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REGULAR ARTICLE

Dynamics of quote and deal prices in the foreign exchange market

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Abstract We empirically investigate price fluctuations of yen-dollar exchange rate using the high-frequency data recorded in the electronic broking system for seven-year period. The distribution of quote price changes has symmetric fat-tails approximated by a power law; however, that of deal price is asymmetrical. The autocorrelation function and diffusion of price changes indicate that quote price exhibits anti-correlation feature in short time scale, whereas deal price is essentially uncorrelated. The bid-ask

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spread shows power-law distribution and long range temporal correlations similar to that observed in absoute price changes.

Keywords Econophysics · Foreign exchange market · Power-law distribution · Anomalous diffusion

JEL Classification C10 · C16 · D40 · G10

The foreign exchange market is the most liquid and largest financial market in the world. Most of spot interbank transactions are executed through the global electronic broking systems such as EBS and Reuters around-the-world. In the yen-dollar exchange rate, the EBS has a strong market share. We analyze quote prices and deal prices of the yen-dollar exchange rate provided by EBS for the seven-year period from January 1999 to December 2005 with a recording frequency of every one second.[1] Although the study of deal price fluctuations has been studied on stock markets, very few studies, to our knowledge, have been used on foreign exchange markets. The data consist of best bid (bid), best ask (ask), deals done on the bid side (deal bid) and deals done on the ask side (deal ask). The bid and ask, representing lowest sell offer and highest buy offer respectively, are recorded at the end of one second time slice. The deal bid that is initiated by seller is recorded as the lowest deal price, and the deal ask that is initiated by buyer as the highest one within one second. Since market makers set credit limit each other, the bid and ask, which are the best quotes available individually, deviate from the best market quotes. Due to the (lack of) credit lines between the pair, reversal situations in which the bid is higher than the ask occur at the probability of less than 1% of the time. In our analysis we exclude data of special days such as weekends, holidays, intervention days, which are obviously different from regular business days. The total number of each bid and ask is about 1.5×10^7 (the average transaction interval is 6.7 s). That of each deal bid and deal ask is about 6.0×10^6 (18 s).

Statistical properties of price fluctuations is important to understand market dynamics. We define the price change as

$$G(t) = P(t + \Delta t) - P(t), \tag{1}$$

where $P(t)$ is the price at time t and Δt is the sampling time interval. There being different behaviors of traders, we find an intraday pattern in the absolute price changes $|G(t)|$ (Fig. 1) similar to the previously reported intraday patterns in bid-ask spread (Ito and Hashimoto 2004, 2006), where the spread is $P_{\mathrm{ask}} - P_{\mathrm{bid}}$ and the midquote is $(P_{\mathrm{ask}} + P_{\mathrm{bid}})/2$. The intraday pattern $A(t_{\mathrm{day}})$ is defined as the absolute price changes at time t_{day} of the day averaged over all N days (Gopikrishnan et al. 1999),

$$A(t_{\mathrm{day}}) = \frac{1}{N} \sum_{i=1}^{N} |\, G^i(t_{\mathrm{day}})\,|, \tag{2}$$

[1] The authors are grateful to EBS for providing a data set for academic purpose.

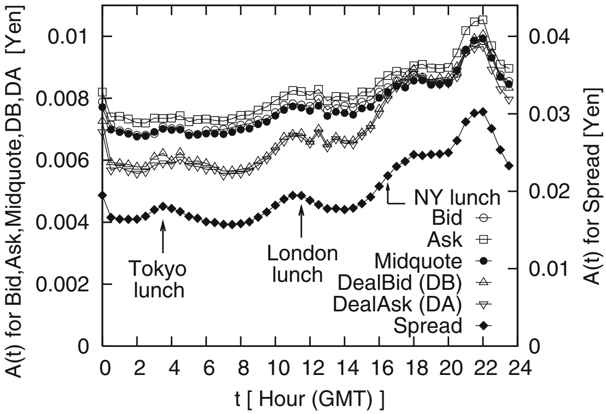


Fig. 1 The intraday pattern of absolute price change for various prices and spread. In our analysis we have used 10-min interval pattern

where t_{day} denotes the time in a day and $G^i(t_{\text{day}})$ is the price changes at time t_{day} in day i. We remove the intraday pattern from $G(t)$ by studying

$$G'(t) = G(t) \,/\, A(t_{\text{day}}). \tag{3}$$

In order to compare the distribution for different prices, we normalize the price changes,

$$g(t) = \frac{G'(t)}{\sqrt{\left\langle G'(t)^2 \right\rangle - \langle G'(t) \rangle^2}}, \tag{4}$$

where $\langle \cdots \rangle$ denotes an average over the entire length of the time series.

Figure 2 shows the cumulative distribution of the normalized midquote changes for $\Delta t = 1$ tick, i.e. the probability that a price change has an absolute value larger than a threshold $|g|$, $P(> |g|)$. For both positive and negative tails of the distribution, we observe a power law behavior

$$P(> |g|) \propto |\, g \,|^{-\alpha} \tag{5}$$

with exponent $\alpha \sim 3$. The positive and negative price distribution are almost identical, that is, the distribution is symmetrical with respect to the direction of price movement. As shown in Figs. 3 and 4, all other distribution of price changes also have similar functional forms, however, these are asymmetrical.

For the following reason, this asymmetry is significant in the deal prices. In stock markets, it is reported that the signs of trades has a long range correlations (Bouchaud et al. 2004; Farmer and Lillo 2004). We can confirm the similar feature in our data that, when the price goes up, the deals tends to be done on the ask side successively and the deal bid hardly recorded. Therefore, the positive price changes of the deal bid in one ticks becomes large since the deal bid cannot follow the rise of the price.

Fig. 2 Cumulative distribution of the midquote changes. Power-law fits using maxiimun likelihood estimation (MLE) (Goldstein et al. 2004) in the region $|g| \geq 2.5$ yield $\alpha = 3.12$ (positive tail) and 3.16 (negative tail)

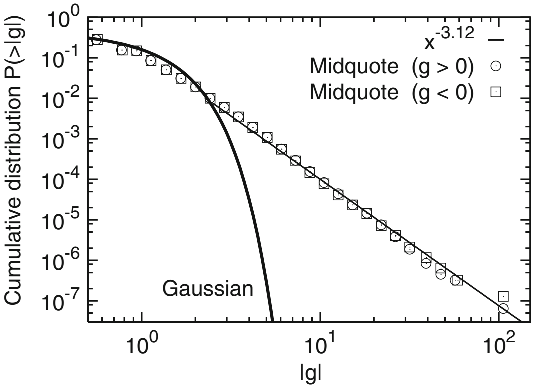


Fig. 3 Cumulative distribution of the bid and ask changes. Power-law fits using MLE in the region $|g| \geq 2.5$ yield $\alpha = 2.93$ (positive tail of bid), 2.8 (negative tail of bid), 2.87 (positive tail of ask) and 3.02 (negative tail of ask)

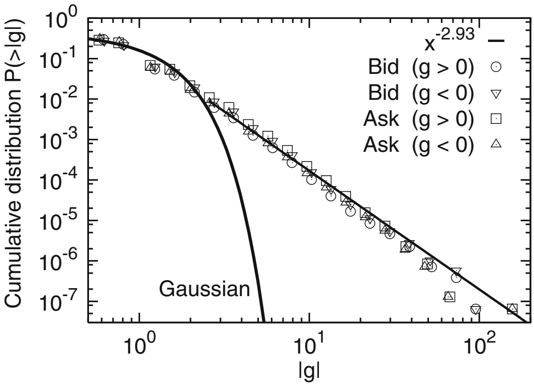
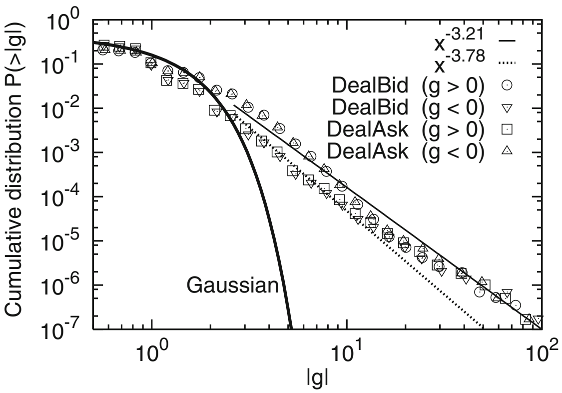


Fig. 4 Cumulative distribution of the deal bid and deal ask changes. Power-law fits using MLE in the region $|g| \geq 2.5$ yield $\alpha = 3.21$ (positive tail of deal bid), 3.78 (negative tail of deal bid), 3.60 (positive tail of deal ask) and 3.18 (negative tail of deal ask)



Next, we extend our analysis for longer time scales. Figure 5 shows the negative tails of the cumulative distribution of the normalized midquote changes for time scales up to 1024 tick (approximately 2 h), using non-overlapping time windows. The distribution retains its power-law form for these time scales. All other distributions also have qualitatively similar feature. The power exponent α varies with the time scale Δt is

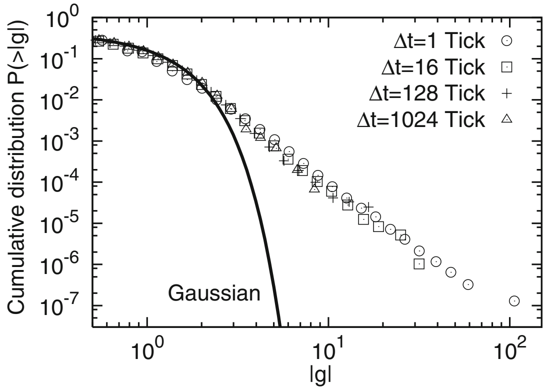


Fig. 5 Cumulative distribution of the midquote changes for negative tail

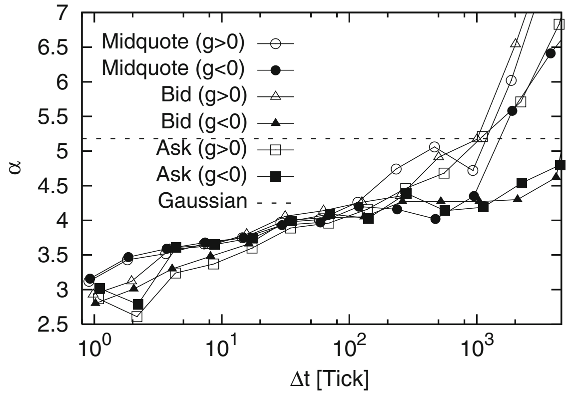


Fig. 6 The values of the exponent α as a function of the time scale Δt for the midquote, bid, and ask. Power-law fits using MLE are done in the region $|g| \geq 2.5$. The exponent value for Gaussian distribution is shown for comparison

shown in Fig. 6 for the midquote, bid, and ask and Fig. 7 for the deal bid and deal ask. For the positive tail of each distribution, the exponent α starts slight rise with increasing Δt, and then increases rapidly after $\Delta t \sim 2\,\text{h}$ (1,000 ticks in Fig. 6 and 500 ticks in Fig. 7), namely, the distribution slowly converges to Gaussian behavior. In contrast, the negative tail of each distribution keeps the power law even for long time scale; for $\Delta t > 100$ ticks the exponent α of the negative tail becomes less than one of the positive (Mizuno et al. 2003).

We next consider the autocorrelation function of price changes per tick, $\langle g(t)g(t+\tau)\rangle$. This correlation of the bid, ask, and midquote takes a negative value for a single tick, and it almost vanishes for longer time intervals. This anti-correlation indicates that if the price goes up at the previous tick, then it is likely to go down at the next tick and vice verse. On the other hand, the autocorrelation of the deal bid and deal ask is almost zero, that is, the fluctuation of the deal price changes is very close to a white noise.

Figure 9 shows the time evolution of standard deviation of price changes[2] $\sigma(dt) = \sqrt{\left\langle (\,P(t+dt) - P(t)\,)^2 \right\rangle}$ as a function of time difference dt. The bid, ask, and

[2] We have also analyzed normalized price changes. However, results are qualitatively the same.

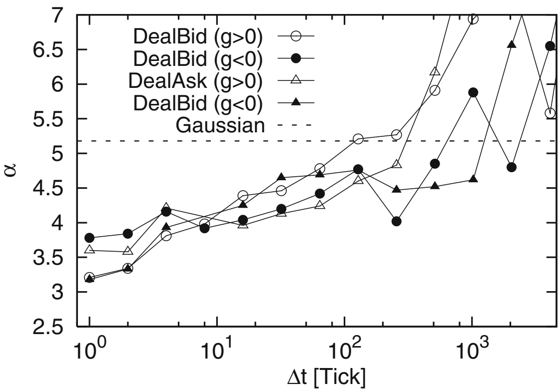


Fig. 7 The values of the exponent α as a function of the time scale Δt for the deal bid and deal ask. Power-law fits using MLE are done in the region $|g| \geq 2.5$. The exponent value for Gaussian distribution is shown for comparison

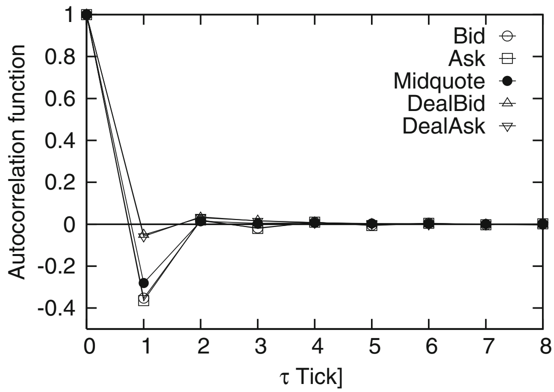


Fig. 8 The autocorrelation function of the price changes $g(t)$

midquote exhibit abnormal diffusion in short time scale, whereas the behavior in long time scale is well approximated by normal diffusion, $\sigma(dt) \propto (dt)^{0.5}$. The characteristic time scale that separates anti-persistence and random walk is a few minutes. In contrast, the deal bid and deal ask are essentially uncorrelated in all time scale, in agreement with Fig. 8. Thus the deal prices are basically expected as a random walk. Similar normal diffusion property can also be confirmed for optimally weighted average of quote prices (Ohnishi et al. 2004).

The lack of linear correlation does not directly imply that price changes are independent random variables. Indeed, for both the quote prices and the deal prices, the autocorrelation function of $|g(t)|$ has power law decay with long persistence as shown in Fig. 10.

We also analyze the fluctuations of the bid-ask spread $S(t)$ in the same way by replacing $G(t)$ with $S(t)$. Quantifying the fluctuations of the spread provides considerable insight into the dynamics of the market liquidity. Figure 11 shows the cumulative distribution of the normalized spread $s(t)$. The positive tail of the distribution follows a power law with exponent $\alpha \sim 3.5$. The autocorrelation function of $s(t)$ shows long range correlations similar to that of $|g(t)|$. These results are qualitatively consistent with recent studies on stock markets (Plerou et al. 2005).

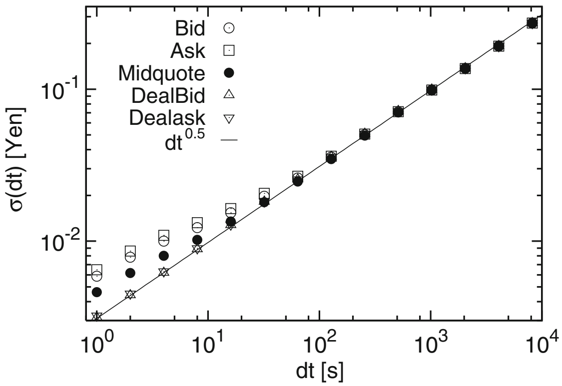


Fig. 9 Time evolution of standard deviation of price changes $\sigma(dt)$

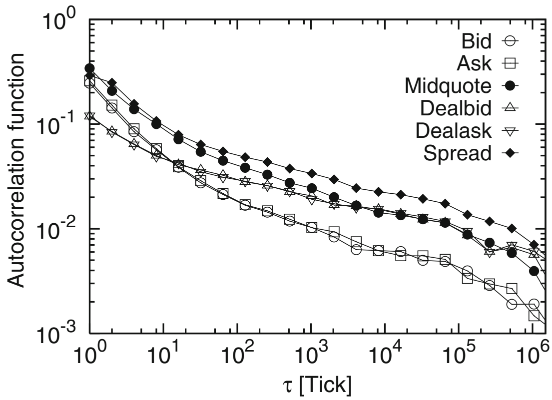


Fig. 10 The autocorrelation function of the absolute price changes $|g(t)|$ and the spread $s(t)$

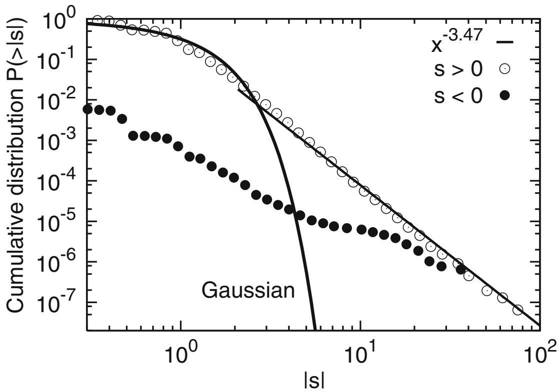


Fig. 11 Cumulative distribution of the spread. Power-law fits using MLE in the region $|s| \geq 2.5$ yield $\alpha = 3.47$

In summary, we have presented a detailed analysis of the fluctuations of price changes. We have found that the distributions of price changes show power law behaviors. Because of the long memory in signs of trades, distributions of the deal prices have asymmetric fat-tails. We have studied the autocorrelation functions and diffusion of price changes. Differing from the quote prices, the deal prices have almost no correlation even in short time scale. Finally, we have shown that the bid-ask spread displays power-law distribution and long range temporal correlations similar to those for the volatility. On the basis of our results, further study on the micro-structural dynamics of price changes is being undertaken.

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