J Econ Interac Coord (2007) 2:195–210
DOI 10.1007/s11403-007-0023-1

ERRATUM

Self-organization of R&D search in complex technology spaces

Gerald Silverberg · Bart Verspagen

Published online: 7 July 2007

Abstract We extend an earlier model of innovation dynamics based on percolation by adding endogenous R&D search by economically motivated firms. The {0, 1} seeding of the technology lattice is now replaced by draws from a lognormal distribution for technology 'difficulty'. Firms are rewarded for successful innovations by increases in their R&D budget. We compare two regimes. In the first, firms are fixed in a region of technology space. In the second, they can change their location by myopically comparing progress in their local neighborhoods and probabilistically moving to the region with the highest recent progress. We call this the moving or self-organizational regime (SO). The SO regime always outperforms the fixed one, but its performance is a complex function of the 'rationality' of firm search (in terms of search radius and speed of movement). The clustering of firms in the SO regime grows rapidly and then fluctuates in a complex way around a high value that increases with the search radius. We also investigate the size distributions of the innovations generated in each regime. In the fixed one, the distribution is approximately lognormal and certainly not fat tailed. In the SO regime, the distributions are radically different. They are much more highly right skewed and show scaling over at least 2 decades with a slope around one, for a wide range of parameter settings. Thus we argue that firm self-organization leads to self-organized criticality.

The online version of the original article can be found under doi:10.1007/s11403-006-0008-5.

G. Silverberg (✉)
MERIT, Postbus 616, 6200 MD Maastricht, The Netherlands
e-mail: gerald.silverberg@merit.unimaas.nl

B. Verspagen
ECIS, Technical University of Eindhoven, PO Box 513, 5600 MB Eindhoven, The Netherlands
e-mail: H.H.G.Verspagen@tm.tue.nl

G. Silverberg · B. Verspagen
IIASA, Schlossplatz 1, 2361 Laxenburg, Austria

Keywords Innovation · Percolation · Search · Technological change · R&D · Clustering · Self-organized criticality

JEL Classification C15 · C63 · D83 · O31

Erratum to: J Econ Interac Coord DOI 10.1007/s11403-006-0008-5

The following paper is being published in two versions, a corrected and revised version (“Erratum”, DOI 10.1007/s11403-007-0023-1) and an erroneous earlier version (DOI 10.1007/s11403-006-0008-5). Readers are advised to completely disregard the erroneous version and only read and cite the corrected one.

On 7 November 2006 the authors notified the editors and Springer-Verlag that they had discovered a subtle programming error that invalidated some and modified other conclusions of the original paper, and requested that the paper be withdrawn from publication pending the preparation of a new version. On 5 December 2006 they submitted a completely revised paper, which was then re-refereed and accepted for publication in place of the original, erroneous version.

However, because the original version had already been published by Springer-Verlag in its Online First service on 22 September 2006, the publishers consider themselves legally obligated to publish the first, erroneous version in both the hardcopy and online JEIC media, and the corrected version as an “Erratum” in the same issue.

1 Introduction

The paradoxical characteristic of innovations is that their nature, significance and date of arrival are intrinsically unknowable in advance—if we knew them in detail, then the innovation would in effect have already happened. It is this property of intrinsic uncertainty that makes innovations so central and different from other factors in the theory of long-term economic evolution. On the assumption of total ignorance and independence, the natural approach would be to regard discrete innovations as generated by a simple stochastic point process such as the time-homogeneous Poisson. At the same time we know that technologies are not picked out of a hat at random times in random orders—to some extent there is a logical order in which they can be discovered, and they build on each other. Modern computers could not exist without a mastery of electronics (although Babbage tried and failed to make a purely mechanical one in the nineteenth century), electronics without a mastery of electricity, and electricity without the metallurgical skills necessary to make wires. Thus we shall argue in the following, based both on empirical evidence and a theoretical model, that the innovation process, while highly uncertain and stochastic, is still more structured in important respects than such a null hypothesis would suggest.

While the study of the statistical properties of the innovation process is scientifically interesting in its own right, it also has important implications for economic theory and innovation management. If innovations are drawn from a highly skewed and even infinite variance process (Pareto), then economic growth may be even more erratic than if they are of constant ‘size’ but generated by a Poisson process (see Sornette and Zajdenweber 1999 on the former case, Silverberg and Lehnert 1996 on the latter). If they are drawn from an infinite variance and even infinite mean process, then R&D risk management and portfolio policy are confronted with such high risk that the standard tools of capital asset management theory are inapplicable (Scherer and Harhoff 2000). Empirical evidence for these pathological means is presented in Silverberg and Verspagen (2004).

Three earlier papers (Silverberg 2002; Silverberg and Verspagen 2003a, 2005) examined a model based on percolation theory that takes several stylized facts about innovation into account. In Silverberg and Verspagen (2003a), we summarized the stylized facts about innovation under three types of ‘clustering’. First (major) innovations tend to be clustered in time: they “are not evenly distributed in time, but ... on the contrary they tend to cluster, to come about in bunches, simply because first some, and then most firms follow in the wake of successful innovation” (Schumpeter 1939, p. 75, see also, Silverberg and Verspagen 2003b). Second, innovations are clustered in ‘technology space’ (a concept that will be operationalized in terms of our model). In the economic literature analyzing the development of technological change there are numerous suggestions that the innovative process follows relatively ordered pathways that can be measured ex post in technology characteristics space. Examples of propositions in this direction are Nelson and Winter’s (1977) *natural trajectories*, Sahal’s (1981) *technological guideposts*, and Dosi’s (1982) *technological paradigms*. Empirically oriented contributions that illustrate the point are, e.g., Foray and Grübler (1990), Saviotti (1996) and Frenken and Leydesdorff (2000). Third, recent literature such as Scherer (1998), Harhoff et al. (1999), Scherer et al. (2000), and Harhoff et al. (2003) suggests that the distribution of innovation sizes, as captured by some measure of economic returns to R&D investment, is highly skewed, with most innovations having low or negative returns but with a highly skewed tail extending into the region of extremely high rates of return. The same tendency can be observed using the data compiled by Trajtenberg (1990) for the ‘value’ of patents proxied by the number of patent citations. These data suggest that the distribution of innovations may follow a power law, at least in the tails (cf. Silverberg and Verspagen 2004).

Our earlier model was aimed at explaining these stylized facts from the simplest possible assumptions regarding the nature of the innovative process. Thus we abstracted from any economically motivated, active search process. The earlier model did have ‘research’ in the form of search in technology space, but the efforts put into this process were completely exogenous, both in terms of their size (“amount of expenditures”) and direction (selection of promising avenues for research). It is the aim of this paper to introduce a more elaborate, economically motivated basis for technological search, and to investigate its implications for the stylized facts of innovation that our model addresses.

The process of technological search will be motivated by two crucial economic factors. The first concerns the way in which firms[1] select the parts of technological

[1] We use this term to describe the abstract agents that operate in our technological space, but our model only addresses the R&D function of these agents. We ignore various aspects of firm behavior that would normally be the subject of economic models, such as production, sales, investment and firm growth.

space they want to search. Each of the firms in the model can only address a (small) part of the technological space (we exogenously set a search radius for all firms), but we now allow the firms a certain latitude in determining the search region themselves. This implies that, contrary to our earlier model, search may now be concentrated in selected neighborhoods, when firms collectively decide to locate their R&D activities there. Other parts of technological space may conversely be abandoned. Although firms can, in principle, move freely through technology space, the model does assume that there are costs associated with this. Although we do not model such costs explicitly, we do assume that relocation of search in technology space is the result of two counteracting tendencies. On the one hand, firms want to move to the places where technological opportunities seem to be largest, but, on the other hand, they also tend to stick to locations that they know from previous experience (something Nelson and Winter's 1977 termed 'local search').

The second assumption consists of introducing a positive feedback resulting from successful innovations. Firms that realize innovations will generate resources to invest in new R&D efforts in the next period proportional to their success. Although in the real world financial markets may also reward (economically) unproven innovations (venture capital), it is fair to say that in many cases 'success breeds success' in innovative activity. This is what Winter (1984) has called a "routinized" regime of innovation.

In Sect. 2 we formally describe this new part of the model, as well as the basic structure retained from the previous version. This section also discusses our approach to modeling technology space, based on percolation theory. Section 3 describes some of the results of the model. Although the model has relatively few parameters, the total parameter space is large, and we have only just begun to analyze it in a systematic way. We focus here on comparing the fixed firm regime with the self-organizational one with moving firms, with respect to several indicators. In particular the size distribution of innovations is shown to change radically between the two regimes. The results are summarized and directions for further analysis are outlined in the concluding section (Sect. 4).

2 The model

2.1 Technology space

Our probabilistic model of innovation is an elaboration of the model in Silverberg and Verspagen (2005). As in the original model, the present model hinges on two essential properties. First, technologies constitute a discrete topological space with a neighborhood structure reflecting their technological interrelatedness, and second, over time technologies can only come 'online' by becoming contiguous to previously operational technologies, even if R&D search takes place in a more 'leapfrogging' or farsighted manner.

For simplicity, consider a lattice, unbounded in the vertical dimension, anchored on a baseline (or space), with periodic boundary conditions in the horizontal dimension.[2]

[2] Thus our space is like a cylinder with the left and right edges pasted together to make them the extreme columns neighbors.

The horizontal space represents the universe of technological niches (metaphorically, one may think of these as technology fields, such mechanics, electronics, chemistry, etc., or as economic sectors or application areas), with neighboring sites (columns) being closely related from a technological point of view. While the technology space is represented here and in the following as one-dimensional, it can easily be generalized to higher dimensions or different topologies. The vertical axis measures an indicator of performance intrinsic to that technology and could also be conceived as multidimensional. For simplicity we will restrict ourselves to a two-dimensional lattice in the following.

A lattice site $\mathbf{a_{ij}}$ can be in one of three states: 0 or not yet discovered, 1 discovered but not yet viable, and 2, discovered and viable. Compared to the original version of the model, the present model has one fewer state. While the original model in Silverberg and Verspagen (2005) also had sites that were technologically impossible (excluded by the laws of nature), the present model does not have such a state. No sites are excluded by the laws of nature, but sites do differ with regard to the difficulty of discovering them. Some sites are easy to discover, others more difficult if not nearly impossible.

A site may become discovered, i.e., move from state 0 to 1, by means of repeated effort by the agents searching in its region of the technology lattice. Agents invest R&D with the aim of discovering the site. Each site on the lattice is randomly initialized with a 'resistance' value, which we denote by $\mathbf{q_{ij}}$. This value is drawn from a lognormal distribution with mean <q> and standard deviation σ. When an agent invests **b** units of R&D with the aim of discovering the site, the resistance value is diminished according to the following rule: $\mathbf{q_{ij,t+1}}. = \mathbf{q}_{ij,t} - \boldsymbol{b}\omega$, where ω is a random variable drawn from a uniform distribution on [0, 1) (this represents the stochastic nature of the R&D process), and the subscripts t and $t + 1$ denote the value of the resistance factor before and after the agent's R&D project. We define an invention at a particular site as the event that $\mathbf{q_{ij}}$ becomes zero or negative. At this point, the site passes into state 1.

A site moves from state 1 to 2, i.e., from discovered to viable (invented to innovated), when there exists a contiguous path of viable (state 2) sites connecting it to the baseline. The neighborhood we shall use is the von Neumann one of the four sites top, bottom, right and left $\{\mathbf{a_{i\pm1,j}}, \mathbf{a_{i,j\pm1}}\}$, with periodic boundary conditions horizontally. The intuition here is that a discovered technology only becomes viable or operational when it can draw on a chain of supporting technologies already in use. In the model, until such a path (chain) exists, the technology is still considered to be under development (state 1)—it is still an invention, not an innovation.

At any point in time t a best-practice frontier (BPF) can be defined consisting of the highest sites in state 2 for each baseline column (of which there are N_c):

$$\mathrm{BPF}(t) = \{(i, j(i)), i = 1, Nc\}, \quad \text{where } j(i) = \max j|ai, j = 2\}. \tag{1}$$

If there is no viable site in column i we set $j(i) = -1$.

2.2 The firm-based R&D process

An innovation is defined as a jump in the *BPF* in the vertical dimension (column) above a single baseline site in a single time period. The size of the innovation, denoted by $\boldsymbol{s}$, is defined as the number of levels (rows) that the frontier has moved upward in that column. The payoff of an innovation will be assumed proportional to $\boldsymbol{s}$. The firm's R&D budget in each period consists of a fixed, exogenous, part, which is equal for all firms and all periods, and a part deriving from the payoffs to innovations in the preceding R&D round of that firm. This is formulated as follows:

$$\boldsymbol{B}_t = \pi_0 + \sum_k \boldsymbol{s}_{k,t-1}\pi, \tag{2}$$

where $\boldsymbol{B}_t$ is the total R&D budget that the firm spends in period t, π_0 is the base part of the R&D budget, $\boldsymbol{s}_{k,t-1}$ is the size of the firm's innovation (if any) in column k that the firm made when it last had a turn at performing R&D, π is the payoff per 'unit' of innovation, and k is summed over all columns in the firm's search neighborhood. If the firm was unsuccessful in its previous R&D round (no innovations were realized), its R&D budget falls to π_0. This autonomous part of R&D financing can be likened to venture capital and ensures that firms do not exit the economy when they fail to produce an innovation in a single period.

Our firms are modeled as decision-making agents that display simple, rule-based behaviors. We only model the R&D function of the firm (i.e., production and sales are outside the realm of our model), according to the pay-off rule specified above. Firm behavior has only two dimensions: allocation of the R&D budget to sites of the lattice that are currently being explored in the firm's R&D neighborhood, and finding a position on the lattice to search from. At a given point in time, the firm operates from a single position (site) in the lattice, but this position potentially changes after each R&D round.

The firm's search neighborhood consists of a (diamond-shaped) neighborhood of radius m in the 'Manhattan' metric induced by the neighborhood relation centered around the firm's present site. This neighborhood contains $2m(m+1)$ points. We assume for simplicity that the total R&D budget $\mathbf{B}$ of each firm is always spent in each R&D period, and distributed equally over all sites in the current neighborhood, irrespective of whether or not they have already been discovered. Thus, the R&D budget available for a single site is $\boldsymbol{b} = \boldsymbol{B}/2m(m+1)$.

Since R&D is aimed at the local environment of the firm, an important element of the model is how firms determine their position in technology space from which R&D is undertaken. Just before a firm starts an R&D cycle to search its local neighborhood, it may move its position on the lattice. We now differentiate between two distinct regimes of firm behavior. In both scenarios, we populate every column on the lattice with exactly one firm at time zero. The number of firms is fixed throughout the simulation (since we only model the R&D function of the firm, and firms always have an R&D budget of at least π_0, they cannot go bankrupt).

If we are in the *fixed firm* regime, the firm simply moves vertically in its 'own' (fixed) column, to the present level of the BPF that it inherits from the last innovator.[3] Thus firms are always located on the BPF, and their search obviously extends to areas beyond as well as below the BPF. In this regime, overlap between search areas of

[3] Thus the term *fixed* firms refers only to the column to which the firm is fixed, not the row.

firms is limited: each firm has its own fixed column, and can extend its search into only a limited number of neighboring columns (the exact number is determined by the exogenous parameter m).

In the *moving firm or self-organized* regime, the firm also first moves vertically to the BPF,[4] but after this step it is allowed to move (as opposed to just search as in the previous regime) into neighboring columns. Although each move is restricted to a radius of m columns in the horizontal dimension in either direction, a sequence of subsequent moves may obviously take the firm to arbitrarily distant columns. We assume that firms have a desire to move to columns that have a higher value of the BPF. Hence, each firm, after having moved vertically to the BPF, also examines the heights of the points on the BPF in the columns within a radius m of its current column. It then decides probabilistically whether to move to one of these columns. After a firm has moved to a column, it is again moved up vertically to the BPF in that column. We calculate probabilities for each column to which a firm can potentially move as follows. For each column j in its m-radius column neighborhood, we calculate a value $u_j = e^{\beta(h_j - h_i)}$, where h_j is equal to column j's BPF technological level (i.e., height on the lattice), h_i is the current level of the firm, and β is an exogenous parameter reflecting the speed of movement to higher sites. We calculate the probability p_{ij} that the firm will move to column j on the BPF from its current column i (or stay where it is, with probability p_{ii}) as

$$p_{ij} = \frac{u_j}{U}, \quad U = \sum_j u_j. \tag{3}$$

By varying β we can make firms more or less 'rational' or responsive to disparities in the state of neighboring columns in technology space. The parameter β plays a role similar to that in the Brock and Hommes (1998) model of routes to chaos in financial market with heterogeneous agents. Columns with higher values of the BPF are assigned a higher probability in this calculation, but the firm may stay where it is or even jump to a lower site with a low probability.

In this scenario, we can anticipate that firms will gravitate to areas of the lattice that correspond to (local) peaks in the BPF. Hence it may be the case that particular regions of technology space (corresponding to local valleys) are left completely unpopulated, at least for a while. Such 'herding' behavior implies that the search areas of firms overlap to a significant extent. This creates a large potential for synergies between firms' efforts, since each firm that undertakes R&D will tend to diminish the resistance value **q** at a site, and this becomes available as an externality for other firms that search in a nearby neighborhood. Also, when other firms in the neighborhood advance the BPF, the current firm is first moved up to the new BPF on its current column before it undertakes R&D. The potential for such synergies and externalities is positively related to the amount of overlap in search areas, and hence is smaller in the case of the fixed firms regime. At the same time, the amount of R&D duplication increases and backward regions of technology space may be abandoned, which may potentially outweigh the synergies on balance.

4 Thus firms benefit from an interfirm technology externality after one innovation period in both regimes.

After the firm moves, it performs R&D in its (new) local neighborhood. Payoffs are awarded (added to the R&D budget of the next period) after this R&D process, and the global *BPF* is updated. The fact that search continues to take place below the BPF means that the paths connecting sites on the frontier to the baseline may shorten over time as 'shortcuts' and missing links are discovered. We regard this as one way of representing *incremental* innovation, but we will not deal with this aspect here.

2.3 Innovation dynamics

A discovered site (state 1) need not initially connect up with the operational network of sites in state 2. It is this fact that permits innovations of variable length (as measured by the jump in the BPF they entail) to occur spontaneously. Thus we obtain a natural explanation of innovation clustering (but of the random kind), as shown in Fig. 1. This happens when a disjoint extended network of discovered but not yet operational (state 1) sites is finally connected to the technological frontier. When this happens (through the 'cornerstone innovation'), the previously disjoint innovation island suddenly forms an 'overhanging cliff' that may advance laterally in subsequent periods. In Fig. 1, for example, firms may now search from the right- (or left-)most peak of the previously disjoint island. Just left and right of this 'promontory' (i.e., in neighboring columns), the BPF may lie much below the BPF level on the promontory itself. When innovation occurs in such neighboring columns, the increment (innovation size) can be much larger than m, the search radius, and is in fact unbounded from above. In the actual simulation runs we observe, such overhanging cliffs that move laterally are the dominant generators of large ('radical') innovations.

The basic unit of time in the model is one R&D cycle by one firm. At the beginning of each cycle, a single firm is drawn randomly from the population. We update the BPF and the states of the lattice sites after that firm has done R&D. We refer to this as sequential updating. In contrast, the previous version of the model had parallel updating, i.e., R&D was performed in each column by a firm and only after all columns had completed R&D was the BPF updated. Obviously, the way in which updating is done

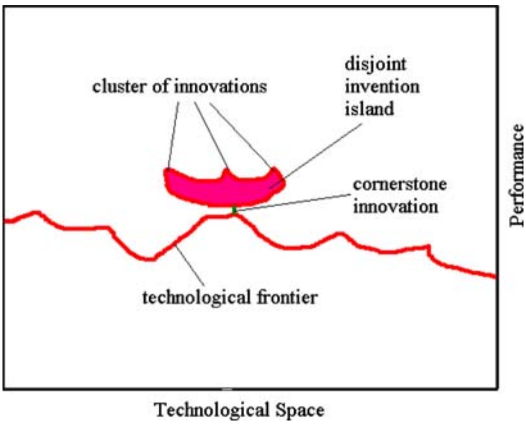

Fig. 1 Clusters of innovations occur when disconnected islands of inventions are joined to the BPF by cornerstone innovations

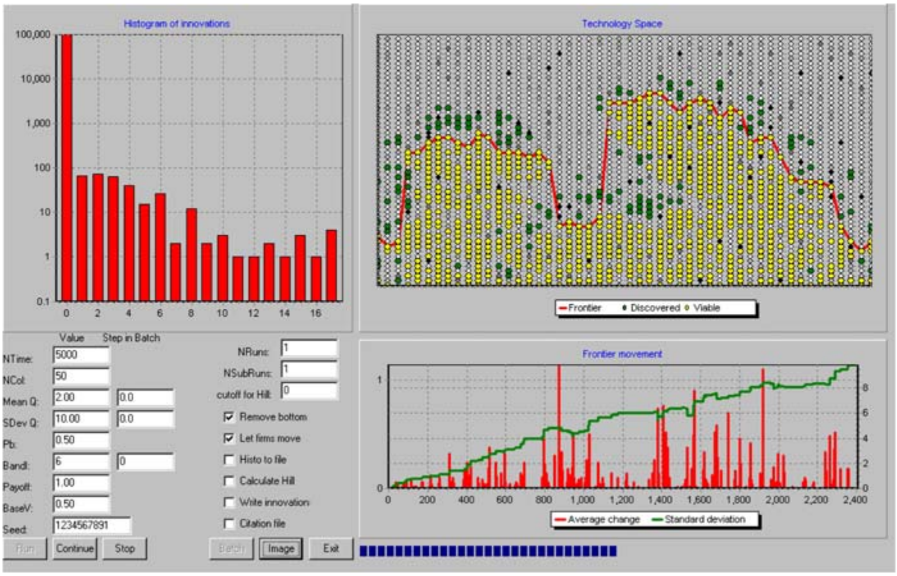

Fig. 2 Screenshot of the computer implementation of the model

(sequentially or in parallel) has a large impact on the amount of synergies and externalities taking place (see the discussion in the previous section). With asynchronous updating, more synergies/externalities are implied.[5]

The computer implementation of the model is illustrated in Fig. 2, which is a screen shot of the user interface in interactive mode. The rectangle on the upper right shows the state of the lattice at this point of time. Grey dots represent undiscovered lattice sites (state 0), with darker colors indicating higher values of **q**. Green sites represent discovered but not yet viable sites (state 1), and yellow sites are viable technologies (state 2), i.e., discovered and connected to the baseline. The red line represents the BPF around which search is taking place in a band of radius 6. A typical pattern is shown of 'overhanging cliffs' of yellow sites on the left and in the middle.

3 Simulation results

We begin by comparing the behavior of the model with firms either evenly distributed over columns and fixed, or self-organizing and locally moving to more attractive sites in the manner described above. Figure 3 shows the average rate of innovation generated per period (defined as the average number of steps that the BPF moves up per period) in runs of 15,000 periods for the two regimes as a function of the mean <q> and standard deviation σ of the lognormal distribution (averaged over five runs per parameter value

[5] While we intend to also study parallel updating in this model, this would require certain additional assumptions about how the fruits of simultaneous discovery are apportioned (e.g., a random winner takes all, or equal sharing) and how the **q** values change with duplicate effort of different firms. A comparison of the two updating schemes is the subject of a forthcoming paper.

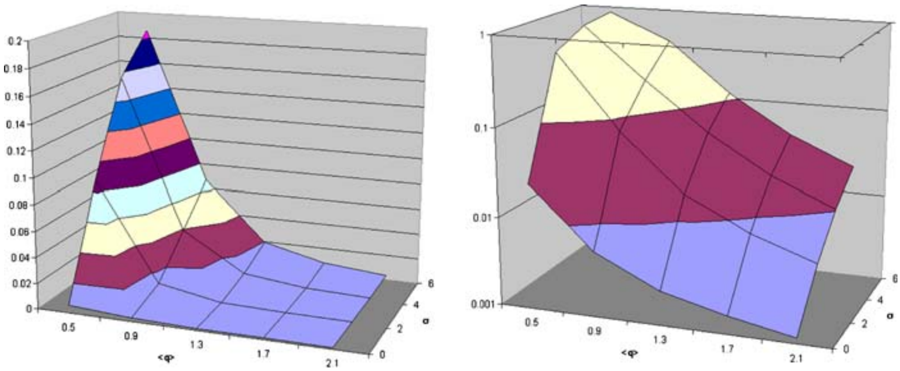

Fig. 3 The innovation rate as a function of the mean and standard deviation of the generating lognormal distribution for fixed (*left*) and moving (*right*) firm regimes

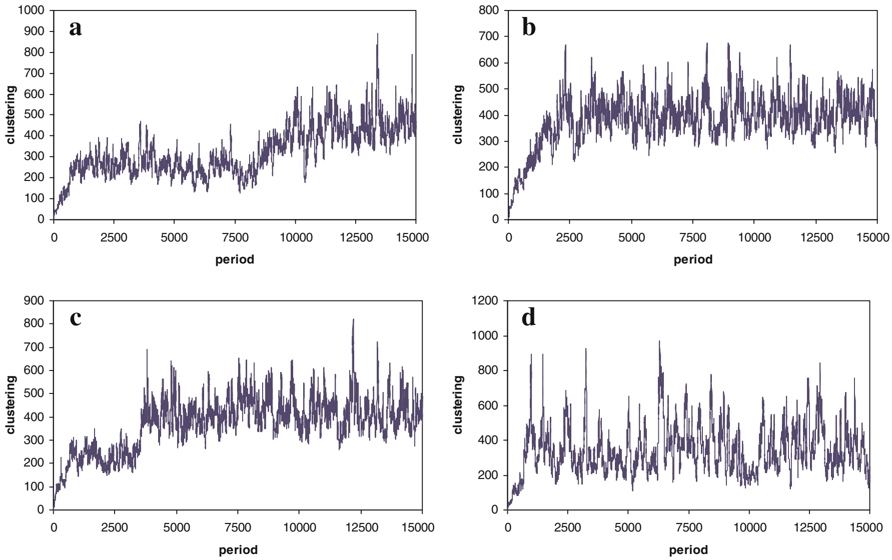

Fig. 4 Clustering index for four runs with $<q>=0.5$, $\pi=1$ and: **a** $\sigma = 1, m = 3$; **b** $\sigma = 2, m = 3$; **c** $\sigma = 4$, $m = 3$; and **d** $\sigma = 1$, $m = 10$. Bimodality appears to be present in **a** and **c**

generated with different random seeds, with $m = 3$ and $\pi = 1$). The SO regime clearly outperforms the fixed-firm one for all parameter values, with the innovation rate declining with $<q>$ (i.e., as average site difficulty increases) and increasing rapidly with σ (as the landscape becomes more rugged and, due to the percolation property, passable valleys form) in both cases.

Figure 4 displays how the firms cluster over time in the self-organized regime by plotting the clustering index d as a function of time, where d is defined as

$$d = \sum_{i=1}^{N_c} n_i^2 - n, \tag{4}$$

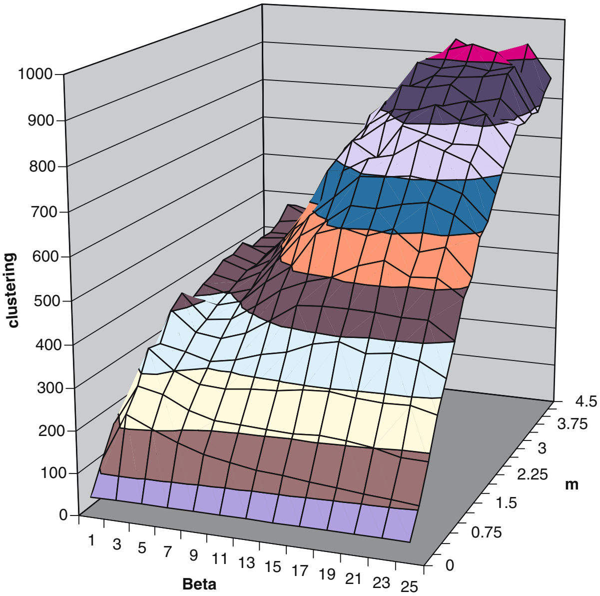

Fig. 5 The clustering index as a function of the search radius m and the speed of relocation parameter β (mean of five runs per value)

with n_i the number of firms in the ith technology column and n the total number of firms (which by assumption is equal to the total number of columns N_c). Figure 5 presents time series of the clustering index for four runs with differing values of σ and m. Common to all is a rapid rise to a steady state value about which rapid and upper-skewed fluctuations take place. This is some evidence for multiple steady states, however, as can be seen in panels (a) and (c). However, the progression always seems to occur from a lower steady state to a higher one, to the extent that one can generalize from this limited evidence. The dependence of clustering on the parameters β and m is shown in Fig. 5. For a critical value of m around 10 and β greater than 1 this begins to rise quite steeply.

The influence of the two 'strategic' parameters m (search radius) and β (speed of local movement along the BPF) on SO firms is more complex (Fig. 6). For low values of β (a value $\beta = 0$ represents completely random movement, irrespective of frontier values in the local neighborhood), the innovation rate is monotonically increasing in m. For higher values of β (0.5 and beyond), this gives way to a trough between very values of m (local search) and global search (when $2m + 1$ approaches the width of the technology space, in the present simulations 50). The reason for this behavior is not entirely clear, but it seems to indicate that the effects of greater search radius (m) are not exclusively advantageous, for example because of 'lock-in' of a large part of the population to a limited part of the total technology space. On the other hand, an increase in β (representing the 'rationality' of firm movement) does have a monotonically positive effect, although this levels off for values of β beyond 0.75–1.

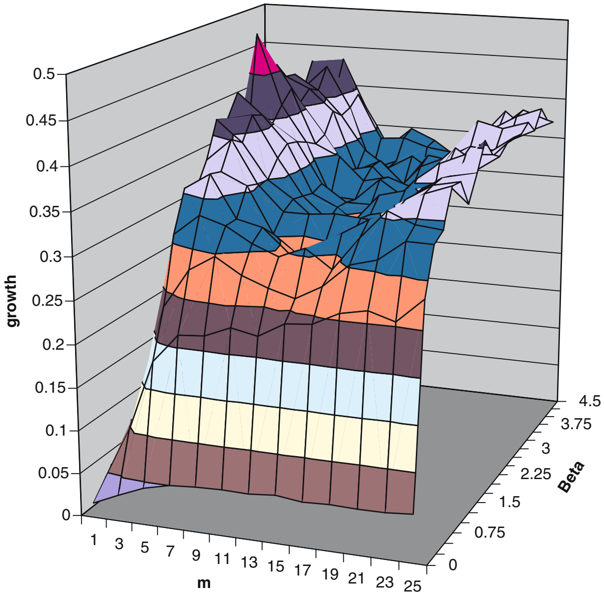

Fig. 6 The innovation rate as a function of the search radius m and the speed of relocation β (mean of five runs per parameter value)

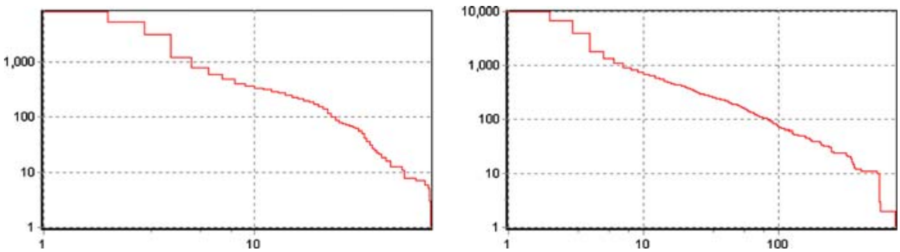

Fig. 7 Pareto plots of innovation size distributions for fixed (*left panel*) and moving (*right panel*) firm regimes. $<q>=0.5$, $\sigma = 1$, $m = 3$, $\pi = 1$

In Fig. 7 we compare the innovation size distributions resulting from a run with fixed firms and the equivalent run with moving ones ($<q>=0.5, \sigma = 1, m = 3, \pi = 1$). These are Pareto plots showing the number of observations greater than or equal to a certain size, on a double-log scale. Pareto-distributed observations will fall on a straight line in such a plot. For fixed firms we observe a definite curvature (indicating that, while the distribution is highly skewed, it is not fat-tailed and more resembles a lognormal distribution). In contrast, in the moving firm regime we observe a striking region of linearity over at least 2 decades of observations. The slope of this curve is almost exactly -1. The tail index α, which is equal to the inverse value of the slope of the size distribution curve, can more properly be calculated by making use of its maximum likelihood estimator due to Hill (1975). This is defined using the largest k

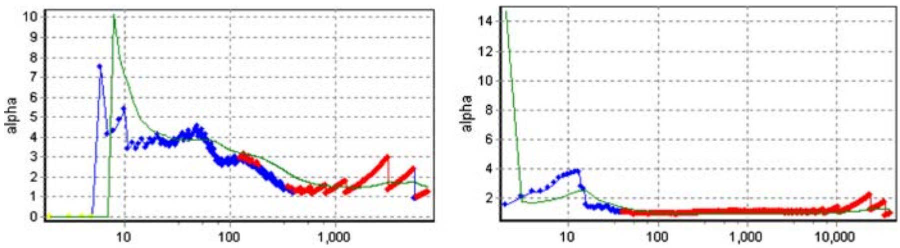

Fig. 8 Hill plots for fixed (*left*) and moving (*right*) firm regimes corresponding to the Pareto plots of Fig. 7. *Blue values* are statistically insignificant, *orange* ones significant at 95% level. *Green line* represents the QQ estimator (see Resnick 2004)

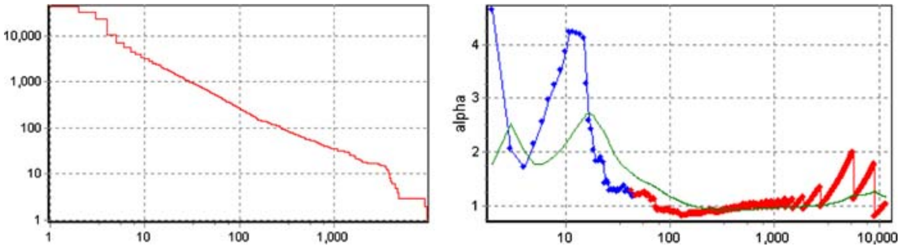

Fig. 9 Pareto plot (*left*) and Hill plot (*right*) for same parameters as Fig. 7 except σ now doubled to 2

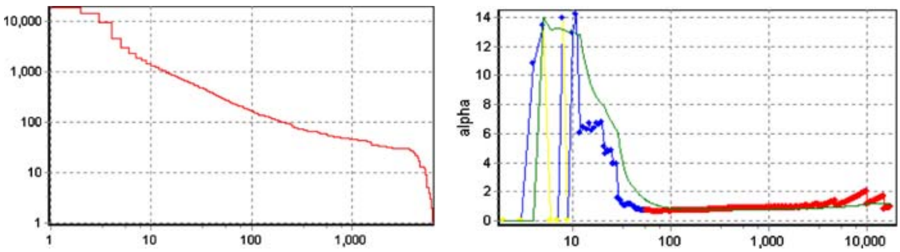

Fig. 10 Pareto plot (*left*) and Hill plot (*right*) for same parameters as Fig. 7 except β now doubled to 2

values of the rank order statistics of the observations as follows:

$$1/\alpha(k,n) = H(k,n) = \frac{1}{k}\sum_{i=1}^{k}(\ln X_{[i]} - \ln X_{[k+1]}). \tag{5}$$

Figure 8 shows the Hill plots of the tail index for the fixed and the moving firm cases, respectively. The lack of a stable plateau of the Hill plot for the former indicates that it is not fat tailed. On the other hand, for the moving firms case, the near-perfect plateau at a value of α around one between the 15th and the 10,000th largest observations is striking. However, there are significant and systematic deviations from linearity for both the smallest and very largest innovations.

What is most remarkable about this scaling behavior in the self-organized regime is that it appears to be insensitive to the values of the principal parameters. The panels of Figs. 9, 10 and 11 display a selection of Pareto and Hill plots for various constellations

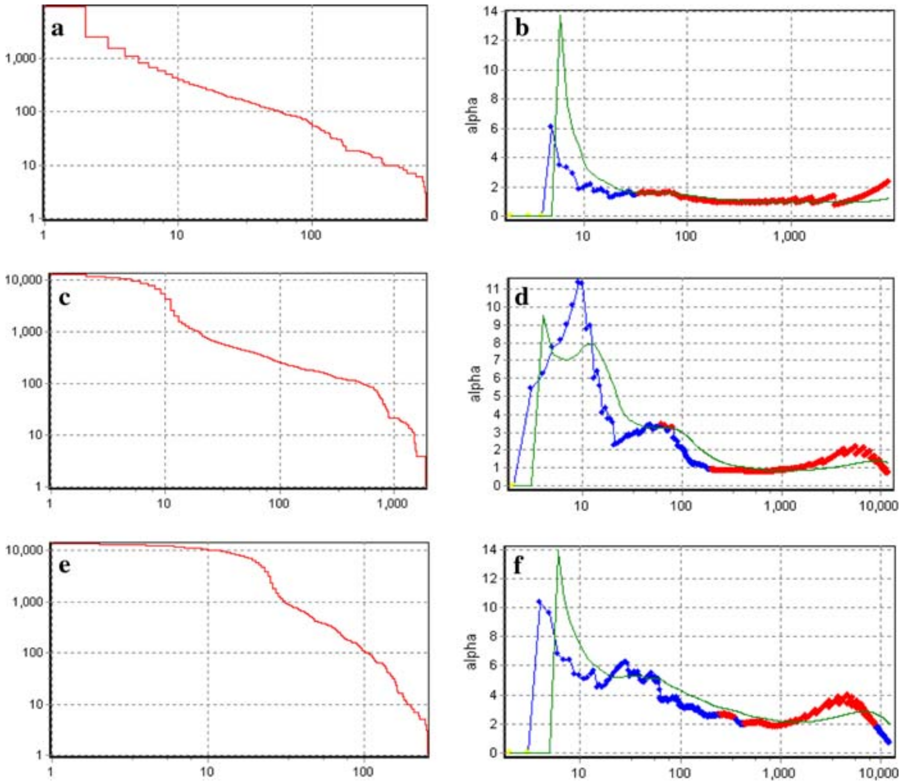

Fig. 11 Pareto plots ($left$) and Hill plots ($right$) for same parameter values as in Fig. 7 while stepping through search radius m: **a**, **b** $m = 1$; **c**, **d** $m = 10$; **e**, **f** $m = 24$ (global search)

of parameter values. It is only when we move to higher values of m that the scaling region shrinks and the estimate of the tail index rises from one to two. Because this regularity emerges without the need to tune the system to a critical value of an exogenous parameter, as would be the case in a pure percolation model, it appears, at least within limits, to be an instance of self-organized criticality. This is perhaps not completely surprising, since in some respects our model resembles the well-known model of interface growth due to Sneppen (1992).[6] Why the tail index always assumes a value of one remains something of a mystery. However, it falls into the same ballpark as the empirical estimates of the tail index of monetary measures of the returns to innovation found in Silverberg and Verspagen (2004), where values near or just below one were observed. Of course our measure of innovation size does not map directly to any of the monetary measures employed in the empirical literature. Nevertheless, this congruence is intriguing.

[6] It differs from the Sneppen model in that interface growth (the advance of the BPF) takes place at sites selected by a local, probabilistic ‘extremal’ rule (firms move only locally and with a probability less than one to the most active previous sites) rather than employing straightforward extremal dynamics.

4 Conclusions and future research

In this paper we have introduced endogenous R&D search by economically motivated firms in a percolation model of innovation dynamics. A previous model without endogenous R&D search has already proven to be useful in explaining some of the stylized facts about innovation, specifically with regard to the temporal clustering of innovation and the skewed nature of innovation size distributions. The introduction of the 'Toyota' landscape in place of the binary percolated technology landscape used in our previous model does not seem to change the basic properties of the model in terms of its ability to generate these stylized facts. However, the ability of firms to move and redeploy their R&D efforts as a result of previous advances on the landscape introduces a quality of self-organization into the model. The most striking change in comparison with fixed firms is a much more highly skewed distribution of innovations and scaling over a considerable range, with a characteristic tail index of one. While this local rationality of moving firms always leads to superior aggregate rates of innovation compared to similar fixed firms, it is not necessarily the case that increasing the radius of search and the speed of relocation always raises that performance. Intermediate values of the search radius at high speeds of relocation can actually diminish innovative performance compared to more local or fully global search. This is probably due to an excessive level of clustering in just a few (but not necessarily the best) regions.

Preliminary research also indicates that the manner in which firm R&D is updated (successive R&D as in the present version vs. concurrent, parallel R&D) may make a significant difference to some of these characteristic features. It may also be the case that the topology of the underlying technology space may play a role. These will be the subjects of future investigation.

Acknowledgments The authors thank an anonymous referee and the guest editor, Sheri Markose, for helpful suggestions.

References

Brock WA, Hommes CH (1998) Heterogeneous beliefs and routes to chaos in a simple asset pricing model. J Econ Dyn Control 22:1235–1274

Dosi G (1982) Technological paradigms and technological trajectories. Res Policy 11:147162

Foray D, Grübler A (1990) Morphological analysis, diffusion and lock-out of technologies: ferrous casting in France and the FRG. Res Policy 19:535–550

Frenken K, Leydesdorff L (2000) Scaling trajectories in civil aircraft 1913–1997. Res Policy 29:331–348

Harhoff D, Narin F, Scherer FM, Vopel K (1999) Citation frequency and the value of patented inventions. Rev Econ Stat 81:511–515

Harhoff D, Scherer FM, Vopel K (2003) Exploring the tail of patented value distribution. In: Grandstrand O (ed) Economics, law and intellectual property. Kluwer, Boston/Dordrecht/London

Hausman J, Hall BH, Griliches Z (1984) Econometric models for count data with an application to the patents-R&D relationship. Econometrica 52:909–938

Hill BM (1975) A simple general approach to inference about the tails of a distribution. Ann Stat 3:1163–1174

Nelson RR, Winter SG (1977) In search of a useful theory of innovation. Res Policy 6:36–76

Resnick S (2004) Modeling data networks. In: Finkenstaedt B, Rootzen H (eds) Extreme values in finance, telecommunications, and the environment. Chapman& Hall, London

Sahal D (1981) Patterns of technological innovation. Addison-Wesley, New York

Saviotti PP (1996) Technological evolution, variety and the economy. Edward Elgar, Cheltenham and Brookfield

Scherer FM (1998) The size distribution of profits from innovation. Ann Econ Stat 49/50:495–516

Scherer FM, Harhoff D (2000) Technology policy for a world of skew-distribution outcomes. Res Policy 29:559–566

Scherer FM, Harhoff D, Kukies J (2000) Uncertainty and the size distribution of rewards from innovation. J Evol Econ 10:175–200

Schumpeter JA (1939) Business cycles: a theoretical, historical and statistical analysis of the capitalist process. McGraw-Hill, New York (page numbers quoted in the text refer to the abridged version reprinted in 1989 by Porcupine Press, Philadelphia)

Silverberg G (2002) The discrete charm of the bourgeoisie: quantum and continuous perspectives on innovation and growth. Res Policy 31:1275–1289

Silverberg G, Lehnert D (1996) Evolutionary chaos: growth fluctuations in a Schumpeterian model of creative destruction. In: Barnett, WA, Kirman A, Salmon M (eds) Nonlinear dynamics in economics. Cambridge University Press, Cambridge

Silverberg G, Verspagen B (2003a) Brewing the future: Stylized facts about innovation and their confrontation with a percolation model. Eindhoven: ECIS Working Paper 80. http://www.tm.tue.nl/ecis/Working%20Papers/eciswp80.pdf

Silverberg G, Verspagen B (2003b) Breaking the waves: a Poisson regression approach to schumpeterian clustering of basic innovations. Camb J Econ 27:671–693

Silverberg G, Verspagen, B (2004) The size distribution of innovations revisited: an application of extreme value statistics to citation and returns measures of patent significance. Maastricht: MERIT Research Memorandum 2004–021. http://www.merit.unimaas.nl/publications/rmpdf/2004/rm2004-021.pdf, forthcoming in J Econ

Silverberg G, Verspagen B (2005) A percolation model of innovation in complex technology spaces. J Econ Dyn Control 29:225–244

Sneppen K (1992) Self-organized pinning and interface growth in a random medium. Phys Rev Lett 69:3539–3542

Trajtenberg M (1990) A penny for your quotes: patent citations and the value of innovations. Rand J Econ 21:172–187

Winter SG (1984) Schumpeterian competition in alternative technological regimes. J Econ Behav Org 5:287–320