Satellite Lab1

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1 Introduction

2 Data Description

2.1 ITRF2008 IGS station

The ITRF is The International Reference Frame, and ITRF2008 is a realization of the International Terrestrial Reference System that uses as input data time series of station positions and EOPs provided by the Technique Centers of the four space geodetic techniques (GPS, VLBI, SLR, DORIS).

In the file "ITRF2008_GNSS.SSC.txt", we can find the coordinates of different stations at epoch 2005.0. and Time is given as the yyyy.yyyy format, which is the number of decimal years:

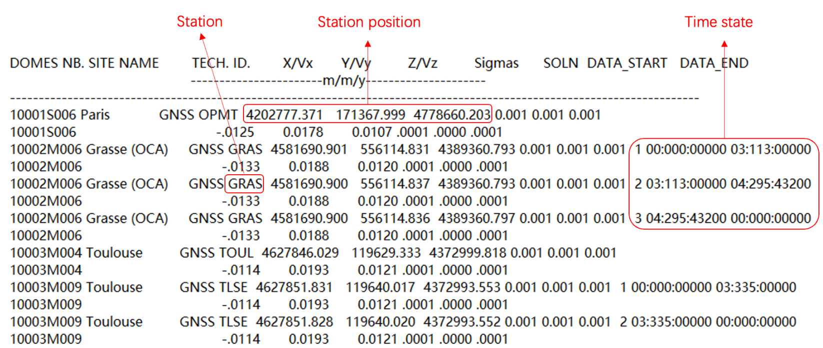

Figure 1: ITRF2008_GNSS.ssc.txt Description

2.2 Station GPS Obversations

We were responsible for the computation of the positions and movements of three measurement stations: KIRU, MORP, and REYK. The locations are illustrated in the following figure:

An example of the observation file for each data set is provided below, including two time formats and XYZ coordinates.

MJD GPS Week Geocentric Cartesian Coordinates (x,y,z)

```
KIRU  A   1    49366.12486 0731 5 .22514209584056605E+7 .86281710792228591E+6 .58854765731352381E+7 .4546590E-2 .3153733E-2 .8558948E-2
KIRU  A   1    49372.49986 0732 4 .22514209625573754E+7 .86281710931147402E+6 .58854765783153027E+7 .4445152E-2 .3220038E-2 .7440252E-2
KIRU  A   1    49379.49986 0733 4 .22514209610558231E+7 .86281710966029135E+6 .58854765743633714E+7 .3647180E-2 .2867647E-2 .6408686E-2
KIRU  A   1    49386.49986 0734 4 .22514209588548429E+7 .86281710788470914E+6 .58854765686460575E+7 .5082105E-2 .4125389E-2 .6495703E-2
KIRU  A   1    49393.24986 0735 4 .22514209570439737E+7 .86281710557302972E+6 .58854765683967089E+7 .4931841E-2 .3570944E-2 .7544502E-2
KIRU  A   1    49400.74986 0736 4 .22514209576969156E+7 .86281710531363264E+6 .58854765698663928E+7 .4235104E-2 .3361212E-2 .5809427E-2
KIRU  A   1    49407.49986 0737 4 .22514209544383213E+7 .86281710564680735E+6 .58854765704803048E+7 .4650871E-2 .3912282E-2 .6691448E-2
KIRU  A   1    49414.49986 0738 4 .22514209554168875E+7 .86281710892977635E+6 .58854765673357984E+7 .3256841E-2 .2874934E-2 .5449907E-2
KIRU  A   1    49421.49986 0739 4 .22514209616359631E+7 .86281710437103314E+6 .58854765726452796E+7 .6094717E-2 .5145147E-2 .7171221E-2
KIRU  A   1    49428.49986 0740 4 .22514209574105954E+7 .86281711162705650E+6 .58854765722047342E+7 .5246855E-2 .3984657E-2 .5913708E-2
KIRU  A   1    49434.49988 0741 3 .22514209631967791E+7 .86281711364467372E+6 .58854765689678248E+7 .3371217E-2 .2967789E-2 .5178104E-2
KIRU  A   1    49442.49986 0742 4 .22514209645643337E+7 .86281711846710742E+6 .58854765758999716E+7 .3923771E-2 .3339545E-2 .5824893E-2
KIRU  A   1    49449.49986 0743 4 .22514209587599267E+7 .86281711157962470E+6 .58854765782895284E+7 .3788400E-2 .3217135E-2 .5796049E-2
KIRU  A   1    49456.62486 0744 4 .22514209637514777E+7 .86281711424310168E+6 .58854765822355244E+7 .4633061E-2 .3850479E-2 .6923458E-2
KIRU  A   1    49463.49986 0745 4 .22514209604567364E+7 .86281711332197592E+6 .58854765825606706E+7 .4195015E-2 .3593613E-2 .5366438E-2
KIRU  A   1    49470.49986 0746 4 .22514209616745543E+7 .86281711298889748E+6 .58854765863939598E+7 .4262998E-2 .3515094E-2 .6085852E-2
```

Figure 2: station.xyz Description

And the file Discontinuities_snx provides the discontinuity information in the positions' time series. The reasons for that include change of antenna and receiver, earthquake and so on. In this example(station: REYK), the discontinuity happened three times due to earthquake, antenna change and unknown reason.

```
REUN  A   1 P 00:000:00000 00:000:00000 V -
REYK  A   1 P 00:000:00000 00:169:56460 P - Earthquake
REYK  A   3 P 00:173:03120 03:117:00000 P - Unknown
REYK  A   4 P 08:073:00000 00:000:00000 P -
REYK  A   5 P 03:117:00000 08:073:00000 P - Antenna Change
REYK  A   1 P 00:000:00000 00:000:00000 V -
REYZ  A   1 P 00:170:00000 00:000:00000 P -
REYZ  A   2 P 00:000:00000 00:170:00000 P - Unknown
REYZ  A   1 P 00:000:00000 00:000:00000 V -
```

Figure 3: Discontinuities in time series

2.3 NUVEL 1A Model

NUVEL(Northeast University Velocity) is a the collective term for geophysical Earth models that describes observable continental movements through a dynamic theory of plaet tectonics.

The "NNR_NUVEL1A.txt" gives the rotation referred to epoch t_0. The file contains the following data, where the leftmost column represents the station name, and in that row, the angular velocity changes in three directions are provided (unit: radians per million years or rad/My).

Plate name	Wx (rad/Ma)	Wy (rad/Ma)	Wz (rad/Ma)
Pacific	-0.001510	0.004840	-0.009970
Cocos	-0.010425	-0.021605	0.010925
Nazca	-0.001532	-0.008577	0.009609
Caribbean	-0.000178	-0.003385	0.001581
South_America	-0.001038	-0.001515	-0.000870
Antarctica	-0.000821	-0.001701	0.003706
Inida	0.006670	0.000040	0.006790
Australia	0.007839	0.005124	0.006282
Africa	0.000891	-0.003099	0.003922
Arabia	0.006685	-0.000521	0.006760
Eurasia	-0.000981	-0.002395	0.003153
North_America	0.005200	-0.003599	-0.000153
Juan_de_Fuca	0.005200	0.008610	-0.005820
Philippine	0.010090	-0.007160	-0.009670
Rivera	-0.009390	-0.030960	0.012050
Scotia	-0.000410	-0.002660	-0.001270

Figure 4: NUVEL_1A.txt Description

2.4 Other data

coast30.mat: coast lines for visualization
crust_ICE4G.mat, crust_ICE5G.mat: Global grids with vertical crustal deformations rates [mm/year]. These matrices are given from 89.5° to −89.5° ellipsoidal latitude and 0.5° to 359.5° longitude.

2.5 Matlab Code

3 Methodology

3.1 Transoformation to LHS

[Geocentric cartesian coordinate system] is a three-dimensional, earth-centered reference system in which locations are identified by their x, y, and z values. The x-axis is in the equatorial plane and intersects the prime meridian (usually Greenwich). The y-axis is also in the equatorial plane; it lies at right angles to the x-axis and intersects the 90-degree meridian. The z-axis coincides with the polar axis and is positive toward the north pole. The origin is located at the center of the sphere or spheroid.

[Local horizontal system] uses the Cartesian coordinates(East,Nort,Up) to represent position relative to a local origin. The local origin is described by the geodetic coordinates.

The initial coordinates are in the geocentric Cartesian coordinate system and need to be transformed into representation in the local horizontal coordinate system. In this project, we use two angles and the ITRF2008 point positions as the original point,

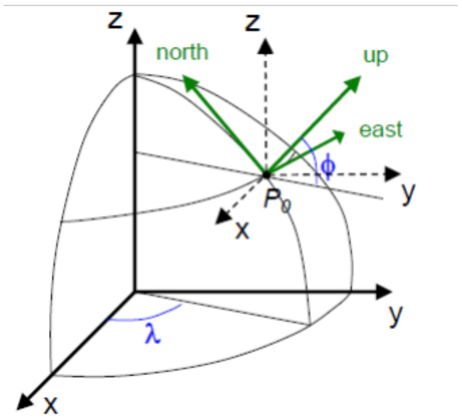

Figure 5: Coordinates Transformation

Calculate the angle according to stations' geodetic coordinates:

$$\lambda = \arctan \frac{y}{x}$$

$$\varphi = \arctan \frac{2}{\sqrt{x^2 + y^2}}$$

Then we can get the rotation matrix:

$$R_2(\delta) = \begin{pmatrix} \cos\delta & 0 & -\sin\delta \\ 0 & 1 & 0 \\ \sin\delta & 0 & \cos\delta \end{pmatrix} \quad R_3(\delta) = \begin{pmatrix} \cos\delta & \sin\delta & 0 \\ -\sin\delta & \cos\delta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Transformation of coordinates:

$$\begin{pmatrix} x_{up} \\ x_{east} \\ x_{north} \end{pmatrix} = R_2(-\varphi^0)R_3(\lambda^0)\left(\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} - \begin{pmatrix} x_1^0 \\ x_2^0 \\ x_3^0 \end{pmatrix}\right)$$

$x^0$are the stations' geodetic coordinates, and x is observations in file '.xyz'. Notice that we also can directly use the longitude and latitude of stations provided in the file "Discontinuities_CONFIRMED.snx".

In terms of velocity, its transformation into LHS only requires multiplication by a rotation matrix.

3.2 Least Square Adjustment for Parameters Estimation

For time series,

$$y(t) = \beta_1 + \beta_2 t + \beta_3 \text{cos}\omega t + \beta_4 \text{sin}\omega t$$

among which β_3 and β_4 are total amplitude of annual, and β_2 is linear trend; so we can build model like:

$$\begin{pmatrix} y(t_1) \\ \vdots \\ y(t_n) \end{pmatrix} = \begin{pmatrix} 1 & t_1 & \cos\omega t_1 & \sin\omega t_1 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & t_n & \cos\omega t_n & \sin\omega t_n \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix}$$

We can simppfit the model like:

$$Y = X\beta + \varepsilon$$

where Y is the observations, X is the design matrix, β is the parameters, and ε is the noise.

According to least square, minimize the noise, derivative the square of noise and set it to zero so we get:

$$\beta = (X^T X)^{-1} X^T Y$$

through this we can get the estimated parameters.

3.3 Model of Plate Tectonics

The movement of any plate on a spherical Earth can be described through a rotation around the Euler pole:

$$\underline{\Omega} = (\Omega_1, \Omega_2, \Omega_3)^T$$

In point $\underline{x}_0 = (x, y, z)^T$ the velocity vector v is obtained by:

$$\underline{v} = \underline{\Omega} \times \underline{x}_0 = \begin{pmatrix} 0 & -\Omega_z & \Omega_y \\ \Omega_z & 0 & -\Omega_x \\ -\Omega_y & \Omega_x & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

3.4 Program Description

4 Results and Analysis

4.1 Time Series, Linear Trend and Residuals

Through the least square adjustment, out group get the time series of these three stations as shown below:

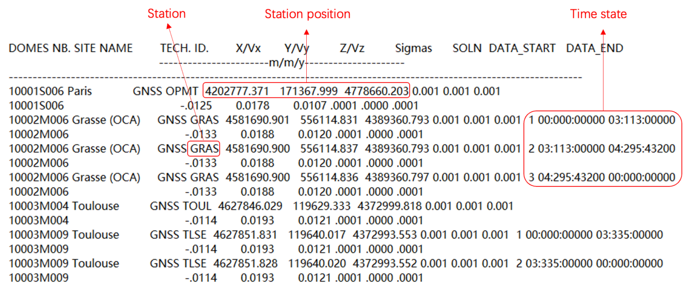

Figure 6: Time Series of KIRU

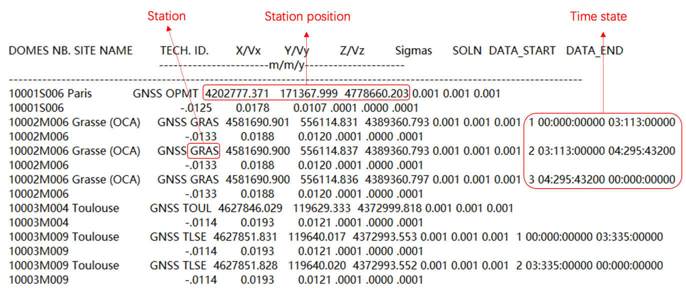

Figure 7: Time Series of MORP

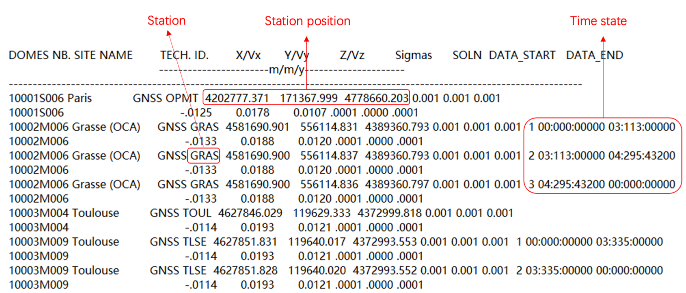

Figure 8: Time Series of REYK

Table 1: Linear Trend of Time Series

Station	Linear Trend(mm/y)		
	UP	EAST	NORTH
KIRU			
MORP			
REYK			

And we can see that the linear trend (millimeter) of KIRU in Up, East and North directions are , , and respectively.

After we minus the linear trend, out group get the residual time series of these three stations as below and the specific vaules are shown in the table 1. At the same time, table 2 shows the total amplitude of non-linear trend (millimeter per year) which can be computed by $\sqrt{\beta_3^2 + \beta_4^2}$.

Plate name	Wx (rad/Ma)	Wy (rad/Ma)	Wz (rad/Ma)
Pacific	-0.001510	0.004840	-0.009970
Cocos	-0.010425	-0.021605	0.010925
Nazca	-0.001532	-0.008577	0.009609
Caribbean	-0.000178	-0.003385	0.001581
South_America	-0.001038	-0.001515	-0.000870
Antarctica	-0.000821	-0.001701	0.003706
Inida	0.006670	0.000040	0.006790
Australia	0.007839	0.005124	0.006282
Africa	0.000891	-0.003099	0.003922
Arabia	0.006685	-0.000521	0.006760
Eurasia	-0.000981	-0.002395	0.003153
North_America	0.005200	-0.003599	-0.000153
Juan_de_Fuca	0.005200	0.008610	-0.005820
Philippine	0.010090	-0.007160	-0.009670
Rivera	-0.009390	-0.030960	0.012050
Scotia	-0.000410	-0.002660	-0.001270

Figure 9: Residual Time Series of KIRU

When we removing the linear trend, our group also remove the constant term in the model($\beta_1 + \beta_2 t$), so that we can see more clearly about the residaul values. After doing this, we can see residual value in millimeter in three directions of three stations.

And obvisously, the residual values of UP direction are much larger than the other two directions, which also are predominant in total residual values.

Plate name	Wx (rad/Ma)	Wy (rad/Ma)	Wz (rad/Ma)
Pacific	-0.001510	0.004840	-0.009970
Cocos	-0.010425	-0.021605	0.010925
Nazca	-0.001532	-0.008577	0.009609
Caribbean	-0.000178	-0.003385	0.001581
South_America	-0.001038	-0.001515	-0.000870
Antarctica	-0.000821	-0.001701	0.003706
Inida	0.006670	0.000040	0.006790
Australia	0.007839	0.005124	0.006282
Africa	0.000891	-0.003099	0.003922
Arabia	0.006685	-0.000521	0.006760
Eurasia	-0.000981	-0.002395	0.003153
North_America	0.005200	-0.003599	-0.000153
Juan_de_Fuca	0.005200	0.008610	-0.005820
Philippine	0.010090	-0.007160	-0.009670
Rivera	-0.009390	-0.030960	0.012050
Scotia	-0.000410	-0.002660	-0.001270

Figure 10: Residual Time Series of MORP

Plate name	Wx (rad/Ma)	Wy (rad/Ma)	Wz (rad/Ma)
Pacific	-0.001510	0.004840	-0.009970
Cocos	-0.010425	-0.021605	0.010925
Nazca	-0.001532	-0.008577	0.009609
Caribbean	-0.000178	-0.003385	0.001581
South_America	-0.001038	-0.001515	-0.000870
Antarctica	-0.000821	-0.001701	0.003706
Inida	0.006670	0.000040	0.006790
Australia	0.007839	0.005124	0.006282
Africa	0.000891	-0.003099	0.003922
Arabia	0.006685	-0.000521	0.006760
Eurasia	-0.000981	-0.002395	0.003153
North_America	0.005200	-0.003599	-0.000153
Juan_de_Fuca	0.005200	0.008610	-0.005820
Philippine	0.010090	-0.007160	-0.009670
Rivera	-0.009390	-0.030960	0.012050
Scotia	-0.000410	-0.002660	-0.001270

Figure 11: Residual Time Series of REYK

4.2 Comparison with the Plate Tectonics Model

Through the NUVEL 1A model, we can get the velocity of three stations, for this model only consider the horizontal movements, so our group only compare the velocity in East and North directions. And the comparison is shown below:

Plate name	Wx (rad/Ma)	Wy (rad/Ma)	Wz (rad/Ma)
Pacific	-0.001510	0.004840	-0.009970
Cocos	-0.010425	-0.021605	0.010925
Nazca	-0.001532	-0.008577	0.009609
Caribbean	-0.000178	-0.003385	0.001581
South_America	-0.001038	-0.001515	-0.000870
Antarctica	-0.000821	-0.001701	0.003706
Inida	0.006670	0.000040	0.006790
Australia	0.007839	0.005124	0.006282
Africa	0.000891	-0.003099	0.003922
Arabia	0.006685	-0.000521	0.006760
Eurasia	-0.000981	-0.002395	0.003153
North_America	0.005200	-0.003599	-0.000153
Juan_de_Fuca	0.005200	0.008610	-0.005820
Philippine	0.010090	-0.007160	-0.009670
Rivera	-0.009390	-0.030960	0.012050
Scotia	-0.000410	-0.002660	-0.001270

Figure 12: Velocity of KIRU

4.3 Comparison of vertical movements

5 Conclusion

Plate name	Wx (rad/Ma)	Wy (rad/Ma)	Wz (rad/Ma)
Pacific	-0.001510	0.004840	-0.009970
Cocos	-0.010425	-0.021605	0.010925
Nazca	-0.001532	-0.008577	0.009609
Caribbean	-0.000178	-0.003385	0.001581
South_America	-0.001038	-0.001515	-0.000870
Antarctica	-0.000821	-0.001701	0.003706
Inida	0.006670	0.000040	0.006790
Australia	0.007839	0.005124	0.006282
Africa	0.000891	-0.003099	0.003922
Arabia	0.006685	-0.000521	0.006760
Eurasia	-0.000981	-0.002395	0.003153
North_America	0.005200	-0.003599	-0.000153
Juan_de_Fuca	0.005200	0.008610	-0.005820
Philippine	0.010090	-0.007160	-0.009670
Rivera	-0.009390	-0.030960	0.012050
Scotia	-0.000410	-0.002660	-0.001270

Figure 13: Velocity of MORP

Plate name	Wx (rad/Ma)	Wy (rad/Ma)	Wz (rad/Ma)
Pacific	-0.001510	0.004840	-0.009970
Cocos	-0.010425	-0.021605	0.010925
Nazca	-0.001532	-0.008577	0.009609
Caribbean	-0.000178	-0.003385	0.001581
South_America	-0.001038	-0.001515	-0.000870
Antarctica	-0.000821	-0.001701	0.003706
Inida	0.006670	0.000040	0.006790
Australia	0.007839	0.005124	0.006282
Africa	0.000891	-0.003099	0.003922
Arabia	0.006685	-0.000521	0.006760
Eurasia	-0.000981	-0.002395	0.003153
North_America	0.005200	-0.003599	-0.000153
Juan_de_Fuca	0.005200	0.008610	-0.005820
Philippine	0.010090	-0.007160	-0.009670
Rivera	-0.009390	-0.030960	0.012050
Scotia	-0.000410	-0.002660	-0.001270

Figure 14: Velocity of REYK