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REGULAR ARTICLE

The Naming Game in social networks: community formation and consensus engineering

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Abstract We study the dynamics of the Naming Game (Baronchelli et al. in J Stat Mech Theory Exp P06014, 2006b) in empirical social networks. This stylized agent-based model captures essential features of agreement dynamics in a network of autonomous agents, corresponding to the development of shared classification schemes in a network of artificial agents or opinion spreading and social dynamics in social networks. Our study focuses on the impact that communities in the underlying social graphs have on the outcome of the agreement process. We find that networks with strong community structure hinder the system from reaching global agreement; the evolution of the Naming Game in these networks maintains clusters of coexisting opinions indefinitely. Further, we investigate agent-based network strategies to facilitate convergence to global consensus.

1 Introduction

Agent-based models and simulations provide invaluable frameworks and tools to gain insight into the collective behavior of social systems (Epstein and Axtell 1996; Challet et al. 2005; Anghel et al. 2004). Opinion spreading and social dynamics

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(Durlauf 1999; Castellano et al. 2008) on regular and random networks are examples of the latter. A large number of studies have investigated models of opinion dynamics (Castellano et al. 2005; Ben-Naim 2005; Deffuant et al. 2000; Hegselmann and Krause 2002; Lorenz 2007, 2008; Krapivsky and Redner 2003; Sood and Redner 2005; Sznajd-Weron and Sznajd 2000; Kozma and Barrat 2008a; Benczik et al. 2008; Antal et al. 2005; Jung et al. 2008) and the dissemination of culture (Axelrod 1997; San Miguel et al. 2005; Mazzitello et al. 2007), while fundamental models for residential and ethnic segregation have also attracted strong interest (Schelling 1971; Zhang 2004; Vinkovic and Kirman 2006; Lim et al. 2007). Most recently, researchers have also turned their focus to models where both the network topology and opinions change over time (Kozma and Barrat 2008a,b). With the availability of empirical data sets and cheap and efficient computing resources, one can implement stylized socio-economic models on empirical social networks, and evolve "artificial societies" (Epstein and Axtell 1996) to study the collective properties of these systems.

Here, we focus on one such stylized model, the Naming Game (NG) (Baronchelli et al. 2006b). The NG is a minimal model, employing local communications, that can capture generic and essential features of an agreement process in networked agent-based systems. For example, in the context of a group of robots (the original application), the NG dynamics mimics the emergence of shared communication schemes (synthetic languages), while in the context of sensor networks, such an agreement process can describe to the emergence of a shared key for encrypted communications. In a system of human agents, the NG can be considered as a minimal model to describe the recent phenomenon of collaborative tagging or social bookmarking (Cattuto et al. 2006, 2007; Golder and Huberman 2006) on popular web portals like Del.icio.us (http://del.icio.us), Flickr (http://www.flickr.com), CiteULike (http://www.citeulike.org), and Connotea (http://www.connotea.org). Another common example is the evolution and spread of coexisting dialects in everyday use (see, e.g., the geographical distribution of "Pop" versus "Soda" for soft drinks in the US (McConchie 2002)). In a broader context, the NG can be employed to investigate the emergence of large-scale population-level patterns arising from empirically supported local interaction rules between individuals.

The common feature in the above examples and applications is that global agreement can emerge spontaneously (without global enforcement) purely as a result of local (e.g., pairwise agent-to agent) communications. The NG has been studied intensively on regular and random complex network models (see Sect. 2). Here, we investigate the evolution of the agreement process in the NG on empirical social graphs. It is well known that empirical social graphs exhibit strong community structure (Girvan and Newman 2002; Newman and Girvan 2004; Onnela et al. 2007; Palla et al. 2005, 2007). It is also known that in networks with community structure, reaching global agreement can be hindered (Lambiotte and Ausloos 2007; Candia and Mazzitello 2008). Here, we investigate the NG precisely from this viewpoint. Further, we analyze strategies to destabilize otherwise indefinitely coexisting clusters of opinions, to reach global consensus of a selected opinion. The later can also be considered as an abstract agent-based marketing approach.

The paper is organized as follows. In Sect. 2, we briefly review recent results on the NG on various regular and complex networks models. In Sect. 3, we present results for the NG on empirical social networks. In particular, we investigate the effect of communities in the underlying static social graphs on the agreement process (typically leading to indefinitely coexisting clusters of opinions). In Sect. 4, we study and analyze node-selection strategies to facilitate the convergence to a global opinion. In Sect. 5, we conclude our paper with a brief summary.

2 Background, model, and prior results on the Naming Game on regular and complex network models

In the simplified version of the NG, agents perform *pairwise* games in order to reach agreement on the name to assign to a *single* object. This version of the NG was investigated on the complete graph (CG) (corresponding to mean-field or homogeneous mixing) (Baronchelli et al. 2006b, 2005), on regular (Baronchelli et al. 2006a), on small-world (SW) (Dall'Asta et al. 2006a; Lin et al. 2006), and on scale-free (SF) networks (Dall'Asta et al. 2006b; Baronchelli et al. 2006c). On a CG, each agent has a chance to meet with all others and compare their current local vocabularies (list of "synonyms") before updating them. On regular networks, agents have only a small number of nearest neighbors with whom they can interact/communicate (e.g., four nearest neighbors in two dimensions). The communication in both cases is "local", in that *pairs of agents* are selected to interact and to update their vocabularies. The basic algorithmic rules of the NG are as follows (Baronchelli et al. 2006a,b). A pair of neighboring nodes (as defined by the underlying communication topology), a "speaker" and a "listener", are chosen at random.[1] The speaker will transmit a word from her list of synonyms to the listener. (If the speaker has more than one word on her list, she randomly chooses one; if it has none, it generates one randomly.) If the listener has this word, the communication is termed "successful", and both players delete all other words, i.e., collapse their list of synonyms to this one word. If the listener does not have the word transmitted by the speaker (termed "unsuccessful" communication), she adds it to her list of synonyms without any deletion. In this paper, we measure time in units during which N agents are selected at random as speakers, where N is the number of agents in the network. The above rules are summarized in Fig. 1.

Here, we considered initial conditions when all agents have an empty vocabulary. Then such an agent, when chosen to be a speaker, invents a *random* word to be transmitted to the listener. (For initial conditions with a single random word per agent, see the Electronic Supplementary Material.) In terms of the number of different words, the evolution of the game will go through stages of growth (due to unsuccessful communications) and stages of elimination (due to successful ones). In all of the above mentioned networks, starting from empty vocabularies, an early time explosion of words is followed by a slow elimination of all synonyms, except one; that is agents come to global agreement on the naming of the object in question.

[1] Note that on strongly heterogeneous (scale-free) graphs, the order whether the listener or the speaker is chosen first, strongly impacts the efficiency toward global agreement. Choosing the listener first at random will increase the chance for selecting a node (as a neighbor) with larger degree for speaker. In turn, hubs will be the most frequent speakers, giving rise to faster convergence to global agreement at a mildly elevated memory cost (Dall'Asta et al. 2006b; Baronchelli et al. 2006c).

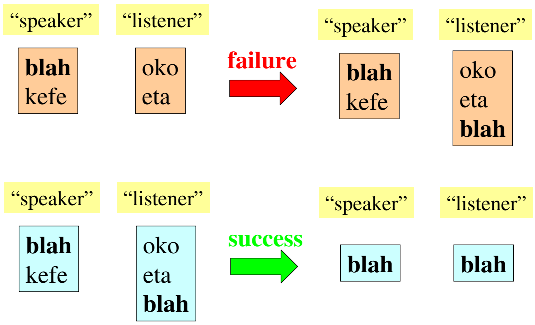

Fig. 1 Schematic rules of the Naming Game (Baronchelli et al. 2006b) as described in the text. If the speaker has more than one word on her list, she randomly chooses one; if it has none, it generates one randomly

Also recently, the NG was studied on homogeneous random geometric graphs (RGGs) (Lu et al. 2006, 2008). RGG is both spatial and random (Meester and Roy 1996; Penrose 2003; Dall and Christemsen 2002). Motivated by the deployment of sensor networks, nodes are randomly scattered in a two-dimensional area, and two nodes are connected if they fall within each others transmission range. Further, pairwise communications were replaced by local broadcasts to capture the essential features of communication protocols in sensor networks. Similar to earlier findings on regular, SW, and SF networks, we found that the NG on RGG with homogeneous node density also leads to global consensus, facilitating an application to autonomous key creation for encrypted communication in a community of sensor nodes (Lu et al. 2006, 2008).

It was found that the NG dynamics on the above networks will lead to global consensus among all agents, i.e., after some time, agents' vocabularies eventually converge to a unique word, the same for all agents (Baronchelli et al. 2006a,b, 2005; Dall'Asta et al. 2006a). The major differences between the NG on CGs (homogeneous mixing) and on low-dimensional networks (such as regular one- or two-dimensional grids, and RGGs) arise in the scaling of the memory need and in the scaling of the time t_c with the number of agents N to reach global agreement. [The memory need in the present context is the typical value of the largest number of words an agent may posses throughout the evolution of the game (Baronchelli et al. 2006a,b).] In CGs, the convergence process to global agreement is fast, $t_c \sim \mathcal{O}(N^{1/2})$ (measured in units of communications per agent), but large memory, $\mathcal{O}(N^{1/2})$, is needed per agent (Baronchelli et al. 2006b). For a regular two-dimensional network or RGG, spontaneous evolution toward a shared dictionary is slow, $t_c \sim \mathcal{O}(N)$, but the memory requirement per agent is much less severe, $\mathcal{O}(1)$ (Baronchelli et al. 2006a). When the NG is implemented on Watts–Strogatz (Watts and Strogatz 1998) SW networks (Dall'Asta et al. 2006a), or when long-range random links are added to the RGG (Lu et al. 2008), the agreement dynamics performs optimally in the sense that the memory needed is small, while the convergence process is much faster than on the respective low-dimensional network, $t_c \sim \mathcal{O}(N^{0.4})$, closer to that of CGs or homogeneous mixing.

The above results, on spatial graphs, can be understood within the framework of coarsening, a well-known phenomenon from the theory of domain and phase ordering in physical and chemical systems (Bray 1994). Starting from empty vocabularies, agents invent words randomly. After time of $\mathcal{O}(1)$ (on average one communication per node), $\mathcal{O}(N)$ *different* words have been created. Following the early-time increase of the number of different words (essentially corresponding to the number of different clusters of agents) $N_d(t)$, through pairwise or local communications, agents slowly reconcile their "differences", and eventually will all share the same word. First, a large number of small spatial clusters sharing the same word develop. By virtue of the slow coalescence of the interfaces separating the clusters, more and more of the small clusters are being eliminated, giving rise to the emergence of larger clusters, eventually leading to one cluster in which all nodes are sharing the same word, i.e., $N_d = 1$ (Baronchelli et al. 2006a; Lu et al. 2008). In domain coarsening, the typical size of domains (each with already agreed upon one word) is governed by a single length scale $\xi(t) \sim t^{\gamma}$ with $\gamma = 1/2$. Thus, in d dimensions the average domain size (inside which all agent share the same word) scales as $\xi^d(t) \sim t^{d\gamma}$ and the total number of *different* words N_d at time t scales as the typical number of domains $N_d(t) \sim N/\xi^d(t) \sim Nt^{-d\gamma}$. Global consensus is reached when $\xi^d(t_c) \sim N$ (or equivalently $N_d(t) \sim 1$), hence the typical time to global agreement scales as $t_c \sim N^{1/(d\gamma)}$.

On SW and SF random network models with no community structure, the long-time behavior of the NG is essentially governed by the mean-field fixed point,[2] and global consensus time scales as $t_c \sim N^{1/2}$ (although with noticeable finite-size corrections) (Dall'Asta et al. 2006a,b; Lu et al. 2008). On the other hand, Dall'Asta et al. (2006b) found that on stylized network models with community structure (composed of fully connected cliques with a single link between cliques) the evolution of the NG runs into long-living meta-stable configurations, corresponding to different co-existing words (different for each clique). Here, we study precisely this later scenario by implementing the NG on static empirical social graphs.

In this work we employ the basic NG where agents have infinite memories (i.e., the number of words they can store on their individual lists is unlimited). For a finite-memory version (Wang et al. 2007) of the model on social networks see the Electronic Supplementary Material.

In passing we note that the issue of the emergence of meta-stable or frozen opinion clusters and fostering consensus have been discussed for models of opinion formation under bounded confidence (Deffuant et al. 2000; Hegselmann and Krause 2002; Lorenz 2007, 2008). In those models, however, community formation or opinion segmentation is the result of the agents' interaction being limited by bounded confidence: an agent can gradually adjust her opinion toward another one's only if their opinions were already sufficiently close to one another to begin with. As a result, opinion segmentation can emerge in networks with no community structure with low-confidence agents. In contrast, the NG dynamics does not require that agents' opinions are sufficiently close in order to potentially interact (i.e., their confidence is unbounded), and as mentioned earlier, the NG dynamics always lead to global consensus on networks *without* community structure. Our motivation here, by studying the NG on empirical social graphs, is to directly study how the community structure of the underlying graphs affects the emergence of meta-stable or long-living opinion clusters.

[2] Here, by the mean-field fixed point, we refer to the characteristic scaling behaviors of the NG on the complete graph (also referred to as homogenous mixing) where each agent can potentially interact with all others (Baronchelli et al. 2006b; Dall'Asta et al. 2006b).

3 The Naming Game on empirical social networks

One of the most important feature of social graphs is their modularity: these networks typically consist of a number of communities; nodes within communities are more densely connected, while links bridging communities are sparse. Since the community structure of empirical networks is often not known a priori, detecting communities in large networks itself is a difficult problem (Palla et al. 2005). A number of current methods for finding community structures utilize various forms of hierarchical clustering, spectral bisection methods (Scott 2000; Newman 2006; Wu and Huberman 2004), and iterative high-betweenness edge removal (Newman and Girvan 2004; Newman 2004; Girvan and Newman 2002). A different approach involves searching for the ground-states of generalized multi-state spin models (corresponding to different opinions) on these networks, such as the q-state Potts model (Blatt et al. 1996; Reichardt and Bornholdt 2004; Kumpula et al. 2007; Fortunato and Barthelemy 2007). Also, recently a novel method has been developed to detect overlapping communities in complex networks (Palla et al. 2005).

The NG, as summarized in the Sect. 2, in low-dimensional networks exhibits slow coarsening, while networks with small-world characteristic (small shortest path, such as in SW and SF networks) facilitate faster (and guaranteed) convergence to a global consensus among nodes. But in all cases, global consensus is reached, provided the network has no heterogeneous clustering or modularity (i.e., community structure).

Here, we study the NG on networks which do exhibit strong community structure. The set of social networks (high-school friendship networks), on which we implemented the NG, were provided by the National Longitudinal Study of Adolescent Health (Add Health).[3] The high-school friendship networks investigated here, were constructed from the results of a paper-and-pencil questionnaire in the AddHealth project (Moody 2001). Here, nodes represent students while the edges are for their mutual relations or friendships. Two students are considered to be friends (thus have a link between them) when one nominates the other as her/his friend and both of them participated in some activities, e.g., talked over the phone, spent the weekend together, etc., in the last 7 days. (for this study, we considered the relationships reciprocal, and associated them with undirected links in the NG). These networks exhibit exponential degree distributions (no hubs), with an average degree of the order of 10. For a baseline comparison we also constructed a Watts–Strogatz (WS) network (Watts and Strogatz 1998) network with the same number of nodes N, average degree $\overline{k}$, and clustering coefficient $\overline{C}$ as the friendship network. The WS network has homogeneous clustering, hence, no community structure.

We selected a few networks with a large number of students (on the order of 1,000) from the available data set. Starting from an empty word list for all agents, both the friendship network and WS network show nearly identical early-time development

[3] This research uses the network-structure data sets from Add Health, a program project designed by J. Richard Udry, Peter S. Bearman, and Kathleen Mullan Harris, and funded by a grant P01-HD31921 from the National Institute of Child Health and Human Development, with cooperative funding from 17 other agencies. For data files contact Add Health, Carolina Population Center, 123 W. Franklin Street, Chapel Hill, NC 27516-2524 (addhealth@unc.edu, http://www.cpc.unc.edu/projects/addhealth/).

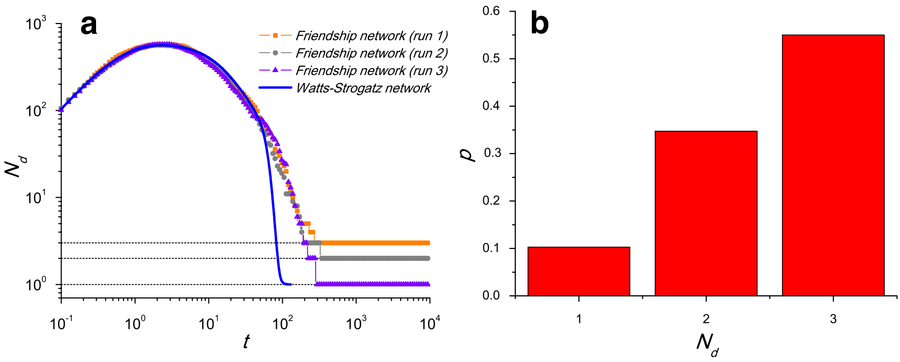

Fig. 2 **a** Number of different words N_d versus time for a friendship network (*thin lines*) and for the Watts–Strogatz network (*bold line*). $N = 1{,}127$, $\overline{k} = 8.8$, and $\overline{C} = 0.067$ for both systems. Results for the WS network are averaged over 1,000 independent realizations. For the high-school friendship network we show three individual realizations (*thin lines*), reaching different final states with $N_d = 1$, $N_d = 2$, and $N_d = 3$ (indicated with *horizontal dashed lines*). Note the log scales on both axes. **b** The probability (relative frequency) of final configurations with N_d different words (opinions) for the same high school friendship network as in (**a**) based on 10,000 independent runs. Statistically, in this particular network, the most likely final configuration exhibits three opinions

of the number of different words N_d. However, the friendship-network simulations exhibit a long-time behavior very different from the ones discussed in Sect. 2, and also from the baseline reference, the NG on the WS network (Fig. 2a). In the late stage of the NG, networked agents without community structure (including the WS network) *always* exhibit a spontaneous evolution toward a shared "dictionary" (or opinion), i.e., a global consensus is reached. In contrast, in the empirical high-school networks, consensus is rarely reached (for long but finite simulation times) (Fig. 2a). For this particular high-school friendship network, performing 10,000 independent runs of the NG with a fixed simulation time of $t = 10^4$ steps, 10, 35, and 55% of these runs, ended up with one, two, and three different words, respectively, in their final configurations (Fig. 2b). Thus, in this network, the most likely (or typical) outcome of the NG is one with three different clusters of opinions. Snapshots taken from the typical evolution of the NG on this network are shown in Fig. 3. In analogy with domain formation in physical systems, we can regard these long-living configurations with coexisting multiple opinions as "meta-stable" ones.

The emergence of different long-living clustered opinions is not unexpected. In fact, the same high-school networks have been analyzed for community structures in a study of friendship segregation along racial lines among high-school students (Moody 2001; Gonzáles et al. 2007). For example, close to the final stage, the time-evolution of the NG on the particular network shown in Fig. 3b exhibits four communities. These four clusters of opinions correspond to segregation along the two-schools involved in the particular network, high-school (HS)—middle-school (MS) pair, and along racial lines, whites students—black students in each. Checking the race and school-grade attribute of the node information in the raw data, we confirmed that the four communities exhibited by the NG in Fig. 3b correspond to black HS (upper left), white HS (upper right), black MS (lower left), and white MS (lower right) students. Then, in the

Fig. 3 Snapshots of the time-evolution of the Naming Game on a high-school friendship network. Initially agents have an empty word list (no opinions). In the snapshots, *different colors* correspond to different words. In the very early stage of the game (**a**), "speakers" with no words has to create one randomly. After a slow but steady coarsening of opinions, in the final stages of the game, the system exhibit relatively long plateaus in the number of different opinions. The corresponding clusters, i.e., agents with the same opinions, can be regarded as communities. For the particular network shown here, in the next to final stage (**b**), the network exhibits four communities. Eventually, two of these communities coalesce, leading to a final configuration (**c**) with three communities

final state (Fig. 3c), only three communities remain; opinions, segregated along the racial line coalesce in the middle-school portion of the students, simply indicating that racial segregation in friendships is weaker in this group, in this particular network set.

Admittedly, the objective of our paper is not to draw over-ambitious conclusions from a social science viewpoint. Instead, we are interested in how the evolution of the NG (a stylized model for opinion formation) is affected by the community structure of the underlying graphs, such as the high-school friendship networks which are well-known to exhibit strong community structure (Moody 2001; Gonzáles et al. 2007). We demonstrated that the outcome of the NG, is strongly affected by the existence of communities in the underlying network. Conversely, at some coarse level, the long-living late-stage meta-stable clusters of words (opinions) reveal important aspects of the community structure of the underlying network. Thus, the NG, together with other stylized models for opinion formation, can not only be used as a tool to understand generic features of spontaneous agreement processes in a network of artificial or human agents, but can also be employed to extract relevant information on the community structure of complex networks (Blatt et al. 1996; Reichardt and Bornholdt 2004; Kumpula et al. 2007; Fortunato and Barthelemy 2007).

4 Engineering consensus in social networks

There are several ways to influence the outcome of social dynamics, e.g., to facilitate the outcome of a specific global opinion that one would prefer the system to achieve (preferred opinion for short). All methods essentially rely on "breaking the symmetry" of the otherwise equivalent coexisting opinions. One possibility is to expose and couple many or all agents to an "external" global signal (analogous to mass media effects) (Mazzitello et al. 2007; Candia and Mazzitello 2008). Alternatively, one can break the symmetry by choosing a small number of well-positioned "committed" agents who will stick to a preferred opinion without deviation. In the next subsection, we investigate this latter scenario first.

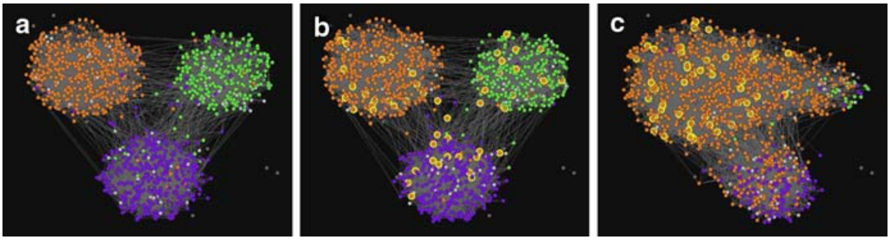

Fig. 4 Snapshots of the Naming Game on a high-school friendship network with committed agents. The system is initialized from a state with three coexisting meta-stable communities [see (**a**)] with a small number of well-positioned committed agents (indicated with *yellow core* around the nodes as indicated in (**b**)). Global consensus (i.e., a single opinion) is reached exponentially fast. Here we employed 50 committed agents, selected according to their degree ranking

4.1 Committing agents

In the simulations, by committed agents we mean an agent who has a fixed opinion which cannot be changed. In the context of the NG, a committed agent has a single word. As a listener, she does not accept any new word from their neighbors, but as a speaker, always transmits her word. Of the three co-existing communities at the end-stage of the NG (Fig. 3c), we choose one community as the one representing the "preferred" opinion, and we "indoctrinate" selected committed agents with this opinion. Figure 4 shows snapshots of the evolution of the NG with committed agents. Initiating the simulations from the final configuration of the original NG (exhibiting three meta-stable opinion clusters), introducing a small number of committed agents yields a relatively fast convergence to the global consensus of the selected opinion.

To quantify this phenomena we investigated the temporal behavior of this agreement process, in particular, its dependence on the method of selecting committed agents and on the number of these selected agents. Among the methods to select committed agents are selecting nodes with the highest degrees (nodes with the highest number of neighbors), with the highest betweenness (likely to bridge different communities), with hop-distance proximity to the core cluster (nodes outside, but no farther than two hops from the core cluster of "preferred" opinion), and for comparison, also selecting committed agents at random.

Our main observation is that once the number of committed agents is sufficient to induce global consensus, it happens *exponentially* fast, independently of the selection method. More precisely, we ran 10,000 realizations of the NG with committed agents. The initial configuration here is the final multi-opinion meta-stable configuration of the original NG with no committed agents (with $N_d = 3$) (Fig. 4). We kept track of the *fraction of surviving runs*, $n_s(t)$, defined as the fraction of runs that have not reached global consensus by time t, i.e., runs that have more than one opinion at time t. (This quantity then can also be interpreted as the probability that a single run has not reached consensus by time t.) We choose committed agents, to maximize their influence in reaching global consensus, according to their ranking in a number of graph theoretical measures. We selected the top M agents according to their degree, shortest-path betweenness centrality (Newman and Girvan 2004; Newman 2004), hop-distance

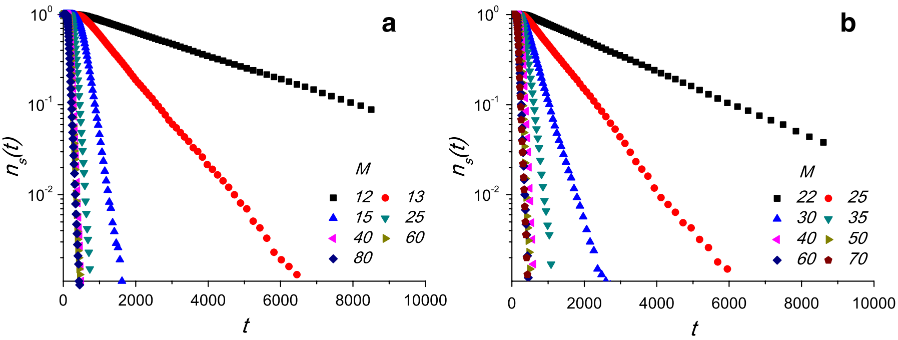

Fig. 5 Fraction of surviving runs as a function of time for varying number of committed agents M when agents are selected according to their (**a**) degree ranking and (**b**) (shortest-path) betweenness ranking. The total number of agents is $N = 1{,}127$. For the degree-based ranking selection method different symbols represent the fraction of surviving runs for 12, 13, 15, 25, 40, 60, and 80 committed agents, from top to bottom. In betweenness selection method the number of committed agents M ranges from 22, 25, 30, 35, 40, 50, 60, to 70, from top to bottom

proximity to the preferred core opinion cluster, or at random, for reference. Figure 5 displays the fraction of surviving runs $n_s(t)$ for the degree and for the betweenness ranking for a number of different committed agents.

A common feature of all methods is that a very small fraction ($f = M/N$) of committed nodes is sufficient to induce global consensus. I.e., there seems to be a very low threshold in f_{c1}, such that for $f > f_{c1}$ the dynamics with committed nodes leads to global agreement. Further, in this case, the fraction of surviving runs (fraction of runs with more than one opinion), $n_s(t)$, in the long-time regime, decays exponentially

$$n_s(t) \propto \mathrm{e}^{-t/\tau}. \tag{1}$$

The time scale of the exponential decay τ, of course, depends on the selection method and the fraction of committed nodes. The inverse time scale $1/\tau$, i.e., the rate at which global consensus is approached is, initially, an increasing function of the number of committed nodes, but it quickly *saturates* and essentially remains constant. This can be seen in Fig. 5, as the slopes of the exponential decays are becoming progressively steeper, up to a certain M, then they remain constant. Thus, there is second characteristic fraction of committed agents, such that for $f > f_{c2}$ the rate of reaching global consensus becomes essentially a constant (saturates).

These three features, (i) small threshold f_{c1} required for global consensus, (ii) exponential decay of $n_s(t)$ if $f > f_{c1}$ (Figs. 5, 6), and (iii) saturation of the rate to reach consensus for $f > f_{c2}$ (Fig. 7), are exhibited by all selection method we considered here. Further, both characteristic values and the gap between them are very small, f_{c1}, f_{c2}, $f_{c2} - f_{c1} \ll 1$. These results are essentially summarized in Fig. 7. The convergence rate for the randomly selected committed nodes is also shown for comparison. On this particular social network, selecting a small number of the nodes with the highest degree works best, followed by the hop-distance proximity (to the core cluster) ranking. (We refer to a strategy as more efficient if the convergence rate

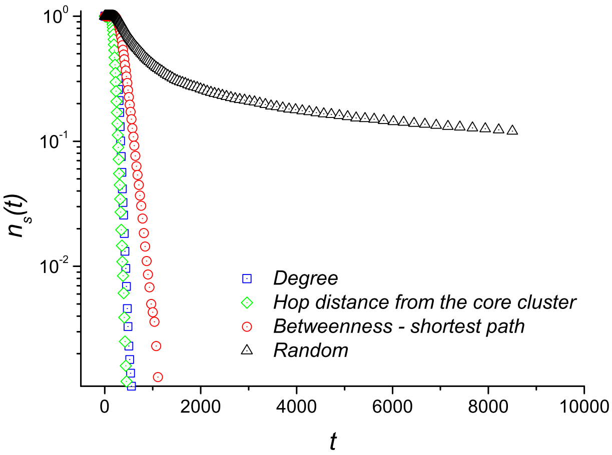

Fig. 6 Fraction of surviving runs as a function of time for different strategies with the same number of committed agents on the same network ($M = 35$, $N = 1{,}127$, $f \simeq 0.031$). The three strategies (selection of committed agents) shown here are based on degree ranking (*squares*), hop-distance proximity to the core cluster (*diamonds*), and shortest-path betweenness (*circles*). For comparison, the result of selecting committed agents randomly is also shown (*triangles*)

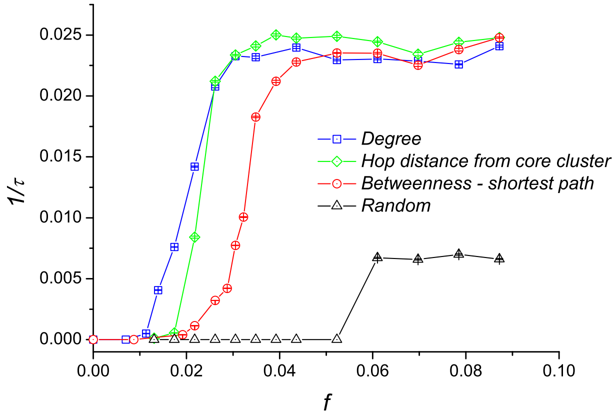

Fig. 7 Convergence rate as a function of the fraction of committed agents $f = M/N$, for different selection methods of committed agents, including the degree ranking (*squares*), hop-distance proximity to core cluster (*diamonds*), and shortest-path betweenness (*circles*). For comparison, the result of selecting committed agents randomly is also shown (*triangles*)

$1/\tau$ is larger for the same fraction of committed agents.) For example, selecting the committed agents according to their degree ranking, $f_{c1} \approx 0.01$ and $f_{c2} \approx 0.03$ (Fig. 7). Selecting committed agents just above this latter fraction is optimal, since the rate of convergence does not improve beyond this value.

In general, the optimal selection method will vary, depending on the community structure of the particular underlying network. However, because we changed the dynamics of the NG by breaking the symmetry of otherwise equivalent opinions, the

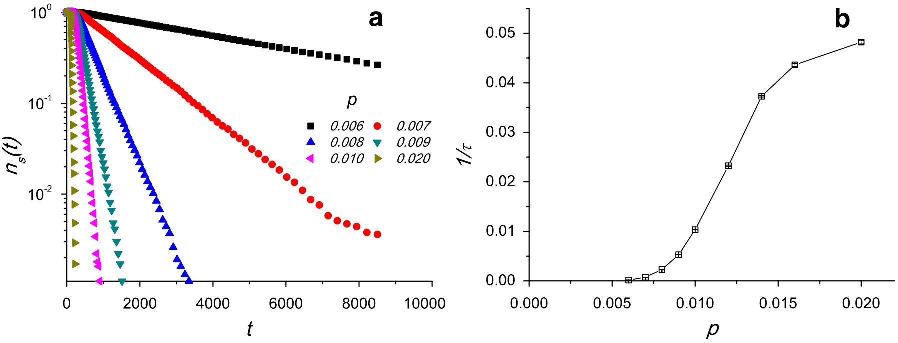

Fig. 8 **a** The fraction of surviving runs as a function of time for several values of the strength of external influence p (p is the probability that in a time step an agent will adopt the fixed externally and globally promoted opinion). **b** Convergence rate to global consensus as a function of the strength of external field p

exponential decay and the saturation of the convergence rate is expected to be generic for a large class of opinion formation models on networks with community structure.

4.2 Global external influence

As mentioned in the introductory paragraph of this section, another natural way of influencing the outcome of the competition among otherwise neutral and meta-stable opinions, is to couple all or a fraction of agents to a global external "signal" [mimicking a mass media effect (Mazzitello et al. 2007; Candia and Mazzitello 2008)]. For comparison, we implemented the NG with an external field (affecting all agents) corresponding to the selected opinion among the three meta-stable ones in the final stage of the NG. Then, similar to the committed-agent approach, we initialize the system with that final meta-stable state with co-existing opinions of the original NG. In the presence of mass media, an agent, when randomly chosen, with probability p will adopt the externally promoted opinion. Otherwise, the usual rules of the game are invoked (i.e., the node, as a speaker, initiates communication with a listener). Our findings indicate that even for extremely small values of p, the fraction of surviving runs (the fraction of runs that have not reached global consensus) decays exponentially, ultimately leading to global order (Fig. 8a). The rate of convergence $1/\tau$ increases monotonically and smoothly with p (Fig. 8b). For application oriented studies, one should associate a cost with the mass-media coupling, and a cost with committing an agent (e.g., finding these nodes and giving them incentives impossible to resists), then perform a relevant cost-benefit comparative analysis for the selection or optimal combination of two approaches.

5 Summary

We studied the Naming Game on social networks. Earlier works have shown that this simple model for agreement dynamics and opinion formation always leads to global consensus on graphs with no community structure. On the other hand, social networks are known to have rich community structure. The Naming Game on such networks exhibits, in the late-stage of the dynamics, several meta-stable coexisting communities; these configurations, in effect, are the computationally observed final configurations.

In the context of models for social dynamics, communities manifest themselves in the context in which distinct stylized opinions (e.g., religions, cultures, languages) have evolved and emerged over time. Clusters of nodes having reached consensus are part of a community, reflecting the inherent community structure of the underlying social networks. Thus, if at the late stages of the social dynamics on the networks, several communities persist (different opinions survive), they are the authentic signatures of the underlying community structure. The Naming Game, together with other similar models for opinion formation, can be employed to probe these properties of complex networks.

We then considered the task of destabilizing the coexisting meta-stable opinions (in order to reach consensus) by selecting the optimal number of committed agents with a preferred opinion, as an alternative to a global external signal (mass media effect). The results implied that a small number of committed agent is sufficient to facilitate an exponential decay toward global consensus of the selected opinion. Further, selecting more agents than a system-specific upper cut-off, yields no improvement in the convergence rate. Hence, there seems to be an optimal number of agents for this task, beyond which it does not pay off to invest in committing more agents. Selecting the committed agents according to their degree, betweenness, or hop-distance proximity to the core cluster of the preferred opinion, all displayed the above qualitative features. Further, they all significantly outperformed committing the same number of agents at random.

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References

Anghel M, Toroczkai Z, Bassler K, Korniss G (2004) Competition-driven network dynamics: emergence of a scale-free leadership structure and collective efficiency. Phys Rev Lett 92:058701 (4 pp)

Antal T, Krapivsky P, Redner S (2005) Dynamics of social balance on networks. Phys Rev E 72:036121

Axelrod R (1997) The dissemination of culture: a model with local convergence and global polarization. J Confl Resolut 41:203–226

Baronchelli A, Dall'Asta L, Barrat A, Loreto V (2005) Strategies for fast convergence in semiotic dynamics. arXiv:physics/0511201

Baronchelli A, Dall'Asta L, Barrat A, Loreto V (2006a) Topology induced coarsening in language games. Phys Rev E 73:015102(R)

Baronchelli A, Felici M, Caglioti E, Loreto V, Steels L (2006b) Sharp transition towards shared vocabularies in multi-agent systems. J Stat Mech Theory Exp P06014

Baronchelli A, Loreto V, Dall'Asta L, Barrat A (2006c) Bootstraping communications in language games: strategy, topology, and all that. In: Cangelosi A, Smith ADM, Smith K (eds) Proceedings of the 6th international conference on the evolution of language, World Scientific, Singapore, pp 11–18

Ben-Naim E (2005) Opinion dynamics: rise and fall of political parties. Europhys Lett 69:671–677

Benczik I, Benczik S, Schmitmann B, Zia R (2008) Lack of consensus in social systems. Europhys Lett 82:48006

Blatt M, Wiseman S, Domany E (1996) Superparamagnetic clustering of data. Phys Rev Lett 76:3251–3254

Bray A (1994) Theory of phase-ordering kinetics. Adv Phys 43:357–459

Candia J, Mazzitello KI (2008) Mass media influence spreading in social networks with community structure. J Stat Mech Theory Exp P07007

Castellano C, Loreto V, Barrat A, Cecconi F, Parisi D (2005) Comparison of voter and Glauber ordering dynamics on networks. Phys Rev E 71:066107

Castellano C, Fortunato S, Loreto V (2008) Statistical physics of social dynamics. arXiv:07103256v1

Cattuto C, Loreto V, Pietronero L (2006) Semiotic dynamics and collaborative tagging. Proc Natl Acad Sci USA 104:1461–1464

Cattuto C, Baldassarri A, Servedio VDP, Loreto V (2007) Vocabulary growth in collaborative tagging systems. arXiv:07043316v1

Challet D, Marsili M, Zhang YC (2005) Minority games: interacting agents in financial markets. Oxford University Press, New York

Dall J, Christemsen M (2002) Random geometric graphs. Phys Rev E 66:016121

Dall'Asta L, Baronchelli A, Barrat A, Loreto V (2006a) Agreement dynamics on small-world networks. Europhys Lett 73:969–975

Dall'Asta L, Baronchelli A, Barrat A, Loreto V (2006b) Nonequilibrium dynamics of language games on complex networks. Phys Rev E 74:036105

Deffuant G, Neau D, Amblard F, Weisbuch G (2000) Mixing beliefs among interacting agents. Adv Complex Syst 3:87–98

Durlauf SN (1999) How can statistical mechanics contribute to social science? Proc Natl Acad Sci USA 96:10582–10584

Epstein JM, Axtell R (1996) Growing artificial societies: social science from the bottom up. MIT Press, Cambridge

Fortunato S, Barthelemy M (2007) Resolution limit in community detection. Proc Natl Acad Sci USA 104:36–41

Girvan M, Newman MEJ (2002) Community structure in social and biological networks. Proc Natl Acad Sci USA 99:7821–7826

Golder S, Huberman BA (2006) Usage patterns of collaborative tagging systems. J Inf Sci 32:198–208

Gonzáles MC, Herrmann HJ, Kertész J, Vicsek T (2007) Community structure and ethnic preferences in school friendship networks. Phys A 379:307–316

Hegselmann R, Krause U (2002) Opinion dynamics and bounded confidence: models, analysis, and simulation. J Artif Soc Soc Simul 5:2

Jung WS, Moon HT, Stanley HE (2008) Dynamics of clusterd opinions in complex networks. J Econ Interact Coord 3:81–88

Kozma B, Barrat A (2008a) Consensus formation on adaptive networks. Phys Rev E 77:016102

Kozma B, Barrat A (2008b) Who's talking first? Consensus or lack thereof in coevolving opinion formation models. Phys Rev Lett 100:158701

Krapivsky PL, Redner S (2003) Dynamics of majority rule in two-state interacting spin systems. Phys Rev Lett 90:238701 (4 pp)

Kumpula JM, Saramaki J, Kaski K, Kertész J (2007) Limited resolution in complex network community detection with Potts model approach. Eur Phys J B 56:41–45

Lambiotte R, Ausloos M (2007) Coexistence of opposite opinions in a network with communities. J Stat Mech Theory Exp P08026

Lim M, Metzler R, Bar-Yam Y (2007) Global pattern formation and ethnic/cultural violence. Science 317:1540–1544

Lin BY, Ren J, Yang HJ, Wang BH (2006) Naming Game on smal-world networks: the role of clustering structure. arXiv:physics/0607001

Lorenz J (2007) Continuous opinion dynamics under bounded confidence: a survey. Int J Mod Phys C 18:1819–1838

Lorenz J (2008) Fostering consensus in multidimensional continuous opinion dynamics under bounded confidence. In: Helbing D (ed) Managing complexity: insights, concepts, applications. Springer Series in Understanding Complex Systems, pp 321–334

Lu Q, Korniss G, Szymanski BK (2006) Naming Games in spatially-embeded random networks. In: Proceedings of the 2006 American association for artificial intelligence fall symposium series, interaction and emergent phenomena in societies of agents. AAAI Press, Menlo Park, pp 148–155

Lu Q, Korniss G, Szymanski BK (2008) Naming games in two-dimensional and small-world-connected random geometric networks. Phys Rev E 77:016111

Mazzitello KI, Candia J, Dossetti V (2007) Effects of mass media and dultural drift in a model for social influence. Int J Mod Phys 18:1475–1482

McConchie A (2002) The great pop vs. soda controversy. http://popvssoda.com:2998/

Meester R, Roy R (1996) Continuum percolation. Cambridge University Press, Cambridge

Moody J (2001) Race, school integration and friendship segragation in America. Am J Sociol 107:679–716
Newman MEJ (2004) Detecting community structure in networks. Eur Phys J B 38:321–330
Newman MEJ (2006) Finding community structure in networks using the eigenvectors of matrices. Phys Rev E 74:036104
Newman MEJ, Girvan M (2004) Finding and evaluating community structure in networks. Phys Rev E 69:026113
Onnela JP, Saramaki J, Hyvonen J, Szabó G, Lazer D, Kaski K, Kertész J, Barabási AL (2007) Structure and tie strength in mobile commnication networks. Proc Natl Acad Sci USA 104:7332–7336
Palla G, Derényi I, Farkas I, Vicsek T (2005) Uncovering the overlapping community structure of complex networks in nature and society. Nature 435:814–818
Palla G, Barabási AL, Vicsek T (2007) Quantifying social group evolution. Nature 446:664–667
Penrose M (2003) Random geometric graphs. Oxford University Press, Oxford
Reichardt J, Bornholdt S (2004) Detecting fuzzy community structures in complex networks with a Potts model. Phys Rev Lett 93:218701
San Miguel M, Eguiluz VM, Toral R, Klemm K (2005) Binary and multivariate stochastic models of consensus formation. Comput Sci Eng 7(6):67–73
Schelling TC (1971) Dynamic models of segregation. J Math Sociol 1:143–186
Scott J (2000) Social network analysis: a handbook. 2nd edn. Sage, London
Sood V, Redner S (2005) Voter model on heterogeneous graphs. Phys Rev Lett 94:178–701
Sznajd-Weron K, Sznajd J (2000) Opinion evolution in closed community. Int J Mod Phys C 11:1157–1165
Vinkovic D, Kirman A (2006) A physical analogue of the Schelling model. Proc Natl Acad Sci USA 19:19261–19265
Wang WX, Lin BY, Tang CL, Chen GR (2007) Agreement dynamics of finite-memory language games on networks. Eur Phys J B 60:529–536
Watts DJ, Strogatz SH (1998) Collective dynamics of small-world networks. Nature 393:440–442
Wu F, Huberman BA (2004) Finding communities in linear time: a physics approach. Eur Phys J B 38:331–338
Zhang J (2004) A dynamic model of residential segregation. J Math Sociol 28:147–170