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REGULAR ARTICLE

Frankfurt Artificial Stock Market: a microscopic stock market model with heterogeneous interacting agents in small-world communication networks

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Abstract We study the relationship between communication network topologies, namely the small-world networks introduced by Watts and Strogatz, and the simulation results of an artificial stock market, here the Frankfurt Artificial Stock Market. Heterogeneous interacting agents communicate their success and trading strategy to their nearest neighbors. A process of information diffusion arises through the adaptive behavior of agents when encountering more successful strategies in their direct neighborhood. We will show that an increasing rewiring probability of the small-world network will lead to higher volatility and distortion within our simulation model. It seems probable that the spatial position of traders within a communication network affects the price building process.

1 Introduction

Networks of social interaction between individuals have been studied intensively since the introduction of small-world networks by Watts and Strogatz (1998). In contrast

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to random networks (Erdös and Renyi 1959), small-world networks seem to mimic the properties of social networks much better. Many examples of empirical findings showed a close fit (Dorogovtsev and Mendes 2003; Newman 2000; Strogatz 2001; Watts 2003). When studying diffusion processes in a network environment the question arises which network topology to use. We chose small-world networks because of their relevance in social studies and the fact that they can be easily generated under controlled conditions by increasing the rewiring probability p. Studying artificial stock markets became an issue in finance since the creation of the Santa Fe Artificial Stock Market (Arthur et al. 1997). Other models introduced various improvements to model the stylized facts (Cont 2001) of capital markets (Levy et al. 2000; Bak et al. 1996; Cont and Bouchaud 2000; Lux and Marchesi 1999). We want to extend this research by introducing a small-world trader network into our Frankfurt Artificial Stock Market (FASM). Our goal is to show how the network properties influence the price building process. This goal mainly derives from the herding behavior of financial investors (Bikhchandani and Sharma 2001). With our simulations we are able to identify a relationship between network topologies and the characteristics of the time series in FASM.

2 Network topology and information contagion

Network topology and information contagion plays a crucial role when modeling the communication between individuals. The spread of rumors, or in our case the diffusion of successful trading strategies, are communicated through the contact between individuals. The structure of the network very much defines how fast information may spread (Newman 2000).

Small-world network models show a lot of similarities to social and other types of real world networks (Wang and Chen 2003). They are characterized by a small average path length and a high degree of clustering:

- *Average path length:* If the distance d_{ij} is the minimal number of edges between node i and node j, the average path length is then defined by the average of the minimal distances between all nodes.
- *Clustering coefficient:* Assuming node i has k_i edges and therefore is connected to k_i nodes. The maximum number of edges between these nodes is $k_i(k_i-1)/2$ if all nodes are connected with each other. The clustering coefficient C_i of node i is defined as the ratio of the existing edges E_i and the maximal number of potential edges. The clustering coefficient C of the network is the average of the clustering coefficients of all nodes.

$$C = \frac{1}{n}\sum_{i=1}^{n} C_i = \frac{1}{n}\sum_{i=1}^{n} \frac{2E_i}{k_i(k_i-1)} \tag{1}$$

The network topology of small-world networks interpolates between regular and random graphs. Starting from a completely regular graph, where each node has four links to its direct neighbors (Fig. 1, left), a rewiring procedure with probability p

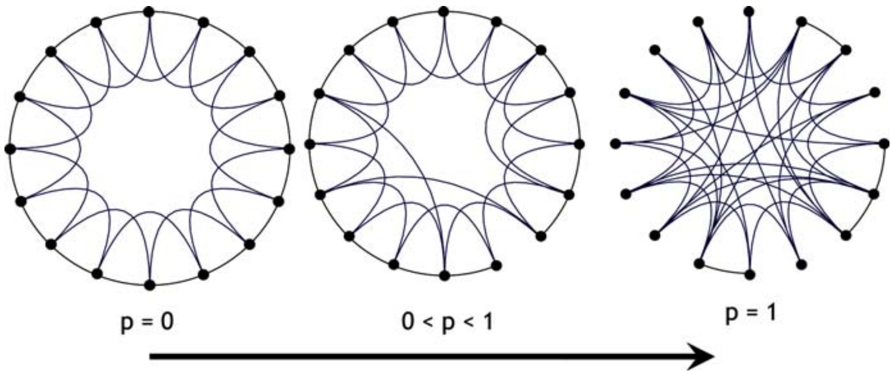

Fig. 1 Generating a small-world network with increasing rewiring probability p

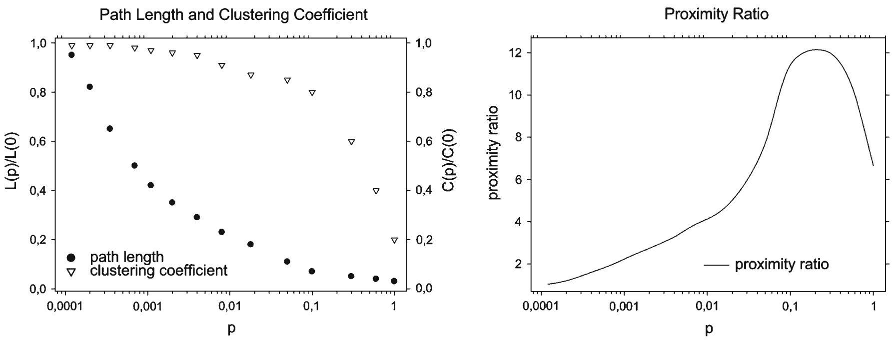

Fig. 2 The small-world effect, fast decay of path length, slow decay of clustering coefficient, proximity ratio (x-axis in log-scale)

takes place. With higher p's more and more links are redirected (Fig. 1, middle) until a random graph emerges (Fig. 1, right).

Watts and Strogatz define a network as a small-world network if L is comparable to a random graph and C is much greater than for a random graph. In Fig. 2 left, normalized path length and clustering coefficient are shown in relation to rewiring probability p. Average path length L decreases much faster than the clustering coefficient C. Walsh (1999) used this phenomenon to define the proximity ratio μ_W:

$$\mu_W = \frac{C(p)/C(0)}{L(p)/L(0)} \tag{2}$$

μ_W is much greater for a network obeying the Watts–Strogatz definition of a small-world network than for a random network (Newman et al. 2006). With $p \gtrsim 0.1$, μ_W becomes significantly greater than 1, indicating that the network is a small-world network (Fig. 2, right).

With a network topology which seems similar to social networks, a process that describes the spread of information among agents is needed. The standard SIS-Model (susceptible–infected–susceptible) is used for modeling the infection process

(Kermack and McKendrick 1927). Nodes representing agents may be in the state "susceptible" or "infected" and are connected to other agents. An infected agent is able to infect his neighbor with probability υ; at the same time an infected agent is able to switch with probability ρ from the state "infected" to the state "susceptible". The propagation rate λ is υ/ρ. There is a critical propagation rate $\lambda_c > 0$ for homogeneous networks, like small-world-networks and random networks. If $\lambda \geq \lambda_c$ the infection stays within the network, if $\lambda < \lambda_c$ the infection disappears (Dorogovtsev and Mendes 2003).

Instead of probabilities we use the profitability of trading strategies within a defined time window. Agents are "susceptible" if they still follow their initial trading strategy, either fundamental or trend oriented. If the agent encounters a superior trading strategy within the neighborhood, a switch may be initiated if an individual threshold is reached and the agent changes to the state "infected". The agent frequently checks the success of its new strategy and may change back to the initial strategy if the new strategy is no longer successful and an individual threshold is reached. In that case the agent falls back to the state "susceptible" and the cycle may repeat again.

3 FASM: Frankfurt Artificial Stock Market

The FASM is not an example of a stock market that tries to generate the universal stylized facts of the capital markets (Cont 2001) with a simple model that is still analytically tractable, as advocated in, e.g., Hommes (2006). The authors have chosen a different approach to loosen the analytic tractability due to a complex system in exchange for a more plausible modeling at the micro level. It is also not intended to achieve a closer match of the stylized facts, but to explore the impact of different communication topologies on price building processes with artificial stock markets. However the stylized facts are taken as a model validation.

The design of the individual *agents* and the *auction mechanism* forms the basis of the desired model. The agents are divided into two separate groups, *fundamental investors* and *trend investors* following the design of other stock market models (Lux and Marchesi 1999). In addition to preceding stock market models, an agent communication network in the form of a small-world network with different rewiring probabilities p is introduced and a new kind of agent type switching algorithm that leads to mimetic contagion will be described. The presentation of the batch double order book used and the order generation process which delivers the necessary limit volume orders concludes the model definition.

All agents are endowed with a random amount of cash and stocks at the beginning of the simulation. However, fundamental agents receive greater wealth than trend investors to make sure that the price deviation from the fundamental price is bound. The agents can only buy or sell if enough cash or stocks are in the portfolio, short selling or lending is not allowed. All agents receive the same but independent communication probability (e.g., 10%, on average agents communicate every 10th day). The time window used for computing the increase in total wealth is the same for all agents (e.g., 20 days). For liquidity purposes a *random trader* is used. It buys and sells daily and randomly with small random volumes to guarantee a minimal trading volume.

The random trader does not communicate with other agents and is not part of the network. The communication network is exogenously generated and the agent types are distributed in clusters over the network. The agent's positions within the network are defined by the network partition and stay the same for all simulation repetitions.

3.1 Behavior of the agents

3.1.1 Fundamental agents

Fundamental investors buy and sell according to the fundamental value of a security. Depending on whether they are under or over valued, the sale or purchase of shares is initiated. If the actual stock price is P_t, the individual threshold τ and the actual fundamental value is P_t^f, the limit order for a fundamental investor is generated as follows:

$$\text{If } P_t^f - \tau > P_t \text{ buy at limit } P_t^f - \tau \tag{3}$$

$$\text{If } P_t^f + \tau < P_t \text{ sell at limit } P_t^f + \tau \tag{4}$$

$$\text{otherwise do nothing}$$

Because of the individual τ each fundamental agent starts trading at a different price deviation.

The fundamental value P_t^f is exogenous and follows a random path:

$$P_{t+1}^f = P_t^f + \eta_{t+1} \tag{5}$$

η_t is a normal independently and identically distributed random process with the standard deviation δ_η and the mean μ_η.

3.1.2 Trend agents

Trend investors just monitor the price. They use moving averages with an individual time window (the sum of the prices of the last x trading days divided by x). The time window for each individual agent is distributed between two values, e.g., 10 and 200 trading days. If the actual price breaks the moving average from below (above) the agents starts buying (selling). Because of the individual time window, all trend agents start trading at a different moving average price. An extra time series of 300 days price history has been generated randomly to allow trend investors to compute moving averages on the first trading day.

3.1.3 Order generation

The order volume is generated relative to the deviation of the actual price from the individual moving average price or the inner value $\pm\tau$ respectively. If the deviation is, e.g., between 0 and 1% one share is traded, between 1 and 2% 3 shares are traded, between 2 and 4% 6 shares, if more than 4% 10 shares are traded (see also Table 2). Additionally each percentage interval may be categorized as a market order. In that case the original limit is erased and a market order with the volume according to the percentage interval is generated. With a market order an agent is willing to trade at the best available price. The order generation models the behavior of investors who want to buy or sell more when the price gets more attractive.

Fundamental agents create a buy and a sell order every trading day. The limit of the buy order is $P^f - \tau$ the limit for the sell order is $P^f + \tau$.

Trend agents create one order per trading day depending on the buy or sell signal of their moving average rule. The limit depends on the price of the previous trading day adjusted by a small positive or negative random percentage number.

3.1.4 Communication and type switching

The agents communicate with their nearest neighbors through a communication network. The number of nearest neighbors depends on the chosen network topology and may vary. The communication probability for each agent defines how often an agent communicates with its neighbors. The communication probability is the same for all agents, but decided individually and independently. The agents communicate their type (trend or fundamental) and their wealth increase in percent over a fixed time window. If one of the nearest neighbors sees a greater wealth increase over several communication events, the agent switches to the same trading strategy used by this nearest neighbor. The number of communication events necessary for a switch is an individual and fixed threshold that the agent initially receives. Since the individual strategy parameter for τ or the moving average time window are not communicated, the agent receives a new random parameter for the trading strategy. By doing so the agent will still be heterogeneous after switching to the new type. The agent still remembers its initial type after the switching. Three cases of further switchings are possible:

1. A nearest neighbor reports a greater wealth increase for an individual number of times. The agent will switch to the neighbors type.
2. If the agent is initially a fundamental agent and has switched to a trend type, it will switch back to the fundamental strategy as soon as the actual price deviates more than an individual multiple of τ from the inner value. We assume that investors are "born" with a natural risk profile. They only deviate from that risk profile if the success of their nearest neighbors exceeds a threshold. Their initial strategy is still examined for extreme misvaluations. If an individual threshold is reached, they return to their initial strategy even if their actual strategy is successful.
3. If the agent is initially a trend agent and has switched to a fundamental agent, it will switch back to the trend strategy as soon as the trend strategy would have been more successful for a threshold number of past trades. As with fundamental investors we assumed that trend investors only temporarily switch to the fundamental strategy if the success of the nearest neighbors is superior for a number of times. They still monitor how successful their initial strategy would have been. If an individual number of past trades have been executed at a price that turned out to be too low, in the case of a sell order, and too high in the case of a buy order,

the agent switches back to its initial type. The idea is that for a number of last trades it may have been better to follow the trend instead of trading according to an inner value.

If a trading strategy (e.g., trend) remains superior long enough, it is possible that the other strategy (e.g., fundamental) will disappear completely within the agent pool. Since the agents still remember their initial type and the switching rules allow the switch back to the initial type, trading strategies may appear again depending on the further development of prices. A required minimum number of agents of each type as used in Lux (2000) is not necessary, but the initial distribution of trend and fundamental agents is more important since it may be reached again several times during the course of a simulation path.

3.1.5 Heterogeneity

Despite the fact that, besides a single random trader, only two types of traders exist, all agents are heterogeneous. Even if the same strategy is used, each strategy has an individual parameter that differentiates each agent. Agents differ in regard of the following parameters:

1. the strategy parameter τ for fundamental traders;
2. the length of the moving average time window for trend agents;
3. an individual threshold number for switching to the superior strategy of the nearest neighbor;
4. an independent random decision for the communication to the nearest neighbor (but with the same probability);
5. an individual threshold to switch back to the initial strategy;
6. the order volume;
7. the order limit;
8. the wealth each agent is endowed with.

All parameters are assigned randomly, but within boundaries.

3.2 Auction method

Bak et al. (1996) used a direct random netting procedure between the agents to settle a new market price. One agent is randomly chosen to look into the market order book. If there are prices within the limits of the desired transaction, the best price is chosen and the order is settled. The disadvantage of this procedure is that the price change is at low volume, because only one buy or sell order is regarded. The advantage is that if no market maker is needed, the problem of active positions is not introduced. Cont and Bouchaud (2000) used a Walrasian auctioneer and Lux and Marchesi (1999) decided for a stochastic process as a market maker. The aforementioned auction mechanisms all have the disadvantage that they are not used at any exchange.

Table 1 Parameter values of fundamental and trend agents

Agents' parameters					
Agent type	#	τ	Time window	Init. cash (million)	Init. stocks
Fundamental	35	0.01–0.1	–	1.5–6.0	500–1,500
Trend	15	–	10–200 days	0.3–1.5	500–2,000

For the sake of realism the authors opted for an auction mechanism that is used every day at the Frankfurt Stock Exchange (http://www.deutsche-boerse.com).[1] It is basically a limit order book that is settled once a day. The resulting price is the price where the maximum volume occurs. It has the advantage that all agents are synchronized and have equal chances to trade and the volume is maximized, so the price is a better indication of the market equilibrium. A price is set once a trading day; with 3,000 trading days used a time series of 3,000 prices will be generated. The view of the time granulation is arbitrary, but is constant and not changed in dependence of volatility as done in Lux (2000).

4 Simulation parameters and results

Complex artificial market models like the FASM have many parameters that influence the results of a simulation run. The focus of this study is to establish a link between network topologies, like the ones with the small-world property, and the simulation results of an artificial market that uses these networks as a communication network for the agents. All other parameters like inner value, number of agents and trading days, agent positions within the network or diffusion speed of information are held constant. Table 1 exhibits the basic parameter values for fundamental and trend agents. The parameter values for the order generation are displayed in Table 2. The percentage interval in the buy/sell signal column denotes the percentage deviation from the price at which the agents start trading (either the moving average price, or $P_t^f \pm \tau$). If the percentage deviation from the price where the buy/sell signal occurred is increasing, the volume of ordered shares rises. The necessary parameter values for the communication are shown in Table 3. All agents share the same communication probability, but decide independently. The wealth increase is computed with a 20 day time window. Nearest neighbors have to communicate a superior wealth increase for 3 times before a switch to the neighbor's type is possible (inertness). The switch back for fundamental agents who changed their type to a trend agent is forced as soon as the deviation of the actual price from the inner value is more than the individual $\tau + 5\%$. Trend agent are forced to switch back to their initial strategy, if the executed price of the last 8 orders has been either too low (sell) or too high (buy) compared to the price of the following trading day. Table 4 summarizes the generation parameter values of the inner value.

[1] It is called the "Einheitskurs" or "Kassakurs" and is used for odd lots (order amounts of less than 100 or 50 shares, depending on the price).

Table 2 Volume of shares ordered by of fundamental and trend agents depending on the strength of the buy/sell signal

Fundamental agents		Trend agents	
Deviation from signal (%)	Vol. in shares	Deviation from signal (%)	Vol. in shares
0–2	1	0–4	5
2–5	2	4–8	8
5–7	5	8–16	30
7–∞	10	16–∞	40

Table 3 Communication parameters for agent type switching

Communication probability	Time window for wealth increase	Switching threshold	Fundamental agent switchback	Trend agent switchback
15%	20	3	$\tau + 5\%$	8 trades

Table 4 Parameters for fundamental value time series parameters generation

P_0^f	σ	μ	No. of trading days
1, 000	0.01	0	3, 000

Simulation runs with identical parameter sets are repeated to achieve better robustness in the results. An example of time series in price, volume, and log returns produced by simulation runs that use a small-world communication network generated with a rewiring probability of $p = 0.2$ is displayed in Fig. 3.

Using IVC[2] 21 small-world networks were generated for p's between 0 and 1. As within other market models like the ones of Cont and Bouchaud (2000) and Lux and Marchesi (1999), the agents were initially placed into clusters within the network environment. In contrast to the mentioned models they do not act in concert, but all exhibit heterogeneous behavior. Figure 4 left shows a small-world communication network with $p = 0$, as in Fig. 1 left circle, in the middle with $p = 0.2$, on the right with $p = 0.6$. The networks exhibit a greater number of rewired links with rising p and in consequence more links between the two clusters. They encourage a greater information exchange between the two groups of agents. The probability of switching to the other type of behavior during the course of the simulation should rise with increasing p's.

Fifty-one agents (initially with 35 fundamental, 15 trend oriented agents and one random trader) and 3, 000 trading days were used, and each simulation was repeated 15 times per parameter set. The communication probability was set to 15%. The inner value was held identical for every run. Box–Whisker-Plots were used to demonstrate the dispersion of the repeated simulations results per probability p. Volatility was measured as average absolute quadratic relative changes as in Westerhoff (2003);

[2] Information Visualization Cyber Infrastructure, Information Visualization Lab at Indiana University, http://iv.slis.indiana.edu.

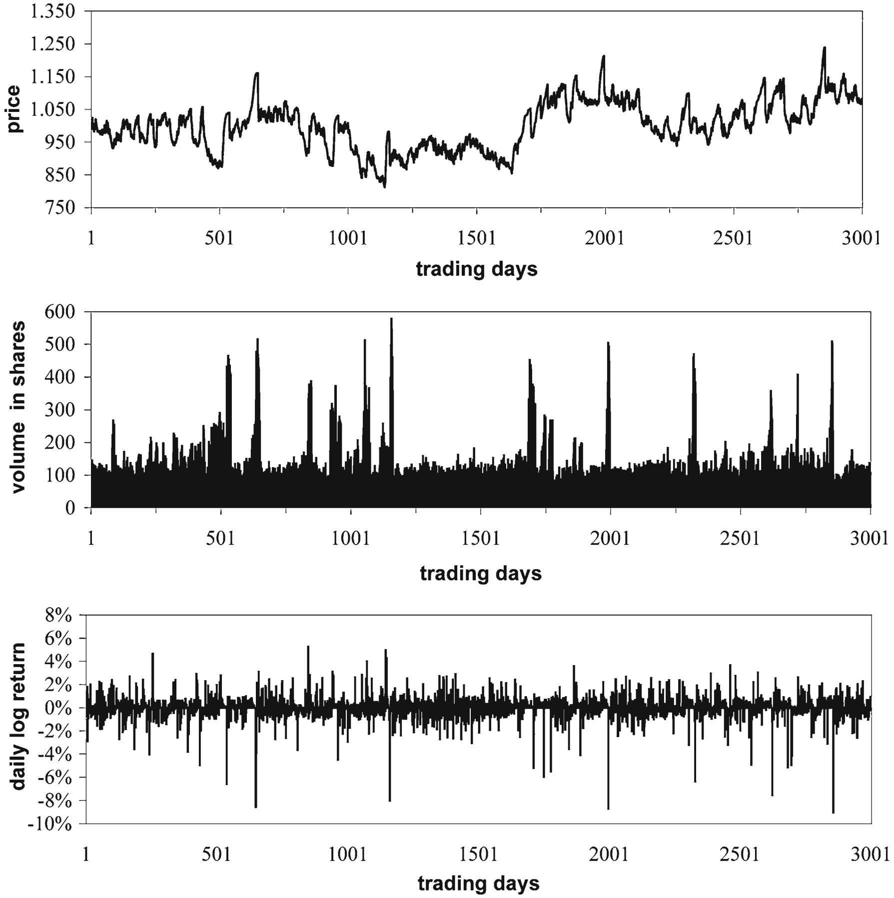

Fig. 3 Price chart with volume and daily log returns. A small-world communication network with $p = 0.2$ rewiring probability has been used

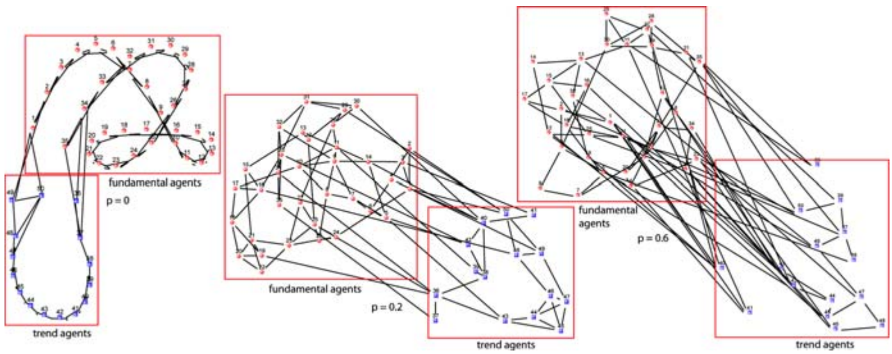

Fig. 4 Initial small-world communication networks with two clusters of agent types (large cluster with 35 fundamental agents, small cluster with 15 trend following agents) with $p = 0$ (*left*), $p = 0.2$ (*middle*) and $p = 0.6$ (*right*)

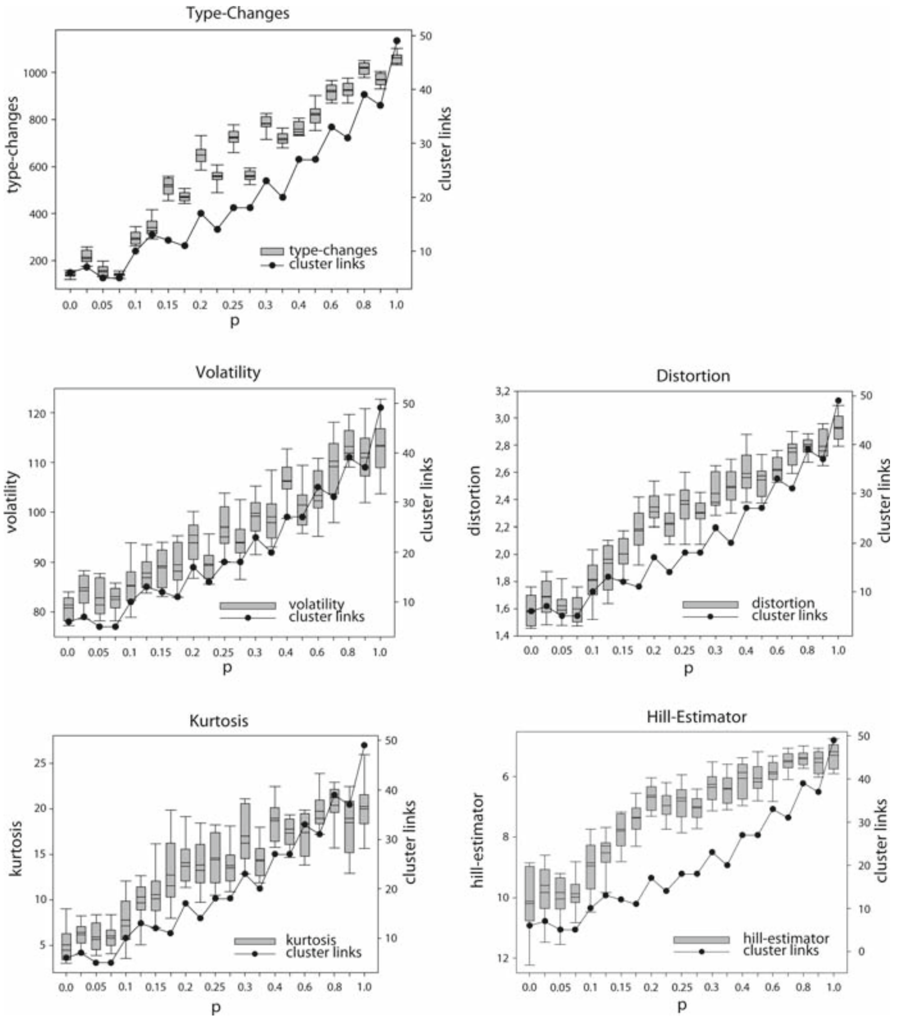

Fig. 5 Simulation results and number of links between type clusters

however, in contrast to Westerhoff quadratic relative changes were chosen to give more weight to large returns:

$$\text{volatility} = \frac{100}{T}\sum_{t=2}^{T}((P_t - P_{t-1})/P_{t-1})^2 \tag{6}$$

Similar the distortion is measured as average absolute relative deviation between price and fundamental value (Westerhoff 2003):

$$\text{distortion} = \frac{100}{T}\sum_{t=2}^{T}\left|(P_t - P_t^f)/P_t^f\right| \tag{7}$$

Figure 5 shows simulation results in relation to network properties. All charts indicate a strong relation between the considered results (box plots) and the number of links (line plot) between the clusters. As mentioned earlier the p's above 0.1 play an especially important role with regard to the small-world property. When dividing the type changes chart in Fig. 5 into 3 segments of the form $0 \leq p < 0.1$, $0.1 \leq p \leq 0.3$ and $0.3 < p \leq 1$, and calculating the gradient of the median of the type changes within those boundaries, it becomes evident that the segment where the small-world property takes place is the segment with the highest slope. This result is also reflected in the other results, namely the volatility, distortion, kurtosis and the Hill–Estimator. It may be assumed that the network topology, and here especially the small-world property, play an important role for the determination of the price building process within an artificial stock market.[3]

5 Conclusion

This work studied the impact of different communication network topologies on the simulation results of an artificial stock market. Small-world networks alongside a diffusion process on networks were presented as a precondition for the experimental setup. Our Frankfurt Artificial Stock Market has been created to especially study network effects and herding behavior in a controlled simulation environment. The goal is to explore what the consequences of network type variations on the time series generated by our simulation model are. Our results indicate that the spatial position of the individual agents within the network alongside the network topology has a considerable effect on the price building process of the simulation model. All considered simulation results such as the number of type changes, the volatility, distortion and kurtosis increase with the probability p, the Hill–Estimator decreases with p and signals thicker tails.

Especially for p's in the range of 10–30%, where Walsh's proximity ratio μ_W reaches high levels, simulation results show a steeper slope than outside of this range. This may be related to the small-world property. We believe that the novelty of our research result consists in the fact that we can show that the spatial positions of traders in the network and the network topology influences the outcome of the price building process. We are confident that future research based on our results will lead to a better understanding of real markets. Our research is currently being extended to include other network types like scale-free networks.

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[3] For further evaluation the system may be downloaded at http://www.finace.org.

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