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REGULAR ARTICLE

Smart predictors in the heterogeneous agent model

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Abstract We extend the original heterogeneous agent model by introducing the concept of smart traders. The idea of smart traders is based on the endeavor of market agents to estimate future price movements. The main result of the simulations is that the probability distribution functions of the price deviations change significantly with an increasing number of smart traders in the model. We also find that the Hurst exponent is significantly increasing with an increasing number of smart traders in the simulations. Hence the introduction of the smart traders concept into the model results in significantly higher persistence of the simulated price deviations.

Keywords Heterogeneous agent model · Market structure · Smart traders · Hurst exponent

JEL Classification C15 · D84 · G14

1 Introduction

The ability to explain stylized facts observed in financial time series, mainly fat tails and volatility clustering (see Lux and Marchesi 2000; Farmer and Joshi 2002;

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Giardina and Bouchaud 2003), is an important feature of heterogeneous agents models (HAM). Two types of agents, fundamentalists and chartists, are typically distinguished in the model. Fundamentalists base their expectations about future asset prices and their trading strategies on market fundamentals and economic factors, such as dividends, earnings, macroeconomic growth, and unemployment rates. Chartists or technical analysts try to extrapolate observed price patterns, such as trends, and exploit these patterns in their investment decisions. A large amount of literature on heterogeneous agent models has developed, e.g. Brock and Hommes (1998), Chiarella and He (2000), more recently LeBaron (2006) and Hommes (2006).

In our previous work—Vosvrda and Vacha (2002, 2003)—we introduced a concept of memory and learning into the model of Brock and Hommes (1998). We also introduced other extensions, such as stochastic formation of beliefs and parameters including memory length. Worst Out Algorithm (WOA), which periodically replaces the trading strategies with the lowest performance was introduced in Vacha and Vosvrda (2005) and Vacha and Vosvrda (2007). In Vacha and Vosvrda (2005) we also showed how the memory length distribution in the agents' performance measure affects the persistence of the simulated price time series.

This paper introduces a new concept—smart predictors. The idea of smart traders is based on the endeavor of market agents to estimate future price movements. By adding smart traders we try to improve the original heterogeneous agents model so it can better approximate real markets. Smart traders are designed to forecast the future trend parameter of price deviations using an information set consisting of past deviations. They are modeled to assume that the price deviations, defined by the model, are an AR(1) process and they use the maximum likelihood estimation method for forecasting. Thus, in our model we use two groups of traders: smart traders and a group of stochastically generated trading strategies which are, moreover, selected by the Worst Out Algorithm. Our main expectation is that the introduction of smart traders will change the simulated market prices significantly.

The structure of the paper is as follows. After the introduction, a heterogeneous agent model which is an extension of the Brock and Hommes model (Brock and Hommes 1998) is introduced. Next part briefly introduces the implementation of smart traders into the heterogeneous agent model framework. The last part of the paper investigates how the presence of smart predictors qualitatively changes the market structure.

2 Model

Presented model is described by system of interacting agents who immediately process new information and adapt their predictions by choosing from a limited number of beliefs (predictors or trading strategies). Each belief is evaluated by a performance measure and agents use it to make a rational choice which depends on the heterogeneity in agent information and subsequent decisions. The first part of this model follows Brock and Hommes (1998), the second part is our extension of the original model.

Consider an asset pricing model with one risky asset and one risk free asset. Let p_t denote the price (ex dividend) per share of the risky asset at time t (random variables at time $t+1$ are denoted in bold) and let $\{\mathbf{y}_t\}$ be the stochastic dividend process of the risky asset. The supply of the risk free asset is perfectly elastic at the gross interest rate R, which is equal to $1+r$, where r is the interest rate. Then the dynamics of the wealth are defined as:

$$\mathbf{W}_{t+1} = RW_t + (\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - Rp_t)\, z_t, \tag{1}$$

where z_t denotes the number of shares of the asset purchased at time t. Let us further consider E_t and V_t as the conditional expectation and conditional variance operators based on the set of publicly available information consisting of past prices and dividends, i.e., on the information set $\mathcal{F}_t = \{p_t, p_{t-1}, \ldots; y_t, y_{t-1}, \ldots\}$. Let $E_{h,t}$ and $V_{h,t}$ denote the beliefs (or forecast) of investor type h about the conditional expectation and conditional variance. Investors are supposed to be myopic mean-variance maximizers, so that the demand $\mathbf{z}_{h,t}$ for the risky asset is obtained by solving the following criterion:

$$\max_{z_{h,t}} \left\{ E_{h,t}\left[\mathbf{W}_{t+1}\right] - \frac{a}{2} V_{h,t}\left[\mathbf{W}_{t+1}\right] \right\}, \tag{2}$$

where the risk aversion coefficient, $a > 0$, is assumed to be the same for all traders. Thus the demand $\mathbf{z}_{h,t}$ of type h for the risky asset has the following form

$$E_{h,t}\left[\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - Rp_t\right] - a\sigma^2 \mathbf{z}_{h,t} = 0, \tag{3}$$

$$\mathbf{z}_{h,t} = \frac{E_{h,t}\left[\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - Rp_t\right]}{a\sigma^2}, \tag{4}$$

assuming that the conditional variance of excess returns is constant for all investor types

$$V_{h,t}\left(\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - Rp_t\right) = \sigma_h^2 = \sigma^2. \tag{5}$$

Let z_t^s be the supply of outside risky shares. Let $n_{h,t}$ be the fraction of investors of type h at time t and it sums up to one. The equilibrium of demand and supply is

$$\sum_{h=1}^{H} n_{h,t} \left\{ \frac{E_{h,t}\left[\mathbf{p}_{t+1} + \mathbf{y}_{t+1} - Rp_t\right]}{a\sigma^2} \right\} = z_t^s, \tag{6}$$

where H is the number of different trader types and. In the case of zero supply of outside shares , i.e., $z_t^s = 0$, the market equilibrium is

$$Rp_t = \sum_{h=1}^{H} n_{h,t} \left\{ E_{h,t}\left[\mathbf{p}_{t+1} + \mathbf{y}_{t+1}\right] \right\}. \tag{7}$$

In a market where all agents have rational expectations, the asset price is determined by economic fundamentals. The price is given by the discounted sum of future dividends

$$p_t^* = \sum_{k=1}^{\infty} \frac{E_t\left[\mathbf{y}_{t+k}\right]}{(1+r)^k}. \tag{8}$$

The fundamental price p_t^* depends upon the stochastic dividend process $\mathbf{y}_t$. In the special case where the dividend process $\{\mathbf{y}_t\}$ is an independent, identically distributed (IID) process, with constant mean $E_t\{\mathbf{y}_t\} = \bar{y}$, the fundamental price is given by

$$p^* = \sum_{k=1}^{\infty} \frac{\bar{y}}{(1+r)^k} = \frac{\bar{y}}{r}. \tag{9}$$

2.1 Heterogeneous beliefs

This part deals with traders' expectations about future prices. As in Brock and Hommes (1998), we assume beliefs about future dividends to be the same for all trader types and equal to the true conditional expectation, i.e.,

$$E_{h,t}\left[\mathbf{y}_{t+1}\right] = E_t\left[\mathbf{y}_{t+1}\right], \quad h = 1, \ldots, H, \tag{10}$$

In the case where the dividend process $\{\mathbf{y}_t\}$ is an IID process, $E_t\{\mathbf{y}_{t+1}\} = \bar{y}$, then all traders are able to derive the fundamental price p_t^* that would dominate in a perfectly rational world. Abandoning the idea of rationality and moving to the real world we allow prices to deviate from their fundamental value p_t^*. For our purposes, it is convenient to work with the deviation x_t from the benchmark fundamental price p_t^*, i.e.,

$$x_t = p_t - p_t^*. \tag{11}$$

In general, beliefs about the future price $E_{h,t}[\mathbf{p}_{t+1}]$ have the following form

$$E_{h,t}\left[\mathbf{p}_{t+1}\right] = E_t\left[p_t^*\right] + f_h\left(x_{t-1}, \ldots, x_{t-L}\right), \quad \text{for all } h, t, \tag{12}$$

where $f_h(x_{t-1}, \ldots, x_{t-L})$ represents a model of the market. A type h trader believes that the market price will deviate from its fundamental value p_t^*. The heterogeneous agent market equilibrium, defined in Eq. 7, can be reformulated in deviations from the benchmark fundamental as

$$Rx_t = \sum_{h=1}^{H} n_{h,t} E_{h,t}\left[\mathbf{x}_{t+1}\right] \equiv \sum_{h=1}^{H} n_{h,t} f_{h,t}. \tag{13}$$

2.2 Selection of strategies

Beliefs are updated evolutionarily. The selection is controlled by endogenous market forces (Brock and Hommes 1997). The fractions of trader types on the market $n_{h,t}$ are given by the multinomial logit probabilities of discrete choice

$$n_{h,t} = \exp\left(\beta U_{h,t-1}\right)/Z_t, \tag{14}$$

$$Z_t = \sum_{h=1}^{H} \exp\left(\beta U_{h,t-1}\right), \tag{15}$$

where $U_{h,t-1}$ is the fitness measure of strategy h evaluated at the beginning of period t. As the fitness measure of trading strategies we use the moving averages of realized profits, where m_h denotes the length of the moving average filter. In a real market this parameter can be interpreted as the memory length or evaluation horizon for trading strategy h. The fitness measure $U_{h,t}$ is defined as

$$U_{h,t} = \frac{1}{m_h} \sum_{l=0}^{m_h-1} \left[(x_{t-l} - Rx_{t-1-l}) \frac{\left(f_{h,t-1-l} - Rx_{t-1-l}\right)}{a\sigma^2} \right]. \tag{16}$$

3 Trading strategies

Basic framework for trading strategies in the heterogeneous agent models was investigated by Brock and Hommes (1998), who proposed simple linear rule with one lag with fixed g_h and b_h:

$$f_{h,t} = g_h x_{t-1} + b_h. \tag{17}$$

We enrich the model by introducing smart traders, who are able to forecast these linear rules and thus are able to forecast the future trend parameter $g_{h,t}$, which is variant in time. Two groups of trading strategies are distinguished. The first group consists of smart trading strategies which use simple linear regression predictions of $f_{h,t}$, and the second group comprises trading strategies which are generated stochastically and selected with the Worst Out Algorithm (WOA) during the simulations. These two groups $f^1_{h,t}$ and $f^2_{h,t}$ will be defined later.

3.1 Worst out algorithm

The idea of the WOA algorithm is to periodically replace the trading strategies with the lowest performance level. WOA makes the model more realistic and yields a more evolutionary design. Without loss of generality, this algorithm is constructed to evaluate and rank the performance of all the trading strategies from the second group after every 40th iteration in descending order. The four strategies with the lowest performance are then replaced by the newly generated strategies. The four new strategies are drawn randomly from the same distribution as the initial set. The use of the WOA in the simulations can significantly change the price time series parameters and modify the behavior of investors on the simulated market—see Vacha and Vosvrda (2005) and Vacha and Vosvrda (2007).

3.2 Smart traders

By adding smart traders we try to improve the original heterogeneous agents model so that it provides a closer description of real markets. The smart traders are agents, who are able to estimate future price movements.

The simplest way to implement this type of market behavior into the Brock and Hommes (1997) model is to use simple linear forecasting techniques. The smart trading strategies thus use maximum likelihood estimation of an AR(1) process to estimate the trend parameter $g_{h,t}$ for the next period. Smart traders thus assume that the deviations x_t follow an AR(1) process, and they base their forecasts of x_{t+1} on the information set $\mathcal{F}_t = \{x_t, x_{t-1}, \ldots, x_{t-k-1}\}$. Then the trading strategies of the smart traders are defined as follows:

$$f^1_{h,t+1} = \widehat{f}_{h,t} = \phi_1 x_{t-1}, \tag{18}$$

where ϕ_1 is the estimated trend $\widehat{g}_{h,t}$. In the simulations, we use various types of smart traders with different lengths k of the information set $\mathcal{F}_t$.

3.3 Stochastic beliefs

The trading strategies of the second group, $f^2_{h,t}$, are generated stochastically. The trend parameter g_h and the bias parameter b_h of trader type h are realizations from the normal distribution $N(0, \sigma^2)$. In this paper we use $N(0, 0.16)$ and $N(0, 0.09)$, respectively. The memory parameter m_h of the trading strategy $f^2_{h,t}$ is a realization from the uniform distribution, specifically $U(1, 100)$. The memory parameter can be interpreted as the evaluation horizon for the trading strategy h.

When $m_h = 1$ for all types h, we get the Brock and Hommes model (Brock and Hommes 1998). If $b_h = 0$ and $g_h > 0$, the investor is called a pure trend chaser. If $b_h = 0$ and $g_h < 0$, the investor is called a contrarian. Moreover, if $g_h = 0$, and $b_h > 0$ ($b_h < 0$), the investor is said to have an upward (downward) bias in his beliefs. In the special case of $g_h = b_h = 0$, the investor is fundamentalist, i.e., the investor believes that the price always returns to its fundamental value.

4 Simulation results

The main purpose of the simulations is to examine the influence of the proposed smart traders concept on the simulated market prices. In this part we describe the methodology of our simulations and summarize the main results. We compare the initial model

Table 1 Lengths of the information sets used by various types of smart traders

Number of smart traders	Length of information set k for each smart trader
1	$\{k_i\}_{i=1}^{1} = \{40\}$
2	$\{k_i\}_{i=1}^{2} = \{20, 60\}$
3	$\{k_i\}_{i=1}^{3} = \{80, 40, 5\}$
5	$\{k_i\}_{i=1}^{5} = \{80, 60, 40, 20, 5\}$
10	$\{k_i\}_{i=1}^{10} = \{100, 80, 60, 50, 40, 30, 20, 15, 10, 5\}$

Table 2 Descriptive statistics

Statistics	0 ST	1 ST	2 ST	3 St	5 ST	10 ST
Mean	0.0042	−0.0083	−0.0006	−0.0003	0.0139	0.0047
Median	0.0072	−0.0136	−0.0097	−0.0054	0.0148	0.0128
Variance	0.2161	0.2378	0.2190	0.2109	0.2285	0.2024
SD	0.4622	0.4833	0.4648	0.4563	0.4746	0.4474
Skewness	−0.0272	−0.0272	0.0488	0.0158	0.0152	−0.0741
Kurtosis	2.5416	2.8067	4.0048	4.5029	3.5775	3.2369
Min.	−2.9724	−6.4122	−6.2246	−10.108	−6.0194	−4.9048
Max.	4.3509	5.4661	10.7469	12.5352	9.5375	4.2615

without smart traders (0ST) with the models with 1, 2, 3, 5, and 10 smart traders (1ST, 2ST, 3ST, 5ST, and 10ST, respectively).

In each simulation we consider 40 trading strategies. For the model without smart traders, all the strategies are second group strategies $f_{h,t}^2$. For the simulation with one smart trader, there is one first group strategy $f_{h,t}^1$ and the remaining 39 strategies are second group strategies $f_{h,t}^2$. For the simulations with 2, 3, 5, and 10 smart traders, there are, respectively, 2, 3, 5, and 10 first group strategies $f_{h,t}^1$ and 38, 37, 35, and 30 second group strategies $f_{h,t}^2$ so they always sum to 40. Table 1 shows the lengths of the information sets, defined by k, used for trend parameter estimation by various types of smart traders.

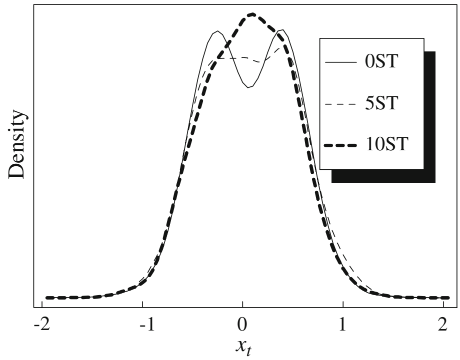
The other parameters of the simulations are fixed to: $\beta = 300$, total number of iterations $N = 15{,}000$, $a\sigma^2 = 1$, $R = 1.1$. Each of the six models has been simulated 45 times to achieve robust results. Table 2 shows the descriptive statistics of the simulated deviations x_t.

For illustration, Fig. 1 shows the kernel estimation[1] of the probability density functions (PDFs) for the following three models: the model without smart traders, the model with 5 smart traders, and the model with 10 smart traders.

The results summary begins with the descriptive statistics of x_t. We can see that the means and variances of x_t do not change with an increasing number of smart traders. The models with 2, 3, 5, and 10 smart traders produce leptokurtic distributions of x_t,

[1] We use the Epanechnikov kernel, which is of the following form: $K(u) = \frac{3}{4}(1 - u^2)\ (|u| \leq 1)$.

Fig. 1 Empirical PDF of x_t for simulated models



while the models without smart traders and with 1 smart trader produce platykurtic distributions. While the values of skewness and kurtosis are the arithmetic means of the 45 simulations, we use the Kruskal–Wallis test (Kruskal and Wallis 1952) to compare the distributions of simulated x_t. This test does not assume normal distribution of the compared sets of data, as the analogous analysis of variance does. The null hypothesis of the test is an equal population of medians against the alternative of an unequal population of medians. Thus, we compare the sets of skewness and kurtosis of all 45 simulations across the six models considered. The Kruskal–Wallis test rejects the null hypothesis of equal medians of the sets at the 5% significance level, thus the skewness and kurtosis of all six models are significantly different and we can conclude that adding smart traders to the original model significantly changes the simulated distributions of x_t. Moreover we can see that with the increasing number of smart traders in the model, distribution changes from bimodal to unimodal. This is very interesting as it points out the fact that smart agents are forcing the distribution of x_t to be unimodal which is more closer to stylized facts of the long-term real stock market returns distributions. On the other hand, more simulations need to be done to make stronger conclusion about this fact.

We continue our analysis by estimating the Hurst (1951) exponent for all the simulated models. We use only the R/S (Range-Scale) statistic, which gives comparable results for financial time series. For a detailed discussion of this topic see Lillo and Farmer (2004) and Taqqu et al. (1995).

Our expectation is that the introduction of smart traders into the simulated market will also increase the Hurst exponent significantly—see Fig. 2. We again use the Kruskal–Wallis test, which rejected the null hypothesis of equal medians of the estimated Hurst exponents at the 1% significance level. However, the Hurst exponent is increasing in non-monotonic manner. We can conclude that an increasing number of smart traders in the model significantly changes the Hurst exponent, moreover from the Fig. 2 there is an apparent growing trend. Thus an increasing number of smart traders in the model might increase the persistence of the simulated market.

Finally, we study the distribution of the trend parameter g_h of the second group trading strategies. The main aim is to find out whether an increasing number of smart traders has a significant impact on the distribution of the trend parameter on the simulated market.

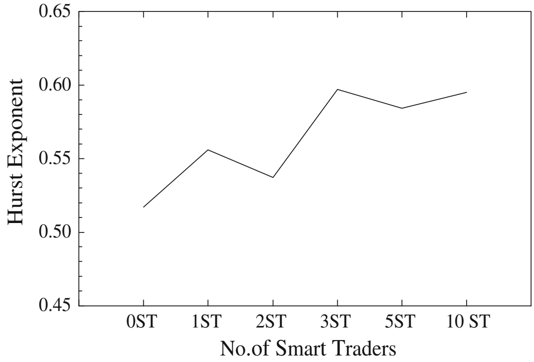


Fig. 2 Estimated hurst exponent

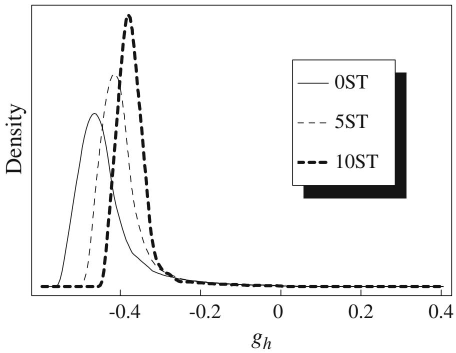


Fig. 3 Empirical PDF of the trend parameters g_h

For the comparison of the simulated sets we again use the Kruskal Wallis test, which strongly rejected the null hypothesis of equal medians of the trend parameters g_h at the 1% significance level. We can conclude that the distributions of trend parameter g_h are significantly different. From Fig. 3 it can be observed that increasing the number of smart traders in the simulation increases the kurtosis of the distribution of the trend parameter, which is confirmed by the Kruskal Wallis test. This means that with a higher number of smart traders the distributions of the trend parameter become more leptokurtic.

5 Conclusion

Introduction of smart traders extends the original heterogeneous agent model. The concept of smart traders improves the model so it can better approximate real markets.

Two groups of traders are considered in the model: smart traders and a group of stochastically generated trading strategies which are, moreover, selected by the Worst Out Algorithm. Smart traders assume that the deviations from fundamental prices are an AR(1) process and they are able to forecast the future trend of price movements using an information set consisting of past prices.

Simulations showed that the probability distribution functions of the price deviations change significantly with an increasing number of smart traders in the model. We also use the Hurst exponent, which measures the persistence of the price deviations, and we find that the Hurst exponent is significantly changing, moreover it has an upward trend with an increasing number of smart traders in the model. This means that the introduction of the smart traders concept into the model results in significantly higher persistence of the simulated price deviations.

Our last finding is that the distributions of the trend parameter from the models with smart traders also differ significantly from the model with stochastic traders only. Increasing the number of smart traders in the simulation increases the kurtosis of the distribution of the trend parameter. This means that with a higher number of smart traders the distributions of the trend parameter become more leptokurtic.

As this paper introduces a new approach for modeling heterogeneous agents, it also opens up considerable scope for further research. The most interesting aspect would be to show the impact of smart traders on the market price.

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References

Brock WA, Hommes CH (1997) A rational route to randomness. Econometrica 66:1059–1095

Brock WA, Hommes CH (1998) Heterogeneous beliefs and routes to chaos in a simple asset pricing model. J Econ Dyn Control 22:1235–1274

Chiarella C, He X (2000) Heterogeneous beliefs, risk and learning in a simple asset pricing model with a market maker. Quantitative Finance Research Group, University of Technology Sydney, Research Paper No. 35

Farmer JD, Joshi S (2002) The price dynamics of common trading strategies. J Econ Behav Organ 49:149–171

Giardina I, Bouchaud JP (2003) Bubbles, crashes and intermittency in agent based market models. Eur Phys J B 31:421–437

Hommes CH (2006) Heterogeneous agent models in economics and finance. In: Tesfatsion L, Judd KL (eds) Handbook of computational economics, agent-based computational economics, vol 2. Elsevier Science BV, Amsterdam, pp 1109–1186

Hurst HE (1951) Long-term storage capacity of reservoirs. Trans Am Soc Civil Eng 116:770–799

Kruskal WH, Wallis WA (1952) Use of ranks in one criterion variance analysis. J Am Stat Assoc 47(260):583–621

LeBaron B (2006) Agent-based Computational Finance. In: Tesfatsion L, Judd KL (eds) Handbook of computational economics, vol 2. Agent-Based Computational Economics. Elsevier Science BV, Amsterdam

Lillo F, Farmer JD (2004) The long memory of the efficient market. Stud Nonlinear Dyn Econom 8:1–33

Lux T, Marchesi M (2000) Volatility clustering in financial markets: a micro-simulation of interacting agents. Int J Theor Appl Finance 3:675–702

Taqqu MS, Teverovsky V, Willinger W (1995) Estimators for long-range dependence: an empirical study. Fractals 3:785–788

Vacha L, Vosvrda M (2005) Dynamical agents' strategies and the fractal market hypothesis. Prague Econ Pap 14:172–179

Vacha L, Vosvrda M (2007) Wavelet decomposition of the financial market. Prague Econ Pap 16:38–54

Vosvrda M, Vacha L (2002) Heterogeneous agent model and numerical analysis of learning. Bull Czech Econom Soc 17:15–22

Vosvrda M, Vacha L (2003) Heterogeneous agent model with memory and asset price behaviour. Prague Econ Pap 12:155–168