

# Satellite Lab1

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## 1 Introduction

## 2 Original Data

### 2.1 ITRF2008 IGS station

The ITRF is The International Reference Frame, and ITRF2008 is a realization of the International Terrestrial Reference System that uses as input data time series of station positions and EOPs provided by the Technique Centers of the four space geodetic techniques (GPS, VLBI, SLR, DORIS).

In the file "ITRF2008\_GNSS.SSC.txt", we can find the coordinates of different stations at epoch 2005.0. and Time is given as the yyyy.yyyy format, which is the number of decimal years:

DOMES NB.	SITE NAME	TECH. ID.	Station position			Sigmas	SOLN	DATA_START	DATA_END
			X/Vx	Y/Vy	Z/Vz				
			m/m/y						
10001S006	Paris	GNSS OPMT	4202777.371	171367.999	4778660.203	0.001 0.001 0.001			
10001S006			-.0125	0.0178	0.0107	.0001 .0000 .0001			
10002M006	Grasse (OCA)	GNSS GRAS	4581690.901	556114.831	4389360.793	0.001 0.001 0.001		1 00:000:00000	03:113:00000
10002M006			-.0133	0.0188	0.0120	.0001 .0000 .0001			
10002M006	Grasse (OCA)	GNSS GRAS	4581690.900	556114.837	4389360.793	0.001 0.001 0.001		2 03:113:00000	04:295:43200
10002M006			-.0133	0.0188	0.0120	.0001 .0000 .0001			
10002M006	Grasse (OCA)	GNSS GRAS	4581690.900	556114.836	4389360.797	0.001 0.001 0.001		3 04:295:43200	00:000:00000
10002M006			-.0133	0.0188	0.0120	.0001 .0000 .0001			
10003M004	Toulouse	GNSS TOUL	4627846.029	119629.333	4372999.818	0.001 0.001 0.001			
10003M004			-.0114	0.0193	0.0121	.0001 .0000 .0001			
10003M009	Toulouse	GNSS TLSE	4627851.831	119640.017	4372993.553	0.001 0.001 0.001		1 00:000:00000	03:335:00000
10003M009			-.0114	0.0193	0.0121	.0001 .0000 .0001			
10003M009	Toulouse	GNSS TLSE	4627851.828	119640.020	4372993.552	0.001 0.001 0.001		2 03:335:00000	00:000:00000
10003M009			-.0114	0.0193	0.0121	.0001 .0000 .0001			

Figure 1: ITRF2008\_GNSS.ssc.txt Description

### 2.2 Station GPS Observations

We were responsible for the computation of the positions and movements of three measurement stations: KIRU, MORP, and REYK. The locations are

illustrated in the following figure:

An example of the observation file for each data set is provided below, including two time formats and XYZ coordinates.

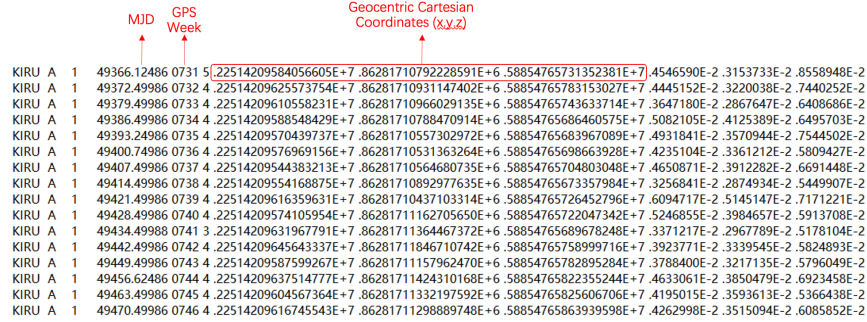


Figure 2: station.xyz Description

And the file Discontinuities\_snx provides the discontinuity information in the positions' time series. The reasons for that include change of antenna and receiver, earthquake and so on. In this example(station: REYK), the discontinuity happened three times due to earthquake, antenna change and unknown reason.

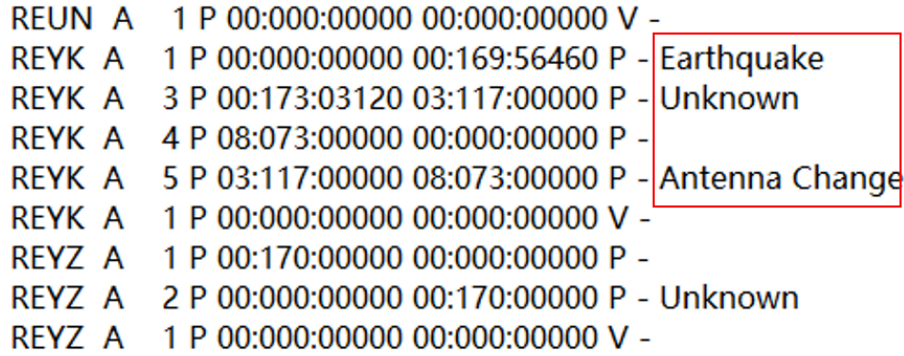


Figure 3: Discontinuities in time series

## 2.3 NUVEL 1A Model

NUVEL(Northeast University Velocity) is a the collective term for geophysical Earth models that describes observable continental movements through a dynamic theory of plaet tectonics.

The "NNR\_NUVEL1A.txt" gives the rotation referred to epoch  $t_0$ . The file contains the following data, where the leftmost column represents the station

name, and in that row, the angular velocity changes in three directions are provided (unit: radians per million years or  $rad/My$ ).

Plate name	Wx (rad/Ma)	Wy (rad/Ma)	Wz (rad/Ma)
Pacific	-0.001510	0.004840	-0.009970
Cocos	-0.010425	-0.021605	0.010925
Nazca	-0.001532	-0.008577	0.009609
Caribbean	-0.000178	-0.003385	0.001581
South_America	-0.001038	-0.001515	-0.000870
Antarctica	-0.000821	-0.001701	0.003706
India		0.006670	0.000040
Australia	0.007839	0.005124	0.006282
Africa		0.000891	-0.003099
Arabia		0.006685	-0.000521
Eurasia		-0.000981	-0.002395
North_America		0.005200	-0.003599
Juan_de_Fuca		0.005200	0.008610
Philippine	0.010090	-0.007160	-0.009670
Rivera		-0.009390	-0.030960
Scotia		-0.000410	-0.002660

Figure 4: NUVEL\_1A.txt Description

## 2.4 GIA Models

## 2.5 Other data

## 2.6 Matlab Code

# 3 Methodology

## 3.1 Transformation to LHS

[Geocentric cartesian coordinate system] is a three-dimensional, earth-centered reference system in which locations are identified by their x, y, and z values. The x-axis is in the equatorial plane and intersects the prime meridian (usually Greenwich). The y-axis is also in the equatorial plane; it lies at right angles to the x-axis and intersects the 90-degree meridian. The z-axis coincides with the polar axis and is positive toward the north pole. The origin is located at the center of the sphere or spheroid.

[Local horizontal system] uses the Cartesian coordinates(East,Nort,Up) to represent position relative to a local origin. The local origin is described by the geodetic coordinates.

The initial coordinates are in the geocentric Cartesian coordinate system and need to be transformed into representation in the local horizontal coordinate system. In this project, we use two angles and the ITRF2008 point positions as the original point,

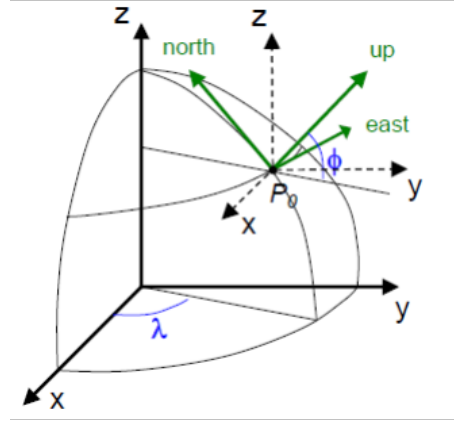


Figure 5: Coordinates Transformation

Calculate the angle according to stations' geodetic coordinates:

$$\lambda = \arctan \frac{y}{x}$$

$$\varphi = \arctan \frac{2}{\sqrt{x^2 + y^2}}$$

Then we can get the rotation matrix:

$$R_2(\delta) = \begin{pmatrix} \cos \delta & 0 & -\sin \delta \\ 0 & 1 & 0 \\ \sin \delta & 0 & \cos \delta \end{pmatrix} \quad R_3(\delta) = \begin{pmatrix} \cos \delta & \sin \delta & 0 \\ -\sin \delta & \cos \delta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Transformation of coordinates:

$$\begin{pmatrix} x_{up} \\ x_{east} \\ x_{north} \end{pmatrix} = R_2(-\varphi^0) R_3(\lambda^0) \left( \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} - \begin{pmatrix} x_1^0 \\ x_2^0 \\ x_3^0 \end{pmatrix} \right)$$

$x^0$  are the stations' geodetic coordinates, and  $x$  is observations in file '.xyz'. Notice that we also can directly use the longitude and latitude of stations provided in the file "Discontinuities\_CONFIRMED.snz".

In terms of velocity, its transformation into LHS only requires multiplication by a rotation matrix.

### 3.2 Least Square Adjustment for Parameters Estimation

For time series,

$$y(t) = \beta_1 + \beta_2 t + \beta_3 \cos \omega t + \beta_4 \sin \omega t$$

among which  $\beta_3$  and  $\beta_4$  are total amplitude of annual, and  $\beta_2$  is linear trend; so we can build model like:

$$\begin{pmatrix} y(t_1) \\ \vdots \\ y(t_n) \end{pmatrix} = \begin{pmatrix} 1 & t_1 & \cos \omega t_1 & \sin \omega t_1 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & t_n & \cos \omega t_n & \sin \omega t_n \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix}$$

We can simpfit the model like:

$$Y = X\beta + \varepsilon$$

where  $Y$  is the observations,  $X$  is the design matrix,  $\beta$  is the parameters, and  $\varepsilon$  is the noise.

According to least square, minimize the noise, derivative the square of noise and set it to zero so we get:

$$\beta = (X^T X)^{-1} X^T Y$$

through this we can get the estimated parameters.

### 3.3 Model of Plate Tectonics

The movement of any plate on a spherical Earth can be described through a rotation around the Euler pole:

$$\underline{\Omega} = (\Omega_1, \Omega_2, \Omega_3)^T$$

In point  $\underline{x}_0 = (x, y, z)^T$  the velocity vector  $\underline{v}$  is obtained by:

$$\underline{v} = \underline{\Omega} \times \underline{x}_0 = \begin{pmatrix} 0 & -\Omega_z & \Omega_y \\ \Omega_z & 0 & -\Omega_x \\ -\Omega_y & \Omega_x & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

### 3.4 Program Description

## 4 Results and Analysis

### 4.1 Time Series and Linear Trend

Through the least square adjustment, our group got the time series of these three stations as shown below:

DOMES NB. SITE NAME	TECH. ID.	Station position			Sigmas	SOLN	DATA_START	DATA_END
		X/Vx	Y/Vy	Z/Vz				
		m/m/y						
-----								
10001S006 Paris	GNSS OPMT	4202777.371	171367.999	4778660.203	0.001 0.001 0.001			
10001S006		-0.125	0.0178	0.0107	0.001 .0000 .0001			
10002M006 Grasse (OCA)	GNSS GRAS	4581690.901	556114.831	4389360.793	0.001 0.001 0.001		1 00:00:00000 03:113:00000	
10002M006		-0.133	0.0188	0.0120	0.001 .0000 .0001			
10002M006 Grasse (OCA)	GNSS GRAS	4581690.900	556114.837	4389360.793	0.001 0.001 0.001		2 03:113:00000 04:295:43200	
10002M006		-0.133	0.0188	0.0120	0.001 .0000 .0001			
10002M006 Grasse (OCA)	GNSS GRAS	4581690.900	556114.836	4389360.797	0.001 0.001 0.001		3 04:295:43200 00:000:00000	
10002M006		-0.133	0.0188	0.0120	0.001 .0000 .0001			
10003M004 Toulouse	GNSS TOUL	4627846.029	119629.333	4372999.818	0.001 0.001 0.001			
10003M004		-0.114	0.0193	0.0121	0.001 .0000 .0001			
10003M009 Toulouse	GNSS TLSE	4627851.831	119640.017	4372993.553	0.001 0.001 0.001	1	00:00:00000 03:335:00000	
10003M009		-0.114	0.0193	0.0121	0.001 .0000 .0001			
10003M009 Toulouse	GNSS TLSE	4627851.828	119640.020	4372993.552	0.001 0.001 0.001	2	03:335:00000 00:000:00000	
10003M009		-0.114	0.0193	0.0121	0.001 .0000 .0001			

Figure 6: Time Series of KIRU

DOMES NB. SITE NAME	TECH. ID.	Station position			Sigmas	SOLN	DATA_START	DATA_END
		X/Vx	Y/Vy	Z/Vz				
		m/m/y						
10001S006 Paris	GNSS OPMT	4202777.371	171367.999	4778660.203	0.001 0.001 0.001			
10001S006		-0.125	0.0178	0.0107	0.001 0.000 0.001			
10002M006 Grasse (OCA)	GNSS GRAS	4581690.901	556114.831	4389360.793	0.001 0.001 0.001		1 00:00:00000 03:113:00000	
10002M006		-0.133	0.0188	0.0120	0.001 0.000 0.001			
10002M006 Grasse (OCA)	GNSS GRAS	4581690.900	556114.837	4389360.793	0.001 0.001 0.001		2 03:113:00000 04:295:43200	
10002M006		-0.133	0.0188	0.0120	0.001 0.000 0.001			
10002M006 Grasse (OCA)	GNSS GRAS	4581690.900	556114.836	4389360.797	0.001 0.001 0.001		3 04:295:43200 00:000:00000	
10002M006		-0.133	0.0188	0.0120	0.001 0.000 0.001			
10003M004 Toulouse	GNSS TOUL	4627846.029	119629.333	4372999.818	0.001 0.001 0.001			
10003M004		-0.114	0.0193	0.0121	0.001 0.000 0.001			
10003M009 Toulouse	GNSS TLSE	4627851.831	119640.017	4372993.553	0.001 0.001 0.001	1	00:000:00000 03:335:00000	
10003M009		-0.114	0.0193	0.0121	0.001 0.000 0.001			
10003M009 Toulouse	GNSS TLSE	4627851.828	119640.020	4372993.552	0.001 0.001 0.001	2	03:335:00000 00:000:00000	
10003M009		-0.114	0.0193	0.0121	0.001 0.000 0.001			

Figure 7: Time Series of MORP

DOMES NB. SITE NAME	TECH. ID.	Station position			Sigmas	SOLN	DATA_START	DATA_END
		X/Vx	Y/Vy	Z/Vz				
		m/m/y						
-----								
10001S006 Paris	GNSS OPMT	4202777.371	171367.999	4778660.203	0.001 0.001 0.001			
10001S006		-0.125	0.0178	0.0107 .0001 .0000 .0001				
10002M006 Grasse (OCA)	GNSS GRAS	4581690.901	556114.831	4389360.793	0.001 0.001 0.001		1 00:00:00000 03:113:00000	
10002M006		-0.133	0.0188	0.0120 .0001 .0000 .0001				
10002M006 Grasse (OCA)	GNSS GRAS	4581690.900	556114.837	4389360.793	0.001 0.001 0.001		2 03:113:00000 04:295:43200	
10002M006		-0.133	0.0188	0.0120 .0001 .0000 .0001				
10002M006 Grasse (OCA)	GNSS GRAS	4581690.900	556114.836	4389360.797	0.001 0.001 0.001		3 04:295:43200 00:000:00000	
10002M006		-0.133	0.0188	0.0120 .0001 .0000 .0001				
10003M004 Toulouse	GNSS TOUL	4627846.029	119629.333	4372999.818	0.001 0.001 0.001			
10003M004		-0.114	0.0193	0.0121 .0001 .0000 .0001				
10003M009 Toulouse	GNSS TLSE	4627851.831	119640.017	4372993.553	0.001 0.001 0.001	1	00:000:00000 03:335:00000	
10003M009		-0.114	0.0193	0.0121 .0001 .0000 .0001				
10003M009 Toulouse	GNSS TLSE	4627851.828	119640.020	4372993.552	0.001 0.001 0.001	2	03:335:00000 00:000:00000	
10003M009		-0.114	0.0193	0.0121 .0001 .0000 .0001				

Figure 8: Time Series of REYK

Table 1: Linear Trend of Time Series

Station	Linear Trend(mm/y)		
	UP	EAST	NORTH
KIRU			
MORP			
REYK			

And we can see that the linear trend of KIRU in Up, East and North directions are , , and respectively.

After we minus the linear trend, our group get the residual time series of these three stations as below and the specific vaules are shown in the table 2.

Plate name	Wx (rad/Ma)		Wy (rad/Ma)	Wz (rad/Ma)
Pacific	-0.001510	0.004840	-0.009970	
Cocos	-0.010425	-0.021605	0.010925	
Nazca	-0.001532	-0.008577	0.009609	
Caribbean	-0.000178	-0.003385	0.001581	
South_America	-0.001038	-0.001515	-0.000870	
Antarctica	-0.000821	-0.001701	0.003706	
Inida		0.006670	0.000040	0.006790
Australia	0.007839	0.005124	0.006282	
Africa		0.000891	-0.003099	0.003922
Arabia		0.006685	-0.000521	0.006760
Eurasia		-0.000981	-0.002395	0.003153
North_America		0.005200	-0.003599	-0.000153
Juan_de_Fuca		0.005200	0.008610	-0.005820
Philippine	0.010090	-0.007160	-0.009670	
Rivera		-0.009390	-0.030960	0.012050
Scotia		-0.000410	-0.002660	-0.001270

Figure 9: Residual Time Series of KIRU

When we removing the linear trend, our group also remove the constant term in the model, so that we can see more clearly about the residaul values. After doing this, we can see residual value in millimeter in three directions of three stations.

Plate name	Wx (rad/Ma)	Wy (rad/Ma)	Wz (rad/Ma)
Pacific	-0.001510	0.004840	-0.009970
Cocos	-0.010425	-0.021605	0.010925
Nazca	-0.001532	-0.008577	0.009609
Caribbean-0.000178	-0.003385	0.001581	
South_America	-0.001038	-0.001515	-0.000870
Antarctica-0.000821	-0.001701	0.003706	
Inida	0.006670	0.000040	0.006790
Australia 0.007839	0.005124	0.006282	
Africa	0.000891	-0.003099	0.003922
Arabia	0.006685	-0.000521	0.006760
Eurasia	-0.000981	-0.002395	0.003153
North_America	0.005200	-0.003599	-0.000153
Juan_de_Fuca	0.005200	0.008610	-0.005820
Philippine 0.010090	-0.007160	-0.009670	
Rivera	-0.009390	-0.030960	0.012050
Scotia	-0.000410	-0.002660	-0.001270

Figure 10: Residual Time Series of MORP

Plate name	Wx (rad/Ma)	Wy (rad/Ma)	Wz (rad/Ma)
Pacific	-0.001510	0.004840	-0.009970
Cocos	-0.010425	-0.021605	0.010925
Nazca	-0.001532	-0.008577	0.009609
Caribbean-0.000178	-0.003385	0.001581	
South_America	-0.001038	-0.001515	-0.000870
Antarctica-0.000821	-0.001701	0.003706	
Inida	0.006670	0.000040	0.006790
Australia 0.007839	0.005124	0.006282	
Africa	0.000891	-0.003099	0.003922
Arabia	0.006685	-0.000521	0.006760
Eurasia	-0.000981	-0.002395	0.003153
North_America	0.005200	-0.003599	-0.000153
Juan_de_Fuca	0.005200	0.008610	-0.005820
Philippine 0.010090	-0.007160	-0.009670	
Rivera	-0.009390	-0.030960	0.012050
Scotia	-0.000410	-0.002660	-0.001270

Figure 11: Residual Time Series of REYK



**4.2 Comparison of horizontal movements**

**4.3 Comparison of vertical movements**

**5 Conclusion**