MASTER'S THESIS

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Chapter 1

Introduction

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Chapter 2

Theoretical Background

2.1 Spare part inventory management

Spare parts are necessary for the restoration of defective industrial systems (Kareem Lawal, 2015). Lack of spare parts can lead to costs of downtime and serious loss of production and reputation. Many industries observe that non-availability of spare parts accounts for a significant portion of the total maintenance costs, especially critical parts. In order to operate effectively, spare parts must be available in sufficient quantities at the right time, and at the right place.

Spare parts inventory management ensure that spare parts are available for maintenance of machinery at the most possible optimum cost. Spare parts need to be stored in sufficient amount to make sure the equipment have enough spare parts for maintenance when some failures happen. If the spare parts are not enough to service maintenance of equipment, unplanned downtime can occur. This not only result costly delays in maintenance, production schedules and order deliveries, but it also increases the risk of personnel injury, and environmental incidents. On the other hand, spare parts should not be overstocked because it may induce extra costs in terms of holding cost and obsolete parts which can be seen as dead inventory (outdated and or no further usage). Figure 2.1 shows the optimal balance between inventory cost and downtime cost.

Inventory management of spare parts requires a balance between benefits (spare parts availability and service level) and costs (warehouse area, working capital, and obsolescence risk). This can be particularly challenging due to the inherent variability of demand levels. Some studies have also raised key questions that must be answered in spare parts management:

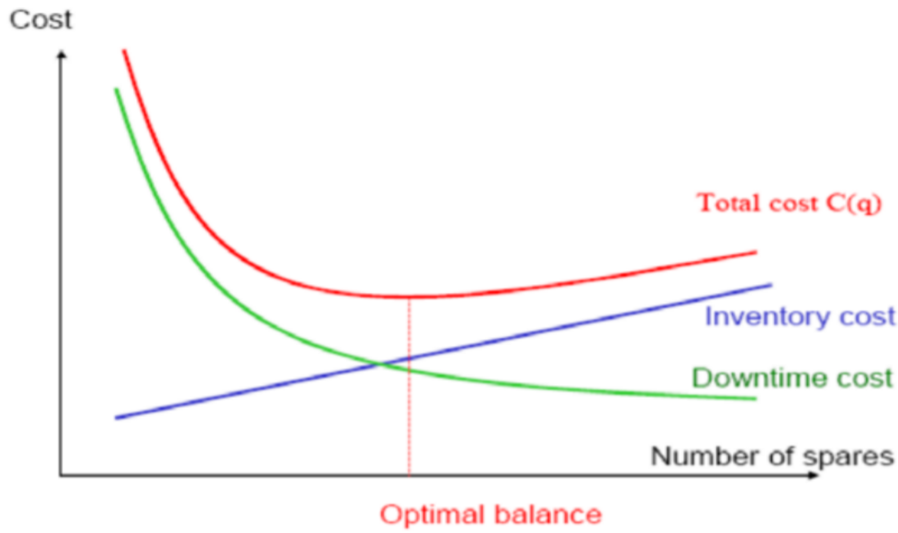

Figure 2.1: The optimal balance between inventory cost and total cost (Kumar et al., 2014)

- Which service parts have to be stocked? How many of them should be stocked? Where should they be stocked? How many of them should be stocked? (Botter Fortuin, 2000)
- Should each spare part be kept in stock? How many spare parts should be ordered at once? How many pieces to keep in stock? When to release a new order? (Bošnjaković, 2010)
- Which spare parts should be stocked? When should they be (re)ordered? How many items should be (re)ordered? (Hu et al., 2018)

As each study have a different objective, the answer to the key questions above depends on each company's goals. Hu et al. (2018) stated that most of the literature discusses two main objectives in this decision-making process: maximizing spares availability and minimizing economic costs.

2.2 Inventory Management Models

In Inventory Management, two of the most popular inventory control models will are worth discussing. Among the most popular inventory control models, two models will be discussed. These include Economic Order Quantity (EOQ) and Reorder Point (ROP).

2.2.1 Economic Order Quantity: EOQ

Economic Order Quantity (EOQ) is a cost-effective method of determining the number of items to be purchased in considering to minimize the total cost of both handling of inventory (Handling Cost) and order processing (Ordering Cost). In 1913 Ford W. Harris developed the EOQ model, and K. Andler and R. H. Wilson were credited with the in-depth analysis and application of the model (Hax Candea, 1984). Many extensions of the EOQ inventory model have been developed by researchers and academicians. This inventory model answers one of the key questions in inventory management: how many products must be ordered?

Assumptions of EOQ that are made by Horngren, Datar, and Rajan (2011):

- For simplicity, the EOQ model assumes that there are only ordering and carrying costs, since these are the most common costs associated with inventory.
- The same quantity of materials is being ordered at each re-order point.
- Demand, ordering costs, carrying costs and lead time are reliably known with certainty.
- The purchasing cost of materials is constant: bulk purchases do not qualify for discounts.
- There is no stock-outs.
- When deciding on the quantity of a purchase order, the costs of quality and shrinkage costs are considered only to the extent that these costs affect ordering or holding costs.

The EOQ formula is used to find the minimum point of total cost where the ordering costs and holding costs are minimized. Figure 2.2 illustrates how the annual ordering costs and holding costs change as the reorder quantity increases; the EOQ is marked as the lowest point of the total cost line.

The total cost function is made of the product cost, the ordering cost and the holding cost. Therefore, the total cost function can be written as follows:

$$\text{Total Costs} = \text{Relevant Ordering Costs} + \text{Relevant Carrying Costs} \tag{2.1}$$

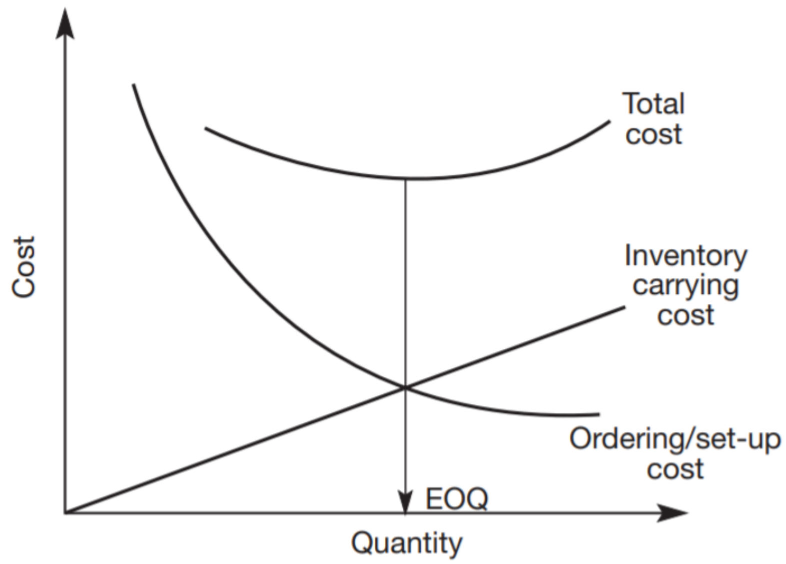

Figure 2.2: Total cost in Inventory control (Christopher, 2011)

Annual relevant ordering costs are made up of the number of purchase orders per year multiplied by the relevant ordering cost per purchase order i.e.,

$$\text{Relevant Ordering Costs } = \left(\frac{D}{Q} \times P\right) \tag{2.2}$$

Annual relevant carrying costs are the cost of holding in stock that is made up of the average inventory in units multiplied by Annual relevant carrying cost per unit i.e.,

$$\text{Relevant Carrying Costs } = \left(\frac{Q}{2} \times C\right) \tag{2.3}$$

Where:

- D = Demand in units for a specified period
- Q = Size of each order (order quantity)
- P = Relevant ordering cost per purchase order
- C = Relevant carrying cost of one unit in stock for the time period used for D

Therefore, Equation 2.1 can be expanded as follows:

$$\text{Total costs } = \left(\frac{D}{Q} \times P\right) + \left(\frac{Q}{2} \times C\right) \tag{2.4}$$

According to Figure 2.2, annual relevant total costs are minimized when relevant ordering costs equal relevant carrying costs. Hence, to solve for EOQ, annual relevant ordering costs are set to be equal to annual relevant carrying costs:

$$\left(\frac{D}{Q} \times P)\right) = \left(\frac{Q}{2} \times C)\right) \tag{2.5}$$

Multiplying both sides of Equation 2.5 by $\frac{2Q}{C}$, we get:

$$Q^2 = \frac{2DP}{C} \tag{2.6}$$

Therefore, EOQ is stated as:

$$Q^* = \sqrt{\frac{2DP}{C}} \tag{2.7}$$

Thus, EOQ is the square root of 2 times the annual Demand times the cost of one Order divided by the annual cost to Hold one unit. Based on the formula, higher demand and/or higher ordering costs can lead to higher EOQ, and a higher cost of carrying can result in lower EOQ.

2.2.2 Reorder Point: ROP

A reorder point (ROP) is a minimum stock level that triggers the reordering of a particular stock item. In other words, it answers the question "when to place an order?" to prevent stock outs and ensure the service level. ROP is made of average demand during lead time and the safety stock. Figure 2.3 represents how the reorder point is influenced by the lead time and order quantity as a function of time. The reorder point formula can be written as follows:

$$\text{Reorder Point} = \text{Demand during lead time} + \text{Safety stock} \tag{2.8}$$

To find demand during lead time, we can simply multiply the lead time (in days) for a product by the average number of units sold daily:

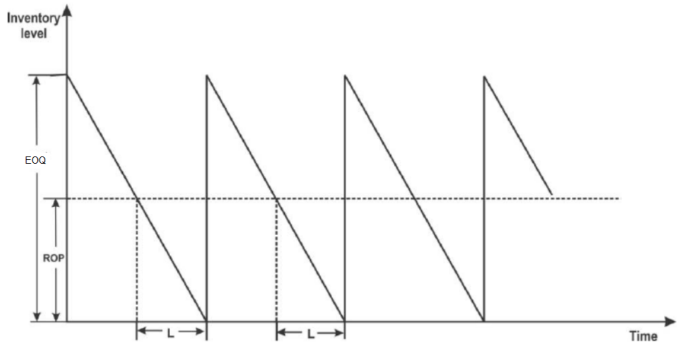

Figure 2.3: Reorder point in the EOQ model (P. O., A., 2017)

$$\text{Demand during lead time} = \text{Lead time} \times \text{Average daily demands} \quad (2.9)$$

Where:

- Lead time: time taken (in days) for supplier to fulfill order
- Average daily demands: the number of demands made in an average day of that particular item

Safety Stock is the additional stock that helps prevent stock-outs. This can be determined by the deviations from the real demand. Figure 2.4 shows the relationship between the level of service and standard deviation of demand with assumptions that the demand of the item is normally distributed and the mean ($\bar{x}$) and standard deviation (σ) are known (Christopher, 2011).

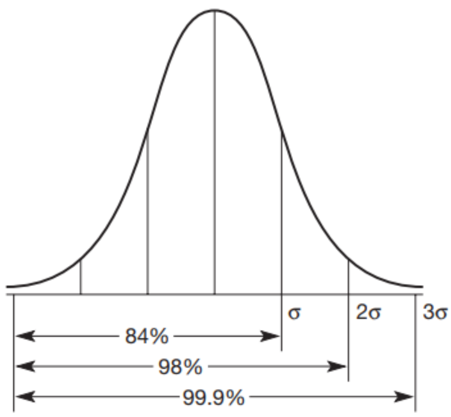

Figure 2.4: Service levels and the normal distribution (Christopher, 2011)

As it can be seen in Figure 2.4, the safety stock level increases in accordance with the desired service level. This effect illustrates in Table 2.1

Table 2.1 Inventory level and service level

Inventory level	Service level
$\bar{x}$	50%
$\bar{x} + \sigma$	84%
$\bar{x} + 2\sigma$	98%
$\bar{x} + 3\sigma$	99.9%

- A service level of 50% would be achieved if the inventory held is equal to the average expected daily demand ($\bar{x}$)) without safety stock.
- A service level of 84% would be achieved if the average expected daily demand ($\bar{x}$)) and safety stock equivalent to one standard deviation of demand (σ) are held.
- A service level of 98% would be achieved if the average expected daily demand ($\bar{x}$)) and safety stock equivalent to two standard deviations of demand (2σ) are held.
- A service level of 99.9% would be achieved if the average expected daily demand ($\bar{x}$)) and safety stock equivalent to three standard deviations of demand (3σ) are held.

2.3 Monte Carlo methods

2.3.1 Introduction

Monte Carlo simulation was introduced in the 1940s by Von Neumann, Fermi, Ulam, and Metropolis while they were working on nuclear weapon projects in the Los Alamos National Laboratory. It was performed to evaluate the probability of the chain reaction needed for an atom bomb to detonate successfully. The scientists involved in this project were big fans of gambling, so they named the simulations Monte Carlo which came from a famous casino in the Monaco resort town.

Monte Carlo Simulation is a computerized mathematical technique that relies on random sampling and statistical analysis to compute the outcomes. In simple terms, this process involves taking input variables based on probability density functions, calculating output, and repeating this process a set number of times. The technique is very closely related to random experiments that perform numerous fictitious experiments with random numbers. The results will be a range of possible outcomes and the probabilities they will occur.

Ulam cited an example of a Monte Carlo simulation in his autobiography. When he was playing solitaires, he found himself asking: "What are the possibilities that the Canfield solitaire with 52 cards would come out successfully?" As he spent a great deal of time trying to estimate them by pure combinatorial calculations, he began to wonder if it would be more effective to simply record the results of 100 games, count the number of events of interest among the all events and calculate a percentage that way.

With the computer's potential, it could easily be programmed to generate random lists representing the 52 cards of a deck, prepare lists representing the different piles, and then simulate the completion of the game. It was capable of performing numerous experiments and assessing their outcomes much faster than a human would take to do so. Based on the outcomes of many repetitions, a Monte Carlo estimate of a chance of success can be determined.

2.3.2 Methodology

According to Williamson et al. (2006), there are five steps to be performed when applying the Monte Carlo simulation as shown in Figure ...

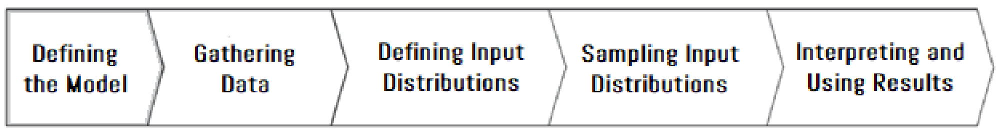

Figure 2.5: Five step of Monte Carlo forecasting (Williamson et al., 2006)

1. **Defining the Model:**

   To begin, the forecaster should define what he or she is willing to forecast (the outputs of the simulation), what is the scope of the analysis model, and what are the appropriate input parameters that should be included in the model.

2. **Gathering Data:**

   Predictive algorithms are based on the right data sample, therefore selecting the appropriate data is a key activity in any well-forecast. Assuming that the exact values of inputs to the model are unknown, data collection will be necessary in order to quantify this uncertainty.

The data set may be referred as offset data since it is different in time and/or place from the subject of the forecast. Tables and graphs are helpful in understanding and visualizing the offset data.

3. **Defining Input Distributions:**

   The offset data could be seen as a guild to define the probability distributions shape (e.g., uniform, triangular, or log-normal), and distribution parameters (e.g., mean, mode variance, standard deviation, or P90 percentile) for the model.

4. **Sampling Input Distributions:**

   Uncertain forecast inputs will be generated through random sampling based on the probability distribution function. Before conducting a Monte Carlo simulation, correlation among input variables should be taken into considerations. Correlation indicates a degree of relationship between two or more uncertain input variables. If the forecaster is not aware of the correlation, it can affect the output variation either by increasing or decreasing the variance.

   The simulation will be carried out hundreds or thousands of times (or more) to give multiple realizations of outcomes. The number of iterations in Monte Carlo studies is shown by Koçak (2019) to have significant impact on its estimation. As iteration number increases, the estimation gets more precise.

5. **Interpreting and Using the Results:**

   As a result, the outputs of a Monte Carlo simulation will be assembled into probability distributions of possible outcomes that provide statistical confidence for decision-making. A set of output parameters for basic statistical analysis are presented as well, including mean (average), variance, standard deviation, P10, P90, and P50 (median).

   Before using the results, a quality-assurance check needs to be performed in order to ensure that the model fulfills its functions and conforms to its specifications. The results of the simulation can be used for decision-making or for further analysis in order to enhance understanding.

2.3.3 Creation of random numbers

In order to perform the Monte Carlo Simulation, random numbers need to be generated. In computing, pseudorandom number generators are generally preferred for programmers who are using Monte Carlo methods. The pseudorandom number generators produce sequences of numbers generated through mathematical formulas, whose properties are approximated by properties of random numbers.

In fact, these numbers are not truly random since the end results obtained are completely determined by an initial value, called the seed value or key. This means if the seed value and the algorithm are known, these seemingly random numbers could be reproduced.

There are so many random number generator algorithms, for instance, Complementary-multiply-with-carry, Linear feedback shift register, multiplicative linear congruential generator, Middle-Square Method algorithm, etc. The Middle-Square Method algorithm will be discussed further as an example.

In 1946, John von Neumann has constructed the mid-square method which is the first pseudorandom generator (Ali-Pacha et al., 2019). The principle of this method is simply described as the following steps:

1. **Choose positive integers as the base number (seed):**

$$u_0 = 0.4872$$

2. **Square the base number:**

$$\sqrt{0.4872} = 0.69799713$$

3. **Take the middle n numbers and add decimal digits in front of it to get the next random number:**

$$u_1 = 0.7997$$

4. **Repeat the process n times:** The sequence of number would be 0.4872, 0.7997, 0.4259, 0.2610, 0.0881, 0.6816, and so on.

Various high-level programming languages comes with standard libraries that provides a pseudorandom generator to generate random numbers. Users can also write his/her own function to create random numbers with any distributions (uniform, Poisson, normal, etc). Figure 2.6 represents random number visualization of 500 random numbers uniformly distributed in (-1, 1).

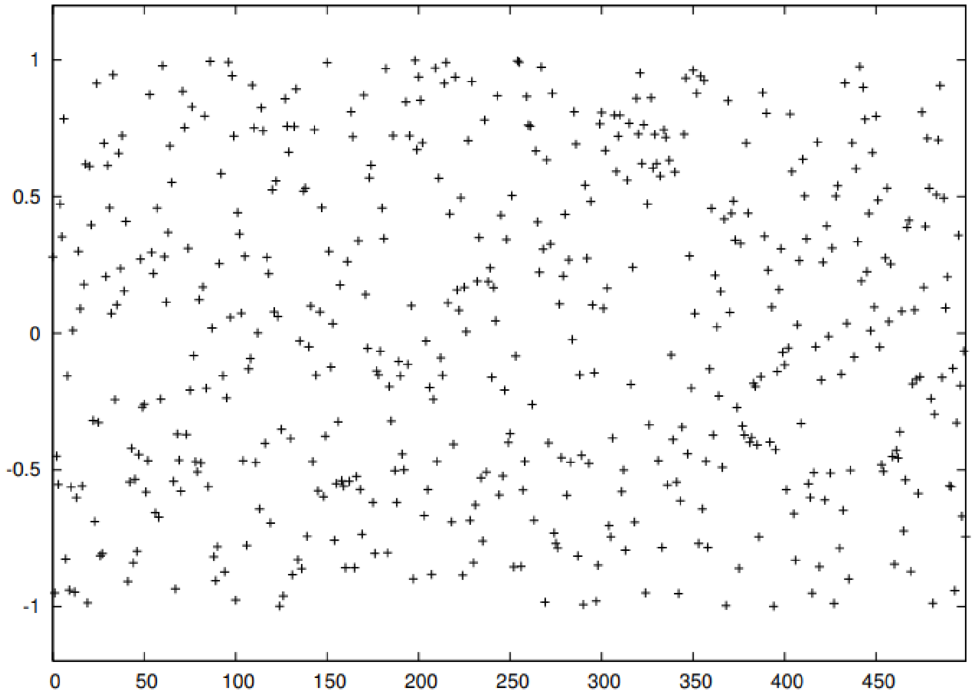

Figure 2.6: Plot of the 500 samples for a uniform probability distribution (-1, 1).

2.4 Holt-Winters' method

Holt-Winters' method is useful for capturing of capturing level, trend and seasonality components. As stated in Hyndman and Athanasopoulos (2018), Holt, C. E. and Winters, P. R. extended Holt's method by applying a triple exponential smoothing — one for the level at time t (l_t), one for the trend at time t (b_t), and one for the seasonal component at time t (s_t). The model requires smoothing parameters: α, β and γ are the coefficients for the level smoothing, the trend smoothing, and the seasonal smoothing respectively. The smoothing parameter restriction is 0 to 1.

Holt-Winters' can be divided into two models of seasonality: additive method and multiplicative method. When seasonal variations are roughly constant throughout the series, the additive method is best used, while the multiplicative method is more effective when the seasonal variations change proportionally to the level of the series.

2.4.1 Holt-Winters' additive model

The additive model is generally stated as follows:

$$\text{Forecasting: } \hat{y}_{t+h|t} = \ell_t + hb_t + s_{t+h-m(k+1)} \tag{2.10}$$

$$\text{Level: } \ell_t = \alpha\left(y_t - s_{t-m}\right) + (1-\alpha)\left(\ell_{t-1} + b_{t-1}\right) \tag{2.11}$$

$$\text{Trend: } b_t = \beta^*\left(\ell_t - \ell_{t-1}\right) + (1-\beta^*)\, b_{t-1} \tag{2.12}$$

$$\text{Seasonality: } s_t = \gamma\left(y_t - \ell_{t-1} - b_{t-1}\right) + (1-\gamma)s_{t-m} \tag{2.13}$$

2.4.2 Holt-Winters' multiplicative method

The additive model is generally stated as follows:

$$\text{Forecasting: } \hat{y}_{t+h|t} = \left(\ell_t + hb_t\right) s_{t+h-m(k+1)} \tag{2.14}$$

$$\text{Level: } \ell_t = \alpha\left(\frac{y_t}{s_{t-m}}\right) + (1-\alpha)\left(\ell_{t-1} + b_{t-1}\right) \tag{2.15}$$

$$\text{Trend: } b_t = \beta^*\left(\ell_t - \ell_{t-1}\right) + (1-\beta^*)\, b_{t-1} \tag{2.16}$$

$$\text{Seasonality: } s_t = \gamma\left(\frac{y_t}{\ell_{t-1} + b_{t-1}}\right) + (1-\gamma)s_{t-m} \tag{2.17}$$

Where:

- $\alpha, \beta, \gamma \in (0, 1)$ which translates to $0 \leq \alpha \leq 1$, $0 \leq \beta \leq 1$, and $0 \leq \gamma \leq 1$.
- m is the period of the seasonality, e.g., m = 4 for quarterly data and m = 12 for monthly data.

2.5 Forecasting accuracy measures

Many studies have adopted various accuracy metrics as evaluation criteria in the recent decades to evaluate the performance of forecasting methods. Shcherbakov et al. (2013) shows that every accuracy measure has its drawbacks and there is no perfect single measure that can be used universally. For this reason, the forecasting accuracy will be analyzed in a combination of both percentage error-based and absolute error-based measures.

2.5.1 Mean Absolute Percentage Error

Mean Absolute Percentage Error (MAPE) is the most widely used measure for checking forecast accuracy. It is calculated by dividing the absolute forecast error in each period by the actual value in that period and then averaging those fixed percentages. This can be expressed as a percentage given as:

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{A_t - F_t}{A_t}\right| \times 100 \tag{2.18}$$

2.5.2 Mean Absolute Error

Mean Absolute Error (MAE) provides an indication of the forecast accuracy by averaging the error term in the forecast for the whole time series. It is given by:

$$MAE = \frac{1}{n}\sum_{t=1}^{n}|A_t - F_t| \tag{2.19}$$

2.5.3 Mean Squared Error

Mean Squared Error (MSE) is calculated by squaring the error values of the forecast and finding the mean of the sum of squared errors. It is given by:

$$MSE = \frac{1}{n}\sum_{t=1}^{n}|A_t - F_t|^2 \tag{2.20}$$

Where:

- n is the number of observations.
- A_t is the actual value at time t,
- F_t is the forecast value at time t.

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