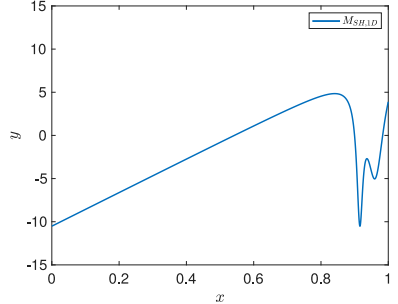
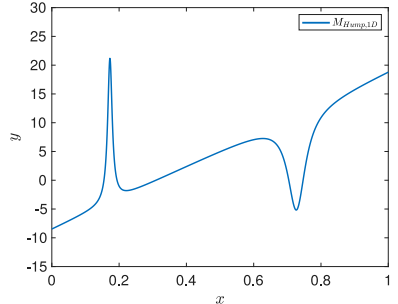
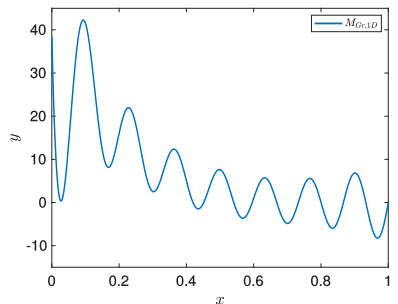
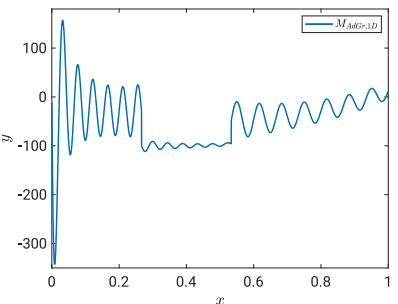


Provided benchmark tests of this library

New users may read the provided readme-file. The following benchmark tests can be called inside the library.

One-dimensional benchmark tests

Test number	Function	Visualization over the normalized parametric domain
P1	for $x \in [-1.5, 5]$, $\mathcal{M}_{SH,1D}(x) = 3x - \frac{0.05}{(x - 4.75)^2 + 0.04} - \frac{0.07}{(x - 4.45)^2 + 0.005} - 6,$	
P2	for $x \in [-0.5, 5]$, $\mathcal{M}_{Hump,1D}(x) = 5x + \frac{0.05}{(x - 0.45)^2 + 0.002} - \frac{0.5}{(x - 3.5)^2 + 0.03} - 6$	
P3	for $x \in [-1.5, 1.0]$, $\mathcal{M}_{Gr,1D}(x) = \frac{60 \sin(6\pi x)}{2 \cos(x)} + (x - 1)^4$	
P4	for $x \in [-1.5, 6.0]$, $\mathcal{M}_{AdGr,1D}(x) = \begin{cases} t_1(x) & \text{if } x \in]0.5, 2.5] \\ t_2(x) & \text{if } x > 2.5 \\ t_3(x) & \text{else} \end{cases}$	

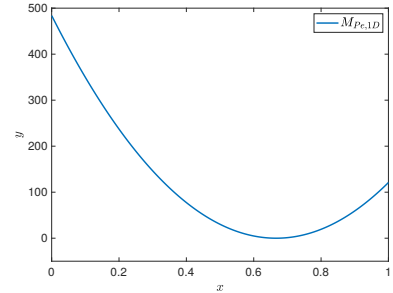
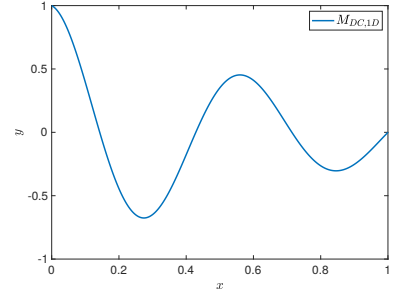
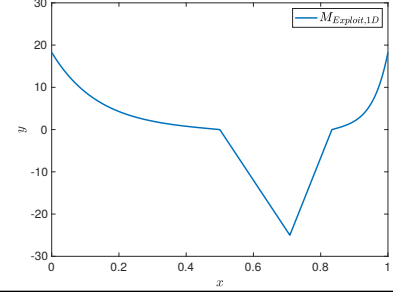
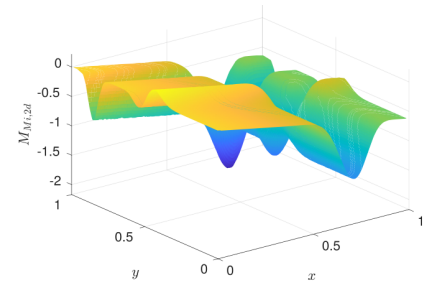
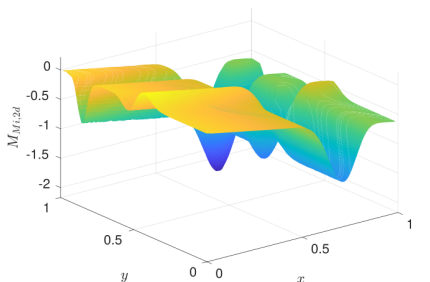
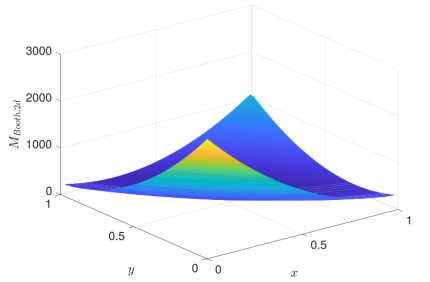
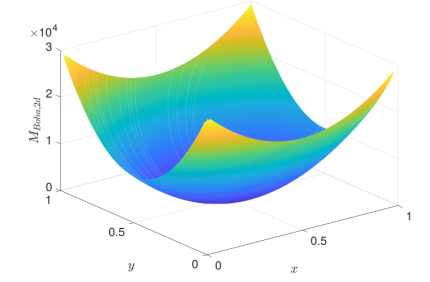
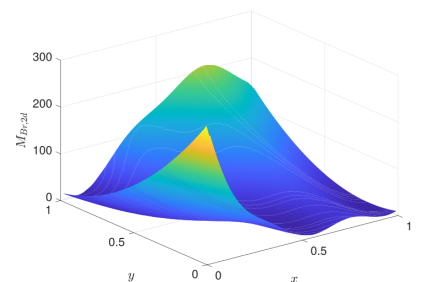
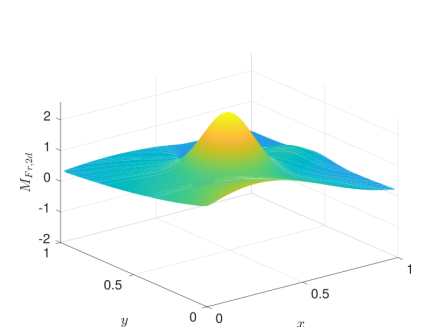
P5	<p>for $x \in [-1, 1]$,</p> $\mathcal{M}_{Pe,1D}(x) = (11(x - 1))^2$	
P6	<p>for $x \in [0, 1]$,</p> $\mathcal{M}_{DC,1D}(x) = \exp\{-1.4x\} \cos(3.5\pi x)$	
P7	<p>for $x \in [-12, 12]$,</p> $\mathcal{M}_{Exploit,1D}(x) = \begin{cases} -\sinh(0.3x), & \text{if } x < 0 \\ -5x, & \text{if } 0 \leq x < 5 \\ (25/3)x - (200/3), & \text{if } 5 \leq x < 8 \\ \sinh(0.9(x - 8)), & \text{else} \end{cases}$	

Table 1 – One-dimensional benchmark functions (**P1** to **P7**)

Two-dimensional benchmark tests

Test number	Function	Visualization over the normalized parametric domain
P8	<p>for $(x_1, x_2) \in [0, \pi]^2$,</p> $\mathcal{M}_{Mi,2D}(\mathbf{x}) = -\sum_{i=1}^3 \sin(x_i) \sin^{20}\left(\frac{ix_i^2}{\pi}\right)$	

P9	<p>for $(x_1, x_2) \in [-0.6, 0.9]^2$,</p> $\mathcal{M}_{DW,2D}(\mathbf{x}) = -\frac{1 + \cos\left(12\sqrt{x_1^2 + x_2^2}\right)}{0.5(x_1^2 + x_2^2) + 2}$	
P10	<p>for $(x_1, x_2) \in [-10.0, 10.0]^2$,</p> $\mathcal{M}_{Booth,2D}(\mathbf{x}) = (x_1 + 2x_2 - 7)^2 + (2x_1 + x_2 - 5)^2$	
P11	<p>for $(x_1, x_2) \in [-100.0, 100.0]^2$,</p> $\mathcal{M}_{Boha,2D}(\mathbf{x}) = x_1^2 + 2x_2^2 - 0.3 \cos(3\pi x_1) - 0.4 \cos(4\pi x_2) + 0.7$	
P12	<p>for $(x_1, x_2) \in [-5.0, 10.0] \times [0.0, 15.0]$,</p> $\mathcal{M}_{Br,2D}(\mathbf{x}) = \left(x_2 - \frac{5.1}{\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6\right)^2 + 10\left(1 - \frac{1}{8\pi}\right)\cos(x_1) + 10$	
P13	<p>for $(x_1, x_2) \in [-0.5, 1.0]^2$,</p> $\begin{aligned} \mathcal{M}_{Fr,2D}(\mathbf{x}) = & 1.875 \exp\left(-\frac{(9x_1 - 2)^2}{4} - \frac{(9x_2 - 2)^2}{4}\right) \\ & + 1.125 \exp\left(-\frac{(9x_1 + 1)^2}{49} - \frac{9x_2 + 1}{10}\right) \\ & + 0.5 \exp\left(-\frac{(9x_1 - 7)^2}{4} - \frac{(9x_2 - 3)^2}{4}\right) \\ & - 1.6 \exp(-(9x_1 - 4)^2 - (9x_2 - 7)^2) \end{aligned}$	

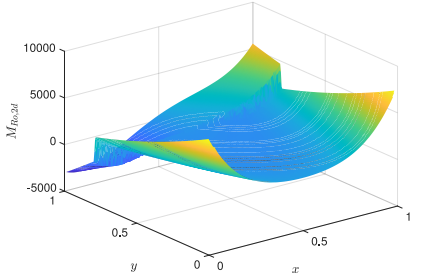
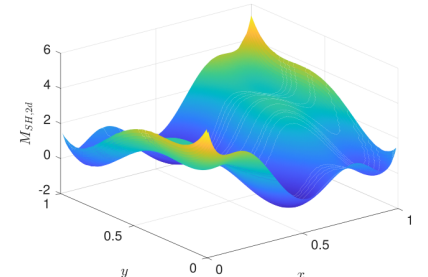
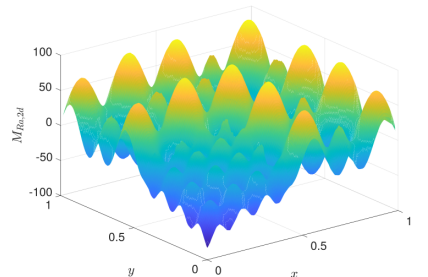
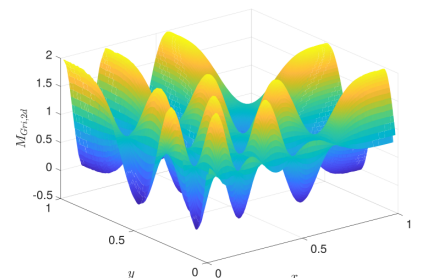
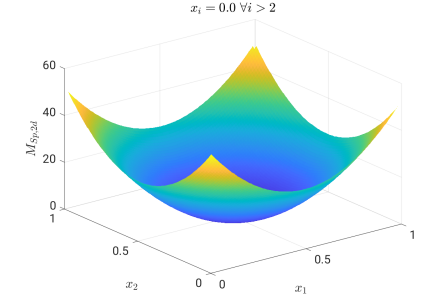
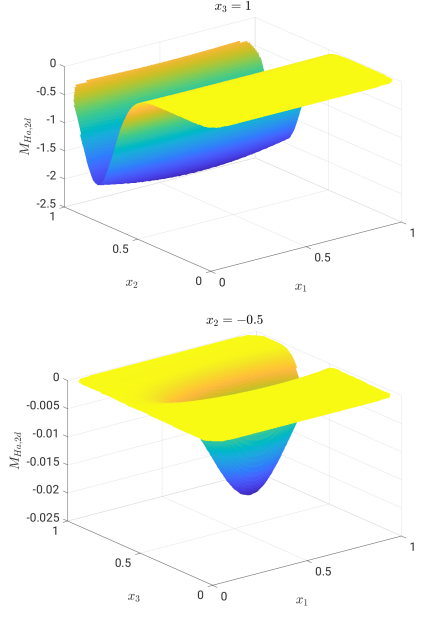
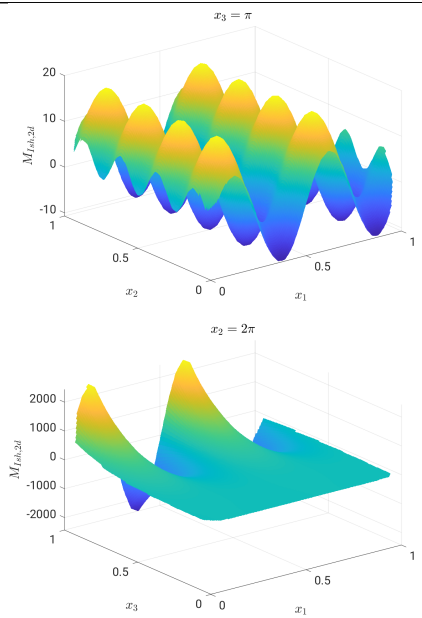
P14	<p>for $(x_1, x_2) \in [-2.5, 2.5]^2$,</p> $\mathcal{M}_{Ro,2D}(\mathbf{x}) = 100 (x_2 - x_1^2)^2 + (x_1 - 1)^2$ $+ \begin{cases} (x_1 - 1)^2 + 700x_1x_2, & \text{if } x_2 > 1.5 \\ (x_1 - 1)^2, & \text{else} \end{cases}$	 <p>A 3D surface plot of the function $M_{Ro,2D}$ over the domain $x, y \in [-2.5, 2.5]$. The vertical axis ranges from -5000 to 10000. The surface shows a complex landscape with a prominent peak near the center and a deep valley along one edge.</p>
P15	<p>for $(x_1, x_2) \in [-2.0, 2.0] \times [-1.0, 1.0]$,</p> $\mathcal{M}_{SH,2D}(\mathbf{x}) = \left(4 - 2.1x_1^2 + \frac{x_1^4}{3}\right)x_1^2$ $+ x_1x_2 + (-4 + 4x_2^2)x_2^2$	 <p>A 3D surface plot of the function $M_{SH,2D}$ over the domain $x, y \in [-2.0, 2.0] \times [-1.0, 1.0]$. The vertical axis ranges from -2 to 6. The surface is smooth and shows a central peak with a saddle-like structure.</p>
P16	<p>for $(x_1, x_2) \in [-6.0, 2.0]^2$,</p> $\mathcal{M}_{Ras,2D}(\mathbf{x}) =$ $\begin{cases} \sum_{i=1}^2 0.2x_i^3 - 10 \cos(2\pi x_i), & \text{if } x_1 < -2.5 \text{ \& } x_2 < -2.5 \\ \sum_{i=1}^2 0.2x_i^3 + 3 x_i - 30 \sin(\pi x_i), & \text{else} \end{cases}$	 <p>A 3D surface plot of the function $M_{Ras,2D}$ over the domain $x, y \in [-6.0, 2.0]^2$. The vertical axis ranges from -100 to 100. The surface is highly oscillatory with many sharp peaks and valleys.</p>
P17	<p>for $(x_1, x_2) \in [-4.0, 0.0]^2$,</p> $\mathcal{M}_{Gri,2D}(\mathbf{x}) = \sum_{i=1}^2 \frac{1}{4000} x_i^3 - \prod_{i=1}^2 \cos\left(\frac{x_i^2}{\sqrt{i}}\right) - 1$	 <p>A 3D surface plot of the function $M_{Gri,2D}$ over the domain $x, y \in [-4.0, 0.0]^2$. The vertical axis ranges from -0.5 to 2. The surface shows a series of sharp, periodic peaks and valleys.</p>

Table 2 – Two-dimensional benchmark tests (**P8** to **P17**)

Higher-dimensional benchmark tests

Test number	Function	Visualization over the normalized parametric domain
P18	<p>for $(x_1, \dots, x_3) \in [-5.12, 5.12\pi]^3$,</p> $\mathcal{M}_{Sp,3D}(\mathbf{x}) = \sum_{i=1}^3 x_i^2$	
P19	<p>for $(x_1, x_2, x_3) \in [-1, 1]^3$</p> $\mathcal{M}_{Ha,3D}(\mathbf{x}) = -\sum_{i=1}^4 \alpha_i \exp\left(-\sum_{j=1}^3 A_{ij}(x_j - P_{ij})^2\right),$ <p>where</p> $\boldsymbol{\alpha} = (1.0, 1.2, 3.0, 3.2)^T$ $\mathbf{A} = \begin{bmatrix} 3.0 & 10 & 30 \\ 0.1 & 10 & 35 \\ 3.0 & 10 & 30 \\ 0.1 & 10 & 35 \end{bmatrix}$ $\mathbf{P} = 10^{-4} \begin{bmatrix} 3689 & 1170 & 2637 \\ 4699 & 4387 & 7470 \\ 1091 & 8732 & 5547 \\ 381 & 5743 & 8828 \end{bmatrix}$	
P20	<p>for $(x_1, x_2, x_3) \in [0, 4\pi]^3$,</p> $\mathcal{M}_{Ish,3D}(\mathbf{x}) = \sin(x_1) + 7 \sin^2(x_2) + 0.05 x_3^3 \sin(x_1),$	

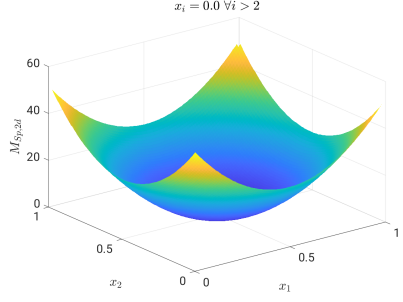
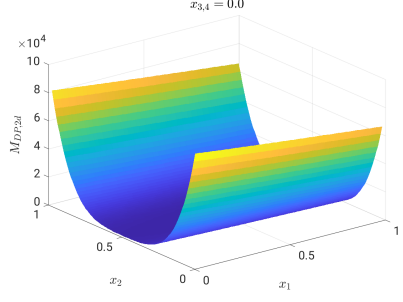
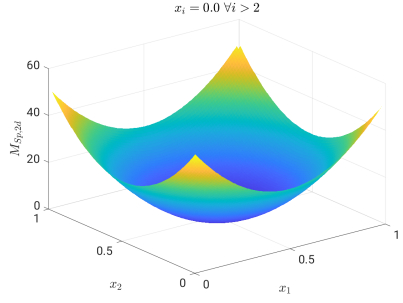
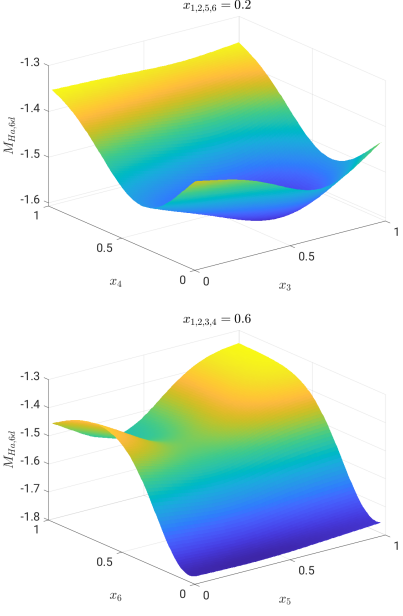
P21	<p>for $(x_1, \dots, x_4) \in [-5.12, 5.12\pi]^4$,</p> $\mathcal{M}_{Sp,4D}(\mathbf{x}) = \sum_{i=1}^4 x_i^2$	 <p>$x_i = 0.0 \forall i > 2$</p>
P22	<p>for $(x_1, x_2, x_3, x_4) \in [-10, 10\pi]^4$,</p> $\mathcal{M}_{DP,4D}(\mathbf{x}) = (x_1 - 1)^2 + \sum_{i=2}^4 i (2x_i^2 - x_{i-1})^2$	 <p>$x_{3,4} = 0.0$</p>
P23	<p>for $(x_1, \dots, x_5) \in [-5.12, 5.12\pi]^5$,</p> $\mathcal{M}_{Sp,5D}(\mathbf{x}) = \sum_{i=1}^5 x_i^2$	 <p>$x_i = 0.0 \forall i > 2$</p>
P24	<p>for $(x_1, x_2, x_3, x_4) \in [0, 1]^6$,</p> $\mathcal{M}_{Ha,6D}(\mathbf{x}) = - \sum_{i=1}^4 \alpha_i \exp \left(- \sum_{j=1}^6 A_{ij} (x_j - P_{ij})^2 \right),$ <p>where</p> $\boldsymbol{\alpha} = (1.0, 1.2, 3.0, 3.2)^T$ $\mathbf{A} = \begin{bmatrix} 10 & 3 & 17 & 3.5 & 1.7 & 8 \\ 0.05 & 10 & 17 & 0.1 & 8 & 14 \\ 3 & 3.5 & 1.7 & 10 & 17 & 8 \\ 17 & 8 & 0.05 & 10 & 0.1 & 14 \end{bmatrix}$ $\mathbf{P} = 10^{-4} \begin{bmatrix} 1312 & 1696 & 5569 & 124 & 8283 & 5886 \\ 2329 & 4135 & 8307 & 3736 & 1004 & 9991 \\ 2348 & 1451 & 3522 & 2883 & 3047 & 6650 \\ 4047 & 8828 & 8732 & 5743 & 1091 & 381 \end{bmatrix}$	 <p>$x_{1,2,5,6} = 0.2$</p> <p>$x_{1,2,3,4} = 0.6$</p>

Table 3 – Higher-dimensional benchmark tests (**P18** to **P24**)