An Introduction to Logic for Knowledge Representation and Neural-Symbolic Computation

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Summary

Introduction

AI Today

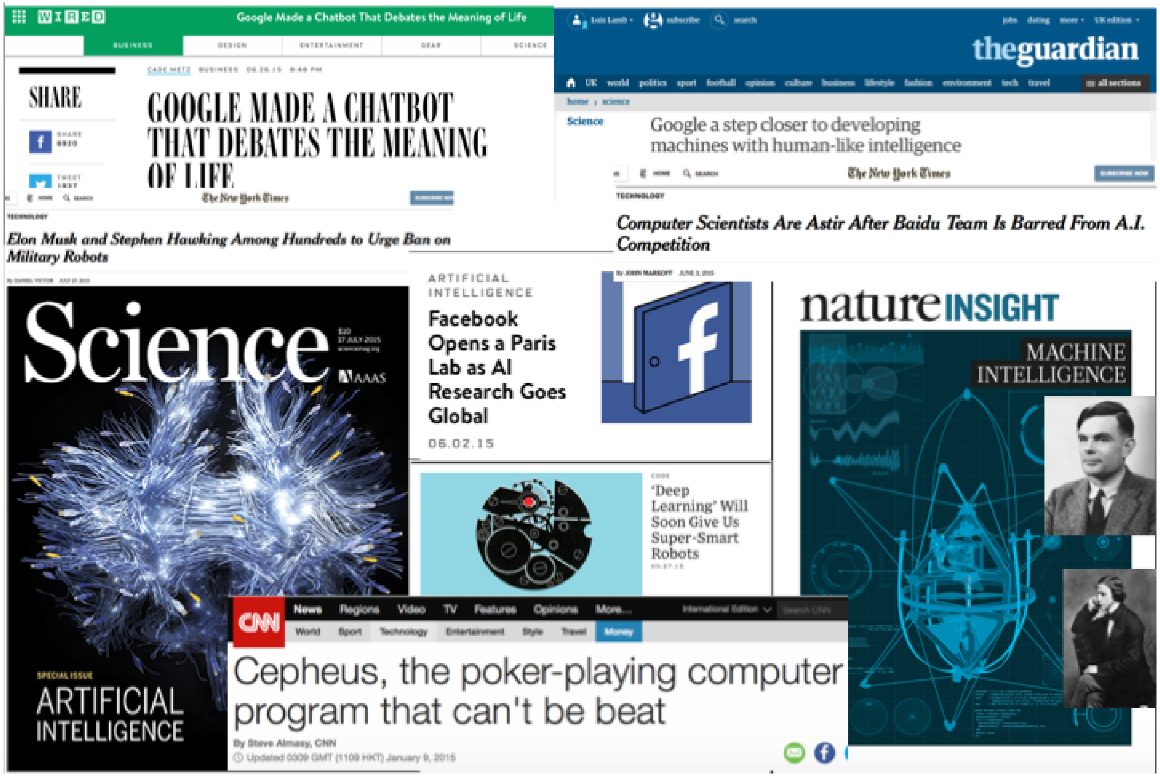


Figure 1: AI, CS Today

Introduction

AI is everywhere

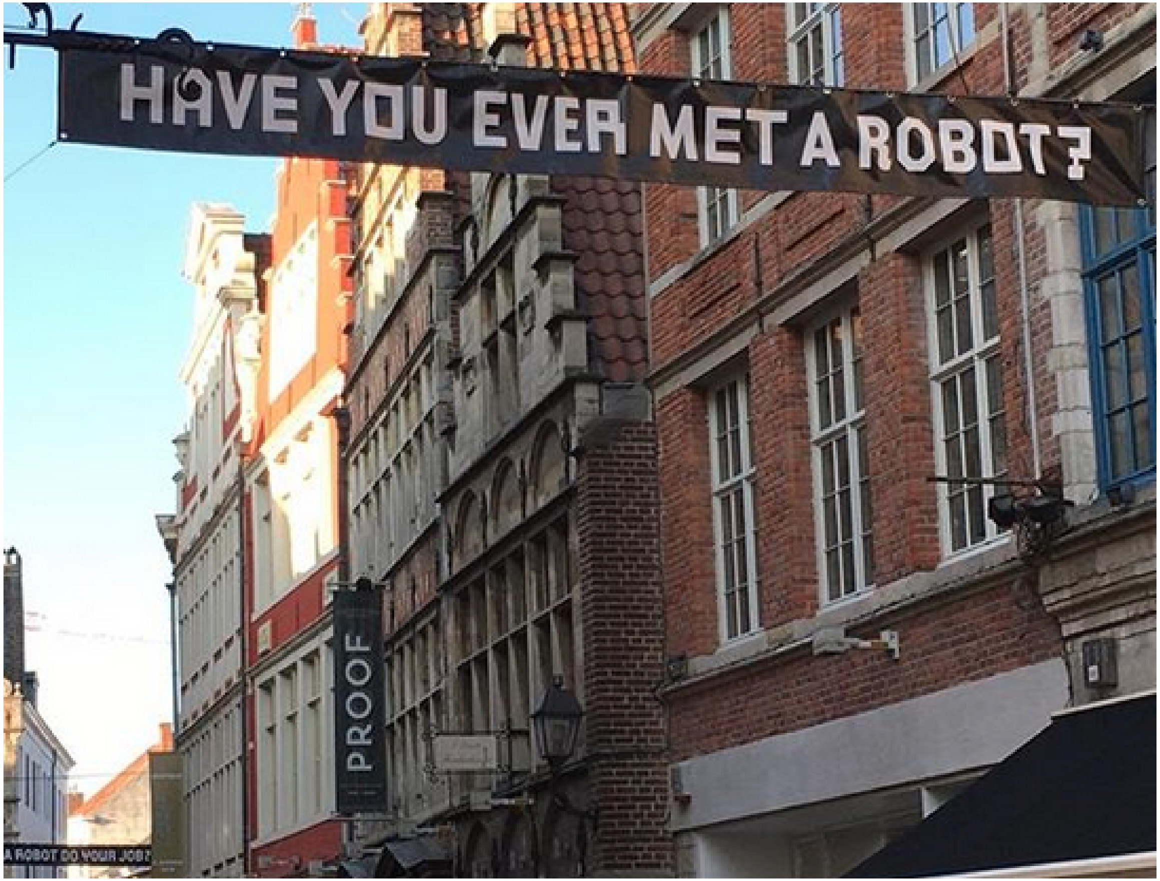


Figure 2: Ghent - Belgium

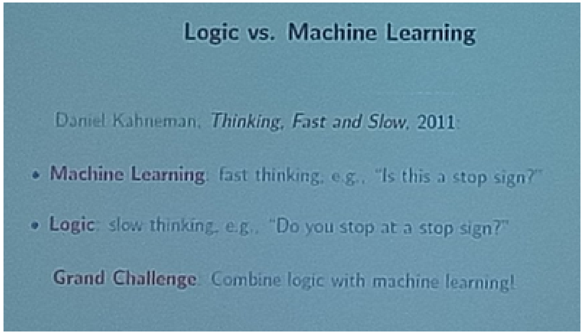
Introduction

Logic is also everywhere...

Figure 3: Logical lift?

Logic and Machine Learning

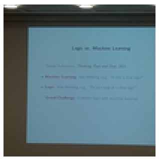
Tweeting - AI research today, 2018
@vardi



Bashar Nuseibeh @BNuseibeh · Jul 4
Thinking, fast and slow ...
[with apologies to #Kahneman, and thanks to @vardi]

Fast thinking: machine learning (e.g. "Is this a stop sign?")
Slow thinking: logic (e.g. "Do you stop at a stop sign?")

#MachineLearning #logic



Ismail Badache ★ @Ismail_badache
Logic Vs. Machine learning
By @vardi

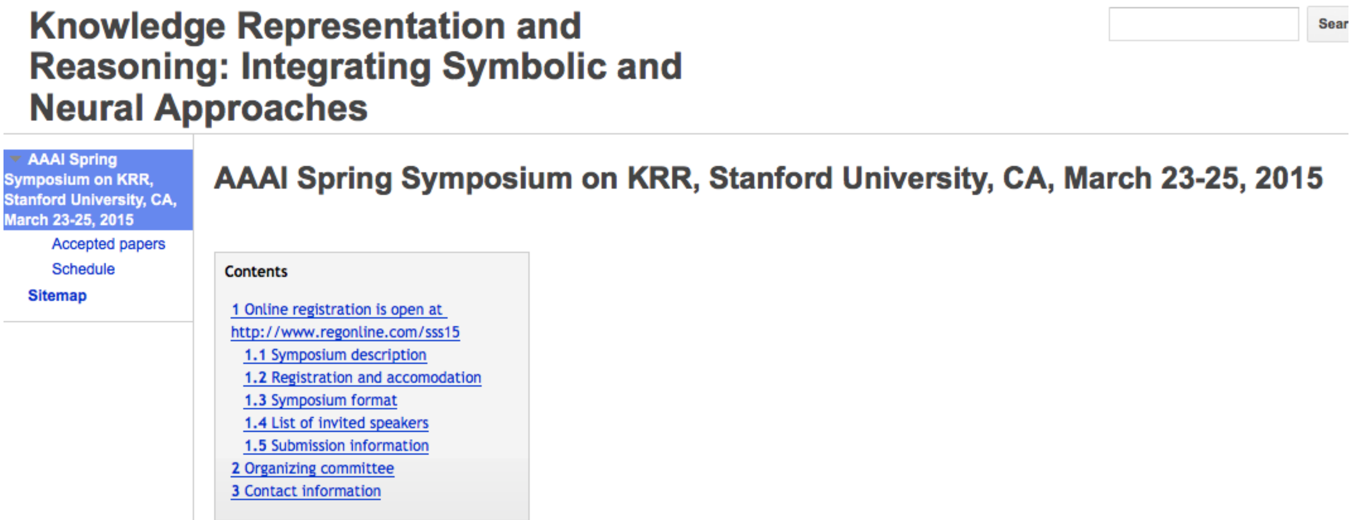
Luis Lamb @luislamb · 8h
Replying to @BNuseibeh @vardi
This is really relevant. Please note the book by @AvilaGarcez and colleagues on Neural-Symbolic Learning and Reasoning

Neural-Symbolic Cognitive Reasoning | Artur S. D'A...
Humans are often extraordinary at performing practical reasoning. There are cases where the human computer, slow as it is, is faster than any artificial intelligence sy...
springer.com

1 2 2

Logic and Machine Learning, 2015

Stanford, March 2015



Knowledge Representation and Reasoning: Integrating Symbolic and Neural Approaches

Sear

AAAI Spring Symposium on KRR, Stanford University, CA, March 23-25, 2015

Accepted papers

Schedule

Sitemap

AAAI Spring Symposium on KRR, Stanford University, CA, March 23-25, 2015

Contents

Logic and Machine Learning, 2017

Royal Society Report

BOX 5

Why are some machine learning systems 'black boxes'?

A neural network is an approach to machine learning in which small computational units are connected in a way that is inspired by connections in the brain. These systems may consist of many layers of neurons: the base layer receives an input from an external source, then each layer beyond it detects patterns in activity from the neurons in the layer beneath, integrates these inputs, and then passes a signal to the next layer[61]. In this way, signals can be passed through many layers, before reaching a top layer where a decision about the input is made. So, if the initial input is an image, the initial signals might come from the pixels of the image, and the top-level decision might be what object is in the image. As such systems learn from data, they strengthen or weaken synaptic connections to make their outputs more accurate.

This approach to processing data means that information across the neural network is highly dispersed, with complicated patterns of connection strength between units, and with potentially many thousands of layers of units. The result is the so-called 'black box' issue: these systems can create highly accurate results, but it is difficult to explain why a result has been obtained.

However, not all machine learning methods use this approach, and alternative approaches can be more readily interpreted.

This is directly related to this topic

Source: **ROYAL SOCIETY report: April 2017**
"Machine learning: the power and promise of computers that learn by example".
ISBN: 978-1-78252-259-1

Introduction

A.M Turing (thinking process/reasoning):
The whole thinking process is still rather mysterious to us, but I believe that the attempt to make a thinking machine will help us greatly in finding out how we think ourselves. Alan Turing, 15 May 1951, “Can Digital Machines Think” BBC.

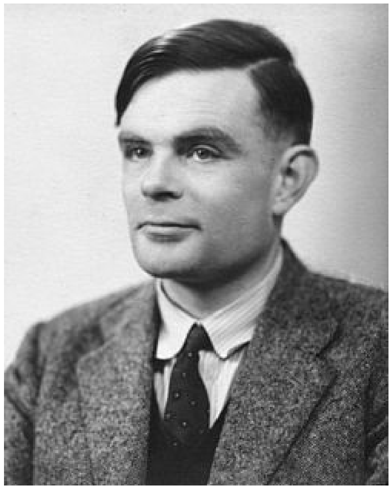


Figure 4: Alan Mathison Turing, FRS (1912-1954)

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- MacTutor History of Mathematics archive (bios, photos, history): http://www-history.mcs.st-and.ac.uk/
- References at the end of the slides.

Impacts of Logic

Lewis Carroll - Charles Lutwidge Dodgson (1832-98).
Alice's Adventures in Wonderland, 1865.
150 years old in 2015.

Figure 5: Lewis Carroll

A bit of history

- Aristotle - syllogisms.
  All men are mortal. All Greeek are men. Therefore, all Greeks are mortals.

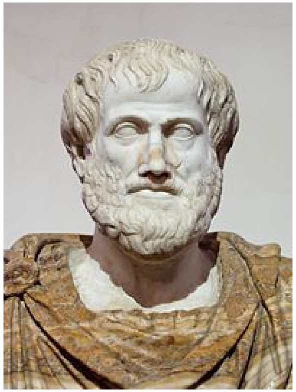


Figure 6: Aristotle 384-322 BC

- Middle ages: Syllogisms/arguments.
- Logicians:
  – Peter of Abelard (Logica ingredientibus, 1121, following de Aristotle)
  – William of Ockham (XXIII Century: Ockam's razor; "De Morgan" style reasoning; three-valued logics)
  – Ramon Lull (XXII/XIII Century: logic/religion; influenced Leibniz).

Gottfried Willhem Leibniz (1646-1716)

Idealized an universal language, *lingua characteristica universalis* to represent human knowledge.

Calculus ratiocinator: universal model of logical calculus, machine execution, algebraic relations.

"Patron saint of Computer Science": Moshe Vardi, in Comm. ACM Editorial, Dec. 2011.

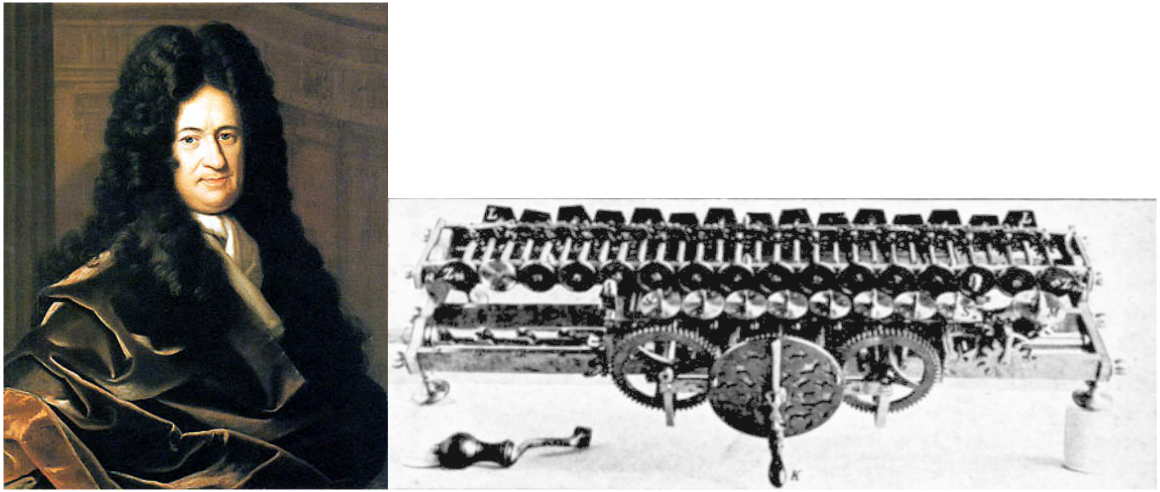


Figure 7: G. W. Leibniz, machine prototype

Images from wikipedia.org, ©public domain

G. Boole; A. De Morgan; G. Frege; C.S. Peirce

- Boole: symbolic logic *An Investigation of the Laws of Thought (1854)*.
- Frege (Begriffsschrift, 1879): First-order logic: objects, predicates, function symbols, logical operations, quantifiers
  $\forall x(Man(x) \rightarrow Mortal(x))$
- De Morgan: laws; relation logic; mathematical induction.
- Peirce: precedes Shannon; deduction, induction, abduction, relational logic (DBs).

Figure 8: XIX Century Logicians: Boole, De Morgan, Frege, Peirce

Whitehead/Russell: *Principia Mathematica* (1910-1913)

- Monumental mathematical logic book: 2000 pgs, 3 volumes.
- Attempt/aimed at deriving all mathematical truths from axioms and logical inference rules.
- Russell: inspired by Frege; identifies paradoxes.
- Introduced type theory

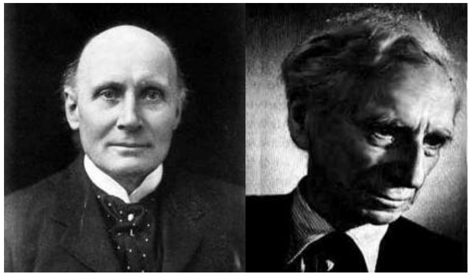


Figure 9: Alfred North Whitehead (1861-1947), Bertrand Russell (1872-1970)

Impacts of Logic in CS

- Church, Turing: formalized algorithm, computability, functional programming (λ-calculus).
- Program stored in a tape (original idea): influenced von Neumann.
- Hilbert Programme: decidability/computability (Church used "calculability": Turing introduced "computability").
- Logic research has a direct contribution to these developments.

Turing Contributions

- Notions of algorithm, effective procedure, computability. [Tur36].

  Although the class of computable numbers is so great, and in many ways similar to the class of real numbers, it is nevertheless enumerable. In ğ8 I examine certain arguments which would seem to prove the contrary. By the correct application of one of these arguments, conclusions are reached which are superficially similar to those of Gödel [1]. These results have valuable applications. In particular, it is shown (ğ11) that the Hilbertian Entscheidungsproblem can have no solution. In a recent paper Alonzo Church[2] has introduced an idea of "effective calculability", which is equivalent to my "computability", but is very differently defined. Church also reaches similar conclusions about the Entscheidungsproblem [3]. The proof of equivalence between "computability" and "effective calculability' is outlined in an appendix to the present paper.

  [1] Uber formal unentscheidbare Satze der Principia Mathematica und verwandter Systeme,I. Monatshefte fur Mathematik und Physik, 38 (1931):173-198.
  [2] An unsolvable problem of elementary number theory, Amer. J Math, 58(1936): 345-363.
  [3] A note on the Entscheidungsproblem. J. of Symbolic Logic 1(1936): 40-41.

Classical Logic

- Logical systems: syntax, semantics, proof theory (proof systems)
- Classical Logics:
  – propositional logics , first-order predicate logics, second order (higher order).
- Propositional logic:
  – atomic statements are propositions: facts that are either true or false at some possible world/situation, static interpretations.
  – Prop. variables $p, q, r...$; connectives $\neg, \vee, \wedge, \rightarrow$.
  – Formulae: $p \rightarrow q \rightarrow (\neg p \vee q)$
  – Expressivity, limited: *all men are mortal*, expressed as an atom.

First-order (predicate) logic

- Allow property specification of structures (graphs, partial orders, groups...), using predicates (relations).
  – Atomic sentences with arguments (terms) - terms denote objects and predicate symbols of n arguments.
  – Variables over objects; quantification existential ($\exists x$) and universal ($\forall x$) over variables
  – $G = (V, E), E \subseteq V^2$
  – Each graph node has at least two neighbours:
  $\forall x \exists y \exists z (\neg(x = y) \wedge E(x, y) \wedge E(x, z))$
  – Relational structures are essentially relational databases ("insight" by Ted Codd (1968), ACM Turing Award).

Propositional Logic: SATisfiability

- One of the great challenges in CS research.
- SATtisfiability of a propositional formula: NP-complete problem.
- Give an arbitrary formula α is there an assignment v such that $v(\alpha) = 1$?
- Steven A. Cook: *The Complexity of Theorem-Proving Procedures* STOC, 1971.

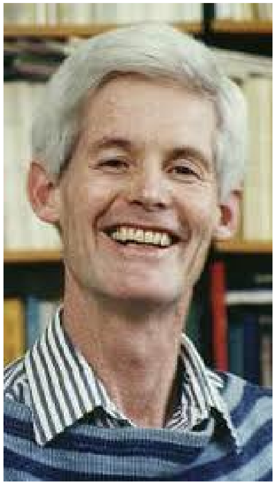


Figure 10: Stephen A. Cook

Non-classical Logics I

- Applications in CS, AI demand other kinds of reasoning.
- Classical logic is monotonic: results, inferences are persistent, not refuted.
- Computing systems are dynamic and interactive: time plays a key role, as do states, transitions, messages.
- Hoare/Dijkstra/Floyd: program logics, precise definitions, science of programming.
- Hoare, Milner [Hoa85, Mil89]: concurrency and interaction demand new computational logics.
- Manna [MW85], Halpern, Immermann, Vardi et al [HHI$^+$01]: *Logic is the calculus of computer science*.

Non-classical Logics II

Figure 11: C.A.R. Hoare (1931-), Robin Milner: Turing Awards (1980, 1991)

- Non-classical logics: widespread use in CS/AI: e.g. Journals, Handbooks, Conferences.
- Turing Awards: M Rabin & D Scott, T. Hoare, R. Milner, J. McCarthy, E. Codd, S. Cook, A. Pnueli, E. Clarke, A. Emerson e J. Sifakis.
- 1990's: Gabbay: Labelled deductive systems[Gab96]: allow the association of added information to formulae.

Non-classical Logics

- In CS/AI, our knowledge about the world/situation/state can be related to the notion of other possible worlds [HHI$^+$01].
- Reasoning about knowledge is relevant to CS/AI: e.g. how process/programs/agents coordinate to run/compute/act on? What do they need to know to compute their tasks? What agents need to know about each other to communicate effectively?
- Non-classical logics offer alternatives/answers to these and other questions: modal, temporal, epistemic, ... logics.

Reasoning/proving/using logics

Reasoning with Propositional and Predicate Logics
Reasoning with Non-classical Logics

Hypothetical reasoning, using hypotheses

- Conditional reasoning is ubiquitous.
- Intensively investigated by philosophers: Stalnaker, Lewis, Pearl, and many others.
- **There are several conditionals**, “if-sentences”.
- *Counterfactuals*: “If Wellington had not beaten Napoleon, someone else would have.”
- *Causal*: “If A then causally B.”
- *Action*: “If A then B is obtained.”
- *Obligation*: “If A then B should (ought to) be the case.”
- *General*: “If A then normally B.”
- Applications: decision theory, game theory and economics, AI.
- Non-classical semantics is sometimes needed: e.g. “If Rome is the capital of Greece, then there are no snakes in Ireland” (Woods, 97)”.

Modal Logic

- Logics of *necessity and possibility*, modern studies by Lewis [Lew18].
- Added operators $\Box$, $\Diamond$ + propositional logics, predicate logics.
- $\Box\phi$: ϕ is necessarily true, and $\Diamond\phi$ states that ϕ is possibly true (equivalent to $\neg\Box\neg\phi$).
- $\Box\phi$: “according to the agent knowledge”, “according to the laws of physics”, “after the run of the program”.
- Hintikka, Kripke *possible world semantcs* [Hin62, Kri63].
- Modelling/reasoning about time, space, beliefs, actions, knowledge; individually or combining them: Fagin, Halpern, Moses, Vardi [FHMV95, Var97].

Temporal Logics, temporal reasoning

Figure 12: London 25 June 2016, Piccadilly Circus

Modal temporal logic

- Modal temporal logic: high impact since Pnueli, *On the temporal logic of programs*, FOCS 1977.
- Program verification, (industrial) hardware verification, database applications, distributed computing.
- Propositional LTL: expressive power of First-order logic over naturals (Gabbay, Pnueli, Shelah, Stavi, 1980, POPL).
- next α (α is *tt* at the next timepoint); eventually α; always α; α until β.

Figure 13: Amir Pnueli (1941-2009): Turing Award, 1996

Modalities

- Alethic:
  $\Box\phi$: ϕ is necessarily true.
  $\Diamond\phi$: ϕ is possibly true (equiv. $\neg\Box\neg\phi$).
- Epistemic modalities
  $K_a\phi$: Agent a knows ϕ.
  $B_a\phi$: Agent a believes that ϕ is true.
- Linear temporal modalities:
  $\bigcirc\phi$ (or $X\phi$): ϕ is true at the next timepoint (state of the world).
  $\Box\phi$ (or $G\phi$): ϕ is always true.
  $\Diamond\phi$ (or $F\phi$): ϕ is possibly (contingently) true in the future (equiv. $\neg\Box\neg\phi$: ϕ will not be always false).
  $\phi\mathbb{U}\psi$: ϕ is true until ψ is true.

Logic and Neural Learning

Valiant: great challenge for CS, J. of the ACM vol. 50, 2003
Building effective models of cognitive computing.
Integration of learning and reasoning.

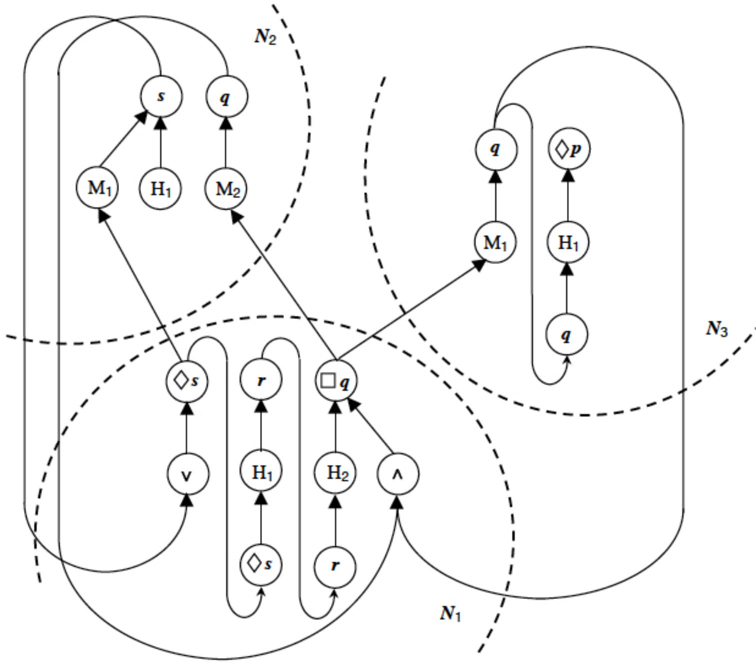
> *"The aim here is to identify a way of looking at and manipulating commonsense knowledge that is consistent with and can support what we consider to be the two most fundamental aspects of intelligent cognitive behaviour: the ability to learn from experience, and the ability to reason from what has been learned. We are therefore seeking a semantics of knowledge that can computationally support the basic phenomena of intelligent behaviour."*

Neural-Symbolic Learning and Reasoning

"Neural-Symbolic Learning and Reasoning seeks to integrate principles from neural networks learning and logical reasoning. It is an interdisciplinary field involving components of knowledge representation, neuroscience, machine learning and cognitive science." [dGBdR+15]

Connectionist Modal Logic

Insight: network ensembles as possible worlds.



Network ensemble which represents

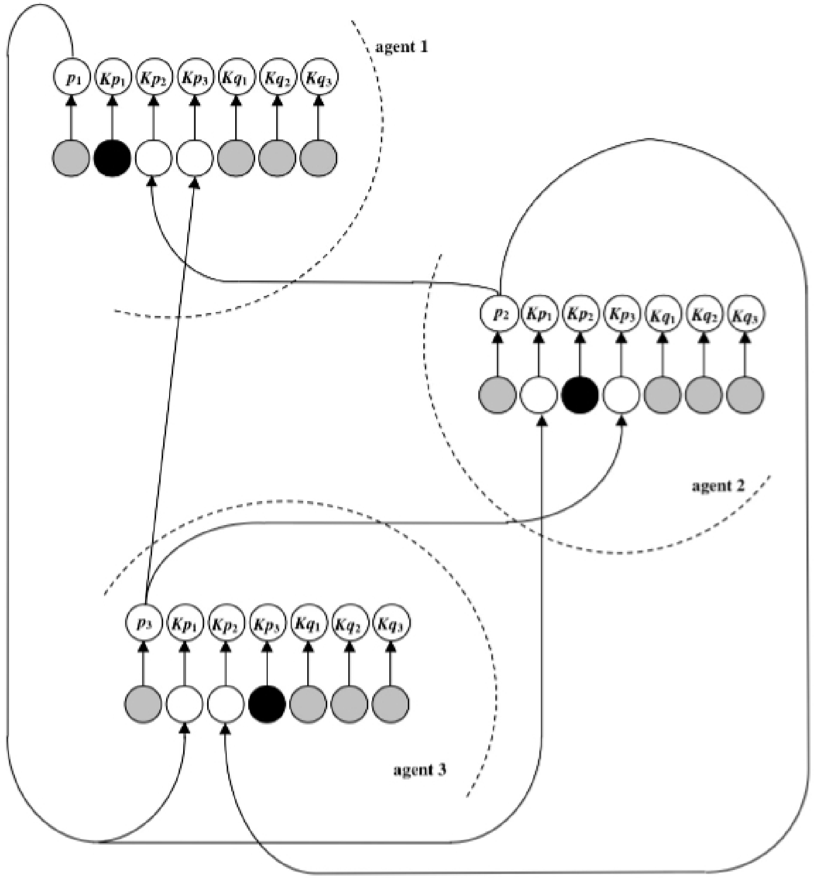
$P = \{r \to \Box q : \omega_1; \Diamond s \to r : \omega_1; s : \omega_2; q \to \Diamond p : \omega_3, R(\omega_1, \omega_2), R(\omega_2, \omega_3)\}$

Thm: For any program P, there exists a neural network ensemble N s.t. N computes the modal fixed point operator of program P. (d'Avila G.+Lamb+Gabbay, Connectionist Modal Logics: Theoretical Computer Science 371(2007):34-53).

CML applications

- Testbeds of distributed systems: muddy children puzzle; wise men puzzle; Monty Hall; dining philosophers... [FHMV95, dGLG07, dLG06, dL06, dLG09].
- Benchmarks of distributed knowledge representation ([FHMV95])

Knowledge interaction in the Muddy Children puzzle



Experimental results: with background knowledge - modal rules: (93% accuracy on the test set).

Temporal Reasoning: Muddy Children Puzzle

Agent knowledge temporal rules

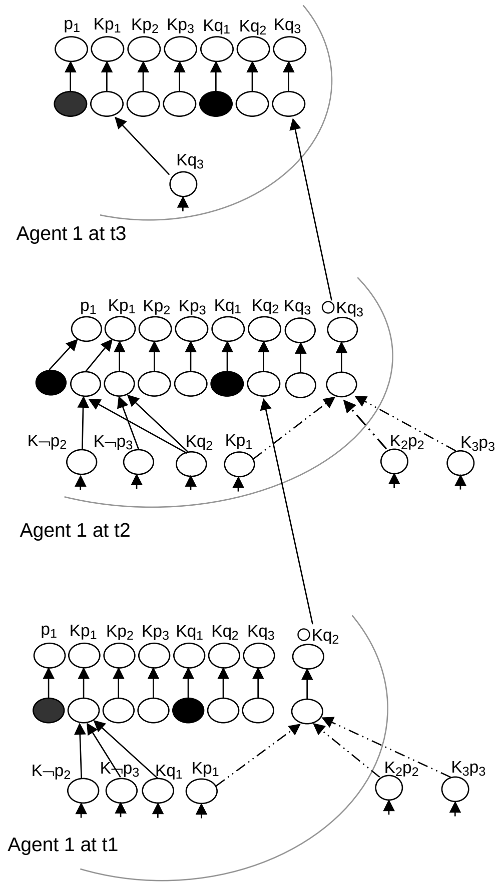
p_i agent i is muddy.

q_k at least k children are muddy.

- Agent 1
  $t_1 : \neg K_1 p_1 \wedge \neg K_2 p_2 \wedge \neg K_3 p_3 \rightarrow \bigcirc K_1 q_2$
  $t_2 : \neg K_1 p_1 \wedge \neg K_2 p_2 \wedge \neg K_3 p_3 \rightarrow \bigcirc K_1 q_3$
- Agent 2
  $t_1 : \neg K_1 p_1 \wedge \neg K_2 p_2 \wedge \neg K_3 p_3 \rightarrow \bigcirc K_2 q_2$
  $t_2 : \neg K_1 p_1 \wedge \neg K_2 p_2 \wedge \neg K_3 p_3 \rightarrow \bigcirc K_2 q_3$
- Agent 3
  $t_1 : \neg K_1 p_1 \wedge \neg K_2 p_2 \wedge \neg K_3 p_3 \rightarrow \bigcirc K_3 q_2$
  $t_2 : \neg K_1 p_1 \wedge \neg K_2 p_2 \wedge \neg K_3 p_3 \rightarrow \bigcirc K_3 q_3$

Thm: For any CTLK Program P, there exists a single hidden layer network ensemble N s.t. N computes the fixed point operator $\bigcirc T_P$ *of P.* [dLG09]

Temporal knowledge evolution



Conclusions

Computational Logic and Human Thinking, How to be artificially intelligent, 2011.
Pointed out the importance of the integration of computational logic and learning in the construction of more realistic AI models.

Figure 14: R.A. Kowalski, NeSy'11 at IJCAI 2011

Conclusions

Figure 15: Leslie Valiant: Turing Award winner 2010

“A fundamental question for artificial intelligence is to characterize the computational building blocks that are necessary for cognition. A specific challenge is to build on the success of machine learning so as to cover broader issues in intelligence. This requires, in particular a reconciliation between two contradictory characteristics – the apparent logical nature of reasoning and the statistical nature of learning.”

https://www.seas.harvard.edu/directory/valiant

Conclusions

Figure 16: Michael Ozer Rabin: Turing Award Winner, 1976

Our field is still in its embryonic stage. It's great that we haven't been around for 2000 years. We are at a stage where very, very important results occur in front of our eyes.
M.O. Rabin, in Denis Shasha and Cathy Lazere: Out of Their Minds: The Lives and Discoveries of 15 Great Computer Scientists, 1995).

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