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REGULAR ARTICLE

Agglomeration in an innovative and differentiated industry with heterogeneous knowledge spillovers

Klaus Wersching

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Abstract This paper introduces an agent-based simulation model to study the technological development, the economic performance of firms and the evolution of agglomerations in a differentiated industry. The analysis is based on the interaction and behavior of firms, which might share knowledge but at the same time are competitors on the goods markets. Firms do not only compete with quantities they can also introduce process and product innovations. The level of knowledge of a firm describes the capabilities to perform innovations. Knowledge can be accumulated by investing in R&D and by knowledge spillover, which depend on geographical and technological proximity. Simulation runs show that there is an incentive to agglomerate in young industries and that geographical proximity enhances innovation, especially the number of product innovations.

Keywords Innovation · Agglomeration · Knowledge spillover · Agent-based simulation · Industry dynamics

JEL Classification D83 · O31 · O32 · R10

1 Introduction

Innovation and technological change are crucial for understanding economic growth. This paper introduces an agent-based simulation model in order to

K. Wersching (✉)
Bielefeld University, Department of Business Administration and Economics,
P.O. Box 10 01 31, 33501 Bielefeld, Germany
e-mail: kwersching@wiwi.uni-bielefeld.de

examine the interplay between technological and geographical location decisions and the evolution of specific knowledge in an industry.

According to Boschma (2005) proximity is seen as the main factor fostering innovation. Two interpretations of proximity, namely geographical and technological proximity, have a major impact on the learning capabilities of firms. Beside investments in Research and Development (R&D) learning allows firms to accumulate knowledge, which is the precondition for generating successful innovations: either to raise productivity through process innovation or to attract new consumer groups with new products through product innovations. Knowledge is inherently different from the more traditional inputs labor, capital, and land: knowledge is intrinsically uncertain, it is asymmetric allocated between economic agents, it is cumulative and can be transmitted voluntary or involuntary (at least over some geographical distance) without loosing any value (see Dosi 1988). In order to catch all relevant effects of innovation the model has to take into account this particular characteristic of knowledge.

The empirical literature suggests that geographical proximity leads to a faster diffusion of knowledge through spatially bounded knowledge spillover. A recent survey by Asheim and Gertler (2005) even claims: "..one simply cannot understand innovation properly if one does not appreciate the central role of spatial proximity and concentration in this process." Based on these observations a core-periphery pattern is used, so that firms can either choose a location in the core or in the periphery. The core may also be understood as a cluster or a local network, where all firms might profit from the knowledge of each other. The concentration of firms in the core leads to higher production cost resulting from shortage of scarce resources. On the other hand, firms can choose a location in the periphery with lower production cost, but then they cannot increase their knowledge with external spillover. If a firm chooses isolation, it will however, not loose their technological competence via involuntary knowledge spillover.

The cognitive or technological distance seems to be significant for the learning process, too. The amount of knowledge a firm is able to use economically is described by the absorptive capacity (see Cohen and Levinthal (1989, 1990)). The concept of absorptive capacity sets a lower bar for the firm's knowledge heterogeneity. But the learning effect is also reduced if a firm wants to absorb very similar knowledge. The heterogeneity of knowledge should be "sufficiently small to allow for understanding but sufficiently large to yield non-redundant, novel knowledge" (Nooteboom (2000, p. 72)). The outcome of the knowledge exchange process could be described as a hump-shaped relation depending on technological proximity. The heterogeneity of knowledge can be expressed by the technological distance, measured by the path between two technologies in a technology space and the technological gap between the knowledge stock of two firms in these technologies. Both elements are relevant for the resulting learning effect through knowledge spillover.

In the past, industry simulation models, which considered knowledge spillover with geographical and/or technological distance were often based on a cellular automata framework, e.g. Verspagen (1993), Keilbach (2000), Brenner

(2001), Caniëls and Verspagen (2001), and Meagher and Rogers (2004) present simulation models with innovation in a spatial landscape. Other approaches like Cantner and Pyka (1998a,b) describe industry dynamic models with heterogeneous knowledge spillover. Here a product market was modeled while focusing either on the absorptive capacity of firms or the selection process with different technologies. Jonard and Yildizoglu (1998) and Zhang (2003) introduce extensions of the traditional Nelson and Winter (1982) model with a technological space and spatially bounded knowledge externalities. Llerena and Oltra (2002) also extend the Nelson–Winter model in order to focus on diversity of innovative strategies. In another simulation model Gilbert et al. (2001) describe the interaction of agents with a specified knowledge base in an innovation network.[1]

This paper combines elements of these simulation studies while concentrating on the interaction on differentiated product markets and strategic location of firms in the sense of geographical and technological distance. The main focus is set on understanding the economic use of knowledge in an evolving industry. The paper is organized as follows. In the next Sect. 2 the model is introduced. The description of the model contains the role of knowledge, the circular technology space, the calculation of knowledge spillover, the market demand and cost structures, as well as the decision making of firms and finally the market clearing. Section 3 shows the setup for the simulation studies. Results of the simulation runs can be found in Sect. 4, where the evolution of an agglomeration and the technological development in different scenarios is presented. The paper closes with the main conclusions in Sect. 5.

2 The model

In order to deal with the effects of geographical and technological proximity an industry-simulation model is introduced. The model is based on Dawid and Reimann (2005a,b) with the extension of heterogeneous knowledge spillover. The production side of the industry is represented by an agent-based model allowing for heterogeneities of location, cost-structures, and strategies concerning production and R&D among the industry firms. The demand side is highly stylized employing the concept of a representative consumer.

2.1 Knowledge and innovations

The knowledge of firms is one of the most important elements of the model. Each firm holds a technological profile, which represents the capabilities for innovations. On the one hand, the company may introduce a new method, which leads to lower production cost, or it presents a better version of an existing product. On the other hand, the firm wants to launch a brand-new product

[1] A broader overview of agent-based simulation models focussing on innovation and the technological development of an industry is presented in Dawid (2006).

in order to meet the needs of new consumer groups. The first part of the technology profile is captured by a knowledge stock for process innovations $\mathrm{RD}_{i,j,t}^{\mathrm{proc}}$ and the second part by a knowledge stock for product innovations $\mathrm{RD}_{i,j,t}^{\mathrm{prod}}$, both depending on the company i, the technology j and the time period t.

Both stock variables can be increased either by own investments in R&D $\left(I_{i,j,t}^{\mathrm{proc}} \text{ or } I_{i,j,t}^{\mathrm{prod}}\right)$ or by knowledge spillover $\left(\mathrm{SP}_{i,j,t}^{\mathrm{proc}} \text{or } \mathrm{SP}_{i,j,t}^{\mathrm{proc}}\right)$, where investments in R&D and spillover are understood as perfect substitutes. The build-up of a knowledge stock for innovations has the property that it is a time consuming process where experiments and knowledge is step by step accumulated over time. It is also assumed, that the return to investment, measured by increases in the knowledge stock, decreases as the company approaches the frontier of $\mathrm{RD}_j^{\max}$. The knowledge starts at zero or at an initialized number in the interval [0,1].

Afterwards the knowledge stock is updated as follows:

$$\mathrm{RD}_{i,j,t}^{\mathrm{proc}} = \mathrm{RD}_j^{\max} - \left(\mathrm{RD}_j^{\max} - \mathrm{RD}_{i,j,t-1}^{\mathrm{proc}}\right) \frac{1+\alpha_i\beta_i\left(I_{i,j,t-1}^{\mathrm{proc}} + \mathrm{SP}_{i,j,t}^{\mathrm{proc}}\right)}{1+\alpha_i\left(I_{i,j,t-1}^{\mathrm{proc}} + \mathrm{SP}_{i,j,t}^{\mathrm{proc}}\right)} \tag{1}$$

Here $\alpha_i > 0$ and $\beta_i > 0$ are firm-specific parameters, which describe the ability of the firm to develop new products and the efficiency of the use of R&D funds. In particular, firm i can each period reduce the gap to the frontier at most by the factor β_i. Equation (1) also represents the cumulative property of knowledge. A rising knowledge stock for process innovations $\mathrm{RD}_{i,j,t}^{\mathrm{proc}}$ leads directly to lower production cost.

The equation for updating the knowledge stock for product innovations can be formulated analogously with the only difference that the upper bound is equal to $\mathrm{RD}_j^{\max} = 1$. In contrast to process innovation, a knowledge stock for product innovation greater than zero does not automatically lead towards a successful product innovation. In fact, the immanent uncertainty with product innovations is captured by a stochastic process, which determines, if a product innovation is successful or not. A product innovation can be either incremental or radical.[2] In the case of a radical innovation a new technology was created which is a little more separated from the others. In order to show this we have to introduce the technology space first.

[2] Two numbers were chosen: u from the uniformly distributed interval $[c, d]$ with $0 < c < d$, and v from the uniformly distributed interval $[d, e]$ with $d < e \leq 1$. If $\mathrm{RD}_{i,j,t}^{\mathrm{prod}} > u$ the firm i was able to introduce a product innovation on the market. If $\mathrm{RD}_{i,j,t}^{\mathrm{prod}} > v$ the new product was a technological breakthrough, which could be interpreted as a radical innovation. Otherwise the product innovation is incremental.

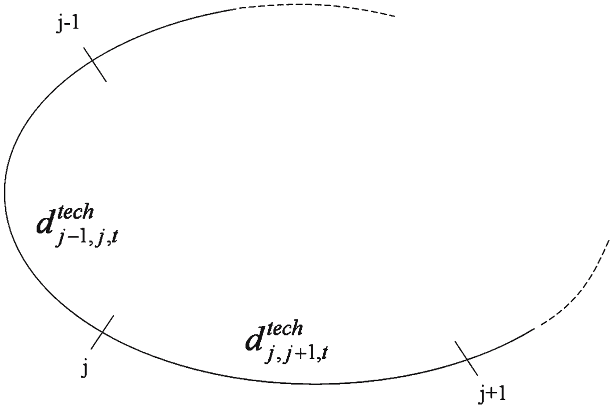

Fig. 1 The technology space as a *circle*

2.2 The technology space

The technology space is interpreted as a circle in tradition of the circular city models in the industrial organization literature (originally introduced by Salop 1979). The idea is that products belonging to a technology j, which marks a certain point on that circle, are horizontally differentiated. The technological distance $d^{\mathrm{tech}}_{j,j+1,t}$ between two technologies j and j+1 is interpreted as the shortest path on the circle. The overall number of existing technologies should be m_t. In Fig. 1 the technological space and the corresponding technological distances are shown.

A successful product innovation adds a new technology on the circular technology space. In the case of incremental innovation the new technology $m_t + 1$ is placed right in the middle between two existing ones. In the case of a radical innovation the circle expands as the new product variant adds significant new features to the product of the industry. The technological space increases in the same level as the size of the market niche, in which the radical product innovation is located.

The innovating firm with a new kind of product stays for τ periods as a monopolist on this new market. After that period other firms can gain specific knowledge in this technology and produce this product variant, too. The initial R&D stock for this new product variant depends on the knowledge of the innovating firm in the neighboring technologies. This fact considers the cumulative structure of knowledge, so that the firm can make use of similar knowledge already accumulated in the firm. Thus the initial knowledge stock for the new technology $\mathrm{RD}^{\mathrm{proc}}_{i,m_t+1,t}$ is the mean of the neighboring technologies of the innovating firm i, but at least a lower bound RD^0 and at most 1. The maximal value $\mathrm{RD}^{\max}_j$, which can be reached in this market should be twice this initial knowledge stock for the new technology. This means that the knowledge in every market can only be doubled.

2.3 Learning through knowledge spillover

Heterogeneity of knowledge is a precondition for learning. The differences in knowledge are interpreted in two ways: First, there are technological distances between different strands of technologies. Second, there is a difference in two knowledge stocks, which represents the knowledge gap. The knowledge gap $t_{ik,jl,t}$ between firm i an k related to different technologies j and l in period t is:

$$t_{ik,jl,t} = \max\left\{\ln\left(\frac{\mathrm{RD}^{\mathrm{proc}}_{k,l,t}}{\mathrm{RD}^{\mathrm{proc}}_{i,j,t}}\right), 0\right\} \tag{2}$$

In this formula the technological gap is only greater than 0, if the other company k has a greater value of the knowledge stock variable. The function is concave.

As mentioned above, the absorptive capacity of a firm is crucial for learning through knowledge externalities. The concept of absorptive capacity is incorporated in the variable $\gamma_{i,t}$ of firm i in t, which is assumed to be the mean value over all m_t technologies. Therefore a firm with a high amount of technological knowledge is able to absorb a higher fraction of external and internal knowledge:

$$\gamma_{i,t} = \frac{\sum_j^{m_t} \mathrm{RD}^{\mathrm{proc}}_{i,j,t}}{m_t} \tag{3}$$

Up to now we presented the technological aspects of proximity and learning. The geographical aspect of learning is captured by the geographical distance $d^{\mathrm{geo}}_{i,t}$ of firm i. The geographical distance is modeled similar to core-periphery models (see e.g. Krugman 1991). There are only two possible locations for firms: in the core (or agglomeration) or outside in the periphery.

$$d^{\mathrm{geo}}_{i,t} = \begin{cases} 0, & \text{in the core;} \\ 1, & \text{in the periphery.} \end{cases} \tag{4}$$

A location in the core leads to a better exchange of tacit knowledge but it might also raise the own production cost because of scarce resources, e.g. through higher office rents or price for building land, higher wages for employees etc. If a company is a technological leader it might also prefer a location in the periphery in order not to let too much knowledge spill over upon competitors.

We now have all ingredients for the calculation of knowledge spillover, which represents learning in the model. Firms i and k can learn from competitors only if both of them are placed in the core, that is $\left(1 - d^{\mathrm{geo}}_{i,t}\right) \cdot \left(1 - d^{\mathrm{geo}}_{k,t}\right) = 1$. But learning can also happen within the firm, where knowledge is transformed from one kind to another. Learning regards all technologies m_t. Based on the formula for knowledge spillover from Verspagen (1993) and Cantner and Pyka (1998a,b) with two kinds of distances $d_{i,t}^{\text{geo}}$ and$d_{j,l,t}^{\text{tech}}$, the technological gap $t_{ik,jl,t}$ and the absorptive capacity $\gamma_{i,t}$ the resulting knowledge spillover for process innovations $\text{SP}_{i,j,t}^{\text{proc}}$ for the technology j and firm i can be written as:

$$\begin{aligned} \text{SP}_{i,j,t}^{\text{proc}} &= \sum_{l=1}^{m_t} \sum_{k \neq i} \left[(1 - d_{i,t}^{\text{geo}}) \cdot (1 - d_{k,t}^{\text{geo}}) \cdot \frac{1}{1 + d_{j,l,t}^{\text{tech}}} \cdot t_{ik,jl,t} \cdot \mathrm{e}^{-\, t_{ik,jl,t}/\gamma_{i,t}} \right] \\ &+ \sum_{l=1}^{m_t} \left[\frac{1}{1 + d_{j,l,t}^{\text{tech}}} \cdot t_{ii,jl,t} \cdot \mathrm{e}^{-\, t_{ii,jl,t}/\gamma_{i,t}} \right] \end{aligned} \tag{5}$$

Analogously the knowledge spillover for product innovations $\text{SP}_{i,j,t}^{\text{prod}}$ can be formulated.

The first term stands for the external knowledge spillover. They are only greater than zero if both firms are in the core. The second term stands for internal knowledge spillover within a firm, which exist independent of the geographical location of the firm. The formula is built in the way that it is maximized if the technological gap equals the absorptive capacity. Any deviations from this point lead to lower knowledge transfer. This represents that learning has less effect if the knowledge is too similar or too different. A higher technological distance $d_{j,l,t}^{\text{tech}}$ between the two regarding technologies j and l reduces the possible learning effect, too.

2.4 Market demand

We consider an industry consisting of n producers. At any point in time t there exist m_t sub-markets within this industry, where each sub-market represents a variant of the product considered. With every technology located on the circular technological space the production of a certain variant is possible. Thus the index j stands for each sub-market as well for the corresponding technology.

Consumers are assumed to have love-for-variety preferences where the representative consumer has a utility function:

$$u_t(X_{1,t} \; \ldots \; X_{m_t,t}) = \left[\sum_{j=1}^{m_t} \left(A_{j,t} \cdot X_{j,t} \right)^b \right]^{1/b} \tag{6}$$

The parameters $A_{j,t}$ denote the current attractiveness of product variant j and $X_{j,t}$ consumption of product variant j. The degree of complementarity between the different product variants is expressed by $b \in [0, 1]$ where values close to zero correspond to complementary goods whereas the variants are perfect substitutes for $b = 1$.

The standard love-for-variety approach assumes equal attractiveness of the variants but in this case the variants should be weighted by the level of attractiveness. The attractiveness $A_{j,t}$ of product variant j depends on the technological distances to the neighbors of j in the technology space. The greater the product of the distances the greater is the market niche and therefore the attractiveness for the consumers:

$$A_{j,t} = d^{\text{tech}}_{j-1,j,t} \cdot d^{\text{tech}}_{j,j+1,t} \tag{7}$$

The utility function in Eq. (6) is maximized subject to the budget constraint:

$$\sum_{j=1}^{m_t} p_{j,t} \cdot X_{j,t} \leq B(t) \tag{8}$$

$B(t)$ denotes the overall amount of money allocated by consumers to purchase goods produced in this industry. We will assume that it increases with the number of product variants, however at a decreasing rate:

$$B(t) = \text{msize}\frac{m_t}{A + m_t} \tag{9}$$

Here msize gives the maximal amount of money that could be allocated to purchase in this industry and A governs how fast the allocated funds grow with increasing overall attractiveness of the sub-markets. By making this assumption we intend to capture the goods produced in this industry do not only compete among themselves but also compete for consumer budget allocation with outside products. All producers in this industry set production quantities for all sub-markets they are in and prices are determined by market clearing. Straightforward calculations yield the following inverse demand curve for a market j:

$$p_{j,t} = \frac{B(t) \cdot \left(A_{j,t}\right)^b}{\left(X_{j,t}\right)^{1-b} \cdot \sum_{l=1}^{m_t} \left(A_{l,t} \cdot X_{l,t}\right)^b} \tag{10}$$

2.5 The Cost Structures of Producers

Each of the n firms in the industry can in every period produce for each of the public sub-markets. $M_{i,t}$ should stand for the set of markets the firm i produces for in period t and by $x_{i,j,t}$ the output quantity of firm i on sub-market j.

The firms production cost are given by:

$$C_{i,t}(x_{i,j,t}) = F_i \cdot \left|M_{i,t}\right| + \sum_{j \in M_{i,t}} \left(c_{i,j,t} \cdot \left(x_{i,j,t}\right)^2\right) \tag{11}$$

The fixed cost F_i are a constant firm specific parameter. For every sub-market the firm produces for fix cost F_i arise. The variable costs $c_{i,j,t}$ depend on two factors: first the amount of knowledge in technology j, and second the geographical location of the firm, because we assume that all production takes place at the location of the firm:

$$c_{i,j,t} = c_{i,j,t}^{\text{proc}} + c_t^{\text{geo}} \tag{12}$$

The first term $c_{i,j,t}^{\text{proc}}$ is in consequence depending on current knowledge and the second term c_t^{geo} on geographical location. An important aspect of this model is the fact that production cost can be decreased over time through process improvements and accumulation of tacit knowledge.

$$c_{i,j,t}^{\text{proc}} = c_{i,j}^{\text{ini}}\left[c_{i,j}^{\min} + \left(1 - c_{i,j}^{\min}\right)\left(1 - \text{RD}_{i,j,t}^{\text{proc}}\right)\right] \tag{13}$$

The scarce resource in the core leads to an increase in the marginal production cost of every firm in the core. If only one firm is in the core no additional cost will occur. The marginal cost for every single output quantity of a firm in the core $\left(d_{i,t}^{\text{geo}} = 0\right)$ is increased by c_t^{geo}. How much the production cost rise depends on the total number of firms in the core and on the parameters R and c^{geo}.

$$c_t^{\text{geo}} = \left(1 - d_{i,t}^{\text{geo}}\right) \cdot (|N_t| - 1)^{c^{\text{geo}}} \cdot R \tag{14}$$

The number of firms in the core should be called $|N_t|$, where N_t is the set of firms, which are located in period t inside the core. The parameter c^{geo} describes the gradient of the geographical cost function.

2.6 Decision making

The decision process of the firms involves three steps: first, to decide on the set of markets the firm intends to service, second to determine the output quantities for these markets, and third to decide on investments in product or process innovations and geographical location.

2.6.1 Market entry and exit

The total number of firms in the industry n is assumed to stay constant. But the number of firms who are active on a certain sub-market is determined endogenously. In this model incumbents do not have the possibility to strategically use their knowledge and market power to prevent firms from penetrating their markets (see the literature about price wars and R&D races, e.g. Reinganum 1983 or Dixit 1988). But in general the entrants have less knowledge for process innovations that results in higher production cost, see Eq. (13). Although the entry is quite easy, entrants have to catch-up with the technological leader in order to gain similar profits. If after entering a market a firm is not able to achieve positive profits, the firm will exit.

The change in the market portfolio a firm holds is modeled as a sequence of rule-based market exit and entry decisions. The exit and entry rules rely on an evaluation of all existing markets carried out at the beginning of each period. It is assumed that at the end of period all firms can observe the average profits on every market and have an idea of the public technology space. In order to keep the model as simple as possible the evaluation for market entry depends only on the average profits on a sub-market and on the technological distance to the own main technological focus. The factors in the evaluation function should be in the interval [0,1]. For this the average profit $\bar{\Pi}_{j,t-1}$ on the market j is divided by the greatest profit a firm made on any sub-market in the last period. The own technological focus l is the technology with the greatest knowledge stock: l from $\max_j \left\{ \mathrm{RD}^{\mathrm{proc}}_{i,j,t} \right\}$. For this the evaluation $v_{i,j,t}$ of a sub-market j is given by:

$$v_{i,j,t} = \left(\frac{\bar{\Pi}_{j,t-1}}{\max_{k,l} \left\{ \Pi_{k,l,t-1} \right\}} \right)^{\delta_{i,\Pi} / (\delta_{i,\Pi} + \delta_{i,T})} \cdot \left(\frac{1}{1 + d^{\mathrm{tech}}_{j,l,t}} \right)^{\delta_{i,T} / (\delta_{i,\Pi} + \delta_{i,T})} \tag{15}$$

The sum of the exponents is chosen to be equal to 1. The exponents are important parameters of the firm's diversification strategy since they represent the weights assigned to profits and technological specialization.

To make the entry decision the firm ranks all available markets[3] it does not currently serve according to their evaluations and determines the best existing non-served market as the entry candidate. The entry candidate is added to the portfolio if $v_{i,j,t} > \kappa_{i,\mathrm{en}}$. The parameter $\kappa_{i,\mathrm{en}}$ is an inertia parameter and represents the aggressiveness of the firm's entry policy. The firm can only enter in one sub-market every period.

The exit decision of the firm is determined solely on the sum of profits of the last τ_{ex} periods. The firm will choose the market with lowest value for $\sum_{\tau}^{\tau_{\mathrm{ex}}} \Pi_{i,j,t-\tau}$ and will exit this sub-market if the sum of the profits is negative: $\sum_{\tau}^{\tau_{\mathrm{ex}}} \Pi_{i,j,t-\tau} < 0$. The knowledge of this specific technology remains in the firm. The firm exits up to one sub-market a period.

2.6.2 Quantity decisions

In order to describe the rules, which govern the quantity decision making of the firm the amount of information firms can use should be mentioned. It is assumed that the aggregate output quantities and the number of firms in all sub-markets at $t-1$ can be observed by all producers including those that were not active in this market. Furthermore, the price elasticities of demand $\varepsilon_{j,t}$ for these quantities are also common knowledge. Each firm has in all periods perfect

[3] After a successful product innovation the innovating firm is monopolist on this market for τ periods and therefore no other firms can enter.

information about the own fixed cost F_i and marginal cost $c_{i,j,t}$ of production of all product variants. Firms, however, do not have perfect information about the exact shape of entire demand function and also do not know other firm's cost structures.

Given the set of sub-markets $M_{i,t}$ firm i tries to maximize their profits by choosing the optimal output quantity $x_{i,j,t}$ in each sub-market:

$$\max_{x_{i,j,t}, j \in M_{i,t}} \left[p_{j,t} \cdot x_{i,j,t} - c_{i,j,t} \cdot \left(x_{i,j,t}\right)^2 - F_i \right], \tag{16}$$

subject to the constraint that current production has to be paid for by the current stock of savings:

$$S_{i,t} \geq F_i \cdot \left|M_{i,t}\right| + \sum_{j \in M_{i,t}} \left(c_{i,j,t} \cdot \left(x_{i,j,t}\right)^2 \right) \tag{17}$$

The corresponding fist order conditions with the Lagrange multiplier $\mu_{i,t} \geq 0$ of the firm's budget constraint and $\mathrm{MR}_{i,j,t}$ as the marginal revenue are:

$$\begin{aligned} p_{j,t} + x_{i,j,t} \cdot \frac{\delta p_{j,t}}{\delta x_{i,j,t}} - 2 \cdot c_{i,j,t} \cdot x_{i,j,t} - 2 \cdot c_{i,j,t} \cdot x_{i,j,t} \cdot \mu_{i,t} \\ = \mathrm{MR}_{i,j,t} - (1 + \mu_{i,t}) \cdot 2 \cdot c_{i,j,t} \cdot x_{i,j,t} = 0 \quad \forall j \in M_{i,t} \end{aligned} \tag{18}$$

Due to the limited information about the demand function and the competitor's production cost, firms cannot simply determine the Nash equilibrium of this quantity setting game. Rather they use some heuristic approximations to determine their output quantity. For setting the quantity output several steps have to be taken.

First, the firms believe that all producers in the sub market j change their output quantity by the same factor λ_j. For this the total estimated output $\hat{X}_{j,t}$ on sub market j is given by: $\hat{X}_{j,t} = \lambda_j \cdot X_{j,t-1}$. Second, the firms assume that the price elasticities are constant: $\hat{\varepsilon}_{j,t} = \varepsilon_{j,t-1}$. Third, the firms expect that all firms change their output in the same way they would do: $\hat{\lambda}_j = \lambda_{i,j,t}$.

Thus, they expect the following prices:

$$\hat{p}_{j,t} = p_{j,t-1} \left(1 + \frac{\lambda_{i,j,t} - 1}{\varepsilon_{j,t-1}} \right) \tag{19}$$

Finally, firms approximate their marginal revenue by the following expression typically used in standard markup pricing formulas:

$$\hat{\mathrm{MR}} = \hat{p}_{j,t} \left(1 + \frac{x_{i,j,t}}{\hat{X}_{j,t} \cdot \varepsilon_{j,t-1}} \right) \tag{20}$$

With this information, firms can calculate their optimal production quantity in each sub-market. For firms that have been in sub-market j in period $t-1$ inserting expressions (18) and (19) into (18) gives the output quantity $x_{i,j,t} = \lambda_{i,j,t} \cdot x_{i,j,t-1}$, where:

$$\lambda_{i,j,t} = \frac{p_{j,t-1}\left(\varepsilon_{j,t-1} - 1\right)\left(X_{j,t-1} \cdot \varepsilon_{j,t-1} + x_{i,j,t-1}\right)}{2c_{i,j,t}\left(1 + \mu_{i,t}\right)x_{i,j,t-1} \cdot X_{j,t-1} \cdot \left(\varepsilon_{j,t-1}\right)^2 - p_{j,t-1}\left(X_{j,t-1} \cdot \varepsilon_{j,t-1} + x_{i,j,t-1}\right)} \tag{21}$$

It becomes obvious from this expression that the actual rates of changes are heterogeneous. A firm which did not produce variant j in period t-1 but added this sub-market in t first tries to estimate the change of output quantity of the incumbents and determines its optimal quantity based on this. The expected rate of change of output of the incumbents in the market is determined analogous to (20) where $x_{i,j,t-1}$ is replaced by the average output of a producer of variant j in period $t-1$.

Finally, there is a minimum quantity $x_{\min} > 0$, which has to be produced by any firm which decided to keep this sub-market in its portfolio. If the result of the quantity calculations is below this level the firm still produces x_{min}. Also in the initial period and every time when a sub-market is founded the quantity $x_{\min}$ is produced by the founder.

2.6.3 Investments in geographical location

Starting from a random location the firms may decide to change their geographical location from core $\left(d_{i,t}^{\text{geo}} = 0\right)$ to periphery $\left(d_{i,t}^{\text{geo}} = 1\right)$ or vice versa. The shifting of the location leads to the sunk cost $I_{i,t}^{\text{geo}} = c_{\text{fix}}^{\text{geo}}$ due to expenses for transferring the production and R&D facilities. These costs are constant in order to keep the model simple.

For the evaluation of the two location alternatives three factors seem to be important. First, the production costs inside the core increase because of scarce resources. Second, the main advantage of a headquarter inside the core lies in the learning effect through spatially transferred knowledge. But knowledge spillover are a threat for the own core competence in knowledge. For this a third factor aims at the own technological leadership. [4]

[4] Because knowledge for product innovations is only a first step to form a technological advantage, the knowledge stock for process innovation $\text{RD}_{i,j,t}^{\text{proc}}$ (and the corresponding spillover $\text{SP}_{i,j,t}^{\text{proc}}$) is the main indicator for a knowledge competence. Thus for the evaluation of the location only this type of knowledge is considered.

The evaluation function can be written as:

$$
\begin{aligned}
v_{i,t}^{\text{geo}} = {} & \left(\frac{1}{|M_{i,t}|}\sum_{j\in M_{i,t}}\frac{c_{i,j,t}\left(d_{i,t}^{\text{geo}}=1\right)}{c_{i,j,t}\left(d_{i,t}^{geo}=0\right)}\right)^{\delta_{i,R}/(\delta_{i,R}+\delta_{i,\text{RD}}+\delta_{i,\text{SP}})} \\
& \left(1-\frac{1}{|R_{i,t}|}\sum_{j\in R_{i,t}}\frac{\text{SP}_{i,j,t}^{\text{proc}}\left(d_{i,t}^{\text{geo}}=1\right)}{\text{SP}_{i,j,t}^{\text{proc}}\left(d_{i,t}^{\text{geo}}=0\right)}\right)^{\delta_{i,\text{SP}}/(\delta_{i,R}+\delta_{i,\text{RD}}+\delta_{i,\text{SP}})} \\
& \left(1-\frac{1}{|M_{i,t}|}\sum_{j\in M_{i,t}}\frac{\text{RD}_{i,j,t}^{\text{proc}}}{\text{RD}_{j,t}^{\text{proc}}}\right)^{\delta_{i,\text{RD}}/(\delta_{i,R}+\delta_{i,\text{RD}}+\delta_{i,\text{SP}})}
\end{aligned}
\tag{22}
$$

The evaluation for the geographical location $v_{i,t}^{\text{geo}}$ of firm i lies in the interval [0,1], where a result of 1 stands for a strong incentive to set the headquarter of the company inside the core. $M_{i,t}$ is the set of sub-markets of firm i in period t. A market j is in the set $R_{i,t}$ if the potential knowledge spillover are greater than zero: $\text{SP}_{i,j,t}^{\text{proc}}\left(d_{i,t}^{\text{geo}}=0\right) > 0$.

As mentioned above the first term considers the marginal production costs depending on the geographical location. If no other firm chooses their location inside the core, the term would become 1. The first term decreases as the number of firms inside the core increases. The second term describes the effect of the knowledge spillover. The numerator is the sum of the internal knowledge spillover, which would occur in every location. The denominator is the sum of the internal and external spillover. If a firm profits a lot from the external spillover, the firm would have an incentive to go inside the cluster. The last term takes acknowledge of the possible loss of a technological core competence. Hereby the firm's i knowledge is divided by the maximum knowledge of another firm in this market where $\text{RD}_{j,t}^{\text{proc}} = \max_i\left\{\text{RD}_{i,j,t}^{\text{proc}}\right\}$. If the potential loss is great the firm would have the incentive to choose a location far away from the other firms, or one minus this term promotes a location in the core.

Like in the evaluation function of market entry, see (15), the firm specific parameters $\delta_{i,R}, \delta_{i,\text{RD}}$ and $\delta_{i,\text{SP}}$ represent the firm strategy. The parameters weight the different terms such that heterogeneous firm strategies can be reproduced. As in the evaluation of markets there exists an inertia parameter $\kappa_{i,S}$. Firms will for example change their geographical location from periphery to core if $v_{i,t}^{\text{geo}} > \kappa_{i,S}$ and if their savings are higher than the shifting cost $S_{i,t} > I_{i,t}^{\text{geo}}$.

2.6.4 Investments in research and development

At the end of a period each firm decides on its investments in product and process innovations. Both investments $I_{i,j,t}^{\text{proc}}$ and $I_{i,j,t}^{\text{prod}}$ increase the corresponding knowledge stocks $\text{RD}_{i,j,t}^{\text{proc}}$, respectively $\text{RD}_{i,j,t}^{\text{prod}}$. The R&D investment quota for product innovation is denoted by q_i^{prod} and the quota for process innovation is q_i^{proc}. The Investments for product innovations are given by $I_{i,j,t}^{\text{prod}} = q_i^{\text{prod}} \cdot \Pi_{i,t}$ and for process innovations by $\sum_j I_{i,j,t}^{\text{proc}} = q_i^{\text{proc}} \cdot \Pi_{i,t}$.

Since process investments lead to a reduction of per unit cost of production the firm allocates these funds to the different sub-markets proportional to an adjusted expression of its current output in each market:

$$I_{i,j,t}^{\text{proc}} = q_i^{\text{proc}} \cdot \Pi_{i,t} \cdot \frac{x_{i,j,t}}{\sum_{k \in M_{i,t}} x_{i,k,t}} \tag{23}$$

A product innovation is seen as an alternative to market entry: in order to extract rents on a profitable market a new market next to the existing one is founded. For this the evaluation function for product innovations $v_{i,j,t}^{\text{prod}}$ is equal to the evaluation of markets, see Eq. (15). The only difference is that now all markets are considered, whereas the decision for market entry only took those markets into account, which were not served by the firm i and entry was not prevented by patent protection. The firm i will invest all his expenditures for product innovations in the market l with the highest evaluation: $I_{i,l,t}^{\text{prod}} = q_i^{\text{prod}} \cdot \Pi_{i,t}$, but only if $v_{i,j,t}^{\text{prod}} > 0$.

2.7 Market clearing

With all given quantity outputs $x_{i,j,t}$ prices and price elasticities can be calculated for all sub-markets. The price for each sub-market is given by expression (10) and the price elasticity of demand can be calculated from the price function. With the given cost functions every firm is able to derive their overall profit $\Pi_{i,t} = \sum_{j \in M_{i,t}} \Pi_{i,j,t}$ from the profits $\Pi_{i,j,t}$ on every sub-market:

$$\Pi_{i,j,t} = x_{i,j,t} \cdot p_{j,t} - F_i - c_{i,j,t} \cdot (x_{i,j,t})^2 \tag{24}$$

All firms start with initial savings of S^0. They can also take debts up to the same level. Every period they earn total profit $\Pi_{i,t}$ but also have to make their investments on process innovations $I_{i,j,t}^{\text{proc}}$ and product innovation $I_{i,j,t}^{\text{proc}}$ and eventually on the change of location $I_{i,t}^{\text{geo}}$. The savings for the next period should be expressed by the following formula while ρ stands for the interest rate:

$$S_{i,t+1} = (1+\rho)S_{i,t} + \Pi_{i,t} - I_{i,t}^{\text{geo}} - I_{i,j,t}^{\text{prod}} - \sum_{j \in M_{i,t}} I_{i,j,t}^{\text{proc}} \tag{25}$$

3 Simulation setup

The parameters of the simulation model can be found in Appendix A. They were chosen in that way that the model was able to find reasonable and robust results. Parameters, which represent the basic industry characteristics were kept constant over all simulated scenarios, but for all firm specific parameters a certain range was defined in order to catch aspects of firm heterogeneity. The firm are heterogeneous in their fixed cost (F_i), capabilities to perform R&D $\left(\alpha_i, \beta_i, q_i^{\text{prod}}, q_i^{\text{proc}}\right)$ and their firm strategies described by the parameters in the evaluation function $\left(\kappa_{i,S}, \kappa_{i,\text{en}}, \delta_{i,T}, \delta_{i,\text{RD}}, \delta_{i,R}, \delta_{i,\text{SP}}, \delta_{i,\Pi}\right)$. The results presented in the next section are based on 100 randomly generated profiles which were created while choosing the parameters from the given interval by a given distribution function. With each of these profiles the model runs for $T = 100$ periods. The results of this runs were averaged in order to get the qualitative impact of the parameters.

The model starts every time with a technology space with $m_0 = 5$ technologies, located on a circle with the technological distances of $d_0^{\text{tech}} = 2$ between those technologies. In total the industry consists of $n = 10$ firms which interact on the product markets. The geographical location of the firms is chosen randomly. Each firm starts with a randomly generated knowledge in one of the technologies and is also an active member of the corresponding sub-market. The starting level is normally distributed around mean RD^0 with the variance σ_0^2. Thus on every sub-market there are exactly two firms active in the first period.

Firms are able to borrow money up to their starting level of savings S^0. If the savings of a firm are less than $-S^0$ the firm is bankrupt. All knowledge of bankrupt firms is lost. Bankrupt firms are replaced in the industry with new savings, same knowledge as the technological leader[5] (but only in one randomly picked market) and random geographical location. The new firm gets new specific parameters, which represent the new strategy. Therefore the total number of firms is constant in the industry but the market structure of the sub-markets is determined endogenously.

4 Results of the simulation studies

This section presents the results of the simulation runs concerning the geographical location decisions of the firms as the industry evolves. The second part shows the resulting technological development in scenarios with constant number of firms located in the core.

4.1 The incentive to agglomerate

Before the discussion about the geographical location decisions of the firms, some properties of the evolving industry are introduced. Figure 2 presents the

[5] The substitution of bankrupt firms is understood as an entry of a new firm in this industry. The firm would only enter if it has, at least in one technology, the same knowledge as a technological leader.

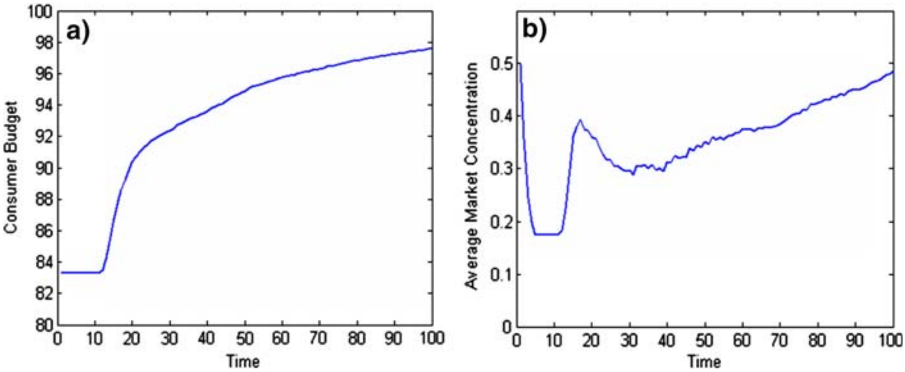

Fig. 2 **a** The evolution of the consumer budget and **b** market concentration in the simulated industry

development of the consumer budget in (a) and the average market concentration, measured as a Hirschmann–Herfindahl index, in (b).

By assumption the consumer budget for products of this particular industry increases with every product innovation. One can see that the additional amount is endogenously reduced as the number of product variants in the industry increases. The evolution of market concentration starts from an initial condition of around 0.5, because two firms with similar output are active on every sub-market. During the first phase, the market concentration decreases to a constant level and then jumps to a higher peak after the first successful introduction of product innovations. Afterwards there is a phase of higher competition, but in the longer run the value increases. A possible interpretation could be that some firms manage to develop competitive advantage in the form of higher knowledge for process and product innovations, what later on results in higher market power and thus raising market concentration.

In order to study the effects of geographical proximity, the number of firms in the core is presented with different cost for the scarce resource, see Fig. 3. All three curves have a similar pattern and show the outcome of 100 simulation runs.

The initial location was chosen randomly and on average five firms start in the core. Firms start with knowledge of only one technology. Because no process innovations were introduced at the start, the additional cost in the core compared to other costs is relatively low. By assumption two firms have a knowledge stock greater than zero in each technology and hence no firm has a huge technological lead. Because of this firms have a strong incentive to join the core at the beginning of the simulations. This incentive is reduced, in particular, after the first introduction of new products around period 20. With successful product innovations the innovating firms become technological leaders and try to keep their status by leaving the core. But afterwards firms again decide to participate in the learning process and enter the core. The relevance of geographical proximity is reduced over the time but on average most firms

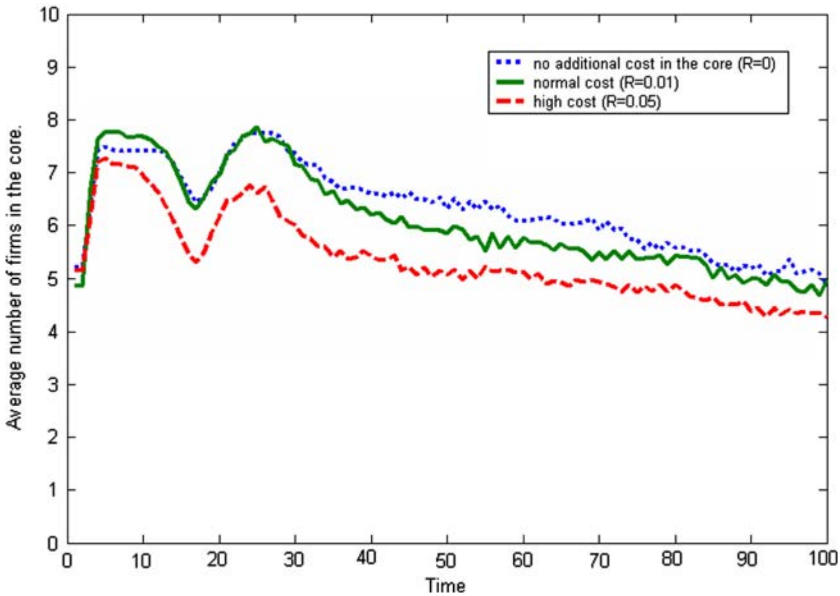

Fig. 3 Mean of firms in the core with different cost for scarce resource

choose a location close to other firms. One possible explanation for the decline of firms in core could lie in the upper bound for process innovations. But the technology space is still increasing with new product innovations, so that new learning possibilities in the core arise. Nevertheless, the overall resulting effect shows that a low geographical distance to competitors is important during a phase of high heterogeneity in knowledge, e.g. during the developing period of an industry, as well as when the industry becomes more mature. But the incentive to agglomerate is reduced over time.

In Fig. 3 three different cases were presented: zero, normal and high additional cost for scarce resources in the core. It can be seen, that even with no extra cost not all firms join core. The possible loss of knowledge through knowledge spillover could be one reason for voluntary isolation. In case of high cost for the scarce resource these cost are around 50% of the average initial cost with half of the firms in the core. [6] This means that a location in core raises the variable cost for every product quantity in that height. Because of process innovations the relative value even increases. As illustrated in the picture even with this high additional cost most firms choose a location with low geographical distance to their competitors.

It can be shown that the attractiveness of a geographical location in the core decreases mostly because of reduced external knowledge spillovers. Firms are initially attracted by high knowledge spillovers from other firms to join core, but

[6] In average the initial cost are $c^{\text{ini}}_{i,j} = 0.5$. With $|N_t| = 5$, $c^{\text{geo}} = 1.2$ and $R = 0.05$ the geographical cost are $c^{\text{geo}}_t = (5-1)^{1.2} \cdot 0.05 = 0.2639$, see Eq. (14).

as learning takes place firms in the core become more similar in their technological profile and the learning effect is reduced. Firms in the periphery cannot profit from spillovers in the core because of too much difference in knowledge. For this the number of firms in core decreases in the long run as the industry evolves for all cases as shown in Fig. 3. These results can be supported by the empirical findings of Audretsch and Feldman (1996), who show that agglomeration tendencies are stronger in the early stages of the industry life cycle. In contrast to them, however, this model doesn't assume a decreasing role of tacit knowledge. Here the reason for a reduced incentive to agglomerate in a mature industry is seen mostly in lower learning possibilities through knowledge spillover: firms in the core are too similar and the potential entrants have too much difference in knowledge in order to benefit from agglomerating. Internal knowledge spillover within each firm, therefore, becomes more important than external knowledge spillover as the industry evolves. Thus, firms in the core leave and firms in the periphery do not enter the agglomeration. Shaver and Flyer (2000) is another empirical work about the geographical location decision of firms. Here, the authors explore an 'adverse selection' effect, that most technologically lagging firms choose to locate in an agglomeration. In the setting of this paper this kind of behavior can be explained in the way: technological leaders firms do not benefit from learning as they fear the loss of knowledge through involuntary knowledge spillovers and therefore choose isolation in the periphery.

4.2 Technological development and agglomeration

The following analysis is based on four scenarios, which differ on the number of firms in the core:

- **0% core**: All firms are always in the periphery. No learning can happen between the firms through knowledge spillover but firms can make use of internal knowledge spillover.
- **50% core**: Half of the firms are located in the periphery, the other half in the core. Firms are not allowed to change their geographical location.
- **Variable**: This scenario represents the standard case with the decision rule for changing the geographical location as described in Sect. 2.6.3. Firms start with a random location and are free to move as they can afford it. [7]
- **100% core**: All firms are always in the core. Firms profit from the externalities arising from knowledge spillover but they also might loose their technological lead pretty fast.

Figure 4 presents boxplots[8] of the total number of product innovations in each simulation run. The total number of product innovations rises sharply as

[7] In average about 60% of firms were in the core, therefore this scenario is placed between 50 and 100% core.

[8] The boxplot function used for presentation was programmed by Ernest E. Rothman.

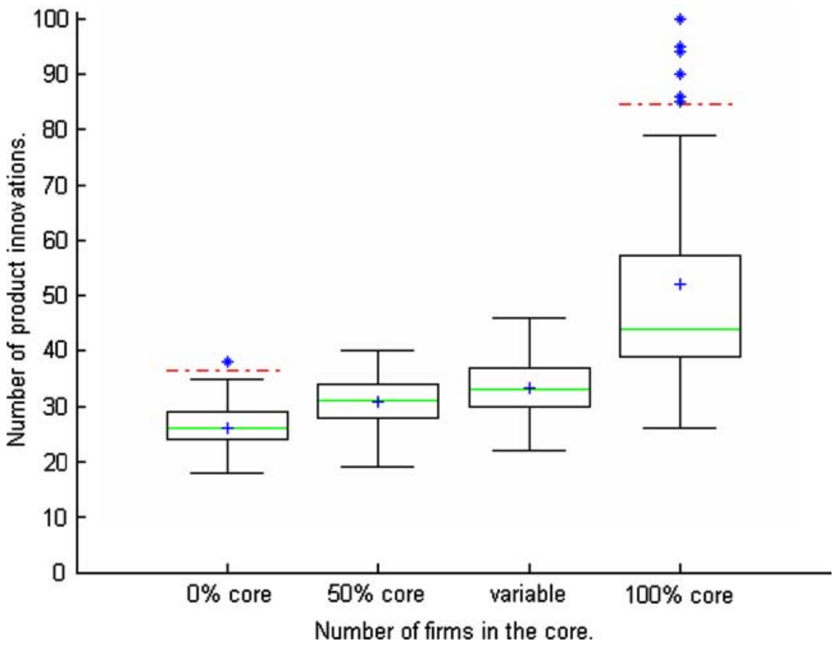

Fig. 4 Boxplots of the total number of successful product innovations in different scenarios

the number of firms in the core increases. The variance and the number of upper outliers increases in the case of all firms are always located in the core. [9] It was also observed that the number of radical product innovation rises with the number of firms, which participate at the learning processes, too. Therefore, it can be concluded that the existence of knowledge flows in the core has positive effects on the number of product innovations developed in the industry compared to a scenario where fewer firms are in the core.

The reached level of the knowledge stock for process innovation is presented in Fig. 5. From this figure two observations can be made: First, the variance declines with the number of firms in the core. The upper bound of 1 for the knowledge stock could be one reason for the smaller variance. Second, the level of the knowledge stock for process innovation seems to reach its maximum in the case with free choice for location. [10] Surprisingly, in contrast to product innovation the maximum level of process innovation in the cases 0% core is not that much lower than in 100% core. [11] From this observation it can be deduced that with fewer product innovations (see Fig. 4) and no learning between firms

[9] A statistical test underlines this results (see Appendix B): with a confidence level greater than 99% the null hypothesis, that the mean in scenario variable is equal or greater than the mean in scenario 100% core, can be rejected.

[10] Statistically this proposition can be supported at 90% confidence level, see Appendix B.

[11] The median increases slightly only from 0.9501 in the first case to 0.9573 in the second case, which is less than 1% change.

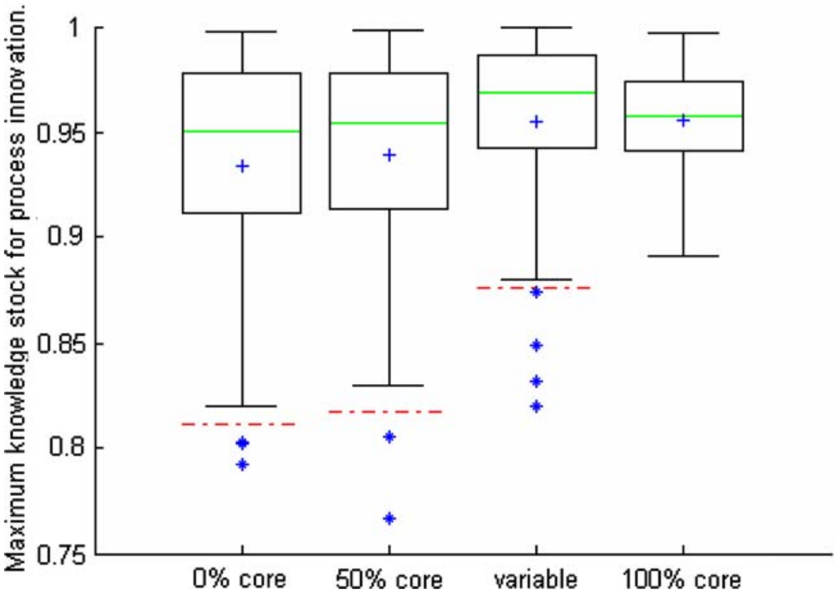

Fig. 5 Boxplots of the maximum knowledge stock for process innovation in different scenarios

almost the same level of process innovation was reached. Only in case of free choice of location the knowledge stock for process innovations was significantly higher.

The last Fig. 6 shows the average level of savings in the different scenarios. The average savings in the case with variable number of firms in the core is smaller than in the case with all firms in the agglomeration. One reason for this is that the firms sunk cost for changing the geographical location are chosen to be very high $\left(c_{\text{fix}}^{\text{geo}} = S_0 \cdot 0.5\right)$ and therefore reduce savings. It can be shown that with low or no sunk cost firms have the highest levels of savings in scenario variable. In all cases where the firms are not allowed to switch location the savings are about the same level. Thus the average profitability of firms does not depend on the number of firms in the core: the average savings in the case of all firms in the periphery or all firms in the core are more or less equal. Therefore the competition between firms, who all have to bear the same cost of congestion, leads to a similar level of accumulated profits in both cases. A similar result was observed in the case where only half of the firms were in the core. [12]

On the one hand, for the profitability of firms the results do not change, but on the other hand, the technological development, measured in the number of product innovations and maximum level of process innovation, does change with the number of firms in the core. This result can be interpreted as a possible

[12] Statistical tests show that with a confidence level greater than 99% the difference of means between the scenarios 0 and 100% core or 50 and 100% core is smaller than 10, see Appendix B.

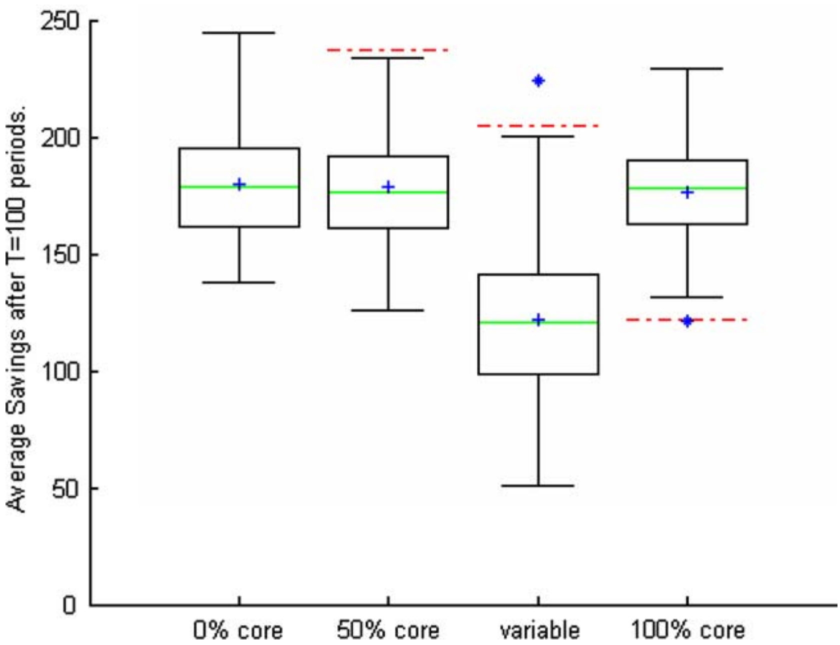

Fig. 6 Boxplots of the average level of savings in different scenarios

justification for economic policy: as firms are indifferent about all located in the core or all in the periphery, economic policy should try to enforce that all firms are located in the core, because this would lead to a faster technological development of the industry.

The question whether firms in agglomerations are more innovative is the topic of a recent empirical study by Beaudry and Breschi (2003), which is related to Baptista and Swann (1998). Whereas Baptista and Swann (1998) found a positive effect of the clustering of firms of the same industry on the probability of a firm to innovate, the result by Beaudry and Breschi (2003) is more ambiguous. Here, the authors could identify positive and negative effects, which depend on the type of firms and employees located in a region. Further analysis shows that an agglomeration of innovative firms in a firm's own industry has a positive and statistically significant influence. A strong presence of non-innovative firms has a negative and statistically significant effect on the firm's innovative performance. Thus, the simulation results, which show that as more innovations occur, the more innovative firms are located in the agglomeration, is borne out empirically.

5 Conclusions

The paper introduces an agent-based simulation model, which considers learning through heterogeneous knowledge spillover. Two factors were discussed in detail: geographical and technological proximity and their impact on innovation. The model takes into account that firms differ in their specific knowledge and firm strategy towards market entry and exit, R&D investments and geographical location.

This model yields the description of the technological development of an industry as well as the evolution of firm specific technological profile. Geographical proximity is important for firms although they have to take into account additional cost of scarce resources. The importance of geographical proximity falls slightly as the industry evolves, but most of the firms still choose to stay inside the core also in a more mature industry.

The comparison of different scenarios shows that the average savings of firms are mainly reduced by location changing cost. As the (average) profitability doesn't change in scenarios with all firms in the core or all in the periphery, the number of firms does matter for the technological development of the industry. With an increasing number of firms, which exchange knowledge, the number of product innovations rises sharply. In case of process innovations an increase of the mean and a reduction of the variance could be observed, but the effect is much clearer with product innovations.

The simulation analysis is limited by the assumed formal representation and parameter settings. For example another interpretation of the demand side, firm strategies or learning processes would probably not lead to the same results. In this paper an example of an industry was analyzed. The calibration of the model to certain industry characteristics would lead to new insights about the process of innovations in different industries. Furthermore, this model concentrates on learning between direct competitors and congestion cost as only agglomeration and de-agglomeration forces. One can introduce other advantages and disadvantages of agglomerations into the model, e.g. knowledge flows to and from suppliers or customers, benefits from specialized inputs or labor market pooling. Another drawback of the model can be seen in the fixed number of firms in the industry. The modeling of industry entry and exit as well as new firm formation, e.g. through spin-offs, are other possible extensions worth considering. One can also think about more complex geographical landscapes, for example several clusters or a cellular automata framework.

Further extensions of the model will concentrate on the aspect of technological distance: What kind of clusters will emerge, technological specialized or diversified? With regard to R&D strategy of firms the question arises, whether it is better to concentrate on the core competence or do firms with more diversified technological profile-earn higher profits? How do the incentives for R&D change through heterogeneous knowledge spillover: can firms profit from their R&D expenses or are they better off as free riders? A first step to get a better picture of firm strategies and their impact on proximity and innovation in this model is taken in Dawid and Wersching (2006). However, there still remain many open questions.

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Appendix

Appendix A: Parameter setting (Tables 1, 2)

Table 1 Fixed parameters

Parameter	Value	Parameter	Value
b	0.5	c	0.93
d	0.94	e	0.95
T	100	m_0	5
n	10	A	1
msize	100	τ	3
τ_{ex}	3	c^{geo}	1.2
R	0.01	RD^0	0.2
σ_0^2	0.001	d_0^{tech}	2
S^0	10	$c_{\text{fix}}^{\text{geo}}$	5
$x_{\min}$	0.1	ρ	0

Table 2 Parameters with range

Parameter	Range	Parameter	Range
$c_{i,j}^{\min}$	0.2–0.4	$c_{i,j}^{\text{ini}}$	0.4–0.6
β_i	0.75–0.85	α_i	3–4
q_i^{proc}	0.08–0.14	q_i^{prod}	$0.4 - q_i^{\text{proc}}$
$\kappa_{i,\text{en}}$	0.25–0.75	$\kappa_{i,S}$	0.1–0.5
F_i	0.2–0.4	$\delta_{i,\Pi}$	0.3–0.4
$\delta_{i,\text{SP}}$	0.3–0.4	$\delta_{i,R}$	0.3–0.4
$\delta_{i,\text{RD}}$	0.3–0.4	$\delta_{i,T}$	0.3–0.4

The parameters in Table 2 were chosen uniformly distributed in the range if not mentioned different.

Appendix B: Statistical tests

In order to present statistical analysis a two-sample Wilcoxon rank sum test is used where this test does not assume that the observations come from normal distributions. The alternative hypothesis is formulated and the results of 100 simulation runs are tested.

1. Average number of product innovations:
   H0: mean for scenario variable $\geq$ mean for scenario 100% core
   H1: mean for scenario variable $<$ mean for scenario 100% core
   Results: $Z = -9.716$, p-value $= 0$.
2. Maximum value of the knowledge stock for process innovation:
   H0: mean for scenario variable $\leq$ mean for scenario 100% core
   H1: mean for scenario variable $>$ mean for scenario 100% core
   Results: $Z = 1.5711$, p-value $= 0.0581$.
3. Average savings:
   H0: |mean for scenario 0% core – mean for scenario 100% core| = 10
   H1: |mean for scenario 0% core – mean for scenario 100% core| $<$ 10
   Results: Z = -2.4495, p-value $= 0.0072$.
4. Average savings:
   H0: |mean for scenario 50% core – mean for scenario 100% core| = 10
   H1: |mean for scenario 50% core – mean for scenario 100% core| $<$ 10
   Results: Z = -3.444, p-value $= 0.0003$.

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