Information Visualization

W11: Direct Volume Rendering

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- W03 4/17 Introduction to Data Visualization
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- W05 4/24 Computer Graphics
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- **W09 5/08 Isosurface**
- **W10 5/09 Exercise (Isosurface Extraction)**
- **W11 5/22 Direct Volume Rendering**
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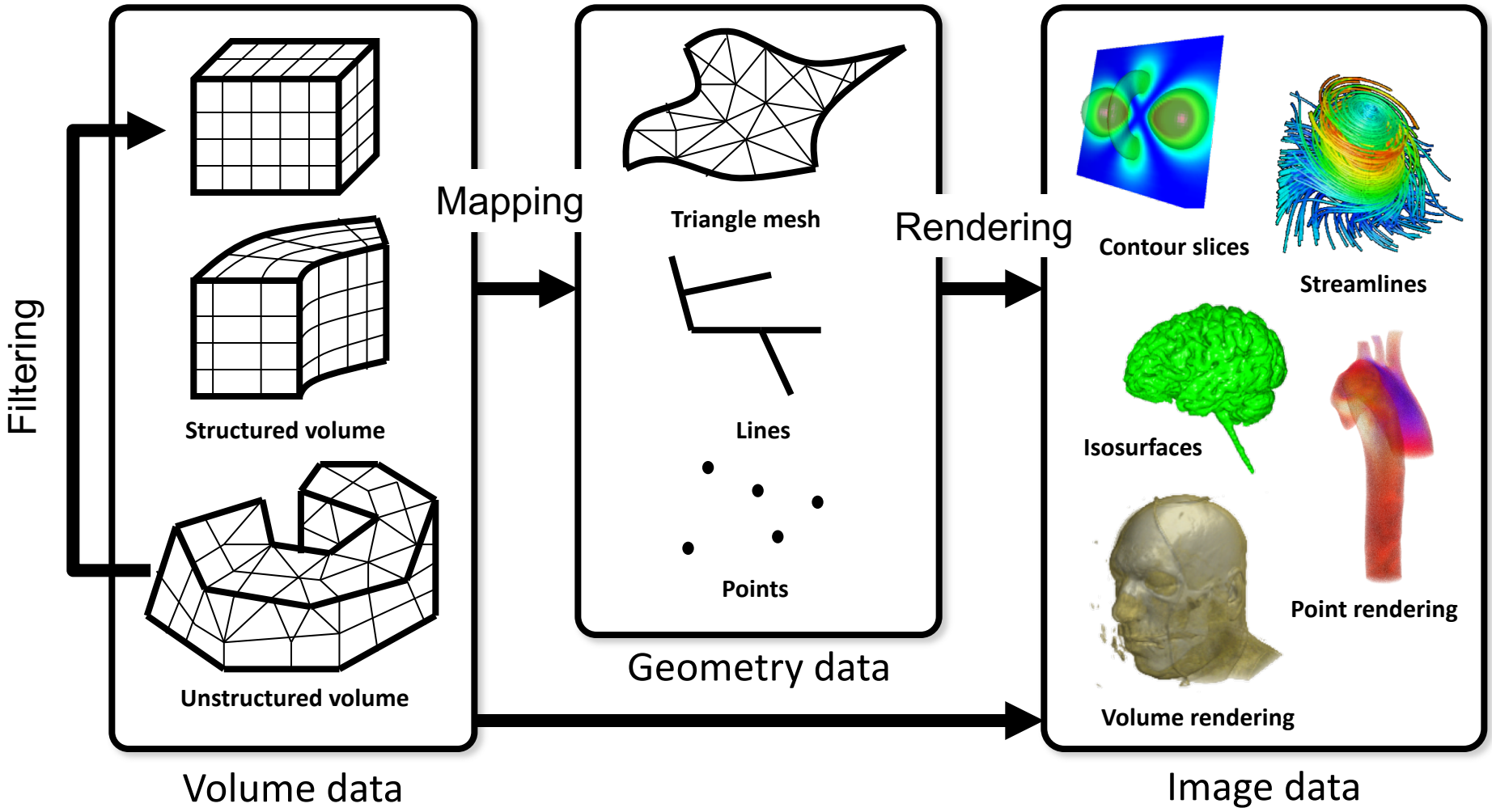
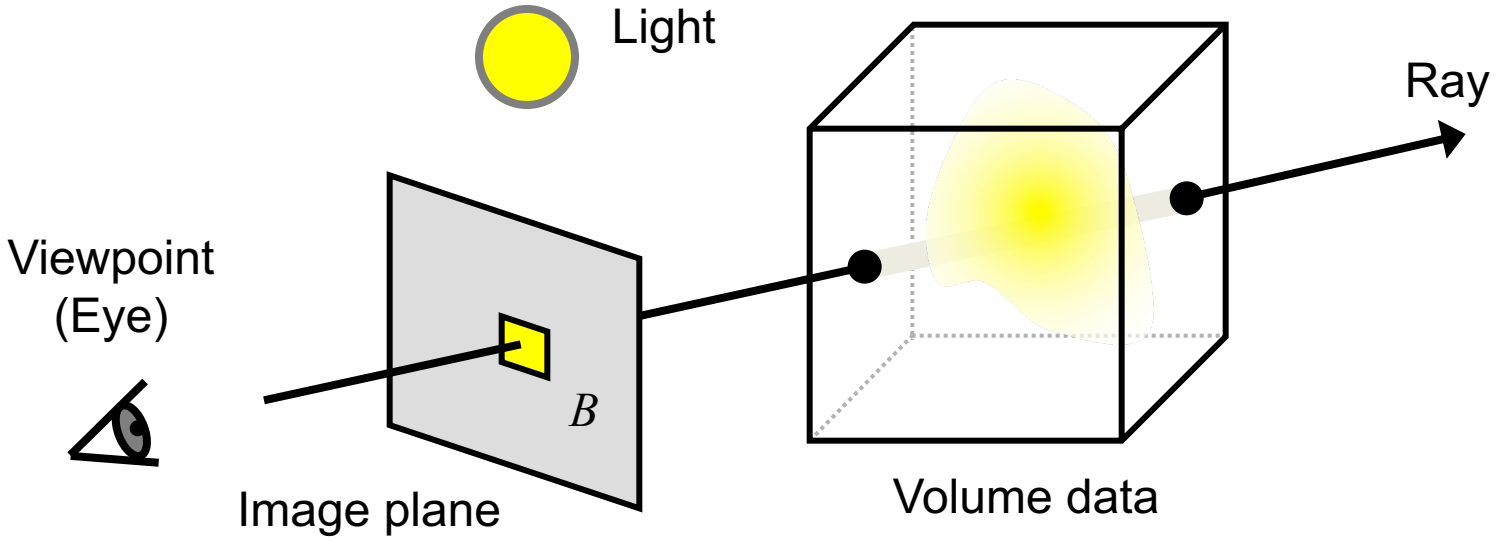
Scientific Visualization



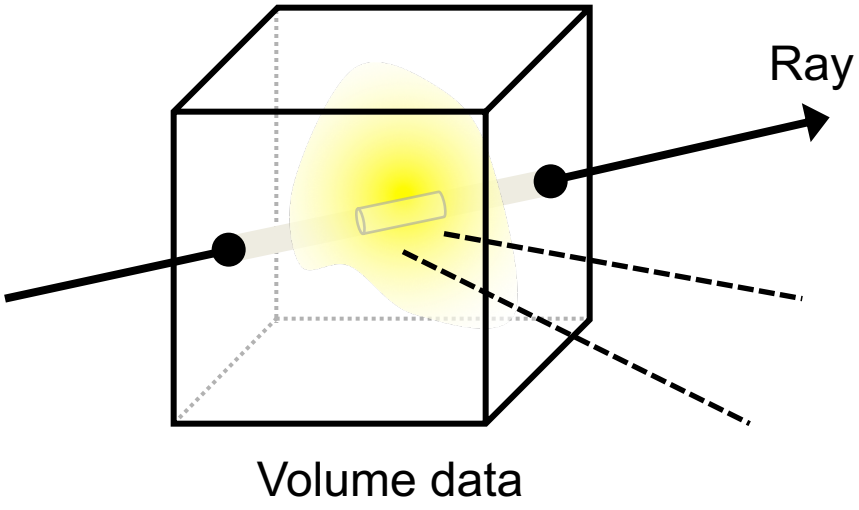
What is volume rendering?

- Volume rendering is a technique to generate images from volume datasets based on the density emitter model.

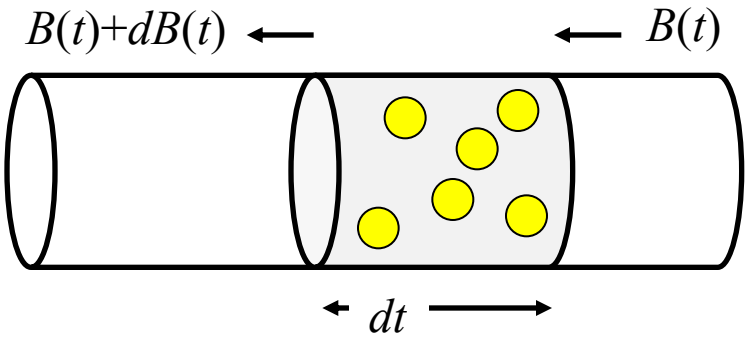


What is volume rendering?

- Density emitter model [Sabella, 1988]



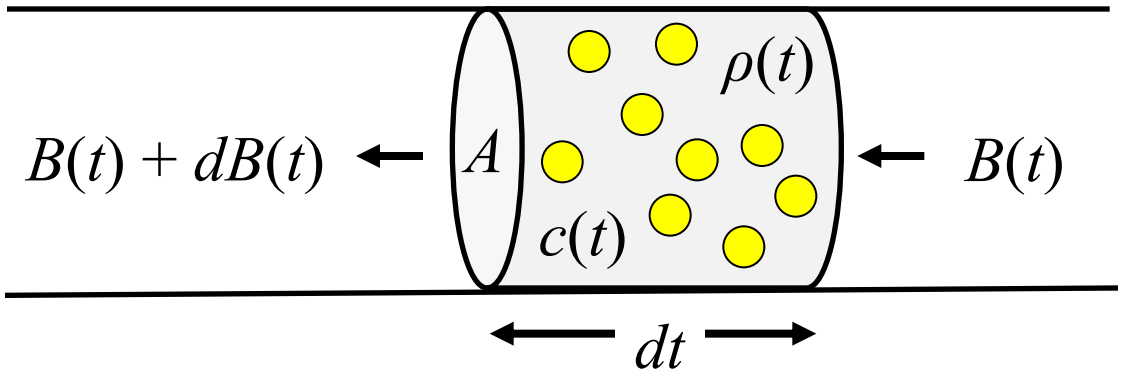
- Emissive and opaque particles are assumed
- Not particle but their densities are modeled



$$B = \int_{t_n}^{t_0} c(t) \times \pi r^2 \rho(t) \times \exp\left(\int_{t}^{t_0} \pi r^2 \rho(\lambda) d\lambda \right) dt$$

Density Emitter Model

- Brightness equation



$B(t)$: Brightness
$\rho(t)$: Particle density
$c(t)$: Luminosity
r : Particle raduis

$$db(t)A = -absorbed + emitted$$

$$= (-B(t) + c(t)) \times \pi r^2 \rho(t) A dt$$

$$\frac{dB(t)}{dt} = (-B(t) + c(t)) \times \pi r^2 \rho(t)$$

Brightness Equation

- Multiplying both sides by a exponential term.

$$\frac{dB(t)}{dt} = \{-B(t) + c(t)\} \times \pi r^2 \rho(t)$$

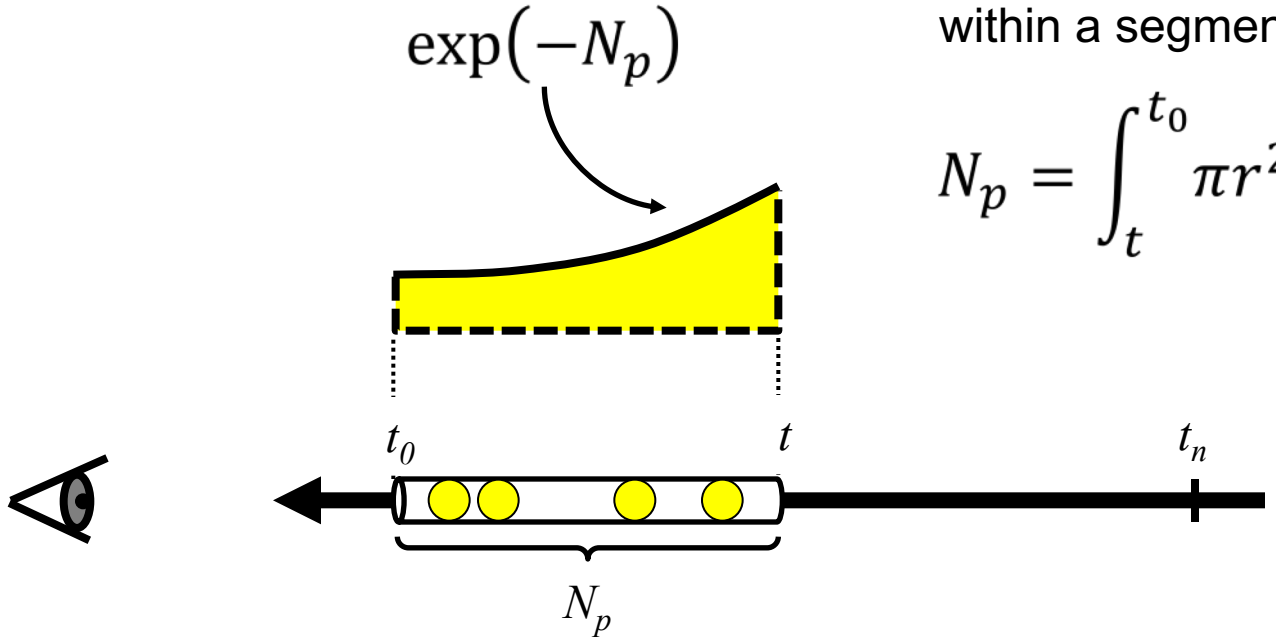
$$\exp\left(\int_{t_n}^{t} \pi r^2 \rho(u) du\right) \frac{dB(t)}{dt} = \exp\left(\int_{t_n}^{t} \pi r^2 \rho(u) du\right) \{-B(t) + c(t)\} \times \pi r^2 \rho(t)$$

$$\Longrightarrow B = B(t_0) = \int_{t_n}^{t_0} c(t) \times \pi r^2 \rho(t) \times \exp\left(-\int_{t}^{t_0} \pi r^2 \rho(\lambda) d\lambda\right) dt$$

Brightness Equation

$$B = \int_{t_n}^{t_0} c(t) \times \pi r^2 \rho(t) \times \exp\left(- \underbrace{\int_{t}^{t_0} \pi r^2 \rho(\lambda) d\lambda}_{\text{Number of particles } (N_p) \text{ within a segment from } t \text{ to } t_0} \right) dt$$

$$N_p = \int_{t}^{t_0} \pi r^2 \rho(\lambda) d\lambda$$



Brightness Equation

- Poisson distribution
  - In the density emitter model, the Poisson distribution has been assumed for the number of particles in a segment.
  - If the expected number of particles in the segment is N_p, then the probability P that there are k particles is represented as follows:

$$P(N = k) = \frac{\exp(-N_p){N_p}^k}{k!}$$

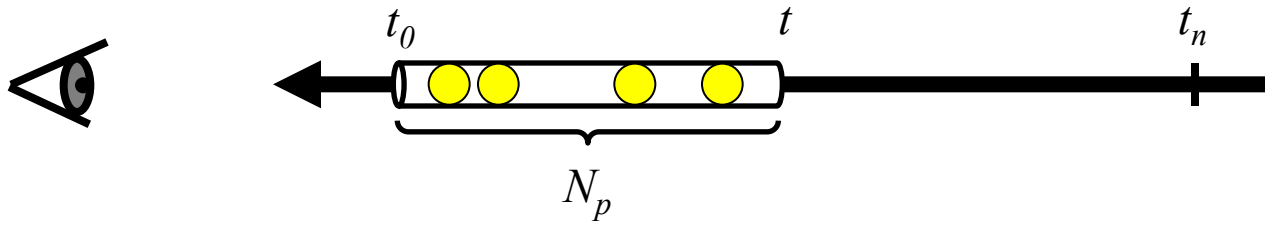
Brightness Equation

- Definition of opacity
  - The probability that there is no particles is defined as transparency

$$0 \leq P(N = 0) = \exp(-N_p) \leq 1$$

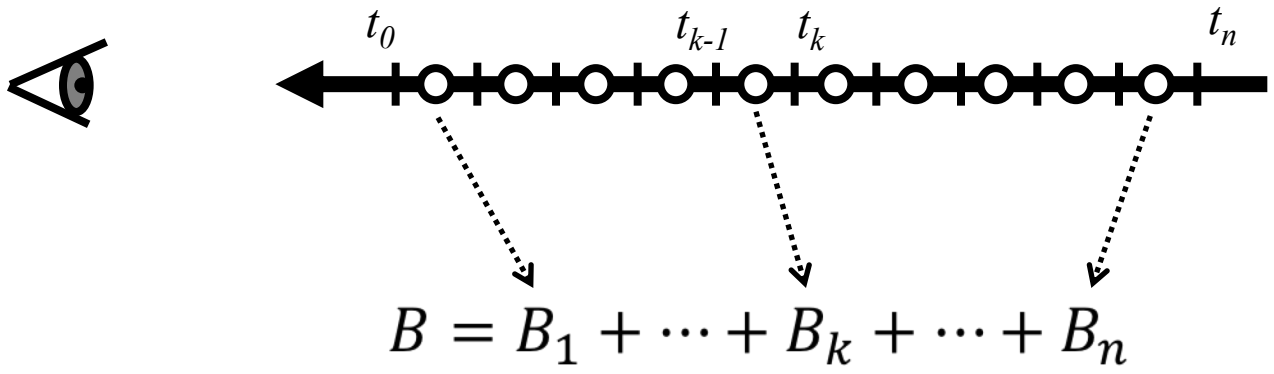
  - The probability that there are more than a particle is defined as opacity

$$0 \leq 1 - P(N = 0) = 1 - \exp(-N_p) \leq 1$$



Brightness Equation

- Discretization
  - Brightness equation is numerically calculated by subdividing a viewing ray into n ray segments.



Brightness Equation

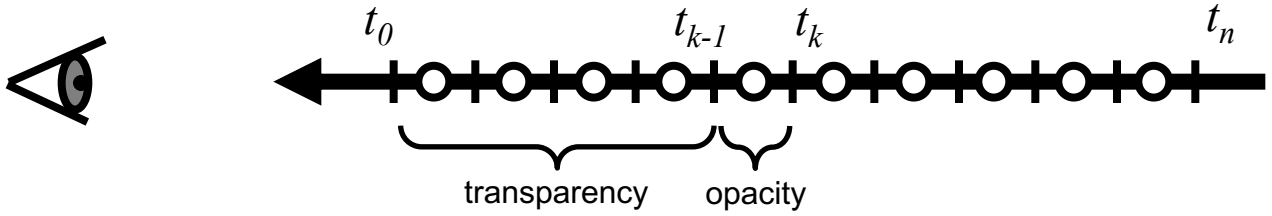
- Numerical Solution

$$B_k = \int_{t_k}^{t_{k-1}} c_k \times \pi r^2 \rho(t) \times \exp\left(-\int_t^{t_0} \pi r^2 \rho(\lambda) d\lambda\right) dt$$

$$= \exp\left(-\int_{t_{k-1}}^{t_0} \pi r^2 \rho(\lambda) d\lambda\right) \int_{t_k}^{t_{k-1}} c_k \times \pi r^2 \rho(t) \times \exp\left(-\int_t^{t_{k-1}} \pi r^2 \rho(\lambda) d\lambda\right) dt$$

$$= \prod_{i=0}^{k-2} \exp\left(-\int_{t_{i+1}}^{t_i} \pi r^2 \rho(\lambda) d\lambda\right) \times c_k \times \left(1 - \exp\left(-\int_{t_k}^{t_{k-1}} \pi r^2 \rho(\lambda) d\lambda\right)\right)$$

transparency opacity

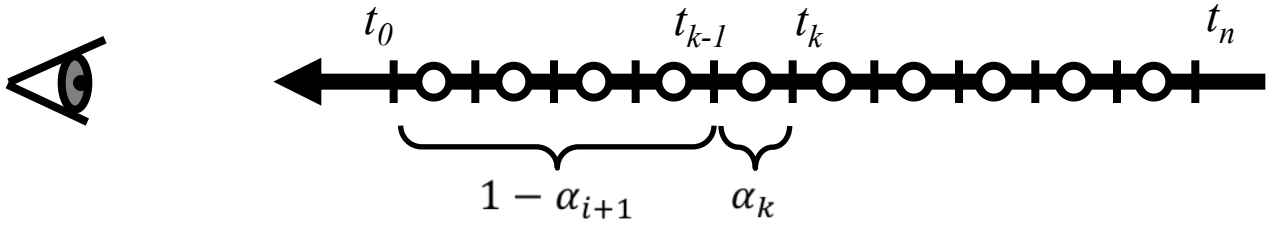


Brightness Equation

- Numerical Solution

$$B_k = \prod_{i=0}^{k-2} \underbrace{\exp\left(-\int_{t_{i+1}}^{t_i} \pi r^2 \rho(\lambda) d\lambda\right)}_{\text{transparency} = 1-\alpha_{i+1}} \times c_k \times \underbrace{\left(1 - \exp\left(-\int_{t_k}^{t_{k-1}} \pi r^2 \rho(\lambda) d\lambda\right)\right)}_{\text{opacity} = \alpha_k}$$

$$= c_k \alpha_k \prod_{i=0}^{k-2} (1-\alpha_{i+1}) = c_k \alpha_k \prod_{i=1}^{k-1} (1-\alpha_i)$$



Brightness Equation

- Opacity correction
  - Opacity along the k-th segment

$$\alpha_k = 1 - \exp(-\int_{t_k}^{t_{k-1}} \pi r^2 \rho(\lambda) d\lambda)$$

$$\approx 1 - \exp(-\pi r^2 \rho_k \times \Delta t)$$

where, Δt is the length of the segment and ρ_k is the density at its center.

  - If an actual length $\Delta t'$ is different from the Δt, the opacity is corrected as follows:

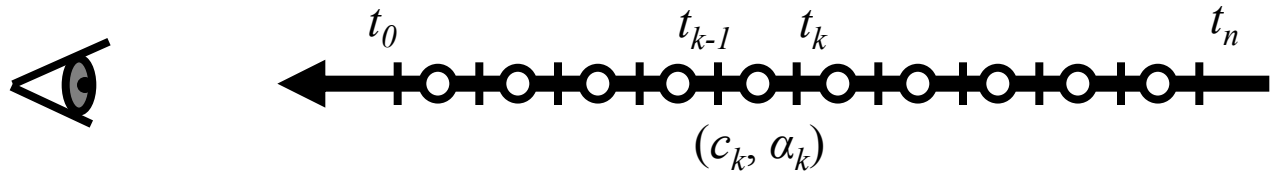
$$\alpha_k{}' = 1 - (1 - \alpha_k)^{\frac{\Delta t'}{\Delta t}}$$

Brightness Equation

- Discretized brightness equation

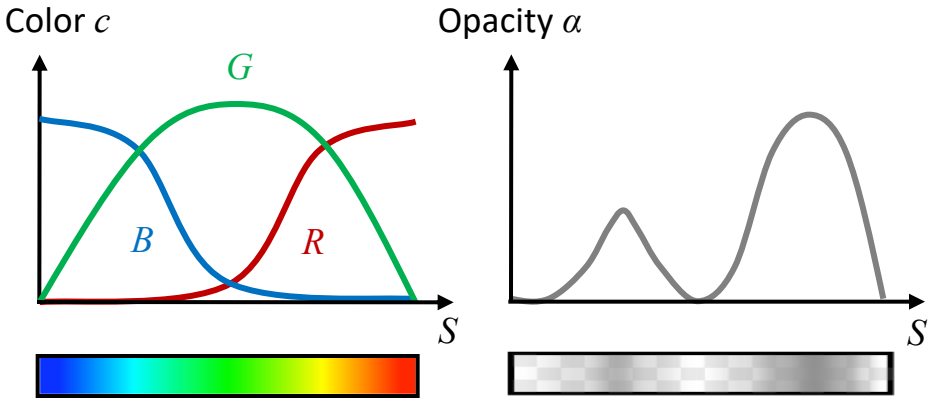
$$B = B_1 + \cdots + B_k + \cdots + B_n$$

$$= \sum_{j=1}^{n} c_j \alpha_j \prod_{i=1}^{j-1} (1 - \alpha_i)$$



Brightness Equation

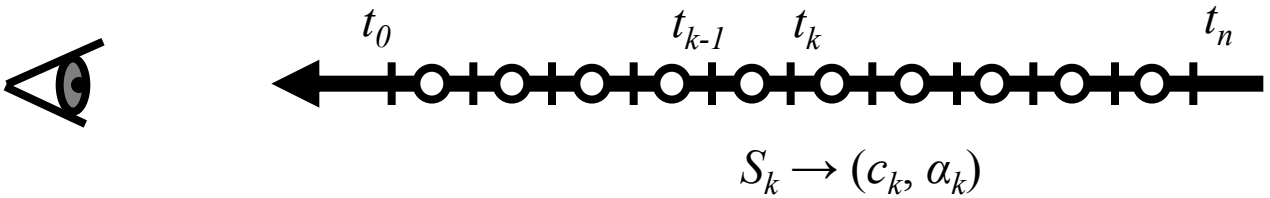
- Transfer function
  - In each segment, a scalar value S is mapped to color c and opacity α with transfer function.



$$S_k \rightarrow c_k = (R_k, G_k, B_k)$$

$$S_k \rightarrow \alpha_k$$

$$B = \sum_{j=1}^{n} c_j \alpha_j \prod_{i=1}^{j-1} (1 - \alpha_i)$$

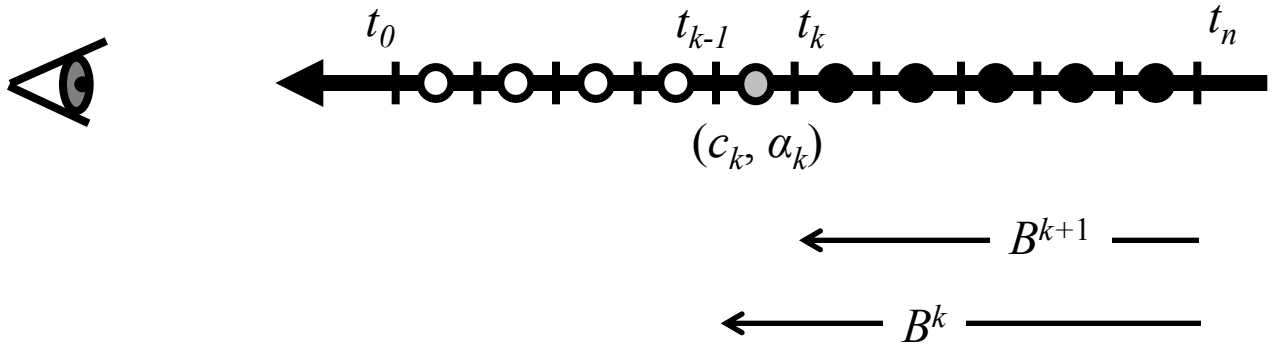


Brightness Equation

- Recurrence formula
  - Back-to-front compositing

$$B^k = c_k \alpha_k + (1 - \alpha_k) B^{k+1}$$

where c_k and α_k are the color and opacity for the k-th segment, and B_k is the accumulated brightness (color) from the back of the volume.



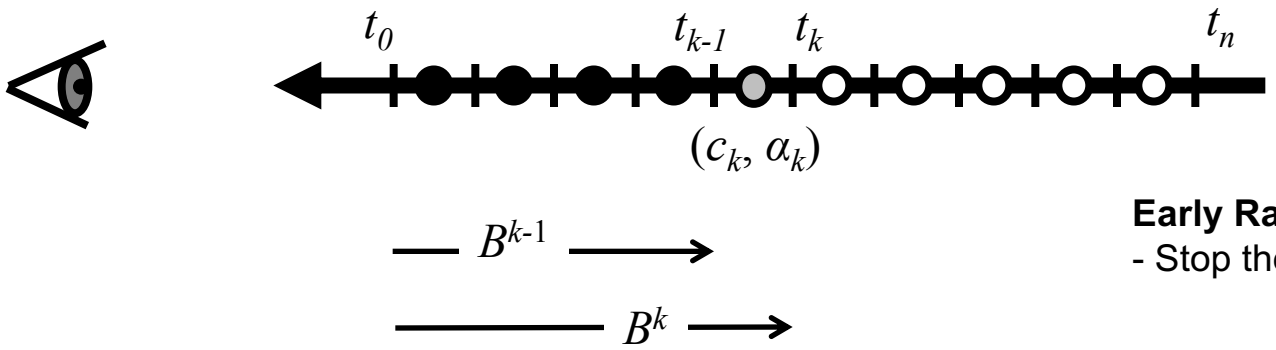
Brightness Equation

- Recurrence formula
  - Front-to-back compositing

$$B^k = c_k \alpha_k \left(1 - A^{k-1}\right) + B^{k-1}$$

$$A^k = \alpha_k \left(1 - A^{k-1}\right) + A^{k-1}$$

where B_k and A_k are the accumulated brightness (color) and opacity from the front of the volume.

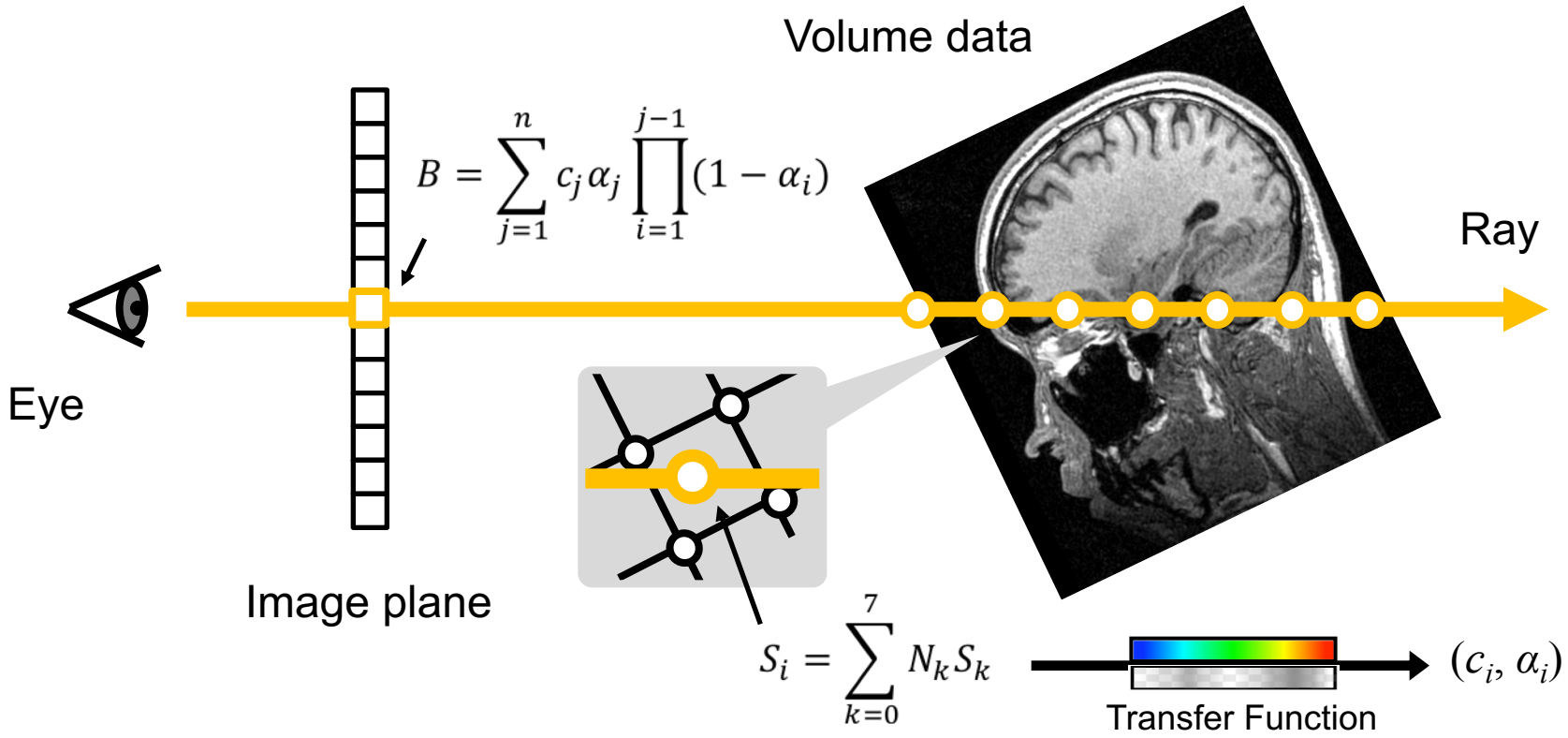


Early Ray Termination
- Stop the calculation when

$$A^k \approx 1$$

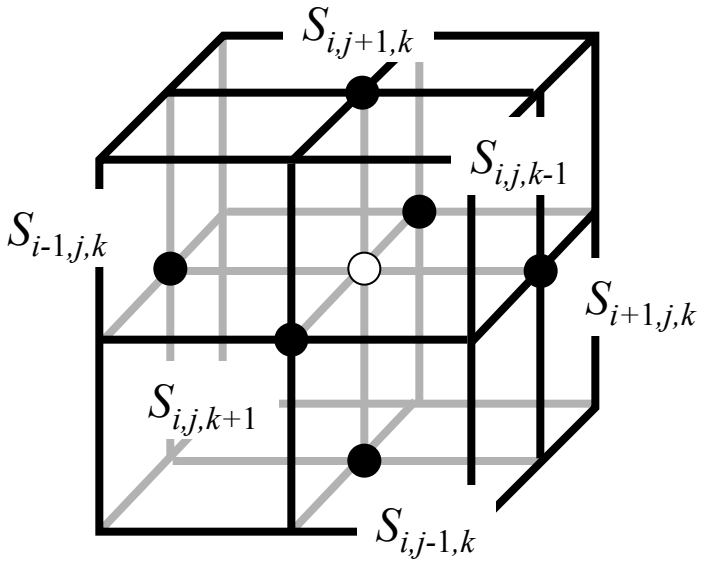
Volume Rendering

- Calculate the color and opacity for each pixel based on the brightness equation



Volume Rendering

- Normal vector $\mathbf{N}$ at the grid point can be calculated as scalar gradient.

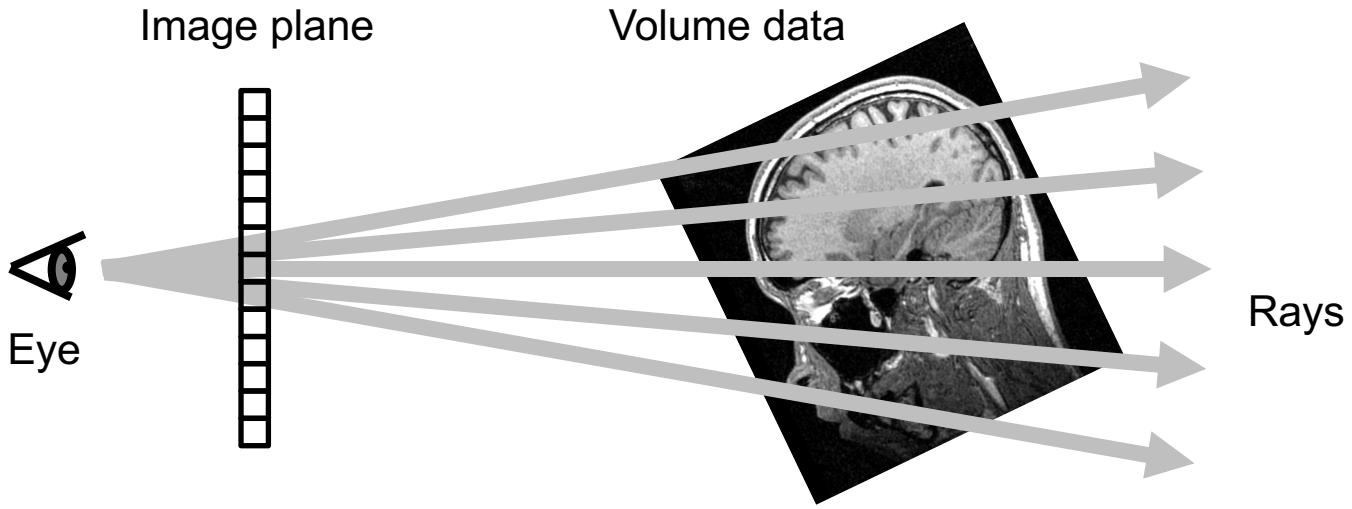


$$\mathbf{N} = \nabla S = \begin{pmatrix} \partial S/\partial x \\ \partial S/\partial y \\ \partial S/\partial z \end{pmatrix}$$

$$= \begin{pmatrix} S_{i+1,j,k} - S_{i-1,j,k}/\Delta x \\ S_{i,j+1,k} - S_{i,j-1,k}/\Delta y \\ S_{i,j,k+1} - S_{i,j,k-1}/\Delta z \end{pmatrix}$$

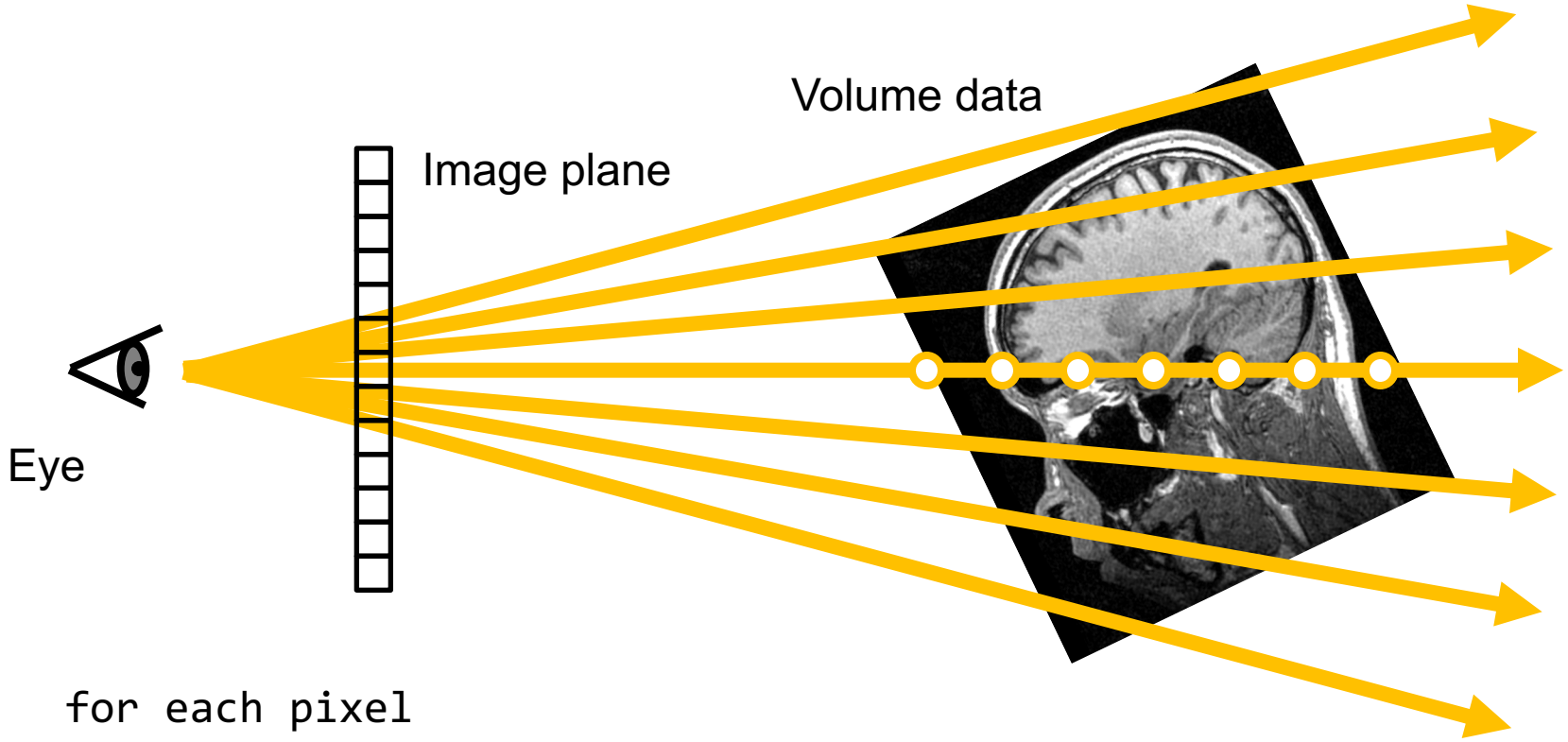
Volume Rendering

- Two approaches of volume rendering techniques
  - Image-order approach
  - Object-order approach



Volume Rendering

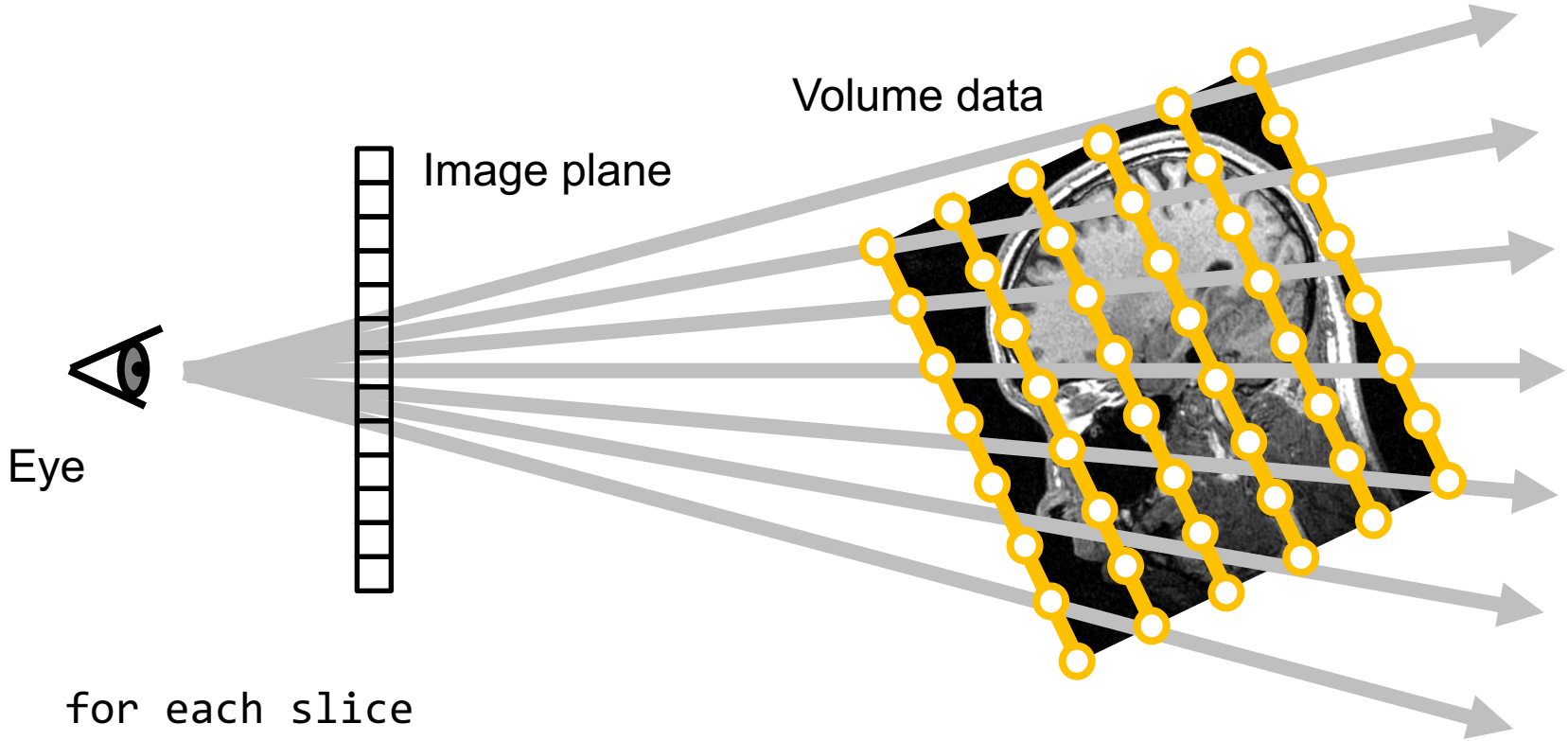
- Image-order approach




```
for each pixel
{
    // Calculation
}
```

Volume Rendering

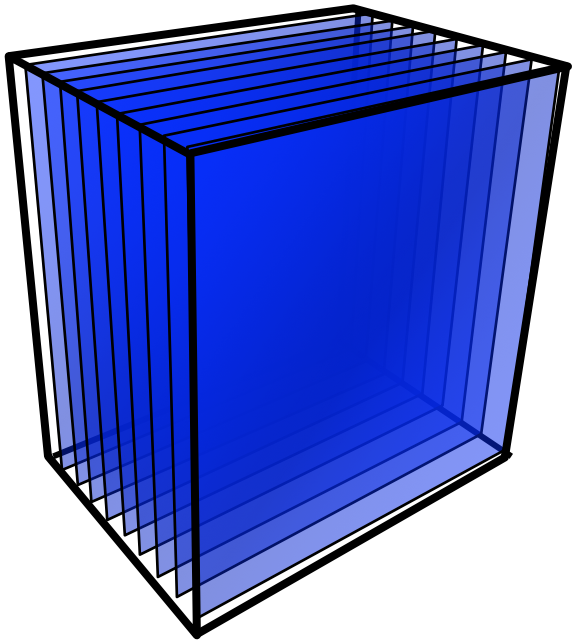
- Object-order approach



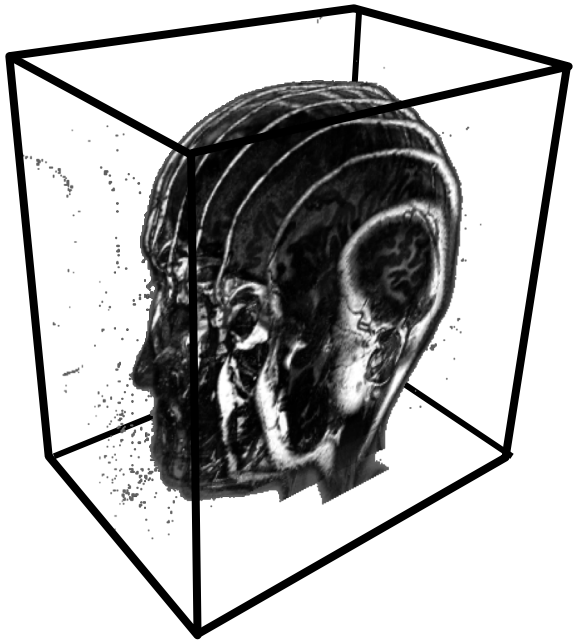

```
for each slice
{
    // Calculation
}
```

Implementation

- Texture-based approach
  - Object-order approach



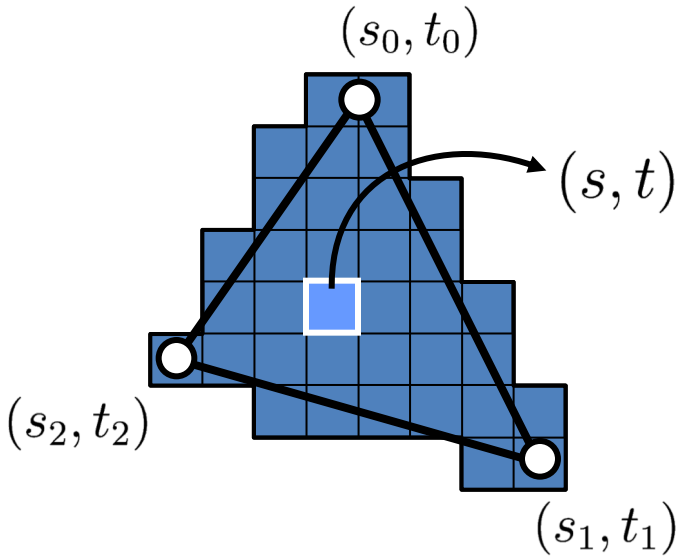
Proxy Geometry

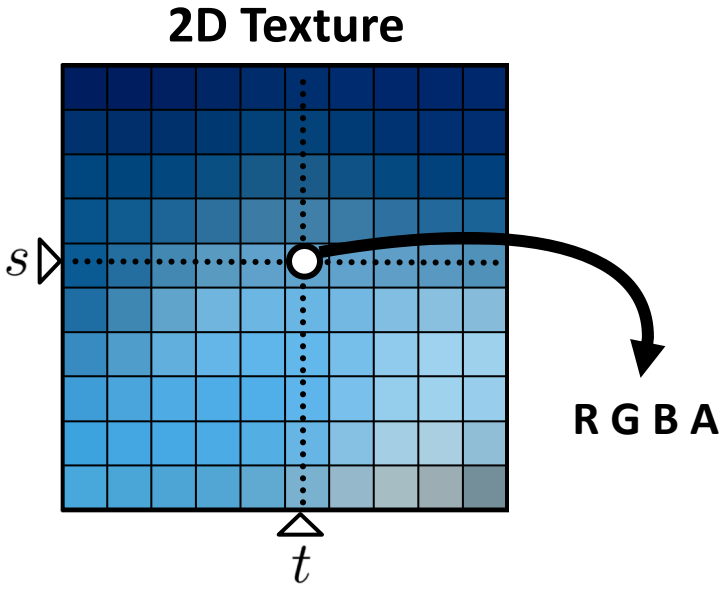


2D Textures

Implementation

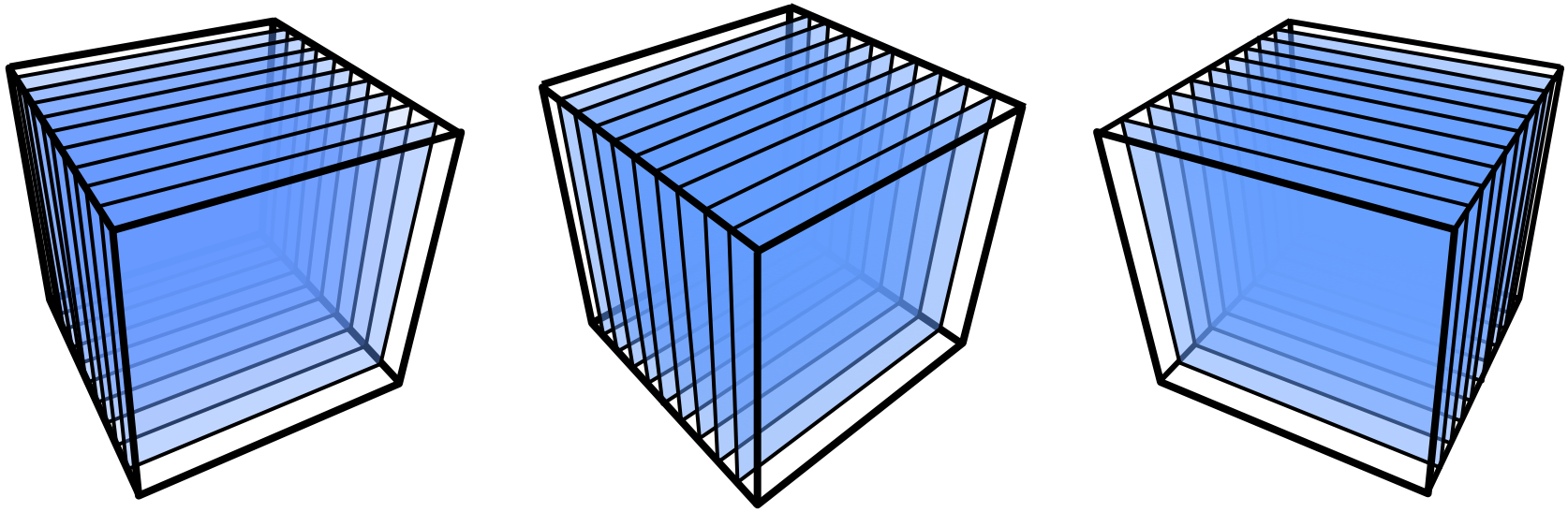
- Texture
  - 1D Texture = 1D array data structure, linear interpolation
  - 2D Texture = 2D array data structure, bi-linear interpolation
  - 3D Texture = 3D array data structure, tri-linear interpolation





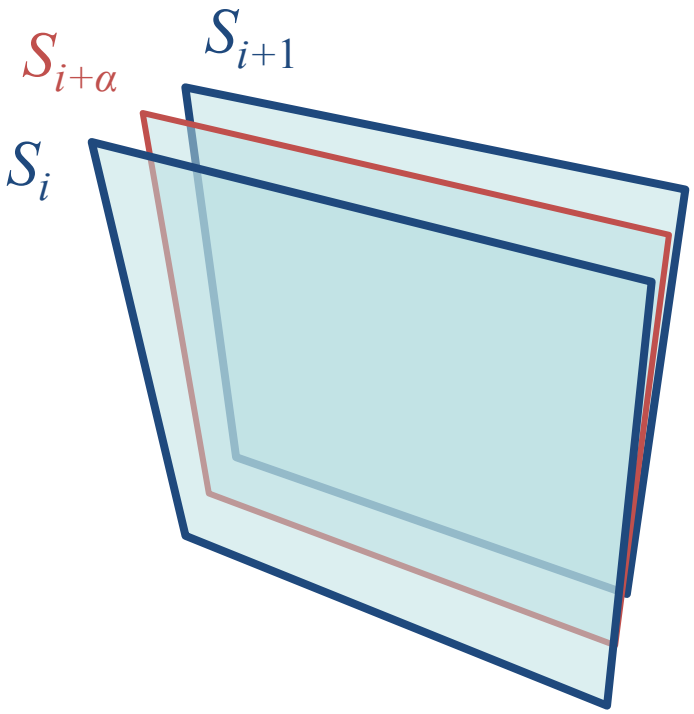
Object-order Approach

- Axis-aligned slices
  - Volume data is stored on GPU as a stack of 2D textures.
  - Three stacks of 2D textures for x,y,z principle directions.



Object-order Approach

- Axis-aligned slices
  - 2D Multi-Textures

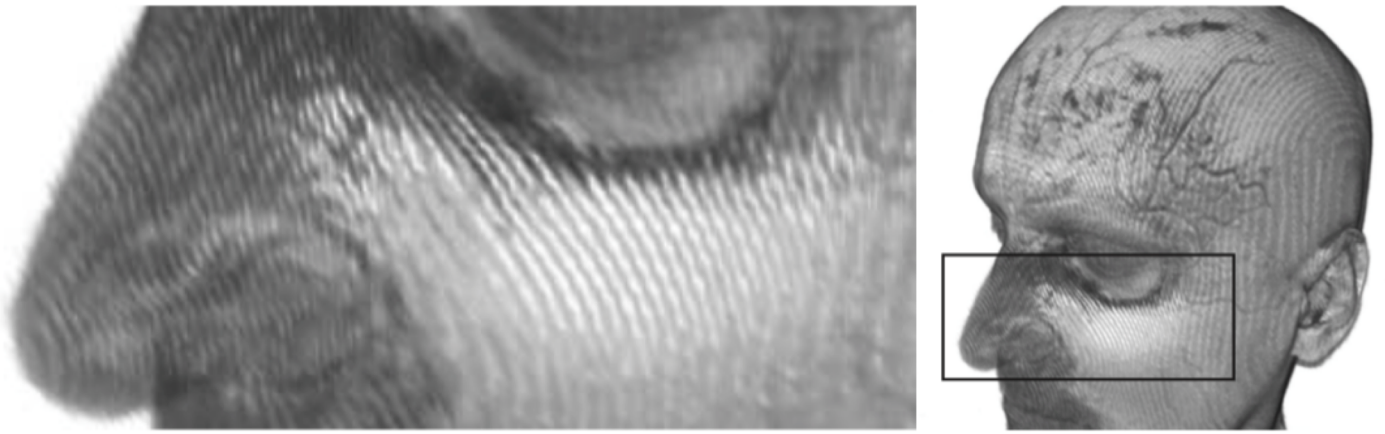


1. Bilinear Interpolation by 2D Texture Unit
2. Blending of two adjacent slice images
   →Trilinear Interpolation

$$S_{i+\alpha} = (1 - \alpha)\, S_i + \alpha\, S_{i+1}$$

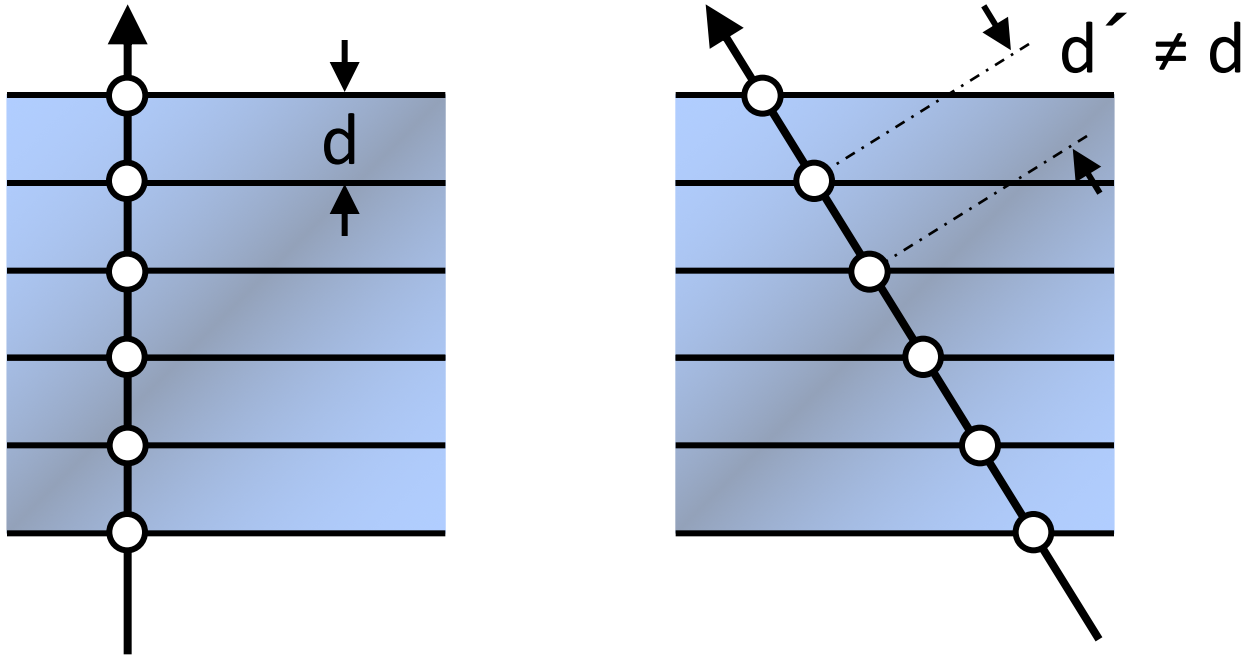
Object-order Approach

- Drawbacks of axis-aligned slices
  - Aliasing artifacts become visible at the edges of the slice polygons.



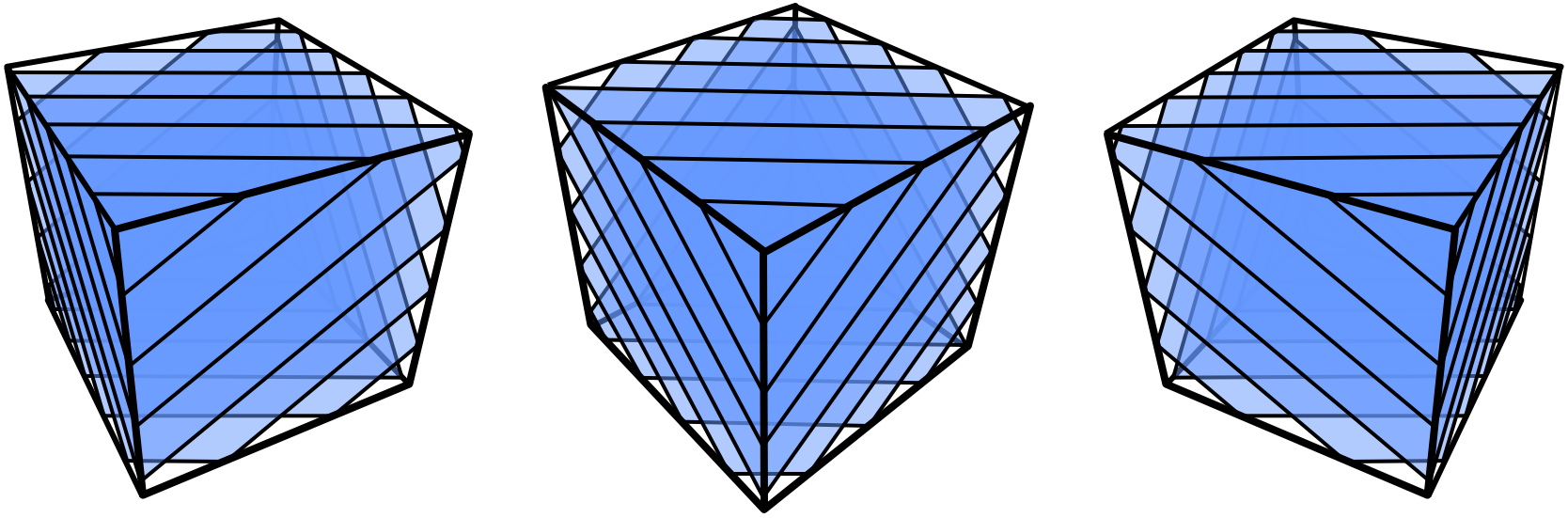
Object-order Approach

- Drawbacks of axis-aligned slices
  - Sampling rate is inconsistent.
  - Color and opacity slightly incorrect.



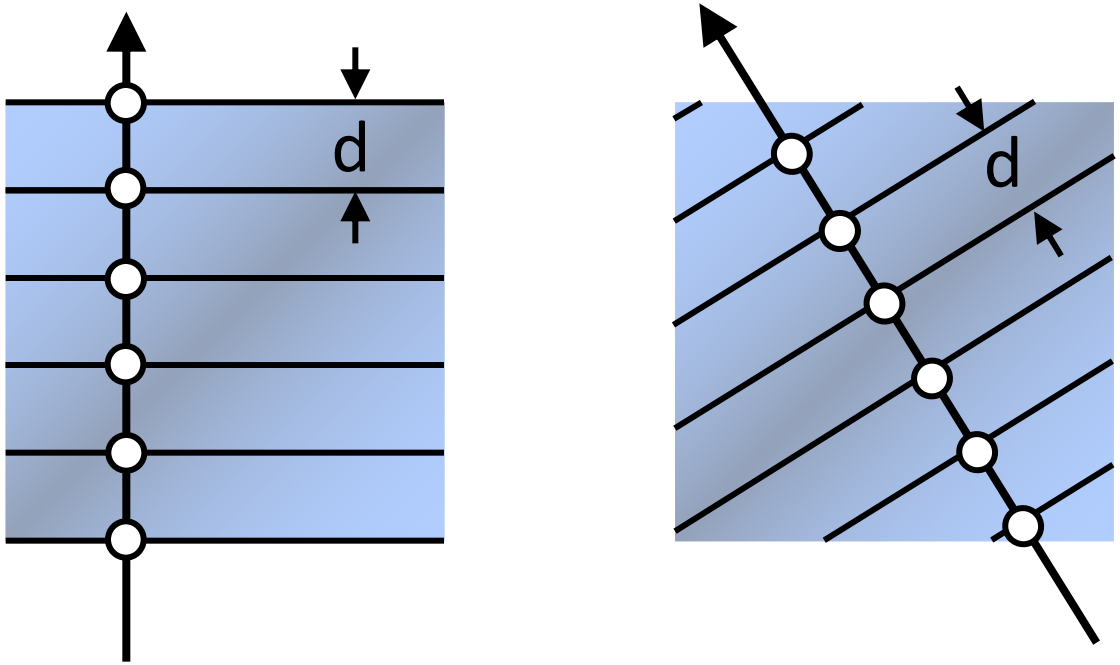
Object-order Approach

- View-aligned slices
  - Volume data is stored on GPU as a 3D texture.
  - Slices parallel to the image plane.



Object-order Approach

- Resampling via 3D Textures
  - Sampling rate is constant.



Object-order Approach

- Drawbacks of view-aligned slices
  - In the case of perspective projection, the sampling rate is still not consistent for all rays.

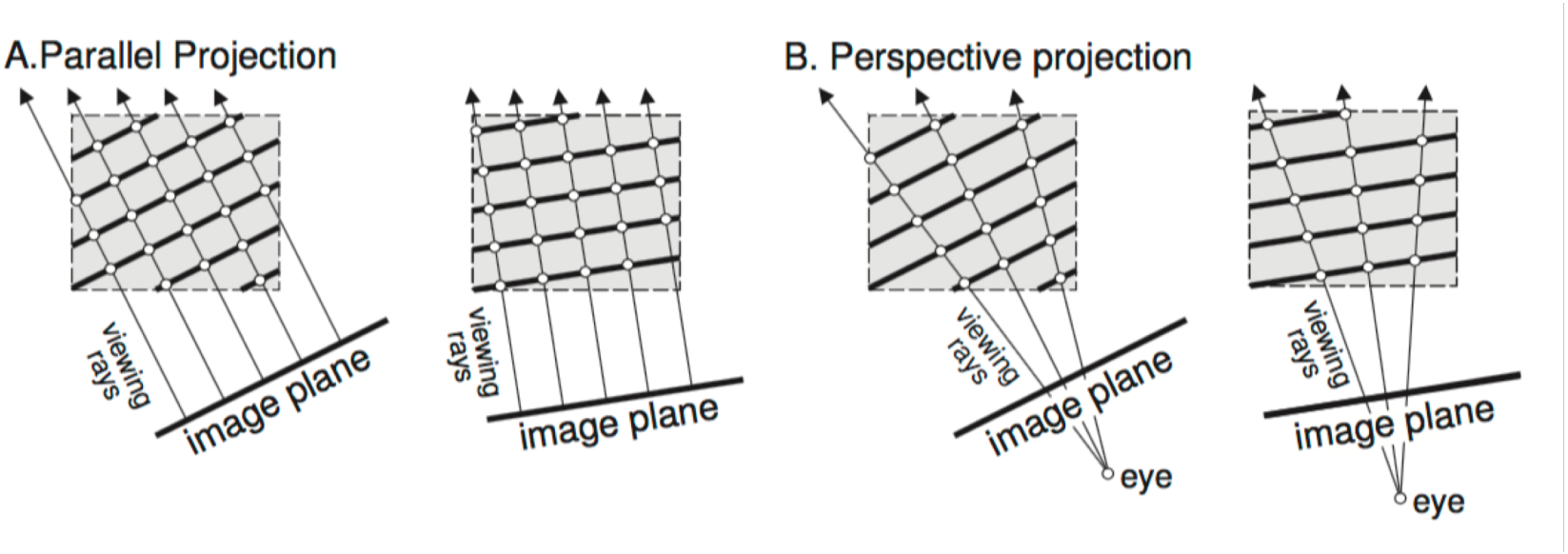


Image-order Approach

- Ray-casting approach
  - Simple implementation
  - Very flexible
  - Correct perspective projection

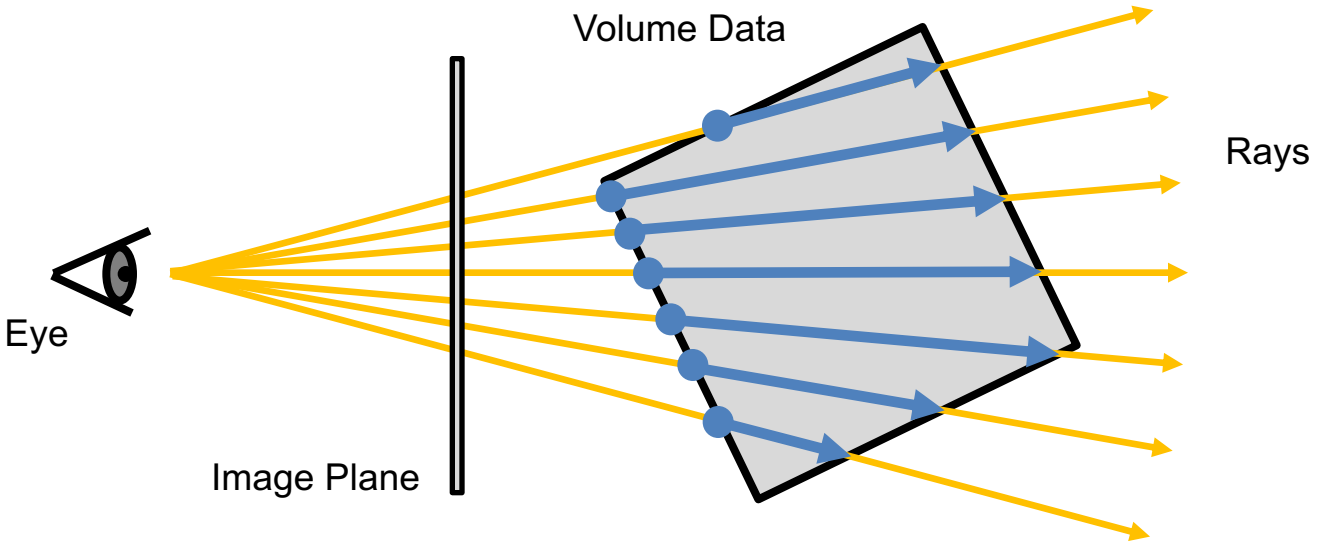
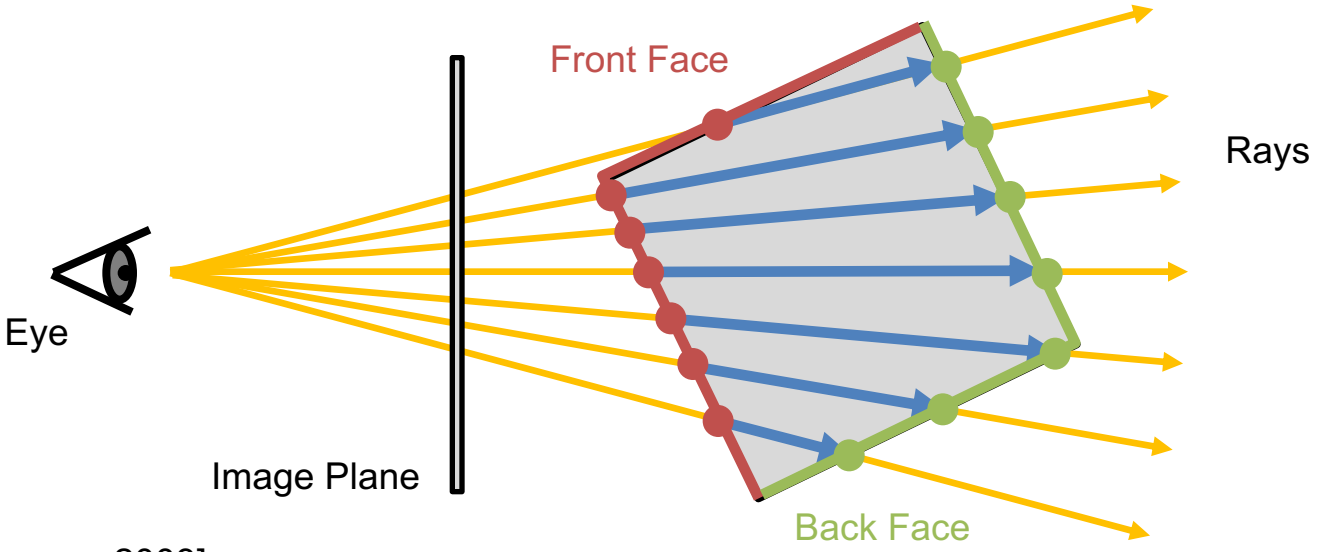


Image-order Approach

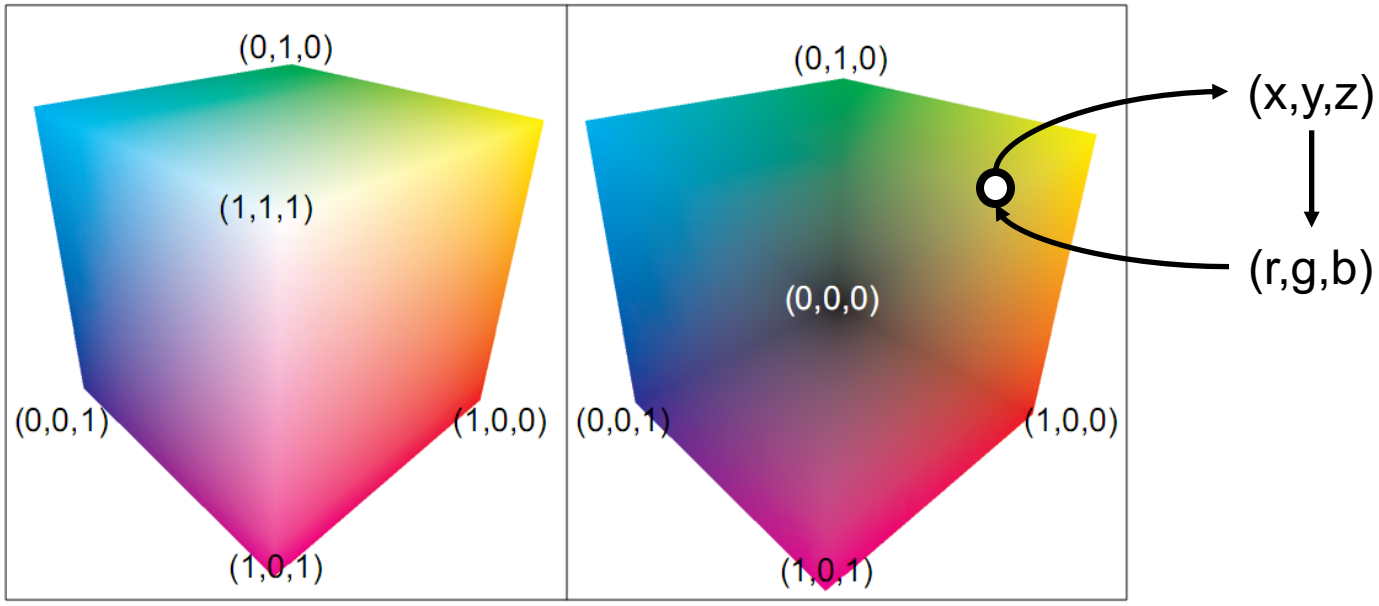
- GPU accelerated ray-casting approach
  - Rasterize front and back faces of volume bounding box.
  - Entry points of ray-casting are located on the front faces.
  - Exit points of ray-casting are located on the back faces.
  - Direction vectors can be calculated by back – front faces.



[Krugger and Westermann, 2003]

Image-order Approach

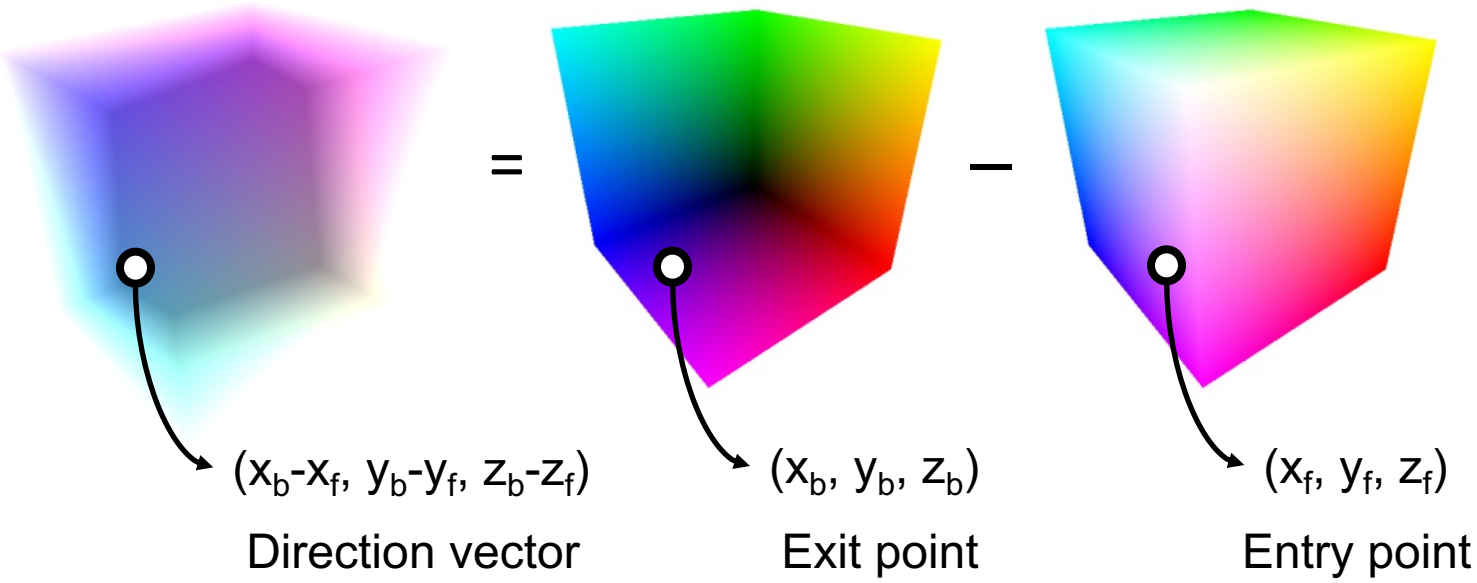
- Positions are stored in 2D Textures for the front and back faces



[Krugger and Westermann, 2003]

Image-order Approach

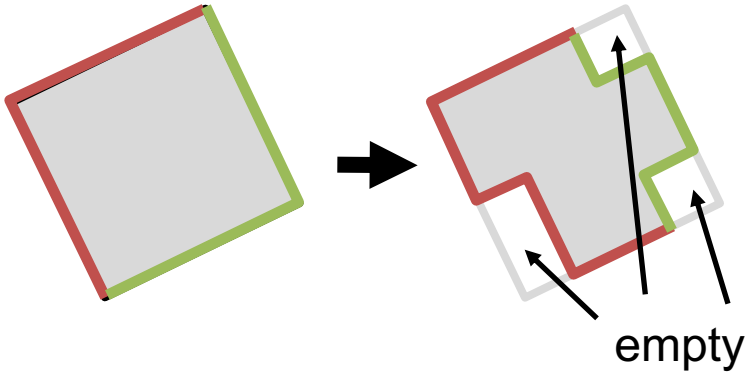
- Ray direction = Back faces – Front faces

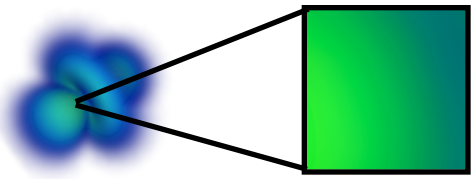


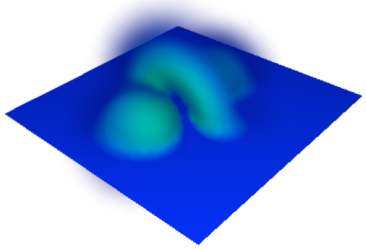
[Krugger and Westermann, 2003]

Image-order Approach

- Optimizations
  - Early Ray Termination
  - Empty Space Skipping
  - …
- Enhancements
  - Moving into the volume
  - Rendering with geometry data
  - …

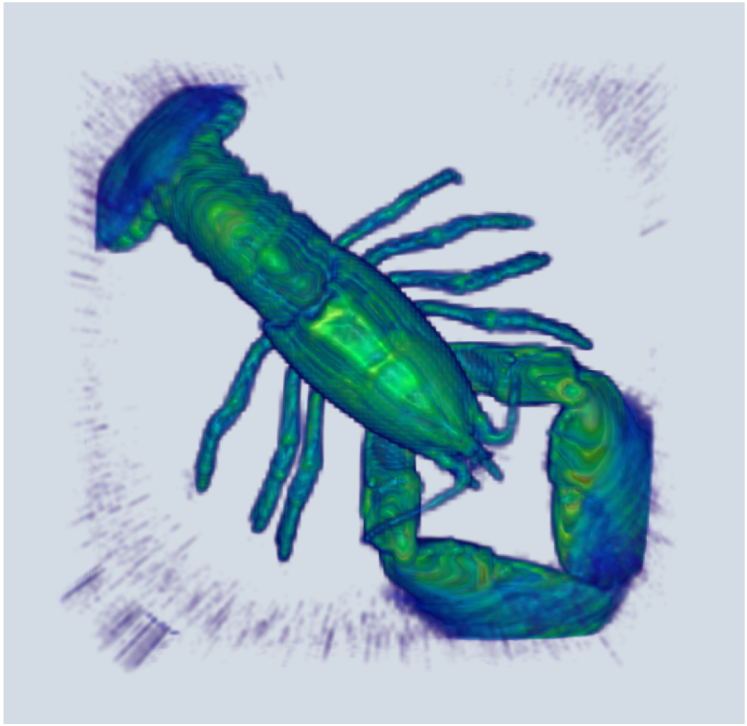






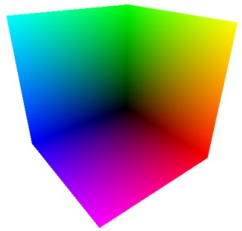
Ex01: Ray-casting Rendering

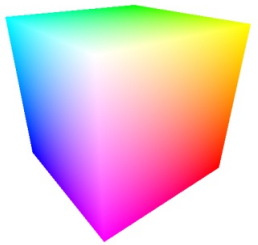
- Rendering a volume data by ray-casting approach.
  - Download
    - w11_main_ex01.js
    - w11_index_ex01.html
    - three.min.js
    - TrackballControl.js
  - Open
    - w11_index_ex01.html



Ex01: Simple Ray-casting

- Two-pass rendering
  - First pass
    - Render the back faces of the bounding box to the off-screen buffer
  - Second pass
    - Render the front faces of the bounding box
    - Perform the ray-casting iterations
    - Draw the final image to the screen





Ex01: Simple Ray-casting

- First pass
  - Render target for the off-screen rendering

```
var exit_texture = new THREE.WebGLRenderTarget(
    screen.width, screen.height,
    {
        minFilter: THREE.LinearFilter,
        magFilter: THREE.LinearFilter,
        wrapS: THREE.ClampToEdgeWrapping,
        wrapT: THREE.ClampToEdgeWrapping,
        format: THREE.RGBFormat,
        type: THREE.FloatType,
        generateMipmaps: false
    }
);
```

Ex01: Simple Ray-casting

- First pass
  - Scene for the off-screen rendering

```
var exit_buffer = new THREE.Scene();
```

  - Off-screen rendering

```
function render() {
    ...
    renderer.render( exit_buffer, screen.camera, exit_texture, true);
    ...
}
```

Ex01: Simple Ray-casting

- First pass
  - ShaderMaterial for the off-screen rendering

```
var bounding_material= new THREE.ShaderMaterial( {
     vertexShader: document.getElementById(‘bounding.vert').textContent;
     fragmentShader: document.getElementById(‘bounding.frag').textContent,
     side: THREE.BackSide
});
```

Ex01: Simple Ray-casting

- First pass
  - Mesh for the off-screen rendering

```
var bounding_geometry = BoundingBoxGeometry( volume );

var bounding_mesh = new THREE.Mesh( bounding_geometry, bounding_material );
```

  - Add the mesh to the scene

```
exit_buffer.add( bounding_mesh );
```

Ex01: Simple Ray-casting

- Second pass
  - ShaderMaterial for the screen

```
var raycaster_metrial = new THREE.ShaderMaterial( {
     vertexShader: document.getElementById( ‘raycaster.vert' ).textContent,
     fragmentShader: document.getElementById( ‘raycaster.frag' ).textContent,
     side: THREE.FrontSide,
     uniforms: {
          volume_resolution:  { type: "v3", value: volume.resolution },
          exit_point: { type: "t", value: exit_texture },
          volume_data: { type: "t", value: volume_texture },
          transfer_function_data:{type: "t", value: transfer_function_texture},
          light_position: {type: “v3", value: screen.light.position }
          camera_position: {type: “v3", value: screen.camera.position }
          background_color: {type: “v3“, value: new THREE.Vector3().fromArray(
screen.renderer.getClearColor().toArray() )
     }
});
```

Ex01: Simple Ray-casting

- Second pass
  - Scene for the on-screen rendering

```
screen.scene
```

  - On-screen rendering

```
function render() {
    ...
    renderer.render( screen.scene, screen.camera );
    ...
}
```

Ex01: Simple Ray-casting

- Second pass
  - Mesh for the on-screen rendering

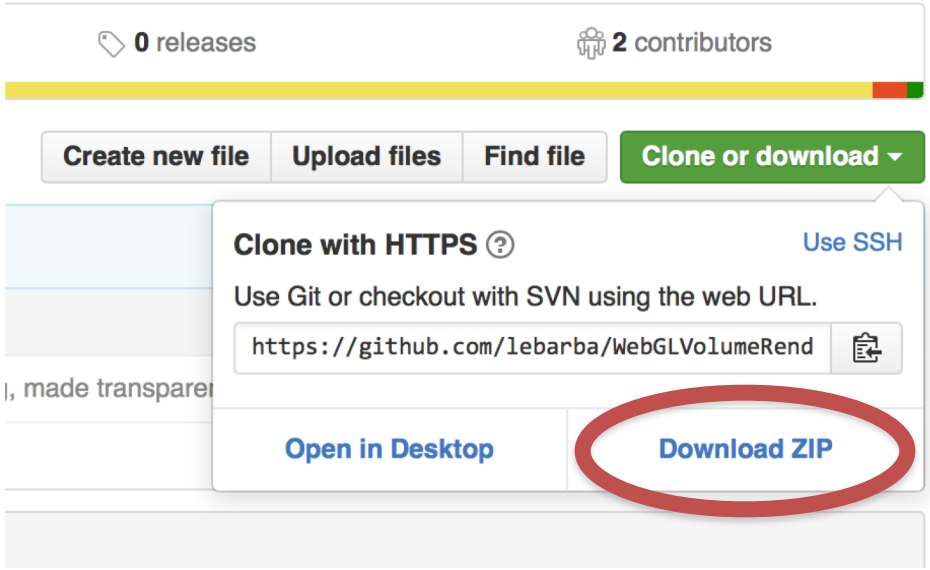
```
var raycaster_mesh = new THREE.Mesh( bounding_geometry, raycaster_material );
```

  - Add the mesh to the scene

```
screen.scene.add( raycaster_mesh );
```

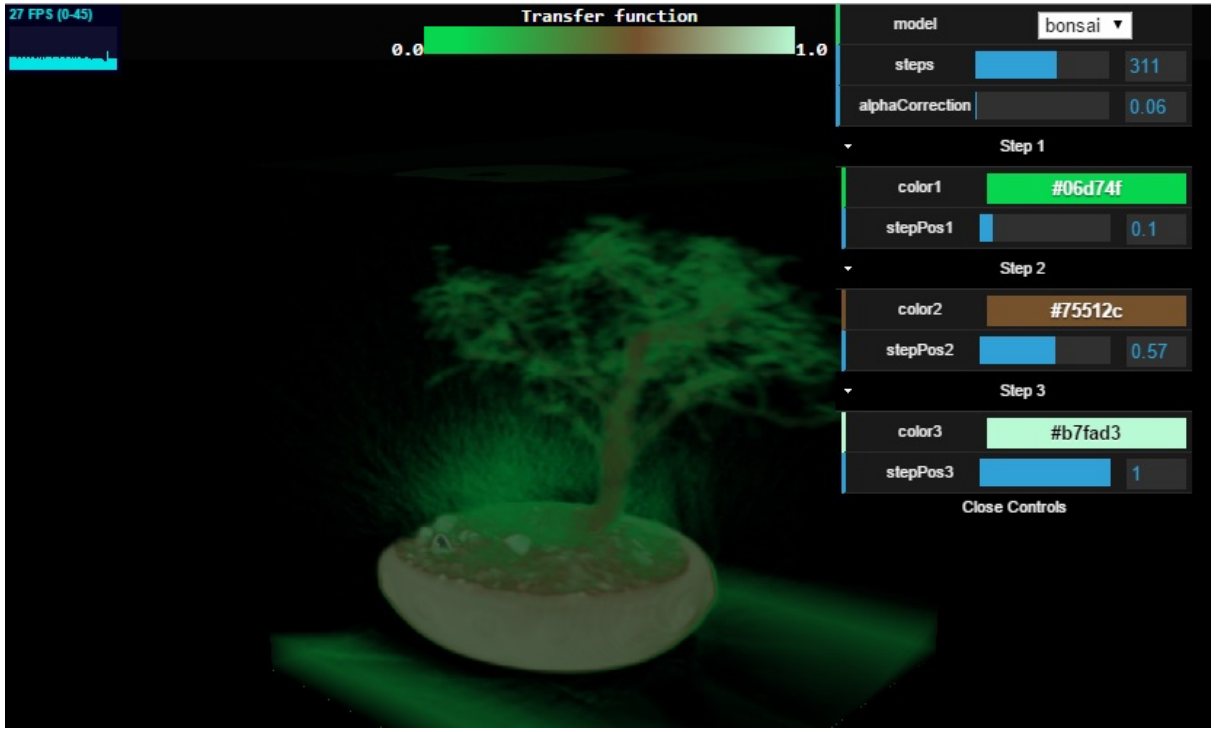
Ex02: Ray-casting with Simple UI

- Rendering a volume data by ray-casting approach.
  - WebGL Volume Rendering
    https://github.com/lebarba/WebGLVolumeRendering
  - Download
    - Clone or download
    - Download ZIP
  - Open
    - Web/Index.html



Ex02: Ray-casting with Simple UI

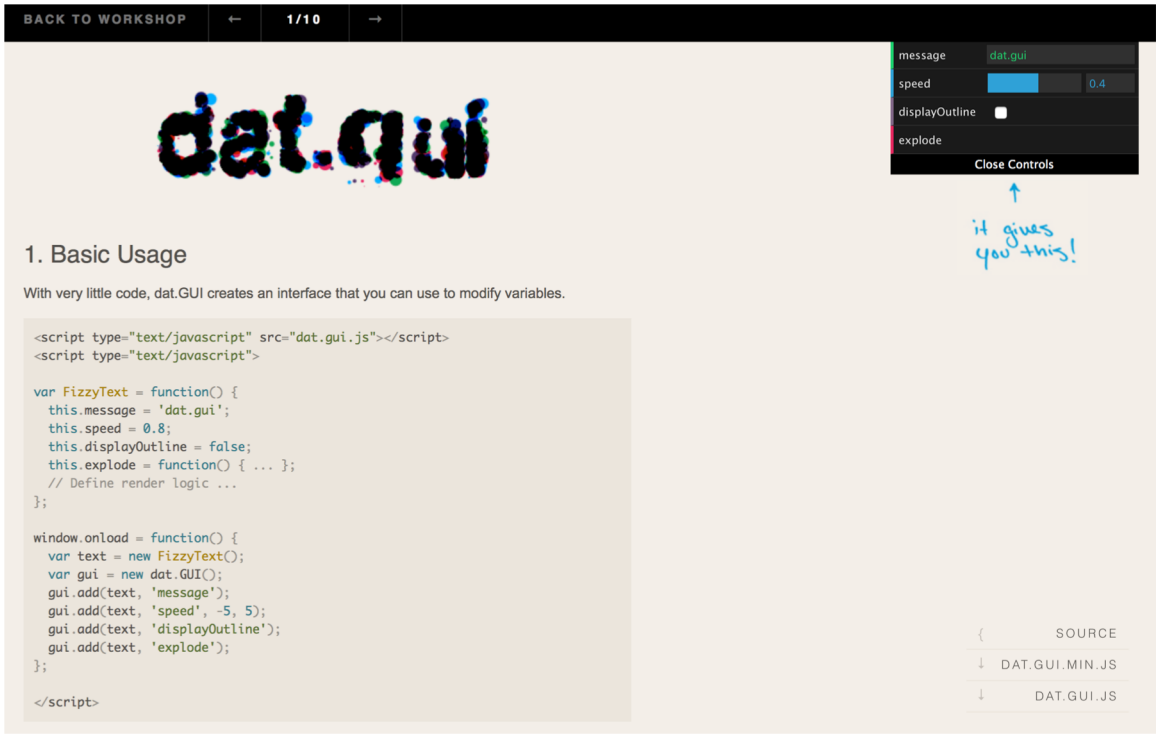
- Rendering a volume data by ray-casting approach.



Ex02: Ray-casting with Simple UI

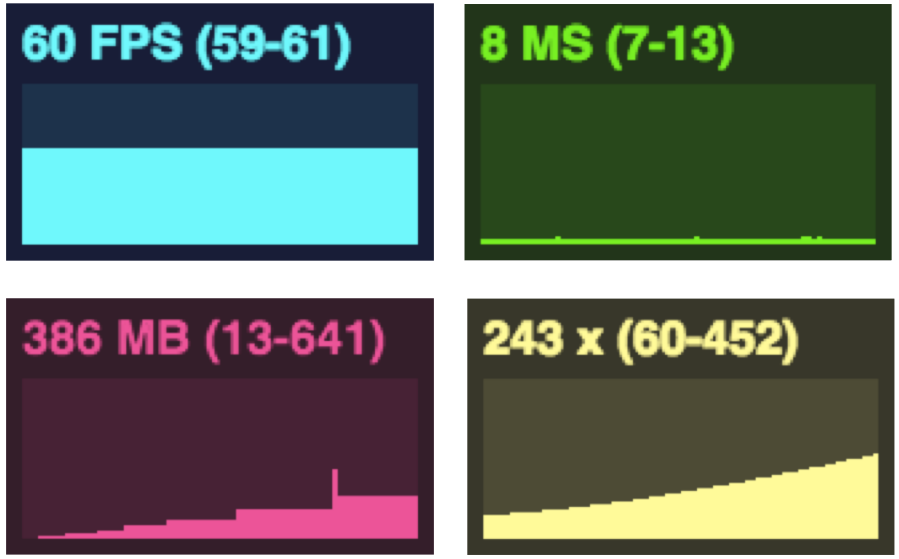
- User Interface
  - dat.UI

    https://workshop.chromeexperiments.com/examples/gui/#1--Basic-Usage



Ex02: Ray-casting with Simple UI

- User Interface
  - stats.js
    https://github.com/mrdoob/stats.js/



Polling

- Take the poll
  - Student ID Number
  - Name