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REGULAR ARTICLE

# Stocks of information in personal consumption: a network model with non-rival borrowing and content overlap

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**Abstract** Stocks of information that cumulate in personal consumption and other non-market contexts have unrecognized welfare implications through properties that include non-rival borrowing and relatively low obsolescence rates. Although non-rival borrowing is emergent in agent interaction, it is typically not included in agent heuristics. Properties such as non-rival borrowing are best studied in networks. Computational studies of both regular networks and small world networks indicate the clustering that these networks tend to. In spite of its cited welfare relevance to distributional inequality, clustering in small world networks has only been examined in terms of the remoteness parameter of the network. We extend a network model of the stock of information to more explicitly represent efficiency-reducing effects that clustering can have through content duplication or overlap and demonstrate the significance these effects can have in computational results. These studies also show the efficiency-increasing effects of non-rival borrowing that continue to be evidenced and the overlap reducing effects that increasing network remoteness can have.

**Keywords** Network stocks of information · Clustering in networks · Small world networks · Distributional inequality

**JEL Classification** 012 · 015 · 090

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## 1 Introduction

A range of studies have represented consumption in policy applications for decision makers in government and corporate settings (e.g., Ben Said et al. 2002; El-Sebaey et al. 2002; Le Bars and Attonaty 2001; Matsatsinis and Samaras 2000; Ramanath and Gilbert 2004). We suggest that consumption is best studied in networks in which it typically occurs. This is particularly important when information is considered as a basis for the competing categories of goods that consumers allocate their resources to, as we will suggest it should be.

The distinction between durable and non-durable goods has historically been a basis for the categorization of goods that consumers hold (e.g., Houthakker and Taylor 1970). There is reason to expect that a categorization of goods that distinguishes information from conventional goods has demonstrable importance to general welfare criteria in personal consumption. A similar contrast has been useful in studies of the firm in (e.g., Adams 1990) but has not been developed in its application to consumers. In studies of the firm, efficiency implications of the use of information are supported by evidence of "spill-overs" from information use (e.g., Caballero and Jaffe 1993). In consumer networks, non-rival borrowing can be considered as a property that corresponds to "spillovers" in the firm's use of information.

Stocks of information and the use of information by households have also been recognized to have social as well as economic importance. Douglas and Isherwood (1996 [1979]), and Bourdieu and Passeron (1979 [1970]) are among the authors who have documented the social advantagement that typically accrues to the possession and use of information, through both its functional and stylized usage. Purcel-Gates' (1996) study of economically disadvantaged households has indicated that even seemingly perfunctionary information use in activities outside the workplace can have notable social and economic effects.

In spite of these indications of the importance of information stocks and information usage in consumption, there have been few direct studies of these stocks and their cumulation. We further observe that most policy applications in the study of consumption have addressed agents as independent entities. Studying networks is of particular importance in personal consumption since that level of aggregation introduces properties that agents are unlikely to include in their decision heuristics.

In the present application, we assume that a welfare objective is to maximize an output measure in which the stock of information is a factor of production. We recognize a coordinate welfare objective in terms of some measure of equality in the distribution of the stock or the output measure in the network that can be examined in terms of clustering in the network. In the discourse to follow, we will give a form to the consumption of information and the stocks that cumulate when consumers are members of networks.

Our results will first show the relationship of a key parameter of the network to commonly observed clustering in the network. We will then extend the basic network model to explicitly represent the content overlap that is typical in the information held by consumers in a network and is a basis for the dysfunctional effects of clustering. We demonstrate effects of content overlap through clustering on criterion variables in computational exercises with our system. We also show how adjustments to the key network parameter as a policy variable can reduce clustering and be welfare increasing.

We begin by giving a form to a network model in which stocks of information that can cumulate in personal consumption are represented. Equations (1)–(5) represent the main equations in a dynamic system in which stocks of information and conventional goods are accumulated by consumers in a network. In Eqs. (1) and (2) of the system, we represent what can be conceptualized as a preference construct that has the effect of internalizing usage history in information and conventional goods. We consider such a construct to be typically operative in consumer agents (see arguments in Bisin and Verdier 2001; Boyer 1983; Caplan 2003; Cowen 1989). We encode a single period lag in the construct but longer lag sequences can be represented. As previously recognized, this representation introduces feedback (e.g., Bisin and Verdier 2001; Boyer 1983; Heal and Ryder 1973) in the system.

$$\hat{\mu}_{t+1}^{(e)} = \tan h\left(c_1\mu_t^{(e)}\right)\left(E_t^{\gamma_t^{(e)}\mu_t^{(e)}}/Z_t^{\gamma_t^{(z)}\mu_t^{(z)}}\right) + \left(\alpha\sum_k M_{jk}\left(g(\mu_{t,k}^{(e)}) - \mu_{t,j}^{(e)}\right)\right) \tag{1}$$

$$\hat{\mu}_{t+1}^{(z)} = \tan h\left(c_2\mu_t^{(z)}\right)\left(Z_t^{\gamma_t^{(z)}\mu_t^{(z)}}/E_t^{\gamma_t^{(e)}\mu_t^{(e)}}\right) + \left(\alpha\sum_k M_{jk}\left(g(\mu_{t,k}^{(z)}) - \mu_{t,j}^{(z)}\right)\right) \tag{2}$$

$$\mu^{(i)} = \hat{\mu}^{(i)}/\left(\hat{\mu}^{(e)} + \hat{\mu}^{(z)}\right), \quad i = e, z. \tag{3}$$

$$E_{t+1} = E_t(1-\rho_e) + c_2 e_t^{\beta_e} + c_3\sum_k R_{jk}E_{t,k} \tag{4}$$

$$Z_{t+1} = Z_t(1-\rho_z) + c_4 z_t^{\beta_z} \tag{5}$$

where $\hat{\mu}^{(i)},\ i = e, z$ are the unnormalized valuing of informational and conventional goods, respectively

$\mu^{(i)},\ i = e, z$ are the normalized value constructs, i.e., $\mu^{(i)} \in (0, 1)$
$E, Z$ are stocks of information and conventional goods, respectively
$M^{(jk)}$ = magnitude of social influence of the $k$th network member on the $j$th member
$R^{(jk)}$ is the magnitude of the borrowing of the $k$th member's information stock by the $j$th member

$$g(\mu) = \begin{cases} +1 & \text{if } \mu > 0 \\ -1 & \text{if } \mu \leq 0 \end{cases}$$

$\rho_e, \rho_z$ are depreciation rates for the respective stocks
$e, z$ are input units of goods to the stocks
$\beta_{(i)}, i = e, z$ are technology parameters of $e, z$, respectively, that indicate returns to scale
$p_e, p_z$ are prices of $e, z$, respectively
$\alpha$, $A$ and $c_1 \cdots c_5$ are scaling constants.

The terms in Eqs. (1) and (2), respectively, (1) the dependency of this construct on its level in a previous period, and an update from the use of stocks of information and conventional goods in the previous period and (2) the influence of contacts with other members of the network. The feedback in the construct introduces interdependence among constructs and multiple equilibria for these constructs. As in other systems with feedback (e.g., Yizhaq et al. 2004), there is an unstable equilibrium or "tipping point" in the preference construct at which small disturbances can send the construct toward different equilibria depending on the direction of the disturbance.[1] This increases the sensitivity of the system to exogenous effects and opens the system to environments.

## 2 The small world consumer network

The consumer networks we study are small world as defined in Watts (1999) and Watts and Strogatz (1998). These authors have indicated the considerable generality in applications and empirically useful parameters of small world networks (SWNs). It has now been shown that many real world networks are SWNs, e.g., the neural network of the worm, *c. elegans*, the power grid of the USA and the collaborative network of film actors, (Watts and Strogatz 1998), social structure in the propagation of disease, (Moore and Newman 1999), and opinion formation (Kuperman and Zanette 2002). Since SWNs can be obtained by randomly rewiring a fraction ($p$) of the connections in a regular network (i.e., a network in which connections are only between neighbors and near-neighbors), they are "partially disordered" networks that lie somewhere between regular ($p = 0$) and completely random ($p = 1$) networks.

### 2.1 The stock of information in the SWN model

The dynamic stocks of information and conventional goods are represented in Eqs. (4) and (5), respectively. The levels of these stocks are depreciated according to $\rho_i$, $i = e, z$, parameters and updated through new goods ($e$, $z$) that the current period budget is allocated to. These goods augment the stocks according to $\beta^{(i)}$ technology parameters in Eqs. (4) and (5). Unlike the case of conventional goods, there can be non-rival borrowing of information from meetings with network members. We represent a simplified form for non-rival borrowing in the initial model in which the $j$th network member augments his or her stock by some small proportion ($R_{jk}$) of the stock of the kth member with whom he or she meets.[2]

[1] The existence of an unstable fixed point and its implication can be directly demonstrated by the following argument. Let $b = \mu^{(e)}/\mu^{(z)}$ and assume $\gamma^{(i)}, i = e$, z and other constants equal 1. Then $b_{t+1} = b_t E^{2\gamma(b_t-1)/\mu^{(z)}}$ will have similar dynamics to the 1D map. $b_{t+1} = b_t \exp(Kb_{t-1})$ for some constant $K = 2\gamma \log(E)/\mu^{(z)} > 0$. This 1D map has an unstable fixed point at $b = 1$ (i.e., the gradient of the RHS at $b = 1$ is $1 + K$ which greater than one and indicates instability). So, initial conditions with $b < 1$ will tend to the (stable) fixed point $b = 0$, which means $\mu^{(e)} \to 0$. Initial conditions with $b > 1$ will tend to infinity, which means $\mu^{(z)} \to 0$.

[2] An initial form for borrowing in the updating of information stocks can be written as

$$c_3 \sum_k R_{jk}(\Sigma(E_{l,k} - E_{l,j})),$$
$$\text{all } E_{l,k} > E_{l,j}$$

where $\ell$ is a running index of $L$ elements of the stock $E$. Only cases where $k > j$ in agent meetings augment the $j$th agent's stock. In this form, transfer is from those who have more information to those who have less.

### 2.2 Budget allocation to information and conventional goods

Consumers are assumed to produce activities that satisfy their objectives from market goods they acquire. At each time period, the output of an activity level ($A_k$) is produced from stocks of information and conventional goods according to technology parameters and their valuing of information-intensive activities. In a Cobb–Douglas form:

$$\underline{A}_k = c_1 Z_k^{\gamma_k^{(z)} \mu_k} E_k^{\gamma_k^{(e)} (1-\mu_k)} \tag{6}$$

where $\underline{A}$ is the level of an activity and the RHS variables and parameters are as previously defined.

Consumers are also assumed to follow a simplifying heuristic for allocation of a fixed budget for each period to the categories of goods under study. This heuristic is approximately the solution to a two period maximization of activity level (A) for the decision variables ($e, z$) under the constraint of a fixed budget. Numerical solutions of the constrained maximization for the non-linear system give us the following approximation to optimal allocation to informational goods:

$$e_k^* = B_k \left( \mu_k - c_p \frac{p_e}{p_z} \right) \tag{7}$$

where $e^*$ is the units of information good that maximizes cumulative activity level

$p_e$, $p_z$ are the prices of units of information and conventional goods, respectively
$c_p$ is a scaling constant.

## 3 Computational studies of system constructs in a SWN

We use computational methods to investigate implications of the network model of information use in consumption for welfare objectives. We begin by considering the relationship of measures of efficiency and inequality in the network to the network parameter of remoteness. In doing this, we recognize that path length and clustering in networks (e.g., Barrat and Weigt 2000; Watts 1999) can be given distinct economic and social interpretations as measures of efficiency and inequality, respectively.

### 3.1 Path length in the network

Path length refers to the number of links in the network that are required to connect each member of the network to every other member of the network. This number is averaged across all network members. Barrat and Weigt (2000) define this in a one-dimensional network as $\ell(N, p) = \frac{1}{N(N-1)} \sum_{i \neq j} d_{ij}$ where $\Sigma d_{ij}$ is the smallest number of links necessary to connect two vertices, $N$ is the number of sites or agents in the network and $p$ is the proportion of connections in the network that are not to neighbors or nearest neighbors.

### 3.2 Clustering in the network

Clustering is the well-recognized property of regular and small world networks for the levels or states of a construct to be related by the proximity of network members (e.g., Dorogovtsev 2004; Jackson and Rogers 2007; Jung et al. 2008). Watts and Strogatz (1998) measure of a clustering-related measure (cliquishness) can be defined as follows. If $c_i$ is the number of neighbors of a vertex $i$, there are a priori $c_i(c_i - 1)/2$ possible links between neighbors. As Barrat and Weigt (2000) show, in a one-dimensional network the ratio of the mean number of links between the neighbors of a vertex and the mean number of possible links can be expressed as $\tilde{C} = \frac{3(k-1)}{2(2k-1)}(1 - p)^3$ where $2k$ is the number of initial connections the agent has.

### 3.3 Remoteness in the SWN

As noted, a SWN is intermediate between a regular network in which all connections of vertices in the network are between neighbors or next-neighbors and a network in which connections of vertices are random. The proportion of vertices that is rewired to randomly selected vertices that are not neighbors or near-neighbors in an SWN is designated as the remoteness ($p$) of the network. The remoteness parameter is the most studied parameter of a SWN and has been shown to have pervasive effects on a range of outcome measures in other applications (e.g., Watts 1999; Watts and Strogatz 1998).

## 4 Method of the computational studies

### 4.1 Definition of the small world network in the model

We study a network with $N^2 = (500^2)$ vertices which were connected in a two-dimensional regular square lattice with periodic boundary conditions. The small world network we study includes both neighbor, near-neighbor and remote connections. Observations of 'remote' connections (agents who are more than a single cell distance from a referent agent) are represented as influencing an agent but at lower rates than neighbors or near-neighbors (agents who are contiguous neighbors or one cell away from contiguous neighbors).

To make this distinction in the parameter for the strength of association operational, connections between neighbors and nearest neighbors are assigned strength $\varepsilon \cdot M_i$ and those between next-nearest neighbors are assigned strength $\varepsilon \cdot M_o$. We also follow the typical assumption that such influence between entities in the network is symmetric (i.e., is the same in magnitude in both directions for any pair of entities) and that there is a global connection strength between entities in the network that can be varied continuously.

### 4.2 Rewiring the connections in a SWN

The standard rewiring technique of Watts and Strogatz (1998) is used to study network connections between agents. A randomly selected fraction ($p$) of the agents in the network will be reconnected so that one of the endpoints of their connections is moved to a new vertex. In our application, the vertex to which it is moved is randomly selected from a uniform distribution over the whole graph.

### 4.3 Dynamics of value construct in the computational model

In the basic implementation of the algorithm for variation across network cells, a set of values $\{\mu_i\}$ is defined, where $i$ ranges over the network member cells from 1 to $N^2$ and $\mu_i$ represents the current level or state of the $i$th edge.[3] At each time step, a synchronous update is performed, with each cell being updated according to the rule

$$\mu_{t+1}^{(i)} = \delta \cdot \frac{1}{n_i} \cdot \sum_{j \in C_i} S_{ij} \left( \nu(\mu_t^{(j)}) - \mu_t^{(j)} \right) \tag{8}$$

where $\delta$ is the update rate, and $n_i$ is the total number of edges connected to the $i$th vertex. $\mathrm{C}_i = 24$ is the number of neighbors and near-neighbors of an agent in the network we study; $\nu(x)$ is defined by:

$$\nu(x) = \begin{cases} 0 & x < 0.5 \\ 0.5 & x = 0.5 \\ 1 & x > 0.5 \end{cases}$$

so as to represent the discrete states of a vertex that an agent is assumed to be at by other agents.

## 5 Path length and clustering as a function of remoteness in the network

In Fig. 1, we show the relationship of the stock of information and the path length and clustering measures to remoteness in the network from our implementation of the computational model. This figure shows that the stock of information is monotonically increasing with remoteness in the SWN. It also shows that the stock of information is more closely related to clustering than to path length across levels of remoteness ($p$) in the network. Whereas path length shows a marked decline at a level of $p$ much less than 0.01, both clustering and the stock of information show little effect of $p$ until $p \geq 0.01$.

## 6 Clustering and content overlap in the stock of information

The relationship of clustering to the stock of information cumulated in the network encourages further consideration of the SWN model of this stock. As we have noted in SWN models, path length and clustering are only parameterized by the remoteness parameter ($p$). We consider this to underrepresent effects that clustering can have in

[3] Since the discrimination of preference in others is more crude than the internal indexing of one's own preference, we represent the judgment of the preference construct of others by an agent to be binary and defined as $\pm 1$.

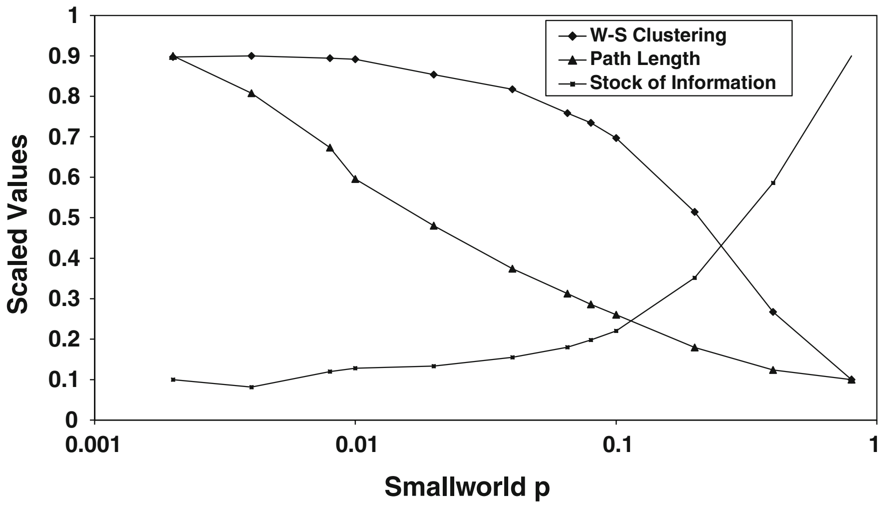


**Fig. 1** Path length, clustering and the stock of information as a function of network remoteness

general and particularly on the stock of information. There is a basis to expect that one of these effects is through content duplication or overlap in this stock. While the increased contact between members of a cluster can increase efficiency in information transfer, it also is likely to increase overlap or redundancy in member information.

We recognize that in the cumulation of stocks of information in a network, some proportion of the information that is exchanged in a meeting between network members duplicates information the members already have (i.e., is redundant) and that this proportion is likely to be higher among members that meet more frequently than among members who meet less frequently. Since clustering increases the likelihood that subsets of network members will meet each other more frequently than they will meet other network members, redundancy or content overlap is likely to be an important mediator of effects that clustering can have on the stocks of information in the network.

Pursuant to this observation, we propose to study clustering in terms of its effects through overlap in the stock of information. After giving a form to content overlap, we will examine the significant effects that clustering can have on the contribution of low obsolescence rate and non-rival borrowing to a measure of consumer efficiency when effects of content overlap are controlled for. We will also directly indicate welfare losses that occur because agents acting as independent entities do not typically include network externalities such as non-rival borrowing in their allocation heuristics. To give a form to what we designate as content overlap in the stock, we will extend the representation we have given to the stock of information in the network in Eq. (4).

## 7 Defining redundancy and content overlap

As used here, content overlap refers to information that is common in the stocks of information of multiple agents and has the consequence of reducing the amount of information that can actually be borrowed in meetings between agents. In fact, repeated meetings of members of a network can be a source of both skilling and redundancy or overlap effects. As used here, skilling refers to shared frames of reference and technical capabilities that facilitate efficient transmission of information between network members. While repeated meetings between network members can be expected to facilitate the transfer of new information between members through increases in skilling, overlap that develops from repeated meetings can be expected to reduce the effective stock. We include both these effects in elaborating a form for the stock of information.

The stylized assumptions we follow in defining forms for skilling and overlap across repeated meetings are that: (1) skilling increases to a maximum with a relatively small number of meetings between agents and (2) overlap continues to increase across a large number of meetings. For computational simplicity, we assume that in the absence of any other effect, overlap eventually decreases the contribution of borrowing in the network to zero.[4]

Since, as we have observed, (1) multiple meetings of a network member with the same other member can be expected to increase the overlap in their stocks faster than meetings with different members, and (2) members of the same cluster in a network generally meet more often than members of different clusters, overlap offers an explicit mechanism through which clustering in a SWN has an effect on the stock of information in a network.

## 7.1 Distance in a network model of the stock of information

So-called "chemical" distance (e.g., Watts 1999), as dimensioned by path length in a network is in terms of the number of links to connect all members of the network. This measure is invariant to the spatial distance between these links. However, in a number of applications, the units of distance that separate network members can have important effects that are not evident in counts of the number of links. For example, when face-to-face meetings typify the network, geographical distance between members will be as or more important than chemical distance to the clustering and overlap that develops in the network. The form for overlap in the stock of members of the network that we define will relate to "geographical" (i.e., spatial) distance.

## 7.2 Dynamics of overlap in the stock of information

Equation (9) introduces forms for the stylized assumptions on skilling and overlap we have described in the stock of information.

[4] In actual networks, we expect that overlap either reduces the stock of information to an increasing small proportion of the total stock. However, the total stock may be increasing faster than the proportion of it that is overlap. The exact growth in the stock in the presence of overlap would depend on the increase in budget, growth of technology and other factors that are exogenous to our model.

$$E_{t+1} = E_t(1-\rho_e) + c_2 e_t^{\beta_e} + \sum_j R_{ij} \cdot F(t,i,j)\gamma_{ij}^{(e)} G(E_j - E_i) \tag{9}$$

where

$R_{ij}$ is the "borrowing" rate

$F(t,i,j)$ is a modifier of the "borrowing" rate through overlap. It parameterizes the time for effects of overlap or redundancy in the stocks of information to reduce borrowing in a meeting between $i$ and $j$ to zero. The closed form we give to this moderator is

$$F(\cdot) = \tanh(c_0 \cdot \text{dist}_{ij} - t)$$

$c_0$ is the time for overlap between neighbors who are distance 1 apart to attain a size such that the transfer from the meeting is zero.

$\text{dist}_{ij}$ is the geographical distance between $i$ and $j$ in a $50^2$ network (max=25)

$\gamma_{ij}^{(e)} = \gamma_{\max,i,j}(1-\exp(-bt))$ the "skill" level of the $i$th agent is communicating information to the $j$th agent and recovering information from the communication of the $j$th agent

- $\gamma_{\max}$ is a maximum skill level that can be attained in a meeting between $i$ and $j$
- $b$ is a rate parameter for skill increase in repeated $ij$ meetings

$G(E_j - E_i)$ is a function that defines the transfer of information as it varies with the difference in the stocks of $j$ and $i$. This function is parameterized by $a_{\min}$ and $a_{\text{scale}}$ and is given the following log form: If $\Delta_{ij} > a_{\min}$, $G = a_{\text{scale}} \log(\Delta_{ij} - a_{\min} + 1)_j$ where $\Delta_{ij} = E_j - E_i$. If $\Delta_{ij} \leq a_{\min}$, $G = 0$.

$a_{\min}$ size of $E_i$ difference that results in zero transfer, i.e., is the greatest difference between $E_j$ and $E_i$ at which there is zero transfer between $j$ and $i$;

$a_{\text{scale}}$ a parameter for the transfer rate.

The first two terms on the RHS of (9) are, as in our previous versions of the form for stock, the obsolescence of the stock in the current period ($\rho_e$) and the allocation of budget to new information goods in the current period, ($e_k$) as exponentiated by the coefficient ($\beta_e$) for returns to scale. The third term on the RHS of (9) represents skilling ($\gamma_{ij}$) and overlap parameters in the F and G functions. This term includes (1) the transfer or "influence" rate ($M$), (2) a time dependent component F that parameterizes the number of periods for the overlap between neighbors who are distance 1 apart in the geographical network to result in zero transfer of information in a meeting ($c_o$), and (3) a distance function $G$, that represents the magnitude of information transfer that occurs between members $i$ and $j$ in a meeting as a function of the differences in the stocks between those members ($E_i - E_j$).

We show an F function for three levels of distance in Fig. 2. As we have described the relationship, the net contribution of the meetings of agents to the stock of information goes to zero as overlap increases for all distances.

The third term on the RHS of Eq. (9) that we add to "borrowing" also includes a function ($G(.)$) for the magnitude of transfer and as it varies with the difference in the

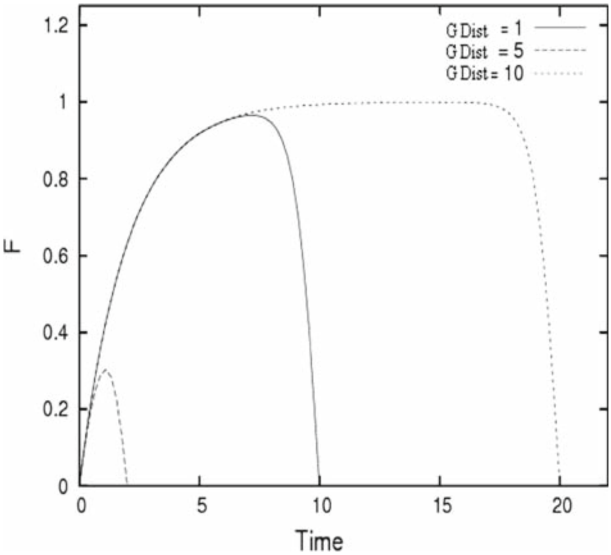


**Fig. 2** Time dependent component of information transfer between members as a function of their geographical distance (G Dist) in the network

stocks of those who meet. The transfer rate is not expected to be constant across all magnitudes of the differences in stock between agents who meet. The form of the $G$ function we implement in Eq. (9) defines the rate of transfer for different signs and magnitudes of the stocks of information in members that meet (i.e., $E_j - E_i = \Delta_{ij}$). We assume that the transfer between the $j$th and $i$th member in a meeting can never be negative and the function is increasing at a decreasing rate. This function is defined as a log function of the $(E_j - E_i)$ difference that results in zero transfer of information and a parameter for the rate of transfer as a function of the $E_j$ and $E_i$ difference when $\Delta_{ij} > a_{\min}$. We show the log form for the function in Fig. 3 with the parameter $a_{\text{scale}} = 1$ in Eq. (9).

## 8 Computational study of the augmented model of the stock of information

### 8.1 Non-rival borrowing and content overlap in an SWN

We will next use computational methods to demonstrate the relationship of parameters of our model that represent fundamental properties we have described in an augmented model of the stock [Eq. (9)] to welfare measures. Our computational procedures track meetings between all pairs of members in the network across multiple time periods and we calculate overlap effects that cumulate in the network from these meetings. In the exercises to follow, we will use mean stock in the network from a fixed budget sequence as an efficiency measure. We will investigate the sensitivity of this output measure to the obsolescence rate, borrowing and network remoteness when content overlap is represented.

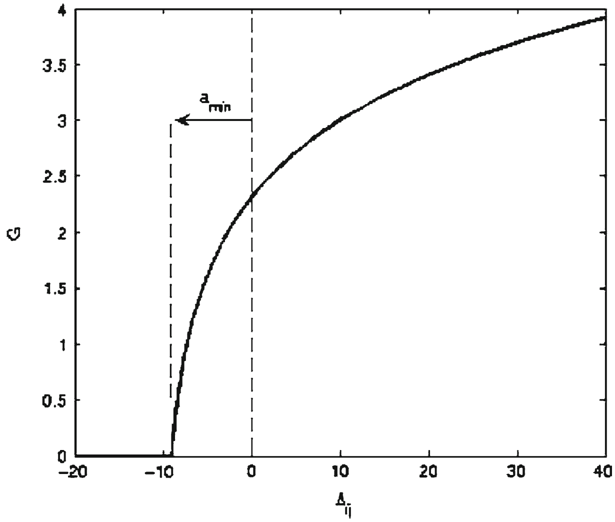


**Fig. 3** Information transfer as a function of the difference in stock between the $i$th and $j$th network members ($a_{\text{scale}} = 1$)

### 8.2 Mean stock of information in the network as an efficiency measure

In these exercises, we substitute Eq. (9) for the stock of information for Eq. (4) in the system we define in Eqs. (1)–(7) and calculate the mean of the stock of information over the time sequence until overlap results in the growth in the stock being zero as a measure of efficiency.

### 8.3 Obsolescence, borrowing and content overlap in the stock of information

The obsolescence rates in Eqs. (4) and (5) index the rate at which the stocks of information and conventional goods, $\rho_e$ and $\rho_z$, respectively, depreciate. In Eqs. (4) and (9), the parameter $R$ represents an initial form for a borrowing rate, i.e., the rate at which information is transferred between members of the network who "meet". As in the previous section $c_o$ is an overlap parameter that indexes the time for overlap to reduce the contribution from information borrowing to zero.

## 9 Results

In Table 1, we show the mean stock of information after adjustment for overlap from a fixed budget as a function of (1) remoteness in the network ($p = 00, 0.01, 0.1, 0.2, 0.3, 0.4$), (2) the borrowing rate in the network ($W = 0, 0.2, 0.4, 0.6$) and (3) the obsolescence rate of information ($\rho_e = 0.2, 0.1, 0.05$) relative to a fixed obsolescence rate for conventional goods ($\rho_z = 0.20$). The means we use in cells of the design are averages of ten runs of the computational model over the interval in which "borrowing" of one agent's information by another goes to zero for each cell.

**Table 1** Mean stock of information ($E$) in the network as a function of small world remoteness ($p$), the borrowing rate ($R$) and obsolescence rate ($\rho_e$) of Information

| $R$ | $\rho_e = 0.20$ | | | | $\rho_e = 0.10$ | | | | $\rho_e = 0.05$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.01 | 0.2 | 0.4 | 0.6 | 0.01 | 0.2 | 0.4 | 0.6 | 0.01 | 0.2 | 0.4 | 0.6 |
| $p$ | | | | | | | | | | | | |
| 0.001 | 2.80 | 7.62 | 13.24 | 19.27 | 3.97 | 15.20 | 26.20 | 37.95 | 6.36 | 30.61 | 52.09 | 75.44 |
| 0.005 | 2.67 | 7.63 | 13.27 | 19.29 | 3.90 | 15.25 | 26.20 | 38.10 | 6.19 | 30.61 | 52.13 | 75.52 |
| 0.01 | 2.57 | 7.66 | 13.26 | 19.27 | 3.67 | 15.25 | 26.25 | 38.10 | 5.97 | 30.65 | 52.16 | 75.48 |
| 0.05 | 2.61 | 7.70 | 13.41 | 19.53 | 3.94 | 15.37 | 26.49 | 38.50 | 6.29 | 30.81 | 52.65 | 76.37 |
| 0.10 | 2.51 | 7.76 | 13.56 | 19.79 | 3.63 | 15.40 | 26.50 | 38.50 | 6.09 | 31.08 | 53.23 | 77.33 |
| 0.4 | 2.67 | 8.14 | 14.40 | 21.27 | 3.76 | 16.30 | 28.40 | 41.85 | 6.26 | 32.53 | 56.53 | 83.07 |
| 0.6 | 2.66 | 8.36 | 14.82 | 22.17 | 3.77 | 16.65 | 29.30 | 43.50 | 6.23 | 33.33 | 58.29 | 86.33 |
| 1.0 | 2.71 | 8.55 | 15.35 | 23.17 | 3.86 | 17.15 | 30.50 | 45.6 | 6.28 | 34.35 | 60.75 | 90.67 |

Obsolescence rate of conventional goods ($\rho_z$) is fixed at 0.20

Results show that for the range in $p$ over 0.001–1.0, the mean stock in the network is monotonically increasing across each of the independent variables for borrowing ($R$), remoteness ($p$) and obsolescence rate of information relative to conventional goods ($\rho_e/\rho_z$). We examine the magnitude of the effects of the independent variables on the mean stock in a regression model.

In Table 2, we report results of a regression of the mean stock of information in the network from a fixed budget sequence on the obsolescence and borrowing rates, remoteness in the network and an overlap parameter. We examine three levels of the overlap parameter ($c_o = 0, 5, 10$) in the independent variables and also include a parameter for the remoteness-overlap interaction.

Results in Table 2 show effects of the independent variables under study on the mean stock of information. The adjusted $R^2$ of the main effects model is 0.93. Signs of the coefficients for the obsolescence/depreciation rate, the borrowing rate, the small world $p$ and the overlap parameter are in predicted directions and statistically significant. As previously, $c_o$ is ordered so that increases in this parameter decrease overlap. Additionally, there is a small but significant interaction between decreasing overlap and increasing remoteness in the network suggesting a return from policy that does both simultaneously.

The large magnitude effect of the obsolescence rate is likely to reflect its functional form as the equivalent of an exponential discount rate in (9). The smaller but significant effects of the R parameter indicate the contribution that "borrowing" in a network can make to the stock of information that cumulates from a fixed budget sequence even when the estimate is adjusted for overlap in the stock.

We also report results of an exercise that directly shows the effects of overlap on the consumer objective of maximizing the output of activity units that they can produce from stocks of information and conventional goods [Eq. (6)]. In Fig. 4, we show that at two levels of the depreciation rate of information relative to a fixed depreciation rate (0.20) for conventional goods and across different levels of

**Table 2** Regression of the mean stock of information in the network on obsolescence rate of information, the borrowing rate, content overlap, and small world remoteness

| | | Unstandardized coefficients | | Standardized coefficients | | |
|---|---|---|---|---|---|---|
| | | B | SE | $\beta$ | $t$ | $p$ |
| 1 | (Constant) | 36.541 | 7.31 | | 4.84 | 0.00 |
| | Depreciation rate of information($\rho_e$) | −310.63 | 21.36 | −0.851 | 14.06 | 0.000 |
| | Borrowing rate ($R$) | 74.38 | 13.32 | 0.320 | 5.05 | 0.000 |
| | Small world $p$ | 7.34 | 3.87 | 0.119 | 1.96 | 0.05 |
| | Overlap ($t$zero $c_o$) | 4.62 | 1.70 | 0.164 | 2.71 | 0.01 |
| | $p \times$ overlap_ $c_o$ | 5.41 | 2.66 | 0.144 | 2.03 | 0.05 |

Dependent variable: mean stock of information for a fixed budget: E
Note that overlap is decreasing as the $c_o$ time-to-zero parameter increases. Consequently, a positive sign for this parameter implies a decrease in the mean stock from overlap

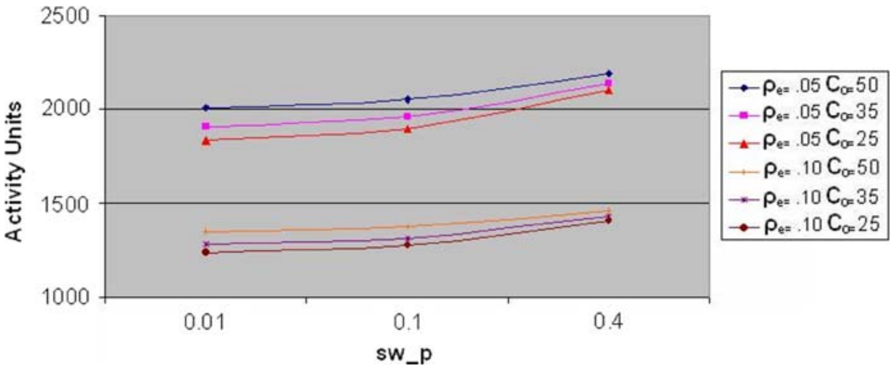


**Fig. 4** Activity units as a function of the obsolescence rate ($\rho_e$), small world $p$ and content overlap ($c_o$)

remoteness in the network, decreases in overlap (i.e., increases in $c_o$, the time that the loss from overlap depreciation reduces growth of the stock to zero), increase the units of activity from a fixed budget sequence. Regression results indicate that the main effect of the overlap parameter ($c_o$) on activity units is statistically significant ($p < 0.05$).

## 9.1 Clustering and network remoteness

Finally in this section, we also directly show the relationship of remoteness in the network to clustering. We graphically show that increasing remoteness can reduce clustering, even in the presence of "high" content overlap (i.e., $c_o = 35$). For this exercise, we use a clustering index with a spatial interpretation as an alternative to WS clustering.

### 9.2 A geometric clustering index

As Watts (1999) and Barrat and Weigt (2000) note, the WS measure of clustering as the density of agents in the same state in common locations of the network is more accurately considered to be a measure of "cliquishness" (e.g., the likelihood of a neighbor of an agents being connected to all other neighbors of the agent). We offer an alternative measure of clustering that has a geometric representation and is closer to constitutive definitions of clustering used by others in studying inequality.

To define a measure of the clustering in the value construct as it results from the spatial evolution of the system, two equilibrium states for the value construct are defined as white ($W$) and blue ($B$). The clustering indices, $C_w, C_b$, for the white ($W$) and blue ($B$) equilibrium states, respectively, are defined as

$$C_i = \frac{ng_i N}{4(n_i)^2}, \quad i = w, b$$

where

| | |
|---|---|
| $n_w$ | is the number of whites ($W$) |
| $n_b$ | is the number of blues ($B$), and |
| $ng_w$ | is the cumulative sum of white (1) neighbors of all white agents |
| $ng_b$ | is the cumulative sum of all blue (0) neighbors of all blue agents |
| $N = n_w + n_b$ | is the total number of agents |

The formula for the above index was chosen so that a completely random set of independent variables would have a clustering index of exactly 1 while a completely clustered set has the clustering index of $N/n_w$.[5]

Figure 5a and b show effects of remoteness on $c_i$ clustering at two levels of remoteness ($p = 0.01$ and 0.40) when other parameters are set in the middle of the range we study. Gini coefficients (Koolman and van Doorslaer 2004) for inequality in the network are 0.422 at $p = 0.01$ and 0.328 at $p = 0.40$. Visual inspection of Fig. 5a and b, and Gini coefficients show the reduction in clustering that remoteness in the network can bring about.

## 10 Welfare-increasing personal consumption: a policy exercise

We have indicated a basis to expect that independent agents will not allocate the proportion of their budgets to categories of goods in information that maximize a welfare criterion of efficiency for the network. This is because their heuristics do not typically include (1) the information they can borrow from others when they use an information good, (2) the borrowing of their new information by others, and (3) network parameters that can facilitate borrowing. We will next report an exercise in which we assume

[5] To establish this, assume that the proportion of whites in the lattice is $p = \frac{n_w}{N}$ where $n_w$ is the number of whites and $N$ is the total number of agents. If their distribution is completely random and independent, the number of "white neighbors of white," would be $ng_w = 4p.n_w = 4p^2N$, and the clustering index $C_w$ would be $C_w = 1$.

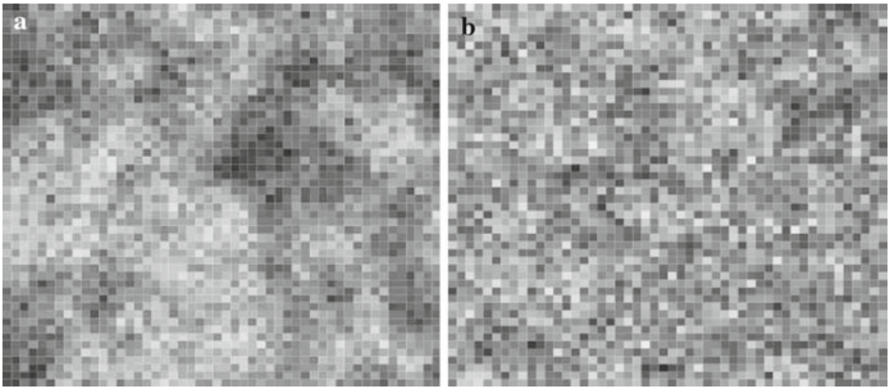


**Fig. 5** Clustering in the stock of information at two levels of remoteness: **a** clustering at $p = 0.01$; **b** clustering at $p = 0.40$

a super-agent can emulate the decision heuristics of agents and include the effects of "non-rival" borrowing and network membership we describe in an optimization heuristic. In this exercise, we illustrate the welfare gain that including information borrowing can offer when a cost-neutral policy variable in the form of a subsidy to the price of information is implemented. Welfare will be defined as the efficiency criterion of units of activities [Eq. (6)] in the network that can be constructed from a fixed budget sequence.

For the exercise, we use the allocation heuristic [Eq. (7)] and dynamic updating of stocks [Eqs. (5), (9)] to generate estimates of the welfare criterion in three cases. The first case is one in which welfare is calculated as a sum of activity units over agents in the network from their independent allocations of budget to information and conventional goods. As indicated, independent agents typically do not include information borrowing and effects of network membership on borrowing in the budget heuristic they use to maximize output. We calculate the units of activity that are constructed from their budget allocation at three levels of obsolescence rate for information at mid ranges of other model parameters.

In the second case, we calculated the estimate of activity units constructed at three levels of the obsolescence rate for information when the super-agent can include information borrowing and network remoteness. In the third case, we assumed that agents allocate their budgets independently in the absence of terms for information borrowing and network remoteness (as in the first case), but that the super-agent implements a policy to reduce the relative price of information by 12% in the network and subtracts the estimated cost of this from the budget sequence available to agents. The allocation agents made to information and conventional goods in the budget heuristic, the updated stocks of information and conventional goods, and the cumulative units of activities were then recalculated under the price subsidy.

In reporting results of these exercises, we define a welfare-deficit as the difference between Case 2: (the units of activities that can be constructed by network members when a super- agent includes the borrowing term in the updates to the stock of information) and Case 1: (units of activities that members of the network acting independently would be able to construct given their allocation decisions). The

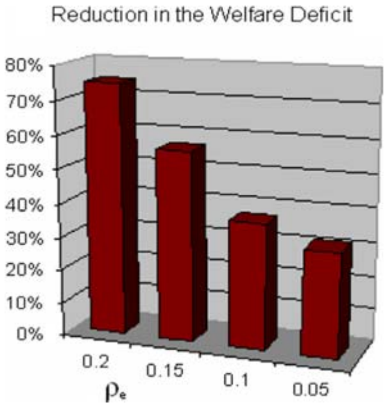


**Fig. 6** Reduction in a welfare deficit from a cost neutral subsidy to the price of information

reductions in the welfare deficit that result from the cost-neutral subsidy to the price of information (Case 3) are shown in Fig 6. Reductions in the welfare deficit are greatest at high rates of information obsolescence but remain significant across the range of rates we study. These results illustrate welfare gains from an implementation of a policy instrument implied by our model of personal consumption in a network.

## 11 Summary and discussion

We offer a network model of personal consumption in which goods are classified as information and conventional goods and note the background of this classification in the study of returns from R&D investment in information by the firm. In studying personal consumption, we define consumer welfare in terms of efficiency and distributional inequality criteria. Network models facilitate the representation of welfare-related properties such as non-rival borrowing of information that consumers do not typically include in their heuristics. We propose that the stocks of information cumulated in personal consumption and other non-market contexts have unrecognized welfare implications through these properties. The networks that we study are small world networks that have been shown to be common forms of many physical and social networks. We expect consumer networks to commonly take this form. The parameters of these networks in our models allow clustering and coordination to be given more explicit forms and directly studied.

Although the tendency toward clustering of interacting agents in both regular and small world forms for networks typically has negative externalities for welfare through decreased efficiency and distributional inequality for information exchange, clustering in SWNs has only been examined in terms of the remoteness parameter of the network. We extend a network model of the stock of information to more explicitly represent efficiency-reducing effects that clustering can have through content duplication or overlap. Clustering implies more frequent meetings between given sets of network members and this in turn can be expected to increase overlap. Since overlap is correspondingly related to the unduplicated stock of information in a network, we propose an explicit form for it and investigate the implications of this form.

The functional form we introduce represents a parameter for the increase in overlap between members at a defined distance in the network and a parameter that defines the transfer of information as it varies with the differences in stock between network members who meet. Our computational results indicate that under a range of model parameters, non-rival borrowing still contributes to the growth of this stock in personal consumption when effects of content duplication are represented. The results for the overlap parameter indicate effects that the process we describe can have in a network model and further supports the importance of borrowing to the stock that cumulates even when the dissipative effect of overlap is represented. We also show the cluster reducing effects that remoteness in a SWN can have with a clustering index that has a spatial interpretation.

In studies of activity units as an output measure in personal consumption and an efficiency criterion, we directly show that reducing overlap is efficiency increasing through this output measure. We exemplify policy effects that our model imply in a computational exercise in which the contribution of a cost-neutral reduction in the price of information to the welfare-related criterion significantly increases the units of the output measure. Increasing remoteness in a SWN as an additional policy direction is facilitated by consumer networks becoming increasingly electronic.

Finally, we note several directions to extend the modeling and computational study. While recognizing that agents engage in more elaborate cognitive processing including multi-period planning under conditions of non-symmetrical influence and multi-period influence of behavioral histories, we have used the forms for effects of contact between network members that have been most often represented in SWNs. Consumer budget allocation to information and conventional goods is also given a standard form in a utility maximization heuristic. We have done this to direct attention to complex effects we model in the network including information borrowing, content overlap and clustering and clustering-reducing parameters. Agents can clearly be endowed with more elaborated cognitive processing and there are now also suggestive works on forms to accomplish this.

One of these is in applications of socio-cognitive theories of reputation (e.g., Conte and Paolucci 2002) in which individual cognition and social propagation are represented. These forms adapted to our model can support studies of emergent effects that are not directly predictable from most network models. In our model, we have also used a single period lag in the preference construct [Eqs. (1), (2)] in representing history effects. These equations can be modified to allow longer lags and more elaborated effects of behavioral histories in the model.

A second direction is to make the model more directly open to effects of environments by giving explicit forms given to exogenous signal and randomness they introduce in networks (e.g., Ward et al. 2002; Wiesenfeld and Moss 1995). A typical representation of these effects would be to add the terms $(A)\sin(\omega t + \alpha_i) + f$ to equations that introduce the influence of network membership to the system [Eqs. (1), (2)]. In these terms, $A$ is an amplitude parameter, $\omega$ denotes location in the phase space of the sin function at time $t$, $\alpha_i$ is an agent-specific displacement parameter and $f$ is a Gaussian noise term parameterized by the standard error of the underlying normal distribution. The signal term represents deterministic exogenous effects such as business or fashion cycles. The magnitude of the displacement or synchronicity parameter $(\alpha_i)$

introduced in this term can be assigned to agents according to defined distributions (e.g., Bahar and Moss 2004; Krawiecki et al. 2000) and introduces a capability to further study economic effects of coordination among network members on clustering and the accumulation of stock. The Gaussian in $f$ represents randomness that environments commonly introduce. Feedback we represent in the networked consumption system we introduce and in other systems in which macroscopic events can influence agents (e.g., Westerhoff and Hohnisch 2007) has the capability to endogenize effects of environments.

A third direction is in the form of the network model. We note that our models can be implemented in alternative network forms. Dynamic network forms (Barabási and Albert 1999; Barabási 2007) are one of the alternatives to the form for SWNs defined in Watts (1999) and Barrat and Weigt (2000) that we use. While Watts SWN networks typically reconfigure a network of a fixed number of nodes according to the remoteness parameter, Barabasi networks add new nodes according to a defined rule for connecting to existing nodes. The power law distribution of the configuration of connections that typically emerge is a highly clustered network. As such, it would be of interest in studying stocks of information and the implementation of policy that ameliorates the welfare reducing effects of clustering we have shown.

We have sought to draw attention to the contribution that the use of information goods in consumption and the stocks that consumers accumulate in their networks from such use can have to welfare criteria. We direct attention to the clustering tendency of networks because of its general relevance to welfare criteria and its particular importance to stocks of information. The elaboration of clustering effects through an explicit form and parameters of content duplication among agents in the accumulation of network stocks of information provides a basis to further parameterize its effects and demonstrate the robustness of non-rival borrowing contributions to welfare. Computational methods provide a means to demonstrate the effects we describe and to assess the adequacy of proposed models when data to accomplish this is limited. Our results indicate that it is timely to elaborate models of consumption that better represent effects of the assortment or composition in goods. There is, in particular a basis to conceptualize welfare contributions of information use by coordinated network agents in consumption and consider policy that recognizes the welfare relevance of the distinction we propose in categories of goods.

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