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REGULAR ARTICLE

Stochastic models of resonating markets

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Abstract This paper describes a way to model a seasonally and irregularly peaking price dynamics, as that originated in commodity and energy markets, using a system of coupled nonlinear stochastic differential equations. The specific case of an electric power market is used to show which microeconomic features this approach is able to model. Critical point analysis is used in a simple way to show how the interaction between dynamic criticality and stochasticity can be used to develop further models, useful to explore more deeply other types of peaking price dynamics.

Keywords Stochastic processes · Power markets · Seasonality · Mean-reversion

JEL Classification C39 · D49 · G19 · L11

1 Introduction

Market price dynamics is usually studied from two different points of view. *Microeconomics* studies the structure of a market and the mechanisms of price formation, often as an aggregated rational choice problem, maybe in discrete time. *Price dynamics modelling* relies more on the direct design of dynamic equations, often in continuous time and in a stochastic setting. The price modelling dynamic equations keep into account the main and more general features detected in the studied market dynamics, and generate time series with the required statistical properties. On the one hand, from a microeconomics point of view, a price dynamics time series is just the quantitative measure of a more complex self-coordinating activity of trading agents, which interact

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under some specific market rules and use prices as feedback. Different market frames correspond to different opportunities. For example, if the agents trade in an exchange, they interact anonymously, and only one price is formed at each market time that can be used for the information feedback. If the agents trade in an over-the-counter market, multiple prices form and agents can use as a feedback only the subset of known prices. Usually, exchanges set a trading protocol in the form of an auction, and the typical stock market continuous-time first-price open-bid single-item double auction is just one among many other possibilities. Depending also on the number of agents that trade in a market, coalitions (allowed or formally forbidden) can form, market power can emerge, and social strategic behavioral patterns become the fabric of the price dynamics. On the other hand, from the modelling point of view, it is very hard to include satisfactorily a market microstructure in the dynamic equations, and agent based simulations are the only feasible way to encode a specific market structure in statistical properties of prices. In any case, a good financial model for a specific market price dynamics should integrate these two perspectives. Successful examples do exist. A first example is the standard stock market model (Samuelson 1964), where a geometric brownian motion embodies the microeconomic efficient market hypothesis but takes also into account the observed exponential growth of equity prices in the long run. A second example is the set of models for the bond market (Bjork 1999), which merge the microeconomic assumption of absence of arbitrage in the short run with the empirically observed interest rates mean reversion in the long run. Basic and advanced models for equity and bond markets, and for the markets for the derivatives written on them, then exist and reliably take into account both microeconomic and dynamic modelling at different levels of sophistication.

Under this perspective, financial models for spot and forward prices in commodity goods or energy markets (Geman 2005) are less satisfactory. From the microeconomic point of view, most spot price commodity models assume market equilibrium, buffering unmatched supply and demand with some storage that can be worth a convenience yield. For some agricultural commodities supply can be seasonal, as seasonal can be demand for energy commodities like heating oil or electricity. Prices adapt to seasonal patterns, usually revert to a long run mean value, and sometimes explode in short-lasting peaks that are in some cases due to stored good shortages or changes in perceived convenience yield in the high season period. From the dynamics modelling point of view, stochastic continuous-time continuous-price dynamic equations that sustain random peaks are difficult to build, and it is even harder to include seasonality and peaks together, especially when peaks have to be triggered following seasonal patterns. This difficulty is chiefly evident in electric power markets, for which the possibility of storage is minimal and *spiking*—as seasonal peaking is called in this context—is endemic but very hard to model.

The aim of this paper is then to discuss a new model-building approach, elaborating on a specific electric power market model introduced in Lucheroni (2007). This approach can help to develop better financial models for seasonal and irregularly peaking commodity markets. The general idea proposed here is that, when seasonality and peaking have to be included together in a model, a system of coupled nonlinear stochastic differential equations can be identified for this purpose, following critical point considerations from dynamical system theory and exploiting a peculiar interplay of stochasticity and nonlinearity. Seasonality can be imposed exogenously or generated endogenously, and random peaking is obtained as a resonance between noise and seasonality. This is the reason why markets exhibiting seasonal and irregularly peaking price patterns will be called here (stochastically) *resonating markets*. It will be shown how the dynamics generated by the chosen equations can be put in direct relation to the dynamics of a single relaxation oscillator, a mechanical analogy hopefully simple enough and useful to discuss the many and promising features of the systems identified in this way. The focus of the paper is kept on the structural properties of the models in relation to the main features of the market price dynamics they have to model, leaving aside mathematical technicalities (discussed in Lucheroni (2007)) and econometric estimates Lucheroni (2009).

The plan of the paper is the following. After this introduction, in Sect. 2, as a working example, a spot power market will be discussed, where the effects of storage and the lack of it, seasonality, strategic behavior and a distinctive auction protocol are very important, and accurate modelling of concurrent seasonality and spiking cannot be escaped. Section 2 is also used to offer striking evidence that there are markets in which prices form in a way dramatically different from that of the stock market, so that it is at least misleading to try to adapt to these markets dynamic models developed for very different settings. In Sect. 3, it will be discussed what the market architecture described in Sect. 2 implies for modelling. Five important issues will be identified, namely load dependency of prices, grid congestion effects, power system memory, mean reversion for prices, presence of a cap price. These issues will be confronted with data taken from the power market operator of Alberta, Canada, the Alberta Electric System Operator (AESO) AESO. In Sect. 4, after a short review of available literature on financial models of power markets, the mathematical structural properties of the model developed in Lucheroni (2007) based on the FitzHugh–Nagumo differential system will be highlighted and confronted with the five modelling issues identified in Sect. 3. In Sect. 5 the structural property of *dynamical criticality* which is at the heart of the model, and its interaction with stochasticity, will be discussed in more detail. It will be shown that more models like for example the Hindmarsh-Rose differential system can be built around this relationship. Section 6 concludes.

2 The power market as an example

On a power system, energy is transferred among *generators* and *load serving entities* by means of a *power grid*. Technically, a set of time varying electric loads, connected to the grid and predictable in their demand to a certain extent, is served by a set of power stations which match instantaneously the aggregated load, burning and transforming fuel into electric energy. In general, energy produced in excess cannot be stored, whereas even small shortages of power can cause the whole system-generators, loads and grid—to shut off into a blackout. Only hydropower plants can alleviate this quantity risk, allowing some energy storage for buffering. Organizationally, power systems can be structured in vertically integrated entities where production and delivery decisions are centralized or, more commonly nowadays, in *power markets*, where production decisions are taken in a way which is decentralized and based on price signals. A vast and consolidated engineering literature (Wood and Wollenberg 1996) studies the problem of the centralized decision-maker within the subjects of *economic dispatch*, where the total cost of burned fuel must be minimized and *unit commitment*, where the profit between proceeds from energy sales and operating costs must be maximized, taking into account together technical constraints and economic convenience. Time constraints are important. A plant that is off can be turned on only after some warm-up time, once it is on it cannot be shut off before a minimum time, and so on. Beside this, different generators have different cost profiles depending on the quantity and type of fuel burned. The time constraints can become conflicting with economic convenience, a conflict that can rise costs in particular times. Cost minimization or profit maximization over a time period for a group of power stations imply a coordinated operations schedule that is called *dispatch*. Mathematically, this schedule is obtained as a solution of a constrained optimization program over a set of discrete times, possibly in the presence of uncertainty, to the extent that load dynamics is uncertain. This optimization must take into account the initial state of the system. Usually, the schedule is set in advance, maybe at noon for the hours from 0 to 23 of the subsequent day, and uses a forecast load. Then, in real time during these 24 h, the predicted production must be corrected more or less heavily depending on contingent events. In decentralized systems (Stoft 2003), economic dispatch and unit commitment are usually substituted by a twofold structure, a financial exchange and a system operator (SO) that sometimes owns the grid. Some markets are limited to the supply side, and the energy demand from individual load serving entities is handled by a single collective buyer.[1] In this case the typical exchange organizes daily a discrete-time first-price closed-bid multiple-item single auction, the so-called *day-ahead* (or spot) market, where generators post unbreakable price-quantity bids to sell units of energy to the exchange for the next individual 24 h, before a deadline (e.g. everyday at 12 p.m., for 24 h starting at 1 a.m. of the next day). In their posts, the generators must also declare how much spare production capacity they have left available. Considering that 'power times time' is energy, with their bids the generators commit themselves to deliver energy for the chosen time spans at their declared prices. The exchange aggregates all bids, computes the aggregated price-quantity supply curve, and for each hour matches it with the forecast demand curve identifying clearing prices. The exchange then sends this information to the SO that checks for generation and transmission technical constraints and in case discards some bids, sets the hourly *forecast price* at which demand matches feasible supply, builds the operating schedule for the forecast load, and dispatches loads and generators, which in this way get to know whether their bids are accepted or not. In case a bid is accepted it becomes committing for the bidding generator, which will be paid the forecast price. The procedure does not end here. A secondary *reserves* market is associated to the spot market, in which the declared spare capacity is offered by each generator and a second auction is held, fixing minute by minute a *real time price* that fluctuates around the forecast price, to invite the generators to supply or to buy power when forecast demand turns out different

[1] Even though the following market frame is taken from the AESO rules, some simplifications are made for clarity and text length reasons. For example, the demand side is represented here by a collective buyer, whereas the AESO market is not limited to the supply side.

from real time demand.[2] Finally, the official hourly *system price* (SP) is computed as the hourly average of the real time price. Often the exchange runs (in parallel with the spot market) a futures market, which uses as a spot reference price the SP.

3 Modelling implications and data

This market frame has at least five important implications for prices. First, *prices depend on load*. Since electricity demand is rigid and seasonal, prices are seasonal, i.e. periodic. For example, during the night the demand is lower than during the day, so that prices are lower during the night and higher during the day. Second, the power grid can become *congested*, because generators don't take into account electricity flow problems in their bidding decisions—this is left to the SO. Because of this network externality, in congested periods forecast prices can become very different from real time prices. Moreover, because of rigidity, loads accept high prices in case of rationing due to congestion. In regard to modelling, this implies that congested and non-congested phases must be modelled in two different ways. Particularly important it is the relation between congestion and periodicity. The congestion mechanism doesn't change during a period, so that there are high demand parts of the period in which congestion risk is higher, and prices are more likely to jump or peak. Third, the system has *memory*, in the sense that each generator has time constraints, and its bids for the next time must take into account the state of the plant at the current time. For example, a coal plant that is off now will take hours to go on. This is very important for the SO, which includes this causality in its dispatch plans and in the prices it forms. Fourth, in the long run, if fuel costs don't change much, if demand and supply grow at the same rate and if the grid is developed at this same rate, *mean reversion* in prices can be expected. That is, the SP oscillates seasonally around a mean reversion level, with peaks that depart from and come back to this level. Fifth, often in these markets a *cap price* is set by the exchange, and prices higher than this cap price are not allowed. Finally, a general consideration about *strategic behavior* should be made. The auction system is implemented because of the assumption that competitive players are expected to be forced to bid at their marginal costs, since such a collective behavior maximizes social welfare. Of course, if the supply side of the market is oligopolistic and/or collusive, strategic behavior can set in, exploiting the market at its own advantage to detriment of social welfare. Very often, the market is indeed oligopolistic, and the congestion times are the most profitable times for exercising market power. Then maximum pressure on prices should be expected on demand crests but not in a systematic way, due to the need of agents to to escape detection of collusive behavior.

All these features can be spotted in SP data. In Fig. 1 the hourly SP series in Canadian dollars (C$) and the hourly actual energy demand in MegaWatt per hour (MWh) are shown for one week, starting from 1 h of a Monday, for the AESO market. Demand has night/day periodicity, during daylight it has morning/afternoon periodicity, and on weekends it is at its week lowest. In longer time series, not shown here, this weekly pattern repeats itself during the year, associated with a longer winter/summer period.

[2] i.e. to produce less than what committed, being paid an extra for this. See Stoft (2003), Chap. 3.6.

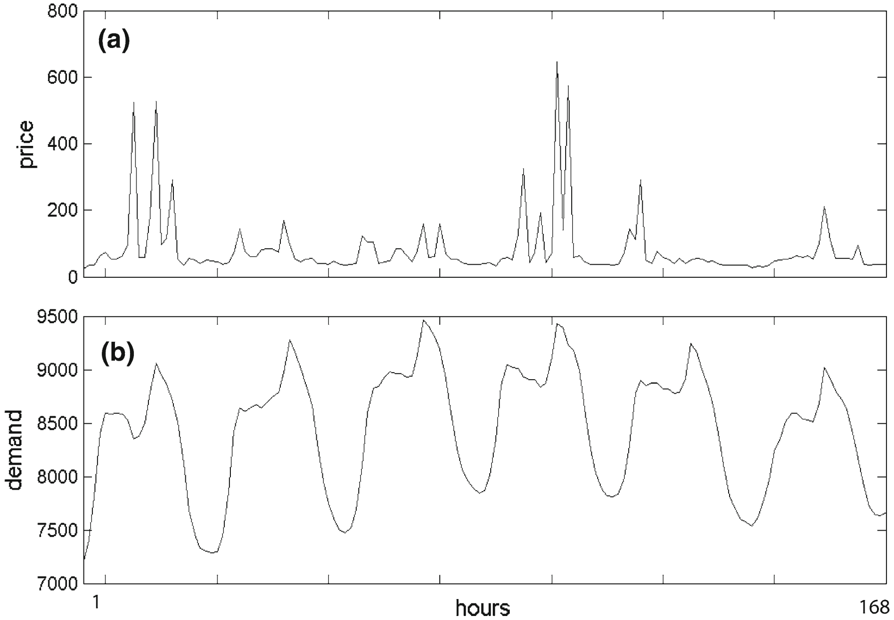

Fig. 1 Alberta power market: one week from Mon Jan-08-2007 to Sun Jan-14-2007, time in hours; **a** SP's in C$, **b** demand in MWh

The SP series has a base (mean reversion) level, follows a night/day pattern, and associates spikes to some demand peaks—then only at daylight. These spikes last very few hours and seem to be fired randomly, since to the same level of demand it does not deterministically correspond spiking. Then a sort of *two regimes dynamics* seems to underly this behavior, some times spiking being active some times not. When spiking is not active, since demand oscillates, prices oscillate with small amplitude around a mean value (but not centered around this value). When spiking is active prices can jump to a random height but then quickly go back to the mean level. In any case, they never cross the cap level, which for the AESO market is 1,000 C$, even though some times they reach it. This behavior has implications for the volatility structure of the dynamics. Once fired, spikes follow a kind of quasi-deterministic path, and from a statistical point of view that appears as a locally increased volatility.

4 Price dynamics models

In the literature about price dynamics modelling Geman (2005), data are not always structured as in Fig. 1. Often demand is not taken in consideration, even though in some papers it is considered very important (Eydeland and Geman 1998; Barlow 2002; Kanamura and Ohashi 2007). In that case, very often the hourly prices without demand are averaged over the 24 h, and a daily price series results which has lost most of its periodicity. This averaged price series can be studied with discrete-time daily GARCH-like models or with continuous-time models. In this latter case, and if price discontinuity is allowed, jump-diffusion non-periodic continuous-time Lévy processes (Geman and Roncoroni 2006; Benth et al. 2008) can be used, with problems with the mean reversion mechanism. As a matter of fact, averaged or not, the SP data show at least two mean reversion time scales, a long run mean reversion and a very short run mean reversion—when prices jump up and then have to jump down to the long run mean reversion level. If price continuity is considered important, as it is in all standard stock market and bond market models (because it is felt essential to develop standard derivative pricing), a possible escape is to build a Markov 2-regimes (or *M*-regimes) dynamics (De Jong 2006; De Sanctis and Mari 2007), allowing the structural parameters of a primary pre-selected market model to jump randomly between 2 (or more) states, thus associating a secondary jump dynamics to the pre-selected dynamics.

The continuous-time continuous-price model discussed in Lucheroni (2007) includes in a natural way all the features exposed in Sect. 3. There are two basic ideas behind this mathematical model, *dynamical criticality* and *its interaction with noise*. If a rigid noisy demand is assumed to be the prices driver, the price series seems to behave like the response of a *nonlinear relaxation oscillator* to a periodic noisy driver. Imagine a children's swing with the seat connected to a horizontal beam by rigid rods. When pushed periodically with a fixed intensity, this system has two possible regimes, the normal swinging phase and the full turn phase, when the swing seat makes a (dangerous) full turn around the beam. There is a critical level of push intensity that separates the two regimes. If a push is applied just below this critical level, the system becomes very reactive to any accidental event that increases even minimally the push. Models of nonlinear oscillators that display these two regimes are available in the physics (Linder et al. 2004) and mathematical biology (Izhikevich 2000) literature. When a deterministic periodic driver is applied to one of these oscillators, tuned just below the oscillator critical level, addition of noise to the driver turns randomly an otherwise small amplitude oscillation into a spike, which then dies as the oscillator reverts to its original small amplitude motion. One of the simplest oscillators that display this behavior is the FitzHugh–Nagumo system (FNS) (FitzHugh 1961; Nagumo et al. 1962), introduced to model neuron cells response to electric stimula. As it is common in finance, a *logprice* process $x(t)$ is associated to the SP process $p(t)$ as

$$x(t) = \ln p(t). \tag{1}$$

The logprice process is modelled in Lucheroni (2007) after the FNS second order stochastic dynamics

$$\epsilon\ddot{x} = (\kappa - 3\lambda x^2 + \epsilon\beta)\dot{x} - (\gamma - \beta\kappa)x - \beta\lambda x^3 - b + f(t) - \sigma(d)\xi, \tag{2}$$

where $\epsilon > 0, \gamma, b, \beta \geq 0, \sigma(d) = \sqrt{2d} > 0$ are constants, ξ indicates the derivative of the Wiener process, i.e. a $\delta(t)$−autocorrelated normal process, $\kappa > 0, \lambda > 0$ and $f(t)$ is a periodic function, like for example $f = A\sin(\omega_f t)$ $(A > 0)$ with frequency ω_f. When b, σ and the coefficients in front of $\dot{x}$ and x^3 are set to zero (and $\gamma > \beta\kappa$), the system looks like a harmonic oscillator, driven by a periodic driver $f(t)$. When the remaining two components, the harmonic oscillator and the deterministic driver, are

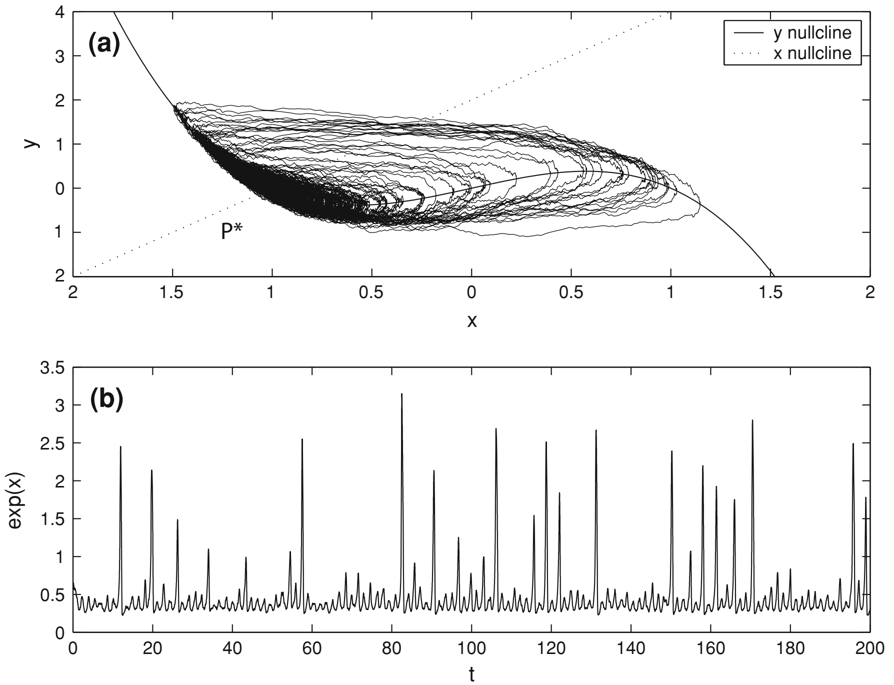

Fig. 2 FitzHugh–Nagumo system, for $\epsilon = 0.15, \kappa = 1, \lambda = 1, \gamma = 2, \beta = 1, b = 2, d = 0.1, A = 2, \omega_f = 4$. P^* is the fixed point for $z = 0$, see text. **a** phase space and nullclines; **b** price $\exp x(t)$ vs. time

taken to *resonance*—for example tuning the driver frequency—the oscillations of $x(t)$ become very large and tend to explode. In the presence of both the relaxation term $\dot{x}$ and the nonlinear term x^3, the driver push is absorbed and the explosive behavior tamed. More interestingly, for small A the system displays small oscillations, but as A crosses a certain critical level, the small oscillations are abruptly substituted by a new type of large oscillations—with a discontinuous jump in oscillation amplitude. This is due to a critical point in the phase space of the FNS, as it will be discussed in Sect. 5. In any case, once this critical level is identified, the deterministic system is ready to be used as a model for the power market. When the full model is set in the small oscillation regime, but *close to the critical point*, addition of noise ($\sigma \neq 0$) introduces random kicks that can be seen as perturbations to the system parameters (specifically, to A), which occasionally take the system to the large oscillation regime. The stochastic kick disappears immediately after having been generated, but the system must go through a full turn before getting back to the small oscillation regime, producing a spike in $x(t)$.

Such a behavior can be seen represented in Fig. 2 where a time series generated by a run of the $p(t)$ dynamics is shown in Fig. 2b, together with a sketch of the system phase space in Fig. 2a. The choice of parameters used for Fig. 2 is motivated in Lucheroni (2007), where the mechanism for a *stochastically resonating spiking* (SRS) and the definition of the *soft* ϵ *regime* are introduced and discussed. Since the FNS was developed as a model for neural spiking, where all spikes have the same height and the synchronization with periodicity is not required, differently from the power market case, no off-shelf neural models have ever been used in finance in this way. But all the microeconomic features exposed in Sect. 3 are present in this model. First, prices depend on load, and it is the load unanticipated part—the noise term—that can trigger spikes. Second, only the crest part of the periodic demand $f - \sigma\xi$ is available for spiking, when noise becomes all important for accessing the large oscillation regime. Congested and non-congested phases are both present in the same model, with no need for Markov switching. Distance from the critical point can be interpreted as *distance from congestion*, where collusive behavior, when sets in, can lead to spiking. Third, the system memory is modelled by use of second order time derivatives. In fact, in discrete times derivatives become differences taken one or more time steps before the evaluation time point. In a certain way, it is the presence of derivatives that makes the dynamics causal. Fourth, both long run and short run mean reversion are included in a very natural way in the model, and the small amplitude oscillations about this mean reversion level are not necessarily symmetric, as in the data. Among other things, this can lead to *volatility clustering*. Fifth, a cap price is implemented statistically, since fluctuations beyond a given level are strongly suppressed.

5 A new modelling approach

A major difference of the dynamics of Eq. (2) from the geometric brownian motion of the standard stock market model or the Ornstein–Uhlenbeck process of the Vasicek bond market model Bjork (1999) are nonlinearity and the derivative order. Second order dynamics is obviously richer than first order, it can sustain oscillations even without periodic driving, and, when driven, its behavior can be very complex. Second order dynamics implies at least two degrees of freedom, as it can be seen rewriting Eq. (2) as the following system of two coupled first order differential equations:

$$\epsilon \dot{x} = g(x) - y \tag{3a}$$

$$\dot{y} = h(x; b) - \beta y + z(t), \tag{3b}$$

where

$$z(t) = -f(t) + \sigma(d)\xi, \tag{4a}$$

$$g(x) = \kappa x - \lambda x^3, \tag{4b}$$

$$h(x; b) = \gamma x + b. \tag{4c}$$

Here b is written as a system parameter in $h(x; b)$, and the stochastic forcing term $z(t)$ is separated, to show that in principle it could be considered an autonomous third degree of freedom coming from an uncoupled dynamics $\ddot{z} = u(\dot{z}, z, t)$. In the absence of $z(t)$ the system is deterministic. Two curves can be defined in the system phase space of coordinates (x, y), the x and y *nullclines*, i.e. the sets of points where respectively $\dot{x} = 0$ and $\dot{y} = 0$. The nullclines cross each other at points $P^* = (x^*, y^*)$ where

$\dot{x} = \dot{y} = 0$. These crossing points P^* are called *fixed points* of the system. Their positions and stability in the system phase space are dynamically very important, since they represent stable or unstable equilibria for the system. In Fig. 2a the phase space for the FNS of Eq. (2) or Eqs. (3) is sketched, together with the nullclines and the system trajectory. When the term $z(t)$ is absent, and for the parameters used to build Fig. 2, Eqs. (3) have only one stable fixed point $P^* = (1, 0)$, and every trajectory ends up in P^*. When a perturbation is present, trajectories tend to stick close to P^*, and the system is subject to small oscillations. The position of P^* depends on the parameter b. Changing b, the position of P^* can be moved to the right, increasing x^* towards zero, until P^* becomes unstable. When P^* is unstable, the FNS sustains only large oscillations, because of its topology. The value b^* (of b) at which the system changes its stability properties is called a *critical point*. A change of stability can be attained even if b is kept fixed in the stable region, since the FNS can be made unstable also by large perturbations due to $z(t)$, which contains periodicity and noise. This possibility of sustaining two different but mutually exclusive regimes, a fundamental one (small oscillations) and a second one (large oscillations) accessible with assistance from noise, is the feature that makes the FNS and any other dynamics with such features interesting for seasonally peaking market modelling.

Building on the ideas discussed up to here, a range of new possibilities for continuous-time continuous-price market modelling, not contemplated in Lucheroni (2007) and originating directly from critical point analysis, comes to mind. Here some possibilities are suggested, certainly not in an exhaustive way. Critical points are classified by the stability changes that they imply when they are crossed. These changes are sometimes called *bifurcations*. 'Dictionaries' exist that try to classify them and to discuss their dynamical properties(Izhikevich 2000). For example, in the setting used for Fig. 2 the FNS is purposely kept close to a subcritical-Hopf bifurcation. This allows stochastically resonating spiking induced by the periodic driver. It is possible to have spiking while *omitting the periodic driver*, looking for a system that oscillates in an autonomous way in its fundamental state, but still has stochastic access to a large oscillation regime. Elaborating on a result reported in Hosaka et al. (2006), a Hindmarsh-Rose type I (HRI) system can be explored, which shows a supercritical-Hopf bifurcation. Changing Eqs. (4) into

$$z(t) = \sigma(d)\xi, \tag{5a}$$
$$g(x) = \kappa x^2 - \lambda x^3, \tag{5b}$$
$$h(x; b) = \gamma x^2 + \gamma x + b, \tag{5c}$$

and after some fixed point considerations, a behavior can be obtained as that shown in Fig. 3, where HRI nullclines and a system trajectory are shown.In this case, the small oscillations are not due to a periodic forcing, even though spikes are occasionally fired by the noise term. The HRI can be used if a time-independent (i.e. autonomous) model is needed, maybe for econometric reasons. Going back to the FNS, if one is interested in modelling SP series *averaged over the 24 h*, where the daily periodicity is washed out, the FNS can be again of help. Even without periodic forcing, i.e. seasonality, the stochastic FNS—kept close to criticality by a suitable choice of b—has two regimes, a

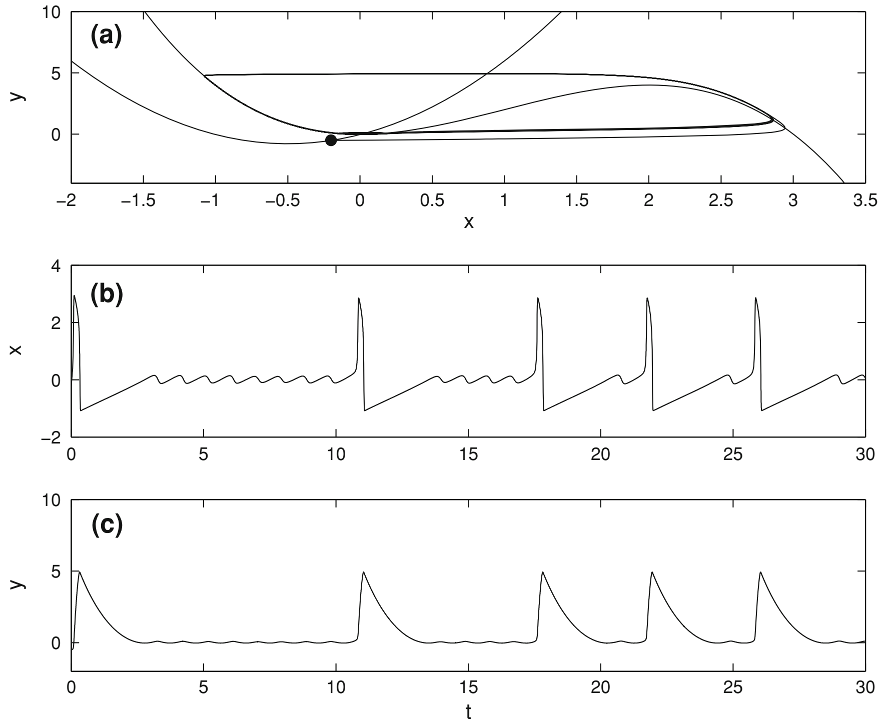

Fig. 3 Hindmarsh Rose I model, for $\epsilon = 0.05, \kappa = 3, \lambda = 1, \gamma = 3, \beta = 1, b = -0.031, d = 0.00001$. **a** phase space and nullclines, **b** $x(t)$, **c** $y(t)$

quiescent one (no firing, just bubbling around the stable point) and a spiking one which is accessed when a stochastic kick is large enough to trigger a full turn in the phase space. Finally, it should be noted that critical point analysis is certainly not limited to second order systems, and *third order* systems with amazing spiking behavior are well known in mathematical biology literature. What can be reminded here is that also *first order* driven nonlinear systems can have a spiking behavior, and this is well known in physics (Linder et al. 2004). This is partly due to the fact that the driver function $z(t)$ can be considered as a degree of freedom on its own, rising to two the actual number of degrees of freedom of a first order dynamical equation. Even more interestingly, spiking behavior can be achieved in first order systems without any periodic driver, as in the 'phase model' (Izhikevich 2000) which combines a stochastic first order dynamic equation with a static transformation equation. This result is linked again to the critical point analysis of a larger system of which the 'phase model' is a limit.

6 Conclusions

Dynamical system theory and fixed point analysis are very widespread and accessible mathematical knowledge fields, so that it seems limiting that uncommon market settings and price formation mechanisms should not be explored using this kind of disciplines. May be their use could not lead to standard financial modelling, but even a quick exploration of the interplay between dynamical nonlinearity and noise can at least lead to stimulating ideas, not to mention better price dynamic modelling. In this paper a commodity market structure was reviewed, that naturally lends itself to being modelled in its seasonal and irregularly peaking price dynamics by a nonstandard financial model as the FitzHugh–Nagumo or the HRI models from the mathematical biology literature. This approach can be extended in two ways, by building more models with the same approach and by using this family of models to study other markets. The author then hopes that the microeconomic analysis, data analysis, and interdisciplinary mathematical discussion presented here are interesting enough to stimulate further discussion along these new directions.

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