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REGULAR ARTICLE

Multi-unit auction format design

Atakelty Hailu · **Sophie Thoyer**

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Abstract Multi-unit auctions are powerful mechanisms for the allocation of numerous resources and production contracts. However, there is uncertainty about the best auction formats when multi-unit auctions are used because there are multiple equilibria and analytical descriptions of optimum strategies are intractable. Empirical studies are scarce and most experimental studies are restricted to two bidders and two units. This paper constructs an agent-based model of bidders to compare the performance of alternative procurement auction formats under circumstances where bidders submit continuous bid supply functions and learn over time to adjust their bids in order to improve their net incomes. The results provide some confirmation of analytical predictions but also indicate that the range of bidding strategies employed is richer than what the theory expects. Bidder coordination for higher prices is possible under both the Vickrey and uniform auctions, especially when the population is heterogeneous and competition low. Judged by budgetary measures, the discriminatory auction is the most expensive when rationing is tight, but this outcome is reversed when rationing is less stringent. Budgetary outcomes can be improved if the auction is organized among more homogeneous bidders.

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A. Hailu (✉)
School of Agricultural and Resource Economics, University of Western Australia,
35 Stirling Highway, M089 Crawley (Perth), WA 6009, Australia
e-mail: ahailu@are.uwa.edu.au

S. Thoyer
ENSAM–LAMETA, 2 place Viala, 34060 Montpellier cedex 1, France
e-mail: thoyer@ensam.inra.fr

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1 Introduction

Governments and private companies are increasingly utilizing multi-unit auctions to allocate assets or to purchase services. Unlike single-unit auctions, multi-unit auctions allow bidders to bid with quantity and price schedules rather than submit single quantity-price bids. Thus, multi-unit auctions help avoid the 'lumpy bid' problem (Tenorio 1993) inherent in single-unit auctions and improve allocation efficiency by allowing bidders to buy or sell multiple units through a bidding process where submitted bids represent supply or demand schedules. Multi-unit auctions are therefore promising mechanisms for the allocation or re-allocation of divisible resources including company stocks, electricity generation contracts, nature conservation contracts, and the buyback of water rights from irrigators in over-extracted river environments.

Single-unit auctions have been extensively studied and the most important theoretical conclusion in the literature, the revenue equivalence theorem (RET), tells us that discriminatory (pay-as-bid) and uniform (or Vickrey) auctions lead to equivalent expected revenues. In the case of multi-unit auctions, however, our knowledge of the relative performance of alternative auction formats is incomplete. The revenue equivalence theorem does not hold. There are very few theoretical results concerning bidding behaviour under multi-unit auctions. The multiplicity of possible equilibria makes an analytical comparison of the different formats and of optimal bidding strategies intractable. Experimental analysis of multi-unit auctions include that of Alsemgeest et al. (1998), List and Lucking-Reiley (2000)[1] Kagel and Levin (2001) and Engelmann and Grimm (2003). These, as we will discuss later, involve simplified frameworks, considering only two- unit, two-person auctions. Competitive pressure and heterogeneity in bidders' characteristics are often overlooked in such formulations. Hence, there is a strong case for computational approaches to further our understanding of multi-unit auction design (Binmore and Swierzbinski, 2000, p 407). Agent-based modeling (ABM) can provide a useful and inexpensive research tool for comparing the relative performance of different auction designs.

This paper constructs an agent-based model to examine the performance of the three standard formats (generalized Vickrey, uniform and discriminatory) for multi-unit auctions. The simulated auction market is cast as a procurement auction where a government agent buys products or services from a population of bidders with private independent values reflecting different opportunity

[1] The List and Lucking-Reiley (2000) paper conducts field experiments by selling postcards in a two-unit, two-person auctions and compares two auction formats.

or production cost structures. Bidders submit supply functions indicating the amount they would be willing to supply at different prices. Bidders use reinforcement learning to update their individual bid functions with the objective of increasing their net incomes.

The paper attempts to answer two questions of particular interest to agencies organizing auctions: the choice of the auction format and the choice of the auction boundaries. The discriminatory (or pay-as-bid) auction has been the format that is used commonly by government agencies. Can outcomes be improved if alternative auction formats are employed? The issue of auction boundaries is relevant in cases where outcomes might be affected by the level of heterogeneity in the bidder population. Is it better to organize several separate auctions each involving more homogeneous bidders or to organize a single auction open to a larger, more heterogeneous population of bidders?

The paper is structured as follows. In the second part, we review the various auction formats and the expected structural properties of equilibrium strategies. We highlight that the generalized Vickrey is the only format for which equilibrium bidding strategies can be analytically calculated (Ausubel, 2005). Then, we describe the agent-based model of boundedly rational bidders revising their strategies using Erev and Roth (1998) reinforcement learning algorithm. In the fourth section, we present the results from the computational experiments and compare bidding behaviours, budgetary outlays and efficiency of allocation for the three auction formats (discriminatory, uniform and generalized Vickrey) and different heterogeneity in bidders' sizes. The performance of an auction is evaluated using two criteria: budgetary outlays and the efficiency of allocation. The simulation results indicate that bidding behaviours exhibit richer patterns than the patterns described by the limited theoretical results that are available. The relative performance of the different auction formats depends on the level of competition[2]. The discriminatory auction has the lowest performance in terms of social cost efficiency. It also performs worse than the other formats except when competition is weak. In the fifth section, we summarize the paper and draw some general conclusions.

2 Multi-unit auctions

The theoretical literature on multi-unit auctions usually distinguishes between sequential and simultaneous auctions of several objects with bidders wanting more than one unit. We concentrate on simultaneous procurement auctions, for multiple identical units, within a symmetric independent private value model.

We also assume that the number of units that the auctioneer wishes to buy is fixed (as opposed to a budget constrained auction or an auction with a downward-sloping demand curve). Since most of the literature on multi-object multiple-bid auctions describes selling auctions, here we describe and

[2] Competition here is simply measured as the ratio of the auctioneer's demand to the aggregate supply of the bidders. It is a measure of the degree of rationing.

summarize briefly what the equivalent predictions would be for a procurement auction in the case of a continuous rather than discrete bid specification. The continuous bid specification can be interpreted as one relating to the purchase of perfectly divisible units and was initially developed by Wilson (1979) who called it "auctions of shares".

In terms of implementations, the three multi-unit formats are extensions of the single unit formats. In all cases, the cut-off price is defined by the intersection of the aggregate supply and demand. All bids below or equal to this cut-off price become winning bids. Thus, the quantity a winning bidder supplies is equal to the amount the bidder is willing to supply at the cut-off price as per her submitted bid curve. A figure illustrating the determination of the cut-off price and the payments is included in the appendix (see Fig. 4). The auction formats differ in bidder payment calculations. In the discriminatory format, each bidder is paid an amount equal to the sum of his actual winning bids. In a uniform-price auction, all units sold earn the cut-off price. Therefore, infra-marginal winners receive payments that are higher than the production costs implied in their bids. In a generalized Vickrey auction, a bidder who sells k units is paid an amount equal to the sum of the k losing bids that would have been included if the bidder was not participating[3]. In other words, each bidder is paid an amount equal to what the auctioneer can save by not having to source those units from the other bidders. Therefore, the amount paid to each bidder is unrelated to their own bids. When there is a single unit to buy, then the generalized Vickrey and the second price auction are the same.

2.1 Theoretical predictions

For multiple-bid auctions, no closed form expressions of the bidding strategies are available in the general case and most authors have therefore focused on the expected structural properties of the equilibrium strategies. Wilson (1979), Back and Zender (1993), Engelbrecht-Wiggan and Kahn (1998), Tenorio (1999) and Ausubel and Cramton (2002), have analyzed the outcomes of different multi-unit auction formats and shown that the revenue equivalence theorem does not extend to the case of multiple-bid auctions. They demonstrate, using different

[3] To illustrate how the Vickrey auction works, let's take an example with two bidders, each wanting to supply a maximum of four units. Say the bidders submit the following bid schedules: (1, 3, 6, 7) and (2, 4, 5, 9). That is, bidder 1 is willing to supply the first unit if she is paid 1, the second unit if she is paid an extra \$3, etc. Similarly, bidder 2 is willing to supply her first unit if she is paid \$2, the second unit if she is paid an extra \$4, etc. The auctioneer's demand is for 4 units. Since the aggregate bid curve prices are (1, 2, 3, 4, 5, 6, 7, 9), the cut-off price is 4 and each bidder would supply 2 units. The payment to bidder 1 would be 14 (i.e. $9 + 5$) while the payment to bidder 2 would be 13 (i.e. $7 + 6$). The cut-off price and what the bidders supply would not change under the uniform and discriminatory auctions. However, the payments to bidder 1 and 2 would be 4 $(1 + 3)$ and 6 $(2 + 4)$ 6, respectively, under the discriminatory while each bidder would receive 8 (i.e. 2×4) under the uniform auction. Of course, the bidding behaviour induced by the different auction formats is not necessarily the same, and (as the results in this paper show) the relative performance of the three formats would not be what is implied in this example.

Table 1 Expected structural properties of equilibrium bidding strategies

Format	Equilibrium strategies and efficiency[7]
Discriminatory	Scope for "high flat supply"[a] and for "supply inflation"[b]. Inefficient allocation
Uniform-price	Supply "inflation", i.e. true entry price but steeper slope. Coordination at a high price equilibrium. Inefficient allocation
Generalized Vickrey	Truthful bidding is a weakly dominant strategy. Efficient allocation

[a] The bid curve is flat, or almost flat, above the true cost curve
[b] Equivalent to demand reduction (or bid shading) in a selling auction: bidding is sincere on the first unit, then there is differential increasing overbidding

but simplified selling auctions[4] the issue of bid shading (or demand reduction) associated with a uniform-price multiple-bid selling auction. Although it is a dominant strategy to bid truthfully for the first unit, it is efficient for bidders to shade their bids for additional units or quantities. Moreover, the amount of bid shading increases with quantities offered. "The reason for this differential shading is that the incentive to win units at any price below marginal value is offset by the incentive to reduce the price paid on infra-marginal units that are won anyway" (Ausubel and Cramton, 2002, p 23). The latter becomes increasingly important when quantities increase, which explains the increasing bid shading. All demonstrations can be carried over with no restriction to the procurement case (i.e. buying auctions) (Table 1). It is also demonstrated that there is an incentive, in a discriminatory format, to submit flatter supply curves than in a uniform price auction. If bidders are risk neutral, submitting entirely flat supply curves is an equilibrium strategy (Back and Zender, 1993), although drastic demand reduction is also a possible outcome (Engelbrecht-Wiggan and Kahn, 1998; Krishna, 2002). Since, there are different classes of equilibrium strategies, it is difficult to analyse how bidders behave and compare the efficiency of the two formats.

Only in the generalized Vickrey auction or its counterpart in the open format [i.e. the ascending auction with "clinched" quantities designed by Ausubel (2005)] is truthful bidding known to be a weakly dominant strategy, resulting in efficient allocation. On the other hand, for uniform and discriminatory formats, only a larger number of bidders can lead to the reduction of strategic behaviour and to more truthful bidding (Ausbel and Cramton 1996; Swinkels 1999).

2.2 Experimental studies

Given both the weakness of the theory on multi-unit auctions and the increasing use of such auctions in economic life, a growing number of researchers have

[4] All demonstrations are made for a model where bidders' values are private and independently distributed and ex-ante symmetric (the distribution of information is uniform across bidders in the pre-auction situation).

tried to investigate bidding behaviour in laboratory environments. Alsemgeest et al. (1998) demonstrate that, in the two-unit, two-bidder case, an ascending clock auction generates less revenue than the uniform sealed-bid auction, due to strategic bid shading. Kagel and Levin (2001) also compare uniform-price sealed bid and open ascending auctions, with a real player with flat demand for two units playing against a robot with unit demand. Their findings also highlight the issue of demand reduction and show that in the open format, bids converge towards equilibrium values more rapidly than in a sealed bid format, as if the "clock could enhance learning". They confirm that the Ausubel format leads to truthful bidding. Engelmann and Grimm (2003) compare bidding behaviour under five auctions formats and for a case where two bidders with flat demand curves compete to buy two units. Their experiments demonstrate that demand reduction is more acute in uniform open than in uniform sealed-bid auctions, and that the Ausubel format eliminates bid shading.

All these experiments are conducted in very simplified settings. Human experiments can also be extremely costly and complicated to run when exploring issues such as competition or heterogeneity amongst bidders. One way to deal with these difficulties is to employ computational or *in silico* experiments, conducted with autonomous "artificial" agents interacting in artificial societies (Tesfatsion, 2002). The starting point of such agent-based models is the specification of agent attributes and behaviours to mimic bidder behaviour.

3 The modelling of bidding strategies with artificial learning agents

Agent-based computational economics (ACE) is the study of artificial societies of interacting autonomous agents that directly emulate the behaviours of individuals, institutions and environmental components that make up the system being studied (Epstein and Axtell 1996; Tesfatsion 2002). Unlike conventional or deductive approaches, the starting point in ACE is the specification of agent attributes and behaviours rather than equations or equilibrium conditions describing the system under study. Therefore, ACE is suited to the study of systems where modelling outcomes can be gainfully enriched through the explicit incorporation of phenomena such as agent heterogeneity, local interactions, networking, inductive learning, as well as through the relaxation of other restrictive assumptions that are normally imposed in theoretical analysis for tractability purposes (Tesfatsion 2002). Studies applying ACE to the analysis of auctions include Andreoni and Miller (1995), Nicolaisen et al. (2001), Bower and Bunn (2001), Bunn and Oliveira (2001), Hailu and Schilizzi (2004) and Koersrindartoto (2005).

3.1 Structure of agent based model

Our auction model has a population of agents participating in a sealed-bid auction organized by a single buyer, the government agent. The government agent has a target or demand level and uses the auction to select bids and determine payments to bidders. Each seller is characterized by a supply function and a capacity. The government agent does not know the true supply functions of the different bidders and makes selection based on submitted supply bid functions. Over time, sellers learn to choose bid functions that maximize their expected net incomes.

Each auction round involves two stages. In the first stage, the government collects bid functions, calculates the residual demand for each bidder and determines the equilibrium quantities bought from each of them. In the second stage, payments to individual bidders are determined according to the auction format in use. Sellers use the results of the auction to compute their net incomes and to update the probabilities with which they choose their bid strategies for the next round. The strategy choice probabilities of a bidder therefore depend on his true opportunity costs as well as on the history of choices he has made and rewards obtained for those choices. Details about the algorithms used in the two stages are presented in the appendix.

For the sake of simplicity, it is assumed that the true supply function of a seller i is linear and can be written as:

$$P_i = a_i^0 + b_i^0 Q_i \quad \text{with } 0 \leq Q_i \leq \text{ms}_i, \text{where ms}_i \text{ is the capacity of bidder } i.$$

This structure implies decreasing returns to scale. One could explore with alternative structures for the supply curve reflecting different degrees of increasing/decreasing returns to scale. To make the scope of the research manageable, we will focus on decreasing returns (upward sloping supply curves) as this is the case that is likely to be empirically more relevant. However, it would be interesting to examine in further research how the results would change under circumstances where the supply curves reflect increasing returns to scale or are greatly different from what is assumed here.

3.2 Seller choice strategies

We make the assumption that the learnt bidding curve is also linear and can therefore be written:

$$\beta_i = a_i^l + b_i^l Q_i \text{ with } \beta_i \ (Q_i) \text{ the strategic bid of player } i.$$

There are, therefore, two dimensions to the seller's choice strategy: intercept choice (a_I^l) and slope choice (b_i^l). The learning algorithm described below will allow bidders to progressively explore different combinations of a and b and to retain the best values based on the performance of past bids. Intercept and slope choices are discretized into seven steps. For the slope parameter, for example, there is a choice of seven values equally spaced between 0 and the maximum slope value implied by the constraint discussed below. The true intercept and slope parameters are included in the choice sets to allow for truthful revelation of supply function parameters.

A constraint is imposed on the choice of learnt bidding strategies so that learnt bids do not lead to losses. This is guaranteed by restricting the allowed or feasible parameter choices (a and b). The range of the learning space is also constrained to avoid bid curves that generate prices that are too high to be considered feasible by bidders. The constraints imposed are: (1) that the learnt supply curve should not fall below the true supply curve; and (2) that the maximum bid price for any bidder is not more than a certain proportion (here 3) of the marginal cost of supply of the most expensive unit by the most expensive supplier. This latter parameter choice does not impose a binding constraint on the learning as the maximum learnt individual prices (and the auction clearing prices) are below the production (opportunity) cost of the most expensive unit. However, the restriction is very useful as it helps avoid unnecessary search by the learning algorithm over portions of the parameter space that are irrelevant.

3.3 The learning algorithm

Over the last several decades, different learning algorithms have been developed. Camerer (2003) provides a review and analyses the relationship between these learning algorithms and how certain variants are special cases of others. The models differ in terms of their information requirements. The reinforcement-learning algorithm (Erev and Roth 1998; Roth and Erev 1995) is chosen for this study as it is particularly suitable for modelling bidding behaviour without requiring that players be knowledgeable about forgone payoffs associated with strategies that they did not select. Moreover, it is an individual learning algorithm and agents do not need to know about their competitors' payoffs, cost structures and other strategic attributes. These features make the algorithm easy to work with and is particularly pertinent when large numbers of players are involved and the assumption of lack of knowledge about other players is a plausible one. "Low rationality" models of reinforcement learning may often provide a more accurate prediction of behavioural outcomes than standard game theoretical analysis. Indeed, adaptive and incremental learning may be the only way in which problems can be solved in complex environments where optimal strategy based on explicit expected utility maximization may not be computationally possible (Markose 2005).

This algorithm has been used in agent-based studies of electricity auction markets (e.g. Nicolaisen et al 2001; Bunn and Oliveira 2001). It is based on the following four principles rooted in the psychology of learning: *the law of effect, the power law of practice, experimentation* and *recency*. *The law of effect* asserts that the tendency to choose an action is strengthened (*reinforced*) or weakened depending upon whether the action produces favourable results or not. This principle implies that choice is probabilistic. *The power law of practice* refers to the fact that learning curves tend to be initially steep. *Experimentation* (or *generalization*) implies that strategies similar to previously chosen successful ones will be employed more often. Experimentation prevents players from quickly being locked into particular choices. *Recency* (or *forgetting*) requires that recent experience has more impact on behaviour than past experience.[5] For the recency parameter, the experimentation parameter as well as a scale parameter, we used the values of (0.1, 0.2 and 9.0) that were suggested by Erev and Roth (1998) based on the success achieved by the algorithm in replicating experimental observations.

4 Simulation results and discussion

The set up of the computational experiments is as follows: Bidding under the Vickrey, uniform and discriminatory pricing auctions were simulated for homogeneous and heterogeneous populations. In each case, the population has a total of six bidders with an aggregate capacity of 12. Bidders have production technologies described by three attributes: an entry cost (a), slope of marginal cost (b) and total capacity (ms). In the homogeneous population, each bidder has a maximum capacity of 2.0 units. The heterogeneous population is composed of two small, two medium and two large bidders. These bidders have maximum capacities of 1.0, 2.0, and 3.0 units, respectively. Auction performance was simulated for different levels of competition by varying the level of demand from the procurement agency from 10 (1.2) to 60% (7.2) of aggregate capacity.

The pure effects on bidding behaviour of heterogeneity in size can be discerned more easily when these effects are not confounded by the effects of economies/diseconomies of scale that accompany changes in size. Therefore, the cost structures for the bidders were chosen so that the opportunity cost curve of a large bidder is exactly like the opportunity costs of three small ones put together while that of a medium one is like two small ones. (For example, if a small bidder is a company with one production plant or a company with one irrigated farm, a large one would be one with three plants or three farms.) For this reason, small, medium and large bidders were assigned supply slopes of 0.6, 0.3, and 0.2, respectively. Entry costs were the same for all bidders and normalized to zero.

As described in Sect. 3, the bidders use reinforcement learning to update their bids through the auction rounds. Since choice under this learning procedure is probabilistic, convergence was defined using two stringent criteria to avoid premature termination of the learning rounds. First, it was required that the probability of choice attached to the most likely strategy be at least 0.5, it was required that this maximum probability be at least three times bigger than the second highest probability if the learning is to be judged as one that has converged on a clear strategy choice. This latter criterion was found to be too strict. Nonetheless, it was met in at least 50% of the cases considering all simulations together. For discriminatory auctions, the convergence criteria are met in at least 90% of the simulation runs for a homogeneous population and

[5] Due to space limitations further details on the model have not been provided here. These details can be requested from the authors.

in more than 70% of the runs for a heterogeneous population. For Vickrey and uniform auctions, the convergence criteria are met at lower demand levels but rarely at high demand levels. When the convergence criteria are not met, the simulation run is allowed to run up to 50,000 rounds. Since the focus in this study is on exploring the possible outcomes under different auction formats, a large number of rounds were allowed for and the convergence criteria made very strict. Below, the observed bidding strategies are presented and discussed. Then, the performances of the three formats are compared.

4.1 Results for the homogenous population of bidders

Bidding strategies under the three auction formats follow different patterns. First, learnt bidding strategies are closest to truthful bidding for the Vickrey auction, followed by the uniform. This is consistent with the theoretical prediction of truthful bidding as a weakly dominant strategy under the Vickrey. Strategies are clearly less sincere under the discriminatory auction. Moreover, bidding under the discriminatory involves less mixing of strategies than under the two other formats.

The insincerity of the bidding can be measured by the mean squared deviations (MSD) from true values of the learnt intercept (entry price) and slope parameters. The deviations for both parameters are lowest under the Vickrey (see Fig. 1). The deviations in the uniform are also similar but slightly higher, especially for the bid slope parameter. The nature of deviations observed in the simulations is consistent with the prediction of supply inflation under uniform auctions from theoretical analysis. The phrase "supply inflation" is used in the literature to denote overbidding that involves a true entry price together with over bidding on subsequent units (i.e. a bid curve that has the same intercept as the true supply curve but a steeper slope). At low demand levels (1.2) bidding in the uniform auction predominantly involves truthful bidding on the first unit with overbidding on subsequent units (higher learnt slopes). As the level of demand increases, inflated entry prices become more frequent leading to a mixture of strategies that involve higher entry prices together with true, higher or lower slopes about 80% of the time. Under the Vickrey, bidding exhibits similar changes in patterns but involves a larger mixture of truthful bidding (especially truthful entry prices).

In the discriminatory auction, on the other hand, high flat bidding is the most dominant strategy at all demand levels. A flattening of the supply curve improves bidder revenue as the prices received for infra-marginal units are brought closer to that of the marginal unit. So under this auction format, there is an incentive for bidders to organize their bids closer to the auction clearing price. This strategy is more successful at lower competition levels where the risk of being undercut by competitors is lower. However, when competition is high (e.g. demand 1.2) supply inflation is an important strategy (with frequency of 30%) as this strategy minimizes the risk of zero gains. The simulations provide

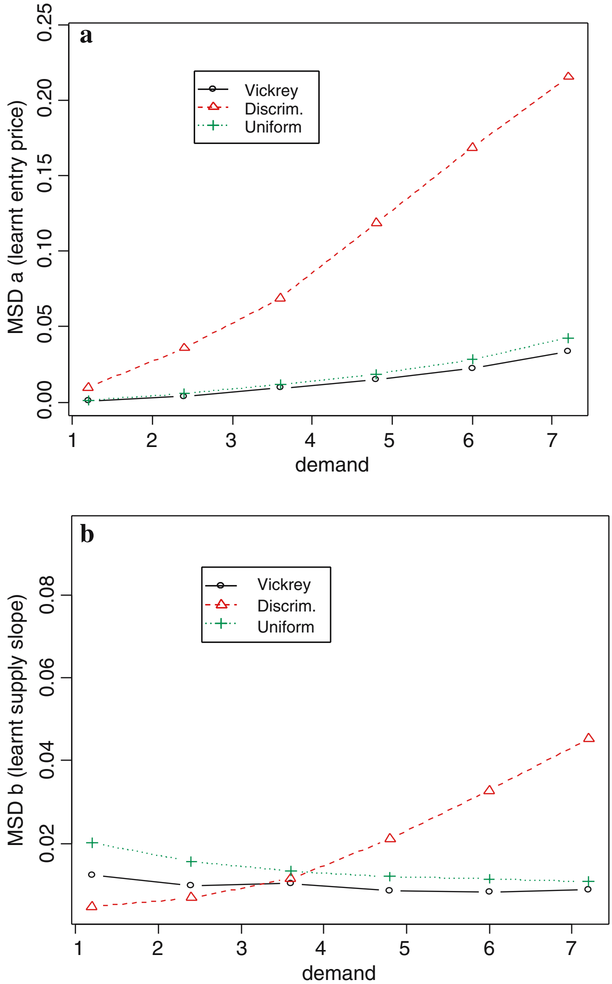

Fig. 1 Mean squared deviations (MSD) of learnt bid intercept and slope values from true values: auction with homogeneous bidders (**a**) MSD for bid intercept (entry price) (**b**) MSD for learnt bid slope

clear evidence of this in the increasing deviations. Figure 2 shows that the learnt supply curves for a bidder under the discriminatory auction assume higher and flatter positions as the level of demand increases.

4.2 Results for a heterogeneous bidder population

Size has more effect on bidding behaviour under the uniform auction and has little or no effect under the discriminatory auction. Bigger bidders undertake more supply inflation activities than their smaller counterparts. Smaller bidder tend to be relatively more truthful thereby “free riding” on the risks taken by the

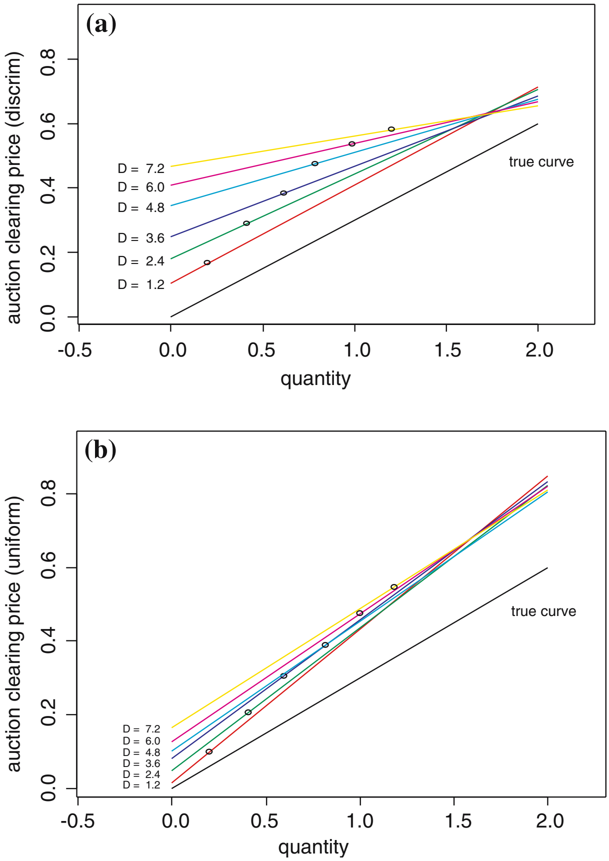

Fig. 2 Learnt bid curves (*dots* indicate quantities supplied by the bidder, D is the level of demand by the auctioneer) for an homogeneous population of bidders (**a**) Discriminatory auction (**b**) Uniform auction

bigger ones who inflate clearing prices under the uniform and also the Vickrey auctions. The propensity to bid more truthfully that is evident at lower competition levels can be explained by the fact that each bidder has a lower probability of being the price setter as the demand increases. Infra-marginal bidders have no incentives to inflate their bids as it does not impact the price they get. Under the discriminatory auction, on the other hand, smaller bidders cannot benefit from a similar "externality" effect and thus exhibit similar bidding behaviour as their bigger counterparts.

In summary, the following general observations can be made about learnt bidding strategies.

1. The Vickrey auction leads to more sincere bidding. This agrees with theoretical analysis which predicts truthful bidding to be a weakly dominant strategy for this auction. It can also be looked at as a confirmation that the learning algorithm used in the simulations does lead to coherent outcomes.

But the theory does not preclude overbidding and coordination at higher prices even under this auction. What our simulations indicate is that it can be a significant phenomenon, especially when there is heterogeneity in size, with bigger suppliers finding it easier to coordinate bidding strategies.

2. Overbidding is the norm under the discriminatory auction. The theoretically predicted high flat bidding (higher entry price and lower or zero slopes) is the most predominant strategy. In fact this strategy is almost the only strategy followed by bidders unless the competition level is very high. When competition is stiff, supply inflation (same entry price with inflated slope) is a better strategy as it helps minimize the risk of zero gains.
3. The uniform auction induces two types of strategies: some truthful bidding and overbidding. The overbidding takes mainly the form of supply inflation when competition is high but increasingly involves higher entry prices with steeper, true or flatter slopes as the level of competition weakens. Overbidding tends to be more predominant among larger bidders. However, the differences in bidding behaviour observed here among big and small bidders are less than what one would expect because the effects of diminishing return on size have been removed through the design of the cost structures for our the experiments.

4.3 Auction performance

The performance of an auction is measured using the following two criteria: budgetary outlay and social production (opportunity) costs. The former measures the monetary transfers from the buyer to the bidders. The latter measures the auction's efficiency. From a social welfare perspective, the auction outcomes are more efficient if the product purchased is sourced from lower cost producers minimizing the social opportunity costs.

The simulated results indicate that the relative social cost efficiency of the three auction formats is not affected by the level of heterogeneity. Performance is equivalent for the two types of bidder population. However, relative efficiency performance depends on the level of competition. For high levels of competition, the three auction formats perform identically. As competition declines, the efficiency of the discriminatory auction becomes inferior to the efficiency of the uniform and the Vickrey, which remain similar. These results are related to the frequencies of truthful bidding displayed under the three formats.

The three auction designs differ more in terms of budgetary performance than in social efficiency outcomes. However, an interesting result is that with a heterogeneous population, the outlay per unit is either equal or superior to the outlay per unit with a homogeneous population. In other words, in all of the three formats, heterogeneity in the bidder population induces equivalent or higher costs for public authorities. This result is a strong argument in favour of organizing auctions gathering homogeneous bidders.

In terms of budgetary performance, the uniform and Vickrey auctions perform equivalently for high levels of competition. Both lead to lower expenses

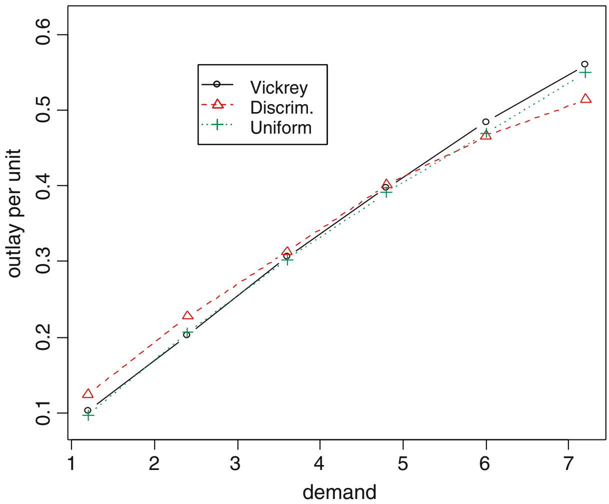

Fig. 3 Budgetary performance (outlay per unit): heterogeneous bidders

than the discriminatory at lower competition levels. However, the uniform auction induces slightly lower outlays than the Vickrey at high demand levels. More important, both loose their advantage over the discriminatory which becomes the least expensive procurement auction at high demand levels (see Fig. 3). This is mainly due to the fact that for higher levels of demand, the clearing price under uniform is nearly as high as under discriminatory (due to the supply inflation strategy of big bidders). Since bidding under discriminatory is not consistently flat, it results in lower total outlay than under the uniform. Vickrey yields to large outlays because sincere bidding declines when the level of competition declines: this format employs a payment scheme which can be extremely costly if bidders coordinate overbidding successfully.

5 Conclusions

Economic theory does not provide an analytical description of the equilibrium bidding strategies under multi-unit uniform and discriminatory auctions. The objective of this paper is to contribute towards filling this knowledge gap by using computational experiments to simulate bidding behaviour and auction performance for three formats: uniform, discriminatory and generalized Vickrey auctions. The experiments are undertaken for six different demand levels, ranging in magnitude from 10 to 60% of aggregate supplier capacity. Results are generated for one homogeneous population and one heterogeneous population of bidders.

Simulations show that bidding under the discriminatory format is dominated by a single strategy (high flat bidding) whereas more mixed strategies are employed by bidders when the auction is uniform or Vickrey. Both these auctions lead to a fair degree of truthful bidding especially on the first unit, with this being more prevalent under the Vickrey. However, as competition weakens, overbidding becomes more frequent even with these formats. The level of overbidding is higher for larger bidders. Thus, the results provide some confirmation of the theoretical predictions for bidding behaviour under these auctions. But they also indicate that behaviour can be dominated by alternative behaviours and measure the degree to which these behaviours prevail over others. For example, for the discriminatory auction a strategy that minimizes the risk of zero gains (supply inflation) is observed in our simulations when demand is low. Similarly, for the uniform and Vickrey auctions, coordination at higher prices is possible with these auctions if competition is weak. Heterogeneity in size has an impact on bidding behaviour in the uniform and Vickrey auctions but almost no effect under the discriminatory as small bidders cannot 'free-ride' on the supply inflation undertaken by their bigger counterparts under this auction.

The analysis of the relative performance of auctions in terms of budget outlays also delivers a strong message. The discriminatory auction, which is commonly used in practice, is in most cases the most expensive for the procurement agency and also in terms of social cost efficiency. The Vickrey and uniform auctions are similar and lead to better efficiency and lower budgetary outlays. However, for lower competition levels, the picture is partly reversed with the discriminatory leading to better budgetary outcomes than the uniform which in turn slightly outperforms the Vickrey. Another important result, which has practical implications, is that outlay per unit bought is always lower with a homogeneous population as compared to an heterogeneous one: this indicates that decision-makers should seek to organize auctions amongst homogeneous bidders (in terms of cost structure and size).

To sum up, this paper has attempted to provide new insights into questions which cannot find responses in theoretical analysis. Moreover, it has described a flexible simulation instrument that can be used to rapidly generate bidding strategies and assess the performance of different multi-unit auction formats for different types of bidding populations. Simulation using agent-based models can be an important policy research and decision support tool.

Appendix

A.1 Auction algorithm

The steps undertaken by bidder and buyer agents in the simulation are presented below

Stage 1 (buyer agent):

1. Invite bids from suppliers.
2. Collect bid functions or schedules submitted by bidders.
3. Construct an aggregate bid curve.

4. Based on demand level, determine cut-off price on aggregate bid curve and then determine how many units each bidder will be supplying.
5. Determine amount of payments for each winning bidder based on the nature of the auction format in effect (see Fig. 4).
6. Announce results informing each bidder the quantity she supplie and the payment she receives. Go to (1) for next auction round.

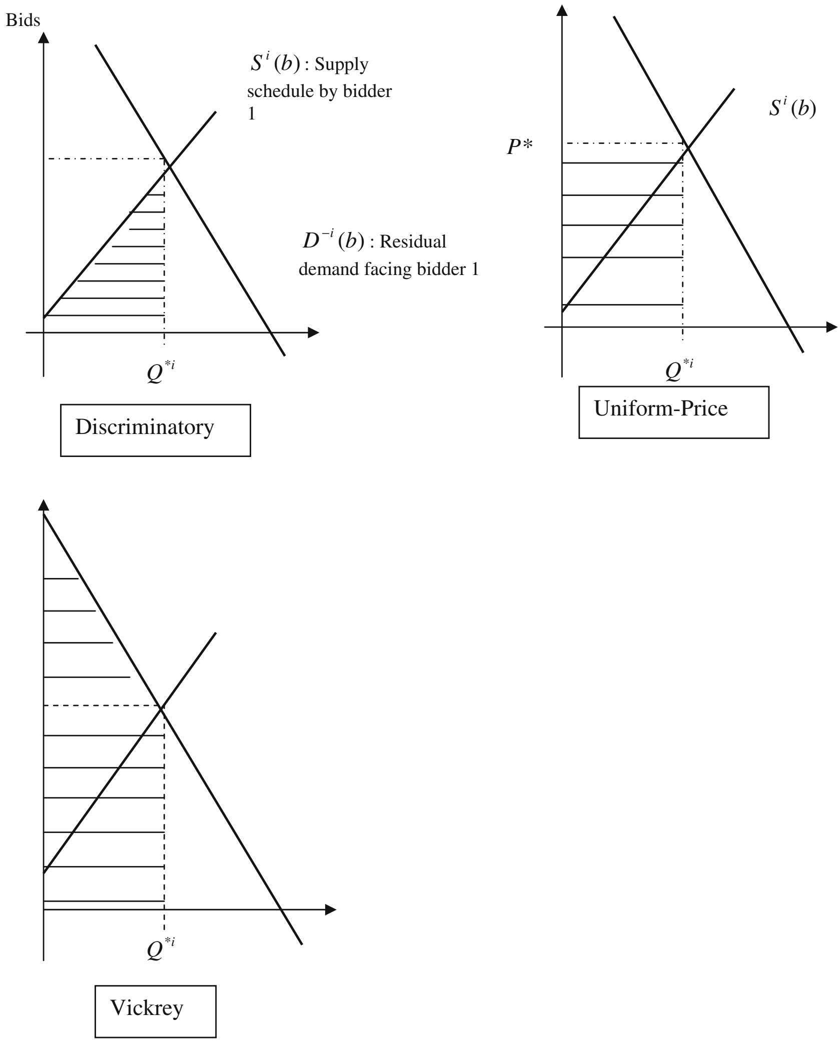

Fig. 4 Determination of payments under different payment rules for multi-unit auctions with continuous bid schedules

Stage 2 (bidder agent):

1. If this is the first round, make all bid function parameter choices equally likely by setting the probability of choice the same for all (i.e. $p_n = 1/N$, where N is the number of learnt a^l and b^l combinations to choose from). Set the initial propensity for any given strategy to the product of the scale parameter and an expected profit from bidding. In our case, the latter is set equal to the cost of supplying an amount equal to the maximum capacity.
2. Choose a learnt a^l and b^l combination to bid with and submit a supply bid curve which has these parameters as the intercept and slope, respectively. The probability that a particular combination is chosen depends on the probability profile over the choice space.
3. Submit chosen bid curve.
4. Wait for auction results from buyer agent (auctioneer).
5. Update the strategy choice propensities based on auction results using the reinforcement algorithm rules allowing for reinforcement, experimentation and recency effects. Start from (1) in the next auction round.

A.2 Payment rules in multi-unit auctions (see Fig. 4)

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