Satellite Geodesy: Lab1 Report

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Contents

1 Introduction

This lab focuses on the geometry and kinematics of the polar region. And our group use the IGS products(station positions) to estimate the time series, horizontal and vertical velocities of three stations: KIRU, MORP, and REYK, whose locations are shown in the following figure: In the final part, we compare these result with the plate tectonics model NUVEL-1A and GIA models.

2 Data Description

2.1 ITRF2008 IGS station

The ITRF is The International Reference Frame, and ITRF2008 is a realization of the International Terrestrial Reference System that uses as input data time series of station positions and EOPs provided by the Technique Centers of the four space geodetic techniques (GPS, VLBI, SLR, DORIS).

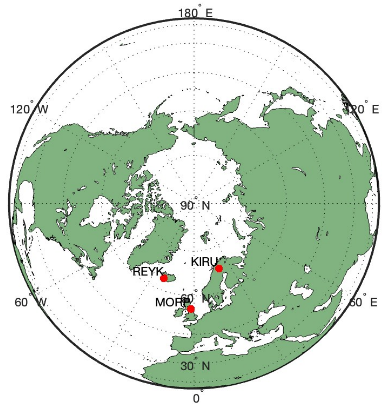
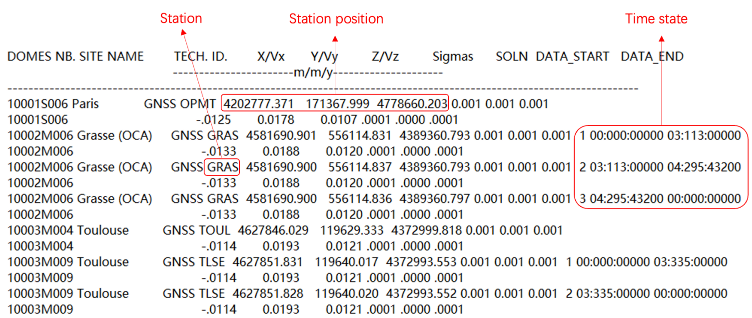


Figure 1: Station Positions

In the file "ITRF2008_GNSS.SSC.txt", we can find the coordinates of different stations at epoch 2005.0. and Time is given as the yyyy.yyyy format, which is the number of decimal years:




```
DOMES NB. SITE NAME        TECH. ID.     X/Vx       Y/Vy        Z/Vz        Sigmas      SOLN  DATA_START   DATA_END
                           ----------------------m/m/y--------------------
-------------------------------------------------------------------------------------------------------------
10001S006 Paris           GNSS OPMT  4202777.371   171367.999  4778660.203 0.001 0.001 0.001
10001S006                       -.0125       0.0178       0.0107 .0001 .0000 .0001
10002M006 Grasse (OCA)    GNSS GRAS  4581690.901   556114.831  4389360.793 0.001 0.001 0.001  1 00:000:00000 03:113:00000
10002M006                       -.0133       0.0188       0.0120 .0001 .0000 .0001
10002M006 Grasse (OCA)    GNSS GRAS  4581690.900   556114.837  4389360.793 0.001 0.001 0.001  2 03:113:00000 04:295:43200
10002M006                       -.0133       0.0188       0.0120 .0001 .0000 .0001
10002M006 Grasse (OCA)    GNSS GRAS  4581690.900   556114.836  4389360.797 0.001 0.001 0.001  3 04:295:43200 00:000:00000
10002M006                       -.0133       0.0188       0.0120 .0001 .0000 .0001
10003M004 Toulouse        GNSS TOUL  4627846.029   119629.333  4372999.818 0.001 0.001 0.001
10003M004                       -.0114       0.0193       0.0121 .0001 .0000 .0001
10003M009 Toulouse        GNSS TLSE  4627851.831   119640.017  4372993.553 0.001 0.001 0.001  1 00:000:00000 03:335:00000
10003M009                       -.0114       0.0193       0.0121 .0001 .0000 .0001
10003M009 Toulouse        GNSS TLSE  4627851.828   119640.020  4372993.552 0.001 0.001 0.001  2 03:335:00000 00:000:00000
10003M009                       -.0114       0.0193       0.0121 .0001 .0000 .0001
```

Figure 2: ITRF2008_GNSS.ssc.txt Description

2.2 Station GPS Obversations

We were responsible for the computation of the positions and movements of three measurement stations: KIRU, MORP, and REYK. The locations are illustrated in the following figure:

An example of the observation file for each data set is provided below, including two time formats and XYZ coordinates.

And the file Discontinuities_snx provides the discontinuity information in the positions' time series. The reasons for that include change of antenna and receiver, earthquake and so on. In this example(station: REYK), the discontinuity happened three times due to earthquake, antenna change and unknown reason.

2.3 NUVEL 1A Model

NUVEL(Northeast University Velocity) is a the collective term for geophysical Earth models that describes observable continental movements through a dynamic theory of plaet tectonics.

MJD GPS Week Geocentric Cartesian Coordinates (x,y,z)

```
KIRU  A   1    49366.12486 0731 5 .22514209584056605E+7 .86281710792228591E+6 .58854765731352381E+7 .4546590E-2 .3153733E-2 .8558948E-2
KIRU  A   1    49372.49986 0732 4 .22514209625573754E+7 .86281710931147402E+6 .58854765783153027E+7 .4445152E-2 .3220038E-2 .7440252E-2
KIRU  A   1    49379.49986 0733 4 .22514209610558231E+7 .86281710966029135E+6 .58854765743633714E+7 .3647180E-2 .2867647E-2 .6408686E-2
KIRU  A   1    49386.49986 0734 4 .22514209588548429E+7 .86281710788470914E+6 .58854765686460575E+7 .5082105E-2 .4125389E-2 .6495703E-2
KIRU  A   1    49393.24986 0735 4 .22514209570439737E+7 .86281710557302972E+6 .58854765683967089E+7 .4931841E-2 .3570944E-2 .7544502E-2
KIRU  A   1    49400.74986 0736 4 .22514209576969156E+7 .86281710531363264E+6 .58854765698663928E+7 .4235104E-2 .3361212E-2 .5809427E-2
KIRU  A   1    49407.49986 0737 4 .22514209544383213E+7 .86281710564680735E+6 .58854765704803048E+7 .4650871E-2 .3912282E-2 .6691448E-2
KIRU  A   1    49414.49986 0738 4 .22514209554168875E+7 .86281710892977635E+6 .58854765673357984E+7 .3256841E-2 .2874934E-2 .5449907E-2
KIRU  A   1    49421.49986 0739 4 .22514209616359631E+7 .86281710437103314E+6 .58854765726452796E+7 .6094717E-2 .5145147E-2 .7171221E-2
KIRU  A   1    49428.49986 0740 4 .22514209574105954E+7 .86281711162705650E+6 .58854765722047342E+7 .5246855E-2 .3984657E-2 .5913708E-2
KIRU  A   1    49434.49988 0741 3 .22514209631967791E+7 .86281711364467372E+6 .58854765689678248E+7 .3371217E-2 .2967789E-2 .5178104E-2
KIRU  A   1    49442.49986 0742 4 .22514209645643337E+7 .86281711846710742E+6 .58854765758999716E+7 .3923771E-2 .3339545E-2 .5824893E-2
KIRU  A   1    49449.49986 0743 4 .22514209587599267E+7 .86281711157962470E+6 .58854765782895284E+7 .3788400E-2 .3217135E-2 .5796049E-2
KIRU  A   1    49456.62486 0744 4 .22514209637514777E+7 .86281711424310168E+6 .58854765822355244E+7 .4633061E-2 .3850479E-2 .6923458E-2
KIRU  A   1    49463.49986 0745 4 .22514209604567364E+7 .86281711332197592E+6 .58854765825606706E+7 .4195015E-2 .3593613E-2 .5366438E-2
KIRU  A   1    49470.49986 0746 4 .22514209616745543E+7 .86281711298889748E+6 .58854765863939598E+7 .4262998E-2 .3515094E-2 .6085852E-2
```

Figure 3: station.xyz Description

```
REUN  A   1 P 00:000:00000 00:000:00000 V -
REYK  A   1 P 00:000:00000 00:169:56460 P - Earthquake
REYK  A   3 P 00:173:03120 03:117:00000 P - Unknown
REYK  A   4 P 08:073:00000 00:000:00000 P -
REYK  A   5 P 03:117:00000 08:073:00000 P - Antenna Change
REYK  A   1 P 00:000:00000 00:000:00000 V -
REYZ  A   1 P 00:170:00000 00:000:00000 P -
REYZ  A   2 P 00:000:00000 00:170:00000 P - Unknown
REYZ  A   1 P 00:000:00000 00:000:00000 V -
```

Figure 4: Discontinuities in time series

The "NNR_NUVEL1A.txt" gives the rotation referred to epoch t_0. The file contains the following data, where the leftmost column represents the station name, and in that row, the angular velocity changes in three directions are provided (unit: radians per million years or rad/My).

2.4 Other data

coast30.mat: coast lines for visualization
crust_ICE4G.mat, crust_ICE5G.mat: Global grids with vertical crustal deformations rates [mm/year]. These matrices are given from 89.5° to −89.5° ellipsoidal latitude and 0.5° to 359.5° longitude.

2.5 Matlab Code

3 Methodology

3.1 Transoformation to LHS

[Geocentric cartesian coordinate system] is a three-dimensional, earth-centered reference system in which locations are identified by their x, y, and z values. The x-axis is in the equatorial plane and intersects the prime meridian (usually Greenwich). The y-axis is also in the equatorial plane; it lies at right angles to the x-axis and intersects the 90-degree meridian. The z-axis coincides with the polar axis and is positive toward the north pole. The origin is located at the center of the sphere or spheroid.

[Local horizontal system] uses the Cartesian coordinates(East,Nort,Up) to represent position relative to a local origin. The local origin is described by the geodetic coordinates.

Plate name	Wx (rad/Ma)	Wy (rad/Ma)	Wz (rad/Ma)
Pacific	-0.001510	0.004840	-0.009970
Cocos	-0.010425	-0.021605	0.010925
Nazca	-0.001532	-0.008577	0.009609
Caribbean	-0.000178	-0.003385	0.001581
South_America	-0.001038	-0.001515	-0.000870
Antarctica	-0.000821	-0.001701	0.003706
Inida	0.006670	0.000040	0.006790
Australia	0.007839	0.005124	0.006282
Africa	0.000891	-0.003099	0.003922
Arabia	0.006685	-0.000521	0.006760
Eurasia	-0.000981	-0.002395	0.003153
North_America	0.005200	-0.003599	-0.000153
Juan_de_Fuca	0.005200	0.008610	-0.005820
Philippine	0.010090	-0.007160	-0.009670
Rivera	-0.009390	-0.030960	0.012050
Scotia	-0.000410	-0.002660	-0.001270

Figure 5: NUVEL_1A.txt Description

The initial coordinates are in the geocentric Cartesian coordinate system and need to be transformed into representation in the local horizontal coordinate system. In this project, we use two angles and the ITRF2008 point positions as the original point,

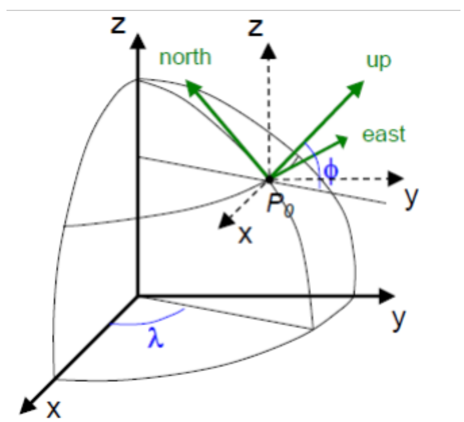


Figure 6: Coordinates Transformation

Calculate the angle according to stations' geodetic coordinates:

$$\lambda = \arctan \frac{y}{x}$$

$$\varphi = \arctan \frac{2}{\sqrt{x^2 + y^2}}$$

Then we can get the rotation matrix:

$$R_2(\delta) = \begin{pmatrix} \cos\delta & 0 & -\sin\delta \\ 0 & 1 & 0 \\ \sin\delta & 0 & \cos\delta \end{pmatrix} \quad R_3(\delta) = \begin{pmatrix} \cos\delta & \sin\delta & 0 \\ -\sin\delta & \cos\delta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Transformation of coordinates:

$$\begin{pmatrix} x_{up} \\ x_{east} \\ x_{north} \end{pmatrix} = R_2(-\varphi^0)R_3(\lambda^0)\left(\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} - \begin{pmatrix} x_1^0 \\ x_2^0 \\ x_3^0 \end{pmatrix}\right)$$

$x^0$are the stations' geodetic coordinates, and x is observations in file '.xyz'. Notice that we also can directly use the longitude and latitude of stations provided in the file "Discontinuities_CONFIRMED.snx".

In terms of velocity, its transformation into LHS only requires multiplication by a rotation matrix.

3.2 Least Square Adjustment for Parameters Estimation

For time series,

$$y(t) = \beta_1 + \beta_2 t + \beta_3 \cos\omega t + \beta_4 \sin\omega t$$

among which β_3 and β_4 are total amplitude of annual, and β_2 is linear trend; so we can build model like:

$$\begin{pmatrix} y(t_1) \\ \vdots \\ y(t_n) \end{pmatrix} = \begin{pmatrix} 1 & t_1 & \cos\omega t_1 & \sin\omega t_1 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & t_n & \cos\omega t_n & \sin\omega t_n \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix}$$

We can simppfit the model like:

$$Y = X\beta + \varepsilon$$

where Y is the observations, X is the design matrix, β is the parameters, and ε is the noise.

According to least square, minimize the noise, derivative the square of noise and set it to zero so we get:

$$\beta = (X^T X)^{-1} X^T Y$$

through this we can get the estimated parameters.

3.3 Model of Plate Tectonics

The movement of any plate on a spherical Earth can be described through a rotation around the Euler pole:

$$\underline{\Omega} = (\Omega_1, \Omega_2, \Omega_3)^T$$

In point $\underline{x}_0 = (x, y, z)^T$ the velocity vector v is obtained by:

$$\underline{v} = \underline{\Omega} \times \underline{x}_0 = \begin{pmatrix} 0 & -\Omega_z & \Omega_y \\ \Omega_z & 0 & -\Omega_x \\ -\Omega_y & \Omega_x & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

3.4 Program Description

4 Results and Analysis

4.1 Time Series, Linear Trend and Residuals

Through the least square adjustment, out group get the time series of these three stations as shown below:

And we can see that the linear trend (millimeter per year) of KIRU in Up, East and North directions are 7.3089, 15.5296 and 14.8375 respectively, that of MORP in Up, East and North directions are 0.4201,

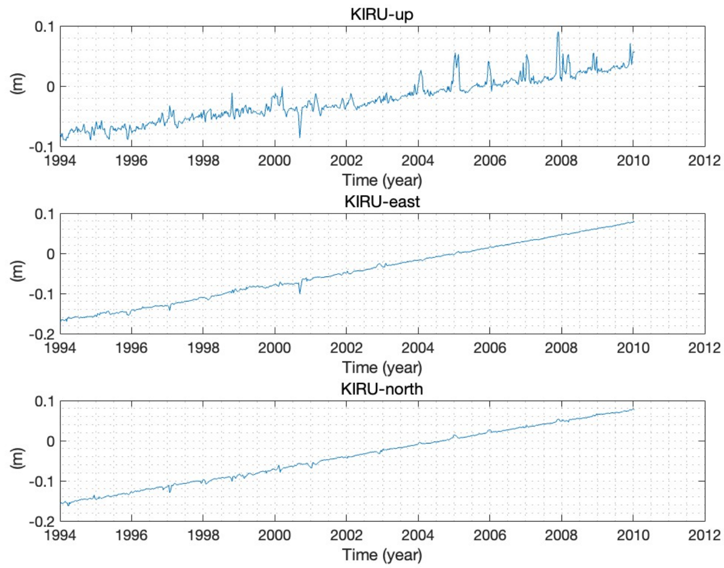


Figure 7: Time Series of KIRU/(meter)

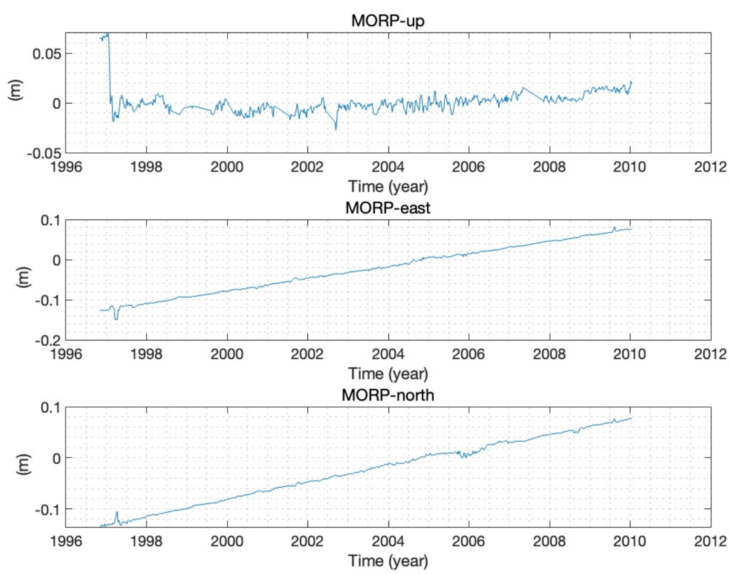


Figure 8: Time Series of MORP(meter)

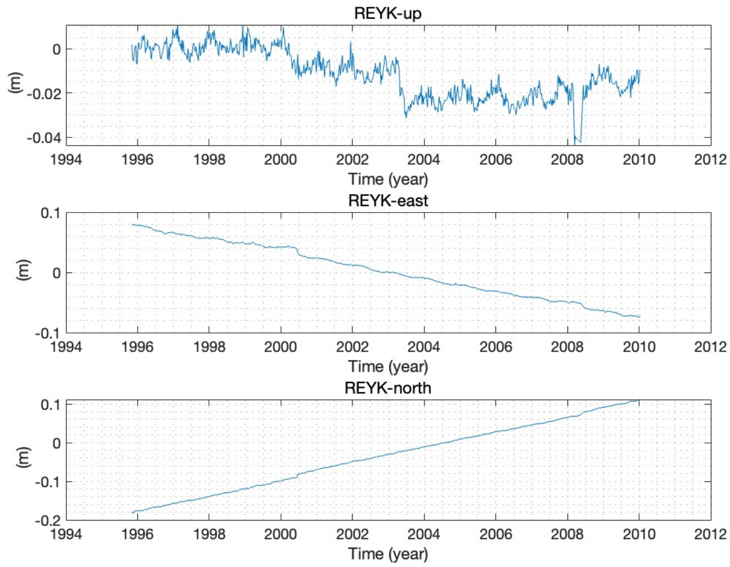


Figure 9: Time Series of REYK(meter)

15.5923 and 15.9229 respectively, and as for REYK, the linear trend in Up, East and North directions are , and respectively.

Table 1: Linear Trend of Time Series

Station	Linear Trend(mm/y)		
	UP	EAST	NORTH
KIRU	7.3089	15.5296	14.8375
MORP	0.4201	15.5923	15.9229
REYK	-2.0306	-10.9442	20.6032

Table 2: Annual Amplitude

Station	Annual Amplitude(mm)		
	UP	EAST	NORTH
KIRU	7.3089	0.3954	0.2587
MORP	3.9678	0.6203	0.0947
REYK	3.0366	1.6303	0.2655

After we minus the linear trend, out group get the residual time series of these three stations as below and the specific vaules are shown in the table 1. At the same time, table 2 shows the total amplitude of non-linear trend (millimeter per year) which can be computed by $\sqrt{\beta_3^2+\beta_4^2}$.

When we removing the linear trend, our group also remove the constant term in the model($\beta_1 + \beta_2 t$), so that we can see more clearly about the residaul values. After doing this, we can see residual value in millimeter in three directions of three stations.

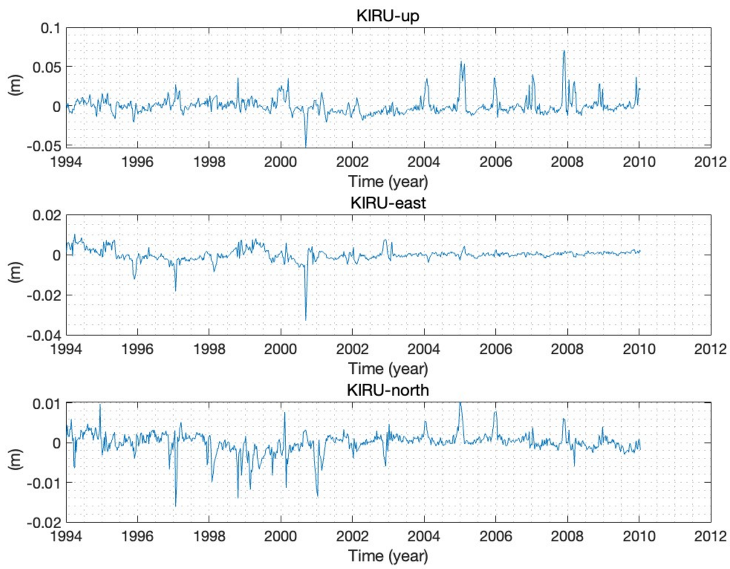


Figure 10: Residual Time Series of KIRU

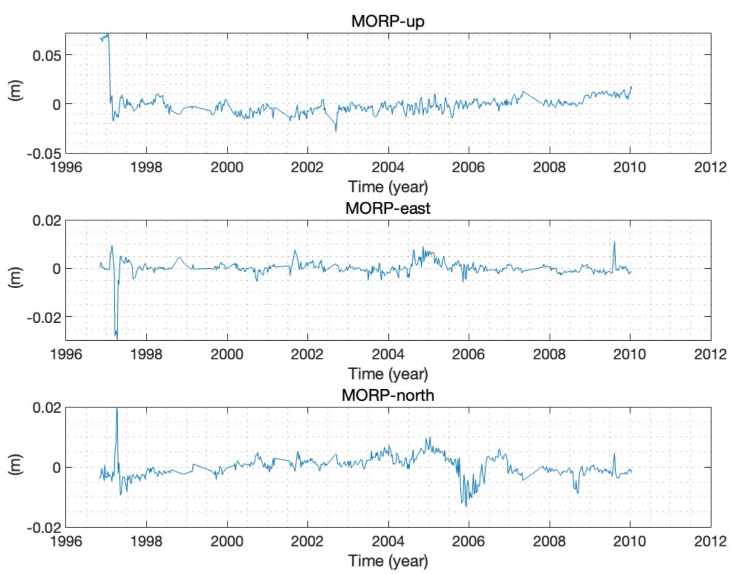


Figure 11: Residual Time Series of MORP

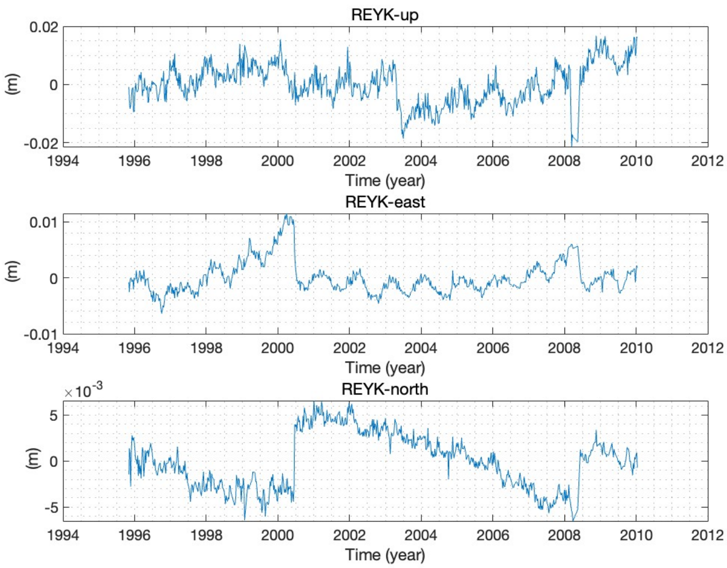


Figure 12: Residual Time Series of REYK

Due to the discontinuities in the observations, our group using the differnet reference coordinates in time series given by the file ”Discontinuities_CONFIRMED.snx”. After separat modeling of different parts of time seies, we can get the residual time series of these three stations as below, where we use different colors to represent different parts:

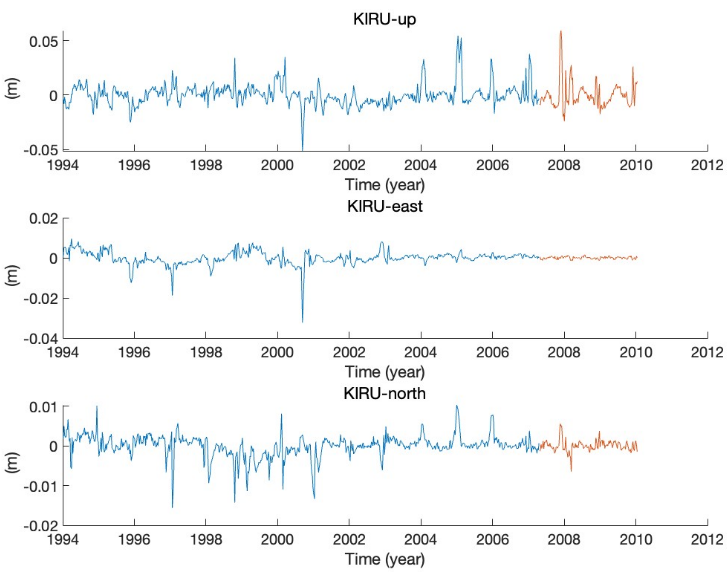


Figure 13: Modified Residual Time Series of KIRU

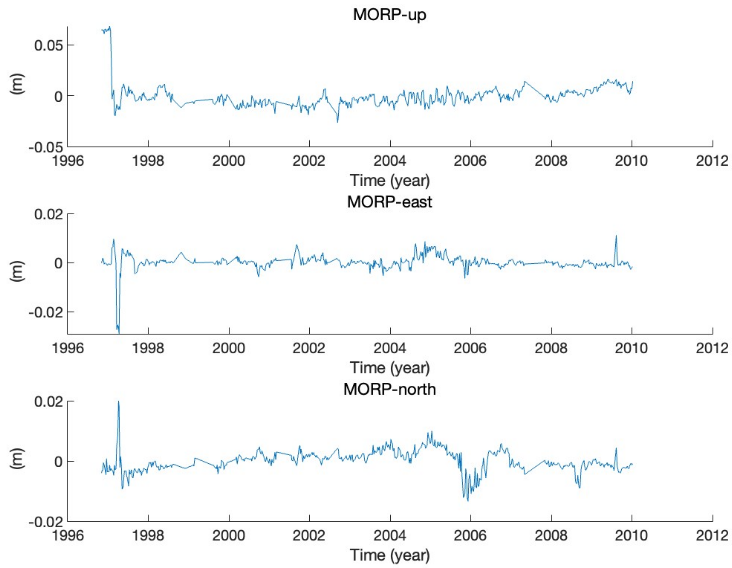


Figure 14: Modified Residual Time Series of MORP

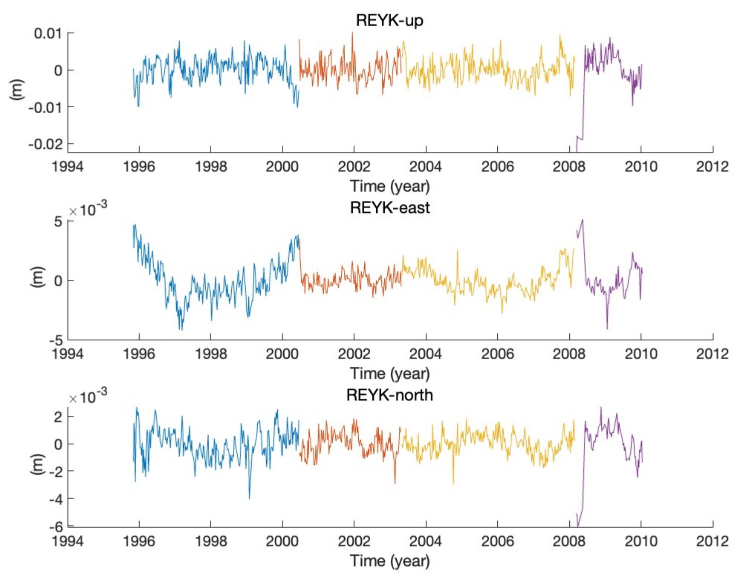


Figure 15: Modified Residual Time Series of REYK

Obviously, the residual values of UP direction are much larger than the other two directions, which also are predominant in total residual values. Through, we know that GPS residuals on the three components are dominated by flicker noise. As for reasons why the amplitudes of UP direction are considerable lareger, We believe this is due to the satellite visibility. Due to the Earth's obstruction, we are unable to receive signals from satellites on the other side of the Earth. In other words, our control of measurements in the up direction is limited to above the horizontal plane, hence the control intensity is certainly lower compared to the other two directions.

Meanwhile, our group also found some anomalies in the residual time series. Firstly, we find there is an annual signal in the all time series, these signals' amplitude are much large for KIRU, but are small(1-2 millimeter) in MORP and REYK. We speculate that the annula signals is caused mainly by the seasonal changes in the atmosphere. Plus, there are some sudden changes, like the one in 1997 of MORP, the one in 2000 of KIRU and the ones in 2000 and 2008 of REYK. We supposed that these changes are caused by the discontinuities in the observations and recording errors or instrument malfunction.

Additionally, we also find that the residual time series of MORP are much more stable than the other two stations, From the map, it can be observed that MORP has the lowest latitude, so we speculate that this is due to stronger interference received by GPS signals in high-latitude areas, which is related to various factors. Firstly, the ionospheric effects are more significant in high-latitude regions. Additionally, particles generated by solar activities are attracted toward the Earth's North Pole, causing electromagnetic interference and making GPS signals more unstable. Secondly, receivers in high-latitude areas receive satellite signals at lower angles, which increases the atmospheric impact on the signals, meanwhile this also causes receiving signals from fewer satellites. Furthermore, much more snow and ice in high-latitude regions strengthens the effects of multipath effect. In summary, various factors contribute to a reduction in the accuracy of position measurements in high-latitude areas.

4.2 Comparison with the Plate Tectonics Model

Through the NUVEL 1A model, we can get the velocity of three stations, for this model only consider the horizontal movements, so our group only compare the velocity in East and North directions.

The comparison is shown below and we can see clearly that the horizontal velocities from NUVEL and GNSS observations differ slightly, sometimes the differences are even considerable like the one in REYK, where velocities in East directions are opposite:

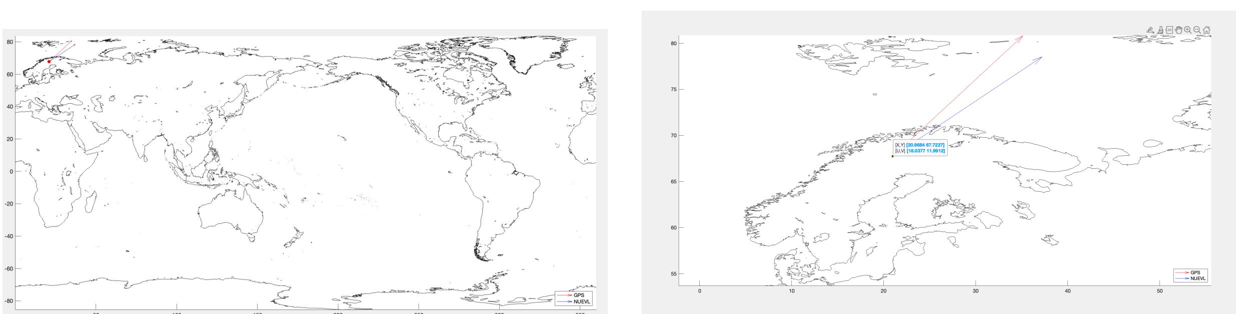


Figure 16: Horizontal Movements Comparison of KIRU

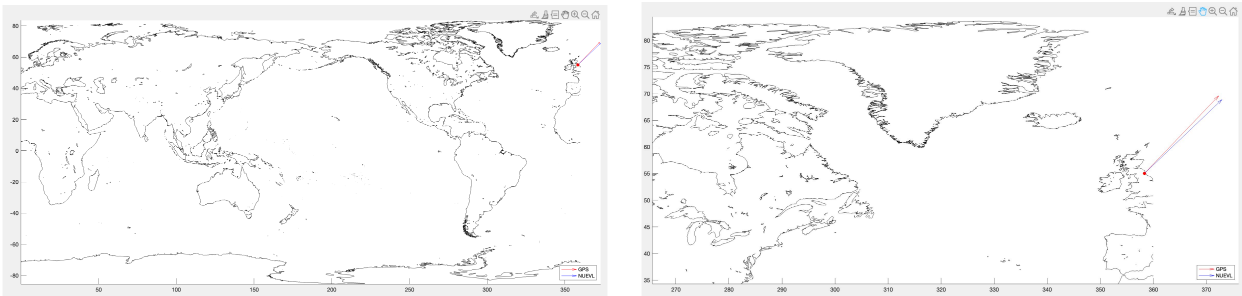


Figure 17: Horizontal Movements Comparison of MORP

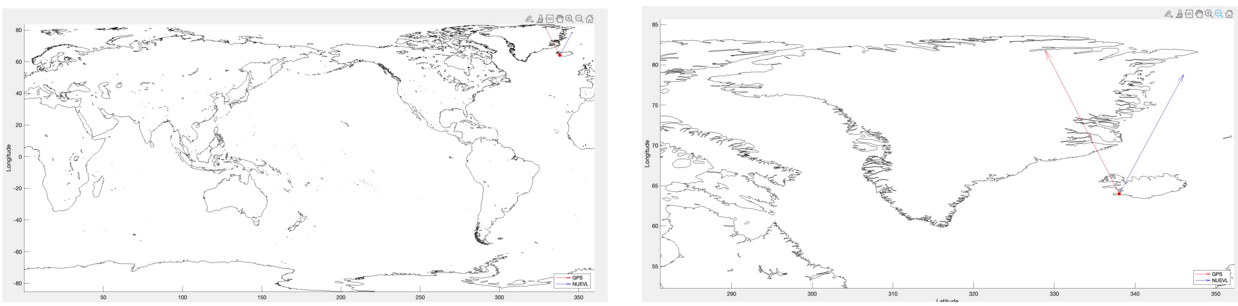


Figure 18: Horizontal Movements Comparison of REYK

And we give the specific values of horizontal velocity(in the East and North axis) as below:

Table 3: Horizontal Movement Rates

Station	Horizontal Velocity/(m/y)			
	GNSS Observation		NUVEL-1A Model	
	EAST	NORTH	EAST	NORTH
KIRU	0.0157	0.0145	0.0180	0.0120
MORP	0.0156	0.0159	0.0162	0.0154
REYK	-0.0104	0.0198	0.00878	0.01646

For the reason that horizontal movements from GNSS observations don't fit very well that from the NUVEL-1A model well, our group think there are several reasons for this: Firstly, both GNSS observations and the data used in the NUVEL model are interfered by various factors. The GPS signal and observations are affected by multipath effects and ionospheric interference, while seismic activities and receiver changes can also impact the observations. Additionally, the NUVEL 1A model has simplified the plate structure and motion, it isn't considered as a highly accurate model now. Also, NUVEL model uses data from multiple GNSS observations, and there will certainly be some offsets compared to our calculation only using data from a single station.

Interestingly, we observed the same phenomenon as in task 1: the MORP with the lowest latitude has better fitting performance compared to the other two high-latitude stations. And the reasons are the same as we mentioned in task1. In addition, the REYK station is located near the boundary of two tectonic plates, which may also be a reason to the significant differences.

4.3 Comparison of vertical movements

In this part, we compare the vertical movements of three stations from GNSS observations and GIA model: ICE-4G and ICE-5G. The visualization of these two models of a global map is here:

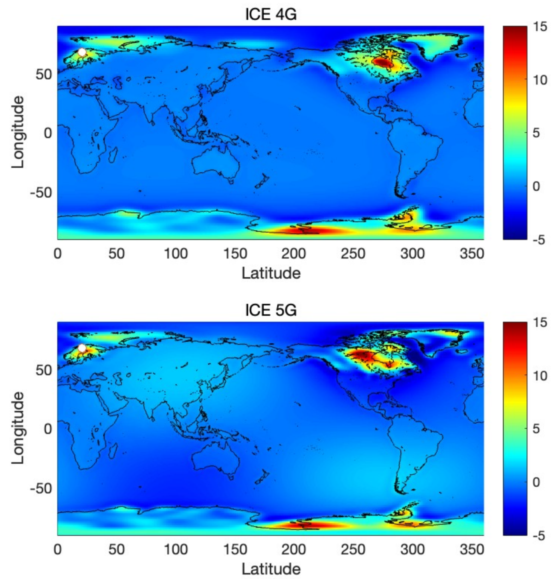


Figure 19: GIA Model

At the same time, out group interpolate the vertical movements rates to the position of three stations, and the results are shown below:

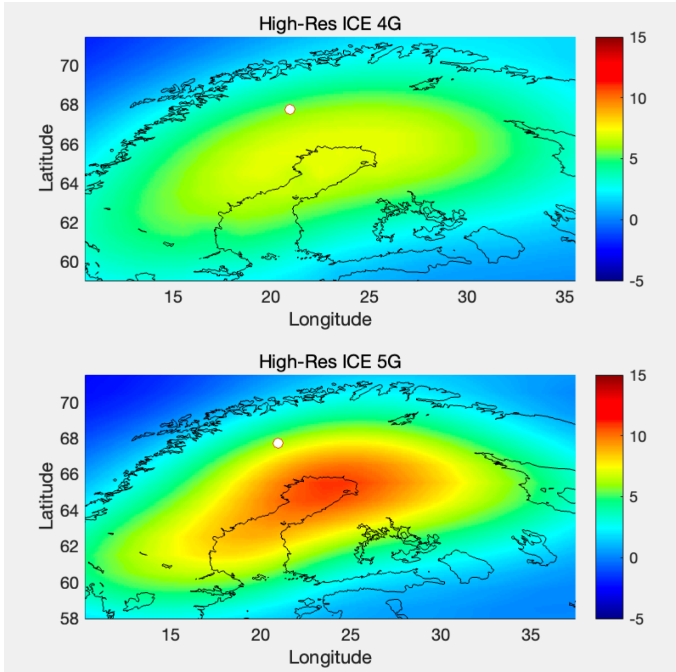


Figure 20: GIA Model Interpolation at KIRU

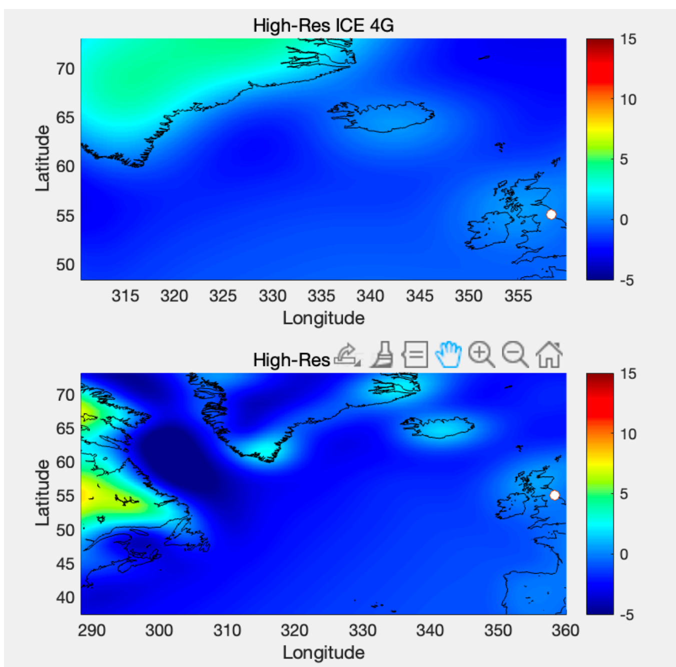


Figure 21: GIA Model Interpolation at MORP

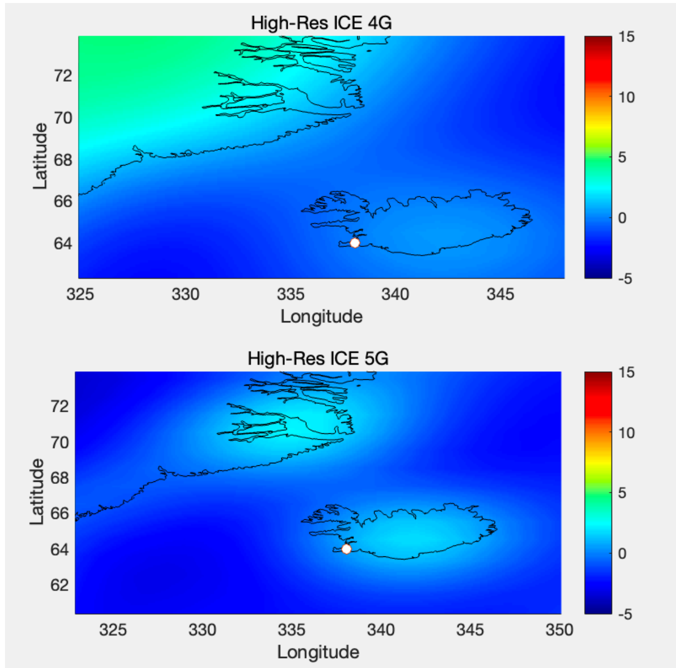


Figure 22: GIA Model Interpolation at REYK

The following table shows the specific values:

Table 4: Vertical Movements Rates

Station	Vertical Movements Rates at IGS stations/(mm/y)		
	GNSS	ICE-4G	ICE-5G
KIRU	6.8840	5.5568	6.1335
MORP	0.0012	0.0140	-0.0408
REYK	0.7212	-0.0464	0.7236

So we can see that, at different stations, the GNSS-obtained data closely align with ICE-5G but still have differences. Meanwhile, ICE-4G data show significant variations with the other two models. Through some materials, it is told that the ICE-4G model is inaccuracy because of incorrect assumptions made during model construction and other factors. On the other hand, ICE-5G is a refined model of the post-LGM global deglaciation process. Therefore, our analysis here focuses more on the differences between GNSS-obtaibed data and the ICE-5G model.

As for reasons, as we have metioned in task 1, the GNSS observations are affected by various factors, including multipath effects, ionospheric interference and so on. Especially, the vertical movements are more sensitive to the satellite visibility, which is limited by the Earth's obstruction. Therefore, the control in the up direction is weaker compared to the other two directions, and there will certainly be some deviation.