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REGULAR ARTICLE

The norm game: punishing enemies and not friends

K. Kułakowski

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Abstract Three mean field models of the norm game are explored analytically. The strategies are: to obey the norm or not and to punish those who break it or not. The punishment, the temptation, the anger and the punishment cost are modeled by four parameters; for the fixed points, only two ratios of these parameters are relevant. For each model, we consider its variant with two mutually punishing groups. We show that all solutions are the same as for the case in one group. This means in particular, that in both groups the amount of defectors is the same.

Keywords Norm game · Mean field · Phase diagram

1 Introduction

The game theory went to the statistical physics (Szabó and Fáth 2007) as a strong mathematical tool of economy (von Neumann and Morgenstern 1944; Schelling 1960), biology (Hofbauer and Sigmund 1998) and social sciences (Axelrod 1997). Initially, it relied on the assumption of rational choice. This condition was later released by the concept of bounded rationality (Simon 1982) and adaptive rather than rational thinking; this should not be understood as 'less logic', but rather 'logic with limited information'. In the never-ending discussion on the applicability of mathematical tools to describe the human behavior, this adaptive thinking is a keyword. Indeed, in many cases the human behavior cannot be explained within the frames of individual rationality. If a society persists longer than the lifetime of its members, it develops ways to induce cooperation and altruism in further generations; examples of

K. Kułakowski (✉)
Faculty of Physics and Applied Computer Science,
AGH University of Science and Technology, al. Mickiewicza 30, 30059 Kraków, Poland
e-mail: kulakowski@novell.ftj.agh.edu.pl

individual sacrifices constitute tradition and serve as social norms. As it was formulated by Elias J. Bickerman, “The first need of any social system is to create incentives to make people do more work than that required by their immediate wants” (Garraty and Gay 1972).

The norm game was introduced by Axelrod (1986) together with a metanorm game, where punishing for non-punishing was included. Axelrod presented some results of the computer simulations, which were later questioned (Galan and Izquierdo 2005) On the other hand, recently reported results on the norm game (Hauert et al. 2007) relied on a definite sets of values of the model parameters. It is then difficult to evaluate to what extent these results are generic. This obstacle is even more painful when we realize that it concerns also the dependence of the results on a particular model, which is many cases is chosen arbitrarily. For a theoretically motivated sociophysicist, the remedy seems to be as follows: build a model as simple as possible; control its assumptions; check how the results depend on the parameters; complications can be introduced only step by step. This procedure should at least make the model clear to those who deal with real data; it is also in harmony with warnings, heard from the professional side of sociology (Firebaugh 2008). On the other hand, the arguments given in our first paragraph suggest that maybe the frames of the game theory are too narrow to describe the social entanglement of individual decisions. Between the ‘get as much as you can’ and the invisible hand of long lasting evolution there is a whole spectrum of motivations. They lead to actions which are less rational than those in economy, but more rational from an individual or social point of view than the narrow set of behaviors of animals. Variable external conditions and individual experience play a role there, but their outcome is not fully determined by the expected payoffs. To resolve the puzzle, one should take into account local traditions, norms and social roles. A sociophysicist cannot talk much about that, then here we treat these aspects as a black box. Our input is the initial distribution of the probabilities of the given strategies; our output is the time dynamics of these probabilities. This dynamics is governed by the fundamental or Master equations; this choice seems to be free from the conceptual limitations of the game theory. To maintain the continuity with the problem set by Axelrod, we keep the term “the norm game”. Punishing of those who do not punish is not considered here.

The aim of this text is to investigate the possibility of an emergence of two mutually punishing groups. This problem is basic for the theory of reference groups, where norms are formulated, expressed and maintained by means of social interactions (Newcomb 1943). According to this theory people decide how to behave having in minds norms of the group where they belong (Cancian 1975). In other words, it is the social group and not a single human being what should be considered as the subject in the problem of norms. Further, any conflict which is not of a purely psychological origin of a single mind has its roots in differences between social norms of two or more social groups. As examples, we can list ethnic clashes in all scales, deep cultural differences, economically based riots, territorial and religious wars and many others (Horowitz 1985; Huntington 1997; O’Meara 2001). All these conflicts are present in the contemporary world. More than often, mutual hostility between the groups leads to the norm of punishing members of an ‘enemy group’ for what is allowed for ‘our group’. For example, punishing can be directed against the merely presence of an 'enemy group' member in the disputed—real or symbolic—area. A search to understand instabilities which could appear in a seemingly homogeneous society is then worthwhile.

Up to our knowledge, the problem of the norm game of two mutually punishing groups was discussed only recently in Prietula and Conway (2007). There, two groups of different sizes have been considered. Obviously, members of the stronger group could punish those from the weaker group more efficiently. There are many examples in history, from Helots and Spartans in ancient Greece to Hutu and Tutsi in Rwanda today, when one group dominates other, and the difference between them is well established. Here, we wonder if the symetry between groups can be broken spontaneously.

Our method here is to simplify the model description of the norm game as much as possible, with a minimal number of parameters. In particular, we reduce chains of reasonings to simple sequences of input and output. For example, an agent is expected to cease punishment if the cost of punishment is large. We introduce only one parameter to describe how the act of punishment reduces the probability of punishing; doing this, we omit the information of the payoffs. In this way, the system dynamics can be expressed by the population equations, what is simpler than the commonly used replication equations (Hofbauer and Sigmund 1998). The purpose of this method is twofold: (i) to reduce the number of parameters. As we demonstrate below, the stable solutions depend only on two parameters. This allows for a complete description of all possible solutions in the form of a phase diagram. (ii) to make the model free from the assumption on the rational choice. We note in passim that the measurement of the payoffs is a weak point of the utility theory (von Neumann and Morgenstern 1944).

Below we present some variants of the model, constructed within these frames. For each variant, we consider its application to the case of two mutually punishing groups. The number of equations and the number of variables are doubled, but in most cases the system cannot be separated into two independent subsystems. It is natural to expect that the number of solutions can be doubled as well. The goal of this work is to prove, that in the case of two groups new solutions do not appear. In particular, the probabilities of breaking the norm and of punishing are the same for one group and for two groups. This result means that the domination of one group over another one cannot appear spontaneously. To dominate, a group has to punish or to evade punishment more efficiently than the other group. More generally, the parameters of the processes involved must be different in different groups. Otherwise the symmetry is preserved and the domination cannot be reached.

2 Those who break the norm can also punish

General frames of this model are the same as all models discussed here. The starting points is the Master equation for breaking the norm and for punishing for it. What is specific in the first model is that we treat these states as statistically independent. This means in particular, that the amount of those who punish among those who obey the norm and those who break it is the same. Intuitively, we could expect that those who break the norm do not punish and the opposite. Policemen who drive drunk provide counterexamples; however, this case cannot be treated as general one. We start from this model to have a reference point.

Let us denote the probability of breaking the norm as z. The time dependence of z is a solution of the Master equation

$$\frac{dz}{dt} = a(1-z) - bzy \tag{1}$$

where y is the probability of punishing, a is the rate of increase of z per an agent because of the gain which we get when we break the norm and b is the opposite rate because of the inhibiting results of the punishment. We can term these rates as 'temptation' and 'punishment'. The latter term is proportional to y, because a punisher is necessary here. Similarly, the time evolution of y is controlled by

$$\frac{dy}{dt} = -cyf(z) + ez(1-y) \tag{2}$$

where the first term on the r.h.s. is responsible for a reduction of y due to the cost of punishing, and the second—for an increase of y because of growing vengeance—the anger, that the norm breaking remains non-punished.

In some respects the result of this anger term is the same as the meta-punishing in the Axelrod model: an increase of the probability of punishment. However, the origin if these two agents are different. While in the Axelrod model the mechanism is the social attitude to punish those who do not punish, the mechanism in Eq. 2 is the natural rate of the probability of punishment, which must be proportional to the number of those who do not punish.

To fix the timescale, we keep $e = 1$ from now on; if we do not, c should be substituted by c/e. It is not clear, if the cost term should depend on z or not. Does the willingness to punish decrease in the absence of those who break the norm? Having no clear answer, we consider two versions of the model: (A1) where $f(z) = z$, (A2) where $f(z) = 1$.

These versions are two limit cases: (A1) means that the decrease of the attitude to punish is proportional to the frequency of events when the norm is broken, and (A2) means that this decrease does not depend on this frequency. The version (A1) seems more plausible. However, if nobody defects for a long time, the norm itself can be less vivid; on the contrary, the notion about punishing can evolve for different reasons, as discussed for example in Dreber et al. (2008); summarizing, it seems reasonable to expect that once our conclusion is true both for (A1) and (A2), it is true also for other cases which are less convenient to be investigated analytically.

In the case (A1) we get an unique stable solution $z(t) = z^*$, $y(t) = y^*$, where

$$z^* = \left(1 + \frac{\phi}{1+c}\right)^{-1} \tag{3}$$

$$y^* = (1+c)^{-1} \tag{4}$$

where $\phi = b/a$. Then the proportion of the observed behaviour of breaking the norm and punishing is $z^* y^*$, the proportion of breaking the norm and not punishing is $z^*(1 - y^*)$ and so on. Note that $z^* + y^* \neq 1$, and in general $z + y \neq 1$.

In the case (A2) we have

$$y^* = \frac{-(1+c) + \sqrt{(1+c)^2 + 4\phi c}}{2\phi c} \tag{5}$$

$$z^* = (1 + \phi y^*)^{-1} \tag{6}$$

The latter equation is true also in the case (A1). If we compare the results of two models (A1) and (A2), we see that they do not differ much. Some graphics are shown in our unpublished preprint, which was the starting point of this work (Kułakowski 2009).

Let us now consider the case when there are two mutually punishing groups r and s. In each of two models (A1) and (A2) there are four equations instead of two, but they are split into two independent pairs. For example, in the model (A1) we have

$$\frac{dz_i}{dt} = a(1 - z_i) - bz_i y_{3-i} \tag{7}$$

$$\frac{dy_i}{dt} = -cy_i f(z_{3-i}) + z_{3-i}(1 - y_i) \tag{8}$$

where $i = 1, 2$. In this model, those who punish in one group do not contact with those who break the norm in the same group. The differences between the solutions can appear if the values of the parameters are different. For example, a reduction of punishing constant b of one group by the other—but not the opposite—can lead to a difference between the solutions z_1 and z_2. If the parameters a, b, c are the same for both groups, such a difference cannot be observed.

3 Only those who obey the norm can punish

Now we are going to discuss another model, a simplified version of the one presented in Hauert et al. (2007), where the roles of defectors and punishers are separated. The simplification is that here we do not discuss the role of loners, and we use the population equations instead of the Moran dynamics (Hauert et al. 2007; Lieberman et al. 2005). The analogy is that in the both formulations the transition probabilities are defined in a Markov process between the homogeneous states of the population.

Main difference between this and the previous formulations (A1, A2) is that now we do not allow to break the norm and to punish simultaneously. Then, now there are three possible strategies: (i) to obey the norm and do not punish, (ii) to obey the norm and punish and (iii) to break the norm and not punish. The probabilities of observing these strategies will be denoted as x, y and z, respectively. Now the normalization condition is $x + y + z = 1$. With this condition we can limit the evolution equations to those for y and z.

Here again we have two models (B1) and (B2), what is due to the same opportunity. The equations of motion are

$$\frac{dz}{dt} = ax - bzy \tag{9}$$

$$\frac{dy}{dt} = -cyf(z) + xz \tag{10}$$

where as before $f(z) = z$ in (B1) or $f(z) = 1$ in (B2). We note that in this model we do not allow to switch directly from y to z or back. This means that all the processes which contribute to changes of y or z do contribute to changes of x.

There are two aspects of this limitation. First is a technical one: to take into account the rates between y and z, we should introduce two new constants to describe the probability flows between these states. Such a generalization crosses with our method to limit the parameter space, as declared in the Introduction. The second aspect is our intention to capture two opposite cases: when there is no correlations between defecting and punishing (A1 and A2), and when these states are possibly far one from another in human mind (B1 and B2). The justification of the latter versions could be that it is perhaps strange to switch directly from the group of punishers to the group of defectors or back; one needs some time to preserve its identity and pass a kind of initiation or a kind of resocialization. However, here again we believe that to prove our result in two opposite cases makes this result more reliable in all intermediate cases between these two.

The solutions show a bifurcation. In the case (B1), a new fixed point appears: $(x^*, y^*, z^*) = (0, 0, 1)$. This means that everybody breaks the norm and nobody punishes. This solution exists in the whole space (a, b, c), but it is stable if and only if $\phi < c$, i.e. if the punishment constant is small enough. There is also another fixed point $(x^*, y^*, z^*) = (0, 1, 0)$, but it is never stable. Third solution is

$$z^* = c/\phi \tag{11}$$

$$y^* = \frac{1 - c/\phi}{1 + c} \tag{12}$$

and it is stable if and only if $\phi > c$. Moreover, out of this range the probability y^* happens to be negative; then this solution is meaningless. However, this bifurcation in the two-dimensional space (z, y) is close to what is termed 'transcritical bifurcation' in the case of one variable (Glendinning 1994).

In the case (B2), the fixed point $(x^*, y^*, z^*) = (0, 0, 1)$ appears again and it is stable in the same range as in (B1), i.e. when $\phi < c$. The fixed point $(0, 1, 0)$ does not exist. The other solution is

$$z^* = \sqrt{c/\phi} \tag{13}$$

$$y^* = \frac{1 - \sqrt{c/\phi}}{1 + \sqrt{c\phi}} \tag{14}$$

and it is stable and meaningful again if and only if $\phi > c$. Comparing the fixed points coordinates in (B1) and (B2) as dependent on the constant c, we see that the largest difference is between the curves of z^*; for (B1) it is linear, and for (B2)—the square root (Kułakowski 2009). Above $c = \phi$, $y^* = 0$ and $z^* = 1$ in both (B1) and (B2).

The case of two mutually punishing groups is described by the equations

$$\frac{dz_i}{dt} = ax_i - bz_i y_{3-i} \tag{15}$$

$$\frac{dy_i}{dt} = -cy_i f(z_{3-i}) + x_i z_{3-i} \tag{16}$$

where the normalization conditions are $x_i + y_i + z_i = 1$ for $i = 1, 2$. On the contrary to the models (A1, A2) punishers do contact with the rest of their group. However, algebraic manipulations show that the fixed points are the same as for the case of one group: $x_1^* = x_2^*$, $y_1^* = y_2^*$, $z_1^* = z_2^*$. For B1 ($f(z) = z$) assuming $z_i^* > 0$ and having eliminated $x_i^* = cy_i^*$ we get $y_i^* = (1 - z_i^*)/(1 + c)$ from the normalization. Further, $cy_i^* = \phi z_i^* y_{3-i}^*$; then $z_1^* z_2^* = (c/\phi)^2$ and $c(1 - z_i^*) = \phi z_i^*(1 - z_{3-i}^*)$. Substituting the former to the latter, we have the same linear equation for z_1^* and z_2^*. This unique solution must reproduce the existing solution for one group. For B2 ($f(z) = 1$) substituting $x_i^* = \phi z_i^* y_{3-i}^*$ to $cy_i^* = x_i^* z_{3-i}^*$ we get $y_1^* = y_2^* \equiv y^*$ and $z_1^* z_2^* = c/\phi$. Then, again from the normalization, $1 - z_i^* - y^* = \phi z_i^* y^*$, what gives $z_1^* = z_2^*$. Assuming $z_1^* = 1$ we get $x_1^* = 0$ from the normalization; then $y_2^* = 0$ and $x_2^* = 0$ from the conditions that at the fixed point $dz_i/dt = dy_i/dt = 0$, and $z_2^* = 1$ again from the normalization. A similar chain of consequences gives $y_2^* = 1$ from $y_1^* = 1$.

The stability of the fixed points is to be checked from the eigenvalues of the Jacobian 4×4. In (B1) and at the fixed point $(0, 1, 0)$ the eigenvalues λ fulfil the condition $\lambda(\lambda + a + b) = \pm ac$; one solution is always positive. This means, that the fixed point is never stable. At the fixed point $(0, 0, 1)$ the secular equation is $(a + \lambda)(1 + c + \lambda) - a = \pm b$; all roots are negative if and only if $ac > b$, what coincides with the case of one group. The same is true also for (B2); the only difference is that the fixed point $(0, 1, 0)$ does not exist. It seems likely, that the fixed point given by Eqs. (11, 12) for (B1) and Eqs. (13, 14) for (B2) remains stable also for two groups, if $\phi > c$. This tentative conclusion is confirmed by our computer simulation; we have not observed any deviation from the stable fixed point. Then again, a difference in the positions of two groups can appear only if the parameters a, b or c are different.

4 Nonlinear anger term

It is justified to expect that our willingness to punish decreases in the case when the norm is broken by the majority. To include this effect to our model, we multiply the anger term in Eq. (16) by $1 - z_{3-i}$. As a consequence we observe a saddle-node bifurcation of the function $z^*(c)$. As before, we consider two models, C1 and C2. In the case of one group we observe a saddle-node bifurcation (Glendinning 1994). The bifurcation point appears at $c = \phi/4$, $z^* = 1/2$ for C1 and at $c = 4\phi/27$, $z^* = 2/3$ for C2. For higher c, only the solution $(x^*, y^*, z^*) = (0, 0, 1)$ is possible; this solution is stable for any c and ϕ. For lower c, i.e. below the bifurcation point, there are two new solutions, but only one with lower z^* is stable. The case was discussed more thoroughly in Kułakowski (2008). Here we are interested only in the case of two mutually punishing groups.

As above. we consider two models, C1 and C2, defined by the equations

$$\frac{dz_i}{dt} = ax_i - bz_i y_{3-i} \tag{17}$$

$$\frac{dy_i}{dt} = -cy_i f(z_{3-i}) + x_i z_{3-i}(1 - z_{3-i}) \tag{18}$$

For C1 we have $f(z) = z$. For $z_i^* \neq 0$ the conditions for the fixed point are

$$x_i^* = \phi z_i^* y_{3-i}^* \tag{19}$$

$$cy_i^* = x_i^*(1 - z_{3-i}^*) \tag{20}$$

Substituting y_i^* from the latter to the former, we get

$$cx_i^* = \phi z_i^*(1 - z_i^*)x_{3-i}^* \tag{21}$$

Once we look for the solution $x_i^* \neq 0$, this gives

$$z_1^* z_2^*(1 - z_1^*)(1 - z_2^*) = \frac{c^2}{\phi^2} \tag{22}$$

Also, from the normalization condition

$$x_i^* + \frac{1}{c}(1 - z_{3-i}^*)x_i^* + z_i^* = 1 \tag{23}$$

what gives direct dependences of x_i^* on z_1^* and z_2^*. Substituting these to Eq. (19) and using Eq. (20) we obtain

$$\frac{z_2^*(1 - z_1^*)}{z_1^*(1 - z_2^*)} = \left[\frac{c + 1 - z_2^*}{c + 1 - z_1^*}\right]^2 \tag{24}$$

The left side of this equation is larger than 1 if $z_2^* > z_1^*$, but then the right side is smaller than 1. Then $z_1^* = z_2^*$ is the only solution.

For C2 we have $f(z) = 1$. In this case we have no analytical proof and we rely on a numerical method. First we express x_i^*, y_i^* as functions of z_i; we get

$$x_i^* = \frac{c(1 - z_i^*)}{c + z_{3-i}^*(1 - z_{3-i}^*)} \tag{25}$$

$$y_i^* = \frac{(1 - z_i^*)z_{3-i}^*(1 - z_{3-i}^*)}{c + z_{3-i}^*(1 - z_{3-i}^*)} \tag{26}$$

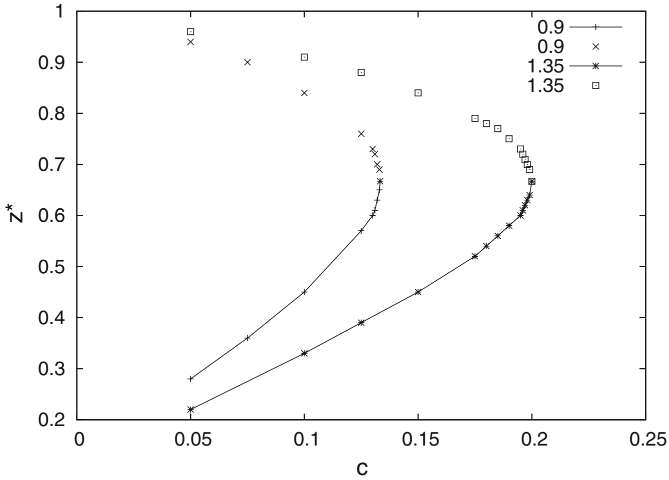

Fig. 1 Numerical results for the unstable (points) and stable (points with lines) fixed points within the model C2 for two groups, for $\phi = 0.9$ and 1.35, against the cost coefficient c. The analytical values of c where the bifurcation appears in the case of one group are $c = 0.13(3)$ and $c = 0.2$, respectively

Next we divide the area (0, 1) × (0, 1) into 10^{10} squares. We assign the values z_1, z_2 to the coordinates of each square and we check, if these z_i^*'s are the solutions of

$$cz_i^{*2}(1 - z_{3-i}) = \frac{\phi(c + z_i^*(1 - z_i^*))}{c + z_{3-i}^*(1 - z_{3-i}^*)} \tag{27}$$

within the numerical accuracy 10^{-4}. The only results are again $z_1^* = z_2^*$. The obtained saddle-node bifurcation point agrees with the analytical evaluation $\phi = 27c/4$, obtained within the model C2 for the case of one group (Kułakowski 2008). The results are shown in Fig. 1.

Both in C1 and C2, the condition $z_i^* = 1$ induces $z_{3-i}^* = 1$. This follows easily from the conditions of the fixed point and the normalization. In C1, $y_i^* = 1$ induces $y_{3-i}^* = 1$ as well. In C2, $y_i^* = 1$ is not a fixed point.

5 Conclusions

We have discussed three models A, B and C, each in two variants. The models (A1, A2) contain the assumption that there is no correlation between obeying or breaking the norm and punishing for breaking it. In the models (B1, B2) we assume that the correlation is strong: nobody who breaks the norm can punish. We expect that any real case is placed between these two extremes. The difference between (A1) and (A2) and between (B1) and (B2) is the same: the relaxation of the probability of punishing does (A1, B1) or does not (A2, B2) depend on the probability that the norm is broken. This difference appears to influence the stationary solution only quantitatively.

When we deal with the social reality, these quantitative details can be of minor importance. The difference between the models (A1, A2) and (B1, B2) is more serious; a separate phase appears where the probability of punishment is zero; on the contrary, everybody breaks the norm. This stationary solution appears if the punishment is not effective when compared to the gain for breaking the norm and the cost of punishing. The boundary of this phase in the space of parameters is the condition $b = ac$, as it was discussed in two previous sections. Then the assumption of the correlation between obeying or not the norm and punishing is important for the model results and it would be desirable to check it experimentally before any model is applied to a given norm. Finally, the difference between the models (B1, B2) and (C1, C2) is that in the latter case, the change of behaviour is discontinuous at the phase transition (Kułakowski 2008). Despite the differences, some results are the same in all models. These are: breaking the norm is more frequent when the cost of punishing increases; simultaneously, the punishment itself is less likely to be observed.

From the mathematical point of view, our result means what follows. In all versions of the model taking into account two groups leads to the doubling of the number of equations. Theoretically, the number of solutions could be doubled as well. However, our result is a proof that this number of solutions does not increase. In other words, the symmetry of the solutions is preserved. To consider non-symmetric colutions, one should introduce different coefficients in the equations for different groups. A similar trick was applied in Sigmund et al. (2001), where the 'bifurcation through reputation' was discussed.

Two patterns of behaviour: to break the norm and to punish, discussed above seem to have some organizing power, attracting collective actions of young men and adult professionals. Both need a cooperation. On the contrary, those who just obey norms but do not punish often prefer individual activities. In well-ruled countries there is no overlap between robbers and cops, at least in a social scale. In others, the power means the ability to break norms and evade being punished. There, the society is inhomogeneous: some obey norms, some do not, and the behaviour is correlated with a place in the social structure.

We made a step towards modeling this inhomogeneities, when discussing two mutually punishing groups. Our result is that the states of the groups are different only if their parameters are different. This fact is reflected in what is observed as the political reality, where political parties struggle to make their control and punishment from outside less effective, exposing simultaneously the events when norms are broken by members of rival parties. Our results suggest, that when the abilities of control and punishment are the same for both parties, the accepted rules of political behavior are obeyed by these parties to the same extent. In this way, our favourite point of view: 'our party is good, the other party is bad' seems to fail. Both sides are equally entangled in the system of mutual control and punishment, using media and/or the impartial prosecutor. We note that this conclusion is valid for the stationary state only, and it does not preclude the possibility that a new party is initially more honest; however, to get the symmetric equilibrium is only a question of time. Another example can be taken from the history of Sicilian families in America, where initially old rules have been observed but an increasing pressure of competing groups resulted in relentless private wars (Puzo 1969). Yet another example is provided by the history of fair play in rugby, where competing clubs gradually accept stronger and stronger faults (Sheard and Dunning 2004). Surely, the list is longer.

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