南开大学 作业纸

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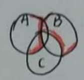
D. 系别＿＿＿＿ 班级＿＿＿＿ 姓名＿＿＿＿ 第　页

1. $(A-B)\cup(B-C)=(A\cup B)-BC$



$(A-B)\cup(B-C)=A\bar{B}\cup B\bar{C}$

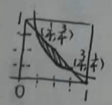
$(A\cup B)-BC=(A\cup B)\overline{BC}=(A\cup B)(\bar{B}\cup\bar{C})=A\bar{B}\cup A\bar{C}\cup B\bar{B}\cup B\bar{C}=A\bar{B}\cup(AB+A\bar{B})\bar{C}\cup\phi\cup B\bar{C}$

$=(A\bar{B}\cup A\bar{B}\bar{C})\cup(AB\bar{C}\cup B\bar{C})=A\bar{B}\cup B\bar{C}$

2. A＝第n次成功之前恰好成功m次，

$p(A)=(1-p)^m p^{n-m}$

3.



$$p=\frac{\int_{\frac14}^{\frac34}(-x+1-\frac{3}{16x})dx}{1\times1}=\int_{\frac14}^{\frac34}(-x-\frac{3}{16x}+1)dx=(-\frac{x^2}{2}+x-\frac{3}{16}\ln x)\Big|_{\frac14}^{\frac34}$$

$$=-\frac12\times(\frac{9}{16}-\frac{1}{16})+(\frac34-\frac14)-\frac{3}{16}(\ln\frac34-\ln\frac14)=\frac14-\frac{3\ln3}{16}$$

4. 证明：若$p(A|B)>p(A|\bar{B})$

有$\dfrac{p(AB)}{p(B)}>\dfrac{p(A\bar{B})}{p(\bar{B})}$，$\therefore p(AB)p(\bar{B})>p(A\bar{B})p(B)$. 即 $p(AB)(1-p(B))>p(A\bar{B})p(B)$

$\therefore p(AB)>p(B)(p(AB)+p(A\bar{B}))$

$=p(A)p(B)$

若$p(B|A)>p(B|\bar{A})$

有$\dfrac{p(AB)}{p(A)}>\dfrac{p(\bar{A}B)}{p(\bar{A})}$，$\therefore p(AB)p(\bar{A})>p(\bar{A}B)p(A)$，即 $p(AB)(1-p(A))>p(\bar{A}B)p(A)$

$\therefore p(AB)>p(A)(p(\bar{A}B)+p(AB))$

$=p(A)p(B)$

得证.

5. A：0白，3红

B：1白，2红

C：至多取到1只白球

$p(C)=p(A)+p(B)$

$$=\frac{C_5^3}{C_8^3}+\frac{C_3^1C_5^2}{C_8^3}=\frac{10+30}{56}=\frac57$$

6.（1）A：没有成对的鞋子

$$p(A)=\frac{C_5^4 2^4}{C_{10}^4}=\frac{80}{210}=\frac{8}{21};$$

（2）B：至少2只可以配成双

$$p(B)=1-p(A)=\frac{13}{21}.$$

7.（1）A：收到信号是1，B：发出信号0，$\bar{B}$：发出信号1

$p(A)=0.6\times0.2+0.4\times0.9=0.48$

（2）~~$p(A|\bar{B})=$~~

$$p(\bar{B}|A)=\frac{p(\bar{B})p(A|\bar{B})}{p(B)p(A|B)+p(\bar{B})p(A|\bar{B})}=\frac{0.36}{\cancel{0.36+0.12}\ 0.48}=0.75$$

8. (1) $P_1 = A_1A_2\cdots A_n$

(2) $P_2 = C_n^1\bar{A}_i\ A_1A_2\cdots A_{i-1}A_{i+1}\cdots A_n$

(3) $P_3 = 1-P_1 = 1-A_1A_2\cdots A_n$

(4) ~~$P_4=$~~

B_0：0件正品

B_1：1件正品

$1-P(B_0)-P(B_1) = 1-\bar{A}_1\bar{A}_2\cdots\bar{A}_n - C_n^1 A_i\bar{A}_1\bar{A}_2\cdots\bar{A}_{i-1}\bar{A}_{i+1}\cdots\bar{A}_n$

9. (1) 第一次拿到新球个数：0, 1, 2, 3

概率 $P_{10} = \frac{C_3^3}{C_{12}^3} = \frac{1}{220}$

$P_{11} = \frac{C_9^1C_3^2}{C_{12}^3} = \frac{27}{220}$

$P_{12} = \frac{C_9^2C_3^1}{C_{12}^3} = \frac{108}{220}$

$P_{13} = \frac{C_9^3}{C_{12}^3} = \frac{84}{220}$

第二次拿球，(第二次2新球)

$P_{202} = \frac{1}{220}\times\frac{C_9^2C_3^1}{C_{12}^3} = \frac{1}{220}\times\frac{108}{220}$

$P_{212} = \frac{27}{220}\times\frac{C_8^2C_4^1}{C_{12}^3} = \frac{27}{220}\times\frac{28}{55}$

$P_{222} = \frac{108}{220}\times\frac{C_7^2C_5^1}{C_{12}^3} = \frac{108}{220}\times\frac{21}{44}$

$P_{232} = \frac{84}{220}\times\frac{C_6^2C_6^1}{C_{12}^3} = \frac{84}{220}\times\frac{9}{22}$

$$P_1 = \frac{1}{220}\times\frac{27}{55}+\frac{27}{220}\times\frac{28}{55}+\frac{108}{220}\times\frac{21}{44}+\frac{84}{220}\times\frac{9}{22}$$

$$= \frac{1377}{3025}$$

(2) $P = \frac{P_{201}}{P_1} = \frac{\frac{27}{220}\times\frac{28}{55}}{\frac{1377}{3025}} = \frac{7}{51}$.

10. 设 A_i 表示第 i 次击中目标.

(1)

$P(A_1\bar{A}_2\bar{A}_3)+P(\bar{A}_1A_2\bar{A}_3)+P(\bar{A}_1\bar{A}_2A_3)$

$= 0.4\times0.5\times0.3+0.6\times0.5\times0.3+0.6\times0.5\times0.7$

$=0.36$

(2) B：至少有一次击中目标.

$P(B) = 1-P(\bar{A}_1\bar{A}_2\bar{A}_3) = 1-0.6\times0.5\times0.3 = 0.91$.