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REGULAR ARTICLE

A comparison of different trading protocols in an agent-based market

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Abstract We compare price dynamics of different market protocols (batch auction, continuous double auction and dealership) in an agent-based artificial exchange. In order to distinguish the effects of market architectures alone, we use a controlled environment where allocative and informational issues are neglected and agents do not optimize or learn. Hence, we rule out the possibility that the behavior of traders drives the price dynamics. Aiming to compare price stability and execution quality in broad sense, we analyze standard deviation, excess kurtosis, tail exponent of returns, volume, perceived gain by traders and bid-ask spread. Overall, a dealership market appears to be the best candidate, generating low volume and volatility, virtually no excess kurtosis and high perceived gain.

1 Introduction

The observation that stock exchanges are run under a diverse set of market architectures and protocols raises some obvious and yet intriguing questions. If we do not content ourself with the explanation that this is due to historic accidents or mere temporal and casual path-dependence, then we could investigate reasons explaining why exchanges developed so differently and which are

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the main differences in the dynamic properties of the prevailing price or return time-series.

We propose a terse model where allocative and informational effects are ruled out in order to be able to ascertain what is causing what, focusing on the price dynamics statistics. The use of agent-based methodology is particularly suited to control for unwanted side-effects that can otherwise blur the results. We develop an artificial market where simple agents trade a risky asset using different market protocols. As they receive no information, have no learning capability and act in a strictly controlled framework,[1] we hope to ascribe unambiguously the effects on price dynamics to the market protocol, which fundamentally is a blind mechanism to translate the demand of traders into actual orders and transactions. Even in the absence of price-discovery considerations or allocative efficiency remarks, this attribution might be of great interest to regulatory agencies or other policy makers in charge to devise and enforce trading rules.

Some important measures of execution quality and price stability can be investigated in this framework, as suggested in Madhavan (2000) and Audet et al. (2001): execution quality can be assessed by quantification of volume, bid-ask spread, liquidity, volatility, fatness of tails of returns distribution, kurtosis, market depth to mention some. Quite often, to borrow Madhavan's words, we would expect from a market some key functions, namely good "provision of liquidity, continuity and price stabilization". Although the overall execution quality of a market is a multidimensional concept, "price stability" in broad sense encompasses a wide set of previously mentioned measures and is quite often a targeted objective of regulators. Excess volatility, kurtosis and tail exponents can be related to relevant financial shocks and to the emergence of bubbles in equity price. The burst of a price bubble can adversely affect the economy and is sometimes deemed responsible of a recession, see Bernanke and Gertler (1999). Institutions should be concerned about price volatility to limit financial fragility of banks and insurance firms whose risk exposure is dependent on price level and volatility, see Vila (2000). Broad stability of markets could also enhance price-discovery and news interpretation as signals can be perceived more easily when the background noise (i.e. the volatility intrinsically generated by the trading protocol) is low.

Different markets could serve to provide different performances with regard to the previously referenced criteria but a comprehensive comparison of different market protocols is, to the best of our knowledge, still lacking in the literature. There are obvious difficulties in comparing markets or institutions in that it's virtually impossible to "repeat" the experiment under different circumstances, as it is often the case in social sciences. If a sequence of trades took place in some market at some time because of some order flow, there is no hope to rewind the film changing the market protocol to see how different the

[1] This setting is suited to describe a calm period in the market, with little informational and liquidity flows and no tensions to be resolved (mergings, IPOs...). Hence the results that we discuss in the sequel might be different from the ones generated in the presence of events like, say, September 11.

outcomes would had been. Clever ideas have been used to overcome these design problems (see Amihud and Mendelson 1987, 1991), using real exchanges data but some of their conclusions can be questioned in several ways.

The effects of one specific market protocol are rather indirectly inferred in Maslov (2000) and LiCalzi and Pellizzari (2003) where simple agents (no information flows, adaptation or herding) interact in a continuous double auction (CDA). Though there is no comparison among different markets, their argument is the following: the behavioral characteristics of the agents are so simple and innocuous that the resulting returns properties can *chiefly* be attributed to the mechanism that forms the price. While there is an appreciable effort to stress the notion that dynamics features of the price time-series could be related to institutional devices rather than to the behavior of agents, as is most often done in the literature see Day and Huang (1990) or Lux and Marchesi (1999),[2] it is clear that their tentative conclusion needs much corroboration. In LiCalzi and Pellizzari (2005) different protocols are compared in their ability to converge to the unique allocative equilibrium.

A good deal of experimental economics papers [see Smith (1988)] have faced the comparison of market protocols directly running real experiments with human subjects. These works often study small scale markets, that can be reproduced in vitro more easily and with reasonable costs. Being the result of real experiments, this body of work is largely immune to the previously mentioned "repeatability" criticisms. Some of the results might be not easily generalized to large-scale financial markets but we feel that other drawbacks could be pointed out. As noted elsewhere, experiments with real subjects might be subtly affected by the skill of the agents, the way in which they have been instructed or their understanding of the problem. Additionally and more importantly for our purposes, many experiments show that the agents learn quite rapidly and naturally to cope more and more effectively with their task. Again, the reported results might not be imputed to the market mechanism alone if agents are adapting and learning in time. These considerations should increase the appeal of running computational experiments to sketch some hopefully clear-cut comparisons among markets, by means of an artificial and controllable agent-based environment.

The work in Audet et al. (2001), Bottazzi et al. (2005) and Cliff (2003) on the contrary, provide direct and explicit comparison of different market institutions. In the first paper, quite sophisticated retail agents are compared on two different market platforms, namely a dealership and a limit order market.[3] The findings by Audet and coauthors are rich and depend both on the features of the learning agents and the dealers, their risk aversion, heterogeneity and

[2] However, an early paper suggesting to investigate the effects of different protocols is LeBaron (2001): "This can be both a curse and a blessing to market designers. On the bad side it opens up another poorly understood set of design questions. However, it may have the beneficial effect of allowing one to study the impact of different trading mechanisms".

[3] This is essentially identical to the batch auction presented in the following section and is not to be confused with a continuous double auction.

number (that is a proxy of market depth). If the utility of the final welfare is used to assess the quality, then the dealership market would be preferred in thin environments or by investors affected by large liquidity shocks, for example.

Cliff (2003) offers a novel way to compare different markets. A continuous space of auctions, including as special cases the usual double auction, the English and the Dutch auctions, is explored in order to minimize Smith's α dispersion index by a genetic algorithm. The fresh idea is that agents, equipped with some sort of machine learning capabilities, *coevolve* optimizing their own profit together with the auction. Hence, the procedure really finds at the same time the best type of agents suited for the most efficient market structure. This is interesting as it can be argued that agents would most probably react to different institutional architectures changing their behavior for self-interested purposes. As far as we know, no other papers investigate the mutual adaptation of traders and market mechanisms.

The paper by Bottazzi et al. (2005) is a careful comparison of three alternative architectures and follows a line similar to our approach. In their model, myopic, quadratic utility maximizing agents trade in different set-ups, namely a Walrasian auction, a batch auction and a 'order-book' mechanism. Ecologies of agents are shaped by the fraction of chartist traders, extrapolating the future from past returns, that coexists with the remaining part of purely noisy investors. As well known, the chartist traders can destabilize the market and the deterministic skeleton of the dynamical system describing the price shows a sequence of slowly exploding bubbles and sudden crashes. The conclusion is that the properties of the time series "are largely shaped by the specific architectures of trading mechanisms" and, to a smaller extent, by the actual traders' ecology. Quite differently, the allocative efficiency is more sensitive to behavioral parameters like the fraction of market orders issued.

In our contribution, we simplify the way agents issue limit orders using a reduced form that avoids the need to specify a full utility function and, more importantly, to devise a necessarily arbitrary procedure to generate some limit orders from it. Moreover, the standard dynamics of prices in our model is homeostatic and does not intrinsically show regular bubbles or crashes that are indeed to be ascribed to the behavior of the agents more than to the trading protocol. Our work differs notably from all the previous papers in that we model much simpler boundedly rational agents and are interested in quantifying the impact of different market types on the macroscopic peculiarities of prices and returns generated by our agents, with the aim to compare price stability and execution quality.

The paper is organized as follows. In Sect. 2 the model is described, splitting the treatment of the (minimalist) behavioral features of the agents from the description of the different market mechanisms: batch auction, CDA, competitive dealership. Section 3 gives some visual evidence that the return time series are diverse and presents the results of the simulations. In the first part, that can be intended as a sort of executive summary, we run a multidimensional scaling analysis to simply and graphically show the effects of distinct trading protocols.

In the second part a more detailed analysis is presented and discussed. Finally, we comment our findings and draw some conclusions.

2 The model

The structural assumptions follow LiCalzi and Pellizzari (2003). The economy we consider has two assets, one bond paying a riskless yearly interest rate r, and one stock. The total amount of stock and cash in the economy is constant[4] because we assume that the interest earned on the bond is spent elsewhere or consumed. However, since the number of active traders may vary over time, the quantities of cash and stock available on the market are not constant. Furthermore, we assume that no new information is ever released. Therefore, except for the number of active traders, the fundamentals of the economy are essentially unchanged over time.

In the following, a detailed description of the artificial agents and the markets where they participate is given.

2.1 The agents

We consider N heterogeneous traders who enter or exit the market independently from each other. They either buy or sell stock in exchange for cash. Short selling or borrowing money are not allowed. Thus, traders are budget constrained both on the buy and the sell side: they can place buy orders only if they have sufficient cash and sell orders only for the stock they possess. Traders' strategies rely on two basic assumptions. First, traders want to buy at low price and sell at high price; second, they estimate the fundamental value v of the stock and this is never revised during the activity period.

When active, trader i is endowed with s_i units of stock and a quantity c_i of cash. Each agent has an investing horizon h_i and expects that the risky asset value will reach the fundamental value v_i (s)he estimates by the time $t + h_i$. Traders wish to maximize their gains over the remaining time span $h_i - t$, which is their activity period. At time t a risk neutral agent buys the stock at price p only if

$$\frac{v}{p} \geq (1+r)^{(h_i - t)}. \tag{1}$$

More generally, agents are risk-averse. Since the bond has a riskless yearly rate of return r while investment in the stock is risky, we model the risk-return trade-off incorporating both a positive yearly risk premium π_i^B to buy and a positive yearly risk premium π_i^S to sell in (1). Hence, traders will buy stock at time t if they expect a return sufficiently higher than the return of the riskless bond, namely if

[4] The dealership is an exception as a small cash quantity is gained by the dealer for each round-trip transaction. Hence the cash in the market is very slowly decreasing in this market.

$$\frac{v_i}{p} \geq \left(1 + r + \pi_i^B\right)^{(h_i - t)}. \tag{2}$$

Consequently, we obtain the highest bid price β_i that an agent is willing to offer at time t:

$$\beta_i(t) = \frac{v_i}{\left(1 + r + \pi_i^B\right)^{(h_i - t)}}. \tag{3}$$

Similarly, the smallest ask price α_i (s)he is willing to offer at time t is

$$\alpha_i(t) = \frac{v_i}{(1 + r + \pi_i^S)^{(h_i - t)}}. \tag{4}$$

In order to ensure that $\beta_i(t) \leq \alpha_i(t)$ we set $\pi_i^B \geq \pi_i^S$ for any i. Hence, if active, agents submit orders as couples (Q_i, P_i), where Q_i is the quantity they are willing to buy/sell and P_i is the relative limit price. We assume that Q_i is strictly positive (negative) for a buy (sell) order. Whenever agents can trade they randomly decide with equal probability if their orders are on the buy or sell side and they trade as much as they can given their budget constraints.

When agents enter the market at time $\bar{t}$, $\alpha_i(t)$ and $\beta_i(t)$ are increasing functions defined in $\left[\bar{t}, h_i\right]$. During this activity period, agents buy stock if the current price p is smaller than $\beta_i(t)$ or sell if it is greater than $\alpha_i(t)$, and hold their position in between. Since the difference between bid and ask price decreases in the residual time $(h_i - t)$, agents are likely to trade more actively towards the end of their activity period, providing liquidity to the market. Notice that the bond provides only the riskless rate of interest r used in the definition of $\alpha_i(t)$ and $\beta_i(t)$, and plays no other role. When $t = h_i$ agents have reached their investment horizon and exit the market keeping their current endowment. These endowments are transferred to new traders entering the market some time later, with new investment horizons and new estimates for the fundamental value of the stock. Therefore, since investment horizons are randomly distributed across agents, a staggered entry of new traders occurs but the total amount of cash and stock is constant (this is only approximately true for the dealership, see Sect. 2.2.3). The time elapsed between the exit of an agent and the subsequent entry of a new trader is a random variable τ. Operationally, when i-th agent exits at h_i, we sample a random τ_i and let a new agent enter at time $h_i + \tau_i$, with the same cash and stock but newly and independently sampled horizon and fundamental value.

In the simulations of the following section, we use the parameter values listed in Table 1. The global parameters are the number N of potentially active agents in a trading session, the riskless interest rate r, assumed constant over time, and the initial stock price p_0. The traders are described by their cash c_i, stock endowment s_i, fundamental value v_i, risk premia π_i^B, π_i^S, activity period h_i and idle time τ_i.

U and exp denote respectively the uniform and the exponential distribution, while '$\sim$' denotes an independent draw from a probability distribution. Global

Table 1 Parameters used in the simulations

	Parameters	Initialization
Global	N	$1,500$
	r	0.02
	p_0	$1,000$
Trader	c_i	$2,000$ (first activation only)
	s_i	1 (first activation only)
	v_i	$\sim U[900, 1100]$
	π_i^B	$\sim U[0, 0.06]$
	π_i^S	$\pi_i^B/2$
	h_i	$\sim t + \lceil \exp(1/250) \rceil$ days
	τ_i	$\sim h_i + \lceil \exp(1/250) \rceil$ days

parameters are set once and for all at time $t = 0$. Traders' initial endowments c_i and s_i are set at the first activation (and then inherited). The other traders' individual parameters are reset at each time traders are activated. While active traders have initially the same endowment of cash and stock, after replacing exiting agents they may later have different endowments. Concerning the risk premium to sell, since bid prices must be smaller than ask prices we must have $\pi_i^B < \pi_i^S$ and we set $\pi_i^S = \pi_i^B/2$ for all $i = 1, \ldots, N$ to reduce the number of parameters. The risk premium to buy π_i^B is in the range from 0 to 3 times the rate of interest, in order to span a large set of attitudes to risk. Finally, whenever needed the fundamental value v is uniformly sampled around $1,000$ with an offset of $\pm 10\%$.

2.2 Market architectures

In this section we describe three market architectures to compare how the price time series generated by the previously described agents are affected by institutional features. The three market protocols we take into account are thus a batch auction (BA), a continuous double auction (CDA) and a dealer market or dealership (DEA) whose descriptions are given below. Some features are common to all trading protocols. Trading is organized in sessions (*days*), in which all active agents submit either one bid or one ask in random order. It is then handy to speak of *years* of 250 trading days.

For concreteness, limit prices are discretized taking the ceiling (floor) of $\alpha(t)$ ($\beta(t)$), the minimum tick is $\Delta = 1$ and all unfulfilled orders are canceled at the end of the trading day. Hence, each day is beginning with no outstanding orders and if an agent was not able to trade, he must resubmit his/her limit orders.

2.2.1 Batch auction

In a Batch Auction agents can submit limit orders (Q_i, P_i), where Q_i is the desired quantity and P_i is the maximum (minimum) price the agent is willing to pay (receive) for buying (selling) one unit of stock. Once all agents with adequate endowments have submitted limit buy and sell orders, a unique clearing price is selected in order to maximize the total quantity of exchanged units. In detail, define the demand and supply at price p as

$$D(p) = \sum_{Q_i > 0, P_i \geq p} Q_i \quad \text{and} \quad S(p) = - \sum_{Q_i < 0, P_i \leq p} Q_i.$$

The clearing price p_t, if it exists, has to be chosen in the set

$$\mathcal{P} = \arg\max_p \min\{D(p), S(p)\}.$$

If $\min\{D(p), S(p)\} = 0, \forall p \in \mathcal{P}$, then no trading takes place, as the biggest bid is smaller than the smallest ask. We then set for convenience $p_t = p_{t-1}$ and there is no exchange of cash and assets.

If on the other hand, $\min\{D(p), S(p)\} > 0$ for some p, we must select one element in $\mathcal{P}$. A common solution in this case is the κ-auction that sets $p_t = \kappa \inf \mathcal{P} + (1 - \kappa) \sup \mathcal{P}$. Observe that prices being quoted discretely, we use a slightly modified version and compute

$$p_t = \lfloor \kappa \min \mathcal{P} + (1 - \kappa) \max \mathcal{P} \rfloor,$$

where $\lfloor x \rfloor$ is the biggest integer not larger than x. Orders of agents that have submitted adequate bids (with $P_i \geq p_t$) and asks (with $P_i \leq p_t$) are then satisfied, possibly with rationing, and cash and stock endowments are accordingly updated.

2.2.2 Continuous double auction

The second market model is a CDA. The agent can submit limit sell or buy orders that are stored in two different books if they cannot find an immediate execution. When an agent issues a buy order, the book containing yet unmatched asks is scanned. If the best ask in the book is smaller than the agent's bid the execution price is the ask in the book and the exchanged quantity is the smallest between the two orders. This procedure is then repeated until the incoming order is fully executed or no compatible ask is found. In this case, the unmatched part of the incoming order is stored for future use in the bid book, with the usual price-time-quantity priority. The same, *mutatis mutandis*, happens when a limit sell order is submitted to the market.

This procedure mimics the following behavior by an agent willing to issue an order at a certain price: (s)he first checks the appropriate book for a compatible price, exchanges units at this price if possible and then submits a limit order for the quantity that has not been exchanged, if needed. While all units are traded for the same price in a BA, in a CDA different prices can be observed in a single trading day (and even in the execution of the same order).

2.2.3 Dealer market

The third market model we consider is a *dealership market* (DEA). Any agent willing to trade must do it through the intermediation of a dealer that quotes the price at which it will buy and sell. In this market the dealer supplies all the liquidity. The agent willing to buy (sell) at price P_i checks for the ask (bid) provided by the dealer. If this is the case, all the units are exchanged at the dealer's quote; else the agent's order remains unfulfilled. After a successful trade, the dealer adjusts immediately his bid and ask prices mechanically increasing (decreasing) both quotes by one tick. This is a rough device to keep the inventory under control, encouraging other traders to take deals on the other side. The bid-ask spread Γ offered by the dealer is then constant and fixed at the beginning of the simulation experiment. The bid-ask spread[5] is the unique distinctive parameter of the dealer and is clearly related to the liquidity of the market. Some reflection shows that the dealer is continuously extracting some cash from the market as a round trip trade produces a gain of $\Gamma - 1$, at the expense of the agents. Hence, this is the only market where the total amount of cash available to traders is (very slowly but) steadily decreasing with time. Moreover, observe that our dealer is completely mechanical and has no strategic behavior. As in the other markets, the role of the dealer could be accomplished by a program as no intelligence or expertise is needed to implement his duties. The quotes available at the closure of one trading day are kept and used to open the following day.

3 Simulation results

Our simulations are not meant to replicate the price dynamics observed in real markets,[6] but to compare the performance of three different mechanisms given the same flow of orders. For all the three architectures a trading session is thought, for convenience, as one day. We collect price dynamics statistics replicating 25 times each market for 2,500 trading days. To avoid transient effects, due for example to the initial dealer's quotes, we discard systematically the first 500 prices to compute returns. Therefore, we are left with summary statistics for 75 time series of 2,000 observations each, corresponding to 8 years of trading data.

We take into account the multidimensional nature of market execution quality collecting data about a set of features possibly occurring with different market architectures. Aiming to discuss price stability and related indicators, see the Introduction, we focused on volatility (standard deviation), excess

[5] We have investigated markets with multiple dealers in a previous version of this paper. The main effect of the presence of many dealers is the reduction of the effective bid-ask spread. However, as this effect can be proxied by decreasing Γ with a unique dealer and the results are qualitatively the same, we describe the simplest situation here.

[6] A more realistic simulation study should take into account, say, the opening auction that could affect the volume and/or the price dynamics. A variety of other institutional devices (like circuit breakers, block trading…) are often implemented in actual markets but not modelled here.

Table 2 Averaged statistics (25 simulations) for each market, based on 2,000 daily returns, standard deviation in brackets. The columns show volatility, kurtosis, tail exponent α, volume, perceived gain and bid-ask spread in the second half of the trading day

Vol	EKurt	TExp	Volume	PercG	Spread
Batch					
3.00%	1.00	−7.62	5.51	174.34	–
(0.063%)	(0.39)	(1.96)	(0.07)	(2.87)	–
CDA					
0.53%	39.36	−2.60	8.19	372.38	3.49
(0.054%)	(26.94)	(0.54)	(0.12)	(6.35)	(0.08)
Dealership					
0.24%	0.067	–	5.28	376.43	[4.00]
(0.005%)	(0.10)	–	(0.08)	(6.86)	–

kurtosis and tail exponent of daily logarithmic returns. When multiple prices are observed in one trading sessions (CDA and DEA), we use as reference price the average the over the whole day. We also report volume of trades, a measure of perceived gain by the agents and the average bid-ask spread prevailing in the second half of the trading day.

In order to get a more meaningful comparison of the various markets, the simulations are run using the same random seed across different protocols. This means that the very *same* random sequence of buy/sell order is used by the agents submitting orders to *different* market mechanisms.

The material of this section is presented in two subsections. The first one is meant to be a brief and compact account of the main results, with emphasis on the overall diversity of price dynamics in different market frameworks. This subsection can be regarded as a sort of executive summary that enhances the most evident facts, postponing a more thorough discussion to the second subsection that contains a detailed analysis of each statistical indicator.

3.1 Executive summary

The summary statistics are reported in Table 2 where the averages over 25 passes (of 2,000 trading days) are shown for each market, with estimated standard deviations in brackets. The columns contain the volatility, excess kurtosis and tail exponent of returns, average volume, perceived gain and bid-ask spread of quotes.

A more detailed description of the statistics is following:

Volatility It is measured as the standard deviation of the returns.

Excess kurtosis It describes the peakedness and fatness of tails of a distribution with respect to the normal one (that has null excess kurtosis). It is computed as $E[(R-\mu_R)^4]/\sigma_R^4-3$, where R is the random return and μ_R, σ_R denote mean and standard deviation.

Tail exponent Assuming the cumulative distribution function of returns is $F(x) \sim 1 - x^{\alpha}$, the exponent provides a widely used measure of the speed of tail decay.

Volume The cumulated number of transactions occurred in every trading day is collected.

Perceived gain For each trading day, it is the cumulated excess gain, computed as the quantity traded times the absolute difference of the limit price order P_j submitted by an agent and the price p_j actually paid/received in the transaction. We have

$$PG_t = \sum_{j \in \mathcal{E}_t} q_j |P_j - p_j|,$$

where PG_t is the perceived gain of day t, the sum is over the set of transactions $\mathcal{E}_t$ of day t and q_j denotes the actual number of traded stocks. Observe that p_j might vary in a single day and even in the execution of a single order in the case of the CDA. The greater PG, the higher is the perception by traders to have made good bargains.

Bid-ask spread The difference between the best ask and the best bid is a very common measure of liquidity in the book-based or quote-driven markets. Clearly, the best ask and bid are simply the dealer's quotes in a dealership. In a CDA, some caution is needed as the spread might be very large at the beginning of a trading day, due to the temporary lack of orders recorded in the book. Hence, we compute, for each day, the average bid-ask spread in the *second* half of the trading session, when the books contain many orders.

The comparison of the three markets (BA, CDA and dealership) shows the following main features:

1. The batch auction produces very volatile returns coupled with low kurtosis and fast tail decay (i.e. extreme returns are rare). The volume ranks in between the CDA and the dealership and the agents perceive a low gain from BA markets;
2. The CDA market has low volatility (one order of magnitude less than the BA) but frequent extreme events. Moreover, very slow tail decay makes it a risky environment. The market generates the biggest amount of trades and is liquid due to the low bid-ask spread. Agents perceive high gain when trading (statistically equal to the one perceived in a dealership);
3. The dealership return dynamics is the least volatile, with insignificant kurtosis (no extreme events) and exponentially fast tail decay. The volume of trades is low but the overall perceived gain is high (it is the biggest on a per-share basis). The bid-ask spread Γ was fixed at four to be somehow close to that of a CDA.

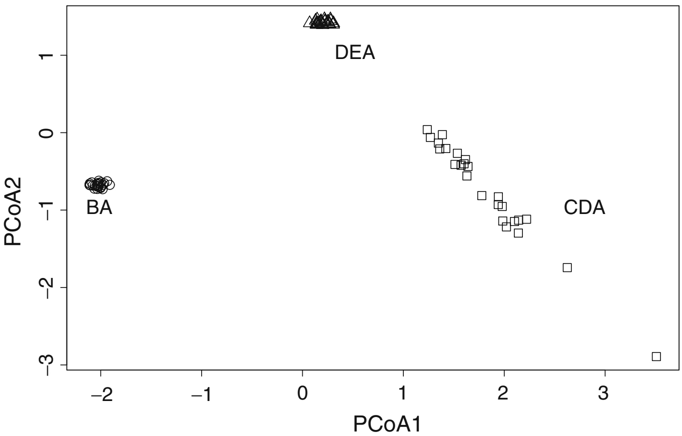

Fig. 1 Multidimensional scaling of simulation data. The planar distance of points in the graph is approximately the same as in the full four-dim space of statistics: BA (*circles*), CDA (*squares*), dealership (*triangles*)

To stress the fact that different market architectures produce markedly dissimilar price dynamics, we process our data (75 vectors in $\mathbf{R}^4$, omitting the tail exponent and spread that are not always defined) in order to reduce their dimensionality to ease graphical comparison. Multidimensional scaling is a well-known technique used to preserve as much as possible the distances between the original datapoints in a lower dimensional subspace, see Cox and Cox (2000) for a full account. Figure 1 shows a planar representation of our simulations where distinct clusters corresponds to the three markets under scrutiny. We used principal coordinate analysis (PCoA), as described in Edwards and Oman (2003). This graph, where the different markets can be clearly distinguished, reinforces the issue that institutional architectures really matter in shaping the multidimensional nature of execution quality and price stability. It also suggests that most (but not all) realizations of a CDA might produce price dynamics closer to a dealership rather than to a batch market. A very similar pictorial result can be obtained by a standard principal component analysis as well.

3.2 A detailed analysis

We give in this section a more complete description of our simulation results.[7]

We show, in Fig. 2, representative time series of the trading price in one simulation for each of the three mechanisms. The scales are the same to ease direct comparison.

A glance at the price time series shows that the CDA returns are leptokurtic. The small number of isolated shocks in the BA explains well the extremely thin

[7] The exemplar Pascal code for the Dealership case is available at `http://www.dma.unive.it/~paolop/papers/DeaAvailable.p`

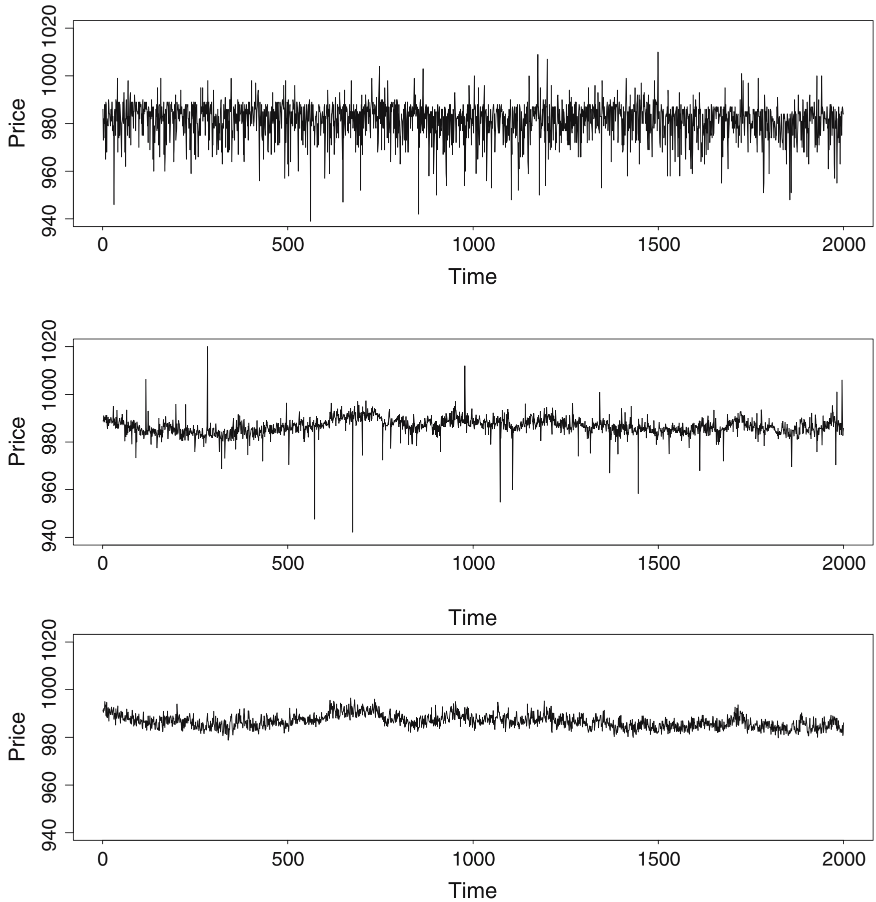

Fig. 2 Daily price time series of batch auction (*first panel*), continuous double auction (*second panel*), dealer market (*third panel*). For the CDA and the DEA, that allow multiple prices in a day, daily averages are shown

tails of its returns distribution, while their complete absence in the DEA makes this protocol rather peculiar. In fact, the support of the price distribution in the dealership is a moving compact interval of length Γ that moves around the mean value. This nicely agrees with the exponentially fast decay of the tails.

Shocks on prices also affect volatility of the returns of the three protocols, as shown in a comparison in Fig. 3. The BA shows an average standard deviation of 3%, while for both the CDA and DEA market the values are much smaller, about 0.5% and 0.2%, respectively. This is evident as well from the price series presented in Fig. 2.

When comparing the excess kurtosis in the distributions of returns one observes that it is not significantly different from zero in the DEA and moderate in a BA, with average values respectively, equal to 0.067 and 1.00, while in the CDA the empirical excess kurtosis has a huge average value of 39.36 (see

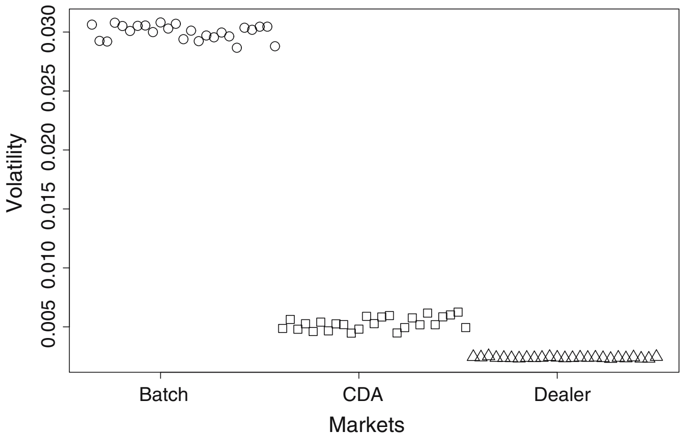

Fig. 3 Comparison of volatilities of returns for BA (*circles*), CDA (*squares*), DEA (*triangles*) markets

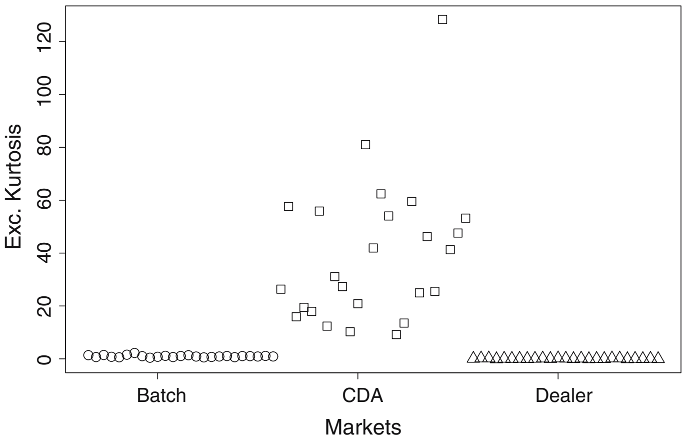

Fig. 4 Excess kurtosis comparison for BA (*circles*), CDA (*squares*), DEA (*triangles*) markets

Fig. 4). This is not surprising since the CDA typically exhibits many big outlying shocks in the price time series.

The tail exponent α is one way to quantify the relevance of extreme events. It is not estimated for the DEA whose returns have nearly compact support. The exponent of the BA distribution is about -7.62, pointing to very fast tail decay, while the exponent of the CDA is -2.63, signalling quite fat tails. Similar values for the tail exponent imply that the theoretical excess kurtosis is infinite, thus explaining huge values for the empirical estimates in the CDA.

The volume in BA and DEA are quite close, 5.51 and 5.28, respectively. The CDA average number of transactions (8.19) is notably bigger, consistently with the observation of excessive volume often found in the literature. We note that a lighter volume in the DEA might in part be justified by the reduction of wealth cashed by the dealer for every round trip transaction. Moreover, to avoid counting twice the same traded unit in a DEA (from one agent to the dealer and then from the dealer to another agent), we compute the volume halving the sum of units bought and sold by the specialist.

In real world, book-based exchanges and dealerships are the most commonly used markets and several authors tried to understand why continuous trading is so widespread. A measure of perceived gain might help to understand this aspect. In fact, from the traders' point of view different trading protocols and, consequently, different available prices, might lead to a subjectively different perception of gain. As we can see in Fig. 5, CDA and DEA have a comparable perceived gain, which is more than twice the value in a BA. Hence, our results might hint at one reason why CDA and DEA markets are preferred.

Table 3 reports the ratio of the perceived gain and the volume of transactions. The dealership strikingly emerges in this evaluation with a per-transaction value much bigger than that of the CDA and more than twice as big as that of the BA. This implies that, between the order-book and the dealership market, the latter might be perceived as a better environment for trading.

In the order-book market, the bid-ask spread (in the second part of the day) is smaller than that observed in case of dealer market. In fact, the CDA shows an average spread of about 3.50, to be contrasted with the constant spread ($\Gamma = 4$) that was fixed in the dealership market.

Finally, in stable markets as ours, high volume means high liquidity and hence, low volatility. The matter is different in real markets where explosive volumes

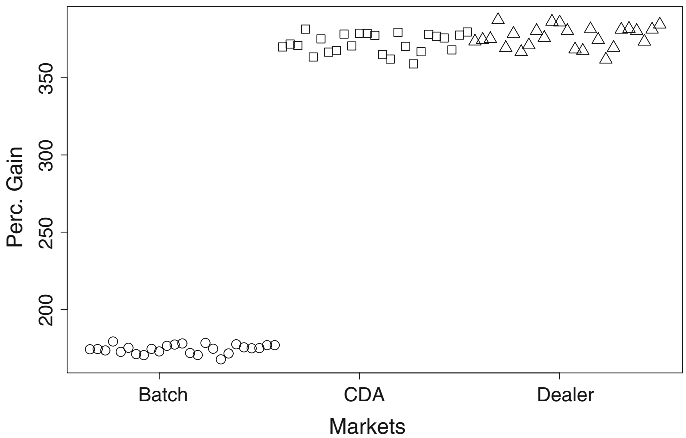

Fig. 5 Comparison of the perceived gain in BA (*circles*), CDA (*squares*), DEA (*triangles*) markets

Table 3 Ratio between the perceived gain and the volume for the three protocols

	PercG/vol
BA	31.64
CDA	45.47
DEA	71.29

are coupled with new information arrival, high uncertainty, strong price movements and high volatility. In summary, the relation we find is reasonable in a calm and stable situation and this does not contrast with the usual claim that high volume is coming with volatility spikes that are appropriate for nervous markets.

4 Conclusion

We have studied three market protocols with respect to “price stability”, studying some relevant indicators like volatility, excess kurtosis and tail exponent of returns, volume, perceived gain and bid-ask spread. Computational experiments are run in a clean agent-based environment that controls for unwanted allocative or informational effects. The markets rank differently depending on the specific performance measure, thus confirming the multidimensional nature of market execution quality. The BA generates the smallest volume but the biggest dispersion in returns, the CDA is a liquid market with rather low standard deviation but has huge excess kurtosis. If one agrees that a regulatory agency might pursue stable, non-volatile, liquid markets with little excess volume and infrequent large shocks then standard deviation, kurtosis, bid-ask spread and tail exponent of price returns should be low. Volume is a more controversial measure, though there might be the general sentiment of too abundant transactions in financial markets. In the framework just sketched, the dealership protocol compares rather favorably to other architectures for various reasons. The volatility is low, virtually no excess kurtosis is present, ultra-fast tail decay is observed and the volume is lower than in a BA (but is clearly dependent on the choice of the spread Γ). As an added bonus, agents might perceive high gains from trading in this market, both in absolute terms and on a per-transaction basis. We may argue that a dealership accommodates the total centralization of a BA with the freedom allowed in a CDA, providing a sort of mediation of two extremes that result in good overall performance. However, this is coming with some cost unique to this market. The dealership functioning is indeed slightly reducing the wealth of the agents as the dealer is cashing a spread on each round-trip transaction, even in the absence of any other transaction fee (that might be present in other mechanisms as well). Summing up, the mild rationality offered by a dealership allows to avoid severe pathologies in price dynamics and, in our view, has few drawbacks and reasonable cost.

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