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REGULAR ARTICLE

Prospect theory and risk appetite: an application to traders' strategies in the financial market

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Abstract According to behavioral finance theories, in this article we develop a dynamic model with heterogeneous traders, where the asset price is determined by the interaction among four different groups of agents: trend reversers, trend followers, risk averters and risk seekers. The main purpose of the study is centered on modeling and testing how the market efficiency changes along with the changes of agent's behavior preference without exogenous influence. Combining with the assumption of risk appetite and prospect theory, focusing on analyzing the rules for selecting strategies, we establish a more reliable and comprehensive dynamic mechanism. In particular, our study suggests that diversified trading strategies will help to realize market efficiency.

Keywords Volatility clustering · Fat tail · Prospect theory · Risk appetite · Market efficiency

1 Introduction

Empirical studies on financial market have shown that the distribution of returns are leptokurtic and have fat tail. The autocorrelation function of return volatility is slowly decaying, which indicates long-term memory effect and market inefficiency. Agent—based models has been applied in many ways to describe the mechanism of such 'anomalies' in financial markets (Follmer 1974; Arthur et al. 1997; Brock and Hommes 1998;

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Farmer and Joshi 2002; Hommes 2002, 2006, to quote only a few). Most of them can reproduce stylized facts with mimic time series and capture desirable statistical properties. Arthur et al. (1997) has founded that when investor revise their expectation rules at a low speed, the market corresponds to efficient market hypothesis, when investors revise their forecast rules more faster, market will self-organize into a more complex pattern and present GARCH effect. Brock and Hommes (1997) utilize the concept of adaptively rational equilibrium to model the decision making process, where agents rational choice between various costly forecasts. Challet and Zhang (1997) propose minority game with agents who choose strategies repeatedly with inductive reasoning, constantly update the 'score' of their strategies and only use the most successful one to make prediction. Lux and Marchesi (1999) has implemented a model consisting of two types of traders, fundamentalists and several chartists. His model demonstrates main stylized facts of empirical asset returns, e.g. heteroscedastic returns with fat-tail and long-term memory in clustered volatility. Chiaralla et al. (2002) analyze the dynamics of asset price based on the fundamentalists-chartists constant relative risk aversion framework.

The present paper aims to develop a dynamic model to describe the relationship between market efficiency and agent's behavior preference without exogenous influence. As traditional dynamic models, we first separate investors into two groups: trend follower and trend reverser. Then combined with prospect theory Kahneman and Tverskey (1979) and risk appetite theory in behavioral finance, investors are risk-seeking over losses and risk-aversion over gains. So in our model, by analyzing agents' investing strategies within risk appetite and trend reverse/trend follower framework, we can examine how the investors' risk appetite and past trends affect the market efficiency. Anderson and Bollerslev (1997) present that the accuracy of volatility forecast can be improved if one moves to analyzing five- min returns. In this paper, we also compare the properties of volatility with different time scales to examine whether the frequency of data do result in different statistic properties.

The structure of the paper is as follows. Section 2 describes the general framework of the model. Section 3 presents simulation data and analytical results. Section 4 concludes.

2 The model

2.1 Basic assumption

We consider a $L \times L$ square lattice where each node i represents an agent. The total number of agent is N, which is fixed, $N = L \times L$. Each agent trades with only one unit of asset at a time. Transactions are not subjected to capital constraints. When the agents choose buying strategy, he has enough money to buy, and when he choose selling strategy, he has at least one unit of asset to sell. Information is fully shared and arrives synchronously; agents make decision at the same time. There is a market maker whose role is to clear the orders, increase the fluid and adjust prices. This was done to ensure that constraints on selling short and buying long were never violated. It also supposes that the market maker always has sufficient inventories to satisfy demand for both money and assets.

The initial state of market is derived randomly. At each step t, a given trader i chooses an action $s_i(t)$ from strategies set $S = \{+1, -1, 0\}$:

$$S_i = \left\{ \begin{array}{ll} +1, \quad if & \text{buy one unit of the asset} \\ 0, \quad if & \text{remain inactive} \\ -1 \quad if & \text{sell one unit of the asset} \end{array} \right\}. \tag{1}$$

N_t^B, N_t^S, N_t^I present the number of agents who choose buying, selling or inactive strategy at step t separately, where $N_t^B + N_t^S + N_t^I = N$. At each time t, agents will remark each strategy according to the past performance, $m_{i,j}(t)$ is the score of strategy j for agent $i$at time t, where $j \in S$.

$P_{i,j}(t)$ is the probability that the agent i will choose strategy j at time t, computed as

$$P_{i,j}\ (t) = \frac{m_{i,j}}{\sum_{j=+1,-1,0} m_{i,j}}. \tag{2}$$

2.2 Strategies

People's attitudes toward risk are divided into risk-aversion, risk-seeking. The utility function of risk aversion is generally assumed to be concave, while for risk seeking, the utility function is assumed to be convex. Kahneman and Tverskey (1979) describe how people make decision when they face the risks and gains. Their studies show that in the case of same expectations, people are more likely to choose the loss of uncertainty rather than face certain risk. This phenomenon is known as loss aversion. In prospect theory, loss aversion refers to people's tendency to strongly prefer avoiding losses to acquiring gains. This leads to risk aversion when people evaluate a possible gain; since people prefer avoiding losses to making gains. Conversely people strongly prefer risks that might possibly mitigate a loss, called risk seeking behavior. According to prospect theory, if an individual investor is risk aversion, he should be inclined to sell a stock that gains money, and if he is a risk seeker, he should be inclined to hold on to a stock that loses money.

This means that people's attitudes toward loss and gain are different; they prefer guaranteed gain and uncertainty of loss. Therefore, when asset prices decline, the market present in a downward trend, investors are more inclined to stop transaction, keep inactive, do nothing, they bet that asset prices will come back 1 day, which resulting in the volume shrinked in a bear market; and when asset price increase, the market present in an upward trend, investors are more active, trading volume is amplified. And empirical studies have shown that in general the trading volume of bull market is greater than that of bear market (Ying 1966; Karpoff 1987). Agents transact more frequently in an upward trend market than in a downward markets. In this model, it embodied as agents tend to choose active strategy of either buying or selling when market increase, and when the market decline, agents are more willing to choose inactive strategy.

Table 1 Mark functions for different types of agents

Type of agent	Marks
Risk aversion & trend reverser	$m_{i,+1} = \{[1 - s_i(t-1) * f(t) * \xi][1 - g(t)]\}$
	$m_{i,-1} = \{[1 + s_i(t-1) * f(t) * \xi][1 + g(t)]\}$
	$m_{i,0} = \{[1 + s_i(t-1) * f(t) * \xi][1 - g(t)]\}$
Risk aversion & trend follower	$m_{i,+1} = \{[1 - s_i(t-1) * f(t) * \xi][1 + g(t)]\}$
	$m_{i,-1} = \{[1 + s_i(t-1) * f(t) * \xi][1 - g(t)]\}$
	$m_{i,0} = \{[1 + s_i(t-1) * f(t) * \xi][1 - g(t)]\}$
Risk seeking & trend reverser	$m_{i,+1} = \{[1 + s_i(t-1) * f(t) * \eta][1 - g(t)]\}$
	$m_{i,-1} = \{[1 - s_i(t-1) * f(t) * \eta][1 + g(t)]\}$
	$m_{i,0} = \{[1 + s_i(t-1) * f(t) * \eta][1 - g(t)]\}$
Risk seeking & trend follower	$m_{i,+1} = \{[1 + s_i(t-1) * f(t) * \eta][1 + g(t)]\}$
	$m_{i,-1} = \{[1 - s_i(t-1) * f(t) * \eta][1 - g(t)]\}$
	$m_{i,0} = \{[1 + s_i(t-1) * f(t) * \eta][1 - g(t)]\}$

Let ω_1 is the weight of risk-seeking agents in the market, and $1 - \omega_1$ is the weight for risk averters. All the agents with different risk appetite will revise the score of each strategy based on the gain or loss from activity of last step, the degree of adjustment is ξ, η, where $0 < \xi < 1, 0 < \eta < 1$.

Define

$$f(t) = \begin{bmatrix} 1 & \text{if} & p(t) > p(t-1) \\ 0 & \text{if} & p(t) = p(t) \\ -1 & \text{if} & p(t) < p(t-1) \end{bmatrix}. \tag{3}$$

Then, agents are further separated into two parties: trend follower, weight is ω_2 and trend reverser with weight $1 - \omega_2$. Trend followers prefer follow with the trend that make profit last step, and trend reverser incline to choose the opposite strategy. Each agent will measure the trend with previous n periods and adjust with$g(t)$,

$$g(t) = \frac{\sum_{i=t-n}^{t-1} f(i)}{n}. \tag{4}$$

The mark functions for each type of agent are shown in Table 1.

2.3 Price and return

According to microeconomic theory, price was influenced by demand and supply. Price changes as the relative amount of demand and supply changes, and price is positive related to surplus demand. Price is also affected by the trading propensity of market. For stagnant market, the volatility of price is smaller than in high active market.

Authors (Hasbrouck 1991; Plerou et al. 2002) confirmed that the relationship between price and volume is a concave functional form. Many empirical studies show that the volatility of price can be strongly explained by trading volume in equity market

(Harris and Gurel 1986; Gallant et al. 1992; Jones et al. 1994; Lo and Wang 2000; Lo et al. 2004). Cornell (1981)'s research on futures market indicated that the volatility of price is positive related to trading volume. Girma and Mougoue (2002) examined the relationship between futures spreads volatility, volume and open interest, and find out trading volume has a considerable explanatory power of future spreads volatility. Then a large literature (Chan and Fong 2000; Chordia et al. 2002) has found that order imbalance is significantly related to contemporaneous market return.

Based on these researches and economic theory, we use the relative strength between buy and sell, the activity of market measured by trading volume as price determinant in our model. In our model, agent trades with market maker. At the end of each trading period, market maker adjusts the asset prices according to the relative buy and sell and the overall trading volume.

Trading volume is

$$V(t) = N_t^B + N_t^S. \tag{5}$$

Then price can be written as

$$p(t+1) = p(t)\left(\frac{N_t^B}{N_t^S}\right)^{\delta}, \quad \text{where} \quad \delta = a\frac{V(t)}{L^2}. \tag{6}$$

Return can be defined as:

$$r(t) = \log\frac{p(t)}{p(t-1)}. \tag{7}$$

Here, L^2 is the number of total agents, which is also the maximum number of asset that can be traded in each step. According to Sect. 2.1, the rules match the facts that market maker react asymmetrically to imbalanced orders placed in periods of high versus low activity in the market. This form is also consistent with the empirically observed positive correlation between absolute return and trading volume.

3 Simulations and results

The outcomes of the model vary as different values of the parameters have been inputed. We centered our analysis on the statistical properties of the probability distribution of asset returns with multi-scaling, the auto-correlation of market volatility, and how the risk attitude of investors affects the market. The total number of agents in this present work is 1,000, $\alpha = 2$.

3.1 Distribution of return with multi-scaling for neutral market

First, we assume that the market is neutral, where $\omega_1 = \omega_2 = 0.5$, adjust the value of ξ and η with different time interval (Table 2). From Table 2, we can notice that for the same degree of adjustment parameters ξ and η, the probabilities for accepting

Table 2 Statistical properties of multi-scaling returns for different time intervals

$\xi = \eta$	0.1				0.5				0.9			
Time interval	5	20	50	100	5	20	50	100	5	20	50	100
Median	0	0	0	0	0	0	0	0	0	0	0	0
Maximum	0.1026	0.0998	0.0989	0.0915	0.0858	0.0989	0.0923	0.0915	0.0783	0.0878	0.0794	0.0673
Minimum	−0.1060	−0.0980	−0.0998	−0.0850	−0.0959	−0.0970	−0.0831	−0.0859	−0.0755	−0.0747	−0.0999	−0.0756
Std. Dev.	0.0257	0.0251	0.0249	0.0247	0.0246	0.0279	0.0230	0.0227	0.0207	0.0178	0.0194	0.0171
Skewness	0.0026	−0.0284	0.0038	−0.0147	−0.0140	0.0102	0.0437	0.0149	0.0248	−0.0141	0.0100	−0.0067
Kurtosis	3.2052	3.0745	2.9988	3.0086	3.09365	2.9126	3.0333	2.9356	3.07732	3.1143	2.9845	3.0436
Jarque-Bera	26.3313	5.4811	0.0363	0.5877	29.5038	11.1021	2.7308	1.5758	33.6748	8.6586	0.3991	1.2985
Probability	0.0000	0.0645	0.9820	0.7454	0.0000	0.0039	0.2553	0.4548	0.0000	0.0132	0.8191	0.5224
Sum	−0.5494	−0.4405	−1.0347	−0.7333	−0.3597	1.1219	−1.4590	0.5042	0.3907	3.6740	0.7433	0.9015
Sum Sq. Dev.	9.9358	9.4775	9.2821	9.1447	4.5448	5.8532	3.9665	3.8672	6.4539	4.7443	5.6244	4.4096
Observations	15,000	15,000	15,000	15,000	15,000	15,000	15,000	15,000	15,000	15,000	15,000	15,000

Gaussian distribution null hypothesis by Jarque-Bera testing are obviously improved as the time interval increases. When investors adapt the marks all the strategies at a low rate (when $\xi = \eta = 0.1$), the market become efficient when take 50 and 100 periods interval. For short intervals (5 or 20 periods), the null hypothesis of Gaussian distribution for returns are refused, all the situations are far away from efficient markets. These indicate that for high-frequency data, the market is inefficient for short-term trading, but when the interval increases, the market become more efficient.

Then comparing the data with different value of ξ and η, it shows that as their values change, the statistical properties of return change slightly. We can infer that the results of our model are not affected too much by the adjustment parameters. This phenomenon indicates that the return distribution is not directly related to the inclination adjustment in short-term strategy.

3.2 Volatility of return

In our model, even in the absence of exogenous shock, there still rise large fluctuations. As shown in Fig. 1, by comparing the extent of volatility with the different time scales, it is obvious that as the time interval increases the fluctuation is amplified dramatically. We observed volatility clustering in all the cases. Volatility clustering indicates the presence of long-term memory effects in returns. By Quantile-Quantile analysis (Fig. 2), we can find that return series have fat tails compared to normal distribution, which reveals the presence of long-term memory. This result sheds light on how the volatility varies and arises. It also reminds us to further research on the true reason for long-term memory and persistent, whether it is caused by the difference of scale or by the structure of the data-generating itself.

3.3 Distribution of return for different composition of investors

ω_1 is the weight of risk-seeking agents in the market, ω_2 is the weight of trend reversers in the market. The p-values of return series for different value of $\omega_1$and ω_2 are shown in Table 3. Keeping ω_1 at a low rate as 0.1, which means 90% of agents are risk aversion, in this case, the market become efficient when the weights of trend reverser and trend follower are similar (where $\omega_2 = 0.4$ or 0.5). Changing the weight ω_2 between trend follower and trend reverser, we find that when one type of them dominate that market, the market become more unstable and inefficient (as $\omega_2 = 0.1$ or 0.9). From Table 3 we can find that when different types of the investors become balanced in the market, the p-values of returns for accepting the null hypothesis of Gaussian distribution are improved, which means the market becomes more efficient with diversified trading strategies.

4 Conclusions

We have exhibited an agent-based model of financial markets based on risk appetite and trend reverse/trend follower framework. This model can explain certain stylized

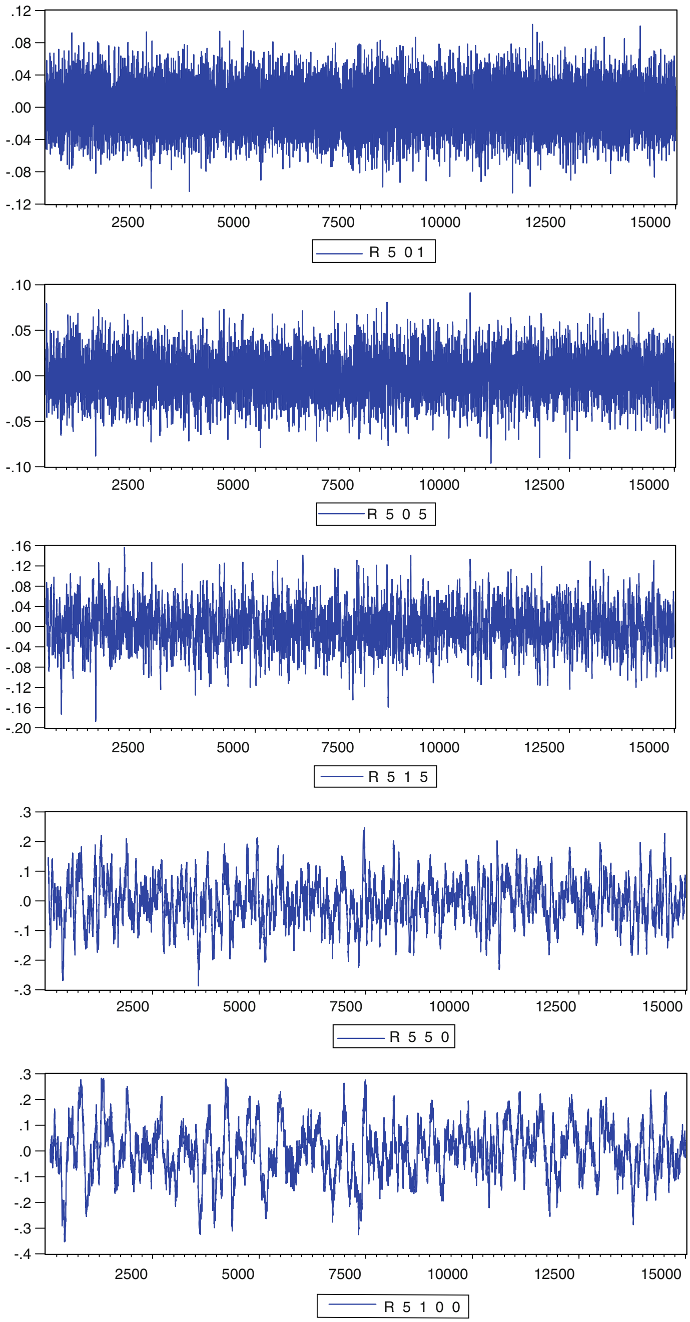

Fig. 1 Volatility of return for $\omega_1 = \omega_2 = 0.5, n = 5$, with different time scales 1 period, 5 periods, 15 periods, 50 periods and 100 periods (E.g. R5100 means $n = 5$, period interval = 100)

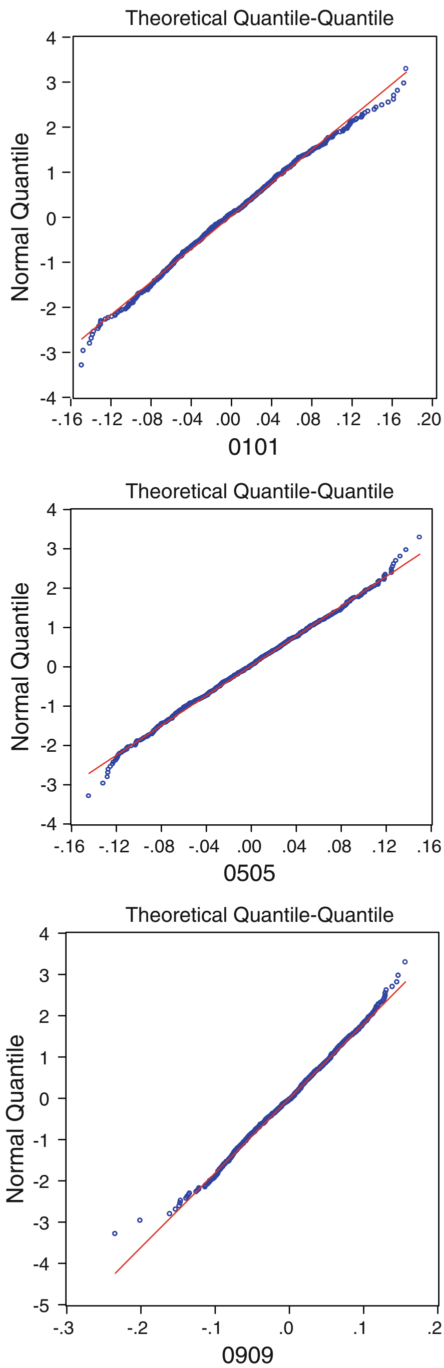

Fig. 2 Q-Q of return for $\omega_1 = \omega_2 = 0.1$, $\omega_1 = \omega_2 = 0.5$, $\omega_1 = \omega_2 = 0.9$ separately (E.g. for horizontal axis 0101 means $\omega_1 = \omega_2 = 0.1$)

Table 3 The p-values of returns for different composition of investors

w1	w2								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	0.029	0.104	0.155	0.850	0.933	0.406	0.441	0.207	0.305
0.2	0.448	0.161	0.234	0.854	0.121	0.321	0.421	0.025	0.004
0.3	0.456	0.278	0.572	0.048	0.745	0.737	0.100	0.713	0.160
0.4	0.185	0.521	0.333	0.764	0.876	0.349	0.932	0.341	0.214
0.5	0.355	0.854	0.835	0.087	0.586	0.178	0.777	0.577	0.097
0.6	0.249	0.017	0.071	0.508	0.675	0.270	0.969	0.373	0.120
0.7	0.197	0.015	0.235	0.381	0.462	0.662	0.157	0.091	0.120
0.8	0.102	0.284	0.184	0.841	0.327	0.653	0.772	0.373	0.104
0.9	0.399	0.995	0.144	0.550	0.301	0.626	0.864	0.524	0.323

facts of stock market return. Our model illustrates the fact that in the absence of exogenous shocks volatility clustering still exists. By examining the volatility clustering with varying time interval and risk appetite, we point out how the microstructure of market influences the volatility of return. Hence, according to this dynamic model, even though all of the information is freely shared and arrives synchronously, and the distribution of return is nearly normal in distribution, the volatility of return still presents clustering and persistence.

Considering the influence of time interval, the results indicate that for high-frequency data, the market is inefficient for short-term trading strategies, but when the interval increases, the market presents to be more efficient.

We adjust different parameters separately, to test how the propensity of market efficiency and returns are affected. The results show that ξ and η, which are the extent of adjustment for agents marking the strategies based on the performance of the last period, do not affects the distribution of return obviously. ω_1 represents risk appetite of the total market. Just when risk aversion and risk seeking investors are balanced, the volatility of return is close to Gaussian distribution. When the market shows risk preference, larger and more fiercely volatility clustering can be observed, and the market is far away from efficient. We can infer that the inclination of risk attitude of the whole market, in some extent, magnifies the volatility. Increase the weight of trend follower or trend reverser makes the market become more unstable. These indicate that the market becomes more efficient with diversified trading strategies.

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