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REGULAR ARTICLE

# The construction of choice: a computational voting model

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**Abstract** Social choice models usually assume that choice occurs among exogenously given and non-decomposable alternatives. On the contrary, choice is often among objects that are constructed by individuals or institutions as complex bundles made up of many interdependent components. In this paper we present a model of object construction in majority voting and show that, in general, by appropriately changing these bundles, different social outcomes may be obtained, depending upon initial conditions and agenda; that intransitive cycles and median voter dominance may be made to appear or disappear; and that decidability may be ensured by increasing manipulability or viceversa.

**Keywords** Social choice · Object construction power · Agenda power · Intransitive cycles · Median voter theorem

**JEL Classification** D71 · D72

## 1 Introduction

The starting point for every argument on individual and social choice is that agents choose among exogenously given and uni-dimensional objects according to their

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preferences. In this paper, we focus on both the "exogenously given" and the "unidimensional" hypotheses.

In particular, we study the case in which "objects", far from being unstructured points in an abstract choice space, are composed of different parts, traits and features that can be instantiated and combined with one another. In this perspective, "objects" can be conceived of as largely under-determined labels that stand for specific compositions of the underlying set of features and dimensions they are composed of. At the same time, we may ask: where do alternatives come from? In answering this question, we try to model situations in which alternatives are endogenously constructed by a social actor that has an "alternatives generation power" which is fulfilled by structuring and instantiating features sets of objects.

Three points are at stake here and they define the subject of this paper. First, as one's preferences might vary as long as the same object receives different instantiations, the power of defining an object by concretely coupling and instantiating its set of features might have a significant relevance with respect to driving and constraining individual choice. Second, there is room for interesting trade-offs to emerge as long as non separabilities (interdependencies) and non-monotonicities exist between different features of the same object. Third, there is an extent to which object construction can lead to specific social outcomes through the selection and categorization of appropriate sets of traits/features.

Broadly speaking, our results are about choice taking place within an institutionally framed scenario which, at a minimum, constructs a set of alternatives. We show that the very construction process is not at all neutral with respect to individual choice and to the selection of social outcomes. In particular, we define some precise tools to investigate the relation between the possibility of aggregating individual preferences, their structure and the existence of some centralized form of power.

### 1.1 A textbook example

Let us consider a simple example of social choice where a group of friends has to decide what to do one evening.

A textbook example would normally begin by assuming a given list of alternatives such as: going to the movies, to a discotheque, to a pub, to a restaurant or having dinner at one's place, and by supposing that individuals in the group have well-defined and complete preferences over these alternatives.

However, under close scrutiny, one realizes that these alternatives are grossly underdetermined labels which stand for combinations of lower-level traits. "Going to the movies" *per se* is far too vague and most subjects would find it hard to express any preference about it, unless it is given a precise content by specifying a list of elements that compose this alternative. Possible elements are for instance: genre, director, actors, title, theater, with whom, at what time, and so on.

Going to the movies and the other alternatives are actually multi-dimensional bundles of components which are likely to possess some non standard properties. First, the set of components is not in general partitioned by the set of alternatives. For instance, the "with whom" and at "what time" elements are likely to appear in each of the above-listed objects, a "type of food" element will be part at least of the going to the restaurant object as well as of the dinner at home object, etc.

Second, non-separabilities, which are often discarded by classical models because of their disturbing analytical consequences, are likely to be the norm when choice is among interdependent bundles. For example, I might prefer Italian to all other possibilities as an instantiation of the "type of food" trait if we are staying at home and I am going to cook, but Italian food might well be one of my least preferred alternatives if we are going to a restaurant in Paris or London. I might very much want Françoise as the only member of the "with whom" trait if the outcome is a tête-a-tête dinner, but if going to the movies is chosen then I might be strongly in favour of watching Lars von Trier's latest masterpiece and in that case I would rather not invite her as she would yawn and complain all the time.

Third, because of non-separabilities and context dependence, the way elements are bundled together, i.e. the way objects are constructed (object construction power), and the order in which they are compared (agenda power) have in general an influence on both individual and social choices. This will be shown in the rest of paper.

Fourth, because of the combinatorial nature of objects, as soon as the number of traits is not minuscule, an exhaustive procedure which requires to vote on all possible combinations of traits (for instance, a pairwise competition between all conceivable couples of alternatives) cannot be completed in a reasonable time. A feasible procedure requires that only a subset of all possible alternatives should be examined: object construction serves this purpose by pre-defining templates that guide the construction of instances of these objects. Only instances of pre-defined 'legal' objects will be considered by the choice procedure. Obviously, the more these templates are finely defined, i.e. the fewer components each template is made of, the fewer alternatives are examined and the more quickly a social choice can be reached. For instance, if every component is voted upon separately, the time needed to reach a decision is linear in the number of components. If, on the contrary, all components are grouped together in one object, the time required to examine all the instances is exponential in the number of components. We will also show that exhaustive voting procedures in which all conceivable combinations of elements are compared are very likely to produce cycles à la Condorcet-Arrow and make the social outcome indeterminate. On the contrary, if social choice is based on objects made up of a small number of components, cycles become less likely and a social outcome can be achieved. However, in this case, many locally optima social outcomes appear and which one is selected depends upon the particular set of objects (object construction power) and the order in which voting takes place (agenda power).

The main issue at hand is that interesting dynamics can emerge from possible clashes between different ways of clustering sets of traits into wholes and more or less separable preferences on the part of agents. Agents, for instance, may or may not be allowed to separately express their preferences on single features as these may be separate objects of choice or may only be jointly considered under a composite category. Agents might indeed have well defined preferences on the objects of their choice once objects are given but still how objects are defined and constructed is a crucial (and very neglected) point, and since agents are compelled to choose on a given object-like categorization of sets of features, none of their single-feature preferences will be reflected in their choices as such.

Our approach here focuses on the way object construction works as an institution with respect to selecting subsets of feasible outcomes. In particular, we view an institution as essentially characterized by some power that defines which set of objects society is called to choose from. Our main focus is on the relations between objects structures and individual preference structures and our main question is about the extent to which object construction can lead to specific social outcomes through the selection and bundling of appropriate sets of features.

In what follows we develop a model of majority voting whereby a plurality of individual agents possesses heterogeneous individual orderings that have to be aggregated into collective outcomes. A well established literature shows that the aggregation of elements into a collective choice is not always straightforward. Arrow (1951) shows that there is no universal voting procedure that aggregates individual preferences into social orderings that satisfy a set of minimal conditions. McKelvey (1979) has proven that under majority rule the stake of agenda manipulation can encompass the entire range of feasible outcomes however individual preferences are defined. Far from being viewed as simple sums of components, aggregation processes do have the potential for unstable, arbitrary, intransitive and chaotic behaviour.

In our model, an institution proposes instantiations of the given objects to agents based on its set of objects. Agents vote according to their preferences and following the majority rule. This voting procedure may enter a cycle or select some locally optimal outcome, depending on the initial condition, on the sequence through which alternatives are presented, and, especially, on how components are aggregated into objects.

We show that under general and plausible (in our setting) conditions, notably if preferences on single components are not fully separable, the outcome is highly dependent upon the set of objects. We show algorithmically that, given a set of individual preferences, by appropriate modifications of the objects we can obtain either a single global optimum or multiple local optima or cycles. In the case of many local optima, by appropriately selecting the starting point and, in some cases, the agenda, any of the local optima can be obtained. We also show that cycles *à la* Condorcet-Arrow (Caritat Marquis de Condorcet 1785; Arrow 1951)[1] may also appear and disappear by appropriately modifying the objects. Finally, we also show that the median voter property (Black 1958; Downs 1957) is dependent upon object construction: by appropriately reconstructing alternatives a winning median agent may be transformed into an outright loser.

It is worth underlining that we show these results in a setting where there is a given and finite set of components and where the set of objects always covers such a set entirely. Different objects are simply different decompositions (not necessarily partitions) of such a set, and the results we obtain show that different decompositions can generate vastly different outcomes. Thus what we show is that there exists something we could call a 'focussing of attention' power, i.e. that in a world where there are potentially infinite choices to be made, a fundamental power is exerted by focussing society's attention on some issues rather than others. In our finite setting all possible issues are always decided upon.

[1] This is a well-known result for which even in the presence of transitive individual preferences, social preferences expressed through some voting rule may be intransitive and generate cycles.

### 1.2 Related literature

We believe that our model captures a neglected aspect of categorization and framing in social choice and that building alternatives based on particular categories confers—to some extent—the power to determine, influence and direct the selection of specific social outcomes. This point seems to be very consonant with some recent work by George Lakoff on the use of frames and metaphors in politics. According to Lakoff (2004):

> "Frames are mental structures that shape the way we see the world. As a result, they shape the goals we seek, the plans we make, the way we act and what counts as a good or a bad outcome of our actions. In politics our frames shape our social policies and the institutions we form to carry out policies. To change our frames is to change all of this. Reframing *is* social change".

To our knowledge this issue of object construction has not been dealt with by economic models. Relatively close to our perspective is the literature on multidimensional voting models (Denzau and Mackay 1981; Kramer 1972; Shepsle 1979). Enelow and Hinich (1983) instead consider a multi-issue case in which each issue is voted sequentially in time and where the agenda induces path-dependency, which might be mitigated by the agents' forecasting abilities.

In particular, Shepsle (1979) presents a model of majority voting in which institutions play a similar role to the one objects have in our own model, i.e. that of limiting the set of outcomes which undergo examination. Two institutional mechanisms are analyzed: jurisdictional restrictions—especially those induced by decentralization and division of labour among decision-making units—and agenda limitations in the possible changes to the current status quo. Both limit the set of attainable outcomes and equilibria (called structure-induced equilibria) and may rule out cycles. There are at least two important differences between this perspective and ours. First, the problem tackled by these papers is essentially the one arising from the sequential interdependency of voting: how we settle an issue today may change how we prefer to settle a related issue tomorrow. In our approach we focus instead on interdependencies generated by how elements interact within the particular objects we are deliberating upon. Second, in Shepsle (1979) restrictions on attainable outcomes are placed by legal and organizational rules that define which outcomes are attainable from the status quo. In our approach instead restrictions are placed by the object construction process exerted by some agent or institution: once an object has been defined all its instances are always generated and compared.

On the grounds of the latter observation, our paper is also related to the recent literature that has begun to analyze decision making when agents group states of the world into coarse categories (Fryer and Jackson 2008; Mullainathan 2000). They show, among other things, that in such circumstances agents can be persuaded, meaning that uninformative messages may influence their decisions (Mullainathan et al. 2008). Our perspective is different and complementary: our objects are not categories based on similarities among the states of the world as in these papers but are human constructs with an internal structure of interdependencies.

Context-dependent voting has also been analyzed by some papers (Callander and Wilson 2006). In these papers, context-dependency refers to the violation of the axiom of Independence of Irrelevant Alternatives (IIA), i.e. the assumption that the preferences expressed by an agent between two outcomes $x_i$ and $x_j$ does not depend on the presence or absence of other outcomes in the choice set. Psychologists and marketing scholars have observed systematic violations of IIA (Kahneman and Tversky 2000). In our model we assume a different form of context dependency, meaning that preferences between two instantiations of a trait or components in general depend upon the value taken by other traits. In the next section we argue why this form of non-separability is very likely to happen in our context of objects made up of interdependent features.

### 1.3 Structure of the paper

The paper is organized as follows. Section 2 presents our formalism, which we use in order to obtain, in Sect. 3, some 'possibility' results. We provide examples in which social outcomes depend upon object construction in that the number and location of social optima and the presence of cycles depend upon the set of objects. In Sect. 4 we show that also standard median voter results are dependent upon the pre-choice object construction activity: we provide an example in which an appropriate choice of objects and initial conditions determines an outcome opposite to the one preferred by the median voter. In Sect. 5 we discuss, by means of computer simulations, the likelihood of such phenomena in randomly generated social decision problems. In particular we show that, in general, cycles are very likely to appear in populations of agents with random preferences when they are asked to vote on all possible alternatives. The likelihood of cycles can be sharply decreased by asking agents to vote on finer rather than coarser objects. On the other hand, using finer objects increases the number of local optima and therefore the manipulability of voting. It appears that decidability can be obtained only if manipulability is also increased: cycles can be avoided and well defined social outcomes can be reached (in a reasonable time) if choices are more 'structured', but this inevitably increases the power to influence the social outcome of those who can determine such a structure. Finally, in Sect. 6 we draw some conclusions.

## 2 The model

We assume that choices are made over social outcome formed by a set of $n$ atomic traits or *components* $F = \{f_1, f_2, \ldots, f_n\}$, each taking a value out of a finite set of possibilities. In order to simplify the notation, we assume that the cardinalities of the sets of possibilities are the same for all components and equal to $\ell \geq 2$. For all components we label possibilities with integers $f_i \in \{0, 1, 2, \ldots, \ell - 1\}$ $\forall i = 1, \ldots, n$. Thus there are $\ell^n$ possible *social outcomes*: $X = \{x_1, x_2, \ldots, x_{\ell^n}\}$.

There exist $h$ individual agents $A = \{a_1, a_2, \ldots, a_h\}$, each characterized by complete individual preferences over the set of social outcomes: given any two outcomes $x_i$ and $x_j$ an agent $a_k$ can always state whether $x_i \succ_k x_j$ or $x_j \succ_k x_i$ or $x_i \approx_k x_j$. We will assume that agents have transitive preferences and therefore that they can (weakly) order social outcomes. No further assumption will be made on agents' preferences: any ordering will be allowed. In particular we will assume that non-separabilities generally characterize preferences on single components.

Individual preferences are aggregated through sincere majority voting. Given a status quo $x_i$ and an alternative $x_j$ agents sincerely vote according to their preferences. Agent $k$ votes for $x_i$ if $x_i \succ_k x_j$, votes for $x_j$ if $x_j \succ_k x_i$ and abstains if $x_i \approx_k x_j$. If $x_j$ obtains the majority of votes (abstentions do not count as votes) it becomes the new status quo, otherwise $x_i$ is kept. We make the hypothesis that this process continues until no other feasible alternative can win against the current status quo. We write $x_j \succ^{\mathfrak{R}} x_i$ if $x_j$ defeats $x_i$ according to this procedure.

Given an initial outcome, the majority voting rule, and a procedure for the generation of alternatives we obtain a social choice process which can either end up with a social optimum or cycle forever among a subset of alternatives.

A further problem may arise from the combinatorial nature of the set of alternatives: the cardinality of the set $X$ is exponential in the number $n$ of features and even for relatively small values of $n$ the number of alternatives may be so large that no realistically feasible voting process can possibly examine all of them. The alternative generation mechanisms also serve the function of narrowing down the number of alternatives to be considered, making decision possible in a feasible time scale.

In our model, a fundamental part of social decision is the pre-voting generative mechanism through which alternatives are generated as instantiations of pre-defined objects and ultimately determines which subset of alternatives undergoes examination. As we shall show, different sets of objects may generate different social outcomes because the subset of generated alternative is different (and some social optima may not belong to some of these subsets) and because the agenda is different. Object construction power appears therefore to be more general a phenomenon than agenda power.

Let $I = \{1, 2, \ldots, n\}$ be the set of indexes and let an *object* $C_i \subseteq I$ be a non-empty subset thereof, we call the *size of object* $C_i$, its cardinality $|C_i|$. We define an *objects-scheme* as a set of objects:

$$C = \{C_1, C_2, \ldots, C_k\} \quad \text{such that} \bigcup_{i=1}^{k} C_i = I$$

Note that an objects-scheme does not necessarily have to be a partition as components may belong to more than one object.

We define the *size of an objects-scheme* as the size of its largest object:

$$|C| = \max\{|C_1|, |C_2|, \ldots |C_k|\}$$

Given an outcome $x_i$ and an objects-scheme $C$, we call instantiation of an object $C_j \in C$, that we denote by $x_i(C_j)$, the substring of length $|C_j|$ containing the components in $x_i$ belonging to object $C_j$:

$$x_i(C_j) = f^i_{j_1} f^i_{j_2} \ldots f^i_{j_{|C_j|}} \quad \text{for all } j_h \in C_j$$

We also use the notation $x_i(C_{-j})$ to indicate the part of the social outcome of length $n - |C_j|$ containing the components of $x_i$ not belonging to object $C_j$.

Two object instantiations can be united by means of the non commutative $\vee$ operator which produces the union of the two instantiations with the first instantiation's components where the two intersect:

$$x(C_j) \vee y(C_h) = z(C_j \cup C_h) \quad \text{where } z_\nu = \begin{cases} x_\nu & \text{if } \nu \in C_j \\ y_\nu & \text{otherwise} \end{cases}$$

We can therefore write $x_i = x_i(C_j) \vee x_i(C_{-j})$ for any $C_j$.

An *agenda* $\alpha = C_{\alpha_1} C_{\alpha_2} \ldots C_{\alpha_k}$ over the objects scheme $C$ is a permutation of the set of objects which states the order in which the objects are examined.

We suppose that if an initial social outcome is (randomly) given[2] then the first object of the agenda is considered and all object instantiations are generated. At every step agents vote the status quo against a new outcome in which the components of the object under consideration are replaced by new object instantiations, whereas all other objects are kept unchanged in their initial values. The outcome obtaining the majority becomes the (new) status quo.

When all instantiations have been examined for the first object in the agenda, the same procedure is repeated for the second, third, ..., $k$-th object in the agenda. As to the stopping rule we can consider two possibilities:

1. objects which have already been settled cannot be re-examined;
2. objects which have already been settled can be re-examined if new social improvements have become possible.

Though less realistic, we will use the latter stopping rule, as the former does not generally lead to optimal outcomes, i.e. outcomes that cannot be further improved by different instantiations of some objects in scheme $C$. Thus the agenda is repeated over and over again until an optimum or a cycle is encountered.

More precisely, we will use the following algorithmic implementation of majority voting:

1. repeat for all initial conditions $x = x_1$ to $x_{\ell^n}$
2. repeat for all objects $C_{\alpha_i} = C_{\alpha_1}$ to $C_{\alpha_k}$ until a cycle or a local optimum is found
3. repeat for $j = 1$ to $\ell^{|C_{\alpha_i}|}$
   - generate an object instantiation $C^j_{\alpha_i}$ of object $C_{\alpha_i}$
   - vote between $x$ and $x' = C^j_{\alpha_i} \vee x(C_{-\alpha_i})$
   - if $x'$ receives the majority of votes (without counting abstentions) it becomes the new current outcome

Given an objects-scheme $C = \{C_1, C_2, \ldots, C_k\}$, we say that an outcome $x_i$ is a *preferred neighbour* of outcome $x_j$ with respect to an object $C_h \in C$ if the following three conditions hold:

[2] In what follows we actually find properties for all possible initial social outcomes.

1. $x_i \succ^{\Re} x_j$
2. $x_i^{\nu} = x_j^{\nu} \forall \nu \notin C_h$
3. $x_i \neq x_j$

Conditions 2 and 3 require that the two outcomes differ only in components belonging to object $C_h$. According to the definition, a neighbour can be reached from a given outcome by voting on a single object.

We call $H_i(x, C_i)$ the set of preferred neighbours of an outcome $x$ for object $C_i$.

A *path* $P(x_i, C)$ from an outcome $x_i$ and for an objects-scheme $C$ is a sequence, starting from $x_i$, of preferred neighbours:

$$P(x_i, C) = x_i, x_{i+1}, x_{i+2}, \ldots \quad \text{with}\, x_{i+m+1} \in H(x_{i+m}, C)$$

An outcome $x_j$ is *reachable* from another outcome $x_i$ and for objects-scheme $C$ if there exists a path $P(x_i, C)$ such that $x_j \in P(x_i, C)$.

A path can end up either in a social (local) optimum, i.e. an outcome which does not have any preferred neighbour, or in a cycle among a set of outcomes which are preferred neighbours to each other. The latter is the well-known case of intransitive social preferences.

The set of *best neighbours* $B_i(x, C_i) \subseteq H_i(x, C_i)$ of an outcome $x$ for object $C_i$ is the set of the socially most preferred outcomes in the set of neighbours:

$$B_i(x, C_i) = \{y \in H_i(x, C_i) \,\text{such that}\, y \succ^{\Re} z \forall z \in H_i(x, C_i)\}$$

By extension from a single object to the entire objects-scheme, we can give the following definition of the set of neighbours for an objects-scheme as:

$$H(x, C) = \bigcup_{i=1}^{k} H_i(x, C_i)$$

An outcome $x$ is a *local optimum* for the objects-scheme $C$ if there does not exist an outcome $y$ such that $y \in H(x, C)$ and $y \succ^{\Re} x$.

Suppose outcome $x_j$ is a local optimum for objects-scheme $C$, we call basin of attraction of $x_j$ for objects-scheme $C$ the set of all outcomes from which $x_j$ is reachable:

$$\Psi(x_j, C) = \{y, \text{ such that}\, \exists P(y, C)\, \text{with}\, x_j \in P(y, C)\}$$

A *cycle* is a set $X^0 = \{x_1^0, x_2^0, \ldots, x_j^0\}$ of outcomes such that $x_1^0 \succ^{\Re} x_2^0 \succ^{\Re} \cdots \succ^{\Re} x_j^0 \succ^{\Re} x_1^0$ and that for all $x \in X^0$, if $x$ has a preferred neighbour $y \in H(x, C)$ then necessarily $y \in X^0$.

**Table 1** Objects and social outcomes

| Order | Agent 1 | Agent 2 | Agent 3 | Agent 4 | Agent 5 |
|---|---|---|---|---|---|
| 1st | 011 | 011 | 011 | 011 | 011 |
| 2nd | 111 | 000 | 010 | 101 | 111 |
| 3rd | 000 | 001 | 001 | 111 | 000 |
| 4th | 010 | 110 | 101 | 110 | 010 |
| 5th | 100 | 010 | 000 | 100 | 001 |
| 6th | 110 | 111 | 110 | 001 | 101 |
| 7th | 101 | 101 | 111 | 010 | 110 |
| 8th | 001 | 100 | 100 | 000 | 100 |

## 3 Objects, local optima and cycles

Having defined the basic characteristics of the paths in the set of outcomes which are generated by voting processes, we are ready to discuss their fundamental properties. Our algorithmic approach allows us to trace all the possible paths and characterize all possible outcomes for every initial condition. We elaborate on previous work: Marengo, Pasquali and Valente (2005) provide a methodology for mapping every decomposition of a finite and discrete search space into possible outcomes in the case where all objects can be re-examined endlessly until no further improvements can be made, while Page (1996) offers similar results in the case where, once decided, an object cannot be re-examined even if improvements become possible later. As already mentioned, we will discuss only the more general case in which all objects can be always re-examined until no further social improvement whatsoever becomes possible.

In this section we show that, in general, social outcomes depend upon the adopted objects scheme and that by appropriately modifying it one can obtain different social outcomes or even the appearance or disappearance of intransitive cycles. In this section we provide "possibility" results, i.e. we show examples of occurrences of such phenomena, and in the next section we will attempt a discussion of their generality and likelihood.

We first show that, in general, different objects-schemes can produce different social outcomes.

Consider first a very simple example in which 5 agents have a common most preferred choice. Table 1 presents their individual preferences, ranked from the most to the least preferred outcome:

It is easy to show that if voting is based upon the objects-scheme $C = \{\{f_1, f_2, f_3\}\}$ the only local optimum is the global one 011 whose basin of attraction is the entire set $X$.

If instead voting is based upon the objects-scheme $C = \{\{f_1\}, \{f_2\}, \{f_3\}\}$ we have the appearance of multiple local optima and agenda-dependence. If for instance the agenda is the sequence $\{f_1\}, \{f_2\}, \{f_3\}$ then 000 is the local optimum whose basin of attraction contains half the possible initial outcomes. For instance, if we start from 110, three out of five agents will vote for changing the first component into a 0: 010 is

**Table 2** Different objects induce different global optima

| Rank | Agent 1 | Agent 2 | Agent 3 |
|---|---|---|---|
| 1st | 011 | 010 | 000 |
| 2nd | 000 | 100 | 110 |
| 3rd | 010 | 101 | 101 |
| 4th | 110 | 011 | 011 |
| 5th | 100 | 000 | 010 |
| 6th | 101 | 110 | 100 |
| 7th | 001 | 001 | 001 |
| 8th | 111 | 111 | 111 |

**Table 3** Cycles in social preferences

| Order | Agent 1 | Agent 2 | Agent 3 |
|---|---|---|---|
| 1st | $x$ | $y$ | $z$ |
| 2nd | $y$ | $z$ | $x$ |
| 3rd | $z$ | $x$ | $y$ |

in fact the best neighbour of 110 for object $\{f_1\}$. Then object $\{f_2\}$ is considered and again the majority (3 out of 5) decide to move to 000. Then no other change can get a majority consensus. If instead the agenda is the sequence $\{f_3\}, \{f_2\}, \{f_1\}$ it is easy to check that the same initial condition 110 will lead to the global optimum 011.

Even stronger cases may be generated where different objects-schemes produce different global optima. Table 2 presents one such example.

In this three agents case it is easy to verify that with the objects scheme $C = \{\{f_1, f_2\}, \{f_3\}\}$ 000 is the (unique) global optimum, while with the different scheme $C = \{\{f_1\}, \{f_2, f_3\}\}$ 011 is instead the (unique) global optimum.

All in all, both multiplicity of social outcomes and agenda-dependence appear to be linked to the specific set of objects which voting is based upon.

Another property of social decision rules is the well-known voting paradox (Caritat Marquis de Condorcet 1785; Arrow 1951): even in the presence of transitive individual preferences, social preferences expressed through some voting rule may be cyclical and therefore social outcomes are indeterminate. In our model this property turns out to be dependent upon the specific scheme of objects through which voting takes place. By appropriately modifying objects, cycles may in fact appear or disappear, holding the set of social outcomes and agents' preferences constant. This "possibility" result may be illustrated by means of an example which is a translation in our formalism of the standard textbook case. Consider the case of three agents and three objects with individual preferences expressed by Table 3.

It is easy to verify that with these individual preferences, social preferences expressed through majority rule are intransitive and cycle among the three objects: $x \succ^{\mathfrak{R}} y$ and $y \succ^{\mathfrak{R}} z$, but $z \succ^{\mathfrak{R}} x$.

Suppose now that $x, y, z$ are three-components objects which we encode according to the following mapping:

$$x \mapsto 000, \quad y \mapsto 100, \quad z \mapsto 010$$

**Table 4** Objects and intransitivity I

| Order | Agent 1 | Agent 2 | Agent 3 |
|---|---|---|---|
| 1st | 000 | 100 | 010 |
| 2nd | 100 | 010 | 000 |
| 3th | 010 | 000 | 100 |
| 4th | 110 | 110 | 110 |
| 5th | 001 | 001 | 001 |
| 6th | 101 | 101 | 101 |
| 7th | 011 | 011 | 011 |
| 8th | 111 | 111 | 111 |

**Table 5** Objects and intransitivity II

| Order | Agent 1 | Agent 2 | Agent 3 |
|---|---|---|---|
| 1st | 100 | 010 | 001 |
| 2nd | 010 | 001 | 100 |
| 3th | 001 | 100 | 010 |
| 4th | 000 | 000 | 000 |
| 5th | 110 | 110 | 110 |
| 6th | 101 | 101 | 101 |
| 7th | 011 | 011 | 011 |
| 8th | 111 | 111 | 111 |

All other combinations of components are dominated by $x$, $y$ and $z$ for all agents and we suppose, for simplicity, that preferences among them are identical across agents. All in all, individual preferences are given in Table 4.

It is easy to verify that if voting is based upon the unique object $C = \{\{f_1, f_2, f_3\}\}$ the voting process always ends up in the cycle among $x$, $y$ and $z$. The same happens if each component is a separate object: $C_a = \{\{f_1\}, \{f_2\}, \{f_3\}\}$.

However, if schemes $C_b = \{\{f_1\}, \{f_2, f_3\}\}$ or $C_d = \{\{f_1, f_3\}, \{f_2\}\}$ are employed, voting always produces the unique global social optimum 010 in both cases. The latter outcome is the most preferred one by agent 3, who can therefore try to have one of these schemes adopted. All other objects-schemes always determine cycles: the social outcomes 000 and 100 which are the ones most preferred by, respectively, agents 1 and 2 cannot be obtained as social optima by any set of objects with this encoding. They could however be obtained with a different encoding.

Consider for instance the following encoding for $x$, $y$, $z$:

$$x \mapsto 100, \quad y \mapsto 010, \quad z \mapsto 001$$

and individual preferences of Table 5.

Once again we obtain cycles when voting is based upon the unique object $C = \{\{f_1, f_2, f_3\}\}$, if instead each component is voted as a separate object: $C = \{\{f_1\}, \{f_2\}, \{f_3\}\}$ we have three local optima: 100, 010, 001 whose basins of attraction depend, both in size and location, upon the agenda. With the objects-scheme

$C = \{\{f_1\}, \{f_2, f_3\}\}$ we have only the two local optima 100 and 010, while $C = \{\{f_1, f_3\}, \{f_2\}\}$ produces the two local optima 010 and 001 and $C = \{\{f_1, f_3\}, \{f_2\}\}$ produces the two local optima 100 and 001.

In Sect. 5 we will show through simulations for populations of agents with random preferences, that these examples can be generalized: voting based upon large objects is very likely to produce cycles, while voting based upon finer objects is unlikely to produce cycles but on the other end is typically characterized by many local optima and path-dependency.

Let us recall also that if $|C|$ is the size of the objects-scheme, the number of pairwise votes needed to find an optimum or a cycle is proportional to $\ell^{|C|}$. Thus small size objects render decidability more likely not only in the sense that cycles are less likely, but also in the sense that a choice may be made in a reasonable time. However, decidability may be obtained only by increasing manipulability because smaller size objects highly increase the number of locally optimal outcomes.

## 4 Objects and the median voter theorem

A relatively trivial consequence of the framework outlined so far is that also the median voter theorem is weakened in a more general setting in which objects can be modified by aggregating or disaggregating basic components. Let us briefly recall that the median voter theorem (Black 1958; Downs 1957) in its stronger version says that if there exists a median voter, his or her most preferred outcome will always beat any other alternative in any pairwise majority vote. Although a median voter might not exist if pairwise voting does not converge to a unique stable outcome but produces a cycle, Duncan Black showed that a sufficient condition for ruling out cycles is that individual preferences are single peaked (Black 1948, 1958).

By applying the framework developed so far we can easily design examples in which we do not have cycles and the median voter's most preferred policy does indeed win a pairwise majority contest for some objects-schemes but not for others, where, on the contrary, he or she might lose on all the objects[3].

Let us provide a simple example in which this happens. Let us suppose that some overall policy can be implemented with 8 possible levels of strength ranked from 0 (the null level) to 7 (the strongest implementation level). There are seven voters, each of whom preferring a different level, with the exception of level 0,[4] which nobody prefers. For all voters the remaining levels are ranked according to their distance from the most preferred one and in case of equal distance, the higher level is preferred to the lower. Individual preferences are summarized in Table 6.

Agent 4 is the median voter, every agent has single peaked preferences and therefore level 4 is the unique social outcome of pairwise voting.

[3] Carrillo and Castanheira (2008) also find a case in which the median voter theorem does not hold, that is when information about the quality of candidates' platforms is not perfect.

[4] We omit an agent preferring level 0 in order to have an odd number of agents and a well-defined median voter.

**Table 6** Median voter theorem, an example: part I

| Order | Agent 1 | Agent 2 | Agent 3 | Agent 4 | Agent 5 | Agent 6 | Agent 7 |
|---|---|---|---|---|---|---|---|
| 1st | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2nd | 2 | 3 | 4 | 5 | 6 | 7 | 6 |
| 3rd | 0 | 1 | 2 | 3 | 4 | 5 | 5 |
| 4th | 3 | 4 | 5 | 6 | 7 | 4 | 4 |
| 5th | 4 | 0 | 1 | 2 | 3 | 3 | 3 |
| 6th | 5 | 5 | 6 | 7 | 2 | 2 | 2 |
| 7th | 6 | 6 | 0 | 1 | 1 | 1 | 1 |
| 8th | 7 | 7 | 7 | 0 | 0 | 0 | 0 |

**Table 7** Median voter theorem, an example: part II

| Order | Agent 1 | Agent 2 | Agent 3 | Agent 4 | Agent 5 | Agent 6 | Agent 7 |
|---|---|---|---|---|---|---|---|
| 1st | 001 | 010 | 011 | 100 | 101 | 110 | 111 |
| 2nd | 010 | 011 | 100 | 101 | 110 | 111 | 110 |
| 3rd | 000 | 001 | 010 | 011 | 100 | 101 | 101 |
| 4th | 011 | 100 | 101 | 110 | 111 | 100 | 100 |
| 5th | 100 | 000 | 001 | 010 | 011 | 011 | 011 |
| 6th | 101 | 101 | 110 | 111 | 010 | 010 | 010 |
| 7th | 110 | 110 | 000 | 001 | 001 | 001 | 001 |
| 8th | 111 | 111 | 111 | 000 | 000 | 000 | 000 |

However, let us now suppose that policy levels are codified by 3 digits binary numbers (Table 7). If voting is based upon the largest object $C = \{f_1, f_2, f_3\}$ the unique social optimum 100, corresponding to level 4, is again always achieved. However if each component is voted as a separate object, i.e. $C = \{\{f_1\}, \{f_2\}, \{f_3\}\}$ we have two local optima: one that corresponds to the median voter's most preferred policy, i.e. 100 and the other that is exactly the opposite of the median voter's most preferred combination of components, i.e. 011. No cycles appear. Thus, with an appropriate combination of objects-schemes and initial conditions, the median voter's inexorable "democratic dictatorship" can be overturned and the median voter transformed into an outright loser of majority vote, also in the absence of any cycle.

Notice that if the number of components increases we can obtain once again an increasing number of local optima. For instance, if we build an analogous binary encoding example with 8 components, 256 possible social outcomes and 255 agents, we obtain a unique social optimum 10000000, corresponding to the median voter's most preferred outcome, if voting is based upon the objects-scheme $C = \{\{f_1, f_2, \ldots, f_8\}\}$; two opposite local optima 10000000 and 01111111 if the two objects $\{f_1, f_2, f_3, f_4\}$ and $\{f_5, f_6, f_7, f_8\}$ are used; and two additional specular local optima, 01111011 and 10000100, if every component is voted separately.

**Table 8** Objects, local optima and cycles ($n = 8$, no. agents $= 99$, 1,000 repetitions)

| Agenda | No. of cases with optima | Average no. of social optima | No. of cases with cycles | Average cycle length |
|---|---|---|---|---|
| $\alpha_1$ | 47 | 1 (0) | 953 | 39.61 (13.88) |
| $\alpha_2$ | 940 | 3.93 (1.45) | 1,000[a] | 4.67 (1.38) |
| $\alpha_4$ | 1,000 | 9.19 (2.33) | 1,000[b] | 4.03 (1.09) |
| $\alpha_8$ | 1,000 | 15.66 (3.05) | 318[b] | 3.11 (0.48) |

[a] Some cases present cycles for some initial conditions and local optima for others
[b] All cases present cycles for some initial conditions and local optima for others; Standard deviations in brackets

## 5 Objects and outcomes with random agents

In the previous sections we have shown that by manipulation of objects we can modify the number and location of social optima and also act upon the possibility that cycles emerge and the median voter dominates.

An interesting and related issue is to try and measure how likely or plausible such phenomena are, that is to ask questions like, e.g.: (a) how many local optima are we likely to encounter? (b) how different and/or distant from each other are such local optima? (c) how does the number and location of local optima change with a modification of objects? (d) how likely are cycles?

Such questions could be addressed either empirically by means for instance of laboratory experiments or theoretically. In this paper we limit ourselves to a preliminary investigation of the latter by means of computer simulations. We simulate in fact the above described voting model for populations of randomly generated agents, i.e. agents whose order relation over the elements of the set $X$ is totally random but always derived from transitive preferences.

In the first benchmark simulation we consider a set of 8 binary components and therefore a space of 256 outcomes, on which a population of 99 random agents vote following the majority rule. All the results we present here and below—unless otherwise specified–are averages over 1,000 repetitions of a simulation all with the same parameters but a different randomly generated population.

We have tested the following agendas:

- $\alpha_1 = \{1, 2, 3, 4, 5, 6, 7, 8\}$
- $\alpha_2 = \{1, 2, 3, 4\}, \{5, 6, 7, 8\}$
- $\alpha_4 = \{1, 2\}, \{3, 4\}, \{5, 6\}, \{7, 8\}$
- $\alpha_8 = \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}, \{7\}, \{8\}$

Table 8 presents a summary of results. The table shows that for agenda $\alpha_1$, that is a single object containing all the components, we almost always have intransitive cycles and that these cycles are rather long (almost 40 different social outcomes on average). Only in about 5% of the randomly generated populations do we obtain a social optimum, which is obviously always achieved by voting based on $\alpha_1$. All in all, intransitive social cycles are the rule in all but a small number of cases.

If instead we take the other extreme, i.e. agenda $\alpha_8$ based on the set of finest objects, in 682 out of 1,000 populations we do not observe cycles, but voting ends in a local optimum. On average there are 15.66 local optima[5] (with SD 3.05). In the remaining 318 cases we observe that voting can end up either on a local optimum or in a cycle, depending upon the initial condition. In particular, in those cases in which we observe cycles, the latter are the outcome in—on average—42.83 (with a large SD of 32.58) out of the 256 possible starting conditions. When they appear, cycles are short, consisting on average in about 3 outcomes. Thus, cycles are not very frequent, but on the other hand, we have a considerable number of local optima, whose selection depends upon the initial condition.

With agenda $\alpha_4$ we always (all 1,000 repetitions) observe the coexistence of cycles and local optima in the same social decision problem, depending upon the initial condition. On average, out of the 256 initial conditions, 128.85 (SD 28.26) lead to a cycle and the remaining to a local optimum. In the latter event, the average number of local optima is 9.19.

Finally, with agenda $\alpha_2$ we observe 60 repetitions in which we observe only cycles for all 256 initial conditions, whereas in the 940 remaining cases cycles appear on average for 206.53 (SD 28.61) initial conditions. The other initial conditions lead to one out of about 4 local optima. Also in this case cycles tend to be short, as they are made up of on average 4.67 outcomes.

To summarize, we observe a very clear trade-off between the presence of cycles and the number of local optima. When large objects are employed, cycles are very likely to occur. The likelihood rapidly drops when increasingly fine objects are employed, but at the same time the number of local optima increases. This implies that a social outcome is determined (and as already mentioned can be reached in a shorter time) but which specific social outcome strongly depends upon the specific objects-scheme employed, the agenda and the initial condition, i.e. the social outcome becomes easily manipulable by an authority with object construction power.

We have also checked whether local optima tend to concentrate in particular parts of the space, that is if, for a single repetition of the simulation, local optima are somehow similar, in the sense that they display at least for some components the same value. All tests reject this hypothesis: the distribution of local optima in the outcome space appears indistinguishable from a randomly generated one.

If we decrease the number of agents we do not observe any difference for the case of one object agenda $\alpha_1$, while for finer objects we observe a slow increase in the number of local optima and a decrease in the frequency of cycles. For instance, with 9 agents and the eight finest objects ($\alpha_8$), the number of local optima increases on average to 16.89 and cycles appear in 284 repetitions, and in those cases on average only 34 initial conditions lead to a cycle. With only three agents the average number of local optima is 20.01 (SD 3.15) and cycles appear in 176 out of 1,000 repetitions, and in the latter only for 30.52 out of 256 initial conditions. A smaller number of agents seems therefore to reduce the likelihood of cycles.

[5] We have also carried out some simulations with 10 and 12 components, where the number of local optima for the finest objects is around 40 and around 150 respectively. The number of local optima rapidly increases with the number of components.

**Table 9** Objects, local optima and cycles ($n = 4$, no. agents $= 99$, 1,000 repetitions)

| Agenda | No. of cases with optima | Average no. of social optima | No. of cases with cycles | Average cycle length |
|---|---|---|---|---|
| $\alpha_1$ | 369 | 1 (0) | 631 | 5.02 (1.78) |
| $\alpha_2$ | 932 | 1.64 (0.69) | 702[a] | 3.87 (1.41) |
| $\alpha_4$ | 988 | 9.19 (2.33) | 75[a] | 3.23 (0.79) |

[a] Some cases present cycles for some initial conditions and local optima for others. Standard deviations in brackets

Finally we can test what happens if we decrease the number of components. Table 9 presents the results of analogous simulations with 99 agents on a "simpler" decision problem with only four components and the three agendas.

- $\alpha_1 = \{\{1, 2, 3, 4\}\}$
- $\alpha_2 = \{\{1, 2\}, \{3, 4\}\}$
- $\alpha_4 = \{\{1\}, \{2\}, \{3\}, \{4\}\}$

Results are in line with those of the previous table. Of course we observe a considerable decrease in the number of local optima and length of cycles due to the vast decrease of the size of the combinatorial search space. We also observe an overall decrease in the occurrence of cycles for all sets of objects.

## 6 Conclusions

Economic theory tends to reduce any act of decision to an act of choice among given alternatives. However, often alternatives are not given exogenously but are themselves the outcome of economic, social and political processes of construction. Consumer choice is among products that are designed by firms; political choice is among candidates and parties that spend enormous energy and resources in building bundles of policies to attract voters, and in committees, boards, councils and the like, those who chair them may strategically frame and "package" choices in such a way as to obtain more favourable outcomes.

One could say that one of the fundamental roles of economic, social and political institutions is precisely the role of constructing the alternatives among which choice is to be made, not only to provide the rules by which choice is to be made. Abba Lerner wrote that "Economics has gained the title of queen of the social sciences by choosing solved political problems as its domain" [(Lerner 1972), p. 259]. Pre-choice object construction is, we argue, one of the sources of political power that economics has overlooked in the quest for a pure theory of decision making as choice. Even when consumers, citizens and members are left totally free to choose among the given alternatives, the authorities possessing the prerogative of building and "giving" such alternatives may strongly influence the outcome of choice.

In this paper we proposed a very simple model which we consider as a first step towards a rigorous analysis of this problem. We made some strong simplifying assumptions, we considered only the simple case of sincere majority voting, and we were able to provide only examples and counter examples but not general theorems. However we believe that the simplicity and the constructiveness of our model can help clarify how the process of alternative generation may strongly influence the outcome of the process of choice. Our simple model allowed us to show that by giving more "structure" to choice, i.e. by constructing smaller objects, institutions may avoid the indeterminacy inherent in many social choice problem, however by doing so they increase their own power to influence the outcome. We also showed that some standard results in social choice theory may be themselves subject to manipulability through alternative construction.

Of course more general and rigorous models are needed to extend and generalize our first results

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