# Identification and Estimation of Treatment and Interference Effects in Observational Studies on Networks[1]

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[1] Source : Forastiere L, Airoldi E M, Mealli F. Identification and estimation of treatment and interference effects in observational studies on networks[J]. Journal of the American Statistical Association, 2021, 116(534): 901-918.

# Interference

- Interference is said to be present when a treatment, exposure, or intervention, on one unit has an effect on the response of another unit [Cox 1958]

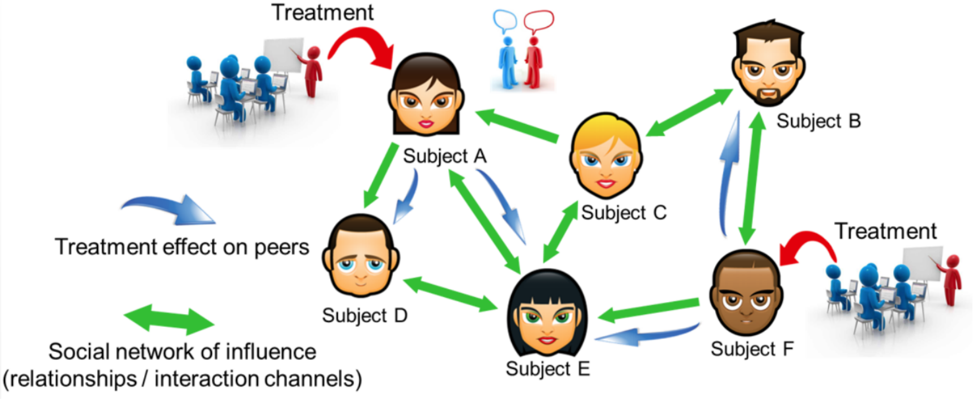


Figure 1: from Kao(2017)

# Interference effect in causal inference

- assume no interference effect [contained in SUTVA,Rubin 1980] ,
- unreasonable and risky (Sobel 2006)
- Experimental studies :
  - cluster randomized trials , two stage randomization (Hudgens and Halloran 2008; Sinclair 2011) , Insulated Neighbors Randomization (Toulis and Kao 2013)
  - test and estimate the total treatment effect ( Rosenbaum 2007), separately test for treatment and spillover effects (Bowers, Fredrickson, and Panagopoulos 2013) , Horvitz–Thompson estimator (Aronow and Samii 2017)

# Interference effect in causal inference

- spillover effect for observational studies
  - parametric multilevel approach (Raudenbush 2006) , three-level generalized hierarchical linear model (Verbitsky-Savitz and Raudenbush 2012) , IPW estimators (Tchetgen Tchetgen and VanderWeele 2012)
- causal inference for observational studies
  - generalized IPW estimator (Liu et al. 2016) , targeted maximum likelihood estimator (TMLE) (Sofrygin and van der Laan 2017) , extended this TMLE estimator to allow for dependence (Ogburn et al 2017)

# Contents

# I. Causal problem and identification

# Notation

- Undirected network $G = (\mathcal{N}, \mathbb{E})$
- Treatment $Z_i \in \{0, 1\}$, outcome $Y_i \in \mathcal{Y}$
- covariates $\mathbf{X}_i \in \mathcal{X}$ ,individual covariates $\mathbf{X}_i^{ind} \in \mathcal{X}^{ind}$,neighborhood covariates $\mathbf{X}_i^{neigh} \in \mathcal{X}^{neigh}$
- For unit i, consider partition $(i, \mathcal{N}_i, \mathcal{N}_{-i})$ , $(Z_i, \mathbf{Z}_{\mathcal{N}_i}, \mathbf{Z}_{\mathcal{N}_{-i}})$ , $(Y_i, \mathbf{Y}_{\mathcal{N}_i}, \mathbf{Y}_{\mathcal{N}_{-i}})$

# Potential Outcomes and Neighborhood Interference

- ***Stable unit treatment on neighborhood value assumption (SUTNVA)***

**Assumption 1 (No multiple version of treatment(consistency))**

$$\text{If } \mathbf{Z}=\mathbf{z} \text{ then } Y_i = Y_i(\mathbf{z})$$

**Assumption 2 (Neighborhood interference)**

Given a function $g_i : \{0,1\}^{\mathcal{N}_i} \to \mathcal{G}_i, \forall i \in \mathcal{N}$, $\forall \mathbf{Z}_{\mathcal{N}_{-i}}, \mathbf{Z}'_{\mathcal{N}_{-i}}$ and $\forall \mathbf{Z}_{\mathcal{N}_i}, \mathbf{Z}'_{\mathcal{N}_i} : g_i(\mathbf{Z}_{\mathcal{N}_i}) = g_i(\mathbf{Z}'_{\mathcal{N}_i})$, the following equality holds :

$$Y_i(Z_i, \mathbf{Z}_{\mathcal{N}_i}, \mathbf{Z}_{\mathcal{N}_{-i}}) = Y_i(Z_i, \mathbf{Z}'_{\mathcal{N}_i}, \mathbf{Z}'_{\mathcal{N}_{-i}})$$

Each unit's potential outcome is subject to two treatments : the *individual treatment* $Z_i$, and *the neighborhood treatment* $G_i$ $[G_i = g_i(\mathbf{Z}_{\mathcal{N}_i}) \in \mathcal{G}_i]$

# Individual and Neighborhood Treatments

- The assignment mechanism is the ***probability distribution of the joint treatment*** in the whole sample, given all covariates and potential outcomes.

$$P(\mathbf{Z}, \mathbf{G} | \mathbf{X}, \{\mathbf{Y}(z, g), z = 0, 1; g \in \mathcal{G}\}) =$$

$$\begin{cases} P(\mathbf{Z} | \mathbf{X}, \{\mathbf{Y}(z, g), z = 0, 1; g \in \mathcal{G}\}) & \text{if} \quad \mathbf{G} = \mathbf{g}(\mathbf{Z}) \\ 0 & \textit{otherwise} \end{cases}$$

# Causal Estimands : Main Effects and Spillover effects

## Average dose-response function (ADRF)

$$\mu(z, g; V) = E[Y_i(z, g) | i \in V] \qquad \forall z \in \{0, 1\}, g \in \mathcal{G}$$

## Main effect $\tau(g)$ and overall main effect $\tau$

$$\tau(g) = \mu(1, g; V_g) - \mu(0, g; V_g)$$
$$\tau = \sum_{g \in \mathcal{G}} \tau(g) P(G_i = g)$$

## Spillover effect $\delta(g; z)$ and overall spillover effect $\Delta(z)$

$$\delta(g; z) = \mu(z, g; V_g) - \mu(z, 0; V_g)$$
$$\Delta(z) = \sum_{g \in \mathcal{G}} \delta(g; z) P(G_i = g)$$

$$TE = \sum_{g \in \mathcal{G}} E[Y_i(Z_i = 1, G_i = g) - Y_i(Z_i = 0, G_i = 0) | i \in V_g] P(G_i = g) = \tau + \Delta(0)$$

# Unconfoundedness of the Joint Treatment

**Assumption 3 (Unconfoundedness of individual and neighborhood treatment)**

$$Y_i(z,g) \perp\!\!\!\perp Z_i, G_i | \mathbf{X}_i \qquad \forall z \in \{0,1\}, g \in \mathcal{G}_i, \forall i$$

**Theorem 1 (Identification of ADRF)**

Under Assumption 1 (no multiple versions of treatment), Assumption2 (neighborhoood interference) , and Assumption 3 (unconfoundedness) , we have

$$E[Y_i(z,g)|i \in V_g] = \sum_{\mathbf{x}\in\mathcal{X}} E[Y_i|Z_i = z, G_i = g, \mathbf{X_i} = \mathbf{x}, i \in V_g] P(\mathbf{X_i} = \mathbf{x}|i \in V_g)$$

# II. Bias analysis

# Bias When SUTVA Is Wrongly Assumed

Under SUTVA ,average (individual) treatment effect : $\tau_{sutva} = E[Y_i(Z_i = 1) - Y_i(Z_i = 0)]$

**Naive estimator**

$$\tau_{\mathbf{X}^\star}^{obs} = \sum_{\mathbf{x} \in \mathcal{X}^\star} (E[Y_i | Z_i = 1, \mathbf{X}_i^\star] - E[Y_i | Z_i = 1, \mathbf{X}_i^\star]) P(\mathbf{X}_i^\star = \mathbf{x})$$

The difference between $\tau$ and the quantity $\tau_{\mathbf{X}^\star}^{obs}$ represents the bias for $\tau$ of a naive approach that neglects interference

# Bias of Naive Estimator When Unconfoudedness Holds

## Theorem 2.A

Let $G = (\mathcal{N}, \mathbb{E}$ be a known social network and lei $\mathcal{N}_i$ be the neighborhood of unit i asa defined by the presence of edges . Let $Z_i \in \{0,1\}$ be a binary treatment assigned to unit i and let $G_i$ be a deterministic function of the subset of the treatment vector $\mathbf{Z}$ in the neighborhood $\mathcal{N}_i$ , that is , $G_i = g_i(\mathbf{Z}_{\mathcal{N}_i})$ , with $g_i : \{0,1\}^{N_i} \rightarrow \mathcal{G}_i$ . If

1. Assumption 1 holds
2. Assumption 2 holds , given function $g_i(\cdot)$ for each unit $i \in \mathcal{N}$
3. Assumption 3 holds conditional on $\mathbf{X}_i^\star$ , that is , $Y_i(z,g) \perp\!\!\!\perp Z_i, G_i | \mathbf{X}_i^\star$, $\forall z \in \{0,1\}, g \in \mathcal{G}_i$

then the following equality holds

$$\tau_{\mathbf{X}^\star}^{obs} = \sum_{\mathbf{x} \in \mathcal{X}^\star} \Big( \sum_{g \in \mathcal{G}} E[Y_i(1,g)|\mathbf{X}_i^\star = \mathbf{x}, i \in V_g] P(G_i = g | Z_i = 1, , i \in X_i^\star = \mathbf{x})$$

$$- E[Y_i(0,g)|, \mathbf{X}_i^\star = \mathbf{x}, i \in V_g] P(G_i = g | Z_i = 0, \mathbf{X}_i^\star = \mathbf{x}) \Big) P(\mathbf{X}_i^\star = \mathbf{x})$$

# Bias of Naive Estimator When Unconfoudedness Holds

## Corollary 1

Under the three conditions of Theorem 2.A and the additional condition

4. $Z_i$ and $G_i$ are independent conditional on $\mathbf{X}_i^\star$ , that is , $Z_i \perp\!\!\!\perp G_i | \mathbf{X}_i^\star$

the following equality holds :

$$\tau_{\mathbf{X}^\star}^{obs} = \tau$$

## Corollary 2

Under the three conditions of Theorem 2.A and $Z_i \not\perp\!\!\!\perp G_i | \mathbf{X}_i^\star$ , an unbiased estimator of $\tau_{\mathbf{X}^\star}^{obs}$ would be biased for the overall main effect $\tau$ , with bisas given by

# Bias of Naive Estimator When Unconfoudedness Holds

## Corollary 2 continued

$$
\begin{aligned}
\tau^{obs}_{\mathbf{X}^\star} - \tau = & \sum_{\mathbf{x} \in \mathcal{X}^\star} \sum_{g \in \mathcal{G}} \Big( E[Y_i | Z_i = 1, G_i = g, \mathbf{X}_i^\star = \mathbf{x}, i \in V_g] \\
& - E[Y_i | Z_i = 1, G_i = g', \mathbf{X}_i^\star = \mathbf{x}, i \in V_g] \Big) \\
& \Big( P(G_i = g | Z_i = 1, \mathbf{X}_i^\star = \mathbf{x}) - P(G_i = g | \mathbf{X}_i^\star = \mathbf{x}) \Big) P(\mathbf{X}_i^\star = \mathbf{x}) \\
& - \sum_{\mathbf{x} \in \mathcal{X}^\star} \sum_{g \in \mathcal{G}} \Big( E[Y_i | Z_i = 0, G_i = g, \mathbf{X}_i^\star = \mathbf{x}, i \in V_g] \\
& - E[Y_i | Z_i = 0, G_i = g', \mathbf{X}_i^\star = \mathbf{x}, i \in V_g] \Big) \\
& \Big( P(G_i = g | Z_i = 0, \mathbf{X}_i^\star = \mathbf{x}) - P(G_i = g | \mathbf{X}_i^\star = \mathbf{x}) \Big) P(\mathbf{X}_i^\star = \mathbf{x})
\end{aligned}
$$

# Bias of Naive Estimator When Unconfoudedness Does Not Hold

**Theorem 2.B**

Under Assumption 1 and 2 , if Assumption 3 does not hold conditional on $\mathbf{X}_i^\star$ ,that is $Y_i(z,g) \not\perp\!\!\!\perp Z_i, G_i|\mathbf{X}_i^\star$ , but holds conditioanl on $\mathbf{X}_i^\star$ and an additional vector of covariates $\mathbf{U}_i \in \mathcal{U}$ , that is , $Y_i(z,g) \perp\!\!\!\perp Z_i, G_i|\mathbf{X}_i^\star, \mathbf{U}_i$ , an unbiased estimator of $\tau_{\mathbf{X}^\star}^{obs}$ would be unbiased for the overall main effect $\tau$ , with bias $\tau_{\mathbf{X}^\star}^{obs} - \tau$ given by

$$
\begin{aligned}
&= \sum_{\mathbf{x}\in\mathcal{X}^\star} \sum_{g\in\mathcal{G}} \sum_{u\in\mathcal{U}} \Big( E[Y_i|Z_i = 1, G_i = g, \mathbf{U}_i = \mathbf{u}, \mathbf{X}_i^\star = \mathbf{x}, i \in V_g] \\
&- E[Y_i|Z_i = 1, G_i = g', \mathbf{U}_i = \mathbf{u}', \mathbf{X}_i^\star = \mathbf{x}, i \in V_g] \Big) \\
&\Big( P(\mathbf{U}_i = \mathbf{u}|, Z_i = 1, G_i = g, \mathbf{X}_i^\star = \mathbf{x}) P(G_i = g|Z_i = 1, \mathbf{X}_i^\star = \mathbf{x}) \\
&- P(\mathbf{U}_i = \mathbf{u}|G_i = g, \mathbf{X}_i^\star = \mathbf{x}) P(G_i = g|\mathbf{X}_i^\star = \mathbf{x}) \Big) P(\mathbf{X}_i^\star = \mathbf{x})
\end{aligned}
$$

# Bias of Naive Estimator When Unconfoudedness Does Not Hold

## Theorem 2.B continued

$$
\begin{aligned}
= \sum_{\mathbf{x}\in\mathcal{X}^\star} \sum_{g\in\mathcal{G}} \sum_{u\in\mathcal{U}} \Big( & E[Y_i | Z_i = 0, G_i = g, \mathbf{U}_i = \mathbf{u}, \mathbf{X}_i^\star = \mathbf{x}, i \in V_g] \\
& - E[Y_i | Z_i = 0, G_i = g', \mathbf{U}_i = \mathbf{u}', \mathbf{X}_i^\star = \mathbf{x}, i \in V_g]\Big) \\
& \Big( P(\mathbf{U}_i = \mathbf{u}|, Z_i = 0, G_i = g, \mathbf{X}_i^\star = \mathbf{x}) P(G_i = g | Z_i = 0, \mathbf{X}_i^\star = \mathbf{x}) \\
& - P(\mathbf{U}_i = \mathbf{u} | G_i = g, \mathbf{X}_i^\star = \mathbf{x}) P(G_i = g | \mathbf{X}_i^\star = \mathbf{x}) \Big) P(\mathbf{X}_i^\star = \mathbf{x})
\end{aligned}
$$

# Bias of Naive Estimator When Unconfoudedness Does Not Hold

## Corollary 3

Under the following conditions

1. Assumption 1 holds and , if
2. Assumption 2 holds given the function $g_i(\cdot)$ for each unit $i \in \mathcal{N}$
3. Assumption 3 holds conditional on $\mathbf{X}_i^\star$ and $\mathbf{U}_i$ , that is , $Y_i(z, g) \perp\!\!\!\perp Z_i, G_i | \mathbf{X}_i^\star, \mathbf{U}_i$ , $\forall z \in \{0, 1\}, g \in \mathcal{G}_i$
4. $Z_i$ and $G_i$ are independent conditioanl on $\mathbf{X}_i^\star$ ,that is , $Z_i \perp\!\!\!\perp G_i | \mathbf{X}_i^\star$

$$
\begin{aligned}
\tau_{\mathbf{X}^\star}^{obs} - \tau = \sum_{\mathbf{x} \in \mathcal{X}^\star} \sum_{u \in \mathcal{U}} \Big( & E[Y_i | Z_i = z, \mathbf{U}_i = \mathbf{u}, \mathbf{X}_i^\star = \mathbf{x}] \\
& - E[Y_i | Z_i = z, \mathbf{U}_i = \mathbf{u}', \mathbf{X}_i^\star = \mathbf{x}] \Big) \\
& \Big( P(\mathbf{U}_i = \mathbf{u} | Z_i = 1, \mathbf{X}_i^\star = \mathbf{x}) - P(\mathbf{U}_i = \mathbf{u} | Z_i = 1, \mathbf{X}_i^\star = \mathbf{x}) \Big) P(\mathbf{X}_i^\star = \mathbf{x})
\end{aligned}
$$

where $z \in \{0, 1\}$

# Bias of Naive Estimator When Unconfoudedness Does Not Hold

## Corollary 4

Under the following conditions

1. SUTVA holds
2. Assumption 3 holds conditional on $\mathbf{X}_i^\star$ and $\mathbf{U}_i$ , that is , $Y_i(z, g) \perp\!\!\!\perp Z_i, G_i | \mathbf{X}_i^\star, \mathbf{U}_i$ , $\forall z \in \{0, 1\}, g \in \mathcal{G}_i$

Bias $\tau_{\mathbf{X}^\star}^{obs} - \tau$ given only by the unmeasured confounder $\mathbf{U}_i$ as in equation from corollary 3

# III.Proposed method

# Generalized Propensity Score

- *joint propensity score* $\psi(z; g; x) = P(Z_i = z, G_i = g | \mathbf{X}_i = \mathbf{x})$
  - Balancing Property $P(Z_i = z, G_i = g | \mathbf{X}_i, \psi(z; g; \mathbf{X}_i)) = P(Z_i = z, G_i = g | \psi(z; g; \mathbf{X}_i))$
  - Conditional unconfoundedness $Y_i(z, g) \perp\!\!\!\perp Z_i, G_i | \psi(z; g; \mathbf{X}_i) \; \forall z \in \{0, 1\}, g \in \mathcal{G}_i$
- *individual propensity score* $\phi(z; x^z) = P(Z_i = z | \mathbf{X}_i^z = \mathbf{x}^z)$
- *neighborhood propensity score* $\lambda(g; z; x^g) = P(G_i = g | Z_i = z, \mathbf{X}_i^g = \mathbf{x}^g)$
- *factorization of joint propensity score* $\psi(z; g; x) = \phi(z; x^z)\lambda(g; z; x^g)$
  - *conditional unconfoundedness*

$$Y_i(z, g) \perp\!\!\!\perp Z_i, G_i | \lambda(g; z; \mathbf{X}_i^g), \phi(1; \mathbf{X}_i^z) \quad \forall z \in \{0, 1\}, g \in \mathcal{G}_i$$

# Propensity Score Based Estimator for Main Effects and Spillover Effects

## unbiased estimator of $\mu(z, g; V)$

- $E\left[E[Y_i|Z_i = z, G_i = g, \psi(z; g; \mathbf{X}_i)]|Z_i = z, G_i = g\right]$
- $E\left[E[Y_i|Z_i = z, G_i = g, \lambda(g; z; \mathbf{X}_i^g), \phi(1; \mathbf{X}_i^z)]|Z_i = z, G_i = g\right]$ (separate joint propensity score)

# Estimation Procedure : Subclassification and GPS

- Derive a subcalssification on the individual propensity score $\phi(1; \mathbf{X}_i^z)$
  (a) estimate $\phi(1; \mathbf{X}_i^z)$ by logistic regression : $Z_i \sim \mathbf{X}_i^z$
  (b) predict $\phi(1; \mathbf{X}_i^z)$
  (c) identify $J$ subclasses $B_j$ by $\phi(1; \mathbf{X}_i^z)$ , where $\mathbf{X}_i^z \perp\!\!\!\perp \mathbf{Z}_i | i \in B_j$

- Within $B_j$ ,estimate $\mu_j(z, g; V_g) = E[Y_i(z, g) | i \in B_j^g]$ , where $B_j^g = V_g \cap B_j$
  (a) estimate model for $\lambda(g; z; x^g)$ by $f^G(g, z, \mathbf{X}_i^g)$
  (b) estimate model for $Y_i(z, g)$ by $f^Y(z, g, \lambda(g; z; \mathbf{X}_i^g))$
  (c) predict $\lambda(g; z; \mathbf{X}_i^g)$ and $Y_i(z, g)$
  (d) estimate $\hat{\mu}_j(z, g; V_g) = \sum_{i \in B_j^g} \hat{Y}_i(z, g) / |B_j^g|$

- Derive the ADRF $\hat{\mu}(z, g; V_g) = \sum_{j=1}^{J} \hat{\mu}_j(z, g; V_g) \pi_j^g$ , where $\pi_j^g = \frac{|B_j^g|}{\nu_g}$

# IV.Simulation studies

# Takeaways for simulation

- Aims
  - validate the analytical derivation of the bias for the main effect when interference is wrongly ruled out
  - show the performance of the proposed estimators in a realistic sample

- Data : We use friendship network data collected through the National Longitudinal Study of Adolescent Health . (29 schools for a total of 16410 students)

- Variables
  - $Z_i$ : denote whether student i was covered or not by some health insurance
  - $Y_i$ : denote the number of days student i missed school because of illness in one given year
  - $\mathbf{X}_i^{ind} = (race_i, grade_i)$ and $\mathbf{X}_i^{neig} = (\frac{\sum_{k \in \mathcal{N}_i} race_k}{N_i}, \frac{\sum_{k \in \mathcal{N}_i} grade_k}{N_i}, N_i)$

# Four scenarios of dependence between $Z_i$ and $G_i$ [2]

- Scenario 1 : $Z_i \perp\!\!\!\perp G_i | \mathbf{X}_i^{ind}$ .
  (treatment generating process : $logit(P(Z_i = 1)) = -18 + 2grade_i + 3race_i$)

- Scenario 2 : $Z_i \perp\!\!\!\perp G_i | \mathbf{X}_i^{ind}, \mathbf{X}_i^{neig}$
  (treatment generating process :
  $logit(P(Z_i = 1)) = -47 + 2grade_i + 4race_i + 3friends.grade_i + 5friends.race_i$)

- Scenario 3 : $Z_i \perp\!\!\!\perp G_i | \mathbf{X}_i^{ind}, \mathbf{N}_i$
  (treatment generating process : $logit(P(Z_i = 1)) = -49 + 3grade_i + 4race_i + 4N_i$)

- Scenario 4 : $Z_i \not\!\perp\!\!\!\perp G_i | \mathbf{X}_i^{ind}, \mathbf{X}_i^{neig}$
  (treatment generating process : $logit(P(Z_i = 1)) = -20 + 2grade_i + 3race_i + 4G_i$)

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[2] In all scenarios but the third, $G_i$ is the proportion of friends with health insurance among the first five best friends. In the third scenario, $G_i$ is the number of "treated" friends among all friends

# Outcome Models

- Outcome Models for Main Effects Simulations

$$Y_i(z,g)|\mathbf{X}_i^{ind} \sim \mathcal{N}(\mu(z,g,\mathbf{X}_i^{ind}),1)$$

$$\mu(z,g,\mathbf{X}_i^{ind}) = 15 - 7\mathbf{I}(\phi(1;\mathbf{X}_i^{ind}) \geq 0.7) - 15z + 3z\mathbf{I}(\phi(1;\mathbf{X}_i^{ind}) \geq 0.7) + \delta g$$

- $X_i^{ind} = [race_i + grade_i]$ , $X_i^g = [race_i, grade_i, friends.race_i, friends.grade_i, N_i]$ , $\delta \in (-5, -8, -10)$ corresponding to a low medium and high level of interference

- main effects : $\tau(g) = -15 + 3\mathbf{I}(\phi(1;\mathbf{X}_i^{ind}) \geq 0.7)\ \forall g \in \mathcal{G} \implies \tau = -15 + 3\mathbf{I}(\phi(1;\mathbf{X}_i^{ind}) \geq 0.7)$

- spillover effects : $\delta(g;z) = \delta g \implies \Delta(z) = \delta E[G_i]\ \forall z = 0, 1$

# Outcome Models

- Outcome Models for Spillover Effects Simulations

$$Y_i(z,g)|\mathbf{X}_i^{ind},\mathbf{X}_i^{g} \sim \mathcal{N}(\mu(z,g,\mathbf{X}_i^{ind},\mathbf{X}_i^{g}),1)$$

$$\mu(z,g,\mathbf{X}_i^{ind},\mathbf{X}_i^{g}) = 15 + friends.grade_i + 7friends.race_i - 10\mathbf{I}(\phi(1;\mathbf{X}_i^{ind}) \geq 0.7) - 10z + \delta g - 10\lambda(g;z,\mathbf{X}_i^{g} + 5g\mathbf{I}(\phi(1;\mathbf{X}_i^{ind}) \geq 0.7) + 3zg$$

- $X_i^{ind} = \left[race_i + grade_i\right]$ , $X_i^{g} = \left[race_i, grade_i, friends.race_i, friends.grade_i, N_i\right]$ , $\delta \in (-5,-8,-10)$ corresponding to a low medium and high level of interference
- main effects : $\tau(g) = -10 + 3g \Longrightarrow \tau = -10 + 3E[G_i]$
  $\tau(g) = -15 + 3\mathbf{I}(\phi(1;\mathbf{X}_i^{ind}) \geq 0.7)\ \forall g \in \mathcal{G} \Longrightarrow \tau = -15 + 3\mathbf{I}(\phi(1;\mathbf{X}_i^{ind}) \geq 0.7)$
- spillover effects : $\delta(g;z) = \delta g 5gE[\mathbf{I}(\phi(1;\mathbf{X}_i^{ind}) \geq 0.7)] - 10\lambda(g;z,\mathbf{X}_i^{g})$

$\Longrightarrow \Delta(z) = \delta E[G_i] + 5E[G_i]E[\mathbf{I}(\phi(1;\mathbf{X}_i^{ind}) \geq 0.7)] - 10E[\lambda(G_i;z,\mathbf{X}_i^{g})] + 3zE[G_i]$

# Main Effects : Bias of Naive Estimator and GPS-Based Estimator

- Table 1 shows the bias computed using formulas in Theorem 2.A and Theorem 2.B, bias resulting from neglecting interference and from adjusting for different sets of covariates : $\mathbf{X}_i^\star = \{\emptyset, \mathbf{X}_i^{ind}, \mathbf{X}_i^z\}$

| Scenario | Interference | Bias($\emptyset$) | Bias($\mathbf{X}_i^{\mathrm{ind}}$) | Bias($\mathbf{X}_i^z$) |
|---|---|---|---|---|
| 1 $(Z_i \perp\!\!\!\perp G_i \mid \mathbf{X}_i^{\mathrm{ind}})$ | Low | −5.977 | −0.045 | −0.045 |
| | Medium | −6.200 | −0.072 | −0.072 |
| | High | −6.323 | −0.090 | −0.090 |
| 2 $(Z_i \perp\!\!\!\perp G_i \mid \mathbf{X}_i^{\mathrm{ind}}, \mathbf{X}_i^{\mathrm{neigh}})$ | Low | −5.749 | −1.636 | −0.034 |
| | Medium | −6.498 | −2.618 | −0.054 |
| | High | −6.998 | −3.273 | −0.068 |
| 3 $(Z_i \perp\!\!\!\perp G_i \mid \mathbf{X}_i^{\mathrm{ind}}, N_i)$ | Low | −4.158 | −1.247 | −0.047 |
| | Medium | −4.792 | −2.079 | −0.075 |
| | High | −5.744 | −3.327 | −0.095 |
| 4 $(Z_i \not\!\perp\!\!\!\perp G_i \mid \mathbf{X}_i^{\mathrm{ind}}, \mathbf{X}_i^{\mathrm{neigh}})$ | Low | −9.504 | −1.414 | −1.415 |
| | Medium | −11.681 | −2.263 | −2.263 |
| | High | −13.132 | −2.829 | −2.825 |

Table 1: Computed bias for $\tau$

# Main Effects : Bias of Naive Estimators and GPS-Based Estimator

- Table 2 reports the mean bias and root mean squared error of six estimators in all scenarios.

| Scenario | Interference | Unadjusted | | Regression $\sim Z_i, \mathbf{X}_i^{\text{ind}}$ | | Subclass $\hat{\phi}(1, \mathbf{X}_i^{\text{ind}})$ | | Regression $\sim Z_i, \mathbf{X}_i^{z}$ | | Subclass $\hat{\phi}(1; \mathbf{X}_i^{z})$ | | Subclass $\hat{\phi}(1, \mathbf{X}_i^{z})$ and GPS | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Bias | RMSE | Bias | RMSE | Bias | RMSE | Bias | RMSE | Bias | RMSE | Bias | RMSE |
| 1 $(Z_i \perp\!\!\!\perp G_i \mid \mathbf{X}_i^{\text{ind}})$ | Low | −5.104 | 5.104 | −3.429 | 3.431 | −0.016 | 0.130 | −3.254 | 3.256 | 0.006 | 0.074 | −0.002 | 0.064 |
| | Medium | −5.278 | 5.278 | −3.529 | 3.532 | −0.021 | 0.149 | −3.256 | 3.258 | 0.007 | 0.090 | 0.002 | 0.067 |
| | High | −5.396 | 5.396 | −3.598 | 3.601 | −0.021 | 0.167 | −3.258 | 3.261 | 0.015 | 0.108 | 0.003 | 0.067 |
| 2 $(Z_i \perp\!\!\!\perp G_i \mid \mathbf{X}_i^{\text{ind}}, \mathbf{X}_i^{\text{neigh}})$ | Low | −5.136 | 5.136 | −3.477 | 3.477 | −1.652 | 1.668 | −2.156 | 2.157 | −0.066 | 0.331 | −0.000 | 0.036 |
| | Medium | −5.884 | 5.884 | −4.521 | 4.521 | −2.620 | 2.631 | −2.318 | 2.319 | −0.097 | 0.511 | 0.003 | 0.039 |
| | High | −6.384 | 6.384 | −5.222 | 5.223 | −3.272 | 3.281 | −2.430 | 2.432 | −0.123 | 0.635 | −0.002 | 0.036 |
| 3 $(Z_i \perp\!\!\!\perp G_i \mid \mathbf{X}_i^{\text{ind}}, N_i)$ | Low | −4.106 | 4.107 | −2.079 | 2.089 | −1.146 | 1.149 | −1.033 | 1.049 | 0.078 | 0.553 | −0.002 | 0.230 |
| | Medium | −4.725 | 4.726 | −2.886 | 2.893 | −1.911 | 1.913 | −1.041 | 1.056 | 0.082 | 0.556 | 0.019 | 0.233 |
| | High | −5.655 | 5.656 | −4.098 | 4.103 | −3.058 | 3.059 | −1.054 | 1.068 | 0.084 | 0.559 | 0.012 | 0.234 |
| 4 $(Z_i \not\perp\!\!\!\perp G_i \mid \mathbf{X}_i^{\text{ind}}, \mathbf{X}_i^{\text{neigh}})$ | Low | −8.670 | 8.670 | −7.104 | 7.104 | −1.443 | 1.445 | −7.076 | 7.076 | −1.738 | 1.740 | 0.000 | 0.105 |
| | Medium | −10.761 | 10.761 | −8.861 | 8.862 | −2.309 | 2.309 | −8.824 | 8.823 | −2.784 | 2.785 | −0.003 | 0.095 |
| | High | −12.154 | 12.154 | −10.030 | 10.030 | −2.883 | 2.884 | −9.985 | 9.985 | −3.477 | 3.478 | 0.010 | 0.090 |

Table 2: Estimation of main effect $\tau$

# Spillover Effects

- Table 3 and Table 4 report the mean bias and root mean squared error of all estimators for spillover effects $\Delta(0)$ and $\Delta(1)$.

| Scenario | Interference | Unadjusted regression $\sim Z_i, G_i$ | | Subclass $\hat{\phi}(1, \mathbf{X}_i^Z)$ | | GPS | | Subclass $\hat{\phi}(1, \mathbf{X}_i^Z)$ and GPS | |
|---|---|---|---|---|---|---|---|---|---|
| | | Bias | RMSE | Bias | RMSE | Bias | RMSE | Bias | RMSE |
| 1 $(Z_i \perp\!\!\!\perp G_i \mid \mathbf{X}_i^{\text{ind}})$ | Low | 1.576 | 1.581 | 2.871 | 2.901 | −1.457 | 1.465 | 0.003 | 0.091 |
| | Medium | 1.568 | 1.574 | 2.838 | 2.872 | −1.462 | 1.472 | 0.005 | 0.114 |
| | High | 1.571 | 1.580 | 2.878 | 2.910 | −1.467 | 1.479 | 0.000 | 0.148 |
| 2 $(Z_i \perp\!\!\!\perp G_i \mid \mathbf{X}_i^{\text{ind}}, \mathbf{X}_i^{\text{neigh}})$ | Low | 3.503 | 3.506 | 4.132 | 5.259 | −0.574 | 0.590 | 0.033 | 0.098 |
| | Medium | 3.506 | 3.510 | 4.037 | 4.933 | −0.579 | 0.599 | 0.041 | 0.118 |
| | High | 3.485 | 3.489 | 4.204 | 5.232 | −0.592 | 0.613 | 0.034 | 0.129 |
| 3 $(Z_i \perp\!\!\!\perp G_i \mid \mathbf{X}_i^{\text{ind}}, N_i)$ | Low | 5.445 | 5.446 | 7.005 | 7.048 | 0.380 | 0.399 | −0.035 | 0.091 |
| | Medium | 5.455 | 5.456 | 7.009 | 7.050 | 0.381 | 0.401 | −0.033 | 0.091 |
| | High | 5.441 | 5.442 | 7.054 | 7.096 | 0.381 | 0.400 | −0.033 | 0.099 |
| 3 $(Z_i \not\perp\!\!\!\perp G_i \mid \mathbf{X}_i^Z, \mathbf{X}_i^{\text{neigh}})$ | Low | 3.002 | 3.002 | 2.577 | 2.584 | −1.201 | 1.202 | 0.071 | 0.111 |
| | Medium | 3.002 | 3.002 | 2.556 | 2.563 | −1.202 | 1.203 | 0.068 | 0.111 |
| | High | 3.005 | 3.005 | 2.548 | 2.555 | −1.200 | 1.201 | 0.066 | 0.111 |

Table 3: Estimation of $\Delta(0)$

# Spillover Effects

| Scenario | Interference | Unadjusted regression $\sim Z_i, G_i$ Bias | RMSE | Subclass $\hat{\phi}(1, \mathbf{X}_i^z)$ Bias | RMSE | GPS Bias | RMSE | Subclass $\hat{\phi}(1, \mathbf{X}_i^z)$ and GPS Bias | RMSE |
|---|---|---|---|---|---|---|---|---|---|
| 1 $(Z_i \perp\!\!\!\perp G_i \mid \mathbf{X}_i^{\text{ind}})$ | Low | 3.195 | 3.196 | 2.870 | 2.878 | 0.432 | 0.441 | 0.003 | 0.055 |
| | Medium | 3.197 | 3.198 | 2.863 | 2.871 | 0.437 | 0.449 | 0.002 | 0.083 |
| | High | 3.198 | 3.200 | 2.869 | 2.877 | 0.434 | 0.452 | 0.004 | 0.103 |
| 2 $(Z_i \perp\!\!\!\perp G_i \mid \mathbf{X}_i^{\text{ind}}, \mathbf{X}_i^{\text{neigh}})$ | Low | 2.483 | 2.485 | 2.842 | 2.862 | 0.386 | 0.393 | 0.001 | 0.059 |
| | Medium | 2.481 | 2.485 | 2.885 | 2.907 | 0.385 | 0.394 | −0.000 | 0.072 |
| | High | 2.473 | 2.477 | 2.882 | 2.919 | 0.379 | 0.391 | −0.005 | 0.088 |
| 3 $(Z_i \perp\!\!\!\perp G_i \mid \mathbf{X}_i^{\text{ind}}, N_i)$ | Low | 3.668 | 3.669 | 5.921 | 5.951 | 0.559 | 0.562 | −0.038 | 0.045 |
| | Medium | 3.668 | 3.668 | 5.944 | 5.975 | 0.558 | 0.561 | −0.040 | 0.046 |
| | High | 3.669 | 3.669 | 5.943 | 5.982 | 0.558 | 0.561 | −0.038 | 0.048 |
| 3 $(Z_i \not\perp\!\!\!\perp G_i \mid \mathbf{X}_i^{z}, \mathbf{X}_i^{\text{neigh}})$ | Low | −3.280 | 3.280 | −5.182 | 5.182 | 0.367 | 0.370 | −0.019 | 0.056 |
| | Medium | −3.277 | −3.277 | −5.182 | 5.182 | 0.368 | 0.371 | −0.021 | 0.057 |
| | High | −3.281 | 3.281 | −5.178 | 5.178 | 0.364 | 0.367 | −0.026 | 0.059 |

Table 4: Estimation of $\Delta(1)$

# Spillover Effects

- Figure 2 depicts the scatterplot of the observed outcomes and the estimated ADRFs $\mu(0, g)$ and $\mu(1, g)$ , $g \in \{0, 0.2, 0.4, 0.6, 0.8, 1\}$ ,for Scenario 2

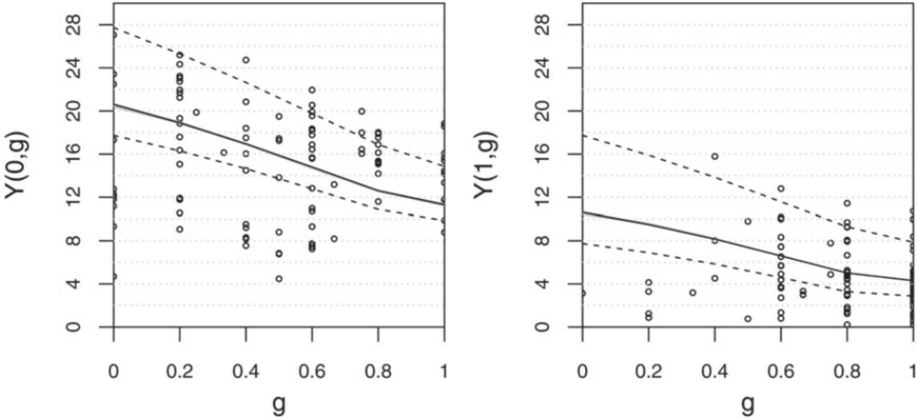


Figure 2: Estimated dose-response function $\mu(0, g)$ and $\mu(1, g)$

# Spillover Effects

- Figure 3 depicts spillover effects $\delta(0, g)$ i.e $\mu(0, g) - \mu(0, 0)$ and spillover effects $\delta(1, g)$ i.e $\mu(1, g) - \mu(1, 0)$

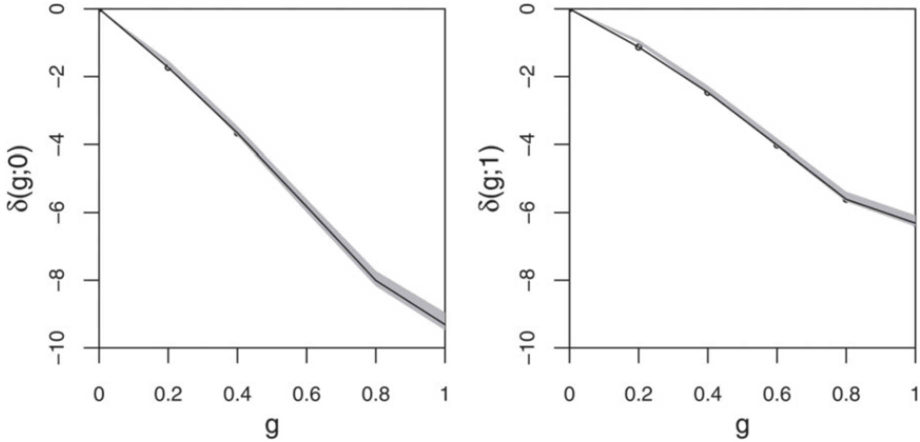


Figure 3: Estimated spillover effects $\delta(g; z)$

# Spillover Effects

- Figure 4 depicts estimated main effects $\tau(g)$ . i.e $\mu(1, g) - \mu(0, g)$ .

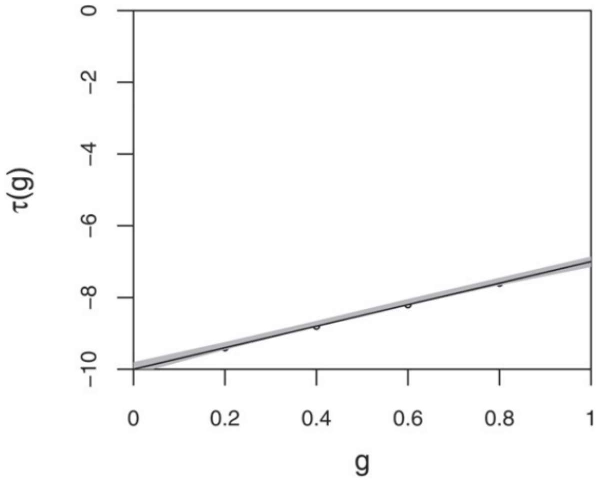


Figure 4: Estimated main effects $\tau(g)$

# V . Conclusion

# Concluding Remarks

- SUTVA $\longrightarrow$ SUTNVA , potential outcome $Y_i(\mathbf{Z}_i, \mathbf{G}_i)$
- simple assignment mechanism $\longrightarrow$ compound assignment mechanism
- naive estimator $\longrightarrow$ Subclassfication and GPS estimator
- Bias sources : $\mathbf{Z}_{\mathcal{N}_i}$, $P(Z_{i,res}, G_{i,res}|\mathbf{X}_i^\star)$
- limitations : network fully known and fixed , model dependent
- future : sensitivity analysis , Bayesian semi-parametric approaches , account for network uncertainty

# Software

- networkinference - Python package by Michael P. Leung
- CausalModel - Python package by Qu, Zhaonan and Xiong, Ruoxuan and Liu, Jizhou and Imbens, Guido
- inferference - R package by Bradley Saul
- clusteredinterference - R package by Brian G. Barkley
- Interference - R package by Georgia Papadogeorgou

Thank You !