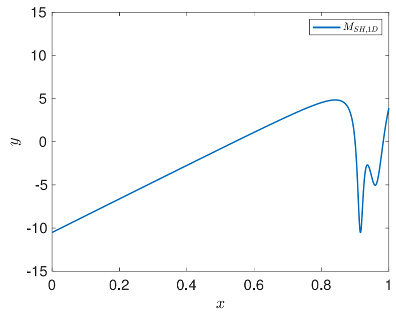
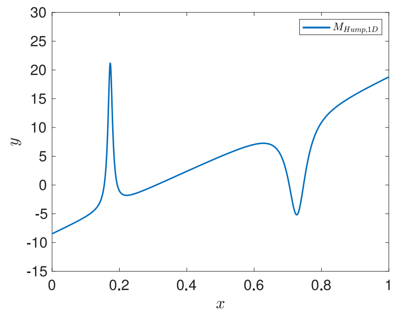
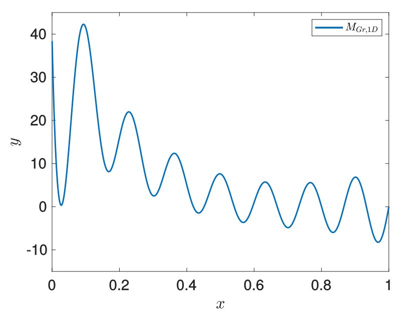
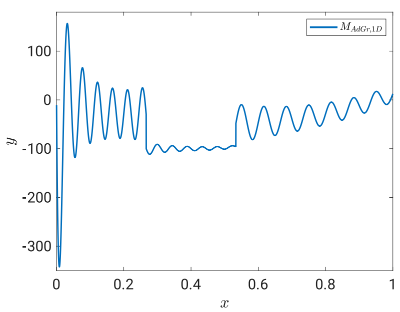
Provided benchmark tests of this library

New users may read the provided readme-file. The following benchmark tests can be called inside the library.

One-dimensional benchmark tests

Test number	Function	Visualization over the normalized parametric domain
P1	for $x \in [-1.5, 5]$, $\mathcal{M}_{SH,1D}(x) = 3x - \frac{0.05}{(x-4.75)^2+0.04} - \frac{0.07}{(x-4.45)^2+0.005} - 6,$	
P2	for $x \in [-0.5, 5]$, $\mathcal{M}_{Hump,1D}(x) = 5x + \frac{0.05}{(x-0.45)^2+0.002} - \frac{0.5}{(x-3.5)^2+0.03} - 6$	
P3	for $x \in [-1.5, 1.0]$, $\mathcal{M}_{Gr,1D}(\boldsymbol{x}) = \frac{60\sin(6\pi x)}{2\cos(x)} + (x-1)^4$	
P4	for $x \in [-1.5, 6.0]$, $\mathcal{M}_{AdGr,1D}(\boldsymbol{x}) = \begin{cases} t_1(\boldsymbol{x}) & \text{if } x \in]0.5, 2.5] \\ t_2(\boldsymbol{x}) & \text{if } x > 2.5 \\ t_3(\boldsymbol{x}) & \text{else} \end{cases}$	

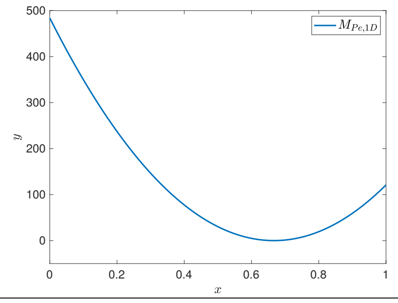
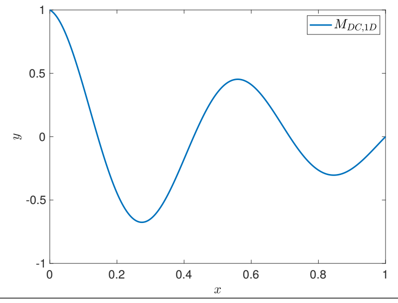
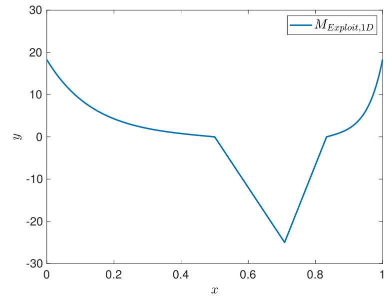
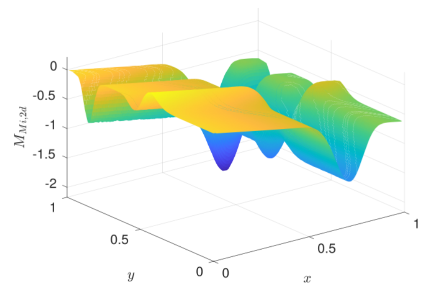
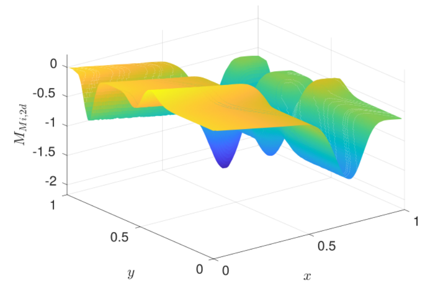
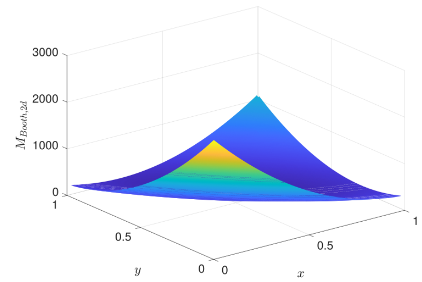
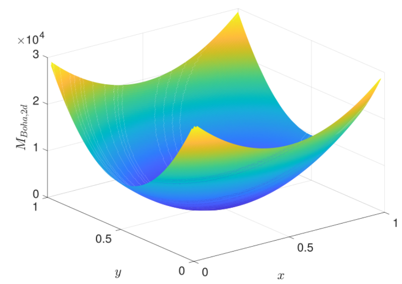
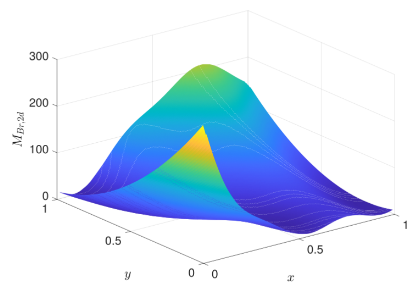
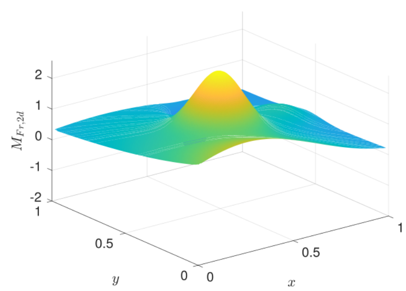
P5	for $x \in [-1,1]$, $\mathcal{M}_{Pe,1D}(x) = (11(x-1))^2$	
P6	for $x \in [0,1]$, $\mathcal{M}_{DC,1D}(x) \quad = \exp\{-1.4x\}\cos(3.5\pi x)$	
P7	for $x \in [-12,12]$, $\mathcal{M}_{Exploit,1D}(x) = \begin{cases} -\sinh(0.3x), & \text{if } x < 0 \\ -5x, & \text{if } 0 \leq x < 5 \\ (25/3)x - (200/3), & \text{if } 5 \leq x < 8 \\ \sinh(0.9(x-8)), & \text{else} \end{cases}$	

Table 1 – One-dimensional benchmark functions (**P1** to **P7**)

Two-dimensional benchmark tests

Test number	Function	Visualization over the normalized parametric domain
P8	for $(x_1, x_2) \in [0,\pi]^2$, $\mathcal{M}_{Mi,2D}(\boldsymbol{x}) = -\sum_{i=1}^{3} \sin(x_i)\sin^{20}\left(\frac{ix_i^2}{\pi}\right)$	

P9	for $(x_1, x_2) \in [-0.6, 0.9]^2$, $\mathcal{M}_{DW,2D}(\boldsymbol{x}) = -\dfrac{1+\cos\left(12\sqrt{x_1^2+x_2^2}\right)}{0.5(x_1^2+x_2^2)+2}$	
P10	for $(x_1, x_2) \in [-10.0, 10.0]^2$, $\mathcal{M}_{Booth,2D}(\boldsymbol{x}) = (x_1 + 2x_2 - 7)^2 + (2x_1 + x_2 - 5)^2$	
P11	for $(x_1, x_2) \in [-100.0, 100.0]^2$, $\mathcal{M}_{Boha,2D}(\boldsymbol{x}) = x_1^2 + 2x_2^2 - 0.3\cos(3\pi x_1)$ $\qquad - 0.4\cos(4\pi x_2) + 0.7$	
P12	for $(x_1, x_2) \in [-5.0, 10.0] \times [0.0, 15.0]$, $\mathcal{M}_{Br,2D}(\boldsymbol{x}) = \left(x_2 - \dfrac{5.1}{\pi^2}x_1^2 + \dfrac{5}{\pi}x_1 - 6\right)^2$ $\qquad + 10\left(1 - \dfrac{1}{8\pi}\right)\cos(x_1) + 10$	
P13	for $(x_1, x_2) \in [-0.5, 1.0]^2$, $\mathcal{M}_{Fr,2D}(\boldsymbol{x}) = 1.875\exp\left(-\dfrac{(9x_1-2)^2}{4} - \dfrac{(9x_2-2)^2}{4}\right)$ $\qquad + 1.125\exp\left(-\dfrac{(9x_1+1)^2}{49} - \dfrac{9x_2+1}{10}\right)$ $\qquad + 0.5\exp\left(-\dfrac{(9x_1-7)^2}{4} - \dfrac{(9x_2-3)^2}{4}\right)$ $\qquad - 1.6\exp\left(-(9x_1-4)^2 - (9x_2-7)^2\right)$	

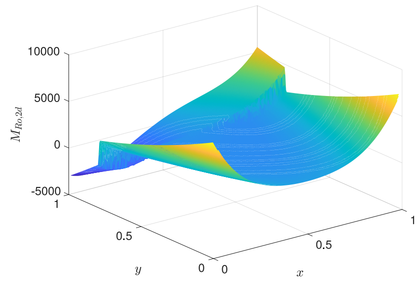
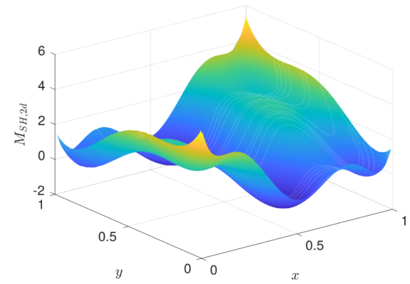
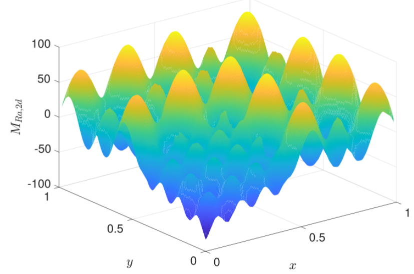
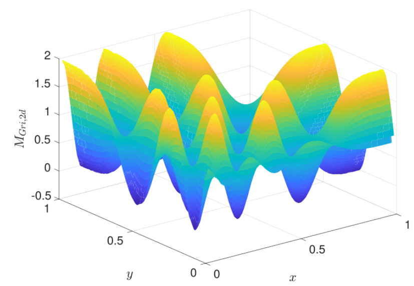
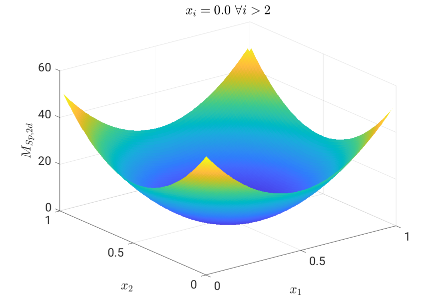
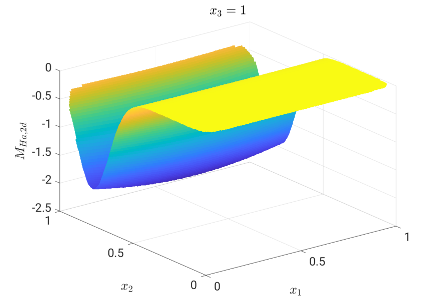
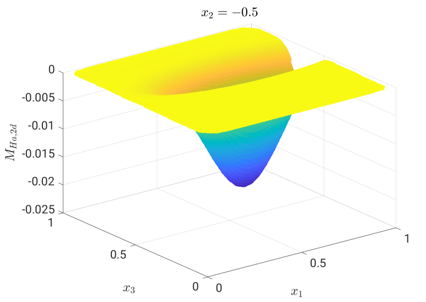
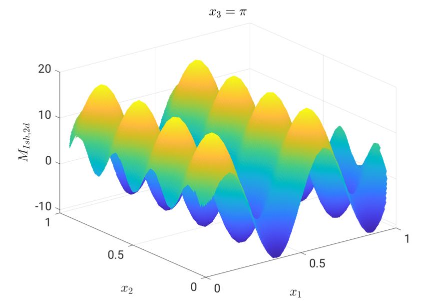
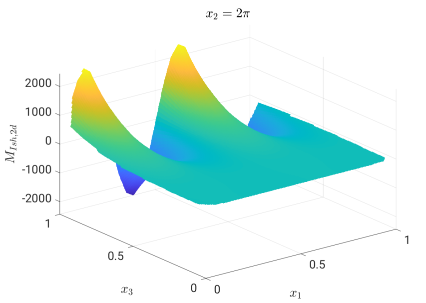
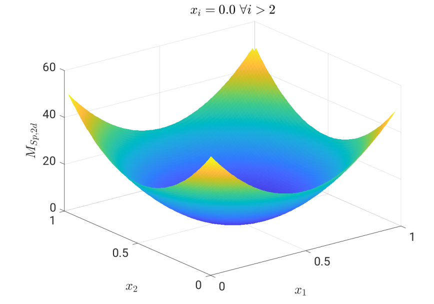
P14	for $(x_1, x_2) \in [-2.5, 2.5]^2$, $\mathcal{M}_{Ro,2D}(\boldsymbol{x}) = 100\left(x_2 - x_1^2\right)^2 + \left(x_1 - 1\right)^2 + \begin{cases} (x_1-1)^2 + 700 x_1 x_2, & \text{if}\, x_2 > 1.5 \\ (x_1-1)^2, & \text{else} \end{cases}$	
P15	for $(x_1, x_2) \in [-2.0, 2.0] \times [-1.0, 1.0]$, $\mathcal{M}_{SH,2D}(\boldsymbol{x}) = \left(4 - 2.1x_1^2 + \frac{x_1^4}{3}\right) x_1^2 + x_1 x_2 + \left(-4 + 4x_2^2\right) x_2^2$	
P16	for $(x_1, x_2) \in [-6.0, 2.0]^2$, $\mathcal{M}_{Ras,2D}(\boldsymbol{x}) = \begin{cases} \sum_{i=1}^{2} 0.2x_i^3 - 10\cos(2\pi x_i), \text{if}\, x_1 < -2.5 \,\&\, x_2 < -2.5 \\ \sum_{i=1}^{2} 0.2x_i^3 + 3\lvert x_i\rvert - 30\sin(\pi\lvert x_i\rvert), \text{else} \end{cases}$	
P17	for $(x_1, x_2) \in [-4.0, 0.0]^2$, $\mathcal{M}_{Gri,2D}(\boldsymbol{x}) = \sum_{i=1}^{2} \frac{1}{4000} x_i^3 - \prod_{i=1}^{2} \cos\left(\frac{x_i^2}{\sqrt{i}}\right) - 1$	

Table 2 – Two-dimensional benchmark tests (**P8** to **P17**)

Higher-dimensional benchmark tests

Test number	Function	Visualization over the normalized parametric domain
P18	for $(x_1,\ldots,x_3) \in [-5.12, 5.12\pi]^3$, $$\mathcal{M}_{Sp,3D}(\boldsymbol{x}) = \sum_{i=1}^{3} x_i^2$$	
P19	for $(x_1,x_2,x_3) \in [-1,1]^3$ $$\mathcal{M}_{Ha,3D}(\boldsymbol{x}) = -\sum_{i=1}^{4} \alpha_i \exp\left(-\sum_{j=1}^{3} A_{ij}(x_j - P_{ij})^2\right),$$ where $\boldsymbol{\alpha} = (1.0, 1.2, 3.0, 3.2)^T$ $$\boldsymbol{A} = \begin{bmatrix} 3.0 & 10 & 30 \\ 0.1 & 10 & 35 \\ 3.0 & 10 & 30 \\ 0.1 & 10 & 35 \end{bmatrix}$$ $$\boldsymbol{P} = 10^{-4} \begin{bmatrix} 3689 & 1170 & 2637 \\ 4699 & 4387 & 7470 \\ 1091 & 8732 & 5547 \\ 381 & 5743 & 8828 \end{bmatrix}$$	 
P20	for $(x_1,x_2,x_3) \in [0, 4\pi]^3$, $\mathcal{M}_{Ish,3D}(\boldsymbol{x}) = \sin(x_1) + 7\sin^{2(x_2)} + 0.05 x_3^3 \sin(x_1)$,	 

<table>
<tr>
<td>P21</td>
<td>for $(x_1, \dots, x_4) \in [-5.12, 5.12\pi]^4$,
$$\mathcal{M}_{Sp,4D}(\boldsymbol{x}) = \sum_{i=1}^{4} x_i^2$$</td>
<td>
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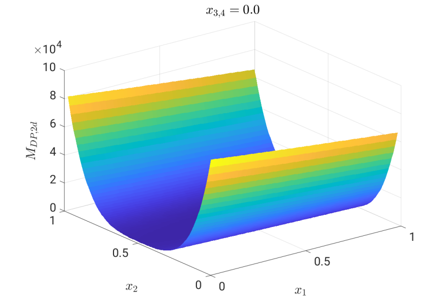
<td>P22</td>
<td>for $(x_1, x_2, x_3, x_4) \in [-10, 10\pi]^4$,
$$\mathcal{M}_{DP,4D}(\boldsymbol{x}) = (x_1 - 1)^2 + \sum_{i=2}^{4} i\left(2x_i^2 - x_{i-1}\right)^2$$</td>
<td>
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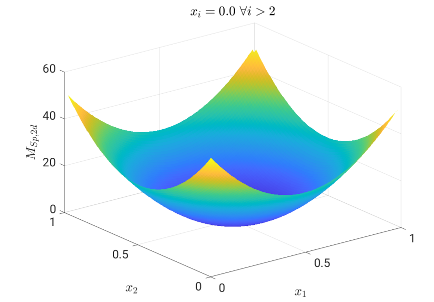
<td>P23</td>
<td>for $(x_1, \dots, x_5) \in [-5.12, 5.12\pi]^5$,
$$\mathcal{M}_{Sp,5D}(\boldsymbol{x}) = \sum_{i=1}^{5} x_i^2$$</td>
<td>
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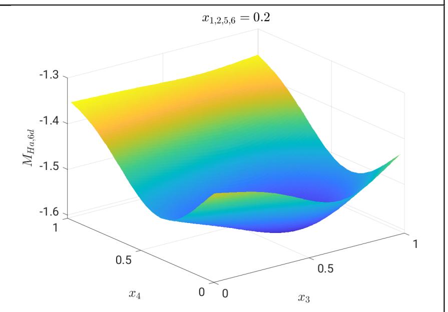
<td>P24</td>
<td>for $(x_1, x_2, x_3, x_4) \in [0, 1]^6$,
$$\mathcal{M}_{Ha,6D}(\boldsymbol{x}) = -\sum_{i=1}^{4} \alpha_i \exp\left(-\sum_{j=1}^{6} A_{ij}(x_j - P_{ij})^2\right),$$
where
$$\boldsymbol{\alpha} = (1.0, 1.2, 3.0, 3.2)^T$$
$$\boldsymbol{A} = \begin{bmatrix} 10 & 3 & 17 & 3.5 & 1.7 & 8 \\ 0.05 & 10 & 17 & 0.1 & 8 & 14 \\ 3 & 3.5 & 1.7 & 10 & 17 & 8 \\ 17 & 8 & 0.05 & 10 & 0.1 & 14 \end{bmatrix}$$
$$\boldsymbol{P} = 10^{-4} \begin{bmatrix} 1312 & 1696 & 5569 & 124 & 8283 & 5886 \\ 2329 & 4135 & 8307 & 3736 & 1004 & 9991 \\ 2348 & 1451 & 3522 & 2883 & 3047 & 6650 \\ 4047 & 8828 & 8732 & 5743 & 1091 & 381 \end{bmatrix}$$</td>
<td>

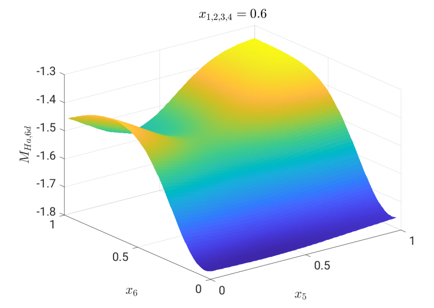

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Table 3 – Higher-dimensional benchmark tests (**P18** to **P24**)