Satellite Lab1

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1 Introduction

2 Orignal Data

2.1 ITRF2008 IGS station

The ITRF is The International Reference Frame, ITRF2008 is the new realization of the International Terrestrial Reference System. The ITRF2008 uses as input data time series of station positions and EOPs provided by the Technique Centers of the four space geodetic techniques (GPS, VLBI, SLR, DORIS). Based on completely reprocessed solutions of the four techniques, the ITRF2008 is expected to be an improved solution compared to ITF2005[1].

In the file "ITRF2008_GNSS.SSC.txt", we can find the coordinates of different stations at epoch 2005.0.

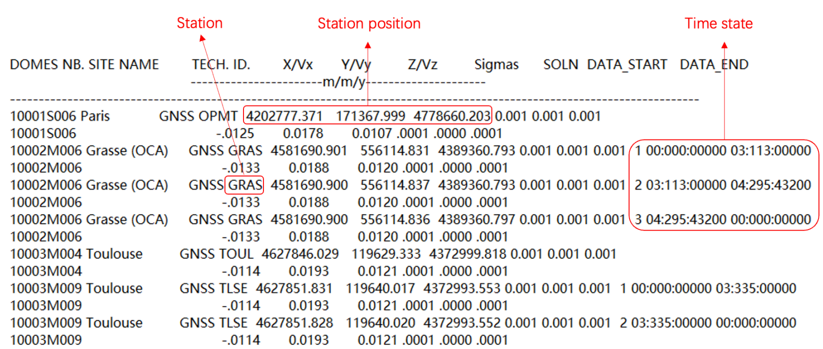


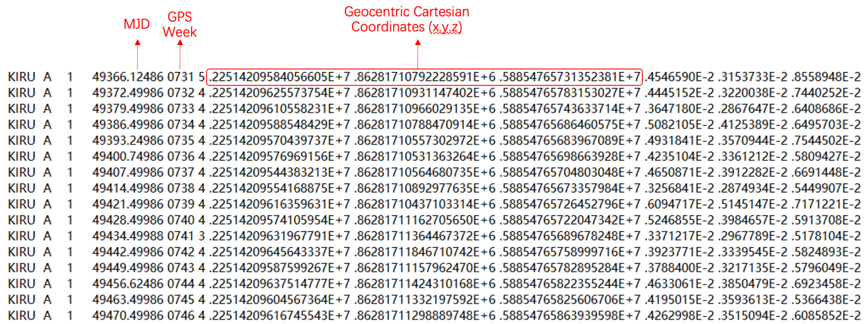
Figure 1: ITRF2008_GNSS.ssc.txt Description

Time state: ?

2.2 Station Obversations

We were responsible for the computation of the positions and movements of three measurement stations: KIRU, MORP, and REYK. The locations are illustrated in the following figure:

An example of the observation file for each data set is provided below, including two time formats and XYZ coordinates.

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KIRU  A   1   49366.12486 0731 5 .22514209584056605E+7 .86281710792228591E+6 .58854765731352381E+7 .4546590E-2 .3153733E-2 .8558948E-2
KIRU  A   1   49372.49986 0732 4 .22514209625573754E+7 .86281710931147402E+6 .58854765783153027E+7 .4445152E-2 .3220038E-2 .7440252E-2
KIRU  A   1   49379.49986 0733 4 .22514209610558231E+7 .86281710966029135E+6 .58854765743633714E+7 .3647180E-2 .2867647E-2 .6408686E-2
KIRU  A   1   49386.49986 0734 4 .22514209588548429E+7 .86281710788470914E+6 .58854765686460575E+7 .5082105E-2 .4125389E-2 .6495703E-2
KIRU  A   1   49393.24986 0735 4 .22514209570439737E+7 .86281710557302972E+6 .58854765683967089E+7 .4931841E-2 .3570944E-2 .7544502E-2
KIRU  A   1   49400.74986 0736 4 .22514209576969156E+7 .86281710531363264E+6 .58854765698663928E+7 .4235104E-2 .3361212E-2 .5809427E-2
KIRU  A   1   49407.49986 0737 4 .22514209544383213E+7 .86281710564680735E+6 .58854765704803048E+7 .4650871E-2 .3912282E-2 .6691448E-2
KIRU  A   1   49414.49986 0738 4 .22514209554168875E+7 .86281710892977635E+6 .58854765673357984E+7 .3256841E-2 .2874934E-2 .5449907E-2
KIRU  A   1   49421.49986 0739 4 .22514209616359631E+7 .86281710437103314E+6 .58854765726452796E+7 .6094717E-2 .5145147E-2 .7171221E-2
KIRU  A   1   49428.49986 0740 4 .22514209574105954E+7 .86281711162705650E+6 .58854765722047342E+7 .5246855E-2 .3984657E-2 .5913708E-2
KIRU  A   1   49434.49988 0741 3 .22514209631967791E+7 .86281711364467372E+6 .58854765689678248E+7 .3371217E-2 .2967789E-2 .5178104E-2
KIRU  A   1   49442.49986 0742 4 .22514209645643337E+7 .86281711846710742E+6 .58854765758999716E+7 .3923771E-2 .3339545E-2 .5824893E-2
KIRU  A   1   49449.49986 0743 4 .22514209587599267E+7 .86281711157962470E+6 .58854765782895284E+7 .3788400E-2 .3217135E-2 .5796049E-2
KIRU  A   1   49456.62486 0744 4 .22514209637514777E+7 .86281711424310168E+6 .58854765822355244E+7 .4633061E-2 .3850479E-2 .6923458E-2
KIRU  A   1   49463.49986 0745 4 .22514209604567364E+7 .86281711332197592E+6 .58854765825606706E+7 .4195015E-2 .3593613E-2 .5366438E-2
KIRU  A   1   49470.49986 0746 4 .22514209616745543E+7 .86281711298889748E+6 .58854765863939598E+7 .4262998E-2 .3515094E-2 .6085852E-2
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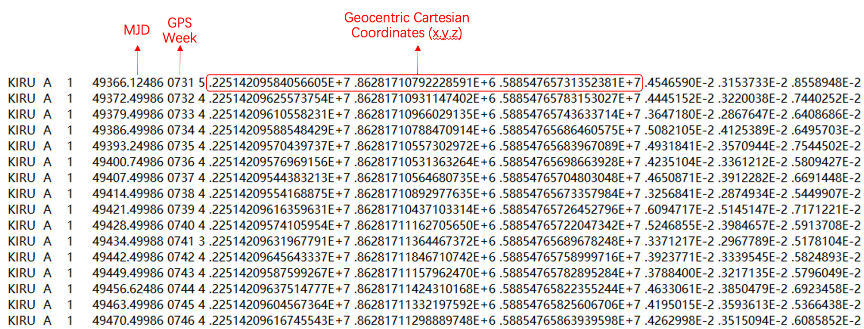
Figure 2: station.xyz.txt Description

2.3 NUVEL 1A Model

NUVEL(Northeast University Velocity) is a the collective term for geophysical Earth models that describes observable continental movements through a dynamic theory of plaet tectonics.

The "NNR_NUVEL1A.txt" gives the rotation referred to epoch t_0. The file contains the following data, where the leftmost column represents the station name, and in that row, the angular velocity changes in three directions are provided (unit: radians per million years or rad/My).


MJD
GPS Week
Geocentric Cartesian Coordinates (x,y,z)

```
KIRU  A   1   49366.12486 0731 5 .22514209584056605E+7 .86281710792228591E+6 .58854765731352381E+7 .4546590E-2 .3153733E-2 .8558948E-2
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```

Figure 3: NUVEL1A.txt Description

2.4 GIA Models

2.5 Other data

2.6 Matlab Code

3 Methodology

3.1 Transoformation to LHS

[Geocentric cartesian coordinate system] is a three-dimensional, earth-centered reference system in which locations are identified by their x, y, and z values. The x-axis is in the equatorial plane and intersects the prime meridian (usually Greenwich). The y-axis is also in the equatorial plane; it lies at right angles to the x-axis and intersects the 90-degree meridian. The z-axis coincides with the polar axis and is positive toward the north pole. The origin is located at the center of the sphere or spheroid.

[Local horizontal system] uses the Cartesian coordinates(East,Nort,Up) to represent position relative to a local origin. The local origin is described by the geodetic coordinates.

The initial coordinates are in the geocentric Cartesian coordinate system and need to be transformed into representation in the local horizontal coordinate system. In this project, we use two angles and the ITRF2008 point positions as the original point,

Calculate the angle according to stations' geodetic coordinates:

$$\lambda = \arctan \frac{y}{x}$$

,

$$\varphi = \arctan \frac{2}{\sqrt{x^2 + y^2}}$$

Then we can get the rotation matrix:

$$R_2(\delta) = \begin{pmatrix} \cos\delta & 0 & -\sin\delta \\ 0 & 1 & 0 \\ \sin\delta & 0 & \cos\delta \end{pmatrix} \quad R_3(\delta) = \begin{pmatrix} \cos\delta & \sin\delta & 0 \\ -\sin\delta & \cos\delta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Transformation of coordinates:

$$\begin{pmatrix} x_{up} \\ x_{east} \\ x_{north} \end{pmatrix} = R_2(-\varphi^0) R_3(\lambda^0) \left(\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} - \begin{pmatrix} x_1^0 \\ x_2^0 \\ x_3^0 \end{pmatrix} \right)$$

$x^0$are the stations' geodetic coordinates, and x is observations in file '.xyz'. Notice that we also can directly use the longitude and latitude of stations provided in the file "Discontinuities_CONFIRMED.snx".

In terms of velocity, its transformation into LHS only requires multiplication by a rotation matrix.

3.2 Least Square Adjustment for Parameters Estimation

For time series,

$$y(t) = \beta_1 + \beta_2 t + \beta_3 \text{cos}\omega t + \beta_4 \text{sin}\omega t$$

among which β_3 and β_4 are total amplitude of annual, and β_2 is linear trend; so we can build model like:

$$\begin{pmatrix} y(t_1) \\ \vdots \\ y(t_n) \end{pmatrix} = \begin{pmatrix} 1 & t_1 & \cos\omega t_1 & \sin\omega t_1 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & t_n & \cos\omega t_n & \sin\omega t_n \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix}$$

We can simppfit the model like:

$$Y = X\beta + \varepsilon$$

where Y is the observations, X is the design matrix, β is the parameters, and ε is the noise.

According to least square, minimize the noise, derivative the square of noise and set it to zero so we get:

$$\beta = (X^T X)^{-1} X^T Y$$

through this we can get the estimated parameters.

3.3 Model of Plate Tectonics

The movement of any plate on a spherical Earth can be described through a rotation around the Euler pole:

$$\underline{\Omega} = (\Omega_1, \Omega_2, \Omega_3)^T$$

In point $\underline{x}_0 = (x, y, z)^T$ the velocity vector v is obtained by:

$$\underline{v} = \underline{\Omega} \times \underline{x}_0 = \begin{pmatrix} 0 & -\Omega_z & \Omega_y \\ \Omega_z & 0 & -\Omega_x \\ -\Omega_y & \Omega_x & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

3.4 Program Description

4 Results and Analysis

4.1 Time Series and Linear Trend

4.2 Comparison of horizontal movements

4.3 Comparison of vertical movements

5 Conclusion

References

[1] https://itrf.ign.fr/en/solutions/ITRF2008