

Identification and Estimation of Treatment and Interference Effects in Observational Studies on Networks¹

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¹[Source](#) : Forastiere L, Airolidi E M, Mealli F. Identification and estimation of treatment and interference effects in observational studies on networks[J]. Journal of the American Statistical Association, 2021, 116(534): 901-918.

Interference

- Interference is said to be present when a treatment, exposure, or intervention, on one unit has an effect on the response of another unit [Cox 1958]

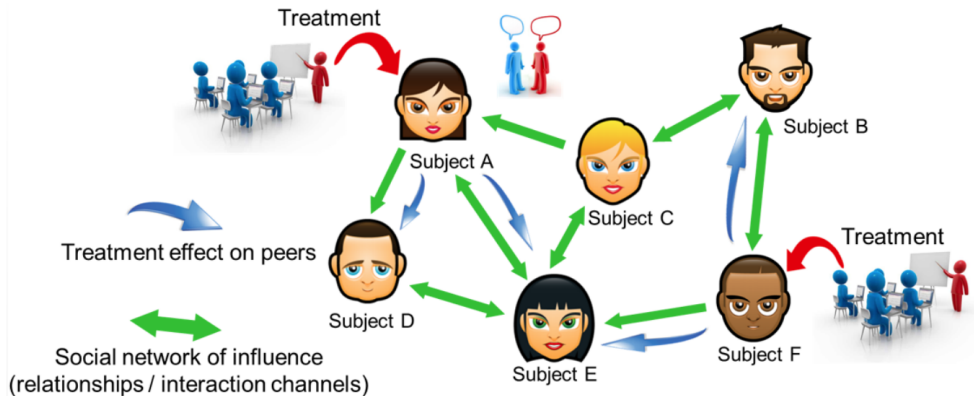


Figure 1: from Kao(2017)

Related literature

- ▶ assume no interference effect [contained in SUTVA, Rubin 1980] ,
- ▶ unreasonable and risky (Sobel 2006)
- ▶ Experimental studies
 - ▶ cluster randomized trials , two stage randomization (Hudgens and Halloran 2008; Sinclair 2011) , Insulated Neighbors Randomization (Toulis and Kao 2013)
 - ▶ test and estimate the total treatment effect (Rosenbaum 2007), separately test for treatment and spillover effects (Bowers, Fredrickson, and Panagopoulos 2013) , Horvitz–Thompson estimator (Aronow and Samii 2017)

Related literature

- ▶ spillover effect for observational studies
 - ▶ parametric multilevel approach (Raudenbush [2006](#)) , three-level generalized hierarchical linear model (Verbitsky-Savitz and Raudenbush [2012](#)) , IPW estimators (Tchetgen Tchetgen and VanderWeele [2012](#))
- ▶ causal inference for observational studies
 - ▶ generalized IPW estimator (Liu et al. [2016](#)) , targeted maximum likelihood estimator (TMLE) (Sofrygin and van der Laan [2017](#)) , extended this TMLE estimator to allow for dependence (Ogburn et al [2017](#))

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I. Causal problem and identification

Notation

- ▶ Undirected network $G = (\mathcal{N}, \mathbb{E})$
- ▶ Treatment $Z_i \in \{0, 1\}$, outcome $Y_i \in \mathcal{Y}$
- ▶ covariates $\mathbf{X}_i \in \mathcal{X}$, individual covariates $\mathbf{X}_i^{ind} \in \mathcal{X}^{ind}$, neighborhood covariates $\mathbf{X}_i^{neigh} \in \mathcal{X}^{neigh}$
- ▶ For unit i , consider partition $(i, \mathcal{N}_i, \mathcal{N}_{-i})$, $(Z_i, \mathbf{Z}_{\mathcal{N}_i}, \mathbf{Z}_{\mathcal{N}_{-i}})$, $(Y_i, \mathbf{Y}_{\mathcal{N}_i}, \mathbf{Y}_{\mathcal{N}_{-i}})$

Potential Outcomes and Neighborhood Interference

- *Stable unit treatment on neighborhood value assumption (SUTNVA)*

Assumption 1 (No multiple version of treatment(consistency))

If $\mathbf{Z}=\mathbf{z}$ then $Y_i = Y_i(\mathbf{z})$

Assumption 2 (Neighborhood interference)

Given a function $g_i: \{0, 1\}^{\mathcal{N}_i} \rightarrow \mathcal{G}_i, \forall i \in \mathcal{N}, \forall \mathbf{Z}_{\mathcal{N}_{-i}}, \mathbf{Z}'_{\mathcal{N}_{-i}}$ and $\forall \mathbf{Z}_{\mathcal{N}_i}, \mathbf{Z}'_{\mathcal{N}_i}: g_i(\mathbf{Z}_{\mathcal{N}_i}) = g_i(\mathbf{Z}'_{\mathcal{N}_i})$, the following equality holds:

$$Y_i(Z_i, \mathbf{Z}_{\mathcal{N}_i}, \mathbf{Z}_{\mathcal{N}_{-i}}) = Y_i(Z_i, \mathbf{Z}'_{\mathcal{N}_i}, \mathbf{Z}'_{\mathcal{N}_{-i}})$$

Each unit's potential outcome is subject to two treatments: the *individual treatment* Z_i , and the *neighborhood treatment* G_i [$G_i = g_i(\mathbf{Z}_{\mathcal{N}_i}) \in \mathcal{G}_i$]

Individual and Neighborhood Treatments

- ▶ The assignment mechanism is the ***probability distribution of the joint treatment*** in the whole sample, given all covariates and potential outcomes.

$$P(\mathbf{Z}, \mathbf{G} | \mathbf{X}, \{\mathbf{Y}(z, g), z = 0, 1; g \in \mathcal{G}\}) = \begin{cases} P(\mathbf{Z} | \mathbf{X}, \{\mathbf{Y}(z, g), z = 0, 1; g \in \mathcal{G}\}) & \text{if } \mathbf{G} = \mathbf{g}(\mathbf{Z}) \\ 0 & \text{otherwise} \end{cases}$$

Causal Estimands : Main Effects and Spillover effects

Average dose-response function (ADRF)

$$\mu(z, g; V) = E[Y_i(z, g) | i \in V] \quad \forall z \in \{0, 1\}, g \in \mathcal{G}$$

Main effect $\tau(g)$ and overall main effect τ

$$\begin{aligned}\tau(g) &= \mu(1, g; V_g) - \mu(0, g; V_g) \\ \tau &= \sum_{g \in \mathcal{G}} \tau(g) P(G_i = g)\end{aligned}$$

Spillover effect $\delta(g; z)$ and overall spillover effect $\Delta(z)$

$$\begin{aligned}\delta(g; z) &= \mu(z, g; V_g) - \mu(z, 0; V_g) \\ \Delta(z) &= \sum_{g \in \mathcal{G}} \delta(g; z) P(G_i = g)\end{aligned}$$

$$TE = \sum_{g \in \mathcal{G}} E[Y_i(Z_i = 1, G_i = g) - Y_i(Z_i = 0, G_i = 0) | i \in V_g] P(G_i = g) = \tau + \Delta(0)$$

Unconfoundedness of the Joint Treatment

Assumption 3 (Unconfoundedness of individual and neighborhood treatment)

$$Y_i(z, g) \perp\!\!\!\perp Z_i, G_i | \mathbf{X}_i \quad \forall z \in \{0, 1\}, g \in \mathcal{G}_i, \forall i$$

Theorem 1 (Identification of ADRF)

Under [Assumption 1](#) (no multiple versions of treatment), [Assumption 2](#) (neighborhood interference), and [Assumption 3](#) (unconfoundedness), we have

$$E[Y_i(z, g) | i \in V_g] = \sum_{\mathbf{x} \in \mathcal{X}} E[Y_i | Z_i = z, G_i = g, \mathbf{X}_i = \mathbf{x}, i \in V_g] P(\mathbf{X}_i = \mathbf{x} | i \in V_g)$$

II. Bias analysis

Bias When SUTVA Is Wrongly Assumed

Under SUTVA ,average (individual) treatment effect : $\tau_{sutva} = E[Y_i(Z_i = 1) - Y_i(Z_i = 0)]$

Naive estimator

$$\tau_{\mathbf{x}^*}^{obs} = \sum_{\mathbf{x} \in \mathcal{X}^*} (E[Y_i|Z_i = 1, \mathbf{x}_i^*] - E[Y_i|Z_i = 0, \mathbf{x}_i^*])P(\mathbf{x}_i^* = \mathbf{x})$$

The difference between τ and the quantity $\tau_{\mathbf{x}^*}^{obs}$ represents the bias for τ of a naive approach that neglects interference

Bias of Naive Estimator When Unconfoundedness Holds

Theorem 2.A

Let $G = (\mathcal{N}, \mathbb{E})$ be a known social network and let \mathcal{N}_i be the neighborhood of unit i as defined by the presence of edges. Let $Z_i \in \{0, 1\}$ be a binary treatment assigned to unit i and let G_i be a deterministic function of the subset of the treatment vector \mathbf{Z} in the neighborhood \mathcal{N}_i , that is, $G_i = g_i(\mathbf{Z}_{\mathcal{N}_i})$, with $g_i : \{0, 1\}^{\mathcal{N}_i} \rightarrow \mathcal{G}_i$. If

1. Assumption 1 holds
2. Assumption 2 holds, given function $g_i(\cdot)$ for each unit $i \in \mathcal{N}$
3. Assumption 3 holds conditional on \mathbf{X}_i^* , that is, $Y_i(z, g) \perp\!\!\!\perp Z_i, G_i | \mathbf{X}_i^*$,
 $\forall z \in \{0, 1\}, g \in \mathcal{G}_i$

then the following equality holds

$$\begin{aligned} \tau_{\mathbf{X}^*}^{obs} = & \sum_{\mathbf{x} \in \mathcal{X}^*} \left(\sum_{g \in \mathcal{G}} E[Y_i(1, g) | \mathbf{X}_i^* = \mathbf{x}, i \in V_g] P(G_i = g | Z_i = 1, \mathbf{X}_i^* = \mathbf{x}) \right. \\ & \left. - E[Y_i(0, g) | \mathbf{X}_i^* = \mathbf{x}, i \in V_g] P(G_i = g | Z_i = 0, \mathbf{X}_i^* = \mathbf{x}) \right) P(\mathbf{X}_i^* = \mathbf{x}) \end{aligned}$$

Bias of Naive Estimator When Unconfoundedness Holds

Corollary 1

Under the three conditions of [Theorem 2.A](#) and the additional condition

4. Z_i and G_i are independent conditional on \mathbf{X}_i^* , that is, $Z_i \perp\!\!\!\perp G_i | \mathbf{X}_i^*$
the following equality holds :

$$\tau_{\mathbf{X}^*}^{obs} = \tau = \sum_{g \in \mathcal{G}} (E[Y_i(1, g) - Y_i(0, g) | i \in \mathbf{V}_g]) P(G_i = g)$$

Corollary 2

Under the three conditions of [Theorem 2.A](#) and $Z_i \not\perp\!\!\!\perp G_i | \mathbf{X}_i^*$, an unbiased estimator of $\tau_{\mathbf{X}^*}^{obs}$ would be biased for the overall main effect τ , with bias given by

Bias of Naive Estimator When Unconfoundedness Holds

Corollary 2 continued

$$\begin{aligned}\tau_{\mathbf{X}^*}^{obs} - \tau &= \sum_{\mathbf{x} \in \mathcal{X}^*} \sum_{g \in \mathcal{G}} \left(E[Y_i | Z_i = 1, G_i = g, \mathbf{X}_i^* = \mathbf{x}, i \in V_g] \right. \\ &\quad \left. - E[Y_i | Z_i = 1, G_i = g', \mathbf{X}_i^* = \mathbf{x}, i \in V_g] \right) \\ &\quad \left(P(G_i = g | Z_i = 1, \mathbf{X}_i^* = \mathbf{x}) - P(G_i = g | \mathbf{X}_i^* = \mathbf{x}) \right) P(\mathbf{X}_i^* = \mathbf{x}) \\ &\quad - \sum_{\mathbf{x} \in \mathcal{X}^*} \sum_{g \in \mathcal{G}} \left(E[Y_i | Z_i = 0, G_i = g, \mathbf{X}_i^* = \mathbf{x}, i \in V_g] \right. \\ &\quad \left. - E[Y_i | Z_i = 0, G_i = g', \mathbf{X}_i^* = \mathbf{x}, i \in V_g] \right) \\ &\quad \left(P(G_i = g | Z_i = 0, \mathbf{X}_i^* = \mathbf{x}) - P(G_i = g | \mathbf{X}_i^* = \mathbf{x}) \right) P(\mathbf{X}_i^* = \mathbf{x})\end{aligned}$$

Bias of Naive Estimator When Unconfoundedness Does Not Hold

Theorem 2.B

Under [Assumption 1](#) and [2](#), if [Assumption 3](#) does not hold conditional on \mathbf{X}_i^* , that is $Y_i(z, g) \not\perp\!\!\!\perp Z_i, G_i | \mathbf{X}_i^*$, but holds conditionally on \mathbf{X}_i^* and an additional vector of covariates $\mathbf{U}_i \in \mathcal{U}$, that is, $Y_i(z, g) \perp\!\!\!\perp Z_i, G_i | \mathbf{X}_i^*, \mathbf{U}_i$, an unbiased estimator of $\tau_{\mathbf{X}^*}^{obs}$ would be unbiased for the overall main effect τ , with bias $\tau_{\mathbf{X}^*}^{obs} - \tau$ given by

$$\begin{aligned} &= \sum_{\mathbf{x} \in \mathcal{X}^*} \sum_{g \in \mathcal{G}} \sum_{\mathbf{u} \in \mathcal{U}} \left(E[Y_i | Z_i = 1, G_i = g, \mathbf{U}_i = \mathbf{u}, \mathbf{X}_i^* = \mathbf{x}, i \in V_g] \right. \\ &\quad \left. - E[Y_i | Z_i = 1, G_i = g', \mathbf{U}_i = \mathbf{u}', \mathbf{X}_i^* = \mathbf{x}, i \in V_g] \right) \\ &\quad \left(P(\mathbf{U}_i = \mathbf{u} | Z_i = 1, G_i = g, \mathbf{X}_i^* = \mathbf{x}) P(G_i = g | Z_i = 1, \mathbf{X}_i^* = \mathbf{x}) \right. \\ &\quad \left. - P(\mathbf{U}_i = \mathbf{u} | G_i = g, \mathbf{X}_i^* = \mathbf{x}) P(G_i = g | \mathbf{X}_i^* = \mathbf{x}) \right) P(\mathbf{X}_i^* = \mathbf{x}) \end{aligned}$$

Bias of Naive Estimator When Unconfoundedness Does Not Hold

Theorem 2.B continued

$$\begin{aligned} &= \sum_{\mathbf{x} \in \mathcal{X}^*} \sum_{g \in \mathcal{G}} \sum_{\mathbf{u} \in \mathcal{U}} \left(E[Y_i | Z_i = 0, G_i = g, \mathbf{U}_i = \mathbf{u}, \mathbf{X}_i^* = \mathbf{x}, i \in V_g] \right. \\ &\quad \left. - E[Y_i | Z_i = 0, G_i = g', \mathbf{U}_i = \mathbf{u}', \mathbf{X}_i^* = \mathbf{x}, i \in V_{g'}] \right) \\ &\quad \left(P(\mathbf{U}_i = \mathbf{u} | Z_i = 0, G_i = g, \mathbf{X}_i^* = \mathbf{x}) P(G_i = g | Z_i = 0, \mathbf{X}_i^* = \mathbf{x}) \right. \\ &\quad \left. - P(\mathbf{U}_i = \mathbf{u} | G_i = g, \mathbf{X}_i^* = \mathbf{x}) P(G_i = g | \mathbf{X}_i^* = \mathbf{x}) \right) P(\mathbf{X}_i^* = \mathbf{x}) \end{aligned}$$

Bias of Naive Estimator When Unconfoundedness Does Not Hold

Corollary 3

Under the following conditions

1. Assumption 1 holds and , if
2. Assumption 2 holds given the function $g_i(\cdot)$ for each unit $i \in \mathcal{N}$
3. Assumption 3 holds conditional on \mathbf{X}_i^* and \mathbf{U}_i , that is , $Y_i(z, g) \perp\!\!\!\perp Z_i, G_i | \mathbf{X}_i^*, \mathbf{U}_i$,
 $\forall z \in \{0, 1\}, g \in \mathcal{G}_i$
4. Z_i and G_i are independent conditionanl on \mathbf{X}_i^* ,that is , $Z_i \perp\!\!\!\perp G_i | \mathbf{X}_i^*$

$$\begin{aligned} \tau_{\mathbf{X}^*}^{obs} - \tau &= \sum_{\mathbf{x} \in \mathcal{X}^*} \sum_{\mathbf{u} \in \mathcal{U}} \left(E[Y_i | Z_i = z, \mathbf{U}_i = \mathbf{u}, \mathbf{X}_i^* = \mathbf{x}] \right. \\ &\quad \left. - E[Y_i | Z_i = z, \mathbf{U}_i = \mathbf{u}', \mathbf{X}_i^* = \mathbf{x}] \right) \\ &\quad \left(P(\mathbf{U}_i = \mathbf{u} | Z_i = 1, \mathbf{X}_i^* = \mathbf{x}) - P(\mathbf{U}_i = \mathbf{u} | Z_i = 0, \mathbf{X}_i^* = \mathbf{x}) \right) P(\mathbf{X}_i^* = \mathbf{x}) \end{aligned}$$

where $z \in \{0, 1\}$

Bias of Naive Estimator When Unconfoundedness Does Not Hold

Corollary 4

Under the following conditions

1. SUTVA holds
2. [Assumption 3](#) holds conditional on \mathbf{X}_i^* and \mathbf{U}_i , that is, $Y_i(z, g) \perp\!\!\!\perp Z_i, G_i | \mathbf{X}_i^*, \mathbf{U}_i$,
 $\forall z \in \{0, 1\}, g \in \mathcal{G}_i$

Bias $\tau_{\mathbf{X}^*}^{obs} - \tau$ given only by the unmeasured confounder \mathbf{U}_i as in [equation](#) from corollary 3

III. Proposed method

Generalized Propensity Score

- ▶ *joint propensity score* $\psi(z; g; \mathbf{x}) = P(Z_i = z, G_i = g | \mathbf{X}_i = \mathbf{x})$
 - ▶ Balancing Property $P(Z_i = z, G_i = g | \mathbf{X}_i, \psi(z; g; \mathbf{X}_i)) = P(Z_i = z, G_i = g | \psi(z; g; \mathbf{X}_i))$
 - ▶ Conditional unconfoundedness $Y_i(z, g) \perp\!\!\!\perp Z_i, G_i | \psi(z; g; \mathbf{X}_i) \quad \forall z \in \{0, 1\}, g \in \mathcal{G}_i$
- ▶ *individual propensity score* $\phi(z; \mathbf{x}^z) = P(Z_i = z | \mathbf{X}_i^z = \mathbf{x}^z)$
- ▶ *neighborhood propensity score* $\lambda(g; z; \mathbf{x}^g) = P(G_i = g | Z_i = z, \mathbf{X}_i^g = \mathbf{x}^g)$
- ▶ *factorization of joint propensity score* $\psi(z; g; \mathbf{x}) = \phi(z; \mathbf{x}^z) \lambda(g; z; \mathbf{x}^g)$
 - ▶ *conditional unconfoundedness*
 $Y_i(z, g) \perp\!\!\!\perp Z_i, G_i | \lambda(g; z; \mathbf{X}_i^g), \phi(1; \mathbf{X}_i^z) \quad \forall z \in \{0, 1\}, g \in \mathcal{G}_i$

Propensity Score Based Estimator for Main Effects and Spillover Effects

unbiased estimator of $\mu(z, g; V)$

- ▶ $E\left[E[Y_i|Z_i = z, G_i = g, \psi(z; g; \mathbf{X}_i)]|Z_i = z, G_i = g\right]$
- ▶ $E\left[E[Y_i|Z_i = z, G_i = g, \lambda(g; z; \mathbf{X}_i^g), \phi(1; \mathbf{X}_i^z)]|Z_i = z, G_i = g\right]$ (separate joint propensity score)

Estimation Procedure : Subclassification and GPS

- ▶ Derive a subclassification on the individual propensity score $\phi(1; \mathbf{X}_i^z)$
 - (a) estimate $\phi(1; \mathbf{X}_i^z)$ by logistic regression : $Z_i \sim \mathbf{X}_i^z$
 - (b) predict $\phi(1; \mathbf{X}_i^z)$
 - (c) identify J subclasses B_j by $\phi(1; \mathbf{X}_i^z)$, where $\mathbf{X}_i^z \perp\!\!\!\perp \mathbf{Z}_i | i \in B_j$
- ▶ Within B_j , estimate $\mu_j(z, g; V_g) = E[Y_i(z, g) | i \in B_j^g]$, where $B_j^g = V_g \cap B_j$
 - (a) estimate model for $\lambda(g; z; \mathbf{x}^g)$ by $f^G(g, z, \mathbf{X}_i^g)$
 - (b) predict $\lambda(g; z; \mathbf{X}_i^g)$ and estimate model for $Y_i(z, g)$ by $f^Y(z, g, \lambda(g; z; \mathbf{X}_i^g))$
 - (c) predict $Y_i(z, g)$
 - (d) estimate $\hat{\mu}_j(z, g; V_g) = \sum_{i \in B_j^g} \hat{Y}_i(z, g) / |B_j^g|$
- ▶ Derive the ADRF $\hat{\mu}(z, g; V_g) = \sum_{j=1}^J \hat{\mu}_j(z, g; V_g) \pi_j^g$, where $\pi_j^g = \frac{|B_j^g|}{v_g}$

IV. Simulation studies

Takeaways for simulation

► Aims

- validate the analytical derivation of the bias for the main effect when interference is wrongly ruled out
- show the performance of the proposed estimators in a realistic sample

► Data : use friendship network data collected through the National Longitudinal Study of Adolescent Health . (29 schools for a total of 16410 students)

► Variables

- Z_i : denote whether student i was covered or not by some health insurance
- Y_i : denote the number of days student i missed school because of illness in one given year
- $\mathbf{X}_i^{ind} = (race_i, grade_i)$ and $\mathbf{X}_i^{neig} = (\frac{\sum_{k \in \mathcal{N}_i} race_k}{N_i}, \frac{\sum_{k \in \mathcal{N}_i} grade_k}{N_i}, N_i)$

Four scenarios of dependence between Z_i and G_i ²

- ▶ Scenario 1 : $Z_i \perp\!\!\!\perp G_i | \mathbf{X}_i^{ind}$.
(treatment generating process : $\text{logit}(P(Z_i = 1)) = -18 + 2\text{grade}_i + 3\text{race}_i$)
- ▶ Scenario 2 : $Z_i \perp\!\!\!\perp G_i | \mathbf{X}_i^{ind}, \mathbf{X}_i^{neig}$
(treatment generating process :
 $\text{logit}(P(Z_i = 1)) = -47 + 2\text{grade}_i + 4\text{race}_i + 3\text{friends.grade}_i + 5\text{friends.race}_i$)
- ▶ Scenario 3 : $Z_i \perp\!\!\!\perp G_i | \mathbf{X}_i^{ind}, \mathbf{N}_i$
(treatment generating process : $\text{logit}(P(Z_i = 1)) = -49 + 3\text{grade}_i + 4\text{race}_i + 4N_i$)
- ▶ Scenario 4 : $Z_i \not\perp\!\!\!\perp G_i | \mathbf{X}_i^{ind}, \mathbf{X}_i^{neig}$
(treatment generating process : $\text{logit}(P(Z_i = 1)) = -20 + 2\text{grade}_i + 3\text{race}_i + 4G_i$)

²In all scenarios but the third, G_i is the proportion of friends with health insurance among the first five best friends. In the third scenario, G_i is the number of “treated” friends among all friends

Outcome Models

► Outcome Models for Main Effects Simulations

$$Y_i(z, g) | \mathbf{X}_i^{ind} \sim \mathcal{N}(\mu(z, g, \mathbf{X}_i^{ind}), 1)$$

$$\mu(z, g, \mathbf{X}_i^{ind}) = 15 - 7\mathbf{I}(\phi(1; \mathbf{X}_i^{ind}) \geq 0.7) - 15z + 3z\mathbf{I}(\phi(1; \mathbf{X}_i^{ind}) \geq 0.7) + \delta g$$

- $X_i^{ind} = [race_i + grade_i]$, $X_i^g = [race_i, grade_i, friends.race_i, friends.grade_i, N_i]$,
 $\delta \in (-5, -8, -10)$ corresponding to a low medium and high level of interference
- main effects :
 $\tau(g) = -15 + 3\mathbf{I}(\phi(1; \mathbf{X}_i^{ind}) \geq 0.7) \quad \forall g \in \mathcal{G} \implies \tau = -15 + 3\mathbf{I}(\phi(1; \mathbf{X}_i^{ind}) \geq 0.7)$
- spillover effects : $\delta(g; z) = \delta g \implies \Delta(z) = \delta E[G_i] \quad \forall z = 0, 1$

Outcome Models

► Outcome Models for Spillover Effects Simulations

$$Y_i(z, g) | \mathbf{X}_i^{ind}, \mathbf{X}_i^g \sim \mathcal{N}(\mu(z, g, \mathbf{X}_i^{ind}, \mathbf{X}_i^g), 1)$$

$$\begin{aligned} \mu(z, g, \mathbf{X}_i^{ind}, \mathbf{X}_i^g) = & 15 + friends.grade_i + 7friends.race_i - 10\mathbf{I}(\phi(1; \mathbf{X}_i^{ind}) \geq \\ & 0.7) - 10z + \delta g - 10\lambda(g; z, \mathbf{X}_i^g + 5g\mathbf{I}(\phi(1; \mathbf{X}_i^{ind}) \geq 0.7) + 3zg \end{aligned}$$

- $X_i^{ind} = [race_i + grade_i]$, $X_i^g = [race_i, grade_i, friends.race_i, friends.grade_i, N_i]$,
 $\delta \in (-5, -8, -10)$ corresponding to a low medium and high level of interference

- main effects : $\tau(g) = -10 + 3g \implies \tau = -10 + 3E[G_i]$

- spillover effects : $\delta(g; z) = \delta g + 5gE[\mathbf{I}(\phi(1; \mathbf{X}_i^{ind}) \geq 0.7)] - 10\lambda(g; z, \mathbf{X}_i^g) + 3zg$
 $\implies \Delta(z) = \delta E[G_i] + 5E[G_i]E[\mathbf{I}(\phi(1; \mathbf{X}_i^{ind}) \geq 0.7)] - 10E[\lambda(G_i; z, \mathbf{X}_i^g)] + 3zE[G_i]$

Main Effects : Bias of Naive Estimator and GPS-Based Estimator

- Table 1 shows the bias computed using formulas in [Theorem 2.A](#) and [Theorem 2.B](#), bias resulting from neglecting interference and from adjusting for different sets of covariates : $\mathbf{X}_i^* = \{\emptyset, \mathbf{X}_i^{ind}, \mathbf{X}_i^Z\}$

Scenario	Interference	Bias(\emptyset)	Bias(\mathbf{X}_i^{ind})	Bias(\mathbf{X}_i^Z)
1 ($Z_i \perp\!\!\!\perp G_i \mathbf{X}_i^{ind}$)	Low	-5.977	-0.045	-0.045
	Medium	-6.200	-0.072	-0.072
	High	-6.323	-0.090	-0.090
2 ($Z_i \perp\!\!\!\perp G_i \mathbf{X}_i^{ind}, \mathbf{X}_i^{neigh}$)	Low	-5.749	-1.636	-0.034
	Medium	-6.498	-2.618	-0.054
	High	-6.998	-3.273	-0.068
3 ($Z_i \perp\!\!\!\perp G_i \mathbf{X}_i^{ind}, N_i$)	Low	-4.158	-1.247	-0.047
	Medium	-4.792	-2.079	-0.075
	High	-5.744	-3.327	-0.095
4 ($Z_i \not\perp\!\!\!\perp G_i \mathbf{X}_i^{ind}, \mathbf{X}_i^{neigh}$)	Low	-9.504	-1.414	-1.415
	Medium	-11.681	-2.263	-2.263
	High	-13.132	-2.829	-2.825

Table 1: Computed bias for τ

Main Effects : Bias of Naive Estimators and GPS-Based Estimator

- Table 2 reports the mean bias and root mean squared error of six estimators in all scenarios.

Scenario	Interference	Unadjusted		Regression $\sim Z_i, \mathbf{X}_i^{\text{ind}}$		Subclass $\hat{\phi}(1, \mathbf{X}_i^{\text{ind}})$		Regression $\sim Z_i, \mathbf{X}_i^Z$		Subclass $\hat{\phi}(1; \mathbf{X}_i^Z)$		Subclass $\hat{\phi}(1, \mathbf{X}_i^Z)$ and GPS	
		Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
1 $(Z_i \perp\!\!\!\perp G_i \mathbf{X}_i^{\text{ind}})$	Low	-5.104	5.104	-3.429	3.431	-0.016	0.130	-3.254	3.256	0.006	0.074	-0.002	0.064
	Medium	-5.278	5.278	-3.529	3.532	-0.021	0.149	-3.256	3.258	0.007	0.090	0.002	0.067
	High	-5.396	5.396	-3.598	3.601	-0.021	0.167	-3.258	3.261	0.015	0.108	0.003	0.067
2 $(Z_i \perp\!\!\!\perp G_i \mathbf{X}_i^{\text{ind}}, \mathbf{X}_i^{\text{neigh}})$	Low	-5.136	5.136	-3.477	3.477	-1.652	1.668	-2.156	2.157	-0.066	0.331	-0.000	0.036
	Medium	-5.884	5.884	-4.521	4.521	-2.620	2.631	-2.318	2.319	-0.097	0.511	0.003	0.039
	High	-6.384	6.384	-5.222	5.223	-3.272	3.281	-2.430	2.432	-0.123	0.635	-0.002	0.036
3 $(Z_i \perp\!\!\!\perp G_i \mathbf{X}_i^{\text{ind}}, N_i)$	Low	-4.106	4.107	-2.079	2.089	-1.146	1.149	-1.033	1.049	0.078	0.553	-0.002	0.230
	Medium	-4.725	4.726	-2.886	2.893	-1.911	1.913	-1.041	1.056	0.082	0.556	0.019	0.233
	High	-5.655	5.656	-4.098	4.103	-3.058	3.059	-1.054	1.068	0.084	0.559	0.012	0.234
4 $(Z_i \not\perp\!\!\!\perp G_i \mathbf{X}_i^{\text{ind}}, \mathbf{X}_i^{\text{neigh}})$	Low	-8.670	8.670	-7.104	7.104	-1.443	1.445	-7.076	7.076	-1.738	1.740	0.000	0.105
	Medium	-10.761	10.761	-8.861	8.862	-2.309	2.309	-8.824	8.823	-2.784	2.785	-0.003	0.095
	High	-12.154	12.154	-10.030	10.030	-2.883	2.884	-9.985	9.985	-3.477	3.478	0.010	0.090

Table 2: Estimation of main effect τ

Spillover Effects

- Table 3 and Table 4 report the mean bias and root mean squared error of all estimators for spillover effects $\Delta(0)$ and $\Delta(1)$.

Scenario	Interference	Unadjusted regression $\sim Z_i, G_i$		Subclass $\hat{\phi}(1, \mathbf{X}_i^Z)$		GPS		Subclass $\hat{\phi}(1, \mathbf{X}_i^Z)$ and GPS	
		Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
1 ($Z_i \perp\!\!\!\perp G_i \mathbf{X}_i^{\text{ind}}$)	Low	1.576	1.581	2.871	2.901	-1.457	1.465	0.003	0.091
	Medium	1.568	1.574	2.838	2.872	-1.462	1.472	0.005	0.114
	High	1.571	1.580	2.878	2.910	-1.467	1.479	0.000	0.148
2 ($Z_i \perp\!\!\!\perp G_i \mathbf{X}_i^{\text{ind}}, \mathbf{X}_i^{\text{neigh}}$)	Low	3.503	3.506	4.132	5.259	-0.574	0.590	0.033	0.098
	Medium	3.506	3.510	4.037	4.933	-0.579	0.599	0.041	0.118
	High	3.485	3.489	4.204	5.232	-0.592	0.613	0.034	0.129
3 ($Z_i \perp\!\!\!\perp G_i \mathbf{X}_i^{\text{ind}}, N_i$)	Low	5.445	5.446	7.005	7.048	0.380	0.399	-0.035	0.091
	Medium	5.455	5.456	7.009	7.050	0.381	0.401	-0.033	0.091
	High	5.441	5.442	7.054	7.096	0.381	0.400	-0.033	0.099
3 ($Z_i \not\perp\!\!\!\perp G_i \mathbf{X}_i^Z, \mathbf{X}_i^{\text{neigh}}$)	Low	3.002	3.002	2.577	2.584	-1.201	1.202	0.071	0.111
	Medium	3.002	3.002	2.556	2.563	-1.202	1.203	0.068	0.111
	High	3.005	3.005	2.548	2.555	-1.200	1.201	0.066	0.111

Table 3: Estimation of $\Delta(0)$

Spillover Effects

Scenario	Interference	Unadjusted regression $\sim Z_i, G_i$		Subclass $\hat{\phi}(1, \mathbf{X}_i^Z)$		GPS		Subclass $\hat{\phi}(1, \mathbf{X}_i^Z)$ and GPS	
		Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
1 $(Z_i \perp\!\!\!\perp G_i \mathbf{X}_i^{\text{ind}})$	Low	3.195	3.196	2.870	2.878	0.432	0.441	0.003	0.055
	Medium	3.197	3.198	2.863	2.871	0.437	0.449	0.002	0.083
	High	3.198	3.200	2.869	2.877	0.434	0.452	0.004	0.103
2 $(Z_i \perp\!\!\!\perp G_i \mathbf{X}_i^{\text{ind}}, \mathbf{X}_i^{\text{neigh}})$	Low	2.483	2.485	2.842	2.862	0.386	0.393	0.001	0.059
	Medium	2.481	2.485	2.885	2.907	0.385	0.394	-0.000	0.072
	High	2.473	2.477	2.882	2.919	0.379	0.391	-0.005	0.088
3 $(Z_i \perp\!\!\!\perp G_i \mathbf{X}_i^{\text{ind}}, N_i)$	Low	3.668	3.669	5.921	5.951	0.559	0.562	-0.038	0.045
	Medium	3.668	3.668	5.944	5.975	0.558	0.561	-0.040	0.046
	High	3.669	3.669	5.943	5.982	0.558	0.561	-0.038	0.048
3 $(Z_i \not\perp\!\!\!\perp G_i \mathbf{X}_i^Z, \mathbf{X}_i^{\text{neigh}})$	Low	-3.280	3.280	-5.182	5.182	0.367	0.370	-0.019	0.056
	Medium	-3.277	-3.277	-5.182	5.182	0.368	0.371	-0.021	0.057
	High	-3.281	3.281	-5.178	5.178	0.364	0.367	-0.026	0.059

Table 4: Estimation of $\Delta(1)$

Spillover Effects

- Figure 2 depicts the scatterplot of the observed outcomes and the estimated ADRFs $\mu(0, g)$ and $\mu(1, g)$, $g \in \{0, 0.2, 0.4, 0.6, 0.8, 1\}$, for Scenario 2

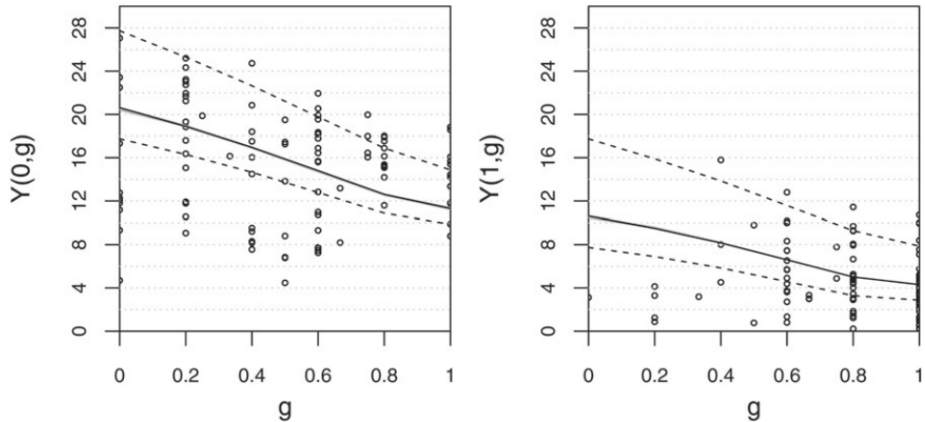


Figure 2: Estimated dose-response function $\mu(0, g)$ and $\mu(1, g)$

Spillover Effects

- Figure 3 depicts spillover effects $\delta(0, g)$ i.e. $\mu(0, g) - \mu(0, 0)$ and spillover effects $\delta(1, g)$ i.e. $\mu(1, g) - \mu(1, 0)$

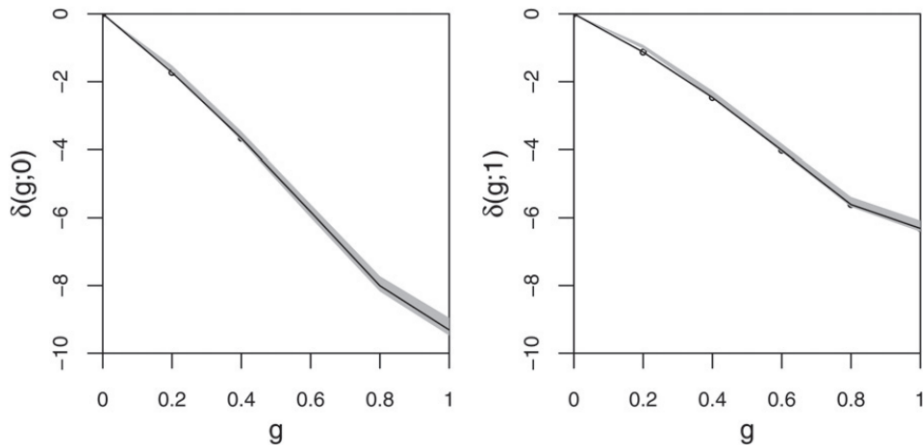


Figure 3: Estimated spillover effects $\delta(g; z)$

Spillover Effects

- ▶ Figure 4 depicts estimated main effects $\tau(g)$. i.e $\mu(1, g) - \mu(0, g)$.

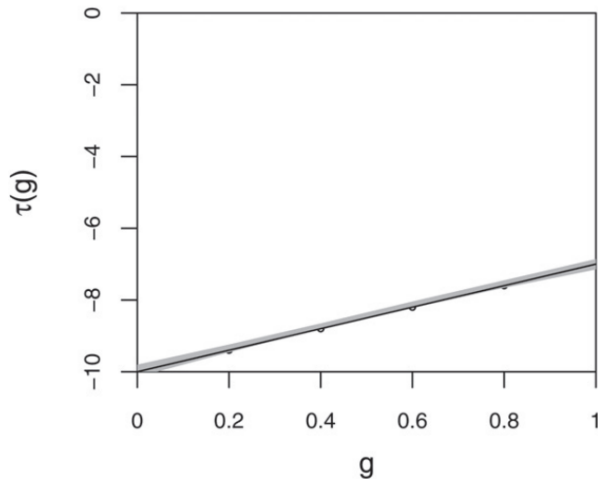


Figure 4: Estimated main effects $\tau(g)$

V . Conclusion

Concluding Remarks

- ▶ SUTVA \longrightarrow SUTNVA , potential outcome $Y_i(\mathbf{Z}_i, \mathbf{G}_i)$
- ▶ simple assignment mechanism \longrightarrow compound assignment mechanism
- ▶ naive estimator \longrightarrow Subclassification and GPS estimator
- ▶ Bias sources : $\mathbf{Z}_{\mathcal{N}_i}, P(Z_{i,res}, G_{i,res} | \mathbf{X}_i^*)$
- ▶ limitations : network fully known and fixed , model dependent
- ▶ future : sensitivity analysis , Bayesian semi-parametric approaches , account for network uncertainty

Software

- ▶ [networkinference](#) - Python package by Michael P. Leung
- ▶ [CausalModel](#) - Python package by Qu, Zhaonan and Xiong, Ruoxuan and Liu, Jizhou and Imbens, Guido
- ▶ [inference](#) - R package by Bradley Saul
- ▶ [clusteredinference](#) - R package by Brian G. Barkley
- ▶ [Interference](#) - R package by Georgia Papadogeorgou

Thank You !