

# Satellite Geodesy: Lab1 Report

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December 9, 2023

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## 1 Introduction

This lab focuses on the geometry and kinematics of the polar region. And our group use the IGS products(station positions) to estimate the time series, horizontal and vertical velocities of three stations: KIRU, MORP, and REYK, whose locations are shown in the following figure: In the final part, we compare these result with the plate tectonics model NUVEL-1A and GIA models.

## 2 Data Description

### 2.1 ITRF2008 IGS station

The ITRF is The International Reference Frame, and ITRF2008 is a realization of the International Terrestrial Reference System that uses as input data time series of station positions and EOPs provided by the Technique Centers of the four space geodetic techniques (GPS, VLBI, SLR, DORIS).

In the file "ITRF2008\_GNSS.SSC.txt", we can find the coordinates of different stations at epoch 2005.0. and Time is given as the yyyy.yyyy format, which is the number of decimal years:

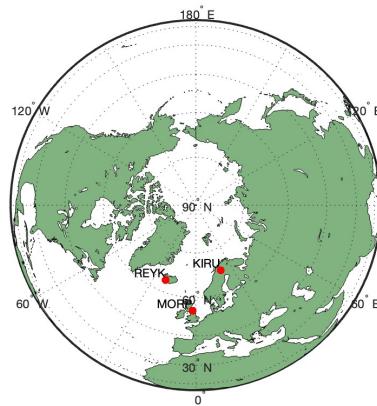


Figure 1: Station Positions

DOMES NB. SITE NAME	TECH. ID.	Station position			Sigmas	SOLN	DATA_START	DATA_END	Time state
		X/Vx -m/m/y	Y/Vy -m/m/y	Z/Vz -m/m/y					
10001S006 Paris	GNSS OPM1	4202777.371	171367.999	4778660.203	0.001	0.001	0.001	0.001	0.001
10001S006		-0.125	0.0178	0.0107	0.001	0.0001			
10002M006 Grasse (OCA)	GNSS GRAS	4581690.901	556114.831	4389360.793	0.001	0.001	0.001	0.001	0.000000:000000 03:113:00000
10002M006		-0.133	0.0188	0.0120	0.001	0.000	0.001		
10002M006 Grasse (OCA)	GNSS GRAS	4581690.900	556114.837	4389360.793	0.001	0.001	0.001	0.001	2 03:113:00000 04:295:43200
10002M006		-0.133	0.0188	0.0120	0.001	0.000	0.001		
10002M006 Grasse (OCA)	GNSS GRAS	4581690.900	556114.836	4389360.797	0.001	0.001	0.001	0.001	3 04:295:43200 00:000:00000
10002M006		-0.133	0.0188	0.0120	0.001	0.000	0.001		
10003M004 Toulouse	GNSS TOUL	4627846.029	119629.333	4372999.818	0.001	0.001	0.001	0.001	0.000000:000000 03:335:00000
10003M004		-0.014	0.0193	0.0121	0.001	0.000	0.001		
10003M009 Toulouse	GNSS TLSE	4627851.831	119640.017	4372993.553	0.001	0.001	0.001	0.001	1 00:000:00000 03:335:00000
10003M009		-0.014	0.0193	0.0121	0.001	0.000	0.001		
10003M009 Toulouse	GNSS TLSE	4627851.828	119640.020	4372993.552	0.001	0.001	0.001	0.001	2 03:335:00000 00:000:00000
10003M009		-0.014	0.0193	0.0121	0.001	0.000	0.001		

Figure 2: ITRF2008\_GNSS.ssc.txt Description

## 2.2 GNSS Observations

We were responsible for the computation of the positions and movements of three measurement stations: KIRU, MORN, and REYK. The locations are illustrated in the following figure:

An example of the observation file for each data set is provided below, including two time formats and XYZ coordinates.

			MJD	GPS Week	Geocentric Cartesian Coordinates (x,y,z)
KIRU A	1	49366.12486	0731	5	.22514209584056605E+7 .86281710792228591E+6 .58854765731352381E+7 .4546590E-2 .3153733E-2 .8558948E-2
KIRU A	1	49372.49986	0732	4	.22514209625573754E+7 .86281710931147402E+6 .58854765783153027E+7 .4445152E-2 .3220038E-2 .7440252E-2
KIRU A	1	49379.49986	0733	4	.22514209610558231E+7 .86281710966029135E+6 .58854765743633714E+7 .3647180E-2 .2867647E-2 .6408686E-2
KIRU A	1	49386.49986	0734	4	.2251420958548429E+7 .86281710788470914E+6 .58854765686460575E+7 .5082105E-2 .4125389E-2 .6495703E-2
KIRU A	1	49393.24986	0735	4	.22514209570439737E+7 .8628171057302972E+6 .58854765683967089E+7 .4931841E-2 .3570944E-2 .7544502E-2
KIRU A	1	49400.74986	0736	4	.22514209576969156E+7 .86281710531363264E+6 .588547656598663928E+7 .4235104E-2 .3361212E-2 .5809427E-2
KIRU A	1	49407.49986	0737	4	.22514209544383213E+7 .86281710564680735E+6 .58854765704803048E+7 .4650871E-2 .3912282E-2 .6691448E-2
KIRU A	1	49414.49986	0738	4	.22514209554168875E+7 .86281710892977635E+6 .588547656537984E+7 .3256841E-2 .2874934E-2 .5449907E-2
KIRU A	1	49421.49986	0739	4	.22514209616359631E+7 .86281710437103314E+6 .58854765726452796E+7 .6094717E-2 .5145147E-2 .7171212E-2
KIRU A	1	49428.49986	0740	4	.22514209574105954E+7 .86281711162705650E+6 .58854765722047342E+7 .5246855E-2 .3984657E-2 .5913708E-2
KIRU A	1	49434.49986	0741	3	.22514209631967791E+7 .86281711364467372E+6 .5885476568976248E+7 .3371217E-2 .2967779E-2 .5178104E-2
KIRU A	1	49442.49986	0742	4	.22514209645643337E+7 .86281711846710742E+6 .58854765758999716E+7 .3923771E-2 .3339545E-2 .5824893E-2
KIRU A	1	49449.49986	0743	4	.22514209587599267E+7 .86281711157962470E+6 .58854765782895284E+7 .3788400E-2 .3217135E-2 .5796049E-2
KIRU A	1	49456.62486	0744	4	.22514209637514777E+7 .86281711424310168E+6 .58854765822355244E+7 .4633061E-2 .3850479E-2 .6923458E-2
KIRU A	1	49463.49986	0745	4	.22514209604567364E+7 .8628171132197592E+6 .58854765825606706E+7 .4195015E-2 .3593613E-2 .5366438E-2
KIRU A	1	49470.49986	0746	4	.22514209616745543E+7 .86281711299889794E+6 .58854765683939598E+7 .4262998E-2 .3515094E-2 .6085852E-2

Figure 3: station.xyz Description

And the file Discontinuities\_snx provides the discontinuity information in the positions' time series. The reasons for that include change of antenna and receiver, earthquake and so on. In this example(station: REYK), the discontinuity happened three times due to earthquake, antenna change and unknown reason.

REUN A	1	P	00:000:00000	00:000:00000	V -
REYK A	1	P	00:000:00000	00:169:56460	P - Earthquake
REYK A	3	P	00:173:03120	03:117:00000	P - Unknown
REYK A	4	P	08:073:00000	00:000:00000	P -
REYK A	5	P	03:117:00000	08:073:00000	P - Antenna Change
REYK A	1	P	00:000:00000	00:000:00000	V -
REYZ A	1	P	00:170:00000	00:000:00000	P -
REYZ A	2	P	00:000:00000	00:170:00000	P - Unknown
REYZ A	1	P	00:000:00000	00:000:00000	V -

Figure 4: Discontinuities in time series

## 2.3 NUVEL 1A Model

NUVEL(Northeast University Velocity) is a the collective term for geophysical Earth models that describes observable continental movements through a dynamic theory of plate tectonics.

The "NNR\_NUVEL1A.txt" gives the rotation referred to epoch  $t_0$ . The file contains the following data, where the leftmost column represents the station name, and in that row, the angular velocity changes in three directions are provided (unit: radians per million years or rad/My).

Plate name	Wx (rad/Ma)	Wy (rad/Ma)	Wz (rad/Ma)
Pacific	-0.001510	0.004840	-0.009970
Cocos	-0.010425	-0.021605	0.010925
Nazca	-0.001532	-0.008577	0.009609
Caribbean	-0.000178	-0.003385	0.001581
South_America	-0.001038	-0.001515	-0.000870
Antarctica	-0.000821	-0.001701	0.003706
Inida	0.006670	0.000040	0.006790
Australia	0.007839	0.005124	0.006282
Africa	0.000891	-0.003099	0.003922
Arabia	0.006685	-0.000521	0.006760
Eurasia	-0.000981	-0.002395	0.003153
North_America	0.005200	-0.003599	-0.000153
Juan_de_Fuca	0.005200	0.008610	-0.005820
Philippine	0.010090	-0.007160	-0.009670
Rivera	0.009390	-0.030960	0.012050
Scotia	-0.000410	-0.002660	-0.001270

Figure 5: NUVEL\_1A.txt Description

## 2.4 Other data

- coast30.mat: coast lines for visualization
- crust\_ICE4G.mat, crust\_ICE5G.mat: Global grids with vertical crustal deformations rates [mm/year]. These matrices are given from  $89.5^\circ$  to  $-89.5^\circ$  ellipsoidal latitude and  $0.5^\circ$  to  $359.5^\circ$  longitude.

## 2.5 Matlab Code

The processing program is named based on IGS station names. Functions include reading the files mentioned upper, calculating parameters, and visualizing the results.

- KIRU.m
- MORP.m
- REYK.m

## 3 Methodology

### 3.1 Transformation to LHS

[Geocentric cartesian coordinate system] is a three-dimensional, earth-centered reference system in which locations are identified by their x, y, and z values. The x-axis is in the equatorial plane and intersects the prime meridian (usually Greenwich). The y-axis is also in the equatorial plane; it lies at right angles to the x-axis and intersects the 90-degree meridian. The z-axis coincides with the polar axis and is positive toward the north pole. The origin is located at the center of the sphere or spheroid.

[Local horizontal system] uses the Cartesian coordinates(East,Nort,Up) to represent position relative to a local origin. The local origin is described by the geodetic coordinates.

The initial coordinates are in the geocentric Cartesian coordinate system and need to be transformed into representation in the local horizontal coordinate system. In this project, we use two angles and the ITRF2008 point positions as the original point,

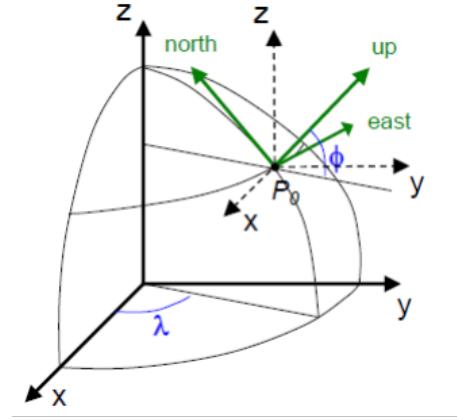


Figure 6: Coordinates Transformation

Calculate the angle according to stations' geodetic coordinates:

$$\lambda = \arctan \frac{y}{x}$$

$$\varphi = \arctan \frac{2}{\sqrt{x^2 + y^2}}$$

Then we can get the rotation matrix:

$$R_2(\delta) = \begin{pmatrix} \cos \delta & 0 & -\sin \delta \\ 0 & 1 & 0 \\ \sin \delta & 0 & \cos \delta \end{pmatrix} \quad R_3(\delta) = \begin{pmatrix} \cos \delta & \sin \delta & 0 \\ -\sin \delta & \cos \delta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Transformation of coordinates:

$$\begin{pmatrix} x_{up} \\ x_{east} \\ x_{north} \end{pmatrix} = R_2(-\varphi^0)R_3(\lambda^0) \left( \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} - \begin{pmatrix} x_1^0 \\ x_2^0 \\ x_3^0 \end{pmatrix} \right)$$

$x^0$  are the stations' geodetic coordinates, and  $x$  is observations in file '.xyz'. Notice that we also can directly use the longitude and latitude of stations provided in the file "Discontinuities\_CONFIRMED.snx".

In terms of velocity, its transformation into LHS only requires multiplication by a rotation matrix.

### 3.2 Least Square Adjustment for Parameters Estimation

For time series,

$$y(t) = \beta_1 + \beta_2 t + \beta_3 \cos \omega t + \beta_4 \sin \omega t$$

among which  $\beta_3$  and  $\beta_4$  are total amplitude of annual, and  $\beta_2$  is linear trend; so we can build model like:

$$\begin{pmatrix} y(t_1) \\ \vdots \\ y(t_n) \end{pmatrix} = \begin{pmatrix} 1 & t_1 & \cos \omega t_1 & \sin \omega t_1 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & t_n & \cos \omega t_n & \sin \omega t_n \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix}$$

We can simppfit the model like:

$$Y = X\beta + \varepsilon$$

where  $Y$  is the observations,  $X$  is the design matrix,  $\beta$  is the parameters, and  $\varepsilon$  is the noise.

According to least square, minimize the noise, derivative the square of noise and set it to zero so we get:

$$\beta = (X^T X)^{-1} X^T Y$$

through this we can get the estimated parameters.

### 3.3 Model of Plate Tectonics

The movement of any plate on a spherical Earth can be described through a rotation around the Euler pole:

$$\underline{\Omega} = (\Omega_1, \Omega_2, \Omega_3)^T$$

In point  $\underline{x}_0 = (x, y, z)^T$  the velocity vector  $v$  is obtained by:

$$\underline{v} = \underline{\Omega} \times \underline{x}_0 = \begin{pmatrix} 0 & -\Omega_z & \Omega_y \\ \Omega_z & 0 & -\Omega_x \\ -\Omega_y & \Omega_x & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

## 4 Results and Analysis

### 4.1 Time Series, Linear Trend and Residuals

Through the least square adjustment, our group get the time series of these three stations as shown below:

And we can see that the linear trend (millimeter per year) of KIRU in Up, East and North directions are 7.3089, 15.5296 and 14.8375 respectively, that of MROP in Up, East and North directions are 0.4201, 15.5923 and 15.9229 respectively, and as for REYK, the linear trend in Up, East and North directions are , and respectively.

Table 1: Linear Trend of Time Series

Station	Linear Trend(mm/y)		
	UP	EAST	NORTH
KIRU	7.3089	15.5296	14.8375
MROP	0.4201	15.5923	15.9229
REYK	-2.0306	-10.9442	20.6032

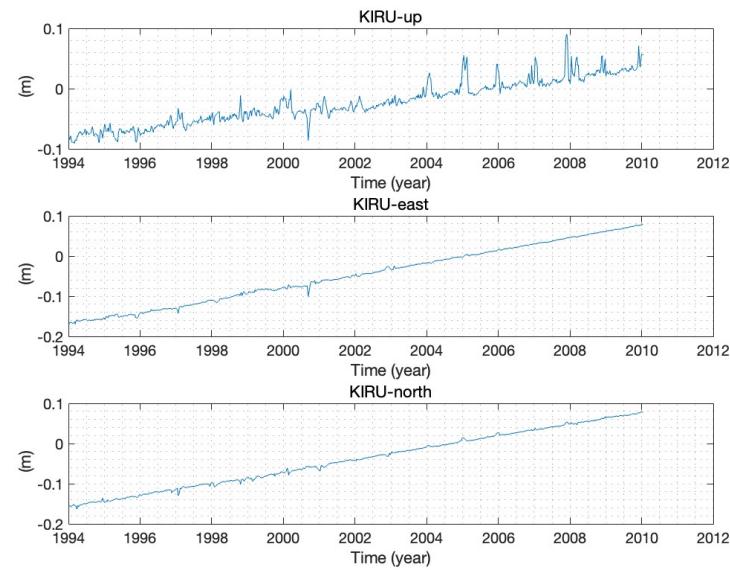


Figure 7: Time Series of KIRU/(meter)

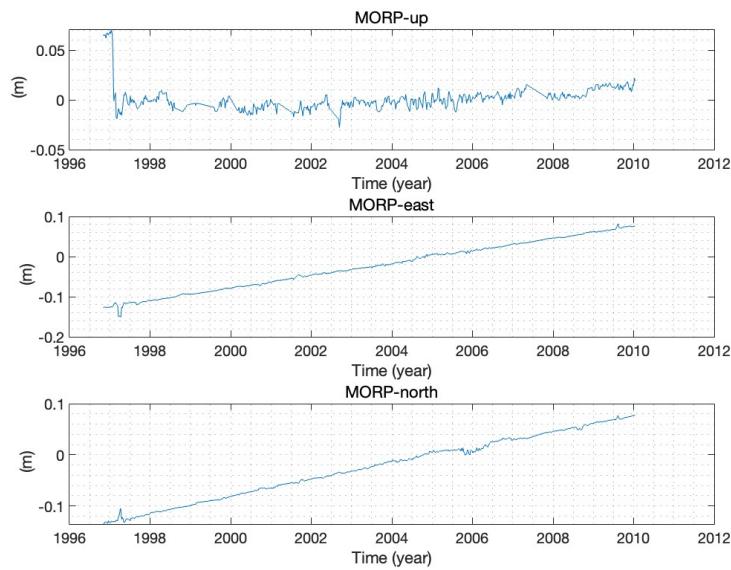


Figure 8: Time Series of MORP(meter)

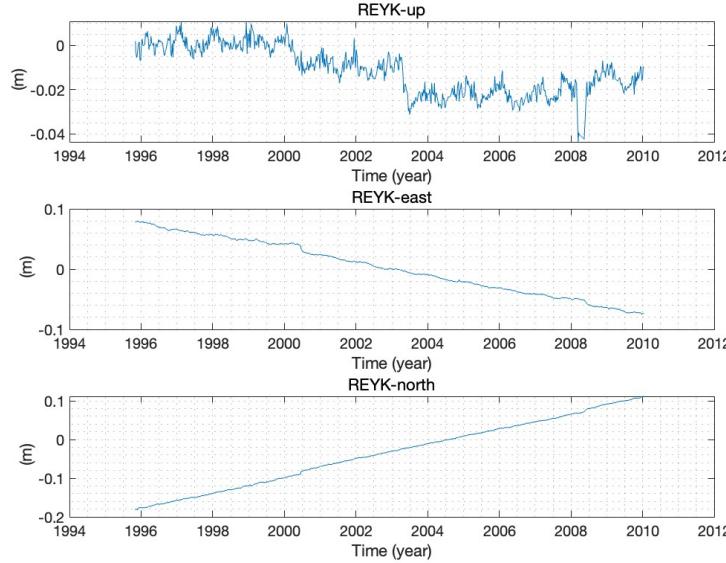


Figure 9: Time Series of REYK(meter)

Table 2: Annual Amplitude

Station	Annual Amplitude(mm)		
	UP	EAST	NORTH
KIRU	7.3089	0.3954	0.2587
MORP	3.9678	0.6203	0.0947
REYK	3.0366	1.6303	0.2655

After we minus the linear trend, our group get the residual time series of these three stations as below and the specific vaules are shown in the table 1. At the same time, table 2 shows the total amplitude of non-linear trend (millimeter per year) which can be computed by  $\sqrt{\beta_3^2 + \beta_4^2}$ .

When we removing the linear trend, our group also remove the constant term in the model( $\beta_1 + \beta_2 t$ ), so that we can see more clearly about the residaul values. After doing this, we can see residual value in millimeter in three directions of three stations.

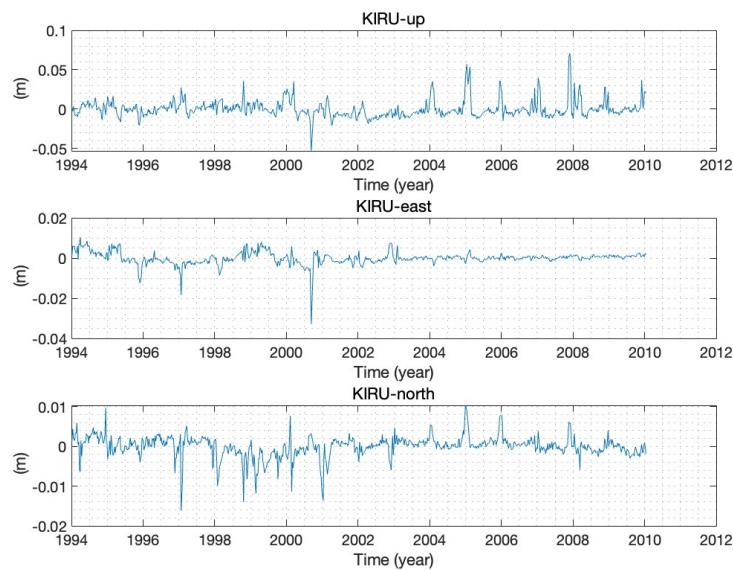


Figure 10: Residual Time Series of KIRU

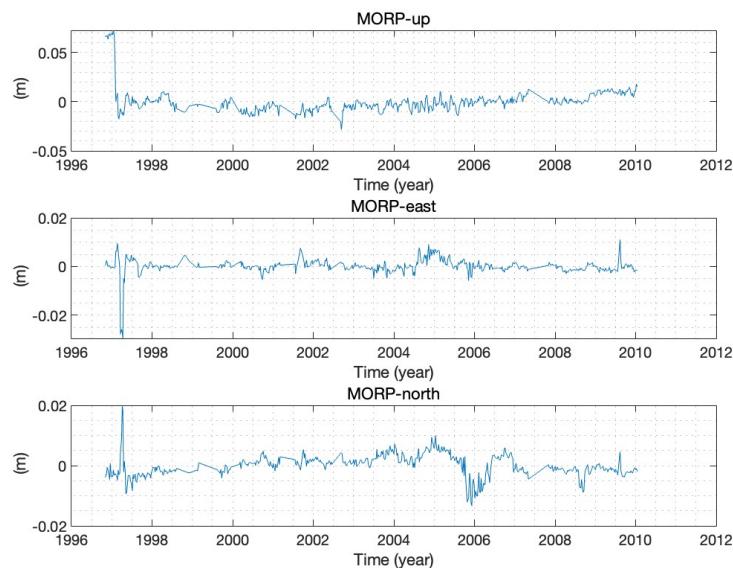


Figure 11: Residual Time Series of MORN

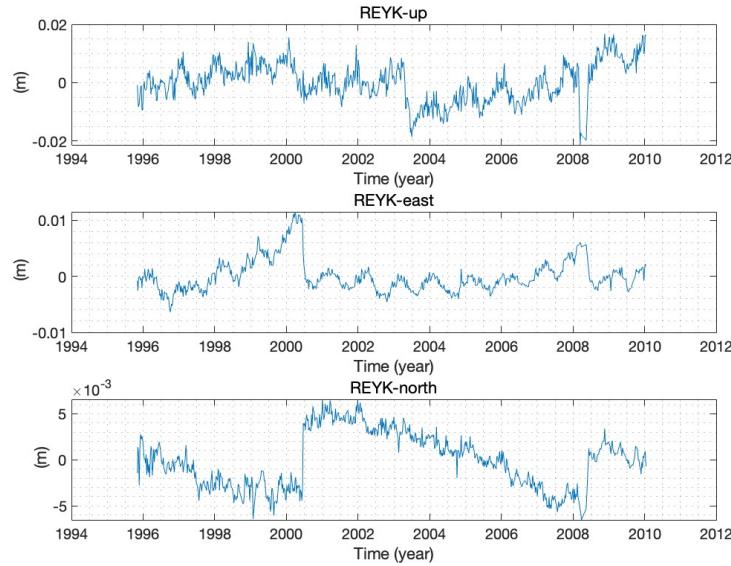


Figure 12: Residual Time Series of REYK

Due to the discontinuities in the observations, our group using the different reference coordinates in time series given by the file "Discontinuities\_CONFIRMED.snx". After separate modeling of different parts of time series, we can get the residual time series of these three stations as below, where we use different colors to represent different parts:

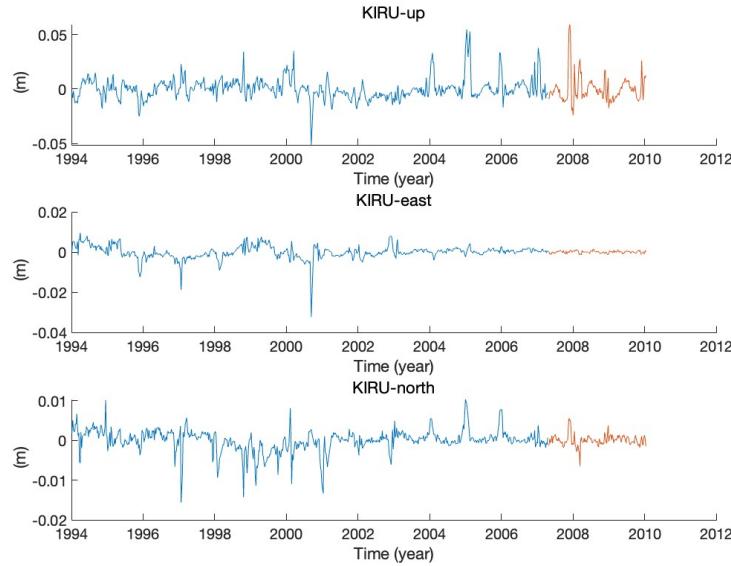


Figure 13: Modified Residual Time Series of KIRU

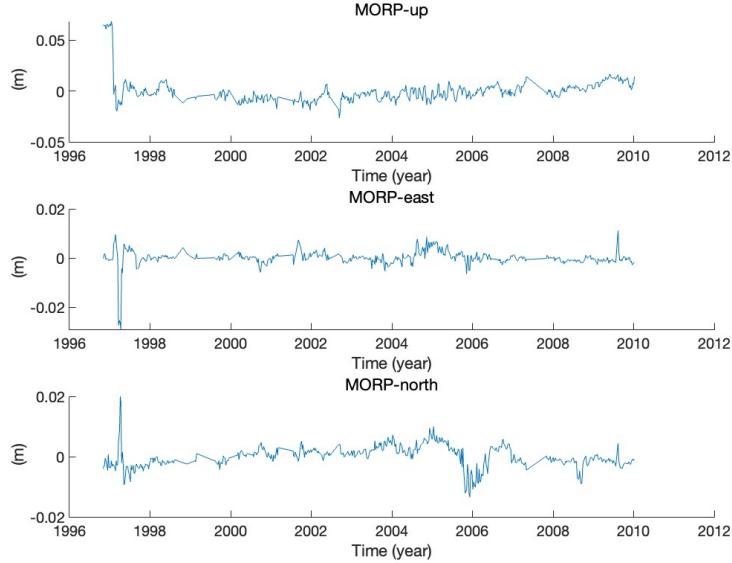


Figure 14: Modified Residual Time Series of MORP

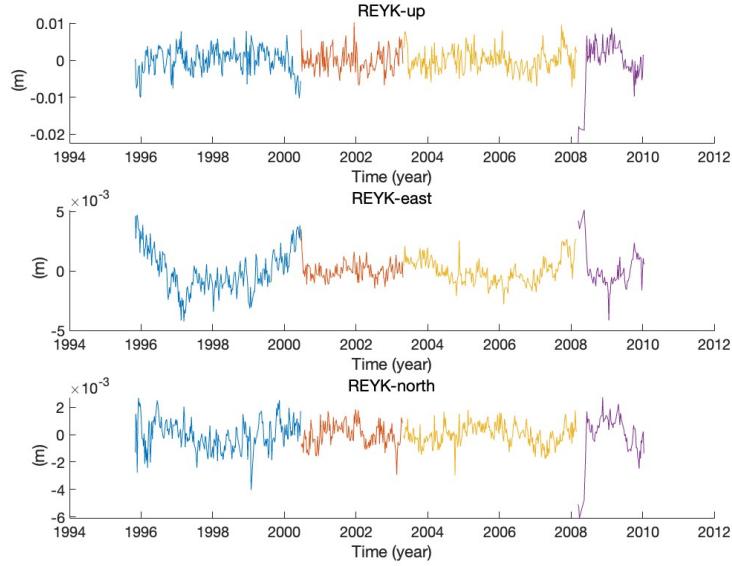


Figure 15: Modified Residual Time Series of REYK

Obviously, the residual values of UP direction are much larger than the other two directions, which also are predominant in total residual values. Through, we know that GPS residuals on the three components are dominated by flicker noise. As for reasons why the amplitudes of UP direction are considerable lareger, We believe this is due to the satellite visibility. Due to the Earth's obstruction, we are unable to receive

signals from satellites on the other side of the Earth. In other words, our control of measurements in the up direction is limited to above the horizontal plane, hence the control intensity is certainly lower compared to the other two directions.

Meanwhile, our group also found some anomalies in the residual time series. Firstly, we find there is an annual signal in the all time series, these signals' amplitude are much large for KIRU, but are small(1-2 millimeter) in MROP and REYK. We speculate that the annula signals is caused mainly by the seasonal changes in the atmosphere. Plus, there are some sudden changes, like the one in 1997 of MROP, the one in 2000 of KIRU and the ones in 2000 and 2008 of REYK. We supposed that these changes are caused by the discontinuities in the observations and recording errors or instrument malfunction.

Additionally, we also find that the residual time series of MROP are much more stable than the other two stations, From the map, it can be observed that MROP has the lowest latitude, so we speculate that this is due to stronger interference received by GPS signals in high-latitude areas, which is related to various factors. Firstly, the ionospheric effects are more significant in high-latitude regions. Additionally, particles generated by solar activities are attracted toward the Earth's North Pole, causing electromagnetic interference and making GPS signals more unstable. Secondly, receivers in high-latitude areas receive satellite signals at lower angles, which increases the atmospheric impact on the signals, meanwhile this also causes receiving signals from fewer satellites. Furthermore, much more snow and ice in high-latitude regions strengthens the effects of multipath effect. In summary, various factors contribute to a reduction in the accuracy of position measurements in high-latitude areas.

## 4.2 Comparison with the Plate Tectonics Model

Through the NUVEL 1A model, we can get the velocity of three stations, for this model only consider the horizontal movements, so our group only compare the velocity in East and North directions.

The comparison is shown below and we can see clearly that the horizontal velocities from NUVEL and GNSS observations differ slightly, sometimes the differences are even considerable like the one in REYK, where velocities in East directions are opposite:

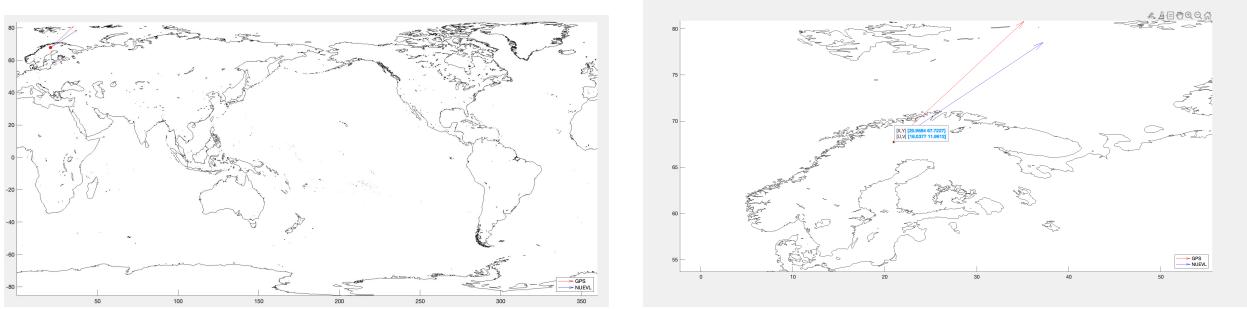


Figure 16: Horizontal Movements Comparison of KIRU

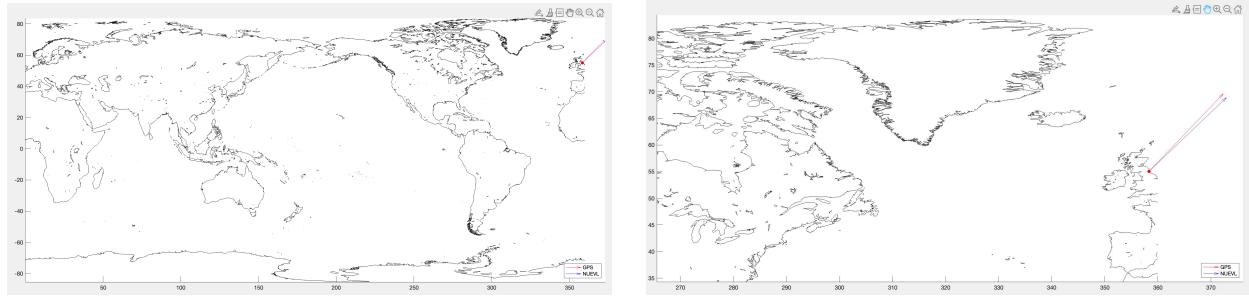


Figure 17: Horizontal Movements Comparison of MORP

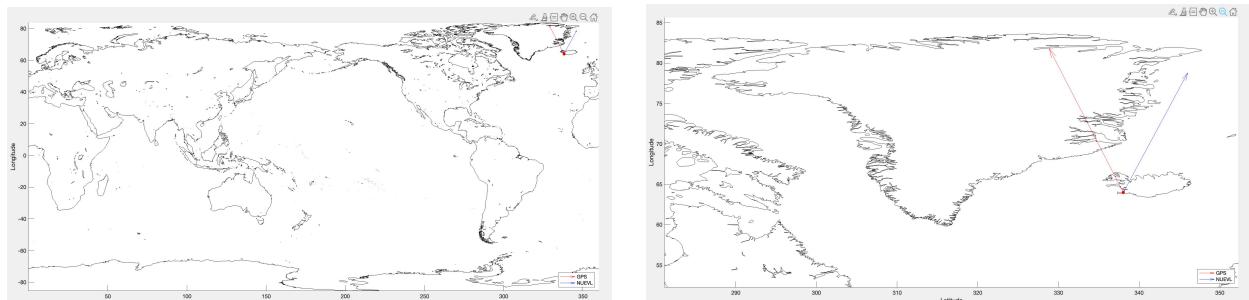


Figure 18: Horizontal Movements Comparison of REYK

And we give the specific values of horizontal velocity(in the East and North axis) as below:

Table 3: Horizontal Movement Rates

Station	Horizontal Velocity/(m/y)			
	GNSS Observation		NUVEL-1A Model	
	EAST	NORTH	EAST	NORTH
KIRU	0.0157	0.0145	0.0180	0.0120
MORP	0.0156	0.0159	0.0162	0.0154
REYK	-0.0104	0.0198	0.00878	0.01646

Station      Horizontal Velocity/(m/y)

	GNSS Observation		NUVEL-1A Model	
	EAST	NORTH	EAST	NORTH
KIRU	0.0157	0.0145	0.0180	0.0120
MORP	0.0156	0.0159	0.0162	0.0154
REYK	-0.0104	0.0198	0.00878	0.01646

For the reason that horizontal movements from GNSS observations don't fit very well that from the NUVEL-1A model well, our group think there are several reasons for this: Firstly, both GNSS observations and the data used in the NUVEL model are interfered by various factors. The GPS signal and observations are affected by multipath effects and ionospheric interference, while seismic activities and receiver changes can also impact the observations. Additionally, the NUVEL 1A model has simplified the plate structure and motion, it isn't considered as a highly accurate model now. Also, NUVEL model uses data from multiple GNSS observations, and there will certainly be some offsets compared to our calculation only using data from a single station.

Interestingly, we observed the same phenomenon as in task 1: the MORP with the lowest latitude has better fitting performance compared to the other two high-latitude stations. And the reasons are the same as we mentioned in task1. In addition, the REYK station is located near the boundary of two tectonic plates, which may also be a reason to the significant differences.

### 4.3 Comparison of vertical movements

In this part, we compare the vertical movements of three stations from GNSS observations and GIA model: ICE-4G and ICE-5G. The visualization of these two models of a global map is here:

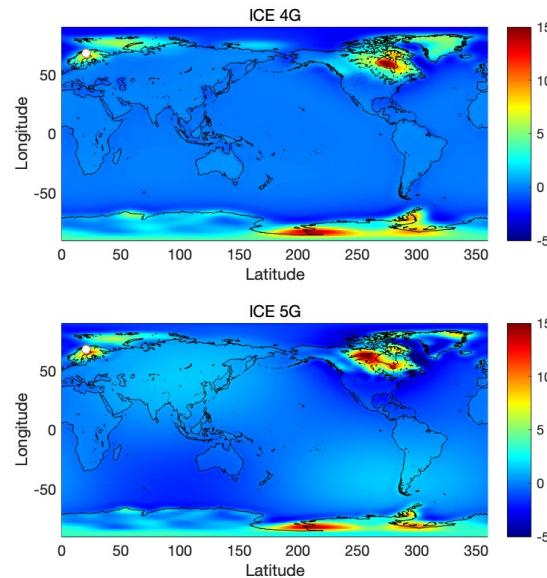


Figure 19: GIA Model

At the same time, our group interpolate the vertical movements rates to the position of three stations, and the results are shown below:

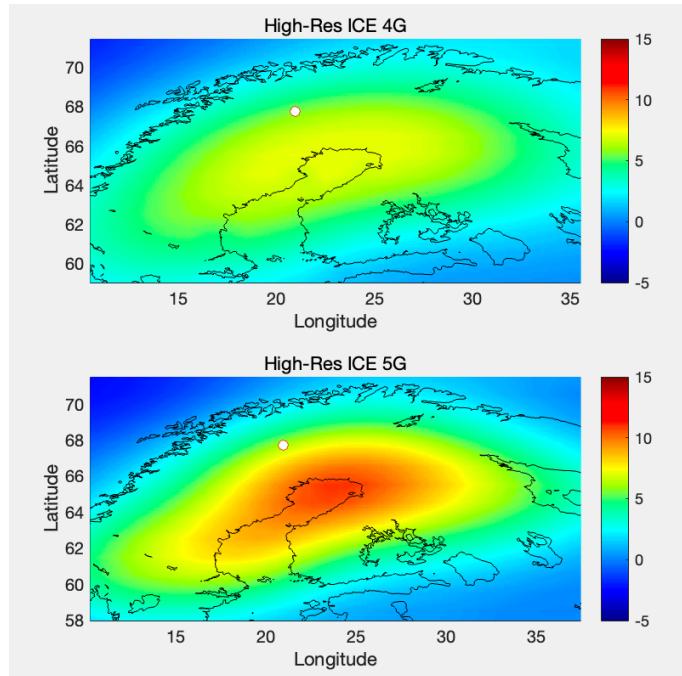


Figure 20: GIA Model Interpolation at KIRU

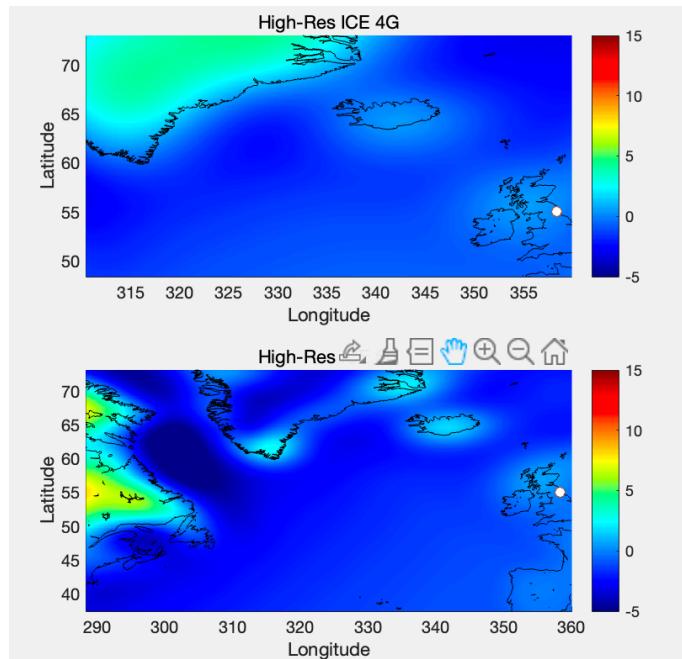


Figure 21: GIA Model Interpolation at MORN

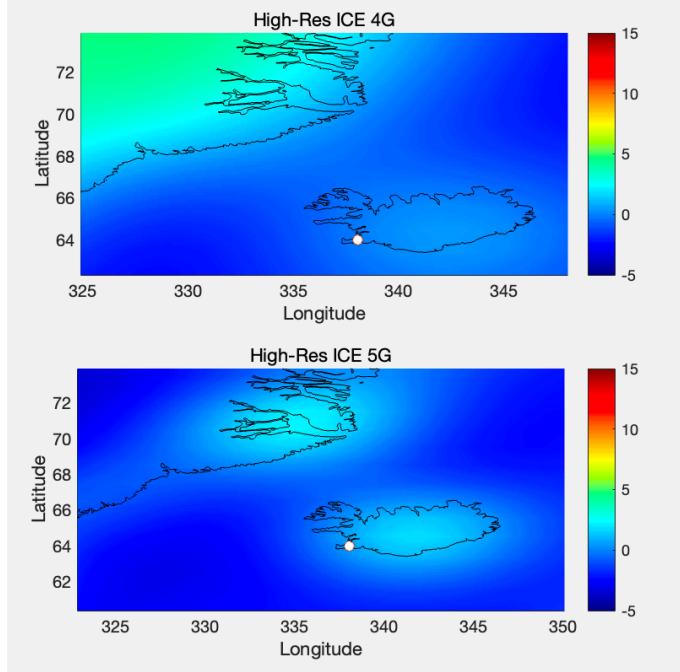


Figure 22: GIA Model Interpolation at REYK

The following table shows the specific values:

Table 4: Vertical Movements Rates

Station	Vertical Movements Rates at IGS stations/(mm/y)		
	GNSS	ICE-4G	ICE-5G
KIRU	6.8840	5.5568	6.1335
MORP	0.0012	0.0140	-0.0408
REYK	0.7212	-0.0464	0.7236

So we can see that, at different stations, the GNSS-obtained data closely align with ICE-5G but still have differences. Meanwhile, ICE-4G data show significant variations with the other two models. Through some materials, it is told that the ICE-4G model is inaccurate because of incorrect assumptions made during model construction and other factors. On the other hand, ICE-5G is a refined model of the post-LGM global deglaciation process. Therefore, our analysis here focuses more on the differences between GNSS-obtained data and the ICE-5G model.

As for reasons, as we have mentioned in task 1, the GNSS observations are affected by various factors, including multipath effects, ionospheric interference and so on. Especially, the vertical movements are more sensitive to the satellite visibility, which is limited by the Earth's obstruction. Therefore, the control in the up direction is weaker compared to the other two directions, and there will certainly be some deviation.

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