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REGULAR ARTICLE

A link between random coefficient autoregressive models and some agent based models

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Abstract An agent based model (ABM), where each agent makes decisions by using the sum of two signals, is proposed. The first is related to the fundamental information while the second comes from trader's idiosyncratic noise. This model entails the switching between two groups called fundamentalist and noise traders. Additionally, if the price impact function is log-linear, then the dynamic of log asset prices belongs to the class of random coefficient autoregressive RCA(p) models, which are known to share important stylized facts of financial prices.

Keywords Artificial financial markets · RCA models

1 Introduction

Modeling financial markets has always been of interest to academic researchers. Several models have been developed for this purpose. The literature can be classified into three groups depending on the hypothesis made about agent decision making. The first group, classical financial models, assumes that agents are fully rational. So their decision respects Von Neumann–Morgenstern or Savage axioms. It is also supposed that all agents have the same views to future expected returns (homogeneous expectation). These assumptions, even if they are not too realistic, allow tractable models. Homogeneous rational expectations with frictionless assumption implies that markets are efficient, in other words, asset prices always reflect all available information. There is no way to earn positive excess returns by using the available information. The arrival of news is the only factor driving security prices. These assumptions would not matter

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if equilibrium prices, which result from these models, match financial time series characteristics. Unfortunately, this is not the case. Since 1980s, some patterns have been detected. For example stock prices are too volatile with respect to their fundamentals, see Shiller (1981). This excess of volatility may be explained by the fact that stock prices overreact to relevant news in the short-term (momentum effects) and exhibit return reversals in the long-term, see De Bondt and Thaler (1985). It is possible to use these patterns to generate significant positive returns, see Jegadeesh and Titman (1993).[1] On the other hand, with homogeneous rational expectations, there is a low to zero trading volume since traders hold the same portfolio, however in practice this is not the case. Trading volume is not small and is positively correlated with volatility. These facts have motivated some researchers to relax one or two of the hypotheses mentioned above. Behavioral Finance relaxes the individual rationality of traders by allowing for both Rational and Noise traders. The latter may survive in the market for a long time because arbitrage opportunities are costly and risky. So Limits to Arbitrage is the first building block of Behavioral Finance. The second is based on insights from psychology field and explains why irrational investors may deviate from subjective expected utility by having correlated trades. We refer to Barberis and Thaler (2003) for a review of the Behavioral Finance foundation. The dynamic prices obtained with these models reproduce the general momentum and long time reversal effects, see for instance Daniel et al. (1998).

The last group is the microscopic (Agent Based) financial markets. This class of models is solvable using computational tools since each trader cannot determine his/her expected returns, which depend on the expectation of the others. Often ABM in financial markets are based on heterogeneous interacting agents (see Lux and Marchesi 2000) or interacting rules with a learning phenomenon (see Arthur et al. 1997). There are other models that work directly on agents' decision making and are not based on a specific learning process. Models are based on herding or other psychological effects, see Iori (2002), Cross et al. (2005).

The aim is to show that random coefficient autoregressive (RCA) models, which have been studied in depth, may be obtained from an ABM with switching phenomena. For some applications of these econometric models, works by Yoon (2003) can be sought and for some theoretical issues, see Nagakura (2009) and references therein.

The paper is divided into four sections. Section 2 describes the ABM. In Sect. 3, some properties of the ABM are derived. Section 4 gives some possible extensions and comments and the last section concludes outcomes of this work.

2 Description of the agent based model

This works considers a market model similar to Ghoulmie et al. (2005). There is a single risky asset whose price is denoted by S_t. The number of traders is M and trading takes place at discrete periods $t_j = j \cdot h$ where $j \in \mathbb{N}^*$ is an integer such that $j \geq 1$ and h is the time step between two transactions. The parameter h is

[1] We recall however that Market efficiency is jointly tested with an equilibrium model. So if the null hypothesis is rejected, it does not mean (necessarily) that markets are inefficient.

measured in terms of minutes or hours. In this case, we may neglect the dividend process and the interest rate model. The asset prices move according to the arrival of news say $I_t,\ t \in \mathbb{T} = \{t_0, t_1, \ldots, t_n, \ldots\}$. At time t_{n-1}, all agents indexed by $k \in \mathbb{J} = \{1, \ldots, M\}$ receive the fundamental information I_{t_n} and they assess it by making some idiosyncratic errors. It is sensible to suppose that the bias is more significant when the market is volatile(i.e trends or big oscillations) rather than stable. To integrate this point, we assume that the trader k evaluates the signal I_{t_n} by

$$R^k_{t_n} = I_{t_n} + \epsilon^k_{t_n}, \quad \epsilon^k_{t_n} = \sigma_k |r_{t_{n-1}}| Z^k_{t_n} \tag{1}$$

where $\epsilon^k_{t_n}$ is the idiosyncratic noise.

In the above expression, $r_{t_{n-1}}$ represents the asset return at time $t_{n-1} = (n-1) \cdot h$ defined by $\ln S_{t_{n-1}} - \ln S_{t_{n-2}}$. The terms $\sigma_k \in \mathbb{R}^*_+, k = 1, \ldots, M$ are supposed to be bounded and $(Z^k_t)_{t \in \mathbb{T}}$ is a Gaussian process.

After deriving $R^k_{t_n}$, the trader makes a decision that is proportional to his/her signal that means $n^k_{t_n} = c_k R^k_{t_n} \in \mathbb{R}$ where c_k is a strictly positive constant. Trader k is called a seller (buyer) at time t_n if $n^k_{t_n}$ is negative (positive) or equivalently if $R^k_{t_n}$ is negative (positive). We make the following assumptions:

Assumption 1 For any $k \in \mathbb{J}$, $(Z^k_t)_{t \in \mathbb{T}}$ and $(I_t)_{t \in \mathbb{T}}$ are white Gaussian processes that are mutually independent with $I_t \sim \mathcal{N}(0, \sigma_I^2)$, $Z^k_t \sim \mathcal{N}(0, 1)$.

Assumption 2 For any $t \in \mathbb{T}$, the random variables $(Z^k_t)_{k \in \mathbb{J}}$ are mutually independent.

Assumption 3 The parameter σ_k given by Eq. (1) is smaller than σ_I (volatility of I_t). $0 < \sigma \le \sigma_k < \sigma_I$.

Before introducing some properties from our ABM, we discuss the above assumptions.

The term $\epsilon^k_{t_n}$, coming from (1) is an idiosyncratic noise that appears when traders assess the fundamental information I_{t_n}. Therefore, $(I_t)_{t \in \mathbb{T}}$ and $(Z^k_t)_{t \in \mathbb{T}}$ must be mutually independent for any trader $k = 1, \ldots, M$ to avoid causality. This is the meaning of the first assumption.

Assumption 2 takes into account the lack of communication between traders. In our model, agents make their decisions without any contact to other traders. That is why the random variables $(Z^k_t)_{k \in \mathbb{J}}$ are supposed to be mutually independent at any time $t \in \mathbb{T}$.

The last assumption introduces bounded rationality.[2] It implies that the bias $\epsilon^k_{t_n}$ is often smaller than I_{t_n} in absolute value. Therefore, the probability of $R^k_{t_n} = I_{t_n} + \epsilon^k_{t_n}$ having the same sign as I_{t_n} is big but less than 1. Consequently, we have the presence of two groups of traders at any time. This assertion is formally shown in the next section.

[2] The fully rationality case (no bias) is obtained by setting $\sigma_k = 0$ for all $k \in \mathbb{J}$.

3 Some properties of the agent based model

The previous section described how each agent makes his/her decision. We now derive the following properties that all hold almost surely (with probability one). The exceptions appear when the asset price remains unchanged ($r_t = 0$ for some $t \in \mathbb{T}$). However this event is negligible since r_t is a continuous distribution in our framework.

Property 1 *At any time $t_n, n \geq 1$ (i.e. $t_n \geq h$), there are buyers and sellers if the number of traders M is sufficiently large.*

Since $n_{t_n}^k = c_k R_{t_n}^k$ with $c_k > 0$, we need only to prove that $P(R_{t_n}^k \geq 0,\ k = 1, \ldots, M)$ and $P(R_{t_n}^k \leq 0,\ \ k = 1, \ldots, M) \to 0$ as $M \to +\infty$. The probability of having only sellers is

$$
\begin{aligned}
P\left(\bigcap_{k=1}^{M}\{R_{t_n}^k \leq 0\}\right) &= P\left(\bigcap_{k=1}^{M}\{R_{t_n}^k \leq 0\}\cap\{I_{t_n} > 0\}\right) + P\left(\bigcap_{k=1}^{M}\{R_{t_n}^k \leq 0\}\cap\{I_{t_n} \leq 0\}\right)\\
&= P\left(\bigcap_{k=1}^{M}\{R_{t_n}^k \leq 0\}\middle|\{I_{t_n} > 0\}\right)P(\{I_{t_n} > 0\}) + P\left(\bigcap_{k=1}^{M}\{R_{t_n}^k \leq 0\}\middle|\{I_{t_n} \leq 0\}\right)P(\{I_{t_n} \leq 0\})\\
&= \prod_{k=1}^{M} P\left(\{R_{t_n}^k \leq 0\}\middle|\{I_{t_n} > 0\}\right)P(\{I_{t_n} > 0\}) + \prod_{k=1}^{M} P\left(\{R_{t_n}^k \leq 0\}\middle|\{I_{t_n} \leq 0\}\right)P(\{I_{t_n} \leq 0\})
\end{aligned}
$$

where the last equation uses at time t_{n-1}, given I_{t_n}, the random variables $R_{t_n}^1, \ldots, R_{t_n}^M$ are independent, see Eq. (1) and Assumption 2. On the other hand, we set

$$A_k = P\left(\{R_{t_n}^k \leq 0\} \mid \{I_{t_n} > 0\}\right), \quad \mathbf{A} = \max\{A_k, k \in \mathbb{J}\}. \tag{2}$$

$$B_k = P\left(\{R_{t_n}^k \leq 0\} \mid \{I_{t_n} \leq 0\}\right), \quad \mathbf{B} = \max\{B_k, k \in \mathbb{J}\}. \tag{3}$$

Assumption 3 implies that $\mathbf{A}$, $\mathbf{B}$ are almost surely in $]0, 1[$ while Assumption 1 gives that $P(I_{t_n} > 0) = P(I_{t_n} \leq 0) = \frac{1}{2}$. So

$$
\begin{aligned}
P\left(\bigcap_{k=1}^{M}\{R_{t_n}^k \leq 0\}\right) &= \frac{1}{2}\left[\prod_{k=1}^{M} P(\{R_{t_n}^k \leq 0\} \mid \{I_{t_n} > 0\}) + \prod_{k=1}^{M} P(\{R_{t_n}^k \leq 0\} \mid \{I_{t_n} \leq 0\})\right]\\
&\leq \frac{1}{2}(\mathbf{A}^M + \mathbf{B}^M) \longrightarrow 0 \quad \text{as } M \longrightarrow +\infty.
\end{aligned}
$$

Similarly, we can show that the probability of having only buyers $P\left(\bigcap_{k=1}^{M}\{R_{t_n}^k \geq 0\}\right)$ tends to zero as M tends to infinity. Therefore at any time, we have buyers and sellers when M is sufficiently large. We may call fundamentalists at time $t_n \in \mathbb{T}$ those such that $R_{t_n}^k$ and I_{t_n} have the same sign ($R_{t_n}^k \cdot I_{t_n} \geq 0$) and noise traders the remaining group ($R_{t_n}^k \cdot I_{t_n} < 0$).

Property 2 (Switching phenomena) *There exists a switching phenomenon between the fundamentalist and noise trader group.*

For any trader k, we have due to Eq. (1)

$$\{R^k_{t_n} \cdot I_{t_n} < 0\} = \Big(\{\sigma_k |r_{t_{n-1}}||Z^k_{t_n}| > |I_{t_n}|\} \cap \{I_{t_n} \cdot Z^k_{t_n} \leq 0\}\Big) \subset \{I_{t_n} \cdot Z^k_{t_n} \leq 0\} = D_{t_n}.$$

Thanks to Assumption 1, the events $D_{t_n} = \{I_{t_n} \cdot Z^k_{t_n} \leq 0\},\ t_n \in \mathbb{T}$ are independent and $P(D_{t_n}) = P(I_{t_n} \geq 0, Z^k_{t_n} \leq 0) + P(I_{t_n} \leq 0, Z^k_{t_n} \geq 0) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$. So

$$P\left(\bigcap_{n=1}^{T} \{R^k_{t_n} \cdot I_{t_n} < 0\}\right) \leq P\left(\bigcap_{n=1}^{T} D_{t_n}\right) = \prod_{n=1}^{T} P(D_{t_n}) = \frac{1}{2^T} \longrightarrow 0, \quad \text{as } T \longrightarrow +\infty. \tag{4}$$

On the other hand,

$$\{R^k_{t_n} \cdot I_{t_n} \geq 0\} = \{R^k_{t_n} \cdot I_{t_n} < 0\}^c \ = \ \Big(\{\sigma_k |r_{t_{n-1}}||Z^k_{t_n}| \leq |I_{t_n}|\} \cup \{I_{t_n} \cdot Z^k_{t_n} > 0\}\Big)$$

where A^c denotes the complementary set of A. We set $c_{\min} = \min\{|r_{t_{n-1}}|, t \in \mathbb{T}\}$ which is strictly positive with probability one since the event $\{c_{\min} = 0\}$ is identical to $\cup_{t \in \mathbb{T}} \{r_t = 0\}$ and thus negligible. Therefore,

$$\begin{aligned}\{R^k_{t_n} \cdot I_{t_n} \geq 0\} &= \{\sigma_k |r_{t_{n-1}}||Z^k_{t_n}| \leq |I_{t_n}|\} \cup \{I_{t_n} \cdot Z^k_{t_n} > 0\} \\ &\subseteq \Big(\{\sigma_k c_{\min} |Z^k_{t_n}| \leq |I_{t_n}|\} \cup \{I_{t_n} \cdot Z^k_{t_n} > 0\}\Big) = F_{t_n}\end{aligned}$$

where $F_{t_n}, t_n \in \mathbb{T}$ are independent events due to Assumption 1. Since the probability $P(F_{t_n})$ is almost surely in $]\frac{1}{2}, 1[$, we have

$$P\left(\bigcap_{n=1}^{T} \{R^k_{t_n} \cdot I_{t_n} \geq 0\}\right) \leq P\left(\bigcap_{n=1}^{T} F_{t_n}\right) = \prod_{n=1}^{T} P(F_{t_n}) \longrightarrow 0, \quad \text{as } T \longrightarrow +\infty. \tag{5}$$

Equations (4) and (5) say that any trader k cannot always remain a noise trader or a fundamentalist when the time horizon is large. So the model may be seen as an ABM with a switching phenomenon.

Property 3 *If the price impact function is log-linear, then the log of asset prices belongs to Random Coefficient Autoregressive* (*RCA*) *models.*

If we have intra daily data, as in our model where $h = t_n - t_{n-1}$ is small, such assumption may be acceptable since the excess demand E_{t_n} will be close to zero and so any non linear function may be well approximated by its first order Taylor expansion, see Farmer and Joshi (2000). The price impact function is defined in this case by $\ln S_{t_n} - \ln S_{t_{n-1}} = \frac{E_{t_n}}{\lambda_M}$ where λ_M is a positive constant which may depend on the number of traders M, see Ghoulmie et al. (2005).

By using Eq. (1), the excess demand E_{t_n} is determined by

$$E_{t_n} = \sum_{k=1}^{M} n_{k,t} = \sum_{k=1}^{M} c_k R_t^k = I_{t_n} \left(\sum_{k=1}^{M} c_k \right) + |r_{t_{n-1}}| \sum_{k=1}^{M} c_k \sigma_k Z_{t_n}^k \tag{6}$$

So

$$\ln S_{t_n} = \ln S_{t_{n-1}} + \frac{E_{t_n}}{\lambda_M} = \ln S_{t_{n-1}} + \tilde{I}_{t_n} + |r_{t_{n-1}}| Z_{t_n} \tag{7}$$

where

$$\tilde{I}_{t_n} = c I_{t_n}, \quad c = \frac{\sum_{k=1}^{M} c_k}{\lambda_M} \quad Z_{t_n} = \frac{1}{\lambda_M} \sum_{k=1}^{M} c_k \sigma_k Z_{t_n}^k \tag{8}$$

Since $|r_{t_{n-1}}| = 1_{\{r_{t_{n-1}} \geq 0\}} (\ln S_{t_{n-1}} - \ln S_{t_{n-2}}) + 1_{\{r_{t_{n-1}} < 0\}} (\ln S_{t_{n-2}} - \ln S_{t_{n-1}})$, Eq. (7) becomes

$$\begin{aligned} \ln S_{t_n} &= \ln S_{t_{n-1}} \left(1 + \left(1_{\{r_{t_{n-1}} \geq 0\}} - 1_{\{r_{t_{n-1}} < 0\}} \right) Z_{t_n} \right) + \tilde{I}_{t_n} \\ &\quad + \ln S_{t_{n-2}} \left((1_{\{r_{t_{n-1}} < 0\}} - 1_{\{r_{t_{n-1}} \geq 0\}}) Z_{t_n} \right). \end{aligned}$$

where $1_A(x)$ is the indicator function of A (takes 1 or 0 when $x \in A$, $x \notin A$, respectively).

It remains to set $a_{t_n}^1 = \left(1 + \left(1_{\{r_{t_{n-1}} \geq 0\}} - 1_{\{r_{t_{n-1}} < 0\}} \right) Z_{t_n} \right)$ and $a_{t_n}^2 = \left(\left(1_{\{r_{t_{n-1}} < 0\}} - 1_{\{r_{t_{n-1}} \geq 0\}} \right) Z_{t_n} \right)$ to obtain the RCA structure for the log of asset prices

$$\ln S_{t_n} = a_{t_n}^1 \ln S_{t_{n-1}} + a_{t_n}^2 \ln S_{t_{n-2}} + \tilde{I}_{t_n} \tag{9}$$

where $(\tilde{I}_{t_n})$ is a white Gaussian process, see Eq. (8) and Assumption 1. Equation (9) is namely a stochastic unit root model, since by using the law of iterated expectations, we have $E(a_{t_n}^1) = 1$ and $E(a_{t_n}^2) = 0$.

The timing of events that gives this dynamic is the following (see Fig. 1 for illustration). At time t_{n-1}, the information flow I_{t_n} is announced for all traders. The next step is the formation of expectations. Based on the past asset prices (state of the market) each investor k forms his/her expectation about the distribution of the next period price by interpreting the information I_{t_n}. He/She makes an idiosyncratic error ϵ_t^k and then receives the signal $R_{t_n}^k$. The latter is used to make a decision by taking $c_k R_{t_n}^k$ where

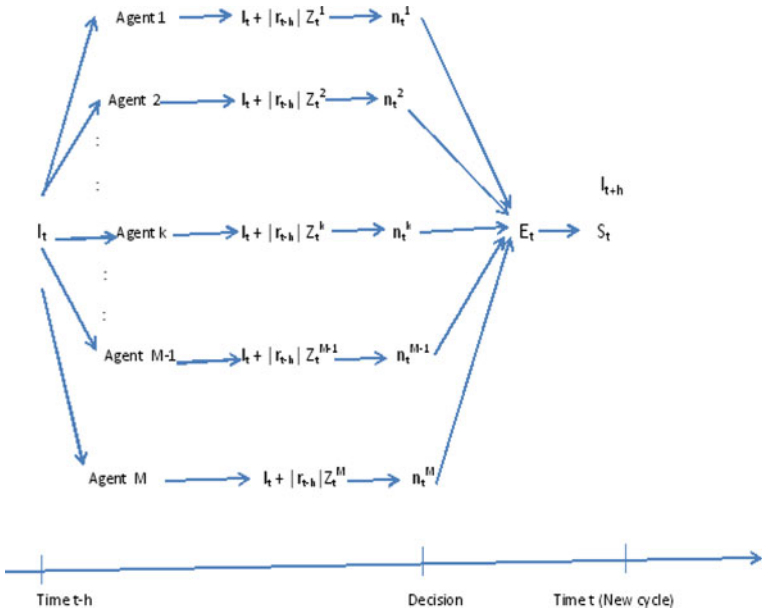

Fig. 1 The timing events of the agent based model (ABM) between $t - h$ and t, $t \in \mathbb{T}$

$c_k > 0$. The excess demand is then determined and the previous price is adjusted by using Eq. (7).[3]

Our ABM extends on the model by Ghoulmie et al. (2005) by relaxing the assumption of threshold behaviors. This extension has the advantage of introducing the presence of buyers and sellers (Property 1) and it also allows determination of the asset return dynamic (Property 3). On the other hand, it is expected that, as an extension, our model will produce the same simulation results. We do not do it here and refer to Yoon (2003) who investigates the properties of a stochastic unit root model.

4 Extension and comments

The previous model may be extended in two ways.

The first extension is with respect to the current volatility. It was defined previously by the last absolute return $|r_{t_{n-1}}|$. Now, it is extended to integrate N previous returns instead of one return and we denote it by $r^N_{t_{n-1}}$. So the script N represents the number of data used to approximate the current volatility. The previous model in Sect. 3 becomes

[3] The market models that use impact functions to determine asset prices are never in equilibrium since $P(E_{t_n} = 0) = 0 \ \forall t_n \in \mathbb{T}$. This is a disadvantage. However, these models are often used, see Ghoulmie et al. (2005), Iori (2002) because their implementation (simulation) takes a little time. See LeBaron (2005) for a survey of other different approaches.

$$\tilde{R}^k_{t_n} = I_{t_n} + \sigma_k r^N_{t_{n-1}} Z^k_{t_n}, \quad r^N_{t_{n-1}} = \sum_{i=1}^{N} \omega_i |r_{t_{n-i}}| \text{ where } N \geq 1, \ \omega_i \geq 0, \quad \sum_{i=1}^{N} \omega_i = 1. \tag{10}$$

where $\tilde{R}^k_{t_n}$ is now the new signal received by trader k.

In this general framework, Eq. (7) becomes

$$\tilde{r}_{t_n} = \ln S_{t_n} - \ln S_{t_{n-1}} = \frac{\tilde{E}_{t_n}}{\lambda_M} = \tilde{I}_{t_n} + r^N_{t_{n-1}} Z_{t_n}$$

where $\frac{\tilde{E}_{t_n}}{\lambda_M} \sim \mathcal{N}(0,\ c^2\sigma_I^2 + \frac{(r^N_{t_{n-1}})^2}{\lambda_M^2} \sum_{k=1}^{M} c_k^2\sigma_k^2)$, see Eq. (8) and Assumptions 1 and 2.

So the dynamic of $(\tilde{r}_{t_n})$ is given by

$$\begin{cases} \tilde{r}_{t_n} = \phi_{t_n} Y_{t_n} \\ \phi^2_{t_n} = c^2\sigma_I^2 + \left(r^N_{t_{n-1}}\right)^2 \frac{\sum_{k=1}^{M} c_k^2\sigma_k^2}{\lambda_M^2} \end{cases} \tag{11}$$

where the random variables Y_{t_n}, $t_n \in \mathbb{T}$ are i.i.d and $Y_{t_n} \sim \mathcal{N}(0, 1)$.[4]

In the simplest case where $N = 1$, we have $(r^N_{t_{n-1}})^2 = (r^1_{t_{n-1}})^2 = (r_{t_{n-1}})^2$, see (10) or Sect. 3. So Eq. (11) says that our RCA structure reduces to an ARCH(1) process[5] for the asset returns, see Engle (1982). Sato and Takayasu (2002) also derived an ARCH process from an ABM where trader rules are deterministic.

There exists other dynamics that are found in the literature. Alfarano (2006) develops in his thesis an ABM where the asset price dynamics are linked to stochastic volatility models when the time step h converges to zero.

Since the RCA process (see also Eq. (11)) is now connected to some ABM, the crucial point that generates the long memory effect of the volatility may be investigated. When agents only use the signal I_{t_n}, they are all fully rational and the decision for the trader k is given by $c_k I_{t_n}$ $(R^k_{t_n} = I_{t_n})$. In this case, it is obvious that asset prices follow a random walk since (I_{t_n}) is a white Gaussian process(no volatility clustering). However, the condition that agents use only I_{t_n} is too restrictive and may be relaxed. We may suppose that agents cannot exactly evaluate the fundamental signal I_{t_n} and then make some overvaluation or undervaluation independently of the market state (current volatility). In this case, their received signal denoted by $\bar{R}^k_{t_n}$ may be modeled for example by $\bar{R}^k_{t_n} = I_{t_n} + \sigma_k Z^k_{t_n}$ where $Z^k_{t_n}$, as previously, satisfies Assumptions 1 and 2. The corresponding excess demand $\bar{E}_{t_n}$ is then given at time t_n by

$$\bar{E}_{t_n} = \sum_{k=1}^{M} n_k = \sum_{k=1}^{M} c_k \bar{R}^k_{t_n} = I_{t_n} \left(\sum_{k=1}^{M} c_k \right) + \sum_{k=1}^{M} c_k \sigma_k Z^k_{t_n}.$$

4 I thank an anonymous referee who provided the formula (11) which is another formulation of our RCA process.

5 Namely, we have $\tilde{r}_{t_n} = \phi_{t_n} Y_{t_n}$, $\phi^2_{t_n} = \alpha_0 + \alpha_1 r^2_{t_{n-1}}$ where $\alpha_0 = c^2\sigma_I^2$, $\alpha_1 = \frac{\sum_{k=1}^{M} c_k^2\sigma_k^2}{\lambda_M^2}$.

So with a log-linear impact price function, we now have

$$\ln S_{t_n} = \ln S_{t_{n-1}} + \frac{\bar{E}_{t_n}}{\lambda_M} = \ln S_{t_{n-1}} + \tilde{I}_{t_n} + Z_{t_n} = \ln S_{t_{n-1}} + \gamma_{t_n}$$

where $\tilde{I}_{t_n}$, Z_{t_n} are defined in Eq. (8) and $(\gamma_{t_n}) = (\tilde{I}_{t_n} + Z_{t_n})$, $t_n \in \mathbb{T}$ is a white Gaussian process, see Eq. (8) and Assumption 1. Therefore, we obtain a random walk theory in the aggregation level. So we need both the presence of heterogeneity (random variables $Z^k_{t_n}, k = 1, \dots, M$) and feedback effects ($r^N_{t_{n-1}}$) with respect to the past asset prices in order to generate persistence in the volatility process.

The second remark is related to the normal distribution. We use only its stability with respect to a sum of independent random variables. So we may replace it by any symmetric α stable laws, say L. In this case, we also obtain an RCA model where the normal distribution of Y_{t_n} in Eq. (11) is now replaced by an L distribution.

Finally, the RCA process, which is precisely a stochastic unit root process, is the result of the combination of both neo-classical and behavioral finance arguments. Traders often follow the sign of the information (efficient market theory) since

$$\begin{aligned} P\left(\{R^k_{t_n} \le 0\} \mid \{I_{t_n} \le 0\}\right) &= P\left(Z^k_{t_n} \le \frac{-I_{t_n}}{|r_{t_{n-1}}|\sigma_k} \,\middle|\, \{I_{t_n} \le 0\}\right) \\ &> P\left(Z^k_{t_n} \ge \frac{-I_{t_n}}{|r_{t_{n-1}}|\sigma_k} \,\middle|\, \{I_{t_n} \le 0\}\right) \\ &= P\left(\{R^k_{t_n} \ge 0\} \mid \{I_{t_n} \le 0\}\right) \end{aligned}$$

So $P\left(\{R^k_{t_n} \le 0\} \,\middle|\{I_{t_n} \le 0\}\right) > P\left(\{R^k_{t_n} \ge 0\}\middle|\, \{I_{t_n} \le 0\}\right)$ where we use $P(X \le a) > P(X \ge a)$ for $X \sim \mathcal{N}(0, 1)$ and $a > 0$ ($a = \frac{-I_{t_n}}{|r_{t_{n-1}}|\sigma_k}$, with $I_{t_n} \le 0$). Similarly, we have $P\left(\{R^k_{t_n} \ge 0\} \,\middle|\{I_{t_n} \ge 0\}\right) > P\left(\{R^k_{t_n} \le 0\}\middle|\, \{I_{t_n} \ge 0\}\right)$.[6]

On the other hand, traders extrapolate the fundamental information when the market is volatile (see Eqs. 1, 10) where the conditional variance of ϵ^k_t depends on $r^N_{t_{n-1}}$. Therefore, the market efficiency question is difficult to analyze. A similar problem has been illustrated in Konté (2010) in a continuous time framework, with two diffusions (X_t) and (Z_t) where (Z_t) was similar to (X_t) but with an additional jump part having zero mean.

5 Conclusion

This paper has proposed an ABM where traders make their decisions without communication. Their behavior is determined by two signals. The first is based on the information flow (I_{t_n}), which is shared by all traders. The second is specific to each trader (idiosyncratic noise) and corresponds to the error made by assessing the fundamental news. We link this second signal to the previous asset returns to emphasize the significance of the bias when the market is volatile. To obtain a realistic model,

[6] The strict inequality is understood almost surely since we can have an equality if $I_{t_n} = 0$.

we impose a small variance for the noise part with respect to the fundamental news. Thus, indicating that traders follow the sign of the information, with some exceptions, resulting in two groups of traders called fundamentalist and noise traders. There is a random switching phenomenon between these two groups. A fundamentalist may become a noise trader and conversely. These results are independent of the structure of the price impact function. If the latter is log-linear that is acceptable for intra daily data, we show that the log prices belong to RCA model. In the special case where the current volatility is approximated by the last absolute return, the RCA model reduces to an ARCH(1) model for the asset returns. After establishing a link between stochastic unit root models and the ABM, the persisting volatility found in financial time series is explained by heterogeneity and feedbacks of traders with respect to recent asset prices. Finally, our ABM may be seen as an extension of the Ghoulmie et al. (2005) model that additionally incorporates the presence of buyers and sellers.

References

Alfarano S (2006) An agent-based stochastic volatility model. PhD thesis, Department of Economics, Christian-Albrechts University of Kiel, Germany

Arthur WB, Holland JH, Palmer BR, Talyer P (1997) Asset pricing under endogenous expectations in an artificial stock market. Econ Notes 26:297–330

Barberis N, Thaler R (2003) A survey of behavioral finance. In: Constantinides GM, Harris M, Stulz RM (eds) Handbook of the economics of finance, (Chap. 18). Elsevier, Amsterdam pp 1053–1128

Cross R, Grinfeld M, Lamba H, Seaman T (2005) A threshold model of investor psychology. Physic A 354:463–478

Daniel K, Hirshleifer D, Subrahmanyam A (1998) Investor psychology and security market under- and overreactions. J Finance 53(6):1839–1885

De Bondt WFM, Thaler R (1985) Does the stock market overreact? J Finance 40:793–805

Engle RF (1982) Autoregressive conditional heteroskedasticity with estimates of the variance of u.k. inflation. Econometrica 50:987–1008

Farmer JD, Joshi S (2000) The price dynamics of common trading strategies. J Econ Behav Organ 49:149–171

Ghoulmie F, Cont R, Nadal J-P (2005) Heterogeneity and feedback in an agent based market model. J Phys Conden Matter 17:1259–1268

Iori G (2002) A micro-simulation traders' activity in the stock market: the rule of heterogeneity, agents' interactions and trade friction. J Econ Behav Organ 49:269–285

Jegadeesh N, Titman S (1993) Returns to buying winners and selling losers: implication for stock market efficiency. J Finance 48:65–91

Konté MA (2010) Behavioural finance and efficent markets: Is the joint hypothesis really the problem? IUP J Behav Finance 7:9–19

LeBaron B (2005) Agent-based computational finance. Handbook of Computational Economics, Volume 2: agent-based computational economics. Elsevier, Amsterdam

Lux T, Marchesi M (2000) Volatility clustering in financial markets: A micro-simulation of interacting agents. Int J Theor Appl Finance 3:675–702

Nagakura D (2009) Testing for coefficient stability of ar(1) model when the null is an integrated or a stationary process. J Stat Plan Inf 139(8):2731–2745

Sato A-H, Takayasu M (2002) Derivation of arch(1) process from market price changes based on deterministic microscopic multi-agent. In: Takayasu H (ed) Proceedings Empirical science of financial fluctuations. Springer, Tokyo, pp 172–178

Shiller RJ (1981) Do stock prices move too much to be justified by subsequent changes in dividends? Am Econ Rev 71:421–436

Yoon G (2003) A simple model that generates stylized facts of returns. Department of Economics, UCSD. Paper 2003-04