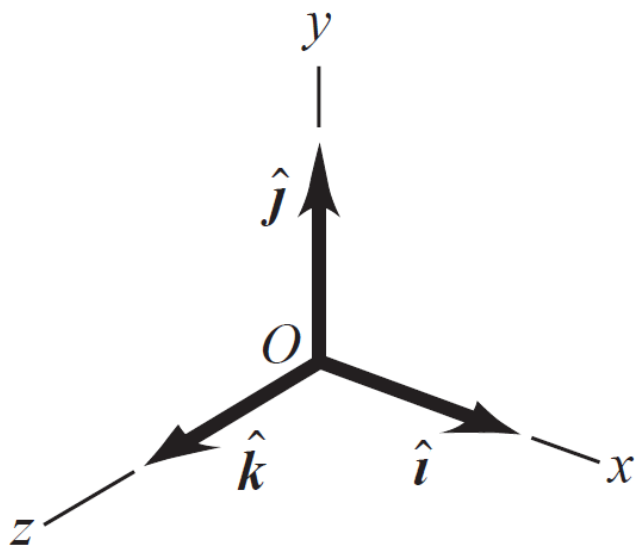


尾对尾放置

$$\vec{A}\cdot\vec{B} = AB\cos\phi = |\vec{A}||\vec{B}|\cos\phi$$



矢量点乘



$$\hat{\imath}\cdot\hat{\imath} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = (1)(1)\cos 0° = 1$$

$$\hat{\imath}\cdot\hat{j} = \hat{\imath}\cdot\hat{k} = \hat{j}\cdot\hat{k} = (1)(1)\cos 90° = 0$$

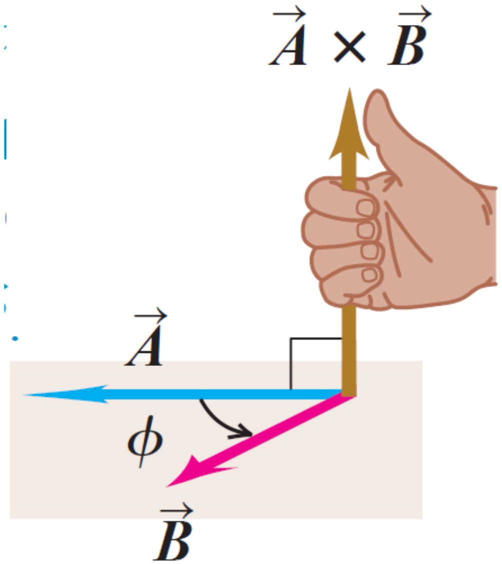
$$\boxed{\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z}$$

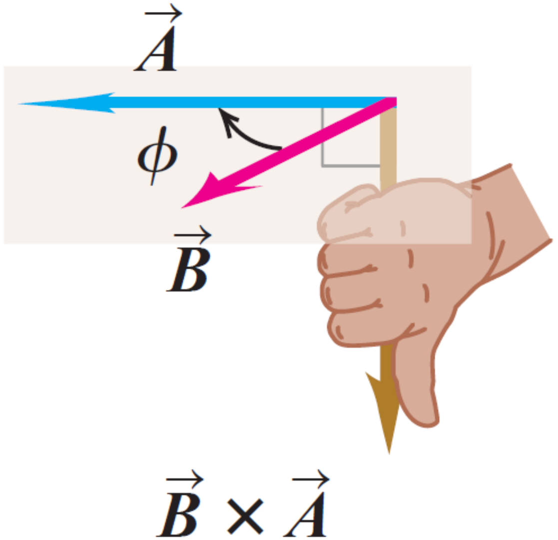
$$\begin{aligned}
\vec{A}\cdot\vec{B} &= (A_x\hat{\imath} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{\imath} + B_y\hat{j} + B_z\hat{k}) \\
&= A_x\hat{\imath}\cdot B_x\hat{\imath} + A_x\hat{\imath}\cdot B_y\hat{j} + A_x\hat{\imath}\cdot B_z\hat{k} \\
&\quad + A_y\hat{j}\cdot B_x\hat{\imath} + A_y\hat{j}\cdot B_y\hat{j} + A_y\hat{j}\cdot B_z\hat{k} \\
&\quad + A_z\hat{k}\cdot B_x\hat{\imath} + A_z\hat{k}\cdot B_y\hat{j} + A_z\hat{k}\cdot B_z\hat{k} \\
&= A_xB_x\hat{\imath}\cdot\hat{\imath} + A_xB_y\hat{\imath}\cdot\hat{j} + A_xB_z\hat{\imath}\cdot\hat{k} \\
&\quad + A_yB_x\hat{j}\cdot\hat{\imath} + A_yB_y\hat{j}\cdot\hat{j} + A_yB_z\hat{j}\cdot\hat{k} \\
&\quad + A_zB_x\hat{k}\cdot\hat{\imath} + A_zB_y\hat{k}\cdot\hat{j} + A_zB_z\hat{k}\cdot\hat{k}
\end{aligned}$$

矢量叉乘

$$\vec{C} = \vec{A} \times \vec{B}$$

$$C = AB\sin\phi$$

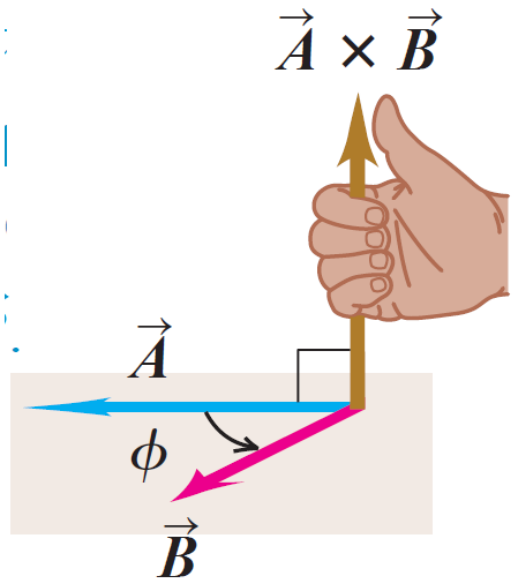


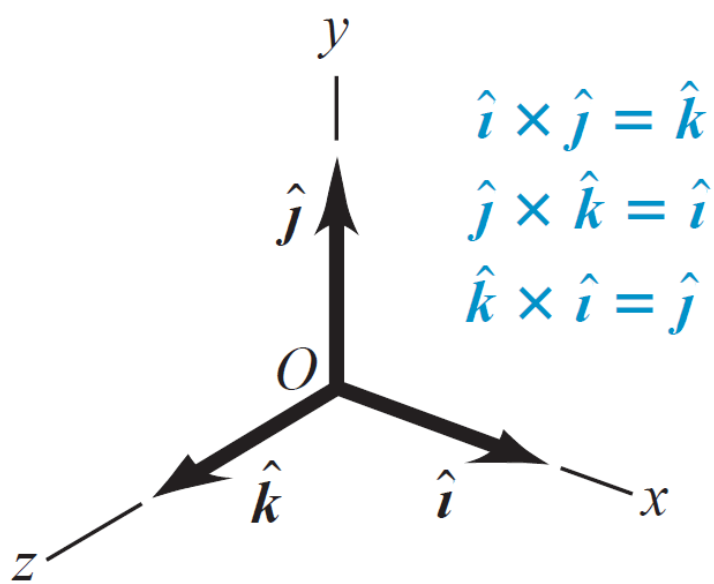


$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

矢量叉乘

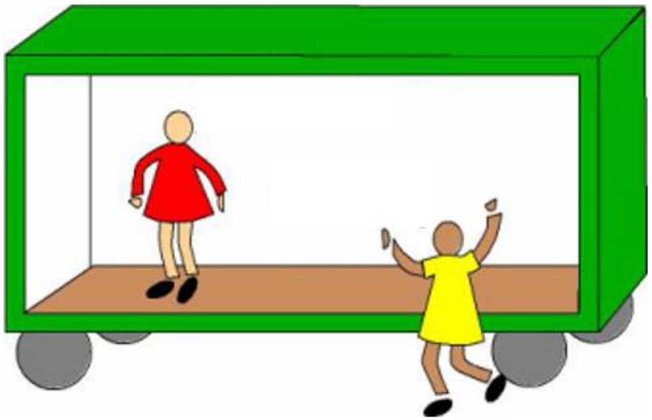
$$\vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k}$$

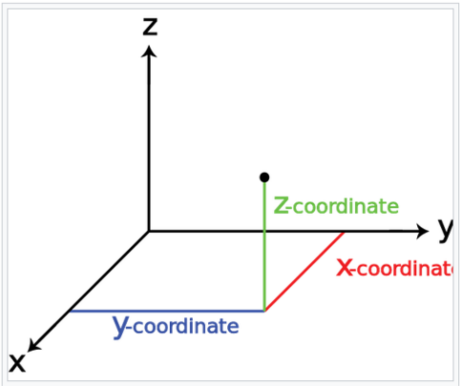




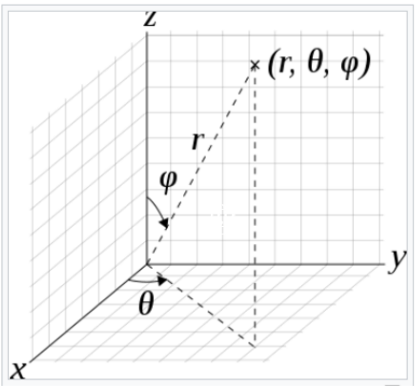
$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

参照系与坐标系





直角坐标系



球坐标系

车为参照系：车上人静止

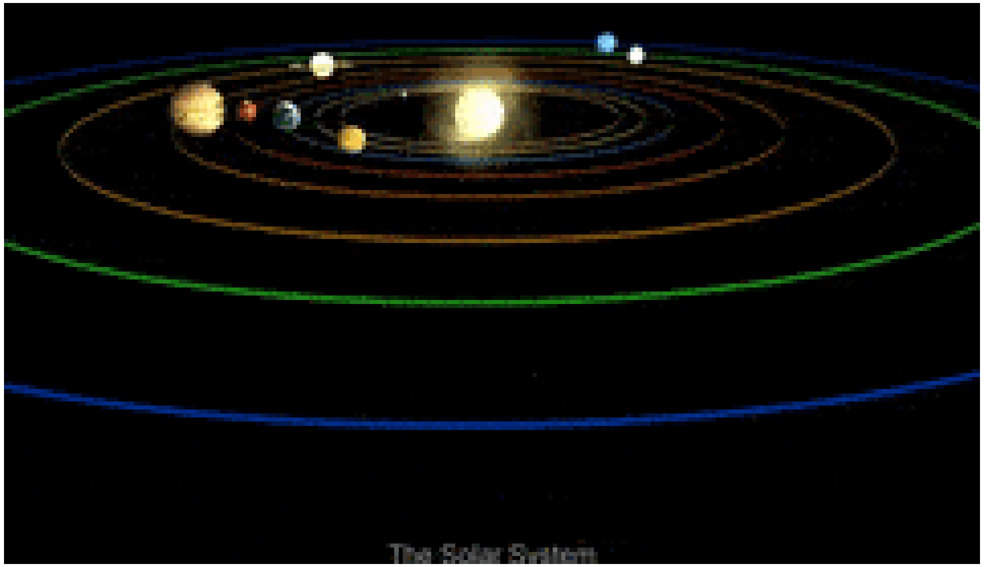
地面为参照系：车上人运动

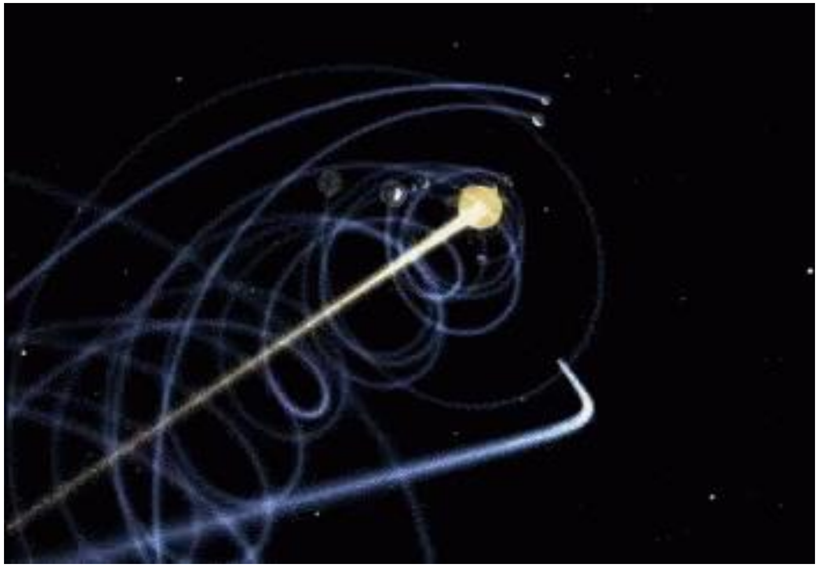
坐标系：参考系中选择一点为原点，取通过原点并标有长度的线为坐标轴，用于定量确定物体在参考系中的位置。

研究运动需要选择一定的参照系

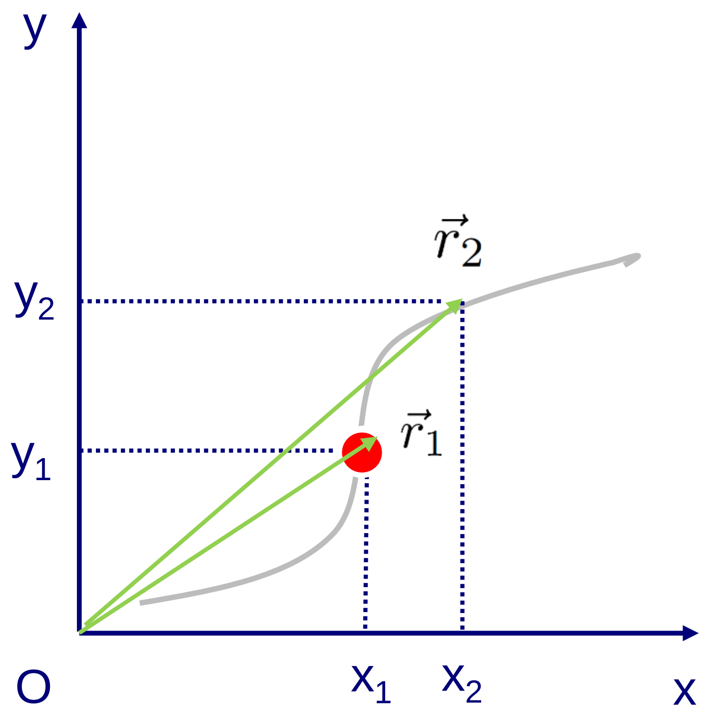
不同参照系下，反映运动的关系不同

太阳系的运动





位置矢量



$$\vec{r}_1 = x_1\hat{i} + y_1\hat{j}$$

$$\vec{r} = x\hat{i} + y\hat{j}$$

$$r = |\vec{r}| = \sqrt{(x^2 + y^2)}$$

位置1： (x_1, y_1)

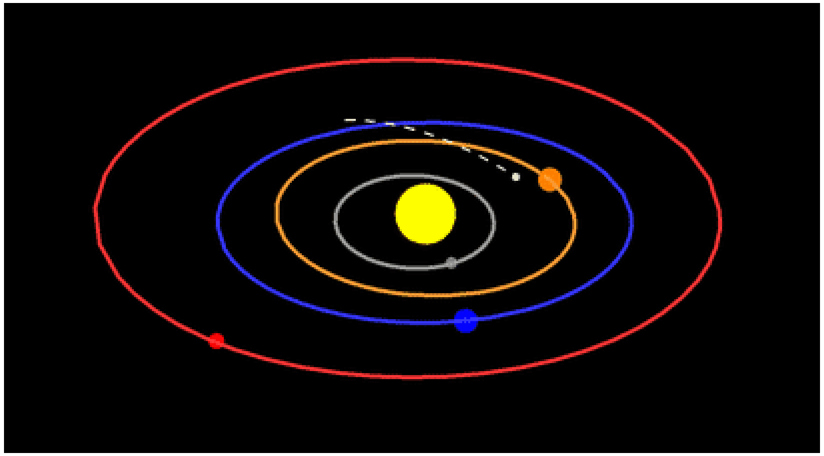
位置2： (x_2, y_2)

运动： $\vec{r} = \vec{r}(t)$

轨道方程

$$\vec{r} = \vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$$

$$\begin{cases} x = x(t) \\ y = y(t) \\ z = z(t) \end{cases}$$



消去时间t得到轨道方程：

x, y, z变量之间的关系，可以体现系统运动的特征

轨道方程

已知，在(x,y)平面内质点轨道方程为

$$x(t) = a\cos wt$$

$$y(t) = b\sin wt$$

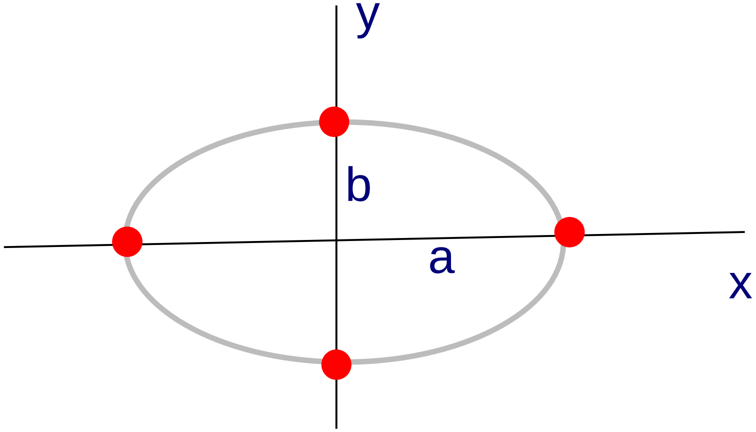
试求其轨道形态及特征。

$$t = 0, x = a, y = 0$$

$$t = \pi / 2\omega, x = 0, y = b$$

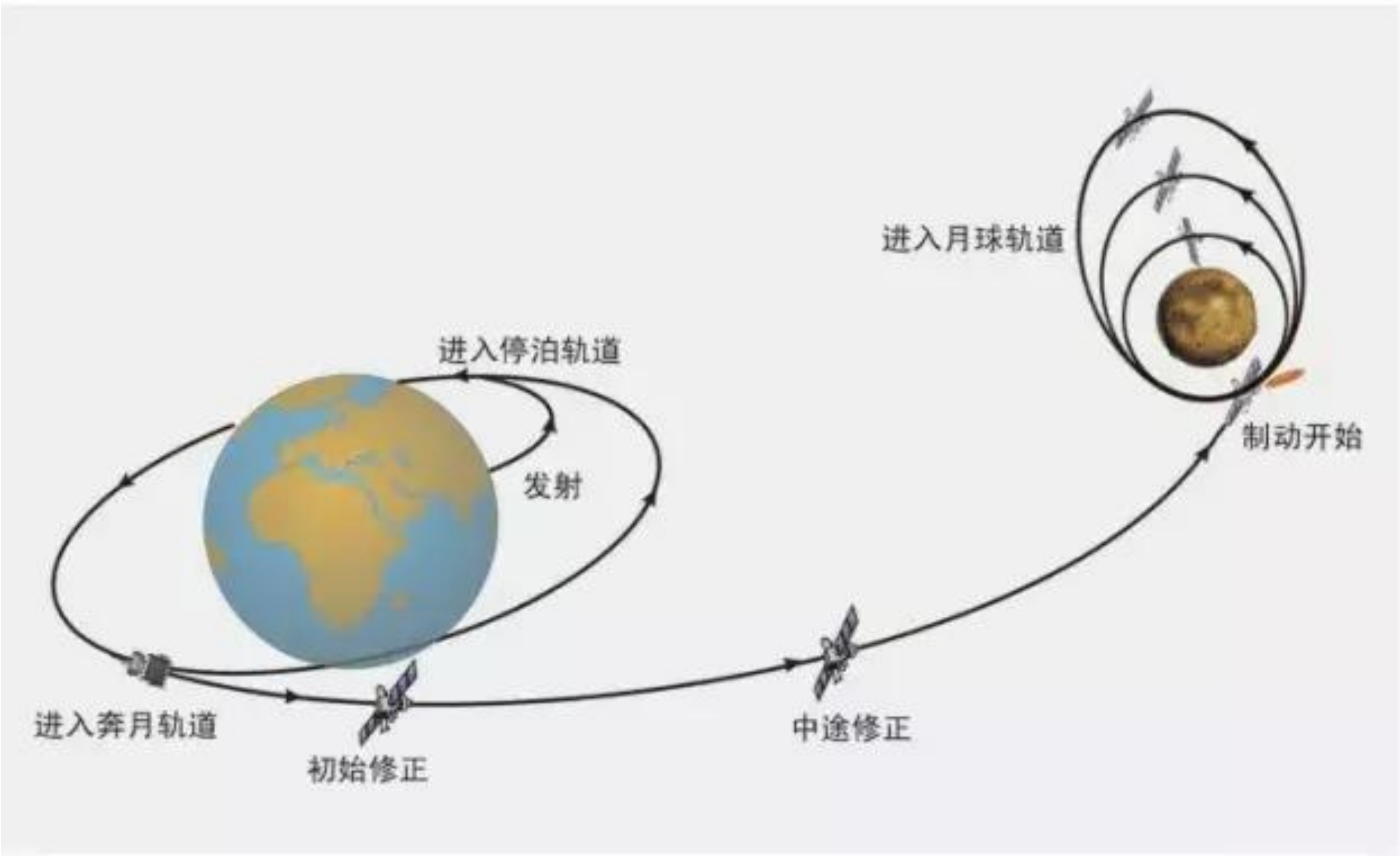
$$t = \pi / \omega, x = -a, y = 0$$

$$t = 3\pi / 2\omega, x = 0, y = -b$$

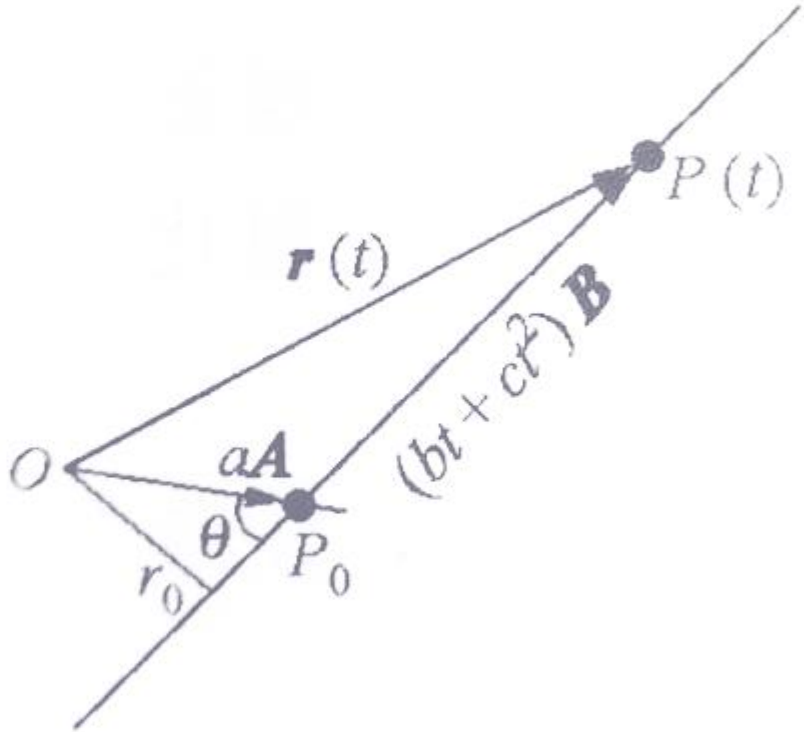


$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

嫦娥二号轨道



轨道方程



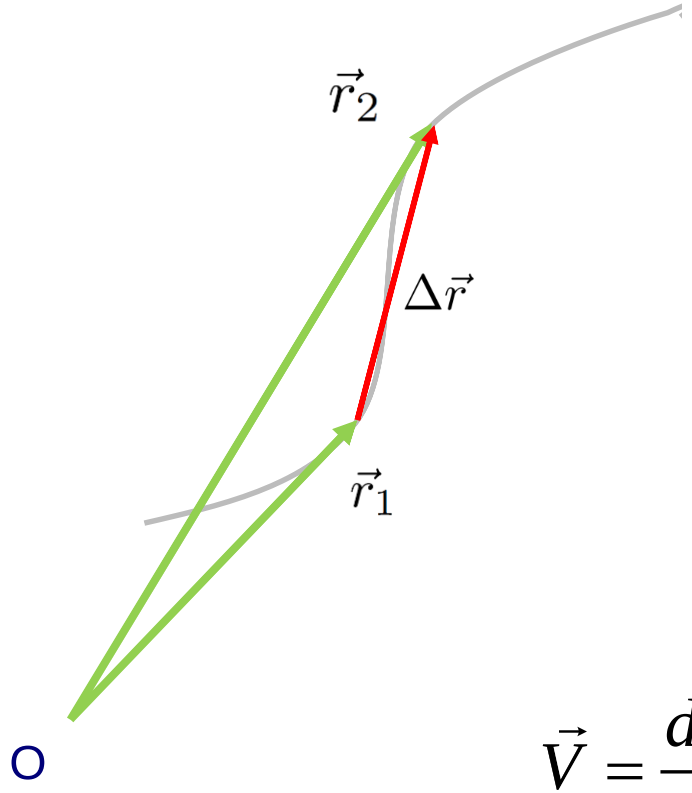
$$\vec{r}(t) = a\vec{A} + b(t + ct^2)\vec{B}$$

轨迹方程为一条直线

速度

速度也是矢量！

$$\vec{V} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$



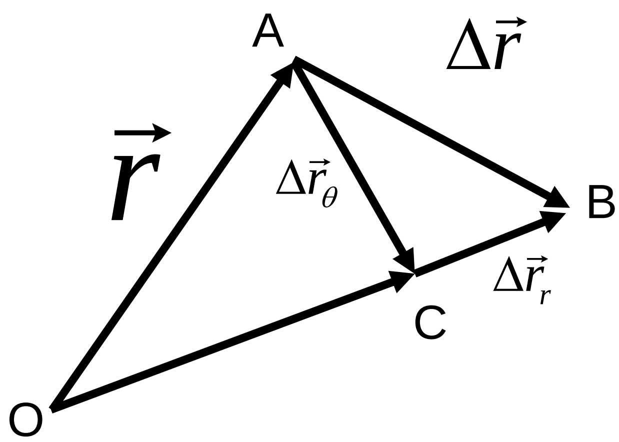
$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

速度分量的形式

$$\vec{r} = x\vec{i} + y\vec{j}$$

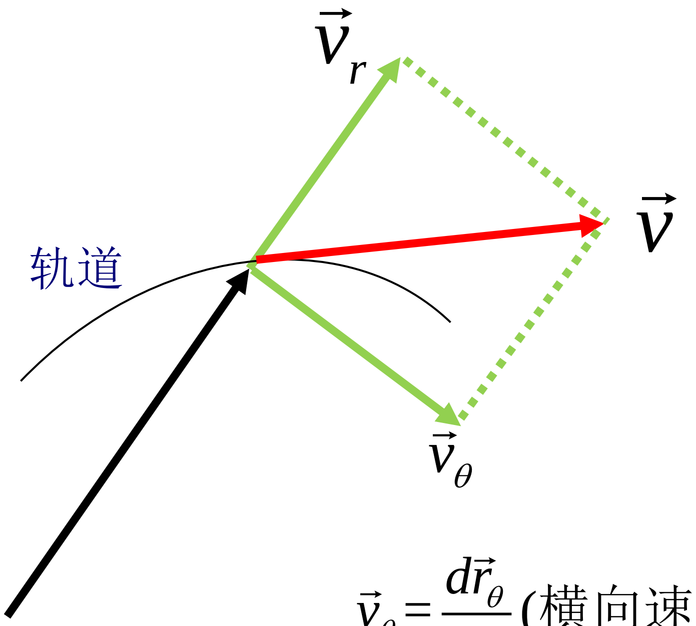
$$\vec{V} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} = v_x\vec{i} + v_y\vec{j}$$

横向速度与径向速度



$$\Delta\vec{r} = \Delta\vec{r}_\theta + \Delta\vec{r}_r$$

横向位移　径向位移

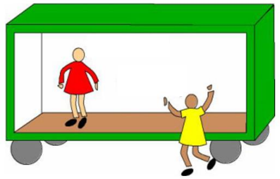


$$\vec{v}_\theta = \frac{d\vec{r}_\theta}{dt}$$ (横向速度）

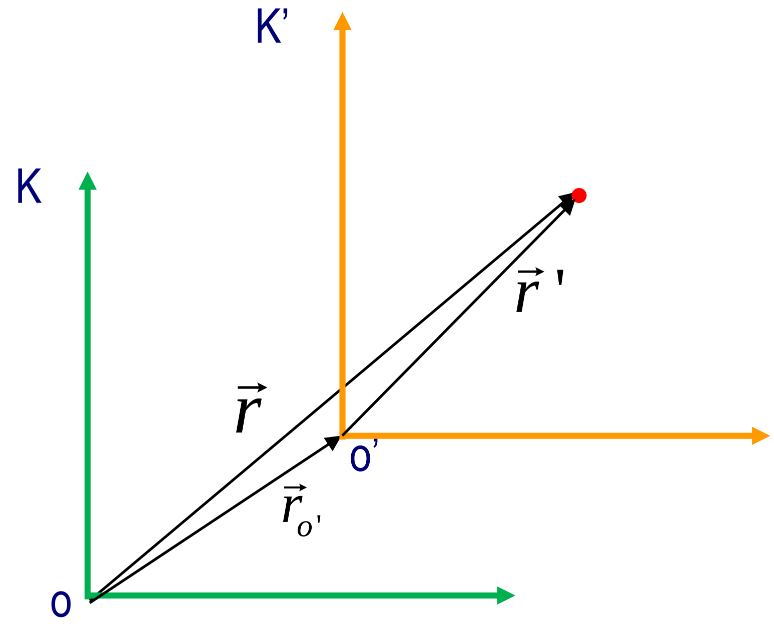
$$\vec{v} = \vec{v}_\theta + \vec{v}_r$$

$$\vec{v}_r = \frac{d\vec{r}_r}{dt}$$ (径向速度）

牵连速度



参照系 K‘相对于参照系K 以速度u移动
质点在K’参照系中以速度v’移动，求质点在K参照系中的速度v。



在K’参照系中： 质点的位置矢量与速度为

$$\vec{r}\,'与\vec{v}\,'=\frac{d\vec{r}\,'}{d\vec{t}}$$

在k参照系中： 质点的位置矢量与速度为

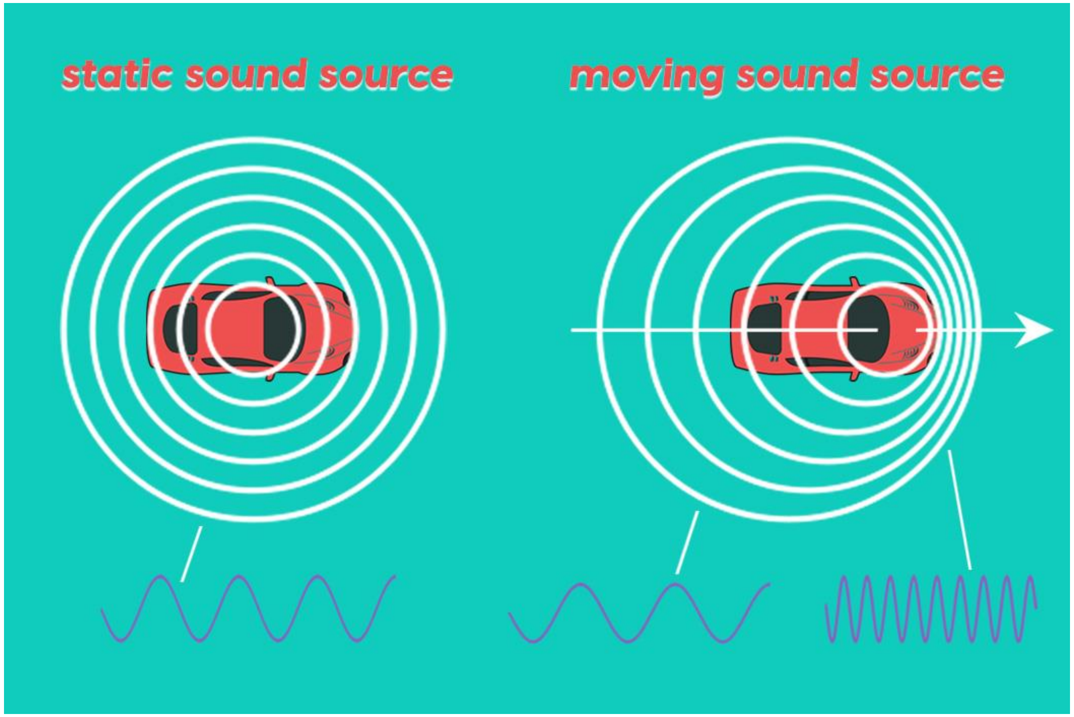
$$\vec{r}与\vec{v}=\frac{d\vec{r}}{d\vec{t}}$$

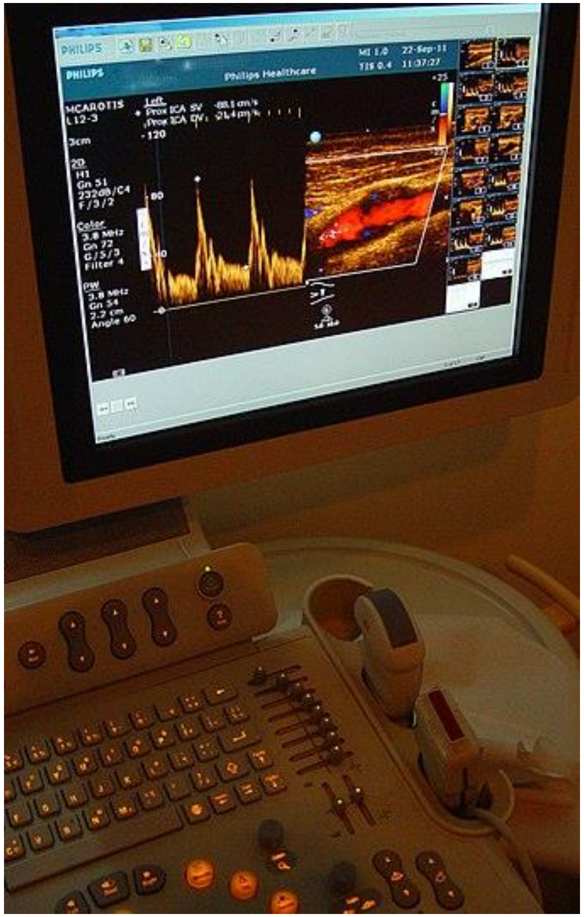
$$\vec{r}=\vec{r}_{o'}+\vec{r}\,'$$

所以：
$$\vec{v}(t)=\frac{d\vec{r}}{dt}=\frac{d(\vec{r}_{0'}+\vec{r}\,')}{dt}=\vec{u}(t)+\vec{v}\,'(t)$$

1. 仅适用于低速牛顿力学体系
2. 如果转动时，u随参考点o’不同而变化

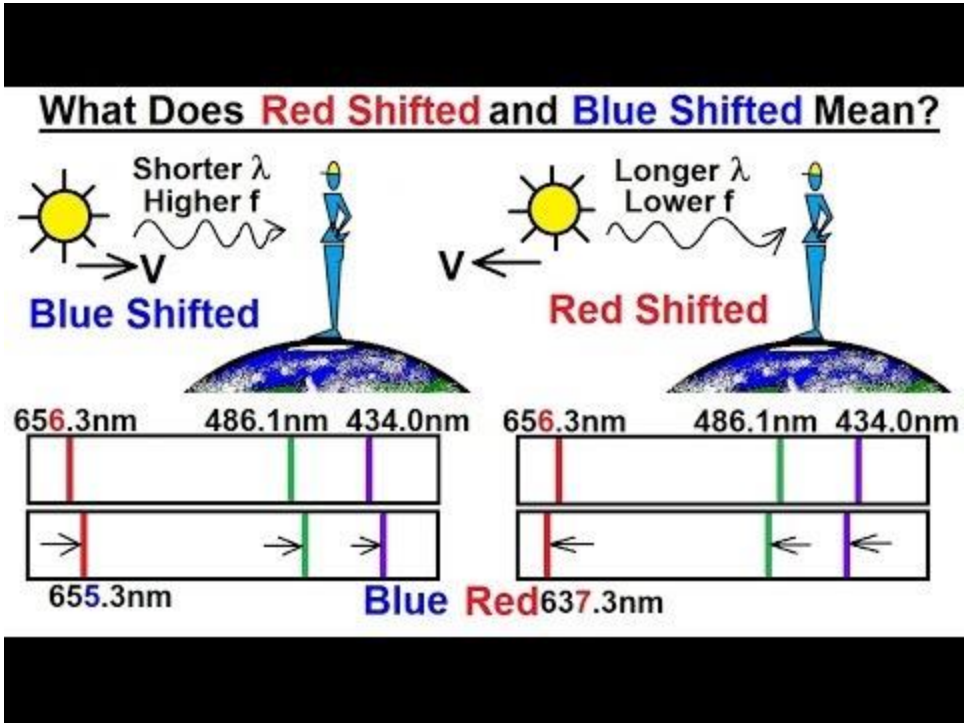
多普勒效应





源相对于接收器运动，频率变化

宇宙的红移



(b)

原子发射谱或吸收谱

谱线位置发生了移动

由多普勒效应： 红移 恒星远离我们

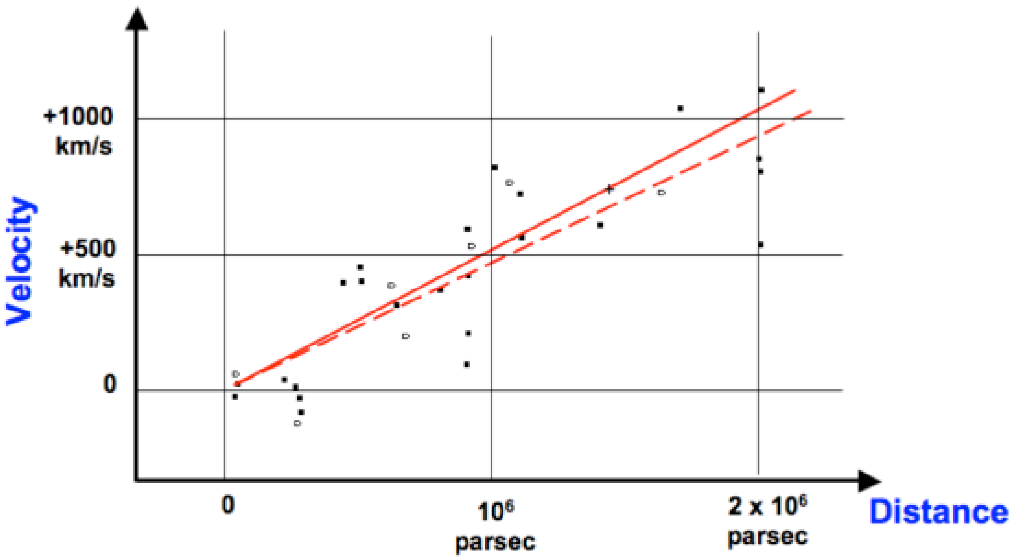
哈勃-勒梅特定律

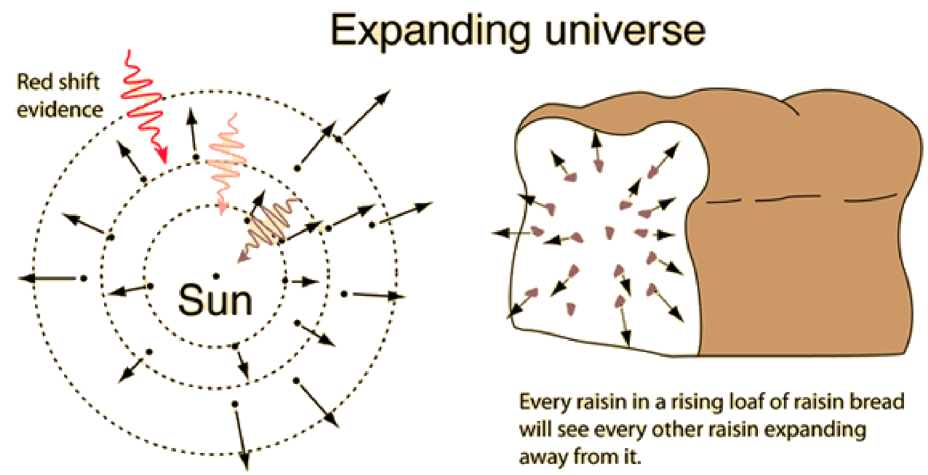
$$v = H_0 r$$

H_0 :哈勃常数

v :退行速度

r :距离

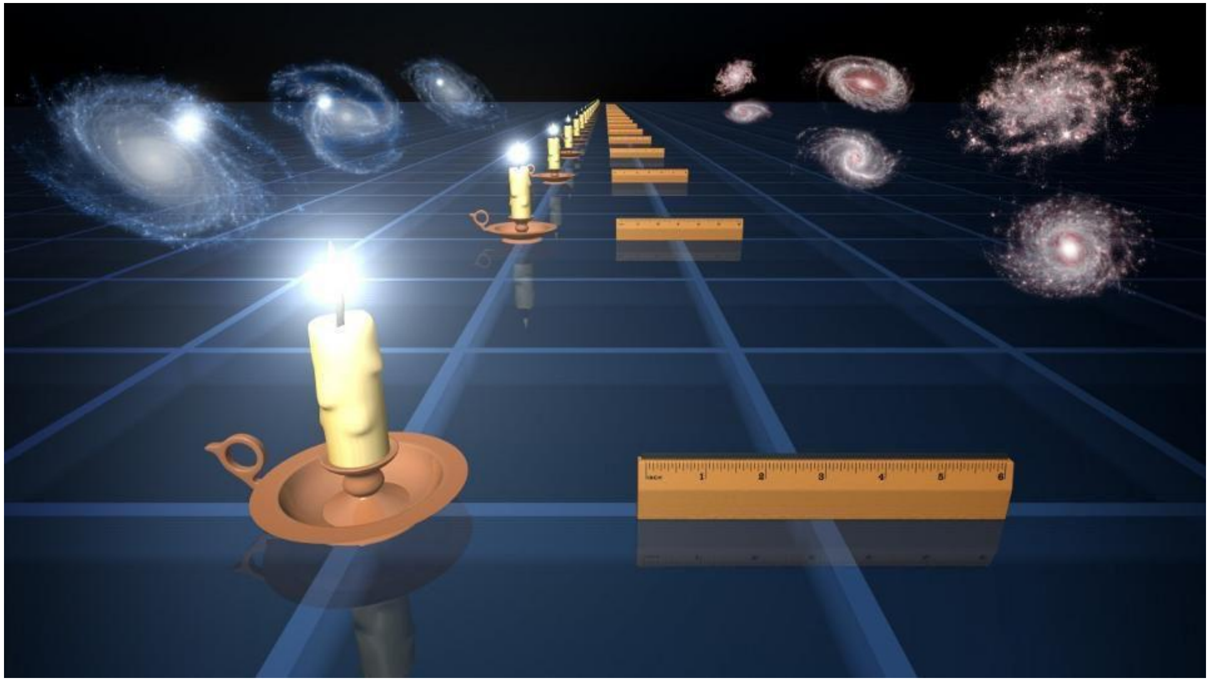


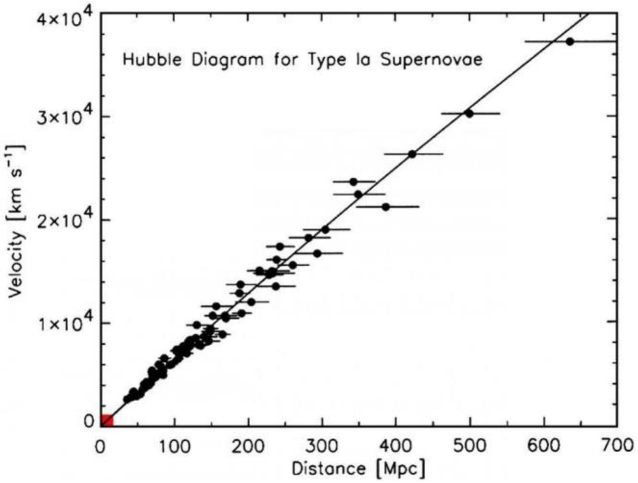


H = 71 km/s/Mpc

宇宙的年龄 ~ $\dfrac{1}{H_0}$

哈勃常数



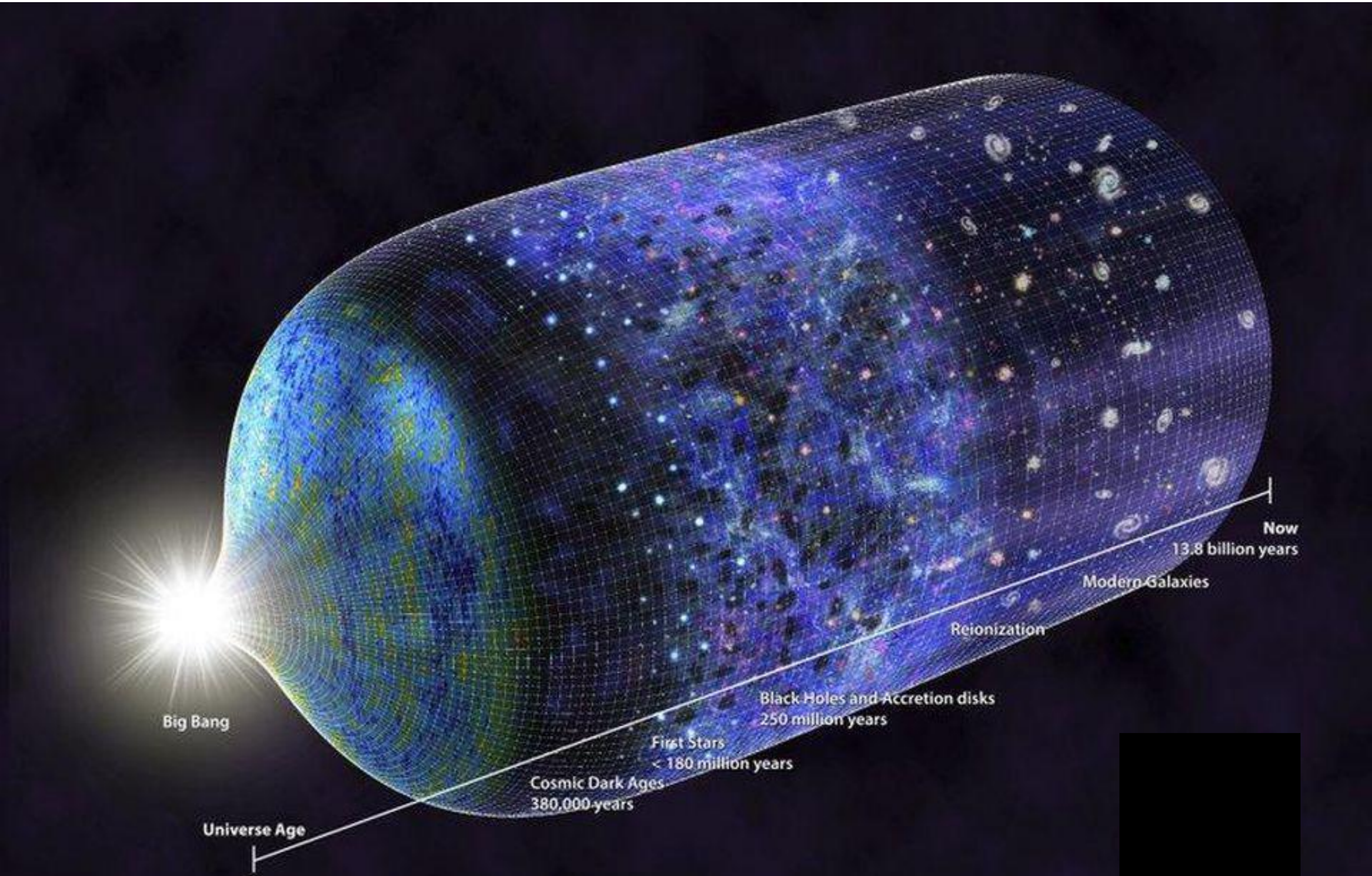


$$v = H_0 r$$

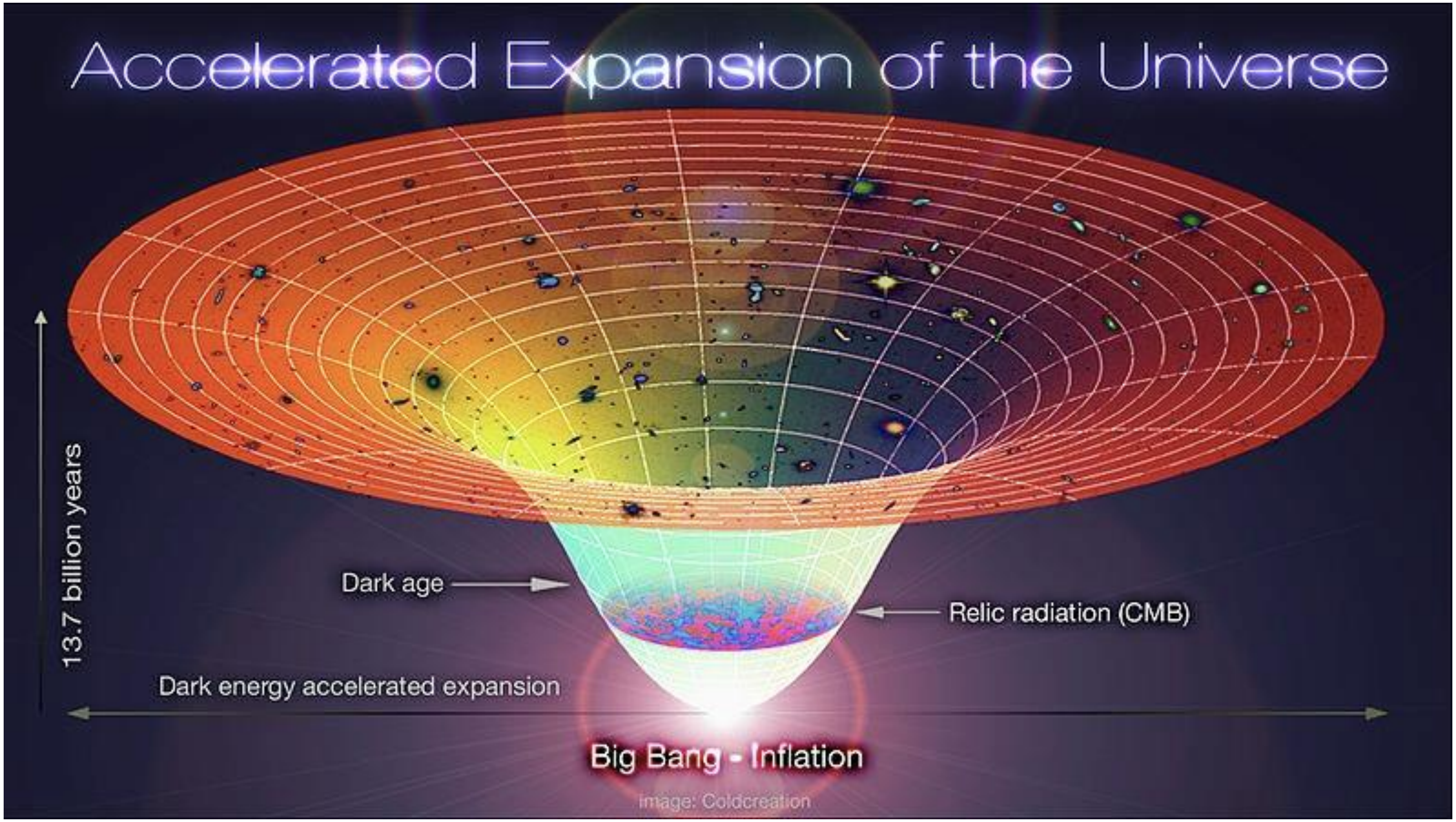
距离r 的测量： 标准烛光法。特定的星体发出的光的亮度是一定的，随距离衰减。

v： 根据红移测得

宇宙的演化


Now
13.8 billion years
Modern Galaxies
Reionization
Black Holes and Accretion disks
250 million years
First Stars
< 180 million years
Cosmic Dark Ages
380,000 years
Big Bang
Universe Age

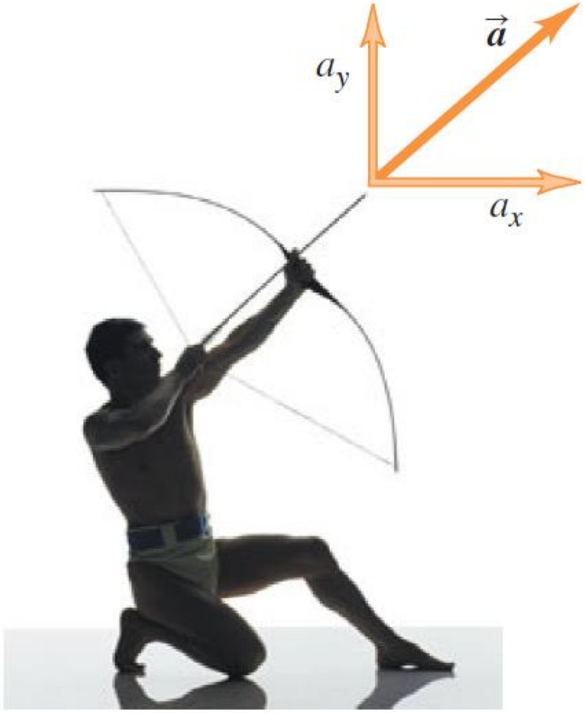
宇宙加速膨胀



Accelerated Expansion of the Universe
13.7 billion years
Dark age
Relic radiation (CMB)
Dark energy accelerated expansion
Big Bang - Inflation
image: Coldcreation

加速度

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2 \vec{r}}{dt^2}$$



$$\vec{F} = m\vec{a}$$

$$F_x = m(dv_x/dt) = m(d^2x/dt^2) = ma_x,$$
$$F_y = m(dv_y/dt) = m(d^2y/dt^2) = ma_y,$$
$$F_z = m(dv_z/dt) = m(d^2z/dt^2) = ma_z.$$