Analysis of Monopoles in Multi-Step Spontaneous Gauge Symmetry Breaking

Fukutarou MIYA

Collaborators: Ryosuke SATO, Soichiro SHIMAMORI

Ongoing Research

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Het Camp

Introduction

Various Beyond Standard Model (BSM) scenarios include monopoles:

- Candidates for dark matter

  ’t Hooft–Polyakov monopoles originating from the dark sector contribute to the relic dark matter abundance

  V.V. Khoze & G. Ro (2014).
  Yang Bai et al. (2014).
  S. Baek et al. (2020)…

- Physics involving axions

  Contributions to axion mass via the Witten effect

  M. Kawasaki et al. (2016).
  A. Banerjee & M.A. Buen-Abad (2025)…

- Possible origin of ultra-high-energy cosmic rays (UHECRs)

  Monopole annihilation and radiation from accelerated monopoles

  E. Huguet & P. Peter (2000)
  Ł. Bratek & J. Jałocha (2025)…

  =Cosmic rays $(> 10^{18}\,\mathrm{eV})$

Can we consider magnetic monopoles within the SM?

Talk Overview

Indeed, The Cho–Maison monopole exists within the SM

However, the energy diverges at the spatial origin.

$$E_0 \propto \int_L^\infty \frac{\mathrm{d}r}{r^2}$$

SM description break down for $E > \dfrac{1}{L}$

➡ We aim to explore a UV completion of the Cho–Maison monopole

Our research

’t Hooft–Polyakov monopole serves as a UV completion of the Cho–Maison monopole.

[1] Y. M. Cho & D. Maison (1996).

Contents

- Review of magnetic monopoles
- UV completion of the Cho–Maison monopole

$$\mathrm{SU(2)_L \times U(1)_Y \longrightarrow U(1)_{EM}}$$

Cho–Maison monopoles

Y. M. Cho & D. Maison (1996).

Hedgehog configuration

$$H = \frac{1}{\sqrt{2}}\rho(r)\xi, \quad \xi = \mathrm{i}\begin{pmatrix} \sin(\theta/2)\,\mathrm{e}^{-\mathrm{i}\varphi} \\ -\cos(\theta/2) \end{pmatrix}$$

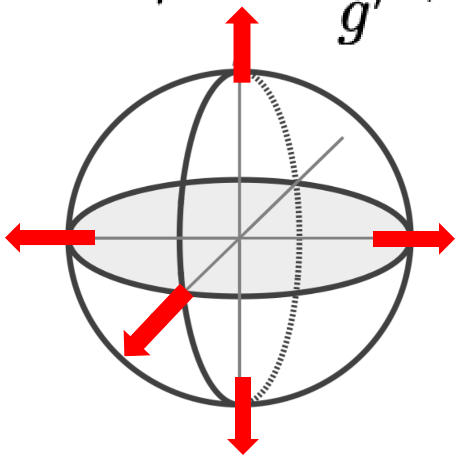
This configuration is undefined at $\theta = \pi$

$$W_\mu^a = \frac{1}{g}\left[f(r) - 1\right] f^{abc}\hat{\phi}^b \partial_\mu \hat{\phi}^c \qquad \hat{\phi} = \xi^\dagger \sigma \xi = -\hat{r}$$

$$B_\mu = -\frac{1}{g'}\left(1 - \cos\theta\right)\partial_\mu \varphi$$

Same configuration as the Dirac monopole

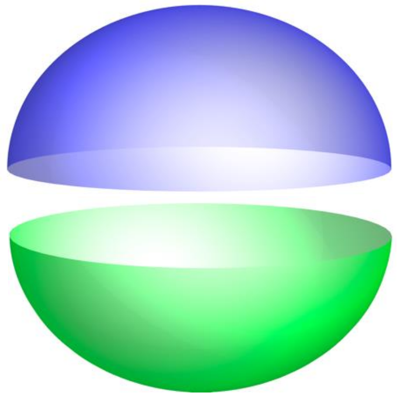
The vacuum manifold can be covered by two coordinate patches.



$$E = E_0 + E_1$$

Energy diverges at the spatial origin

$$\boxed{E_0 \propto \int_0^\infty \frac{1}{r^2}}$$



Existence of solutions: [Y. Yang (2001)]

Stability of solutions: [R. Gervalle & M.S. Volkov (2022)]

- 't Hooft–Polyakov monopole

$$\mathrm{SU}(2) \to \mathrm{U}(1)$$

- Is a nonsingular configuration
- Covers vacuum with a single patch
- Has a finite energy solution

Stability is guaranteed

$$\langle \Phi \rangle : S^2_\infty \to S^2_{\text{vacuum}}$$

$$\Pi_2(\mathcal{M}_{\text{vac}}) \neq \{e\}$$

The Standard Model does not contain the 't HP monopole.

$$\Pi_2\left(\frac{\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}}{\mathrm{U}(1)_\mathrm{EM}}\right) = \{e\}$$

- Cho–Maison monopole

$$\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y} \longrightarrow \mathrm{U}(1)_\mathrm{EM}$$

- Is a singular configuration
- Covers vacuum with two patches
- Energy diverges at the spatial origin

Key Question

How should we understand the behavior of the Cho–Maison monopole at high energies?

At high energies: ’t Hooft–Polyakov monopole
At low energies: Cho–Maison monopole

→ Our research investigates a model that reproduces this scenario.

UV Completion of the Cho–Maison Monopole

J. C. Pati & A. Salam (1974).

- Pati-Salam Model

$$\mathrm{SU}(4)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R}$$
$$\downarrow \langle \Phi \rangle$$
$$\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$$
$$\downarrow \langle H \rangle$$
$$\mathrm{SU}(3)_\mathrm{C} \times \mathrm{U}(1)_\mathrm{EM}$$

Hierarchy of VEV

$$\langle \Phi \rangle \gg \langle H \rangle$$

Conditions for the Existence of the 't HP Monopole

$$\Pi_2(G/H) = \mathbb{Z}$$

In this effective theory, the monopole appears as a Cho–Maison monopole.

Goal

Confirm that the scalar field $H(x)$ behaves as a 't HP Monopole in the high-energy regime.

Next Steps

- Assume a hedgehog configuration and solve the equations of motion
- Investigate the behavior of the scalar field $H(x)$ in high- and low-energy regions

● Pati-Salam Model

$$V(H,\Phi) = -\mu_\Phi^2 \,\mathrm{Tr}\,\Phi^\dagger\Phi - \mu_H^2 \,\mathrm{Tr}\,H^\dagger H + +\lambda_1 \left(\mathrm{Tr}\,\Phi^\dagger\Phi\right)^2 + \lambda_2 \,\mathrm{Tr}\,\Phi^\dagger\Phi\,\Phi^\dagger\Phi + \lambda_3 \left(\mathrm{Tr}\,H^\dagger H\right)^2 + \lambda_4 \,\mathrm{Tr}\,H^\dagger H\,H^\dagger H$$

Hedgehog configuration $\hat{r}^a = \frac{x^a}{r} = \left(\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta\right)$

Scalar field

$$\Phi(4,1,2) \quad \Phi \;\to\; \Phi' \;=\; U_4\,\Phi\,U_R^\dagger \quad \langle\Phi\rangle = v_\Phi \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Phi(x) = v_\Phi \chi(r) \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \cos\theta & e^{-\mathrm{i}\varphi}\sin\theta \\ e^{\mathrm{i}\varphi}\sin\theta & -\cos\theta \end{pmatrix}$$

$$H(1,2,2) \quad H \to H' = U_\mathrm{L} H U_\mathrm{R}^\dagger \quad \langle H\rangle = v_H \begin{pmatrix} 0 & \\ & 1 \end{pmatrix}$$

$$H(x) = v_H h(r) \left(1_2 + \frac{x^a\sigma^a}{r}\right) = v_H h(r) \begin{pmatrix} 1+\cos\theta & e^{-\mathrm{i}\varphi}\sin\theta \\ e^{\mathrm{i}\varphi}\sin\theta & 1-\cos\theta \end{pmatrix}$$

Gauge field

$$G_{4i}^a = \frac{1}{g_4}\left(-f_4(r)+1\right)\,\varepsilon_{aij}\frac{x^j}{r^2}$$

$$W_{\mathrm{R}i}^a = \frac{1}{g_\mathrm{R}}\left(-f_\mathrm{R}(r)+1\right)\,\varepsilon_{aij}\frac{x^j}{r^2}$$

$$W_{\mathrm{L}i}^a = \frac{1}{g_\mathrm{L}}\left(-f_\mathrm{L}(r)+1\right)\,\varepsilon_{aij}\frac{x^j}{r^2}$$

$$G_{40}^a = W_{\mathrm{L}0}^a = W_{\mathrm{R}0}^a = 0$$

$$\begin{gathered} \mathrm{SU}(4)_\mathrm{C}\times\mathrm{SU}(2)_\mathrm{L}\times\mathrm{SU}(2)_\mathrm{R} \\ \downarrow\ \langle\Phi\rangle \\ \mathrm{SU}(3)_\mathrm{C}\times\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)_\mathrm{Y} \\ \downarrow\ \langle H\rangle \\ \mathrm{SU}(3)_\mathrm{C}\ \times\ \mathrm{U}(1)_\mathrm{EM} \end{gathered}$$

$$\langle\Phi\rangle \gg \langle H\rangle$$

Assume this spherically symmetric field behavior and solve the EoM

- # Pati-Salam Model

$$\Phi(x) = v_\Phi \chi(r) \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \cos\theta & e^{-\mathrm{i}\varphi}\sin\theta \\ e^{\mathrm{i}\varphi}\sin\theta & -\cos\theta \end{pmatrix}$$

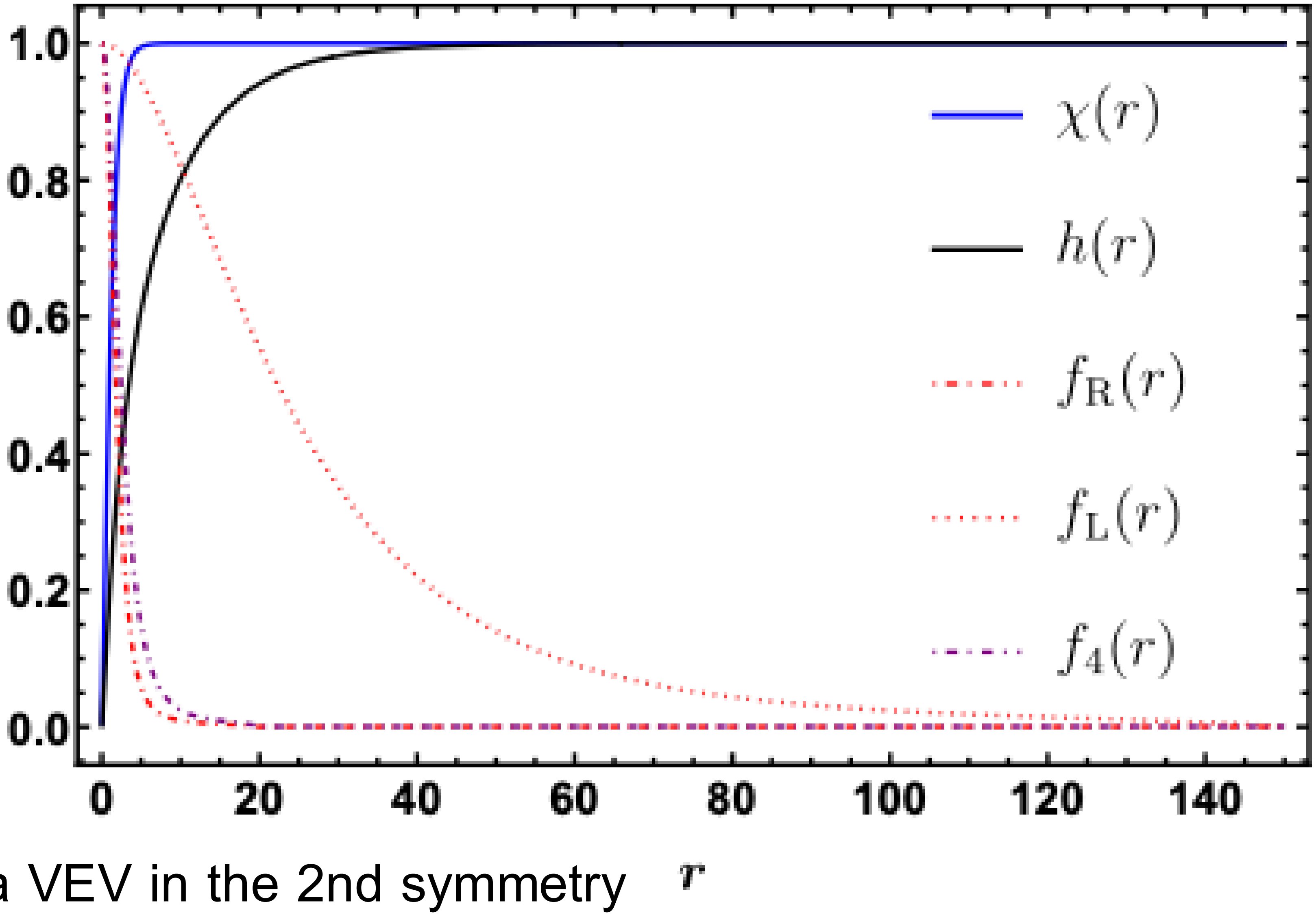
$$H(x) = v_H h(r) \begin{pmatrix} 1+\cos\theta & e^{-\mathrm{i}\varphi}\sin\theta \\ e^{\mathrm{i}\varphi}\sin\theta & 1-\cos\theta \end{pmatrix}$$

$$G^a_{4i} = \frac{1}{g_4}\left(-f_4(r)+1\right)\varepsilon_{aij}\frac{x^j}{r^2}$$

$$W^a_{\mathrm{R}i} = \frac{1}{g_\mathrm{R}}\left(-f_\mathrm{R}(r)+1\right)\varepsilon_{aij}\frac{x^j}{r^2}$$

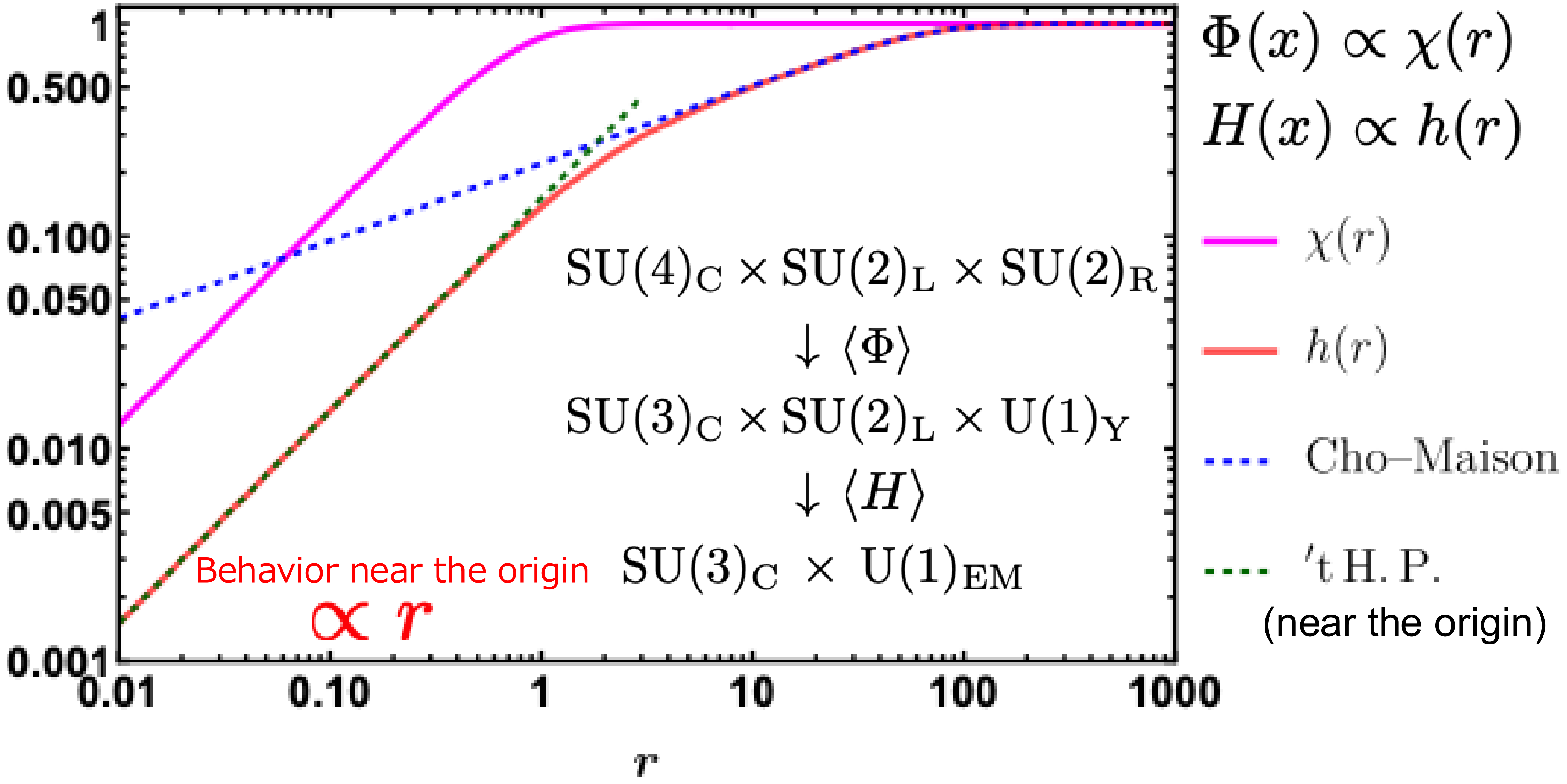
$$W^a_{\mathrm{L}i} = \frac{1}{g_\mathrm{L}}\left(-f_\mathrm{L}(r)+1\right)\varepsilon_{aij}\frac{x^j}{r^2}$$

Solutions to the EoM

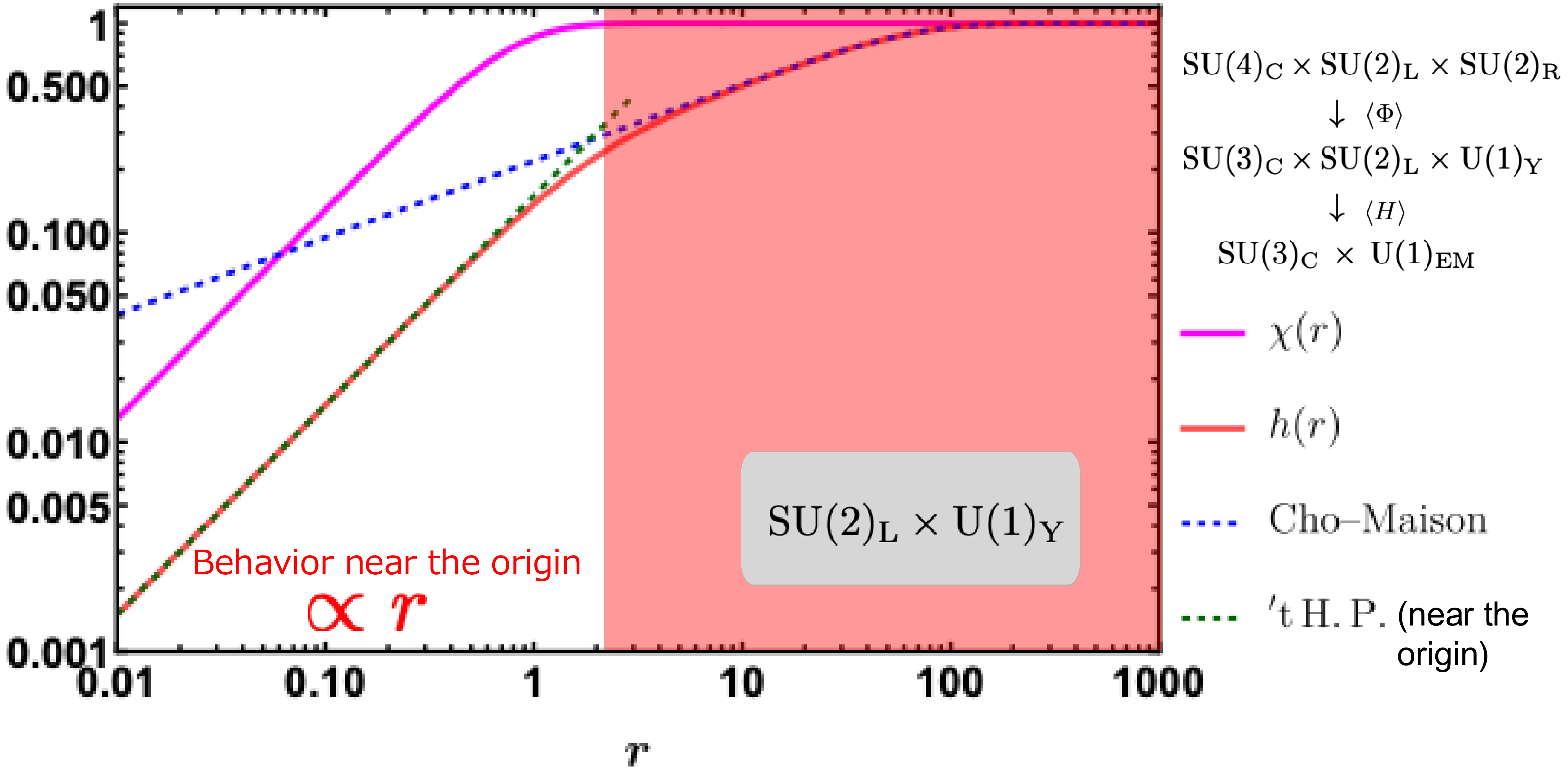


How does $h(r)$, acquiring a VEV in the 2nd symmetry breaking, behave at high energies?

● Pati-Salam Model



● Pati-Salam Model



● Summary

– Cho–Maison monopole exists within the Standard Model, but its high-energy behavior is not described by the SM

– We studied the Pati–Salam model realizing the scenario where the ’t Hooft–Polyakov monopole provides a UV completion of the Cho–Maison monopole.

● Future Directions

– Phenomenological applications

– Classification of gauge symmetry breaking patterns and scalar representations reproducing the Cho–Maison monopole

– Systematic organization of the relation between gauge groups and magnetic charge