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REGULAR ARTICLE

Impact of investor's varying risk aversion on the dynamics of asset price fluctuations

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Abstract While investors' responses to price changes and their price forecast have been identified as one of the major factors contributing to large price fluctuations in financial markets, our study shows that investors' heterogeneous and dynamic risk aversion (DRA) preferences may play a more critical role in understanding the dynamics of asset price fluctuations. We allow an agent specific and time-dependent risk aversion index in a popular power utility function with constant relative risk aversion to construct our DRA model in which we made two key contributions. We developed an approximated closed-form price setting equation, providing a necessary framework for exploring the impact of various agents' behaviors on the price dynamics. The dynamics of each agent's risk aversion index is modeled by a bounded random walk with a constant variance, and such dynamics is incorporated in the price formula to form our DRA model. We show numerically that our model reproduces most of the "stylized" facts observed in the real data, suggesting that dynamic risk aversion is an important mechanism for understanding the dynamics of the financial market and the resultant financial time series.

Keywords Agent-based model · Dynamic risk aversion · Asset price fluctuation · Volatility clustering · Dynamics of financial markets · Financial time series

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1 Introduction

There have been many attempts to construct models of financial markets for understanding the dynamics and key statistical features of financial time series. It remains a great challenge, however, due to the inherent complexity of the financial market, to develop a parsimonious market model that can reproduce all the key "stylized" facts observed in real financial data and provide insights into the market mechanism for the emergence of these stylized facts. One of the promising approaches is agent-based modeling, which has reproduced and explained the emergence of some of the "stylized" facts. The agent-based modeling provides an ideal framework for investigating the impact of investors' behaviors on the price dynamics from many different perspectives; it has become an indispensable tool for understanding the price dynamics of financial markets.[1] With agent-based modeling, one can model and investigate, for example, how investors make their price forecasts and how their price forecasts influence price fluctuations (Arthur et al. 1997; DeLong et al. 1990; Levy et al. 2000), how investors respond to price change (Caldarelli et al. 1997; Lux and Marchesi 1999), and how investors form and change their market beliefs (Barberis et al. 1998; LeBaron et al. 1999). In this paper we use agent-based models to study how investors' fluctuating risk preferences affect the price dynamics. This important issue has not been fully explored.

Financial markets present many important and challenging problems. First, the market consists of intelligent, competing and heterogeneous agents who, with different beliefs in the market, different abilities to acquire and process market information, and mutually conflicting interests, try to make investment decision for their own benefits. Second, each agent's decision depends on his estimate of price expectations of other agents who also make their own estimates; this precludes expectations being formed by deductive means and leaves inductive reasoning as the only choice (Arthur et al. 1997). A market of agents employing inductive reasoning often exhibits irrational herding behavior (Bak et al. 1997; Cont and Bouchaud 2001) resulting in excessive price fluctuations or sometimes market bubbles and crashes. In such market agents' sentiment and degrees of risk aversion play a critical role in determining its price dynamics. Third, agents can learn and therefore adapt their strategies dynamically to improve their performance; this exacerbates the unpredictability of the markets. The change in an investor's strategy or behavior can be the cause or the result of investors' changing sentiment (Barberis et al. 1998), represented by (pessimistic) under-reaction or (optimistic) over-reaction to market dynamics driven by arrival of new information and changing market macro/micro-envi-

[1] For a more comprehensive and updated review the reader is referred to Tesfatsion (2006), agent-based computational economics; LeBaron (2006), agent-based computational finance; and Hommes (2006), Heterogeneous agent-based models in economics and finance.

ronment. These characteristics of financial market (competing with different beliefs and conflicting interests, interdependence of price expectations, and unpredictable changes of risk aversion) may lead to the formation of so-called noise trader behaviors. DeLong et al. (1990) have found this a (behavioral) source for price to diverge significantly from the fundamental value in financial markets – the so called "noise trading effect". A good model, needless to say, must successfully address these challenges; more importantly, it must be able to produce simulation time series that can capture the key "stylized" empirical facts observed in real financial time series.

This paper reports our efforts in constructing such model. In particular we are interested in exploring the underlying causes of the dynamics of financial market and the resultant financial time series. In addition to the efforts mentioned above, there have been a few more candidate theories suggested, in an effort to explain the underlying mechanism for the emergence of the stylized fact observed in financial time series. These include, for example, increased but exogenously fixed frequency for updating forecasting models in the form of 'retraining' of the adaptive agents (Arthur et al. 1997); endogenous switching resulting in herd behavior (Boswjk et al. in Press); and the Red Queen endogenous retraining of adaptive investors which requires investors to 'retrain' when their wealth falls below the average (Markose et al. 2005), etc. Here we try to explore an alternative mechanism, investors' dynamic risk aversion (DRA), for the excess price fluctuations and volatility clustering in financial time series.

Although the underlying causes of investor's time-varying risk aversion may be very complex and therefore remain unclear so far, there have been many instances of great market impacts (arguably) caused by such time-varying risk aversion of investors. One well-known example is "The Black Monday" on October 19, 1987, when the Dow Jones Industrial Average (DJIA) fell 22.6%. Such huge one-day decline was not confined to the United States but was mirrored all over the world. By the end of October, stock markets in Australia had fallen 41.8%, Canada 22.5%, Hong Kong 45.8%, and the United Kingdom 26.4%. Although there is no consensus on why such crashes happened, the panic theory (Leland and Rubinstein 1988) and an argument based on investors' increased perception of stock market risk (Rubinstein 2000) give a reasonable explanation. It is reasonable to attribute these crashes to investors' changed risk aversion (which led to their changed price forecast as well) as there has not been identifiable significant change on the market fundamentals. In addition, economic fundamentals cannot be changed so radically within such short period of time. If one (like most of theoretical frameworks do) assumes a normally distributed error for return forecasting at aggregated level, then the changed risk aversion is the most likely cause for such large price movement.

It is beyond the scope of this paper to compare time varying risk aversion to a loss aversion strategy that comes from insurance industry where the regulation mandates a very strict loss protection rule. When the market moves unfavorably, forced-selling must be done immediately in order to protect the guaranteed return, as for the case of an insurance plan such as constant proportion portfolio insurance (CPPI) (Black and Perold 1992). Can such practice be considered to be a case of changed risk aversion as the portfolio manager's risk aversion experiences a jump when the portfolio is at or below the guaranteed value? To detail and analyze all the possible underlying causes why investors dynamically change their risk aversion preferences may deserve a separate research. This is not the focus of the paper. Our primary interest in the paper is to study whether such dynamic risk aversion is valid alternative mechanism responsible for excess price fluctuations and volatility clustering in financial time series.

We outline here our model of interacting heterogeneous agents. We first consider a baseline model, in which the agents use past price information to form their sets of future price expectations. The agents are adaptive as their price expectations are not based on one particular estimator but are determined by their best-performance estimators which may change from time to time. The agents also use their erroneous stochastic beliefs (DeLong et al. 1990) in the market to make price adjustment on their best forecasts, which are assumed to be normally distributed.

In our baseline model, we assume that all agents have a decreasing absolute, but constant relative, risk aversion (DARA and CRRA) utility function, $U(c) = (c^{1-\gamma}-1)/(1-\gamma)$. The existing agent based models of stock market, such as the Santa Fe artificial stock market model, typically use a constant absolute risk aversion (CARA) utility function, $U(c) = -\mathrm{e}^{-\lambda c}$, for the price setting equation can be easily derived under such utility. As the focus of our paper is on risk aversion, we choose to use the well accepted (DARA) power utility function. Although a simple analytic formula for the demand function under this utility is not available, we have been able to derive a general functional form with a rather good approximation. Like the SFI market model, our baseline model market with a choice of the parameters corresponding to normal market conditions, exhibits some excess volatility, but not to the extent of the volatility observed in real markets. In addition, there is little enhancement of volatility clustering at high volatility regime, which is observed in real market data (Chen et al. 2005).

By simply allowing investors to change their risk aversion attitudes, we obtain excess volatility and volatility clustering in very good agreement with real market data. The implication is clear: dynamic risk aversion (instead of fixed constant risk aversion) is directly responsible for the excess volatility and the associated clustering. Specifically, in our DRA model, which is built from our baseline model, all agents have the power utility functions $((c^{1-\gamma}-1)/(1-\gamma))$ but with different and time-varying risk aversion indices (degrees), $\gamma_{i,t}$, which we assume to follow an independent bounded random walk with a constant variance δ^2. We will show that the magnitude of excess volatility is directly related to δ^2. With such DRA our model market exhibits most of the important statistical characterization of real financial data, such as the "stylized" facts related to excess volatility (Bouchaud and Potters 2000; Brock and LeBaron 1996; Cont 2001; Fama 1969; Mandelbrot 1963; Mantegna and Stanley 1999) and volatility clustering (Baillie et al. 1996; Chen et al. 2005; Chou 1988; Engle 1982; Fama 1965; Mandelbrot 1963; Poterba and Summers 1986; Schwert 1989).

We have also studied the impact of the dynamic risk aversion on the market dynamics using a few other baseline models, including the SFI market model, and we found similar results. This suggests that our results on dynamic risk aversion are rather generic.

The paper is organized as follows: The next section contains a derivation of the price equation under the power utility function for risk aversion agents. Section 3 describes our baseline model with constant risk aversion. Section 4 introduces our DRA model. Section 5 reports the results from numerical simulations of the model. Section 6 tests an alternative DRA scheme. Section 7 considers a DRA model built with the SFI market model as the baseline model. The last section summarizes.

2 Demand function and price setting of the model

2.1 Asset demand with consumption-based model

We consider a market of N heterogeneous agents who form their subjective expectations inductively and independently based on their investment strategies. There are two assets, a risky stock paying a stochastic dividend with a limited supply of N shares,[2] and a risk-free bond paying a constant interest rate, r, with infinite supply. All agents have the same form of power utility function, $U(c_t;\gamma_{i,t}) = \frac{c_t^{1-\gamma_{i,t}}-1}{1-\gamma_{i,t}}$, but they have their own time-dependent risk aversion indices denoted by $\gamma_{i,t}$. At each time step t, every agent decides how to allocate his wealth between the risk-free bond and the risky stock. Since the values for both the dividend payment and the stock price at the next period $t+1$ are unknown random variables, the investors can only estimate the probability of various outcomes. Assume each agent's estimation at time t of the next step's price and dividend is normally distributed with the (conditional) mean and variance, $E_{i,t}[p_{t+1}+d_{t+1}]$ and $\sigma^2_{i,t}$ $(i = 1,2,\ldots,N)$ respectively. It can be shown, by optimizing the total utility, that the demand of agent i for holding the share of the risky stock is approximately

$$D_{i,t} = \frac{E_{i,t}[p_{t+1}+d_{t+1}] - p_t(1+r)}{\gamma_{i,t}\sigma^2_{i,t}p_t(1+r)} \tag{1}$$

where p_t is the stock price at time t, $\gamma_{i,t}$ is agent i's index (degree) of risk aversion, and $\sigma^2_{i,t}$ the conditional variance of price estimation errors.

The market clearing condition: $\sum_i^N D_{i,t+\tau} = \sum_i^N D_{i,t} = N$ can be used to determine the current market price and relate the price at time $t+\tau$, $p_{t+\tau}$ to the price at time t, p_t:

[2] In practice the number of shares is never the same as the number of agents; here we set the two numbers the same for the sake of convenience and setting them different does not change the results.

$$p_t = \frac{\sum_i^N \frac{E_{i,t}[p_{t+1} + d_{t+1}]}{\gamma_{i,t}\sigma_{i,t}^2(1+r)}}{N + \sum_i^N \frac{1}{\gamma_{i,t}\sigma_{i,t}^2}} \tag{2}$$

and

$$p_{t+\tau} = \frac{\sum_i^N \frac{E_{i,t+\tau}[p_{t+\tau+1}+d_{t+\tau+1}]}{\gamma_{i,t+\tau}\sigma_{i,t+\tau}^2(1+r)} - \sum_i^N \frac{1}{\gamma_{i,t+\tau}\sigma_{i,t+\tau}^2}}{\sum_i^N \frac{E_{i,t}[p_{t+1}+d_{t+1}]}{\gamma_{i,t}\sigma_{i,t}^2(1+r)} - \sum_i^N \frac{1}{\gamma_{i,t}\sigma_{i,t}^2}} p_t \tag{3}$$

It can be seen from the demand and price equations that the degree of the agent's risk aversion plays an important role.

2.2 Approximation of demand and price equations

We now show the derivation of the above equations. Assume at time t agent i's consumption is $c_{i,t}$, and he invests a portion x of his current consumption in the risky asset. His total utility function defined over the current and future values of consumption is

$$U(c_{i,t}, c_{i,t+1}, \gamma_{i,t}) = U(c_{i,t}, \gamma_{i,t}) + U\left(\frac{c_{i,t+1}(x)}{R_f}, \gamma_{i,t}\right) \tag{4}$$

where the consumption at time $t+1$ can be written as

$$c_{i,t+1}(x) = c_{i,t}[(1-x)R_f + x\tilde{R}_{i,t+1}] \tag{5}$$

Here $R_f = 1 + r$ is the gross risk-free return and $\tilde{R}_{i,t+1}$ the gross return on the risky asset. Agent i determines the amount of his investment on the risky asset, x, by maximizing his total utility, Eq. (4). The maximization problem can be written as

$$\begin{aligned} &\max_x E_t\left[U(c_{i,t}, \gamma_{i,t}) + U\left(\frac{c_{i,t+1}(x)}{R_f}, \gamma_{i,t}\right)\right] \\ &\equiv \max_x E_t\left[U\left(\frac{c_{i,t+1}(x)}{R_f}, \gamma_{i,t}\right)\right] \end{aligned} \tag{6}$$

The last equality follows because the utility $U(c_{i,t}, \gamma_{i,t})$ is known at time t and it does not contain x.

The power utility function is given by

$$U(c_{i,t}; \gamma_{i,t}) = \frac{c_{i,t}^{1-\gamma_{i,t}} - 1}{1 - \gamma_{i,t}} \tag{7}$$

Substituting Eq. (5) into Eq. (7) and then inserting it back to Eq. (6), the maximization is now given by

$$\max_x \frac{c_{i,t}^{1-\gamma_{i,t}}}{1-\gamma_{i,t}} E_t\{[1+\tilde{r}_{i,t+1}x]^{1-\gamma_{i,t}}\} \tag{8}$$

where $\tilde{r}_{i,t+1} = \frac{\tilde{R}_{i,t+1}-R_f}{R_f}$ is the present (discounted) value of the *net* return at next time step t+1. Assuming that agent i's prediction errors of $\tilde{r}_{i,t+1}$ are (conditionally) normally distributed:

$$\tilde{r}_{i,t+1} = E_t(\tilde{r}_{i,t+1}) + z_{i,t} = r_{i,t}^e + z_{i,t} \tag{9}$$

where $r_{t,i}^e = E_t(\tilde{r}_{i,t+1})$ is the conditional expected net return, at time t, of the next time step; and the error of estimation is $z_{i,t} \sim \mathcal{N}(0, \sigma_{i,t}^2)$. The maximization becomes

$$\begin{aligned} &\max_x E_t\left\{(1+xr_{i,t}^e + xz_{i,t})^{1-\gamma_{i,t}}\right\} \\ &= \max_x \left\{\frac{1}{\sqrt{2\pi\sigma_{i,t}^2}} \int_{-\infty}^{\infty} \mathrm{e}^{-\frac{z_{i,t}^2}{2\sigma_{i,t}^2}} f(z_{i,t}; x, \gamma_{i,t}) \mathrm{d}z_{i,t}\right\} \end{aligned} \tag{10}$$

where $f(z_{i,t}; x, \gamma_{i,t}) = (1 + xr_{i,t}^e + xz_{i,t})^{1-\gamma_{i,t}}$. In writing down the above equation we implicitly assumed $(1 + xr_{i,t}^e + xz_{i,t}) > 0$, which is a necessary requirement for the power function to be a valid utility measure. The above integral can be further simplified to

$$\frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \mathrm{e}^{-z_{i,t}^2} f(\sqrt{2}\sigma_{i,t} z_{i,t}; x, \gamma_{i,t}) \mathrm{d}z_{i,t} \tag{11}$$

This Gaussian integral cannot be evaluated analytically, but it can be approximated by an expansion based on the roots of Hermite Polynomial $H^{(n)}(\xi)$ as

$$\begin{aligned} &\int_{-\infty}^{\infty} \mathrm{e}^{-z_{i,t}^2} f(\sqrt{2}\sigma_{i,t} z_{i,t}; x, \gamma_{i,t}) \mathrm{d}z_{i,t} \\ &\quad = \sum_{k=1}^{n} \lambda_k^{(n)} f(\sqrt{2}\sigma_{i,t}\xi_k^{(n)}; x, \gamma_{i,t}) \end{aligned} \tag{12}$$

where $\lambda_k^{(n)} (k = 1, 2, \ldots, n)$ are the coefficients of the summation, and $\xi_k^{(n)}$ are the roots of nth Hermite polynomial $H^{(n)}(\xi)$. Performing the maximization by setting the derivative with respect to x equal to zero, we obtain the following equation:

$$\sum_{k=1}^{n} \lambda_k^{(n)} \left[1 + \left(r_{i,t}^e + \sqrt{2}\sigma_{i,t}\xi_k^{(n)}\right) x\right]^{-\gamma_{i,t}} \times \left(r_{i,t}^e + \sqrt{2}\sigma_{i,t}\xi_k^{(n)}\right) = 0 \tag{13}$$

Since $|r^e_{i,t}| \ll 1$, $\xi^{(n)}_k \sim \mathcal{O}(1)$, and $\sigma_{i,t} \ll 1$ for a typical time step of one day or shorter, the above can be approximated as

$$\sum_{k=1}^{n} \lambda^{(n)}_k \left[1 - \gamma_{i,t}(r^e_{i,t} + \sqrt{2}\sigma_{i,t}\xi^{(n)}_k)x\right] \times \left[r^e_{i,t} + \sqrt{2}\sigma_{i,t}\xi^{(n)}_k\right] = 0 \tag{14}$$

Here we consider an approximation with $n = 2$. Note that $\lambda^{(2)}_1 = \lambda^{(2)}_2 = \lambda$, $\sqrt{2}\xi^{(2)}_1 = -\sqrt{2}\xi^{(2)}_2$ $(= 1)$, the optimal demand of agent i of risky stock can then be obtained as

$$\begin{aligned} D_{i,t} = x_{i,t} &= \frac{r^e_{i,t}}{\gamma_{i,t}[(r^e_{i,t})^2 + \sigma^2_{i,t}]} \approx \frac{r^e_{i,t}}{\gamma_{i,t}\sigma^2_{i,t}} \\ &= \frac{E_{i,t}[p_{t+1} + d_{t+1}] - (1+r)p_t}{\gamma_{i,t}\sigma^2_{i,t}(1+r)p_t} \end{aligned} \tag{15}$$

which is Eq. (1). In writing down the above approximation we assume $(r^e_{i,t})^2 \ll \sigma^2_{i,t}$, which is certainly true when the time step is one day or shorter.

To get a more accurate approximation of the demand function, higher order Hermite polynomial roots are needed in the summation approximation to the integral in Eq. (11). It can be shown that, the demand function x_i in a higher order approximation is exactly the same as Eq. (15), except for an overall constant factor.

3 The baseline model

3.1 Price prediction

To use the demand and price setting function derived in the previous section, one still needs to incorporate each agent's prediction of the payoff at the next time step, $E_t(p_{t+1} + d_{t+1})$. We assume all agents use the past price information for price forecasting. The simplest way is to calculate a moving average of the available past prices and use it as a proxy of price forecasting for the next time step t+1. Since investors may have different investment horizons and evaluation strategies, they may use different time lags for their calculation of the moving average of past prices, implying that they have heterogeneous memory lengths (Levy et al. 1994). We consider each agent has his own M sets of predictors so that he can choose the best one for forecasting the price at the next time step. Each price predictor $E_{i,j}(p_{t+1} + d_{t+1})(i = 1,2,\ldots,N; j = 1,2,\ldots,M)$ is made of a moving average of past $L_{i,j}$ prices with a subjective erroneous stochastic adjustment:

$$\begin{aligned} E_{i,j,t}(p_{t+1}+d_{t+1}) &= MA_{i,j,t} \\ &= MA_{i,j,t-1}\left(1-\frac{1}{L_{i,j}}\right)+\frac{1}{L_{i,j}}(p_t+d_t)+\varepsilon_{i,j} \end{aligned} \tag{16}$$

where $\varepsilon_{i,j} \sim \mathcal{N}(0, \sigma_{p+d})$ is Gaussian random variable.

The conditional variance of the estimation errors, $\sigma^2_{i,t}$, is assumed to update with a moving average of the squared forecast error:

$$\sigma^2_{i,j,t} = (1-\theta)\sigma^2_{i,j,t-1} + \theta[(p_t+d_t) - E_{i,j,t-1}(p_t+d_t)]^2 \tag{17}$$

where θ $(0 < \theta \ll 1)$ is a weighting constant.

3.2 Dividend process

The dividend process is assumed to be a random walk:

$$d_t = d_{t-1} + r_d + \epsilon_t, \tag{18}$$

where ϵ_t is an i.i.d. Gaussian with zero mean and variance σ_d; r_d is the average dividend growth rate. Note that the dividend process in a real stock market may be more complicated than what we assumed here and it may vary from stock to stock. But our results are not sensitive to the choice of a dividend process.

3.3 The price setting equation of the baseline model

For heterogeneous agents with fixed constant risk aversion, the demand function Eq. (1) and price setting Eq. (2) and (3) can be written as

$$D_{i,t} = \frac{E_{i,t}[p_{t+1}+d_{t+1}] - p_t(r+1)}{\gamma_i \sigma^2_{i,t}(1+r)p_t} \tag{19}$$

and

$$p_t = \frac{\sum_i^N \frac{E_{i,t}[p_{t+1}+d_{t+1}]}{\gamma_i \sigma^2_{i,t}(1+r)}}{N + \sum_i^N \frac{1}{\gamma_i \sigma^2_{i,t}}} \tag{20}$$

$$p_{t+\tau} = \frac{\sum_i^N \frac{1}{\gamma_i}\left(\frac{E_{i,t+\tau}[p_{t+\tau+1}+d_{t+\tau+1}]}{\sigma^2_{i,t+\tau}(1+r)} - \frac{1}{\sigma^2_{i,t+\tau}}\right)}{\sum_i^N \frac{1}{\gamma_i}\left(\frac{E_{i,t}[p_{t+1}+d_{t+1}]}{\sigma^2_{i,t}(1+r)} - \frac{1}{\sigma^2_{i,t}}\right)} p_t \tag{21}$$

It can be see that the risk aversion indices of the agents play the role of weighting factors in the price setting equations.

If the agents are homogeneous in risk aversion, the above can be further simplified to

$$D_{i,t} = \frac{E_{i,t}[p_{t+1} + d_{t+1}] - p_t(r+1)}{\gamma\sigma_{i,t}^2(1+r)p_t} \tag{22}$$

and

$$p_t = \frac{\sum_i^N \frac{E_{i,t}[p_{t+1} + d_{t+1}]}{\sigma_{i,t}^2(1+r)}}{N\gamma + \sum_i^N \frac{1}{\sigma_{i,t}^2}} \tag{23}$$

$$p_{t+\tau} = \frac{\sum_i^N \left(\frac{E_{i,t+\tau}[p_{t+\tau+1}+d_{t+\tau+1}]}{\sigma_{i,t+\tau}^2(1+r)} - \frac{1}{\sigma_{i,t+\tau}^2}\right)}{\sum_i^N \left(\frac{E_{i,t}[p_{t+1}+d_{t+1}]}{\sigma_{i,t}^2(1+r)} - \frac{1}{\sigma_{i,t}^2}\right)} p_t \tag{24}$$

From the above equations we see that the constant risk aversion index γ only affects the overall level of demand but not its fluctuations. It also does not contribute to the price fluctuations (between the time t and time $t+\tau$). Thus in the case of homogeneous and constant risk averse agents, the main source of the price fluctuations is from the investors' price forecasting. The baseline model does not incorporate investor's changing sentiment. As a consequence, the price fluctuation is expected to be very limited and we will subsequently show that this is indeed the case.

4 Model with dynamic risk aversion

4.1 Heterogeneous and dynamic risk aversion

To extend the baseline model, we allow agents to have heterogeneous risk aversion indices (degrees), which vary with time. This reflects the fact that in a real financial market investors have different risk preferences and the investors' sentiment change with time. We assume that the risk aversion index of each agent follows an independent bounded random walk with a constant variance δ^2:

$$\gamma_{i,t} = \gamma_{i,t-1} + \delta z_{i,t}, \quad \gamma_{i,t} \in [\gamma_0, \gamma_u] \tag{25}$$

where $z_{i,t}$ is an i.i.d. Gaussian variable with mean zero and unit variance for agent i, $\gamma_0(>0)$ is the lower boundary, and $\gamma_u(>\gamma_0)$ the upper boundary. It is easy to relate the value of the index at time $t+\tau$ to the value at time t:

$$\gamma_{i,t+\tau} = \gamma_{i,t} + \delta \sum_{t=1}^{\tau} z_{i,t} = \gamma_{i,t} + \delta S_{i,\tau} \tag{26}$$

where $S_{i,\tau} = \sum_{t=1}^{\tau} z_{i,t}$ is the change of risk aversion index of agent i from time t to time $t+\tau$, which can be either positive or negative.

It is worthwhile to note that in real markets, the dynamics of investors' risk aversion attitudes may be more complicated than a simple random walk process we assume here. However, simplifying and idealizing of the real situation helps us to stay focused on the main purpose of investigating the impact of investors' fluctuating risk aversion on the price dynamics.

4.2 Price setting equation with dynamic risk aversion

Upon substitution of Eq. (26) into Eq. (3), we have

$$p_{t+\tau} = \frac{\sum_i^N \frac{1}{(\gamma_{i,t}+\delta S_{i,\tau})}\left(\frac{E_{i,t+\tau}(p_{t+\tau+1}+d_{t+\tau+1})}{\sigma^2_{i,t+\tau}(1+r)} - \frac{1}{\sigma^2_{i,t+\tau}}\right)}{\sum_i^N \frac{1}{\gamma_{i,t}}\left(\frac{E_{i,t}(p_{t+1}+d_{t+1})}{\sigma^2_{i,t}(1+r)} - \frac{1}{\sigma^2_{i,t}}\right)} p_t \tag{27}$$

Comparing Eq. (27) to Eq. (21) ($\gamma_{i,t} = \gamma_{i,t+\tau} = \gamma_i$) for the case of fixed constant risk aversion, we see that there is an extra term, $\delta S_{i,\tau}$, in the price setting equation in the case of DRA. Since $S_{i,\tau}$ can be either positive or negative and its value changes with time, $\gamma_{i,t} + \delta S_{i,\tau}$ deviates from $\gamma_{i,t}$ and fluctuates with time. Unlike the price impact induced by the fluctuation of the error in agents' price estimation, the impact of agent's fluctuating risk aversion has a non-linear effect on the price dynamics as it lies in denominator of the formula. As a result, such non-linear effect may introduce excess price fluctuations. Since $S_{i,\tau} \sim \mathcal{N}(0, \sqrt{\tau})$, for $\sqrt{\tau}\delta \approx 1$ and $\gamma_{i,t} \approx 1$, $\gamma_{i,t+\tau}(= \gamma_{i,t}+\delta S_{i,\tau})$ and $\gamma_{i,t}$ can differ substantially; this, aggregately, results in a large deviation of $p_{t+\tau}$ from p_t. Our numerical results, presented in the next session, clearly show that it is this risk aversion dynamics that gives rise to the excessive price fluctuations and the associated volatility clustering that are consistent with the empirical observation in the real market.

4.3 The range of DRA indices

We now examine the range of possible relative risk aversion indices. The choice of the range is important for modeling investors' decision-making; it has big impact on the price dynamics, as it directly affects investors' demand of the risky asset. The lower the index, the less risk-averse the investor is (thus the higher the demand of risky asset); and vice versa. Thus the risk-aversion attitude has great impact on the price dynamics through its influence on the demand. Some empirical and experimental studies reported that for a "typical" investor, the value of the risk-aversion index γ is in the range of 0–2 (Friend and Blume 1975; Mehra and Prescott 1985; Levy et al. 2000). Some authors (Mehra and Prescott 1985) used a value of risk aversion index with an upper limit of 10 in their treatment of the issue of the "Equity Premium Puzzle". However, to "explain" the "Equity Premium Puzzle" of NYSE over 50 years of US postwar period, one needs a relative risk aversion index of *250* if a consumption-based model is used (Cochrane 2005)! These empirical results show that it is better to model the risk aversion with a range of indices, instead of a fixed value. The range we specified consists of an upper bound and a lower bound for the random walk describing the dynamics of DRA indices.

5 The simulation results and analysis

5.1 The setup

In our simulation we choose the number of agents $N = 200$, the number of predictors each agent has, $M = 2$. Setting different number of agents produces similar results. The initial risk aversion indices $\gamma_{i,0}$ are all set to 1.0 for the baseline model and are set to $\gamma_{i,0} \in [0.2, 4]$ for the model with DRA. The bounds for the index of DRA are $\gamma_{i,t} \in [10^{-5}, 20]$, the risk-free interest rate is $r = 5\%$, the dividend growth rate is $r_d = 2\%$, and the weighting coefficients for the variance of estimation is $\theta = 1/250$. The lags used in the price estimators are $L_{i,j} \in [2, 250]$, and we set $\sigma_{p+d} = 1\%$ for all agents.

5.2 Simulation price and trading volume

Let us first take a look at how excess price fluctuations emerge from a dynamic risk aversion process. Figure 1 shows the simulation time series of the price and the trading volume generated from the model with fixed constant risk aversion (CRA) and the model with dynamic risk aversion (DRA).

From the figure we see clearly that the DRA leads to increased fluctuations in both the price and the trading volume. To have both qualitative and quantitative picture of the impact of the DRA on the price dynamics, we examine the key stylized facts in the following subsections.

5.3 Autocorrelation function

One of the stylized facts observed in real financial data is that the autocorrelation functions (ACF) for the squared or absolute-valued returns usually start with a low value (from ρ_1), but decay very slowly with increase of the time lag. For an almost-Gaussian process, the values of its ACFs for absolute-valued return are close to zero. In Fig. 2, we compare the ACFs of absolute-valued returns for the series generated by our baseline model (with CRA) and the DRA model, the series of real data (DJIA and S&P500 Index), and Gaussian process.

The figure clearly shows that while the ACFs (for the absolute-valued returns) generated from our baseline model (CRA) is close to that of Gaussian process (Gauss), the results generated from our DRA model are very close to the real financial data (DJIA, S&P500).

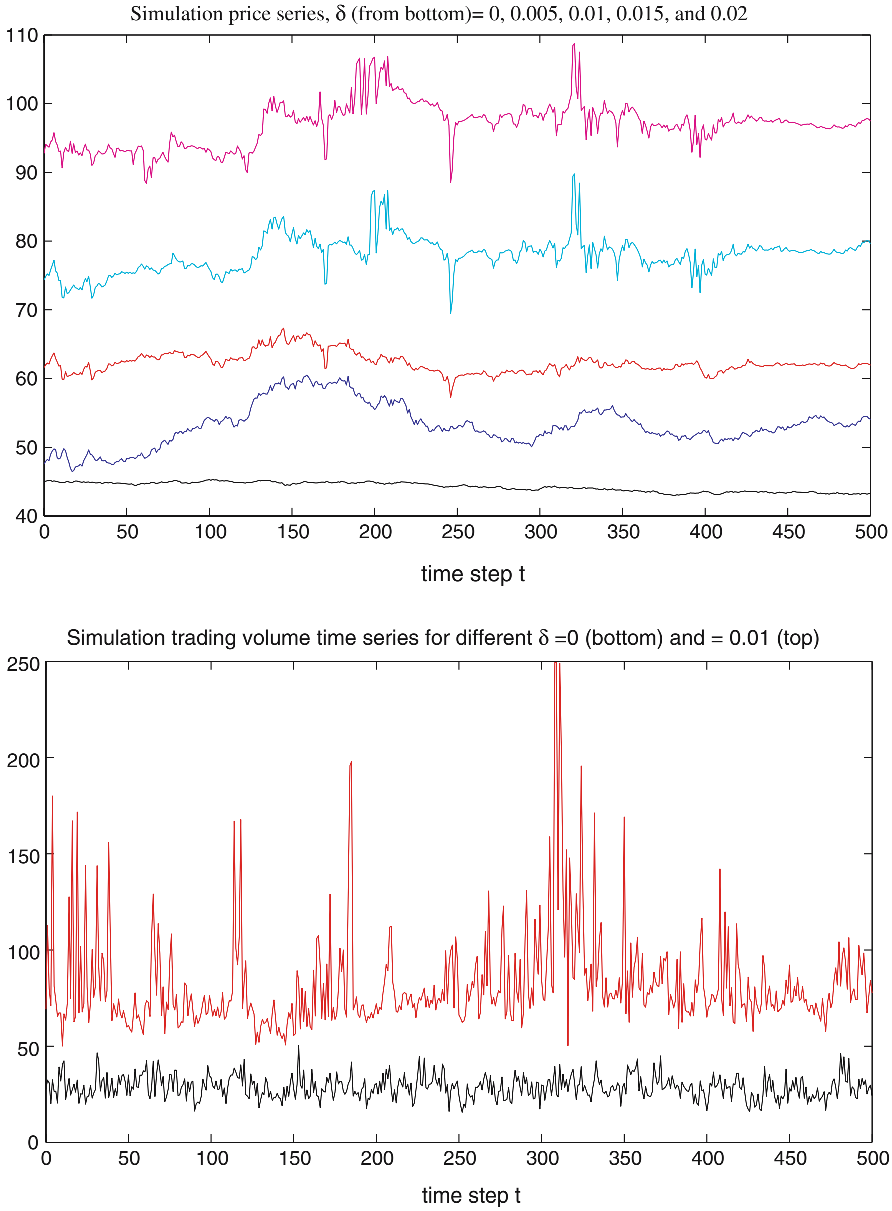

Fig. 1 The time series of price and trading volume from the models with constant ($\delta = 0$) and dynamic ($\delta \neq 0$) risk aversion. For the sake of clarity, the time series with different δs were vertically shifted, e.g., the trading volume for δ =0.01 was upward shifted by 50

5.4 Excess volatility

The second key stylized facts we examine is the excess volatility (or fat-tails) of returns, which measures the price fluctuation of real financial series. In Fig. 3

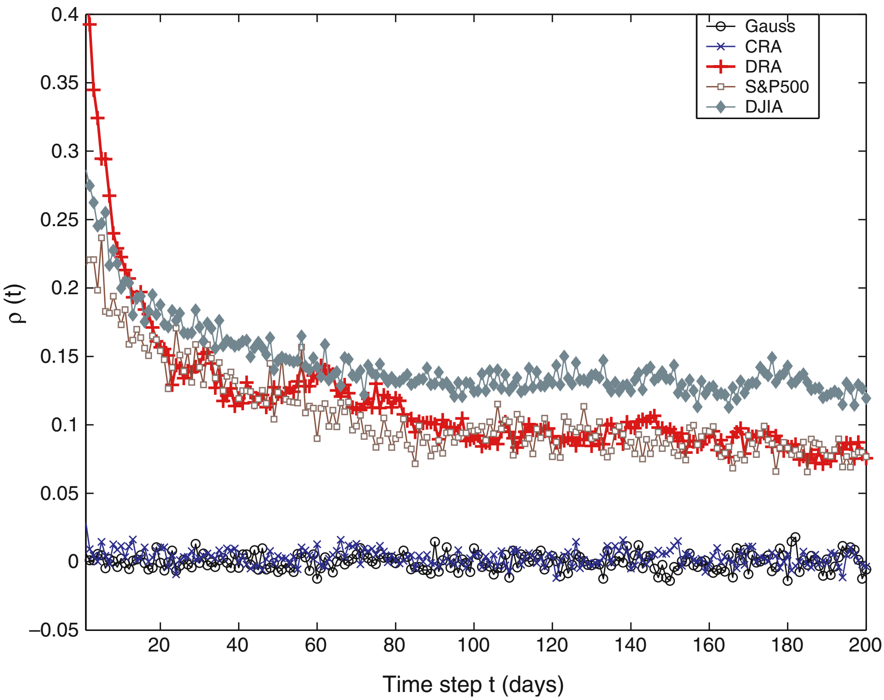

Fig. 2 ACFs of different time steps for (absolute-valued) return series of: the baseline model (CRA), the DRA model, DJIA Index, S&P500 Index, and random walk process (Gauss)

we plot the distributions of risk-adjusted returns (in different time-steps) from our baseline model and the DRA model with the parameters set according to a normal market condition. For comparison, we also plot the return distribution of DJIA and the return distribution generated by a simple Gaussian process.[3]

These plots show that the return distributions from our DRA model are very close to that of real (DJIA) data. On the other hand, the return distributions from the baseline CRA model are almost the same as those of the Gaussian process. We can see that the return distribution of the DRA model have clear power-law tails and such tails are in very good agreement with that of real market (DJIA) data. However, there is no such fat-tail observed from the distributions of the CRA model. Thus, it is clear that the dynamic risk aversion leads to excess volatility or a fat-tail in the return distribution, which is one of the most important characteristic of real financial time series Fama (1965), Mandelbrot (1963). From these plots, we can also see that the time scaling property of DRA model is also consistent to that of the real (DJIA) data, *i.e.*, the shorter the time period is, the fatter the tail of the distribution. However such

[3] We only display the right part of the return distribution to make a clear presentation and doing so does not reduce too much information from the whole distribution as the left part is roughly symmetric to the right.

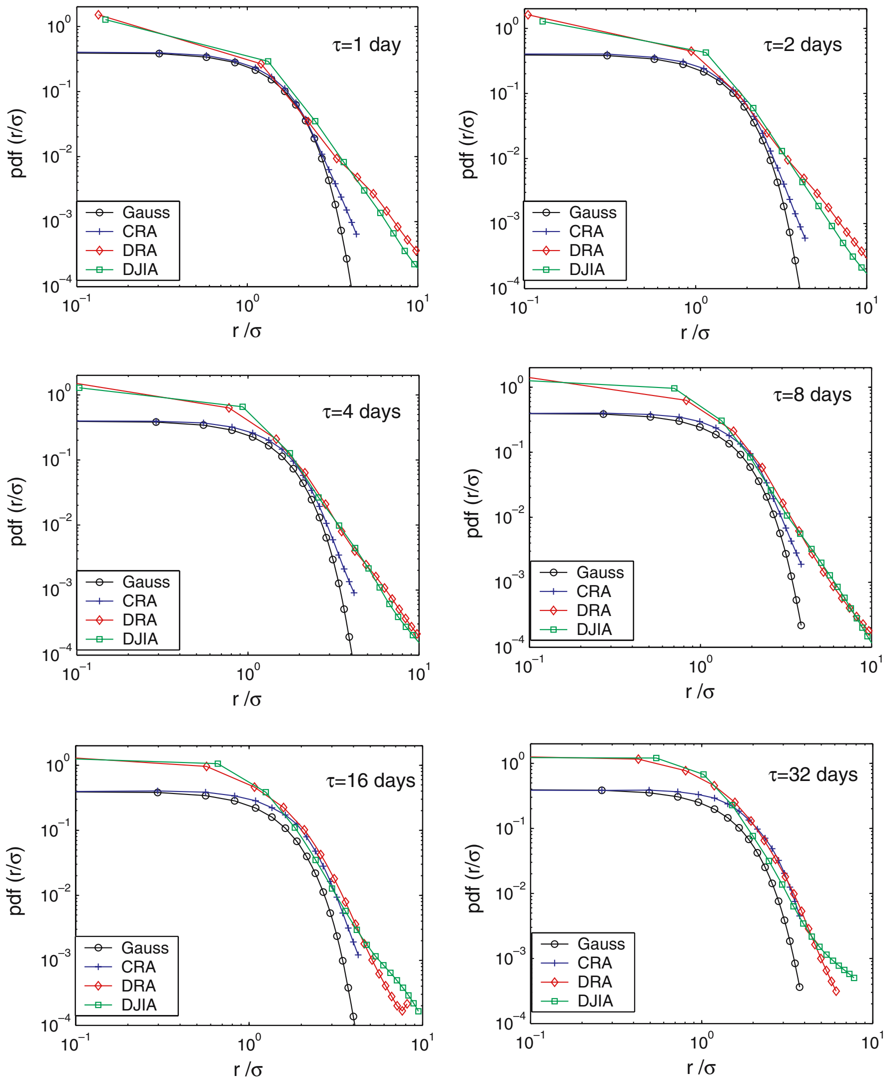

Fig. 3 Risk-adjusted return distributions for the CRA model (δ=0), the DRA model (δ=0.004), Gaussian process (Gauss) and real data of DJIA. Here r is the return and σ, the standard deviation of the return, which measures the volatility (or risk) of financial assets

empirical scaling property has not been observed from the CRA model. These observations suggest that the DRA could be responsible for the excess price fluctuations in financial markets.

To further examine the fat tail of the distribution, we plot, in Fig. 4, the Kurtosis as a function of the square root of variance of time-varying risk aversion, δ.

From these plots, we see that the risk aversion dynamics can change the return distribution significantly from a Gaussian distribution (which has K=3.0).

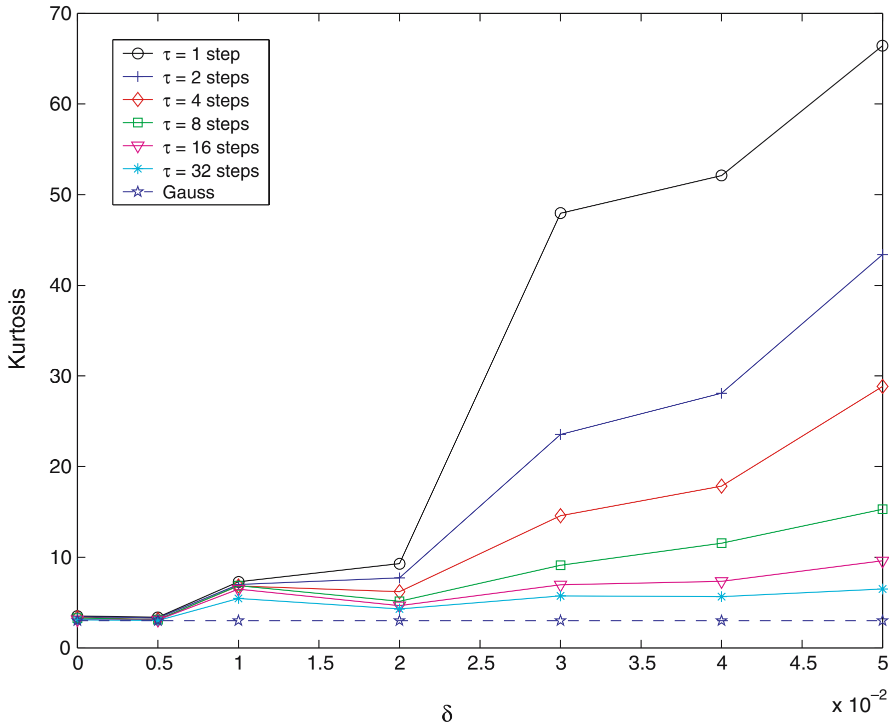

Fig. 4 Kurtosis of simulation price series for different variables δ of the DRA index

Table 1 Some statistics for DRA model (δ=0.01) and DJIA

Lag (τ)	SD	Skewness	Kurtosis
Model $\delta = 0.01$			
1	0.015004	−0.454483	48.406593
2	0.018313	−0.372956	35.233093
4	0.021685	−0.460952	26.051126
8	0.026179	−0.492277	17.309658
16	0.031840	−0.424137	11.376849
32	0.039121	−0.327192	7.444087
DJIA			
1	0.010876	−1.190497	40.2330
2	0.015710	−1.100639	29.9739
4	0.022213	−0.950496	18.9213
8	0.032103	−0.989078	13.6606
16	0.046404	−0.995792	12.0792
32	0.066602	−0.706621	9.5892

In addition, the smaller the lag τ, the larger the Kurtosis generated; this is consistent with the empirical observations in real financial data. To have a more quantitative picture, we display in Table 1, some more key statistics including the Kurtosis values, standard deviations, and the skewness of the return distributions for the DRA model and DJIA real market data.

Here again, all the statistics numbers show that the DRA model is indeed able to generate most of the statistics that are in consistent to that of real market data.

5.5 Volatility clustering

Volatility clustering is another important characteristics of financial time series. Here we use the conditional probability measure developed recently by Chen et al. (2005) to examine the volatility clustering. The method uses the return distribution conditional on the absolute return in the previous period to describe a functional relation between the variance of the current return and the absolute return in the previous period. If the volatility in asset returns is clustered, it will be proportional to the volatility in the previous period, the proportionality constant reflects the strength of volatility clustering.

Figure 5 shows how the current volatility depends on the volatility of the previous period for a random walk process, the baseline market model, DRA model, and DJIA daily data.

From the figures we see clearly that the baseline market model with fixed constant risk aversion (CRA) produces very low volatility clustering [the curve is flat if there is no volatility clustering, such as the case with the random walk model (Gauss)]. In contrast, the volatility clustering from the DRA model is significantly higher, and it is very close to the one generated from DJIA daily prices. The plots also show that, for our DRA model, the smaller the time period (step) of the return, the stronger the volatility clustering; and vice versa; this is consistent with the empirical observations of real market data.

6 An alternative DRA model

We have studied the impact of dynamic risk aversion on asset price dynamics and have shown that the DRA could explain the ubiquitously observed "stylized facts", such as the large traded volumes, the excess price fluctuation and volatility clustering. As our main focus in the current paper is to illustrate the impact of a time-varying risk aversion, we have chosen a simple random walk process for describing the dynamics of risk aversion in the previous section. In this session we examine if an alternative dynamic scheme of time-varying risk aversion can produce similar results.

6.1 Forecasting error-based risk aversion dynamics

The dynamics of our (original) DRA model is described by Eq. (26) where a simple bounded random walk process is applied. Here we consider another simple scheme for the DRA process (we call it DRA^+),[4] based on an assumption

[4] The authors thank Sheri Markose and the anonymous referee for suggesting this model.

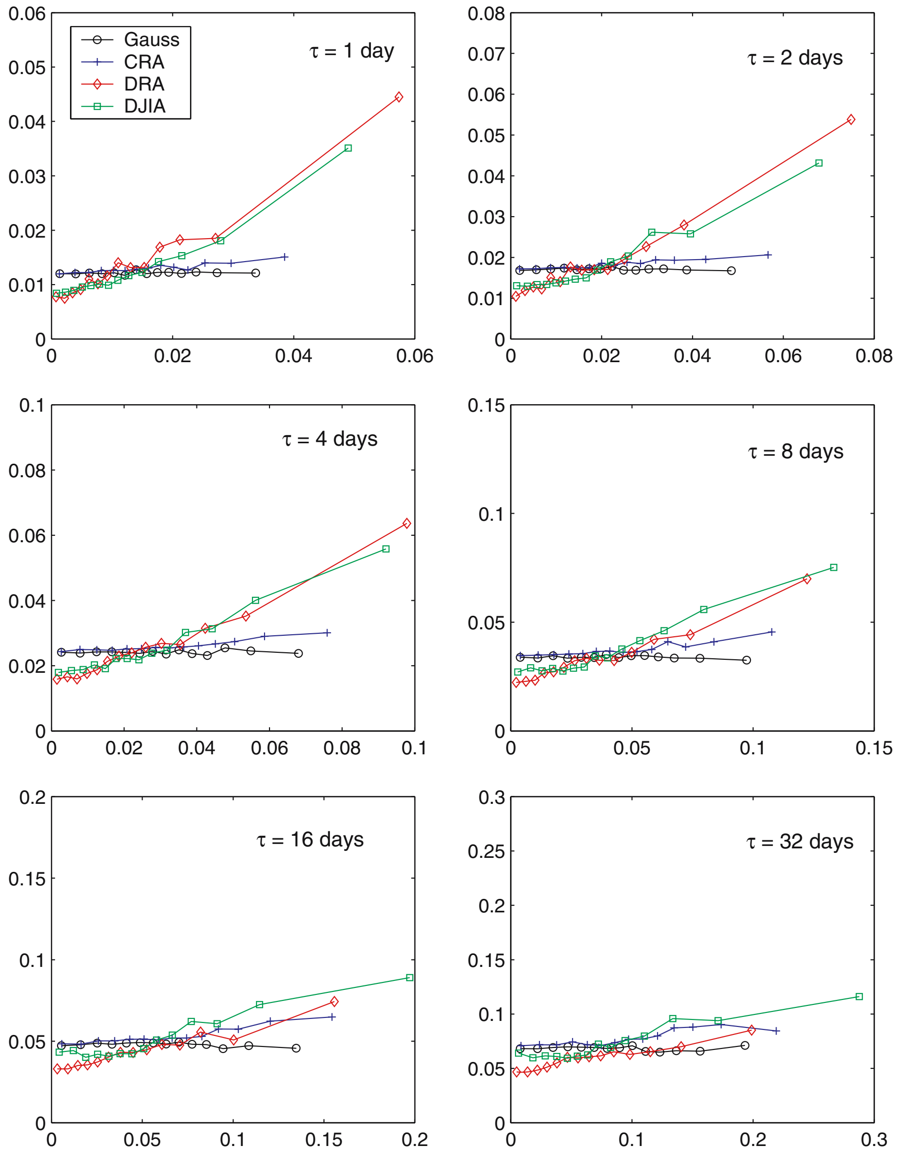

Fig. 5 The volatility clustering measured by the standard deviation of current return versus the absolute return of previous period

that the dynamics of risk aversion depends on the agent's accuracy of price forecasting. This scheme can be described as

$$\gamma_{i,t} = \gamma_{i,t-1} + \beta\varepsilon_{i,t}, \quad \gamma_{i,t}, \in [\gamma_0, \gamma_u] \tag{28}$$

where $\varepsilon_{i,t} = [\hat{p}_i(t-1) - p(t-1)]/p(t-1)$ is the forecasting error of agent i, over the last period; here $p(t-1)$ and $\hat{p}_i(t-1)$ are the realized price of the system and the predicted price of agent i respectively; β is a (constant) parameter which determines how fast the agents change their risk preference in DRA^+ model. It is simple to relate the value of the index at time $t+\tau$ to the value at time t:

$$\gamma_{i,t+\tau} = \gamma_{i,t} + \beta \sum_{t=1}^{\tau} \varepsilon_{i,t} = \gamma_{i,t} + \beta S^+_{i,\tau} \tag{29}$$

where $S^+_{i,\tau} = \sum_{t=1}^{\tau} \varepsilon_{i,t}$ is the change of risk aversion index of agent i from time t to time $t+\tau$, which can be either positive or negative.

6.2 The simulation results of the DRA^+ model

Substituting Eq. (29) into the Eq. (3), we obtain price dynamics equation for DRA^+ model in exact the same form as of Eq. (27) except that the risk aversion evolves differently in the DRA^+ from the original DRA model. Thus we use the Eq. (27) to test our DRA^+ model.

To examine the price impact of the DRA^+ model, we set all system parameters to be the same as in the DRA. We have done the numerical test for the DRA^+ model and found that indeed the DRA^+ model produces similar results as the DRA model. Some of the key results are shown in Fig. 6.

Although the micro-dynamics of the two models are different (in terms of how the agents change their risk aversion), yet the statistical properties of the numerical results from the DRA^+ model are similar to those of the DRA model. This is not surprising, as the agent's price forecasting error $\varepsilon_{i,t}$ is also random and uncorrelated [Eq. (16) of baseline model]. $S^+_{i,\tau}$ is thus follows a similar random process as in the original DRA model.

Even though the risk aversion dynamics in DRA and DRA^+ are rather simple, it still helps to generate the excess price volatility and strong volatility clustering. Mathematically, this is due to the fact that the risk aversion index enters in the denominator of the price formula [see Eq. (27)], and as a result, it causes highly non-linear and non-Gaussian price dynamics. On the other hand, with the CRA, the denominator of the price formulus remains constant, the non-Gaussian process could not be generated by the variable (described by a simple Gaussian process) contained in the numerator. Thus the baseline CRA model fails to produce substantial different dynamics from the random walk process. Most systems previously studied need agents to "switch regimes" in order to generate enough excess volatility. Dynamical risk aversion produces excess volatility and volatility clustering without resorting to such regime switch.

In the next session we explore the question, does the price impact of DRA depend on how the baseline model is constructed?

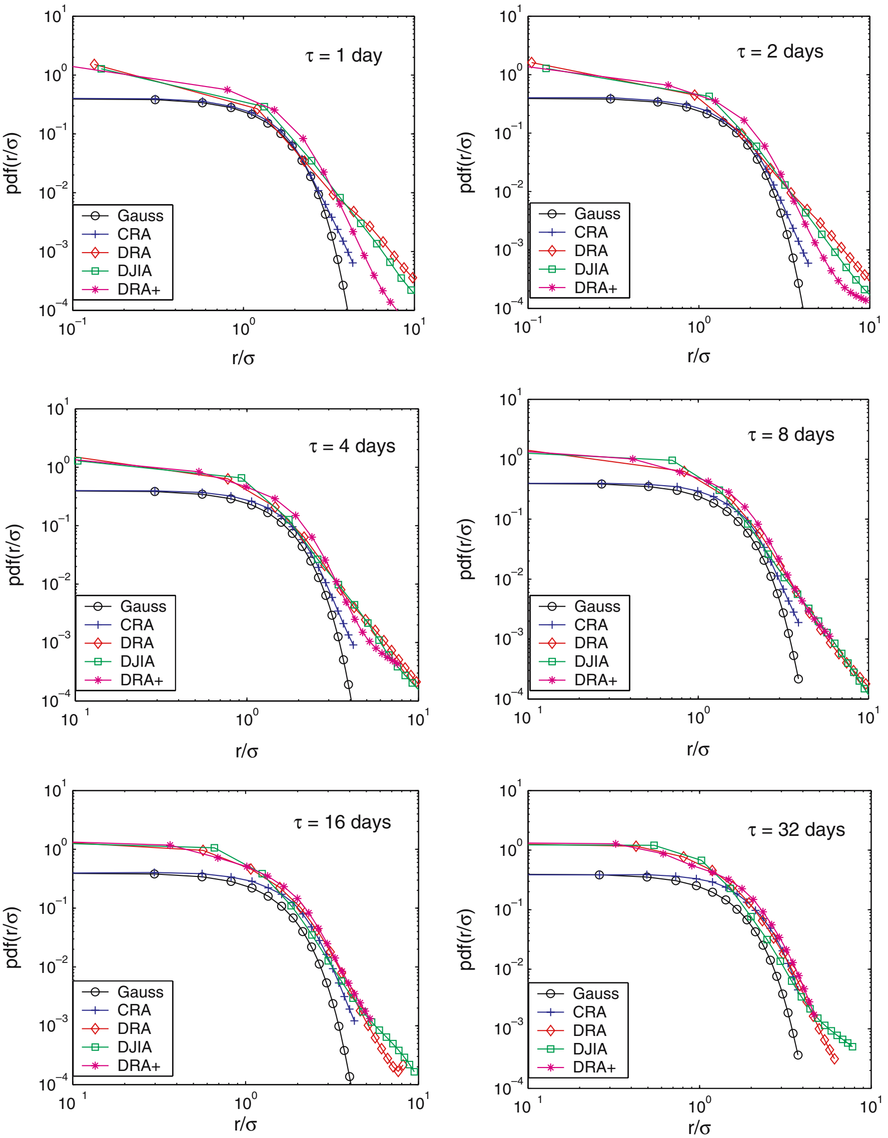

Fig. 6 Risk-adjusted return distributions for the CRA model (δ=0), the DRA model (δ=0.004), the DRA^+ model ($\beta = 0.004$), Gaussian process (Gauss) and real data of DJIA. Here r is the return and σ, the standard deviation of the return, which measures the volatility (or risk) of financial assets

7 The SFI market model with dynamic risk aversion

7.1 Brief introduction to SFI market

To test the impact of DRA on other baseline model, we use the well-known Santa Fe model of artificial market (Arthur et al. 1997; LeBaron et al. 1999).

We first give a brief summary of the SFI market below. In the SFI market a constant absolute risk aversion (CARA) utility function ($U(c_t, \gamma) = -e^{c_t\gamma}$) is assumed for each agent, the demand and price setting equations in fact have similar forms as in Eq. (2) and (3).

The dividend process is assumed to be an AR(1) process:

$$d_t = \bar{d} + \rho(d_t - \bar{d}) + \varepsilon_t \tag{30}$$

where ε_t is an i.i.d. Gaussian with zero mean and variance σ_e

The SFI market assumes that each of the N (=25) agents at any time possesses M (=100) linear predictors and uses those that best matches the current market state and have recently proved most accurate. Each predictor is a linear regressor of the previous price and dividend, $E(p_{t+1} + d_{t+1}) = a(p_t + d_t) + b$; it uses a market state "recognizer" vector consisting of J (=12) elements, each taking a value of either 0, 1 or #(match any market states). The market status is described by a state vector consisting of J binary elements, each taking value of either 1 (its specified market condition exists) or 0 (the market condition does not exist). The elements of market state can represent any important market discriminative information, including macro-/micro-economic environment, summary of fundamentals, and market temporal trends, etc. At each time step, only those predictors which match all their J elements to the corresponding J elements of market status are eligible to be used and are called "active" predictors.

The variance of estimation $\sigma^2_{i,t}$ for each agent is assumed to update with a moving average of squared forecast error, defined in Eq. (17).

Agents learn to improve their performance by discarding the worst (20%) predictors and developing new predictors via a genetic algorithm. This ensures the market some dynamics.

In the SFM, the market conditions are specified as:

1–6 elements represent "current price $\times$ interest rate / dividend $>$ 0.25, 0.5, 0.75, 0.875, 1.0, 1.125.
7–10 elements describe "Current price $>$ 5-period moving average of past prices (5-period MA), 10-period MA, 100-period MA, 500-period MA.
11th element always 1;
12th element always 0;

The regressor's parameters $\{a, b\}$ are set to be randomly and uniformly distributed within the ranges: $a \in (0.7, 1.2)$ and $b \in (-10, 19.002)$.

The risk-free interest rate is set to 5%, and for the dividend process: the auto-regression coefficient $\rho = 0.95$, $\bar{d} = 10$, and $\sigma_e = 0.0745$.

The weighting coefficients for the variance of estimation, $\theta = 1/75, 1/150$ for faster and slower learning respectively. The genetic algorithm is invoked (on average) every $T_e = 250$ periods (faster learning) and 1000 periods (slower learning). For more detailed justification for choosing the parameter values, (see LeBaron et al. 1999).

With these setups, we have checked that the simulation stock price time series and its statistical properties generated are in fact similar to those of our baseline model.

7.2 Numerical results of SFI market model with DRA

The dynamics of risk aversion can be similarly incorporated into the SFI market model. Figure 7 plots the simulation price time series for different DRA variance δ^2. These plots show clearly the impact on price fluctuations from the DRA.

To have a quantitative picture on how the excess volatility emerges from the DRA, we plot in Fig. 8 the functional relation of the Kurtosis vs variable δ. The results are very similar to those plotted in Fig. 4, which were obtained with our much simpler baseline model.

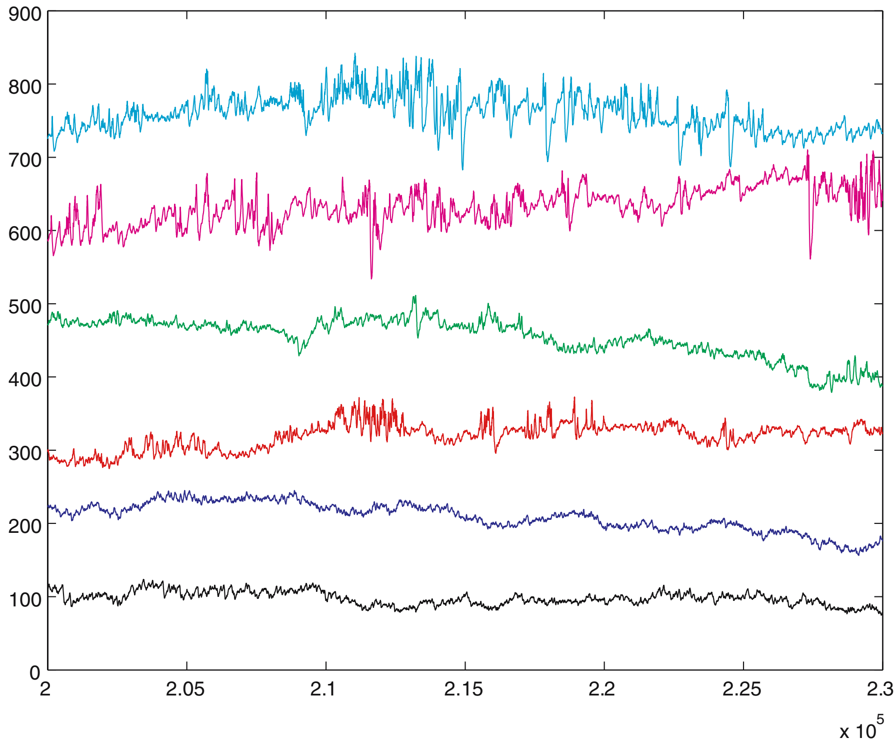

Fig. 7 Simulation price series for different variances of DRA processes with Santa Fe model. The model parameters are $\theta = 1/75$ and $T_e = 250$. The prices for different δs have been vertically shifted to make the comparison clearer. From the bottom to top, the price series are respectively for $\delta = 0, 0.01, 0.02, 0.03, 0.04$ and 0.05

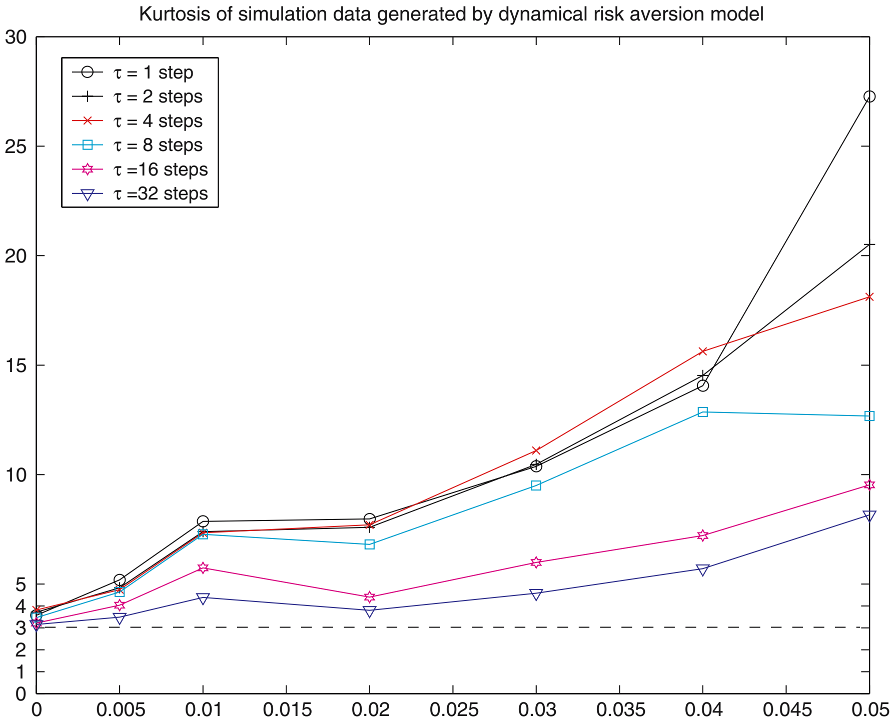

Fig. 8 Kurtosis of simulation price series for different variables, δ, of DRA processes with Santa Fe model. The model parameters are $\theta = 1/75$ and $T_e = 250$. The data were obtained by averaging over 50 independent runs

Table 2 The power-law tail indices for different processes

Lag (τ)	DJIA	DRA	CARA	Gauss
1	3.5	3.2	6	N/A
2	3.6	3.3	6	N/A
4	3.7	3.4	6	N/A
8	3.8	3.5	6	N/A
16	4.0	3.7	6	N/A
32	3.9	3.9	6	N/A

To give a further quantitative view, we here display, in Table 2, the power-law tail indices for SFI (CARA) model, our DRA model, DJIA, and Gaussian distribution.[5]

The numerical values of the power-law tail indices for SFI-DRA model are close to that of DJIA, while such values for SFI-CARA are far different. As a

[5] Note that here the power-law tail indices are obtained from the plots directly and thus they should be regarded as coarse-grain values. One can obtain more accurate values for the indices by running a simple OLS regression of the logarithm of the cumulative probability on a constant or using some other simple methods (Newman 2005). However an accurate measure of power-law indices is neither essential nor warranted here with the sizes of the data we are dealing with.

matter of fact that the validness of the power-law tails for SFI-CARA model are questionable as they are very similar to that of the Gaussion process which admits no power-law tail at all.

We have checked other key features and found that the SFI-DRA model gives the similar results as that reported in Sect. 5 where our DRA model is based on a much simpler baseline model. This suggests that DRA is indeed an alternative mechanism for the emergence of the key stylized facts, and the impact of DRA does not depend on the structures of the baseline models. Therefore the price impact of investors' DRA we have studied is generic.

8 Summary

We have presented a simple multi-agent model of a financial market and show that the dynamic risk aversion (DRA) we introduced could be an alternative mechanism in understanding the dynamics of financial markets and the stylized facts of financial time series. We assume that the index of DRA follows a simple independent bounded random walk with a constant variance δ^2. We demonstrate that such dynamics is directly responsible for excess volatility and the associated volatility clustering. We compare the numerical results from our model with the DJIA daily data and show that the simulation data reproduce most of the "stylized" facts, such as excess volatility (measured by fat tail and high peak of return distribution), volatility clustering measured by conditional return distribution. We have also tested the DRA on the Santa Fe market model and obtain similar results. This suggests that the impact of the DRA of heterogeneous agents on price dynamics does not depend on the structure of the particular baseline model used. The degree of excess volatility is essentially controlled by the parameter δ. Thus δ can be used as a key market sentiment parameter, in conjunction with the other market indicators such as average return r and the average volatility σ_0, to characterize the financial market. We hope that our results presented here will provide new insights into the dynamics of asset price fluctuations governed by investors' fluctuating sentiments.

Due to the lack of clear evidence on how agents vary their risk aversion, the random walk DRA used in the paper may be regarded as a first step in understanding the impact of varying risk aversion on the dynamics of financial markets. Some human subject experiments on this should be conducted, in which various hypothesis or conjectures about the changes in risk aversion can be tested and their implications in the artificial stock market model can be investigated. Such dynamic risk aversion can depend on a sudden change of investment sentiment triggered by market psychology shift related to the herding effect of the agents, on the abrupt change in risk aversion of the fund managers due to strict risk control measure in pension fund or life insurance industry, as well as on how successful agents are in forecasting. All these warrant our future study.

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