J Econ Interact Coord (2011) 6:157–170
DOI 10.1007/s11403-011-0081-2

REGULAR ARTICLE

Oligopoly firms with quantity-price strategic decisions

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Received: 21 October 2010 / Accepted: 25 July 2011 / Published online: 9 August 2011
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Abstract An agent-based model is used to determine market equilibrium with price-setting firms in an oligopoly market. The agent-based model is designed to match the experimental rules that Brandts and Guillen (J Ind Econ 55:453–474, 2007) used with human subjects. Their model uses posted prices and advance production of a perishable good. When the marginal cost is zero, the analytical Bertrand solution is almost perfect competition. When the marginal cost is nonzero, the game does not have a theoretical equilibrium in pure strategies. The agent-based model results show that with one or two firms, prices are at or near the monopoly level, which matches the human experiments. With four firms, prices are always at the perfectly competitive level when particle swarm optimization is used. Results using a genetic algorithm, however, are noisier than those using the particle swarm optimization, and the genetic algorithm falls short of the competitive solution. The triopoly market changes from mostly monopoly to a price in between monopoly and perfect competition when a marginal cost is added. The computerized agents tend to overproduce so that profits are negative in the three- and four-firm cases when production is costly. While the prices in the simulation are close to those observed in experiments with human subjects, the inefficiency due to overproduction is much greater in the agent-based model results. This result suggests that human agents are able to reach solutions, perhaps through social norms, that are missed by the simple agent-based rules used here.

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Keywords Agent-based artificial market · Oligopoly · Particle swarm optimization

JEL Classification C72 · L13 · Q13

1 Introduction

The classical Bertrand model states that for price-setting firms, the Nash equilibrium is the perfect competition level, and the solution does not depend on the number of firms. With the classical Cournot model, however, firms choose quantities rather than price, and firms have market power. In contrast to theoretical work, experiments of finitely repeated games with human subjects show prices deviating from perfect competition or even colluding to the monopoly level in duopoly and triopoly markets under either price or quantity competition (Dufwenberg and Gneezy 2000; Brandts and Guillen 2007). Brandts and Guillen (2007) design experiments by letting firms choose price and quantity simultaneously. For both duopoly and triopoly markets, Brandts and Guillen find that competitors usually learn to collude at the monopoly level or only one firm survives, which results in a monopoly market. Suetens and Potters (2007) find significantly more tacit collusion in Bertrand than in Cournot markets after reexamining experimental results of four previous studies (Fouraker and Siegel 1963; Huck et al. 2004; Davis 2002; Altavilla et al. 2006). Huck et al. (2004) find that experimental markets with more than three competitors are more likely to be competitive than oligopolistic.

One question of interest is whether the extra market power found with human experiments is due to participants making tacit agreements or whether it is due to differences in the designs of the human experiments. We focus on the Brandts and Guillen (2007) study.

To aid in interpreting the Brandts and Guillen (2007) experimental results and to validate the agent-based approach, this research studies the behavior of quantity-price competition firms with an agent-based computational model. Recent work with agent-based models has mostly considered Cournot oligopoly models and finds that agents can find the Cournot oligopoly solution, but results depend on the learning rule used (Waltman and Kaymak 2008; Kimbrough and Murphy 2009; Anderson and Cau 2009). The Cournot oligopoly problem is a much simpler problem for agents to solve than the price competition model where small changes in prices can cause large changes in sales. Past research with agent-based models in economics uses genetic algorithms (GA) (Arifovic 1994, 1996; Axelrod 1987; Vriend 2000) and reinforcement learning algorithms (RL) (Erev and Roth 1998; Kutschinski et al. 2003). We use a new particle swarm optimization (PSO) method developed by Zhang and Brorsen (2009) who adapted the original PSO algorithm of Eberhart and Kennedy (1995) to solve agent-based models. Equilibrium is found for markets with one to four firms. Each firm operates in a set of parallel markets, and each firm uses PSO to solve its own optimization problem. We also compute results using GA to determine the sensitivity of results to the algorithm used. The PSO proves to be far superior to the GA when there is a clear equilibrium.

2 Description of oligopoly model

We follow the Brandts and Guillen (2007) experimental design. Since our work is tied so closely to theirs, we begin with a detailed description of their work.

2.1 Brandts and Guillen (2007)

In Brandts and Guillen (2007), consumers have a box-shaped demand with a maximum demand quantity of 100[1] and a constant willingness to pay of \$100 per unit. Firms must decide both quantity and price in advance without knowing the other firms' choices. If the firm with the lowest price produces less than 100 units, the remaining demand is met by the other firms. Consumers will typically consume 100 units, but there is often unsold quantity. If two firms have the same price, the available demand is shared in proportion to the quantity produced. Each firm can produce up to the market maximum demand quantity of 100, and produces an integer quantity with no fixed cost and a constant marginal cost of \$50. Firms decide both production quantity and selling price strategy simultaneously. Once the price is determined, it cannot be changed.

As Brandts and Guillen (2007) note, their game has no equilibrium in pure strategies. Nevertheless, two key benchmarks are available. The Bertrand or perfect competition outcome is a price of \$50/unit. The monopoly outcome, which in this case is the same as the Cournot outcome, is \$100/unit.

Brandts and Guillen (2007) simulated 50 rounds in an attempt to find a long-run equilibrium. Nevertheless, they found that two of their fourteen two-firm markets and two of nine three-firm markets had not converged after 50 rounds. They call these outcomes fights. The other two outcomes that emerged are monopoly due to the other firms going bankrupt or in some cases firms colluded. In the case of two firms, the collusion was typically at the monopoly price level, but in the case of three firms, the collusion was sometimes at a price less than the monopoly price.

2.2 Our model

Our agent-based model allows considering additional scenarios that were not considered in Brandts and Guillen (2007). First, we consider a case where marginal cost is zero, while Brandts and Guillen only considered a marginal cost of \$50. With a marginal cost of zero, the Bertrand model applies, and the Bertrand solution is a price of \$1. Since prices are discrete, sellers have no incentive to cut prices to zero. Second, we do not impose bankruptcies, and so we do not end up with a monopoly. Third, we consider an example with four firms, while Brandts and Guillen only considered duopoly and triopoly. Fourth, when two firms post the same price, Brandts and Guillen allocated sales based on firms' percentages of total production, while we split sales

[1] Brandts and Guillen let demand quantity be 102 with three firms so that the quantity is divisible by three, but we leave it at 100 in all cases.

evenly. Note that firms do not have any incentive to produce items that cannot be sold. Finally, while Brandts and Guillen sought to find long-run equilibrium, they could only consider a finite game with 50 rounds. The agent-based model allows us to consider many more rounds so that our results should represent a long-run equilibrium. In fact, since the computerized agents have no information about when the game will end, we consider our approach to be an infinitely repeated game, which we merely quit observing at some point. If interpreted as an infinite game, then collusion is theoretically possible, and any solution between Bertrand and monopoly can be a Nash equilibrium. Brandts and Guillen found some small decreases in prices at the end of their game, but in most cases equilibrium had been reached by 50 rounds.

3 Agent-based simulation model

We simulate markets with and without a marginal cost (*mc*) for production. Under each marginal cost scenario, we simulate markets with different numbers of sellers, which are monopoly, duopoly, triopoly, and a four-seller market. Eight alternative settings occur in total. The market and algorithm parameters used in the simulations are listed in Table 1. One simulation round contains multiple iterations. Within each

Table 1 PSO parameters in the artificial market simulation

Parameter	Symbol	Value
Market parameters		
Number of sellers	M	1 for monopoly market
		2 for duopoly market
		3 for triopoly market
		4 for 4-seller market
Maximum demand quantity		100
Maximum willingness to pay		\$100
Particle swarm optimization (PSO) algorithm parameters		
Intercept of inertia weight in Eq. (5) of PSO	β_0^w	1.5
Slope of inertia weight in Eq. (5) of PSO	β_1^w	0.5
Self and global confidence factors of PSO	$c_1 = c_2$	1
Number of parallel market	K	20
Maximum iteration of one simulation round	$t_{\max}$	2,000
Number of simulation round		100
Genetic algorithm (GA) parameters		
Strategy population size	K	100
Loop per iteration	L	40
String bit	B	15
Elitism rate	$\varepsilon_t = \beta^{\varepsilon} \cdot t/t_{\max}$	$\beta^{\varepsilon} = 10\%$
Crossover rate	$\chi_t = \beta^{\chi} \cdot (t_{\max} - t)/t_{\max}$	$\beta^{\chi} = 76\%$
Mutation rate	$\mu_t = \beta^{\mu} (t_{\max} - t)/t_{\max}$	$\beta^{\mu} = 0.33\%$
Ranking selection parameter	$r_{\max} = 1.1, r_{\min} = 0.9$	

round, agents interact repeatedly and learn to find the best response strategy. Iterations continue until the market reaches equilibrium or meets the maximum iteration constraint of 1,000 (2,000 for the four-firm case). Once equilibrium is reached, the average value of the last 20 iterations for the market price and the agent's strategies are reported as the market equilibrium values. Due to the randomness of the learning path, each setting is run 100 rounds with different randomly initialized starting strategies for all agents. The mean and standard deviation of the market equilibrium values of the 100 rounds are used to characterize market equilibrium. The theoretical monopoly price is 100, and the theoretical perfect competition price is $100 - mc$. Thus, the perfect competition price is \$100 with no marginal cost and \$50 with a \$50 marginal cost.

3.1 Particle swarm optimization

The idea of PSO came from the way flocks of birds, fish, or other animals share information to avoid predators and to find food. With an agent-based model, each seller has a separate "flock of birds" that does not share information with the flocks of the other agents. The approach used here closely follows that of Zhang and Brorsen (2009, 2010).

K parallel markets are used, and the agents each have their own clones in every parallel market. Although they have the same behavioral rules, agents may take different strategies in each parallel market since the initialized random values are different. In the simulation, sellers dynamically change their marketing strategies with the PSO algorithm, but buyers simply purchase the product with the lowest price.

Since other agents continuously change prices and/or quantities, the previous local best solutions may not be the best for the current period. Thus, the past best locals of each clone are retested under the current market environment, and the best fit is chosen as the current best local among past best locals and the current strategy.

3.1.1 Particle swarm optimization algorithm

At the start of each period, sellers select strategies $\mathbf{x_i} = [x_i^p, x_i^q]', i = 1, \ldots, N$, simultaneously. Here the superscript p and q indicate price and quantity. The prices are discrete, and one dollar is the minimum price increment. The firm with the lowest price satisfies the demand up to its production quantity. Then the next lowest firm sells its products and so on. If more than one firm sets the lowest price, they split the demand quantity evenly.

For each clone, the initial price and quantity are selected randomly from a uniform distribution. The change in parameters from one iteration to the next is determined by the adjustment velocity, $v_{i,k,t}^{\Gamma} \in [-1, +1]$, where the superscript Γ indicates bid price (p) or production quantity (q), the subscripts k and i indicate the kth parallel market and ith agent, while t represents the iteration. The velocity change of a strategy parameter for a clone is a function of the local best solutions, $p_{i,k,t}^{\Gamma,l} \in [0, 1]$, and agent i's global best solution among all parallel markets, $p_{i,t}^{\Gamma,g} \in [0, 1]$. The superscripts l and g indicate local and global.

In every simulation step, each new choice variable of the ith agent in the kth parallel market is updated as:

$$x^{\Gamma}_{i,k,t+1} = x^{\Gamma}_{i,k,t} + v^{\Gamma}_{i,k,t}, \tag{1}$$

and the velocity is modeled as:

$$v^{\Gamma}_{i,k,t+1} = wv^{\Gamma}_{i,k,t} + c_1 u_1 \left(p^{\Gamma,l}_{i,k,t} - x^{\Gamma}_{i,k,t}\right) + c_2 u_2 \left(p^{\Gamma,g}_{i,t} - x^{\Gamma}_{i,k,t}\right), \tag{2}$$

where $v^{\Gamma}_{i,k,t}$ is the velocity, $u_j \in [0,1]$, $j = 1, 2$ are uniformly distributed random numbers, c_1 and c_2 are learning parameters, and w is an inertia weight factor.

The new best local is chosen from the best locals of the previous L periods and the strategy $\mathbf{x}_{i,k,t}$ of the current period:

$$\mathbf{p}^{l}_{i,k,t} = \arg\max \left\{\pi_k \left(\mathbf{p}^{l}_{i,k,t-1}\right), \ldots, \pi_k \left(\mathbf{p}^{l}_{i,k,t-L}\right), \pi_k \left(\mathbf{x}_{i,k,t}\right) \middle| \mathbf{x}_{i' \neq i,k,t} \right\}, \tag{3}$$

where i'indicates opponents, and $\pi_k\left(\mathbf{x}_{i,k,t}\right)$ is profit. The different past best locals of each agent in the last L periods are reevaluated under the current t period's economic environment by holding other agents' strategies in this period unchanged. The strategy with the highest profit is the new local best. The best global parameter is selected from the best locals:

$$\mathbf{p}^{g}_{i,t} = \arg\max \left\{\pi_1 \left(\mathbf{p}^{l}_{i,1,t}\right), \pi_2 \left(\mathbf{p}^{l}_{i,2,t}\right), \ldots, \pi_K \left(\mathbf{p}^{l}_{i,K,t}\right)\right\}. \tag{4}$$

A large inertia weight provides larger exploration than a smaller one. We let w start with a higher value at the beginning and then decrease w as the optimization proceeds:

$$w_t = \beta^{w}_0 + \beta^{w}_1 \left(t_{\max} - t\right)/t_{\max}, \tag{5}$$

where both β^{w}_0 and β^{w}_1 are constants, $t_{\max}$ is the maximum number of iterations, and t is the current iteration. Similarly, we set c_1 and c_2 in Eq. (2) as:

$$c_{1,t} = c_{2,t} = \beta^{c_1}_0 + \beta^{c_1}_1 \left(t_{\max} - t\right)/t_{\max}, \tag{6}$$

where both $\beta^{c_1}_0$ and $\beta^{c_1}_1$ are constants.

Zero diversity in the strategies of all parallel markets for every agent can be used to signal the stopping point for PSO. Diversity diminishes with time, which causes the same strategy to dominate among all parallel markets. In our simulation, for all agents, if the variance of the strategy parameters in the population is less than 0.01% and the mean deviation of the parameters for the most recent 10 iterations is less than 0.01%, we say the algorithm is converged.

3.1.2 Summary of simulation with PSO

We test three oligopoly markets (duopoly, triopoly, and 4-seller market) and a monopoly market. For each market structure, two marginal cost scenarios (zero and \$50) are considered. Sellers choose bid price or quantity, bid for product simultaneously, and update the combinatorial strategy set with PSO at the end of each period.

We design the artificial markets with PSO by setting up $K = 20$ parallel markets, and sellers and their clones trade in all markets simultaneously and independently. The retest iteration number$L$ is chosen as 10. The simulation steps are described as follows.

(i) For the first L beginning iterations, we randomly initialize strategy set $\mathbf{x}$ for all sellers in every parallel market. We choose the quantity and price $x^{\Gamma}_{i,k,t} \in U\,[0,\ 100]$ and the movement velocities $v_{i,k,t} = 0$ for $i = 1, \ldots, M, \Gamma = p, q, k = 1, \ldots, K$, and $t = 1, \ldots, L$.
(ii) Sellers update their quantity ratios and price, respectively as defined in Eqs. (1) and (2).
(iii) Within each parallel market, consumers first purchase from the current lowest price firm up to the firms' produced quantity. If more than one firm has the same price, then a sharing rule is used. Any remaining demand goes to the second lowest price firm up to its production level and so on.
(iv) After the first L iterations, each firm retests the past L best locals under the current economic environment and compares their performance with that of the current strategy. The parameter set with the highest profit is the new best local as shown in Eq. (3).
(v) Following Eq. (4), the best fit among all best locals is the best global.
(vi) If the market does not reach equilibrium, the PSO returns to step (ii).

3.2 Genetic algorithm

GA is a general-purpose optimization method based loosely on Darwinian principles of biological evolution, reproduction, and the survival of the fittest (Goldberg 1989). GA maintains a pool of candidate solutions called a population and repeatedly modifies them. At each step, GA selects candidates from the current population to be parents and uses them to produce children for the next generation. Over successive generations, the population evolves toward an optimal solution.

3.2.1 Genetic algorithm operators and parameters

In GA, a seller's price and quantity represent chromosomes. Each chromosome is represented by a B bit binary string so the bits can be viewed as genes. The kth price or quantity of firm i in period $t$can be stated as a string of length B:

$$\left\langle a^{B}_{i,k,t}, a^{B-1}_{i,k,t}, \ldots, a^{1}_{i,k,t} \right\rangle, \tag{7}$$

where $a_{i,k,t}^{b} \in \{0, 1\}$ is taken at the bth position in the string, $b \in \{1, 2, \ldots, B\}$. The binary string can be decoded into a decimal integer using

$$d_{i,s,t} = \sum_{b=1}^{B} \left(a_{i,k,t}^{b} 2^{b-1} \right). \tag{8}$$

The maximum value is $d_{\max} = \sum_{b=1}^{B} 2^{b-1}$. For example, if a string contains 4 bits, a binary code "0101" can be decoded to decimal value: $d_i = 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 5$, and the maximum binary code of this string "1111" is 15. Thus a firm using "0101" as its strategy will have a quantity of 5/15 of the maximum quantity. We use 15 bits as the string length in the simulation.

Sellers' decision rules are updated using four genetic operators, *elitism, reproduction, crossover,* and *mutation*. Elitism can very rapidly increase GA's performance, because it prevents the loss of the best solution to date. In our price-quantity competition game, the profit difference between strategies could be large. To avoid one high-profit strategy dominating the next generation with profit proportional selection, ranking selection is used as the reproduction method.

Elitism copies a few of the best strategies from the current K strategies to the new population with an elitism rate ε. If $\varepsilon = 10\%$ and $K = 100$, this means the 10 best strategies are copied from the old population to the new one. The rest are chosen with linear ranking selection.

Reproduction chooses chromosomes as parents from the old strategy population. The ranking selection method ranks an agent's strategies in its population 1 to K from worst to best according to profit ($K =$ population size). If more than one strategy has the same profit, the strategies are randomly ranked. The selection probabilities of the strategies x_k ($k = 1, \ldots, K$) are given by

$$p\left(x_k\right) = \frac{1}{K} \left(r_{\min} + \frac{\left(r_{\max} - r_{\min}\right)\left(rank\left(x_k\right) - 1\right)}{K - 1} \right), \tag{9}$$

where p is the probability of strategies being chosen as new ones, $r_{\max} + r_{\min} = 2$, and $1 \leq r_{\max} \leq 2$. We choose $r_{\max} = 1.1$ and $r_{\min} = 0.9$, so that $\sum p\left(x_k\right)$ equals 1.

Crossover selects genes from two parent chromosomes and creates a new strategy. All new strategies selected with elitism and ranking selection methods are randomly matched as a group of parent chromosomes. With crossover, a switch point of the chromosome string is randomly chosen, and bits before and after this point are exchanged to generate two new chromosomes.

Mutation takes place after a crossover is performed with a probabilityμ. This mutation is to prevent falling into a local optimum. In the binary encoding method, mutation changes the bits of the new strategy from 1 to 0 or 0 to 1 with the mutation rateμ.

Like PSO, we can also use niching methods in GA:

$$\varepsilon_t = \beta^{\varepsilon} \cdot t / t_{\max}, \tag{10}$$

$$\chi_t = \beta^{\chi} \cdot \left(t_{\max} - t\right) / t_{\max}, \tag{11}$$

and

$$\mu_t = \beta^{\mu} \cdot (t_{\max} - t) / t_{\max}, \tag{12}$$

where ε_t, χ_t and μ_t indicate elitism, crossover, and mutation rate, βs are constant, $t_{\max}$ is the number of maximum iterations of the one simulation round. Zhang and Brorsen (2009) show that the chosen parameters give relatively accurate simulation results in the Cournot game when $\beta^{\varepsilon} = 10\%$, $\beta^{\chi} = 76\%$, and $\beta^{\mu} = 0.33\%$ separately. We adopt this setting during the simulation.

3.2.2 GA simulation procedure for price-quantity competition game

At the beginning of the simulation, sellers' strategies are randomly generated. Returns are calculated for each strategy. A new population is generated from the current one with the following procedure. First, the εS highest-return strategies are copied to the new population as elites. Then, $(1 - \varepsilon)\ S$ strategies are chosen with ranking selection methods from the whole population of the old generation and are randomly matched and crossed over. Mutation is performed for the new strategies except the elites.

Table 2 PSO simulation results of price-quantity strategic sellers without marginal cost

Market structure	Seller	Statistic	Market price ($)	Price ($)	Production quantity	Sold quantity	Profit ($)
Monopoly	Seller 1	Mean	100	100	100	100	9,997
		SD	0	0	0	0	17
Duopoly	Seller 1	Mean	100	100	100	49	4,899
		SD	1	1	0	49	4,833
	Seller 2	Mean		100	100	51	5,013
		SD		1	0	49	4,844
Triopoly	Seller 1	Mean	50	51	100	30	1483
		SD	6	6	1	45	2,230
	Seller 2	Mean		51	99	38	1,906
		SD		6	1	48	2,433
	Seller 3	Mean		51	100	32	1,594
		SD		6	1	45	2,292
Four-seller	Seller 1	Mean	1	1	94	25	22
		SD	0	0	9	15	10
	Seller 2	Mean		1	93	25	23
		SD		0	10	14	12
	Seller 3	Mean		1	93	25	22
		SD		0	8	15	9
	Seller 4	Mean		1	95	25	21
		SD		0	6	17	10

With zero marginal cost, the perfect competitive and monopoly price levels are $0 and $100, respectively

Table 3 PSO simulation results of price-quantity strategic sellers with $50 marginal cost

Market structure	Seller	Statistic	Market price ($)	Price ($)	Production quantity	Sold quantity	Profit ($)
Monopoly	Seller 1	Mean	100	100	100	100	5,000
		SD	0	0	0	0	5
Duopoly	Seller 1	Mean	100	100	100	56	5,550
		SD	1	0	0	49	4,833
	Seller 2	Mean		100	100	44	4,368
		SD		1	0	49	4,835
Triopoly	Seller 1	Mean	67	67	99	42	−2,157
		SD	4	4	1	48	3,202
	Seller 2	Mean		67	99	29	−3,070
		SD		4	1	44	2,949
	Seller 3	Mean		68	99	29	−3,022
		SD		4	1	45	2,952
Four-seller	Seller 1	Mean	53	57	79	27	−2,534
		SD	3	8	24	39	2,128
	Seller 2	Mean		56	82	21	−2,996
		SD		5	24	36	1,990
	Seller 3	Mean		57	84	24	−2,962
		SD		7	19	38	1,961
	Seller 4	Mean		57	82	28	−2,653
		SD		8	24	41	2,104

With $50 marginal cost, the perfect competitive and monopoly price levels are $50 and $100, respectively

4 Simulation results

The mean and standard deviation of the equilibrium strategy parameters for each scenario are shown in Tables 2, 3, 4, and 5. Tables 2 and 3 provide results using the PSO algorithm, and Tables 4 and 5 are results using the GA.

With zero marginal cost, market prices are always at the monopoly level for markets with one or two sellers (Table 2). The Bertrand equilibrium is reached for four sellers, and the equilibrium is located between monopoly and perfect competition for three sellers. Huck et al. (2004) obtain similar results with quantity competition experiments with human subjects. They also find that the number of sellers affects the competitive behavior; two sellers tend to collude, and four sellers tend to be competitive.

Table 3 shows that when there is a marginal cost, the duopoly sellers still end up at the monopoly level (often because one of them drops out), but the triopoly sellers compete to a price closer to the perfectly competitive level than to the monopoly level. Brandts and Guillen (2007) found that with human subjects also often found the monopoly price equilibrium either through collusion or through bankruptcy of one of the firms. The average market price of $53 in the four-seller market is lower than the triopoly market price of $67. Brandts and Guillen (2007) found an average

Table 4 GA simulation results of price-quantity strategic sellers without marginal cost

Market structure	Seller	Statistic	Market price ($)	Price ($)	Production quantity	Sold quantity	Profit ($)
Monopoly	Seller 1	Mean	98	98	97	97	9,529
		SD	4	4	5	5	652
Duopoly	Seller 1	Mean	65	68	76	44	2,929
		SD	15	22	24	35	2,519
	Seller 2	Mean		64	78	54	3,527
		SD		22	23	35	2,679
Triopoly	Seller 1	Mean	41	53	74	34	1,411
		SD	14	24	24	36	1,734
	Seller 2	Mean		49	74	39	1,617
		SD		23	22	33	1,637
	Seller 3	Mean		52	71	27	1,101
		SD		24	27	32	1,512
Four-seller	Seller 1	Mean	33	46	65	31	989
		SD	13	24	25	35	1,278
	Seller 2	Mean		49	62	30	1,062
		SD		23	28	32	1,295
	Seller 3	Mean		54	63	18	602
		SD		23	27	28	987
	Seller 4	Mean		49	65	22	671
		SD		25	28	29	1,061

With zero marginal cost, the perfect competitive and monopoly price levels are $0 and $100, respectively

triopoly price of $80, which is due partly to forcing bankruptcies. However, it may also be due to humans being able to reach tacit agreements that the computerized agents could not.[2] Also, the agents are boundedly rational in that they look only at what rule would have worked well the last time. Human agents are likely better able to be forward looking and consider the reaction of other firms. But, the fact that many human subjects went bankrupt suggests that many of them could also be boundedly rational.

In both the three- and four-firm case in Table 3, firms produce much more than they can sell. Brandts and Guillen (2007) found firms overproduced by about 20% in the three-firm case, but we find that firms overproduce by nearly 100%. The computerized agents are clearly overly optimistic about how much they can sell. The mistakes that the computerized agents make are similar to the hindsight bias observed in human experiments (in this case the computerized agents overestimate their ability to predict what the other agents will do).

[2] There are many reasons to suggest caution in interpreting the results of human experiments (Smith 2009; Gintis 2009). Human actors have subjective beliefs about how others will act, and they can be influenced by social norms such as an expectation of reciprocity.

Table 5 GA simulation results of price-quantity strategic sellers with $50 marginal cost

Market structure	Seller	Statistic	Market price ($)	Price ($)	Production quantity	Sold quantity	Profit ($)
Monopoly	Seller 1	Mean	100	100	94	94	4,710
		SD	0	0	9	9	447
Duopoly	Seller 1	Mean	84	84	73	47	348
		SD	13	19	20	31	2,442
	Seller 2	Mean		85	76	52	486
		SD		18	18	31	2,553
Triopoly	Seller 1	Mean	67	69	56	33	−592
		SD	18	23	26	27	1,963
	Seller 2	Mean		72	58	35	−652
		SD		24	25	29	2,084
	Seller 3	Mean		72	60	33	−861
		SD		23	25	27	1,901
Four-seller	Seller 1	Mean	60	67	50	25	−941
		SD	18	17	24	25	1,608
	Seller 2	Mean		70	47	22	−963
		SD		18	24	24	1,680
	Seller 3	Mean		67	52	30	−856
		SD		20	25	29	1,696
	Seller 4	Mean		66	47	22	−1,083
		SD		21	25	26	1,743

With $50 marginal cost, the perfect competitive and monopoly price levels are $50 and $100, respectively

The GA solutions in Tables 4 and 5 are similar, but less extreme. The GA is less able to reach the corner solutions of Bertand or monopoly. The GA also has less overproduction, but it still has negative profits in the three- and four-firm cases when production is costly. GAs are notoriously slow optimization algorithms, so the inability of the GA to find the optimal solution in the given number of rounds is not surprising. But, the GAs do not suffer as much from excessive optimism about how product can be sold as does the PSO.

Figure 1 shows individual runs to illustrate the pricing behavior evolution of sellers with no marginal cost settings. This figure shows that monopoly and four-sellers markets reach equilibrium faster than duopoly and triopoly markets. In this example, all sellers have the same pricing strategy. When the number of sellers is less than four, they reach the monopoly price; otherwise they compete to the perfectly competitive level.

5 Conclusions

An agent-based model is used to study oligopoly with posted prices and advance production of a perishable good. Sellers must choose both price and quantity. Using

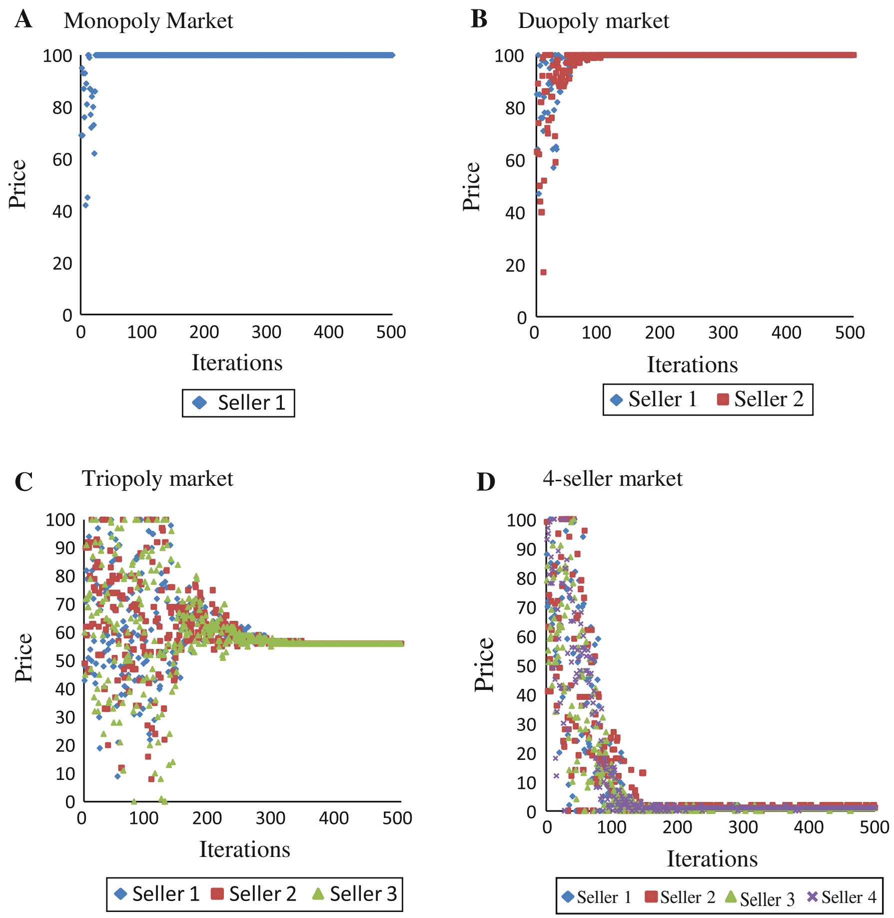

Fig. 1 Sellers' pricing behavior without marginal cost ($c^C = 0$)

particle swarm optimization (PSO) and zero marginal cost, sellers reach nearly the monopoly level with one or two sellers, compete to the Bertrand solution of nearly perfect competition when the market contains four sellers, and average a price in between monopoly and perfect competition with three sellers. When a marginal cost is added, triopoly prices are closer to the Bertrand solution than the monopoly price, but the solutions with one and two firms are still monopoly, and four firms still show near perfect competition.

Simulations using a genetic algorithm (GA) also find prices decrease with an increase in number of firms, but the GA is less able to find the corner solutions of monopoly or Bertrand equilibrium. While the PSO was much more efficient in reaching equilibrium than the GA, the results do not necessarily extend to the other numerous variations of GAs.

Sellers tend to have excess production quantity. Brandts and Guillen (2007) experiment with human subjects also found overproduction, but much less than observed here with the agent-based models. Humans may benefit from social norms that are not available to the computerized agents, or some people may consider the possible reaction of other agents. Also, Brandts and Guillen (2007) imposed bankruptcy which will reduce overproduction since firms that greatly overproduce go bankrupt.

The agent-based model results match the human experiments rather than the theory. Thus, the bounded rationality (Simon 1991) of the agent-based models offers a possible explanation of why human experiments with two or three firms lead to market power that is not predicted by the Bertrand theory.

Acknowledgments This article benefited from the helpful comments of Chris Boyer, James Fain, Jayson Lusk, and Clem Ward. The research was partially supported by the Oklahoma Agricultural Experiment Station Hatch project OKL02170 and by a grant from project 211 (phase III) of the Southwestern University of Finance and Economics.

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