

# Satellite Lab1

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## Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Data Description</b>	<b>1</b>
2.1	ITRF2008 IGS station . . . . .	1
2.2	Station GPS Observations . . . . .	2
2.3	NUVEL 1A Model . . . . .	2
2.4	Other data . . . . .	3
2.5	Matlab Code . . . . .	3
<b>3</b>	<b>Methodology</b>	<b>3</b>
3.1	Transformation to LHS . . . . .	3
3.2	Least Square Adjustment for Parameters Estimation . . . . .	5
3.3	Model of Plate Tectonics . . . . .	5
3.4	Program Description . . . . .	5
<b>4</b>	<b>Results and Analysis</b>	<b>5</b>
4.1	Time Series, Linear Trend and Residuals . . . . .	5
4.2	Comparison with the Plate Tectonics Model . . . . .	7
4.3	Comparison of vertical movements . . . . .	10
<b>5</b>	<b>Conclusion</b>	<b>10</b>

## 1 Introduction

This lab focuses on the geometry and kinematics of the polar region. And our group use the IGS products(station positions) to estimate the time series, horizontal and vertical velocities of three stations: KIRU, MORP, and REYK, whose locations are shown in the following figure: In the final part, we compare these result with the plate tectonics model NUVEL 1A and GIA models.

## 2 Data Description

### 2.1 ITRF2008 IGS station

The ITRF is The International Reference Frame, and ITRF2008 is a realization of the International Terrestrial Reference System that uses as input data time series of station positions and EOPs provided by the Technique Centers of the four space geodetic techniques (GPS, VLBI, SLR, DORIS).

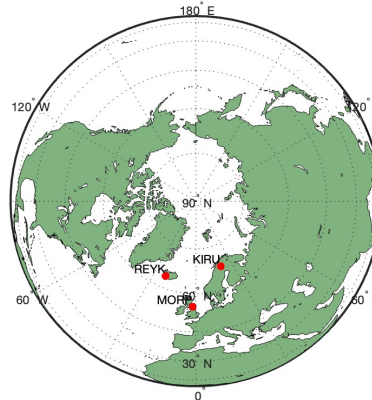


Figure 1: Station Positions

In the file "ITRF2008\_GNSS.SSC.txt", we can find the coordinates of different stations at epoch 2005.0. and Time is given as the yyyy.yyyy format, which is the number of decimal years:

DOMES NB.	SITE NAME	TECH.	ID.	X/Vx	Y/Vy	Z/Vz	Sigmas	SOLN	DATA_START	DATA_END
				m/m/y						
10001S006	Paris	GNSS	OPMT	4202777.371	171367.999	4778660.203	0.001 0.001 0.001			
10001S006				-.0125	0.0178	0.0107 .0001 .0000 .0001				
10002M006	Grasse (OCA)	GNSS	GRAS	4581690.901	556114.831	4389360.793	0.001 0.001 0.001	1	00:00:00000	03:113:00000
10002M006				-.0133	0.0188	0.0120 .0001 .0000 .0001				
10002M006	Grasse (OCA)	GNSS	GRAS	4581690.900	556114.837	4389360.793	0.001 0.001 0.001	2	03:113:00000	04:295:43200
10002M006				-.0133	0.0188	0.0120 .0001 .0000 .0001				
10002M006	Grasse (OCA)	GNSS	GRAS	4581690.900	556114.836	4389360.797	0.001 0.001 0.001	3	04:295:43200	00:000:00000
10002M006				-.0133	0.0188	0.0120 .0001 .0000 .0001				
10003M004	Toulouse	GNSS	TOUL	4627846.029	119629.333	4372999.818	0.001 0.001 0.001			
10003M004				-.0114	0.0193	0.0121 .0001 .0000 .0001				
10003M009	Toulouse	GNSS	TLSE	4627851.831	119640.017	4372993.553	0.001 0.001 0.001	1	00:000:00000	03:335:00000
10003M009				-.0114	0.0193	0.0121 .0001 .0000 .0001				
10003M009	Toulouse	GNSS	TLSE	4627851.828	119640.020	4372993.552	0.001 0.001 0.001	2	03:335:00000	00:000:00000
10003M009				-.0114	0.0193	0.0121 .0001 .0000 .0001				

Figure 2: ITRF2008\_GNSS.ssc.txt Description

## 2.2 Station GPS Observations

We were responsible for the computation of the positions and movements of three measurement stations: KIRU, MORP, and REYK. The locations are illustrated in the following figure:

An example of the observation file for each data set is provided below, including two time formats and XYZ coordinates.

And the file Discontinuities\_snx provides the discontinuity information in the positions' time series. The reasons for that include change of antenna and receiver, earthquake and so on. In this example(station: REYK), the discontinuity happened three times due to earthquake, antenna change and unknown reason.

## 2.3 NUVEL 1A Model

NUVEL(Northeast University Velocity) is a the collective term for geophysical Earth models that describes observable continental movements through a dynamic theory of plaet tectonics.

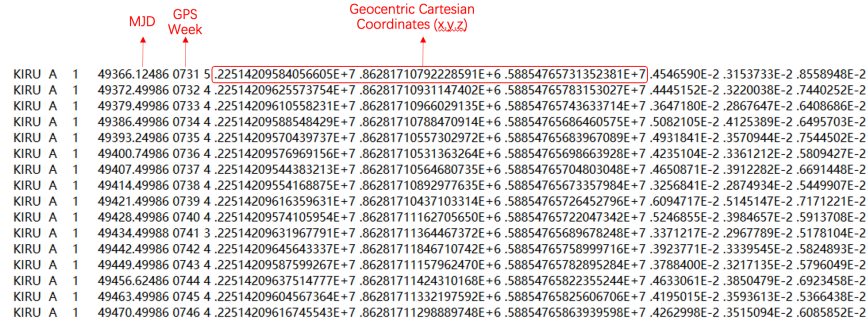


Figure 3: station.xyz Description

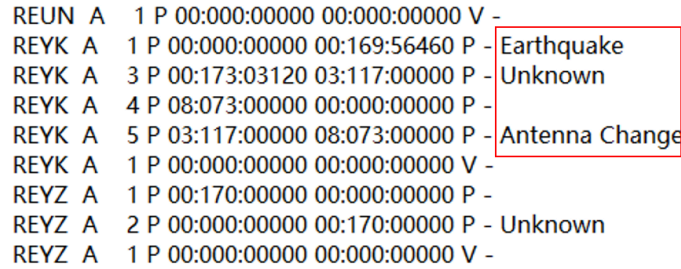


Figure 4: Discontinuities in time series

The "NNR\_NUVEL1A.txt" gives the rotation referred to epoch  $t_0$ . The file contains the following data, where the leftmost column represents the station name, and in that row, the angular velocity changes in three directions are provided (unit: radians per million years or  $rad/My$ ).

## 2.4 Other data

coast30.mat: coast lines for visualization

crust\_ICE4G.mat, crust\_ICE5G.mat: Global grids with vertical crustal deformations rates [mm/year]. These matrices are given from  $89.5^\circ$  to  $-89.5^\circ$  ellipsoidal latitude and  $0.5^\circ$  to  $359.5^\circ$  longitude.

## 2.5 Matlab Code

# 3 Methodology

## 3.1 Transoformation to LHS

[Geocentric cartesian coordinate system] is a three-dimensional, earth-centered reference system in which locations are identified by their x, y, and z values. The x-axis is in the equatorial plane and intersects the prime meridian (usually Greenwich). The y-axis is also in the equatorial plane; it lies at right angles to the x-axis and intersects the 90-degree meridian. The z-axis coincides with the polar axis and is positive toward the north pole. The origin is located at the center of the sphere or spheroid.

[Local horizontal system] uses the Cartesian coordinates(East,Nort,Up) to represent position relative to a local origin. The local origin is described by the geodetic coordinates.

Plate name	Wx (rad/Ma)	Wy (rad/Ma)	Wz (rad/Ma)
Pacific	-0.001510	0.004840	-0.009970
Cocos	-0.010425	-0.021605	0.010925
Nazca	-0.001532	-0.008577	0.009609
Caribbean	-0.000178	-0.003385	0.001581
South_America	-0.001038	-0.001515	-0.000870
Antarctica	-0.000821	-0.001701	0.003706
India	0.006670	0.000040	0.006790
Australia	0.007839	0.005124	0.006282
Africa	0.000891	-0.003099	0.003922
Arabia	0.006685	-0.000521	0.006760
Eurasia	-0.000981	-0.002395	0.003153
North_America	0.005200	-0.003599	-0.000153
Juan_de_Fuca	0.005200	0.008610	-0.005820
Philippine	0.010090	-0.007160	-0.009670
Rivera	-0.009390	-0.030960	0.012050
Scotia	-0.000410	-0.002660	-0.001270

Figure 5: NUVEL\_1A.txt Description

The initial coordinates are in the geocentric Cartesian coordinate system and need to be transformed into representation in the local horizontal coordinate system. In this project, we use two angles and the ITRF2008 point positions as the original point,

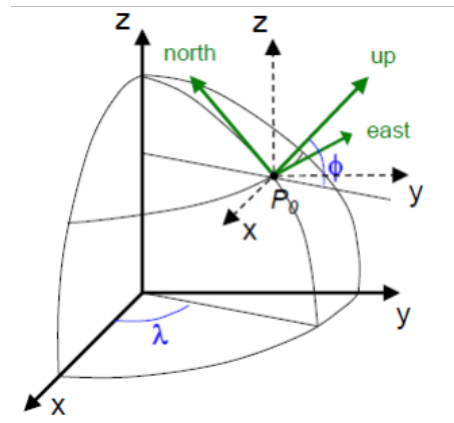


Figure 6: Coordinates Transformation

Calculate the angle according to stations' geodetic coordinates:

$$\lambda = \arctan \frac{y}{x}$$

$$\varphi = \arctan \frac{2}{\sqrt{x^2 + y^2}}$$

Then we can get the rotation matrix:

$$R_2(\delta) = \begin{pmatrix} \cos \delta & 0 & -\sin \delta \\ 0 & 1 & 0 \\ \sin \delta & 0 & \cos \delta \end{pmatrix} \quad R_3(\delta) = \begin{pmatrix} \cos \delta & \sin \delta & 0 \\ -\sin \delta & \cos \delta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Transformation of coordinates:

$$\begin{pmatrix} x_{up} \\ x_{east} \\ x_{north} \end{pmatrix} = R_2(-\varphi^0)R_3(\lambda^0) \left( \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} - \begin{pmatrix} x_1^0 \\ x_2^0 \\ x_3^0 \end{pmatrix} \right)$$

$x^0$  are the stations' geodetic coordinates, and  $x$  is observations in file 'xyz'. Notice that we also can directly use the longitude and latitude of stations provided in the file "Discontinuities\_CONFIRMED.snz".

In terms of velocity, its transformation into LHS only requires multiplication by a rotation matrix.

### 3.2 Least Square Adjustment for Parameters Estimation

For time series,

$$y(t) = \beta_1 + \beta_2 t + \beta_3 \cos \omega t + \beta_4 \sin \omega t$$

among which  $\beta_3$  and  $\beta_4$  are total amplitude of annual, and  $\beta_2$  is linear trend; so we can build model like:

$$\begin{pmatrix} y(t_1) \\ \vdots \\ y(t_n) \end{pmatrix} = \begin{pmatrix} 1 & t_1 & \cos \omega t_1 & \sin \omega t_1 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & t_n & \cos \omega t_n & \sin \omega t_n \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix}$$

We can simpfit the model like:

$$Y = X\beta + \varepsilon$$

where  $Y$  is the observations,  $X$  is the design matrix,  $\beta$  is the parameters, and  $\varepsilon$  is the noise.

According to least square, minimize the noise, derivative the square of noise and set it to zero so we get:

$$\beta = (X^T X)^{-1} X^T Y$$

through this we can get the estimated parameters.

### 3.3 Model of Plate Tectonics

The movement of any plate on a spherical Earth can be described through a rotation around the Euler pole:

$$\underline{\Omega} = (\Omega_1, \Omega_2, \Omega_3)^T$$

In point  $\underline{x}_0 = (x, y, z)^T$  the velocity vector  $\underline{v}$  is obtained by:

$$\underline{v} = \underline{\Omega} \times \underline{x}_0 = \begin{pmatrix} 0 & -\Omega_z & \Omega_y \\ \Omega_z & 0 & -\Omega_x \\ -\Omega_y & \Omega_x & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

### 3.4 Program Description

## 4 Results and Analysis

### 4.1 Time Series, Linear Trend and Residuals

Through the least square adjustment, out group get the time series of these three stations as shown below:

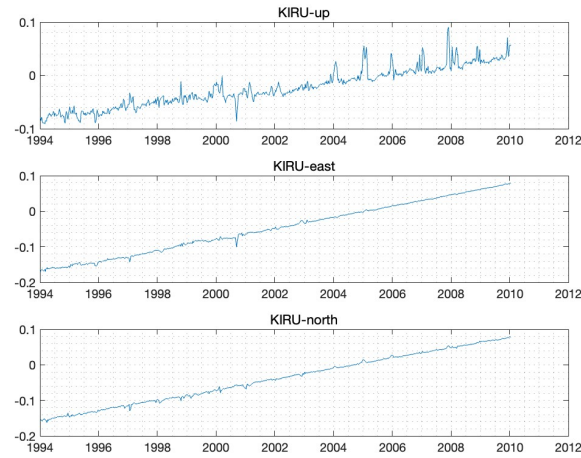


Figure 7: Time Series of KIRU/(meter)

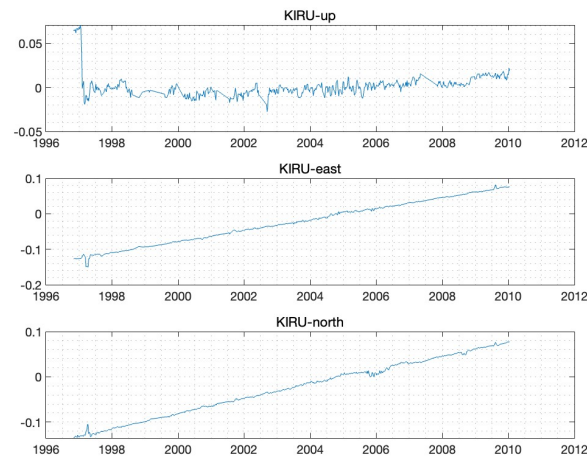


Figure 8: Time Series of MORP(meter)

DONES NB.	SITE NAME	Station	TECH. ID.	Station position	X/Vx	Y/Vy	Z/Vz	Sigmat	SOLN	DATA_START	DATA_END	Time state
					m/m/y							
10001S006	Paris	GNSS	OPMT	4202777.371	171367.999	4778660.203	0.001	0.001	0.001			
10001S006				-0.125	0.0175	0.0107	0.001	0.000	0.001			
10002M006	Grasse (OCA)	GNSS	GRAS	4581690.901	556114.831	4389360.793	0.001	0.001	0.001	1 00:00:00000	03:113:00000	
10002M006				-0.133	0.0188	0.0120	0.001	0.000	0.001			
10002M006	Grasse (OCA)	GNSS	GRAS	4581690.900	556114.837	4389360.793	0.001	0.001	0.001	2 03:113:00000	04:295:43200	
10002M006				-0.133	0.0188	0.0120	0.001	0.000	0.001			
10002M006	Grasse (OCA)	GNSS	GRAS	4581690.900	556114.836	4389360.797	0.001	0.001	0.001	3 04:295:43200	00:000:00000	
10002M006				-0.133	0.0188	0.0120	0.001	0.000	0.001			
10003M004	Toulouse	GNSS	TOUL	4627846.029	119629.333	4372999.818	0.001	0.001	0.001			
10003M004				-0.114	0.0193	0.0121	0.001	0.000	0.001			
10003M009	Toulouse	GNSS	TLSE	4627851.831	119640.017	4372993.553	0.001	0.001	0.001	1 00:000:00000	03:335:00000	
10003M009				-0.114	0.0193	0.0121	0.001	0.000	0.001			
10003M009	Toulouse	GNSS	TLSE	4627851.828	119640.020	4372993.552	0.001	0.001	0.001	2 03:335:00000	00:000:00000	
10003M009				-0.114	0.0193	0.0121	0.001	0.000	0.001			

Figure 9: Time Series of REYK(meter)

And we can see that the linear trend (millimeter per year) of KIRU in Up, East and North directions are 7.3089, 15.5296 and 14.8375 respectively, that of MORP in Up, East and North directions are 0.4201, 15.5923 and 15.9229 respectively.

Table 1: Linear Trend of Time Series

Station	Linear Trend(mm/y)		
	UP	EAST	NORTH
KIRU	7.3089	15.5296	14.8375
MORP	0.4201	15.5923	15.9229
REYK			

After we minus the linear trend, our group get the residual time series of these three stations as below and the specific vaules are shown in the table 1. At the same time, table 2 shows the total amplitude of non-linear trend (millimeter per year) which can be computed by  $\sqrt{\beta_3^2 + \beta_4^2}$ .

When we removing the linear trend, our group also remove the constant term in the model( $\beta_1 + \beta_2 t$ ), so that we can see more clearly about the residaul values. After doing this, we can see residual value in millimeter in three directions of three stations.

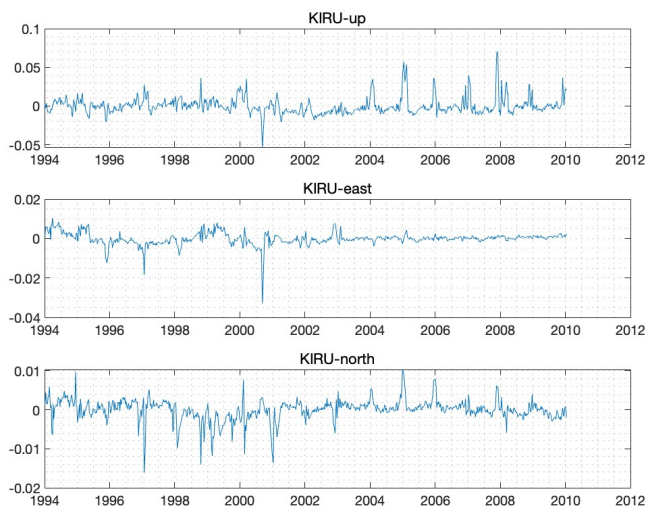


Figure 10: Residual Time Series of KIRU

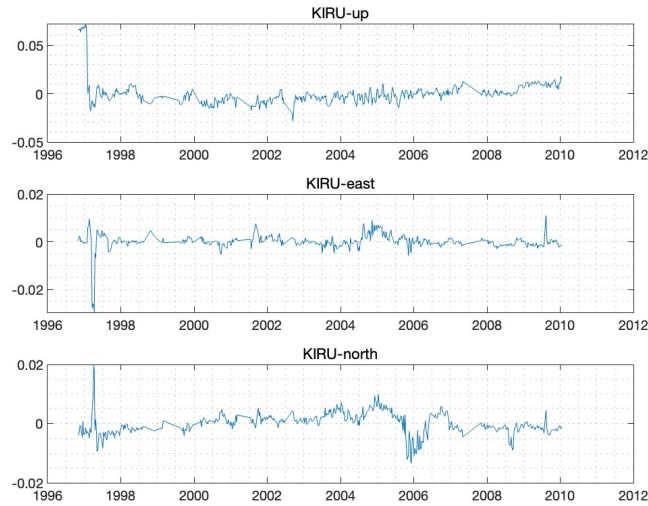


Figure 11: Residual Time Series of MORP

Plate name	Wx (rad/Ma)	Wy (rad/Ma)	Wz (rad/Ma)
Pacific	-0.001510	0.004840	-0.009970
Cocos	-0.010425	-0.021605	0.010925
Nazca	-0.001532	-0.008577	0.009609
Caribbean	-0.000178	-0.003385	0.001581
South_America	-0.001038	-0.001515	-0.000870
Antarctica	-0.000821	-0.001701	0.003706
Inida	0.006670	0.000040	0.006790
Australia	0.007839	0.005124	0.006282
Africa	0.000891	-0.003099	0.003922
Arabia	0.006685	-0.000521	0.006760
Eurasia	-0.000981	-0.002395	0.003153
North_America	0.005200	-0.003599	-0.000153
Juan_de_Fuca	0.005200	0.008610	-0.005820
Philippine	0.010090	-0.007160	-0.009670
Rivera	-0.009390	-0.030960	0.012050
Scotia	-0.000410	-0.002660	-0.001270

Figure 12: Residual Time Series of REYK

And obviously, the residual values of UP direction are much larger than the other two directions, which also are predominant in total residual values.

## 4.2 Comparison with the Plate Tectonics Model

Through the NUVEL 1A model, we can get the velocity of three stations, for this model only consider the horizontal movements, so our group only compare the velocity in East and North directions. And the comparison is shown below:



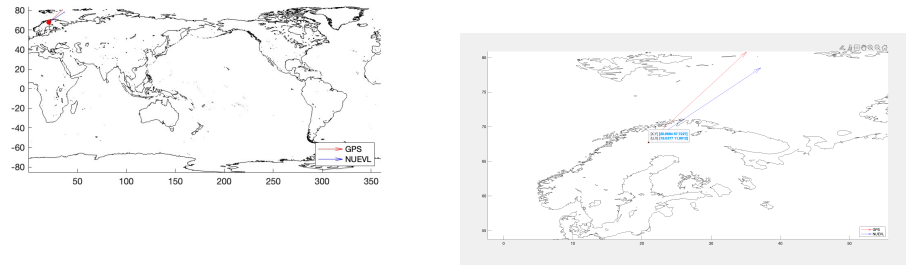


Figure 13: Velocity of KIRU

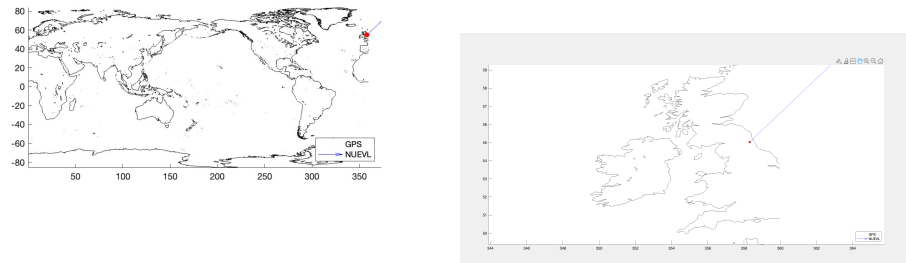


Figure 14: Velocity of MORP

Plate name	Wx (rad/Ma)	Wy (rad/Ma)	Wz (rad/Ma)
Pacific	-0.001510	0.004840	-0.009970
Cocos	-0.010425	-0.021605	0.010925
Nazca	-0.001532	-0.008577	0.009609
Caribbean	-0.000178	-0.003385	0.001581
South_America	-0.001038	-0.001515	-0.000870
Antarctica	-0.000821	-0.001701	0.003706
India	0.006670	0.000040	0.006790
Australia	0.007839	0.005124	0.006282
Africa	0.000891	-0.003099	0.003922
Arabia	0.006685	-0.000521	0.006760
Eurasia	-0.000981	-0.002395	0.003153
North_America	0.005200	-0.003599	-0.000153
Juan_de_Fuca	0.005200	0.008610	-0.005820
Philippine	0.010090	-0.007160	-0.009670
Rivera	-0.009390	-0.030960	0.012050
Scotia	-0.000410	-0.002660	-0.001270

Figure 15: Velocity of REYK

#### 4.3 Comparison of vertical movements

### 5 Conclusion