J Econ Interact Coord (2009) 4:213–220
DOI 10.1007/s11403-009-0055-9

REGULAR ARTICLE

Microscopic spin model for the stock market with attractor bubbling on scale-free networks

Andrzej Krawiecki

Received: 11 September 2008 / Accepted: 27 May 2009 / Published online: 10 June 2009

Abstract A multi-agent spin model for changes of prices in the stock market based on the Ising-like cellular automaton with interactions between traders randomly varying in time is investigated by means of Monte Carlo simulations. The structure of interactions has topology of scale-free networks with degree distributions obeying a power scaling law with various scaling exponents. The scale-free networks are obtained as growing networks where new nodes (agents) are linked to the existing ones according to a preferential attachment rule with an initial attractiveness ascribed to each node. In certain ranges of parameters, depending on the exponent in the degree distribution, the time series of the logarithmic price returns exhibit intermittent bursting typical of volatility clustering, and the tails of the distributions of returns obey a power scaling law with exponents comparable to those obtained from the empirical data. The distributions of returns show also dependence on the number of agents, in particular in the case of networks with the scaling exponents of the degree distributions typical of the social and communications networks.

Keywords Econophysics · Multi-agent models of financial markets · Complex networks

PACS 89.65.Gh · 89.75.Hc · 05.45.-a

1 Introduction

It is well known that stock price fluctuations, measured by logarithmic price returns $G_{\Delta t}(t)$ over time Δt, defined as $G_{\Delta t}(t) = \ln S(t) - \ln S(t - \Delta t)$, where $S(t)$ is the

A. Krawiecki (✉)
Faculty of Physics, Warsaw University of Technology, Koszykowa 75, 00-662 Warsaw, Poland
e-mail: akraw@if.pw.edu.pl

stock price at time t, exhibit interesting statistical properties (Mantegna and Stanley 1999; Bouchaud and Potters 1999; Levy et al. 2000; Voit 2003; Liu et al. 1999; Gopikrishnan et al. 1999; Miranda and Riera 2001; Stanley et al. 2001; Gabaix et al. 2003). For example, the empirical distributions of returns show power-law tails for a wide range of Δt, $\rho(G_{\Delta t}) \propto G_{\Delta t}^{\alpha-1}$; as a result, also cumulative distributions $F(z) \equiv \Pr(G_{\Delta t} > z)$ exhibit power-law scaling $F(z) \propto z^{\alpha}$ with $\alpha < -2$, outside the stable Lévy regime $\alpha \in [-2, 0]$ (Liu et al. 1999; Gopikrishnan et al. 1999; Miranda and Riera 2001; Stanley et al. 2001; Gabaix et al. 2003). The time series $G_{\Delta t}(t)$ have intermittent character, with bursts of high volatility $v(t)$ (defined here for simplicity as $v(t) = |G_{\Delta t=1}(t)|$) separated by quiescent phases (volatility clustering). Many such "stylized facts" about the stock price fluctuations can be reproduced in microscopic multi-agent simulations using various models (Lux and Marchesi 1999; Sznajd-Weron and Sznajd 2000; Cremer 1997; Chowdhury and Stauffer 1999; Iori 1999; Kaizoji 2000; Bornholdt 2001; Sornette and Zhou 2006; Krawiecki et al. 2002; Krawiecki 2005; Yang et al. 2006; Duarte Queirós et al. 2007; Bartolozzi and Thomas 2004).

In particular, in Krawiecki et al. (2002), Krawiecki (2005), Yang et al. (2006), Duarte Queirós et al. (2007) and Bartolozzi and Thomas (2004), a model for stock price fluctuations was proposed, based on a two-state Ising-like cellular automaton with heat bath dynamics and interactions between agents randomly varying in time, where the price returns are proportional to the magnetization. So far, with the exception of Bartolozzi and Thomas (2004) and Bartolozzi et al. (2005), Monte Carlo (MC) simulations of this model have been mostly performed under the assumption of global (all-to-all) interactions between agents, so that the mean-field approximation was exact in the thermodynamic limit. In the latter approximation the model is reduced to a generic model in which the magnetization exhibits attractor bubbling, a predecessor of on-off intermittency in the presence of additive noise (Ashwin et al. 1994; Venkataramani et al. 1996). However, it is well known that many real networks of social interactions are complex networks, e.g., scale-free (SF) networks in which the distributions of the degrees of nodes $p(k)$ (the distributions of the number of edges k linked to each node) obey a power scaling law $p(k) \propto k^{-\gamma}, \gamma > 2$ (Albert and Barabási 2002). Thus, in order to take into account the real structure of interactions between agents, in Bartolozzi et al. (2005) results of MC simulations were presented of the above-mentioned model on a Barabási–Albert network, which is a scale-free network with the scaling exponent $\gamma = 3$ (with nodes of the network corresponding to agents' locations and edges to randomly varying in time interactions between them); besides, such features of real social networks as a high clustering coefficient (high probability that a node linked to a given node is also linked to its neighbours) and indecision of agents (modelled using a three-state Ising cellular automaton) were taken into account. In this paper MC simulations of the model are performed on scale-free networks with various scaling exponents $\gamma < 2$. It is shown that for a range of the parameters of the model the time series of the price returns show volatility clustering and their distributions exhibit power-law tails with the scaling exponent α dependent on γ as well as on the number of agents in the network N. In particular, for increasing γ and N such "stylized facts" occur even in the case of weak time-dependent interactions between agents.

2 The model

The microscopic model for the market from Krawiecki et al. (2002), Krawiecki (2005), Yang et al. (2006), and Duarte Queirós et al. (2007), modified to take into account the complex structure of the network of interactions between agents, is defined as follows. Consider $i = 1, 2, \ldots, N$ agents (spins), located in the network nodes, with orientations $\sigma_i(t) = \pm 1$, corresponding to the decision to sell (-1) or to buy $(+1)$ a share of a traded stock or commodity at discrete time steps t. The orientation of the agent i at time t depends on the local field

$$I_i(t) = \frac{1}{\langle k \rangle} \sum_{j \in \mathcal{K}_i} A_{ij}(t)\sigma_j(t-1) + h_i(t), \tag{1}$$

where the summation extends over the neighbourhood $\mathcal{K}_i$ of the node i, i.e., all nodes j connected with the node i by an edge (reflecting the social or communication network of agent i), $\langle k \rangle$ is the average number of neighbors (attached edges) per node, $A_{ij}(t)$ are time dependent interaction strengths among agents, and $h_i(t)$ is an external field reflecting the effect of environment (e.g., access to external information which can differ between agents). During the MC simulations the states of agents are updated synchronously according to the probabilistic rule analogous to that in the well-known heat bath dynamics, which reflects the human uncertainty in decision making,

$$\sigma_i(t) = \begin{cases} 1 & \text{with probability} \quad q \\ -1 & \text{with probability} \quad 1-q, \end{cases} \tag{2}$$

where $q = 1/\{1 + \exp[-2I_i(t)]\}$.

The interaction strengths and external fields change randomly in time,

$$\begin{aligned} A_{ij}(t) &= \begin{cases} A\xi(t) + a\eta_{ij}(t) & \text{if there is an edge between } i, j \\ 0 & \text{otherwise} \end{cases}, \\ h_i(t) &= h\zeta_i(t) \end{aligned} \tag{3}$$

where, for simplicity, it is assumed that $\xi(t)$, $\eta_{ij}(t)$, and $\zeta_i(t)$ are random variables with no correlation between nodes and in time, uniformly distributed in the interval $(-1; 1)$ (in particular, for any i, j the variables $\eta_{ij}(t)$ and $\eta_{ji}(t)$ are not correlated, thus, in general, $A_{ij}(t) \neq A_{ji}(t)$, which reflects the fact that the agents in the market do not perceive each other in a similar way; thus, though the links between agents are symmetric, the strength of interactions between them is not symmetric). In Eq. (3) A is a measure of the randomly varying average interaction strength between agents, a is a measure of the random fluctuations of the network of interactions, and $h \ll 1$ is a measure of the random fluctuations of the environment. The form of the local field (1) resembles that used in the social impact theory of opinion formation (Latané 1981; Weidlich 1991; Helbing 1995), but with time-dependent interactions between agents.

The price dynamics $S(t)$ is governed by a fundamental equation $dS/dt = \beta x(t) S(t)$, where the magnetization $x(t) = N^{-1} \sum_{i=1}^{N} \sigma_i(t)$ is proportional to the difference between demand and supply, and β is a proportionality constant, assumed henceforth as $\beta = 1$ without loss of generality. After discretizing time this yields the time dependence of $S(t)$ as

$$\ln S(t) = \ln S(t-1) + x(t). \tag{4}$$

Thus, the price returns are proportional to the imbalance between supply and demand, $G_{\Delta t=1} = \ln S(t) - \ln S(t-1) = \beta x(t)$.

It is known that SF networks can be constructed using the preferential attachment growing procedure (Yule 1925; Simon 1955; Price 1965; Barabási and Albert 1999). The complex networks considered in this paper are SF networks with the degree distributions obeying power scaling laws with different exponents, obtained in the following way (Dorogovtsev et al. 2002). First, a small number $m+1$ of fully connected nodes is fixed. Then, step by step, new nodes are added, and each new node is connected to existing nodes with m edges according to the following probabilistic rule: Probability of linking to a node i is $p_i = (k_i + B)/\sum_i (k_i + B)$, where k_i is the actual degree of the node i, $\sum_i k_i$ is the actual number of edges in the whole network, and $B > -m$ is a tunable parameter representing the initial attractiveness of each node. The growth process is continued until the total number of nodes N is reached, when the network structure is frozen. For large N, this preferential attachment rule results in the network with the mean degree of nodes $\langle k \rangle = 2m$ and the degree distribution $p(k) \propto k^{-\gamma(B,m)}$ for $k \gg B$, with $\gamma(B,m) = 3 + B/m$; for small k the distribution $p(k)$ deviates from the power scaling law. In particular, for $B = 0$ the original Barabási–Albert network with $\gamma = 3$ (for the whole range of k) is recovered.

3 Results

Exemplary time series $x(t)$ obtained from the MC simulations of the model with $B = 0$ ($\gamma = 3$, the Barabási–Albert network), $a = 0$ and various A are shown in Fig. 1a. As the parameter A is increased, the time series $x(t)$ bear more and more resemblance to the empirical time series of returns known from real financial markets. In particular, the magnetization exhibits intermittent bursts, consisting of short periods of high volatility separated by quiescent phases. The time series $x(t)$ are qualitatively similar to those obtained from the MC simulations of the model with global interactions between agents or from iterations of the corresponding mean-field equation with attractor bubbling (Krawiecki et al. 2002; Krawiecki 2005; Yang et al. 2006). Besides, for large enough A the the tails of the distributions of returns $\rho(x)$, $x > 0$, obey a power scaling law $\rho(x) \propto x^{\alpha-1}$, possibly with exponential cutoffs close to the maximum possible value of the magnetization, at $x \approx \mathcal{O}(1)$. The exponent α depends on A (Fig. 1b). The above-mentioned results are consistent with those reported in Bartolozzi et al. (2005).

Qualitatively similar results can be obtained for different parameters $m \geq 2$, N, $B \neq 0$ (i.e., $\gamma \neq 3$), and for $a > 0$, $h > 0$. In particular, the distributions of returns can possess power-law tails with the scaling exponent α dependent on all above-mentioned parameters (Fig. 2a–c). The distributions of returns characterized by $\alpha < -2$ (in the empirically significant range) have been obtained from the MC simulations

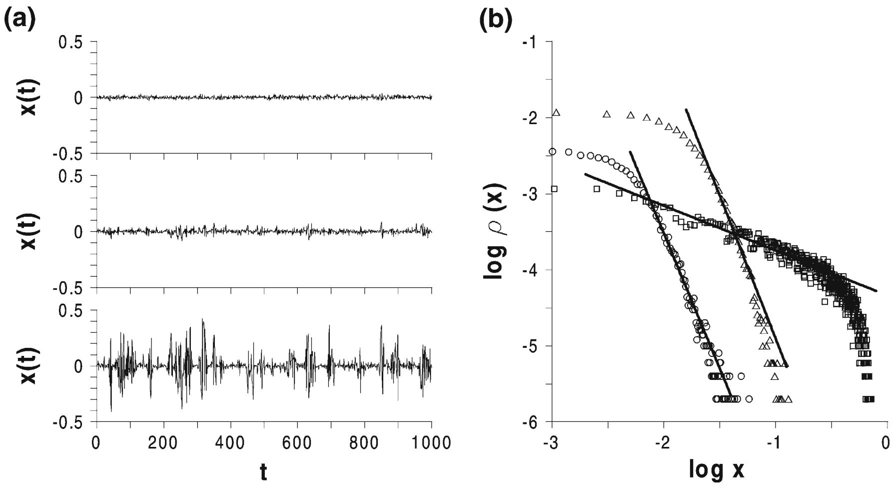

Fig. 1 **a** Time series of the logarithmic price returns $x(t)$ for the model with $a = h = 0$, $m = 10$, $B = 0$ ($\gamma = 3$, the Barabási–Albert network), $N = 10^4$, and $A = 0.5$ (*top*), $A = 0.8$ (*middle*), $A = 1.2$ (*bottom*). **b** Distributions of returns $\rho(x)$ (*symbols*) and fits to the power scaling law $\rho(x) \propto x^{\alpha-1}$ (*solid lines*) for the model with $N = 10^5$, $A = 0.6$, $\alpha = -2.55$ (*circles*), $N = 10^5$, $A = 1.6$, $\alpha = 0.46$ (*squares*), $N = 10^4$, $A = 0.7$, $\alpha = -2.63$ (*triangles*), and other parameters as in **a**. The distributions in **b** are averaged over 10 independent realizations of the network

of the model on SF newtorks with $B \geq 0$ ($\gamma \geq 3$) and with large number of agents N (Fig. 2b, c). In the case $-m < B < 0$ [i.e., for SF networks with $2 < \gamma < 3$, typical of many networks of social interactions (Albert and Barabási 2002)] even for $N = 10^5$ the intervals of the power-law scaling of $\rho(x)$ were too narrow to obtain reliable values of α in the empirically significant range $\alpha < -2$, which are thus not depicted in Fig. 2a. The effect of the external field with $h \ll 1$ in Eq. (1) on the time series and distributions of returns is negligible, and in the MC simulations of a model with finite N it is equivalent to that of thermal fluctuations. The intermittent character of the time series $x(t)$ and the power-law tails $\rho(x) \propto x^{\alpha-1}$ are observed also for $a > 0$, even in the case $a \gg A$; however, in contrast with the results from the MC simulations of the model with global interactions between agents, for a finite mean degree of nodes $\langle k \rangle = 2m$ the exponent α decreases with a (Fig. 2d).

The scaling exponent α in the power-law tails of the distributions of returns shows significant dependence on the number of nodes (interacting agents) in the network N (Fig. 2a–c): for other parameters of the model fixed, α increases with N. Such dependence of α on N has not been observed in the model with global interactions between agents. This result resembles dependence of the critical temperature for the ferromagnetic transition on the number of nodes (interacting spins) in the Ising model on SF networks (Aleksiejuk et al. 2002; Aleksiejuk 2002; Bianconi 2002; Leone et al. 2002; Iglói and Turban 2002; Herrero 2004). However, in the latter case the logarithmic or power-law dependence of the critical temperature on N occurs only in the case of the Barabási–Albert network with $\gamma = 3$ or SF networks with $2 < \gamma < 3$, respectively, and the reason for this dependence can be traced back to the presence of hubs (highly connected nodes) whose number increases with lowering the exponent γ

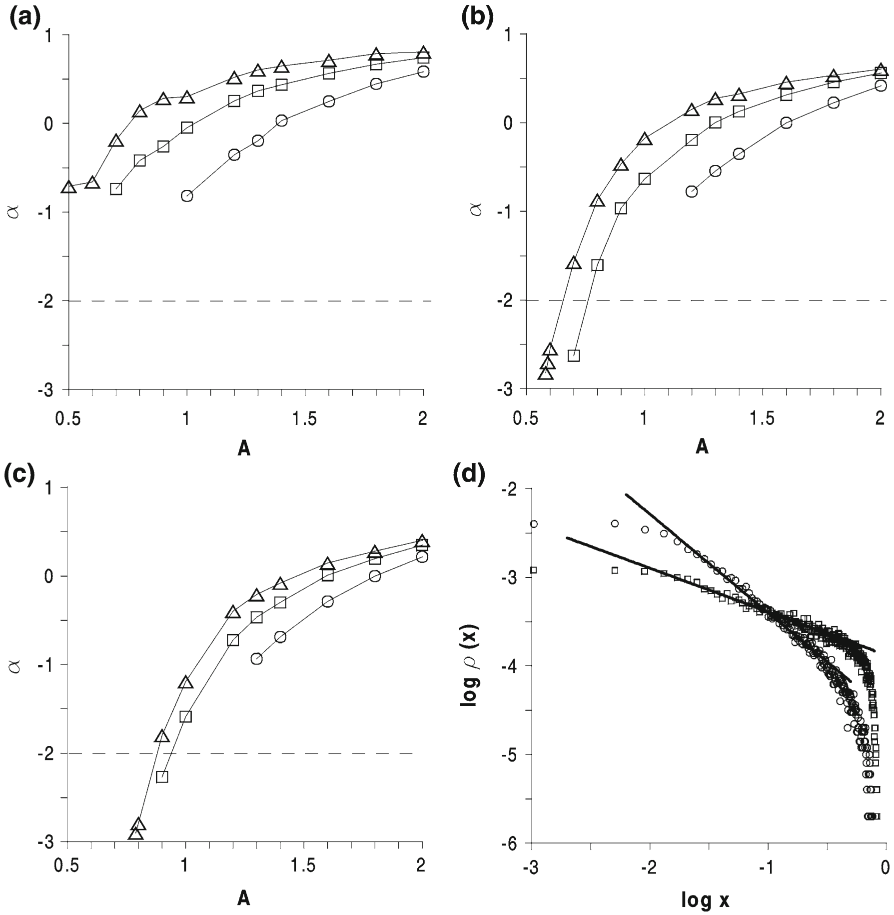

Fig. 2 **a–c** The exponents α in the power scaling law $\rho(x) \propto x^{\alpha-1}$ for the distributions of returns versus A for the model with $a = h = 0$ on SF networks with $m = 10$, **a** $B = -4$ ($\gamma = 2.6$), **b** $B = 0$ ($\gamma = 3$, the Barabási–Albert network), **c** $B = 10$, ($\gamma = 4$), and $N = 10^3$ (*circles*), $N = 10^4$ (*squares*), $N = 10^5$ (*triangles*); **d** Distributions of returns $\rho(x)$ (*symbols*) and fits to the power scaling law $\rho(x) \propto x^{\alpha-1}$ (*solid lines*) for the model with $h = 0$, $m = 10$, $B = 0$ ($\gamma = 3$, the Barabási–Albert network), $N = 10^4$, $A = 2.0$, and $a = 0.5$, $\alpha = 0.51$ (*squares*), $a = 2.0$, $\alpha = -0.11$ (*circles*). All results are averaged over 10 independent realizations of the network

(Aleksiejuk et al. 2002; Aleksiejuk 2002; Bianconi 2002; Leone et al. 2002; Iglói and Turban 2002; Herrero 2004). Here, Fig. 2c suggests that α depends on the number of interacting agents also in the model on SF networks with $\gamma > 3$, at least for the investigated range of N, though this dependence seems weaker than that for the model on SF networks with $2 < \gamma < 3$ (Fig. 2a, b).

In general, for decreasing exponent γ and increasing number of agents N the power-law tails of the distributions of returns comparable with empirical ones are observed for decreasing A, i.e., for diminishing strength of interactions between agents. For small γ and large N most of the nodes belong to large clusters containing hubs. Hence, large fluctuations in the time series of returns occur due to synchronized flips of the magnetization in the clusters of weakly interacting agents, as observed in the MC simulations of the model on a network with a hierarchy of clusters of active traders (Bartolozzi and Thomas 2004).

4 Conclusions

The model for the stock price fluctuations based on the Ising-like cellular automaton on scale-free networks with heat bath dynamics and interactions between agents randomly varying in time was studied by means of the MC simulations. For a range of parameters the time series of the logarithmic price returns (proportional to magnetization) exhibit intermittent bursting typical of volatility clustering and the tails of the distributions of returns obey a power scaling law with the exponents which can be comparable to the empirically observed ones. The latter exponents show significant dependence on the number of interacting agents (network nodes) N and the exponent γ in the distribution of the degrees of nodes. As a result, for decreasing γ and increasing N the distributions of returns which obey the power scaling law with a given exponent α are obtained for decreasing A, i.e., for diminishing strength of interactions. Hence, if the real SF structure of interactions between agents is taken into consideration the model predicts that certain “stylized facts” about the stock price fluctuations can be observed on a market with many agents even due to weak time-dependent interactions between them.

Acknowledgments This work has been supported by the European Science Foundation’s Project STOCHDYN and by Polish Ministry of Science and Higher Education (Grant ESF/ 275/ 2006).

References

Albert R, Barabási A-L (2002) Statistical mechanics of complex networks. Rev Mod Phys 74:47
Aleksiejuk A, Hołyst JA, Stauffer D (2002) Ferromagnetic phase transition in Barabási–Albert networks. Phys A 310:260
Aleksiejuk A (2002) Microscopic model for the logarithmic size effect on the Curie point in Barabási–Albert networks. Int J Modern Phys C 13:1415
Ashwin P, Buescu J, Stewart IN (1994) Bubbling of attractors and synchronisation of chaotic oscillators. Phys Lett A 193:126
Barabási A-L, Albert R (1999) Emergence of scaling in random networks. Science 286:509
Bartolozzi M, Thomas AW (2004) Stochastic cellular automata model for stock market dynamics. Phys Rev E 69:046112
Bartolozzi M, Leinweber DB, Thomas AW (2005) Stochastic opinion formation in scale-free networks. Phys Rev E 72:046113
Bianconi G (2002) Mean field solution of the Ising model on a Barabási–Albert network. Phys Lett A 303:166
Bouchaud J-P, Potters M (1999) Theory of financial risk. Cambridge University Press, Cambridge
Bornholdt S (2001) Expectation bubbles in a spin model of markets: intermittency from frustration across scales. Int J Mod Phys C 12:667
Chowdhury D, Stauffer D (1999) A generalized spin model of financial markets. Eur Phys J B 8:477
Cremer J (1997) A generalization of spin glass theory in marketing: the coordination of interdependent country markets. Phys A 246:377
de Solla Price DJ (1965) Networks of scientific papers. Science 149:510

Duarte Queirós SM, Curado EMF, Nobre FD (2007) A multi-interacting-agent model for financial markets. Phys A 374:715
Dorogovtsev SN, Goltsev AV, Mendes JFF (2002) Ising model on networks with an arbitrary distribution of connections. Phys Rev E 66:016104
Gabaix X, Gopikrishnan P, Plerou V, Stanley HE (2003) Understanding the cubic and half-cubic laws of financial fluctuations. Phys A 324:1
Gopikrishnan P, Plerou V, Nunes Amaral LA, Meyer M, Stanley HE (1999) Scaling of the distribution of fluctuations of financial market indices. Phys Rev E 60:5305
Helbing D (1995) Quantitative sociodynamics. Kluwer Academic, Dordrecht
Herrero CP (2004) Ising model in scale-free networks: a Monte Carlo simulation. Phys Rev E 69:067109
Iori G (1999) Avalanche dynamics and trading friction effects on stock market returns. Int J Modern Phys C 10:1149
Iglói F, Turban L (2002) First- and second-order phase transitions in scale-free networks. Phys Rev E 66:036140
Kaizoji T (2000) Speculative bubbles and crashes in stock markets: an interacting-agent model of speculative activity. Phys A 287:493
Krawiecki A, Hołyst JA, Helbing D (2002) Volatility clustering and scaling for financial time series due to attractor bubbling. Phys Rev Lett 89:158701
Krawiecki A (2005) Microscopic spin model for the stock market with attractor bubbling and heterogeneous agents. Int J Modern Phys C 16:549
Latané B (1981) The psychology of social impact. Am Psychol 36:343
Leone M, Vázquez A, Vespignani A, Zecchina R (2002) Ferromagnetic ordering in graphs with arbitrary degree distribution. Eur Phys J B 28:191
Levy H, Levy M, Solomon S (2000) Microscopic simulations of financial markets. Academic Press, New York
Liu Y, Gopikrishnan P, Cizeau P, Meyer M, Chung-Kang Peng, Stanley HE (1999) Statistical properties of the volatility of price fluctuations. Phys Rev E 60:1390
Lux T, Marchesi M (1999) Scaling and criticality in a stochastic multi-agent model of a financial market. Nature 397:498
Mantegna RN, Stanley HE (1999) An introduction to econophysics: correlations and complexity in finance. Cambridge University Press, Cambridge
Miranda LC, Riera R (2001) Truncated Lévy walks and an emerging market economic index. Phys A 297:509
Stanley HE, Amaral LAN, Gabaix X, Gopikrishnan P, Plerou V (2001) Similarities and differences between physics and economics. Phys A 299:1
Sznajd-Weron K, Sznajd J (2000) Opinion evolution in closed community. Int J Modern Phys C 11:1157
Sornette D, Zhou W-X (2006) Importance of positive feedbacks and overconfidence in a self-fulfilling Ising model of financial markets. Phys A 370:704
Simon HA (1955) On a class of skew distribution functions. Biometrika 42:425
Venkataramani SC, Hunt BR, Ott E, Gauthier DJ, Bienfang JC (1996) Transitions to bubbling of chaotic systems. Phys Rev Lett 77:5361
Voit J (2003) The statistical mechanics of financial markets, 2nd edn. Springer, Heidelberg
Weidlich W (1991) Physics and social science. The approach of synergetics. Phys Rep 204:1
Yang J-S, Chae S, Jung W-S, H-T Moon (2006) Microscopic spin model for the dynamics of the return distribution of the Korean stock market index. Phys A 363:377
Yule UG (1925) A mathematical theory of evolution, based on the conclusions of Dr. J. C. Willis, F.R.S. Philos Trans Royal Soc Lond Ser B 213:21 (Containing papers of a biological character)