Correlation Judgment and Visualization Features: A Comparative Study
Supplementary Materials

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OVERVIEW

These supplementary materials extend the analyses and results in the original paper at breadth and in depth simultaneously. Specifically, we present

Appendix A: the more detailed and strict definitions of all visual features on Page 1;
Appendix B : the additional figures to illustrate the extreme values and colinearity between correlation value r and visual features on Page 1;
Appendix C: the numeric results of the visual feature models from judgments as well as the results of other statistical metrics on Page 1;
Appendix D: a complete derivation of the substitution and the linear assumptions on Page 2;
Appendix E : the additional results for the residuals analysis on Page 2;
Appendix F : a complete integration to infer objective perception from the three JND models (linear, non-linear, and power) on Page 2.
Appendix G: the results of analyzing other visual features on Page 3.

APPENDIX A
COMPUTATION OF VISUAL FEATURES

In Section 3 of the main body, we provide a conceptual description of each candidate visual features. Here, we provide a more detailed and strict definition of each of the visual features (see Table A1, Page 7). We denote with:
N: the number of points, where N=100,
(x_i, y_i), the points, where i=1..100,
(x'_i, y'_i), the points after 45° clockwise rotation,
(x^*_j, y^*_j), the points within 3 standard deviations after 45° clockwise rotation,
(x^c_h, y^c_h), the points on the convex hull and H is the total number of points on the convex hull,

$\bar{x} = \dfrac{\sum x_i}{N}$,

$\bar{y} = \dfrac{\sum y_i}{N}$,

$s_x^2 = \dfrac{\sum (x_i - \bar{x})}{N-1}$, the variance of the points on x-axis,

$s_y^2 = \dfrac{\sum (y_i - \bar{y})}{N-1}$, the variance of the points on y-axis,

r, the correlation coefficient between the x_i and the y_i,

$A = s_y^2$, following the definition of prediction ellipse in [1],

$B = -r s_x^2 s_y^2$,

$C = s_x^2$,

$D = 2\dfrac{N-1}{N(N-2)} s_x^2 s_y^2 (1-r^2) F_{\frac{\alpha}{2},2,N-2}$, where denotes the critical value of the Fisher distribution with 2 and $N-2$ degrees of freedom, for a significance level α,
max and min, the maximum or minimum of all the values specified,
z_α, the critical value of the standard normal distribution at the given α level,
mod, the modulo operation to find the remainder after division,
d, the Euclidean distance from a point to a line,
sd, the standard deviation of the given set of values,
q_α, the αth percentile of the given set of values,
t_i, the local density (defined via k-Nearest Neighbors) of a point, i=1..100,
p_u, the pairwise distance, u=1..$N(N-1)$,
e_v, the edge on the minimum spanning tree, v=1..$N-1$.

APPENDIX B
SCATTERPLOT MATRIX AND CORRELATION MATRIX

In Section 4 of the main body, we present a few examples to illustrate the collinearity and extreme values. Here, we present the full correlation matrix and the scatterplot matrix between Δr and all Δv before removing extreme values. Please see Figure B1, Page 8.

APPENDIX C
VISUAL FEATURE MODELS USING PAIR JUDGMENTS

In Section 4 of the main body, we present the graphical results of a few statistical metrics, such as odds ratios, to illustrate the selection of visual features. In this appendix, we provide the precise numberic results and the results measured by other prevalent statistical metrics. Please see Table C1, Page 9.

Appendix D
Derivation of Substitution

In Section 5 of the main body, we present a substitution technique to derive the correlation model from a visual feature model and concisely explain the intuition beyond the substitution technique. Here, we present the mathematical derivation process, using the power transformation as an example.

The relation between perception and the intensity of correlation is written as

$$JND_r = f_1(r) \tag{1}$$

Using power transformation and the modeling technique in the main body of this paper, f_1 is

$$f_1 : JND_r^{\omega_1} = \beta_{0,1} + \beta_{1,1}r + \beta_{2,1}a + \beta_{3,1}ar + U \tag{2}$$

Similarly, the relation between JND of a visual feature and the intensity of the visual feature is written as

$$JND_v = f_2(v) \tag{3}$$

In the power transformation, f_2 is:

$$f_2 : JND_v^{\omega_2} = \beta_{0,2} + \beta_{1,2}v + \beta_{2,2}a + \beta_{3,2}av + U \tag{4}$$

The transformed JNDs are assumed to follow a normal distribution, so that the relation between the two set of transformed JNDs (f_3) can be written as

$$f_3 : JND_v^{\omega_2} = \beta_{0,3} + \beta_{1,3}JND_r^{\omega_1} + U \tag{5}$$

With the observation that the relation between r (correlation) and v (the visual feature) can be linear (see Figure D1), we have f_4

$$f_4 : v = \beta_{0,4} + \beta_{1,4}r \tag{6}$$

The key point is to transform the intercept and slope in f_2 (Equation 4) into f_1 (Equation 1) using f_3 and f_4, and therefore we discard a and ax. Plugging f_3 (Equation 5) and f_4 (Equation 6) into f_2 (Equation 4), we have

$$\beta_{0,3} + \beta_{1,3}JND_r^{\omega_1} + U = \beta_{0,2} + \beta_{1,2}(\beta_{0,4} + \beta_{1,4}r) + U \tag{7}$$

We can further simplify the above equation into

$$JND_r^{\omega_1} = \frac{\beta_{1,2}\beta_{0,4} + \beta_{0,2} - \beta_{0,3}}{\beta_{1,3}} + \frac{\beta_{1,2}\beta_{1,4}}{\beta_{1,3}}r \tag{8}$$

The intercept and the slope in the above equation are compared to the coefficients in the original model of correlation ($\beta_{0,1}$ and $\beta_{1,1}$ in Equation 1, respectively).

The linear assumption we make for mapping between a visual feature and correlation is illustrated in Figure D1. While it is validate to other non-linear forms, we use the linear form for three reasons:

1) It simplifies the computation and yields fair results. Note that these visual features appear to be very different, but a linear approximation generally yields good fits, especially on the region we have tested (r=[0.3, 0.8]).
2) It avoids the discrepancies between different forms. If we use varying forms in different models and obtain different performance in substitution, it is impossible to tell whether the difference is raised by this mapping or the different JND models. A linear form is a baseline for comparison.
3) This mapping between objective visual feature and objective correlation doesn't affect the choice of modeling technique based on JNDs in subjective space. In fact, the non-linearity between the visual feature and correlation is very likely to be the reason that the JND models are non-linear: participants are using non-linear proxies so that the resulting perception is non-linear. Our argument is that it is not that participants' eyes and brains are doing a secondary non-linear transformation, but rather, they are directly using non-linear proxies.

Appendix E
Additional Results of Regression

In Sections 5 and 6 of the main body, we use an overall detrended Q-Q plot to illustrate the skewness and non-normality in residuals. Here, we provide the residual plots presented in Kay and Heer's [2] and all detrended Q-Q plots for different r levels. Please see Figure E1 and Figure E2.

Appendix F
Inferring Subjective Perception from JND Models

In Sections 5 and 6 of the main body, we focus on JND models, modeling the discrimination in comparing correlation tasks. On Fechner's assumption [**?**], [3], [4], [5], it is possible to infer from the subjective perception by integration. We present the inferring and integrating operations for the three JND models in this section to derive the subjective (perceived) stimulus.

For the linear model, we have

$$JND = k(I + b) \tag{9}$$

where I is the stimuli (i.e., r, visual feature). This equation can be rewritten as

$$\frac{1}{k}\frac{JND}{I+b} = constant \tag{10}$$

On Fechner's assumption that JND represents the change in objective stimulus (I) that results in a fixed change in 6. perceived stimulus (P), this can be written as

$$\frac{1}{k}\frac{\Delta I}{I+b} = \Delta P \tag{11}$$

This equation can be rewritten as

$$\frac{1}{k}\frac{dI}{I+b} = c\, dP \tag{12}$$

where c is the proportionality constant. On integration this yields

$$P = \frac{1}{ck}log(I + b) + C \tag{13}$$

Similarly, for the log-linear model, we have

$$log(JND) = k(I + b) \tag{14}$$

We can rewrite this as

$$\frac{JND}{e^{k(I+b)}} = constant \tag{15}$$

On Fechner's assumption, the above equation turns in

$$\frac{dI}{e^{k(I+b)}} = c\,dP \tag{16}$$

On integration this yields

$$P = -\frac{1}{ck}e^{-k(I+b)} + C \tag{17}$$

Lastly, for the power model, we have

$$JND^{\omega} = k(I+b) \qquad (\omega \neq 1) \tag{18}$$

We can rewrite it as

$$\frac{JND}{(k(I+b))^{a}} = constant \tag{19}$$

where a=ω^{-1}. On Fechner's assumption, this turns in

$$\frac{dI}{(k(I+b))^{a}} = c\,dP \tag{20}$$

On integration the above equation yields

$$P = \frac{k^{-a}}{c(1-a)}(I+b)^{(1-a)} + C \tag{21}$$

The Equations 13, 17, and 21 are inferred models of subjective (perceived) stimulus and objective stimulus. They are final results presented in the main body of our paper.

Appendix G
Modeling Results of Other Visual Features

We present the modeling results of the other features. These features are

- *the standard deviation of all perpendicular distances to the regression line* (*dist_line_sd*, the example we use in the main body),
- *the area of the prediction ellipse* (*ellipse_area*),
- *the length of the minor axis of the prediction ellipse* (*ellipse_minor*),
- *the length of the perpendicular side of the confidence bounding box* (*conf_bounding_box_perp*),

Please see Table G1, Page 11 for the linear models using mean observations, Table G2, Page 11 for the linear models using individual observations, Table G3, Page 12 for the log-linear models using individual observations, and Table G4, Page 12 for the power models using individual observations.

References

[1] M. B. L. Rocchi, D. Sisti, M. Ditroilo, A. Calavalle, and R. Panebianco, "The misuse of the confidence ellipse in evaluating statokinesigram," *Italian Journal Of Sport Sciences*, vol. 12, no. 2, pp. 169–172, 2005.

[2] M. Kay and J. Heer, "Beyond weber's law: A second look at ranking visualizations of correlation," *IEEE Transactions on Visualization and Computer Graphics*, vol. 22, no. 1, pp. 469–478, 2016.

[3] R. A. Rensink and G. Baldridge, "The perception of correlation in scatterplots," *Computer Graphics Forum*, vol. 29, no. 3, pp. 1203–1210, 2010.

[4] E. B. Goldstein, *Sensation and perception*. Thomson Brooks/Cole Publishing Co, 1996.

[5] R. A. Rensink, "The nature of correlation perception in scatterplots," *Psychonomic Bulletin & Review*, pp. 1–22, 2016.

TABLE A1: The Definitions of the Visual Features

No.	Notation	Description	Definition
1	$ellipse\_minor$	the minor axis of the prediction ellipse	$ellipse\_minor = \sqrt{\dfrac{2D}{A + C - \sqrt{(A-C)^2 + 4B^2}}}$ (see the beginning of Appendix A for the definitions of A, B, C, and D)
2	$ellipse\_major$	the major axis of the prediction ellipse	$ellipse\_major = \sqrt{\dfrac{2D}{A + C + \sqrt{(A-C)^2 + 4B^2}}}$
3	$ellipse\_area$	the area of the prediction ellipse	$ellipse\_area = \pi * ellipse\_minor * ellipse\_major$
4	$ellipse\_ratio$	the ratio of the minor axis to the major axis	$ellipse\_ratio = \dfrac{ellipse\_minor}{ellipse\_major}$
5	$ellipse\_ratio\_inverse$	the reciprocal of $ellipse\_ratio$ (the ratio of the major axis to the minor axis)	$ellipse\_ratio\_inverse = \dfrac{ellipse\_major}{ellipse\_minor}$
6	$bounding\_box\_perp$	the side of the bounding box that is perpendicular to the regression line	$bounding\_box\_perp = max(y_i') - min(y_i')$
7	$bounding\_box\_para$	the side of the bounding box that is parallel to the regression line	$bounding\_box\_para = max(x_i') - min(x_i')$
8	$bounding\_box\_area$	the area of the bounding box	$bounding\_box\_area = bounding\_box\_perp * bounding\_box\_para$
9	$bounding\_box\_ratio$	the ratio of the perpendicular side to the parallel side	$bounding\_box\_ratio = \dfrac{bounding\_box\_perp}{bounding\_box\_para}$
10	$bounding\_box\_ratio\_inverse$	the reciprocal of $bounding\_box\_ratio$	$bounding\_box\_ratio\_inverse = \dfrac{bounding\_box\_para}{bounding\_box\_perp}$
11	$conf\_box\_perp$ (dropped)	the side of the confidence box that is perpendicular to the regression line	$conf\_box\_perp = 2 * z_{.025} * sd(y_i')$
12	$conf\_box\_para$ (dropped)	the side of the confidence box that is parallel to the regression line	$conf\_box\_para = 2 * z_{.025} * sd(x_i')$
13	$conf\_box\_area$ (dropped)	the area of the confidence box	$conf\_box\_area = conf\_box\_perp * conf\_box\_para$

TABLE A1: The Definitions of the Visual Features

No.	Notation	Description	Definition
14	$conf\_box\_ratio$ (dropped)	the ratio of the perpendicular side to the parallel side of the confidence box	$conf\_box\_ratio = \frac{conf\_box\_perp}{conf\_box\_para}$
15	$conf\_box\_ratio\_inverse$ (dropped)	the reciprocal of conf_box_ratio	$conf\_box\_ratio\_inverse = \frac{conf\_box\_para}{conf\_box\_perp}$
16	$conf\_bounding\_box\_perp$	the side of the confidence bounding box that is perpendicular to the regression line	$conf\_bounding\_box\_perp = max(y_j^*) - min(y_j^*)$
17	$conf\_bounding\_box\_para$	the side of the confidence bounding box that is parallel to the regression line	$conf\_bounding\_box\_perp = max(x_j^*) - min(x_j^*)$
18	$conf\_bounding\_box\_area$	the area of the confidence bounding box	$conf\_bounding\_box\_area = conf\_bounding\_box\_perp * conf\_bounding\_box\_para$
19	$conf\_bounding\_box\_ratio$	the ratio of the perpendicular side to the parallel side of the confidence bounding box	$conf\_bounding\_box\_ratio = \frac{conf\_bounding\_box\_perp}{conf\_bounding\_box\_para}$
20	$conf\_bounding\_box\_ratio\_inverse$	the reciprocal of $conf\_bounding\_box\_ratio$	$conf\_bounding\_box\_ratio\_inverse = \frac{conf\_bounding\_box\_para}{conf\_bounding\_box\_perp}$
21	$convexhull$	the area of the convex hull	$convexhull = \frac{1}{2}\sum_{h=0}^{H} x_h^c(y_{h+1}^c - y_{h-1}^c)$, with $h(mod\ H)$
22	$dist\_line\_avg$	the average of all the perpendicular distances to the regression line	$dist\_line\_avg = \frac{1}{N}\sum_{i=1}^{N} d_i = \frac{1}{N}\sum_{i=1}^{N}\frac{\lvert ax_i + by_i + c\rvert}{\sqrt{a^2+b^2}} = \frac{1}{N}\sum_{i=1}^{N}\frac{\lvert x_i - y_i\rvert}{\sqrt{2}}$
23	$dist\_line\_sd$	the standard deviation of all the perpendicular distances to the regression line	$dist\_line\_sd = sd(d_i)$
24	$dist\_line\_skewness$	the skewness measure of all the perpendicular distances to the regression line	$dist\_line\_skewness = \frac{q_{90}(d_i) - q_{50}(d_i)}{q_{90}(d_i) - q_{10}(d_i)}$, q is quantile
25	$dist\_line\_avg\_inverse$	the average of all the inverse of the perpendicular distances to the regression line	$dist\_line\_avg\_inverse = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{d_i} = \frac{1}{N}\sum_{i=1}^{N}\frac{\sqrt{2}}{\lvert x_i - y_i\rvert}$
26	$dist\_line\_sd\_inverse$	the standard deviation of all the inverse of the perpendicular distances to the regression line	$dist\_line\_sd\_inverse = sd(\frac{1}{d_i})$
27	$dist\_line\_skewness\_inverse$	the skewness measure of all the inverse of the perpendicular distances to the regression line	$dist\_line\_skewness\_inverse = \frac{q_{90}(\frac{1}{d_i}) - q_{50}(\frac{1}{d_i})}{q_{90}(\frac{1}{d_i}) - q_{10}(\frac{1}{d_i})}$

TABLE A1: The Definitions of the Visual Features

No.	Notation	Description	Definition
28	$pairwise\_dist\_max$	the maximum of all pairwise distances between the points	$pairwise\_dist\_max = max(\sqrt{(x_{i1} - x_{i2})^2 + (y_{i1} - y_{i2})^2})$
29	$pairwise\_dist\_avg$	the average of all pairwise distances between the points	$pairwise\_dist\_avg = \frac{1}{N(N-1)} \sum_{u=1}^{N(N-1)} p_u = \frac{1}{N(N-1)} \sum_{i_1=1}^{N-1} \sum_{i_2=2}^{N} (\sqrt{(x_{i1} - x_{i2})^2 + (y_{i1} - y_{i2})^2})$
30	$pairwise\_dist\_sd$	the standard deviation of all pairwise distances between the points	$pairwise\_dist\_sd = sd(p_u)$
31	$pairwise\_dist\_skewness$	the skewness measure of all pairwise distances between the points	$pairwise\_dist\_skewness = \frac{q_{90}(p_u) - q_{50}(p_u)}{q_{90}(p_u) - q_{10}(p_u)}$
32	$pairwise\_dist\_50$	the 50th percentile (median) of all pairwise distances between the points	$pairwise\_dist\_50 = q_{50}(p_u)$
33	$pairwise\_dist\_75$	the 75th percentile of all pairwise distances between the points	$pairwise\_dist\_75 = q_{75}(p_u)$
34	$pairwise\_dist\_95$	the 95th percentile of all pairwise distances between the points	$pairwise\_dist\_95 = q_{95}(p_u)$
35	$knn3\_avg$	the average of all the local density values of 3 nearest neighbors (k=3)	$knn3\_avg = \frac{1}{N} \sum_{i=1}^{N} t_i = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{k} (\sqrt{(x_i - x_{i_1})^2 + (y_i - y_{i_1})^2} + \sqrt{(x_i - x_{i_2})^2 + (y_i - y_{i_2})^2} + \sqrt{(x_i - x_{i_3})^2 + (y_i - y_{i_3})^2})$
36	$knn5\_avg$	the average of all the local density values of 5 nearest neighbors (k=5)	(see $knn3\_avg$)
37	$knn7\_avg$	the average of all the local density values of 7 nearest neighbors (k=7)	(see $knn3\_avg$)
38	$knn9\_avg$	the average of all the local density values of 9 nearest neighbors (k=9)	(see $knn3\_avg$)
39	$knn3\_sd$	the standard deviation of all the local density values of 3 nearest neighbors (k=3)	$knn3\_sd = sd(t_i)$

TABLE A1: The Definitions of the Visual Features

No.	Notation	Description	Definition
40	$knn5\_sd$	the standard deviation of all the local density values of 5 nearest neighbors (k=5)	(see $knn3\_sd$)
41	$knn7\_sd$	the standard deviation of all the local density values of 7 nearest neighbors (k=7)	(see $knn3\_sd$)
42	$knn9\_sd$	the standard deviation of all the local density values of 9 nearest neighbors (k=9)	(see $knn3\_sd$)
43	$knn3\_skewness$	the skewness measure of all the local density values of 3 nearest neighbors (k=3)	$knn3\_skewness = \frac{q_{90}(t_i) - q_{50}(t_i)}{q_{90}(t_i) - q_{10}(t_i)}$
44	$knn5\_skewness$	the skewness measure of all the local density values of 5 nearest neighbors (k=5)	(see $knn3\_skewness$)
45	$knn7\_skewness$	the skewness measure of all the local density values of 7 nearest neighbors (k=7)	(see $knn3\_skewness$)
46	$knn9\_skewness$	the skewness measure of all the local density values of 7 nearest neighbors (k=9)	(see $knn3\_skewness$)
47	mst_avg	the average of all the edges on the minimum spanning tree	$mst\_avg = \frac{1}{N-1}\sum_{v=1}^{N-1} e_v$
48	mst_sd	the standard deviation of all the edges on the minimum spanning tree	$mst\_sd = sd(e_v)$
49	$mst\_skewness$	the skewness measure of all the edges on the minimum spanning tree	$mst\_skewness = \frac{q_{90}(e_v) - q_{50}(e_v)}{q_{90}(e_v) - q_{10}(e_v)}$

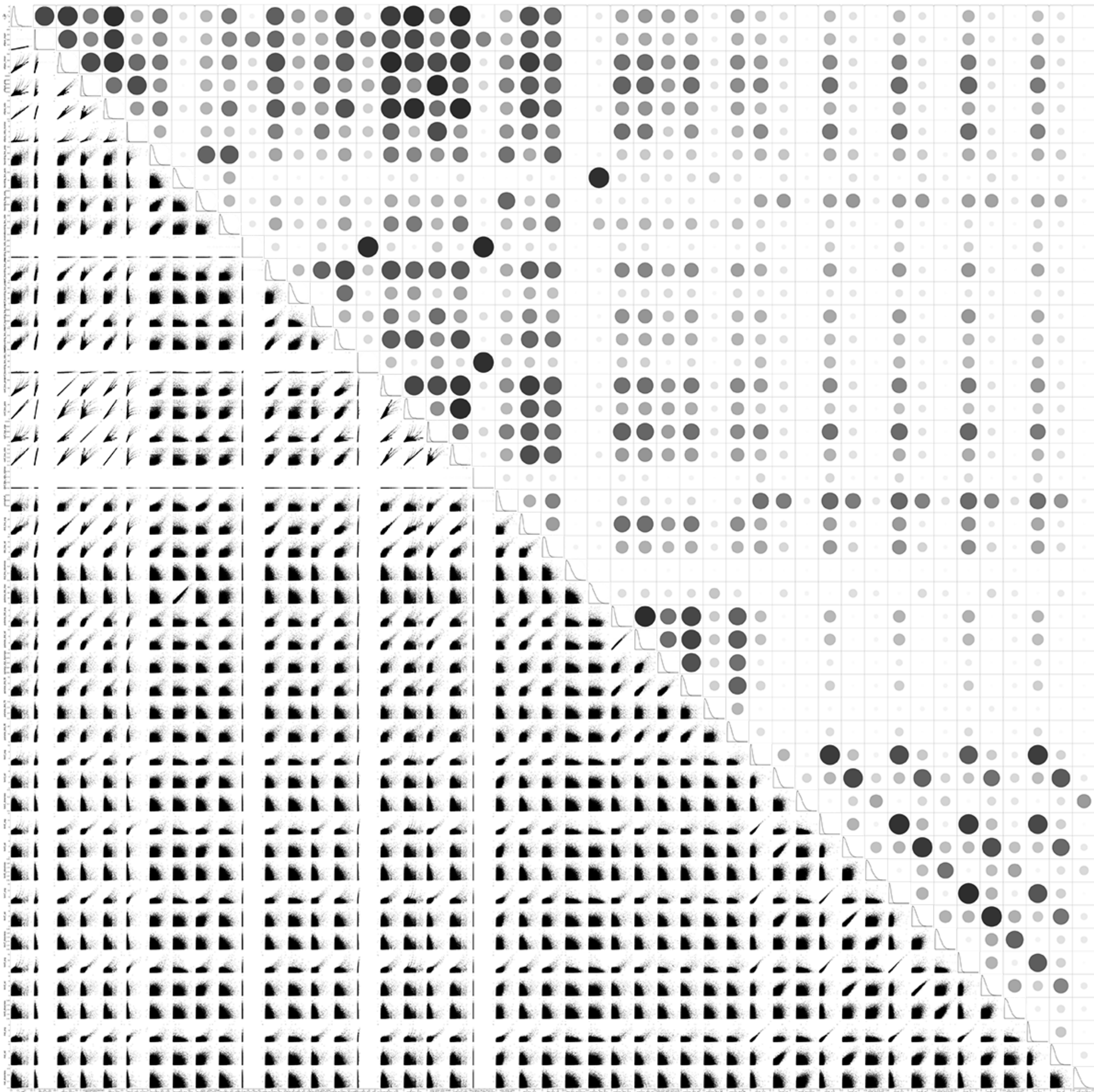

Fig. B1. The correlation matrix and the scatterplot matrix to illustrate the extreme values and collinearity.

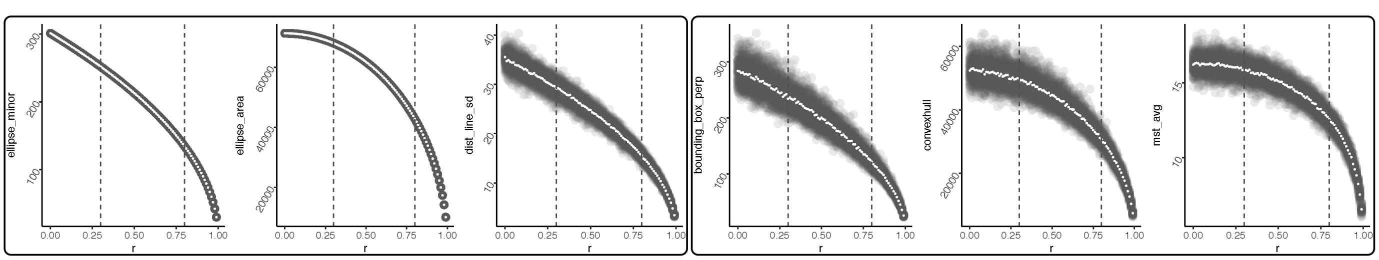

Fig. D1. Linear assumption between a visual feature and correlation: we run simulation for each visual feature and correlation. For simplicity, we present small simulations using a step of 0.01 with 100 points at each step.

Pearson correlation coefficients between r and each visual feature in these plots are: -.98, -.92, -.97, -.96, -.91, and -.87, respectively. Note that we use the term "Pearson correlation coefficients" to avoid confusion.

Pearson correlation coefficients on r=[0.3, 0.8] are: -1.00, -.99, -.96, -.91, -.88, and -.87, respectively.

Note that these figures differ from Figure B1 and Figure 5 in the main body of our paper, where y-axis is $\Delta r/\Delta v$. Additionally, a feature that is linearly correlated with r doesn't necessarily guarantee it is a feature that can more closely align with participants' judgments than r and vice versa.

TABLE C1

The models of correlation and visual features

Group	Models	Coeffiecent	Std. Error	Odds Ratio	95% CI	z-value	p-value	Residual Deviance	AIC	BIC	Likelihood ratio test with the null model	R²	fitted(△r) ~ m	fitted(m) ~ △r
null	1	0.00	0.024	1.00	[0.95, 1.05]	-0.01	.99	9911.90	6621.60	6629.45	-	.0000	-	-
baseline	r	0.30	0.048	1.35	[1.23, 1.48]	6.32	< .001							
	approach / a	-0.10	0.047	0.90	[0.82, 0.99]	-2.16	.03	9840.70	6581.50	6612.89	χ2 = 46.11, p < .001	.0082	-	-
	△r	0.26	0.051	1.30	[1.18, 1.44]	5.11	< .001							
visual feature models	△ellipse_major	0.24	0.053	1.27	[1.14, 1.40]	4.49	< .001	9847.50	6586.40	6617.79	χ2 = 42.56, p < .001	.0076	z = 9.41, p < .001	z = -11.38, p < .001
	△ellipse_minor	0.31	0.048	1.37	[1.25, 1.51]	6.53	< .001	9823.70	6572.80	6604.20	χ2 = 56.15, p < .001	.0100	z = -9.14, p < .001	z = 4.97, p < .001
	△ellipse_area	0.42	0.054	1.53	[1.38, 1.70]	7.83	< .001	9800.20	6563.30	6594.75	χ2 = 65.60, p < .001	.0117	z = -12.81, p < .001	z = 2.40, p = .02
	△ellipse_ratio	0.25	0.05	1.28	[1.16, 1.41]	4.94	< .001	9843.10	6583.50	6614.95	χ2 = 45.40, p < .001	.0081	z = 2.25, p = .02	z = -3.27, p = .001
	△ellipse_ratio_inverse	0.37	0.071	1.45	[1.27, 1.67]	5.23	< .001	9833.60	6584.70	6616.15	χ2 = 44.20, p < .001	.0079	z = -4.46, p < .001	z = -5.91, p < .001
	△bounding_box_perp	0.20	0.048	1.23	[1.12, 1.35]	4.25	< .001	9849.60	6586.00	6617.38	χ2 = 42.97, p < .001	.0077	z = -3.03, p = .00	z = -8.66, p < .001
	△bounding_box_para	-0.06	0.048	0.94	[0.85, 1.03]	-1.32	.19	9866.10	6597.80	6629.23	χ2 = 31.12, p < .001	.0056	z = -1.99, p = .05	z = -166.32, p < .001
	△bounding_box_area	0.06	0.048	1.06	[0.97, 1.17]	1.31	.19	9866.10	6597.60	6629.04	χ2 = 31.31, p < .001	.0056	z = 0.57, p = .57	z = -54.89, p < .001
	△bounding_box_ratio	0.22	0.049	1.24	[1.13, 1.37]	4.43	< .001	9848.10	6585.30	6616.70	χ2 = 43.65, p < .001	.0078	z = -3.66, p < .001	z = -8.06, p < .001
	△bounding_box_ratio_inverse	0.30	0.063	1.35	[1.20, 1.53]	4.78	< .001	9841.40	6584.50	6615.95	χ2 = 44.40, p < .001	.0079	z = -5.55, p < .001	z = -9.85, p < .001
	△conf_bounding_box_perp	0.27	0.048	1.31	[1.20, 1.44]	5.67	< .001	9835.10	6577.40	6608.79	χ2 = 51.56, p < .001	.0092	z = -4.23, p < .001	z = -2.12, p = .03
	△conf_bounding_box_para	0.00	0.048	1.00	[0.92, 1.10]	0.10	.92	9867.80	6599.20	6630.63	χ2 = 29.73, p < .001	.0053	z = 1.61, p = .11	z = -741.66, p < .001
	△conf_bounding_box_area	0.21	0.048	1.24	[1.13, 1.36]	4.41	< .001	9848.20	6584.90	6616.32	χ2 = 44.03, p < .001	.0079	z = -3.89, p < .001	z = -9.43, p < .001
	△conf_bounding_box_ratio	0.20	0.05	1.22	[1.11, 1.35]	4.05	< .001	9851.30	6587.90	6619.29	χ2 = 41.06, p < .001	.0073	z = 0.22, p = .83	z = -6.24, p < .001
	△conf_bounding_box_ratio_inverse	0.31	0.063	1.36	[1.21, 1.55]	4.89	< .001	9839.70	6584.60	6615.97	χ2 = 44.38, p < .001	.0079	z = -4.08, p < .001	z = -7.03, p < .001
	△convexhull	0.17	0.048	1.19	[1.08, 1.30]	3.57	< .001	9855.00	6589.30	6620.71	χ2 = 39.64, p < .001	.0071	z = -2.47, p = .01	z = -14.09, p < .001
	△dist_line_avg	0.15	0.047	1.17	[1.06, 1.28]	3.25	.001	9857.20	6591.20	6622.60	χ2 = 37.75, p < .001	.0067	z = 3.31, p < .001	z = -10.50, p < .001
	△dist_line_sd	0.32	0.048	1.38	[1.26, 1.52]	6.77	< .001	9821.00	6568.90	6600.31	χ2 = 60.05, p < .001	.0107	z = -8.74, p < .001	z = -0.84, p = .40
	△dist_line_skewness	0.01	0.047	1.01	[0.92, 1.11]	0.30	.76	9867.80	6599.20	6630.59	χ2 = 29.76, p < .001	.0053	z = -1.84, p = .07	z = -7986.79, p < .001
	△pairwise_dist_max	-0.06	0.048	0.94	[0.86, 1.03]	-1.24	.21	9866.30	6598.00	6629.39	χ2 = 30.96, p < .001	.0055	z = -2.48, p = .01	z = -233.62, p < .001
	△pairwise_dist_avg	0.25	0.049	1.28	[1.17, 1.41]	5.06	< .001	9841.70	6581.70	6613.08	χ2 = 47.27, p < .001	.0084	z = -4.62, p < .001	z = -6.02, p < .001
	△pairwise_dist_sd	0.24	0.048	1.27	[1.16, 1.40]	4.95	< .001	9842.90	6582.30	6613.73	χ2 = 46.62, p < .001	.0083	z = -4.30, p < .001	z = -6.11, p < .001
	△pairwise_dist_skewness	0.17	0.048	1.19	[1.08, 1.30]	3.58	< .001	9855.00	6590.00	6621.42	χ2 = 38.93, p < .001	.0070	z = -2.37, p = .02	z = -12.36, p < .001
	△pairwise_dist_50	0.22	0.048	1.25	[1.14, 1.37]	4.62	< .001	9846.30	6584.10	6615.47	χ2 = 44.88, p < .001	.0080	z = -3.82, p < .001	z = -7.50, p < .001
	△pairwise_dist_75	0.06	0.048	1.06	[0.97, 1.17]	1.26	.21	9866.30	6598.30	6629.70	χ2 = 30.65, p < .001	.0055	z = -1.03, p = .30	z = -104.12, p < .001
	△pairwise_dist_95	0.17	0.048	1.18	[1.08, 1.30]	3.52	< .001	9855.40	6590.50	6621.93	χ2 = 38.42, p < .001	.0069	z = -1.93, p = .05	z = -11.88, p < .001
	△knn3_avg	0.13	0.048	1.14	[1.04, 1.25]	2.73	.01	9860.40	6593.60	6625.01	χ2 = 35.34, p < .001	.0063	z = -1.32, p = .19	z = -21.79, p < .001
	△knn3_sd	0.00	0.048	1.00	[0.91, 1.09]	-0.07	.94	9867.80	6599.20	6630.64	χ2 = 29.71, p < .001	.0053	z = 0.42, p = .67	z = -1289.52, p < .001
	△knn3_skewness	-0.06	0.047	0.94	[0.86, 1.03]	-1.25	.21	9866.30	6598.30	6629.71	χ2 = 30.64, p < .001	.0055	z = -62.95, p < .001	z = -4215.75, p < .001
	△knn5_avg	0.20	0.049	1.22	[1.11, 1.35]	4.12	< .001	9850.60	6587.10	6618.54	χ2 = 41.81, p < .001	.0075	z = -3.90, p < .001	z = -11.51, p < .001
	△knn5_sd	0.00	0.048	1.00	[0.91, 1.09]	-0.06	.95	9867.80	6599.20	6630.64	χ2 = 29.71, p < .001	.0053	z = 0.44, p = .66	z = -1406.51, p < .001
	△knn5_skewness	-0.03	0.047	0.97	[0.88, 1.06]	-0.72	.47	9867.30	6598.90	6630.31	χ2 = 30.04, p < .001	.0054	z = -55.26, p < .001	z = -20669.62, p < .001
	△knn7_avg	0.23	0.049	1.26	[1.14, 1.39]	4.67	< .001	9845.60	6584.10	6615.49	χ2 = 44.86, p < .001	.0080	z = -4.89, p < .001	z = -8.83, p < .001
	△knn7_sd	-0.02	0.047	0.98	[0.89, 1.08]	-0.39	.70	9867.70	6599.20	6630.57	χ2 = 29.78, p < .001	.0053	z = 0.24, p = .81	z = -385.17, p < .001
	△knn7_skewness	-0.11	0.047	0.90	[0.82, 0.98]	-2.33	.02	9862.40	6595.60	6627.04	χ2 = 33.31, p < .001	.0060	z = -3097.30, p < .001	z = -34602.50, p < .001
	△knn9_avg	0.24	0.05	1.27	[1.15, 1.40]	4.77	< .001	9844.50	6583.50	6614.91	χ2 = 45.44, p < .001	.0081	z = -4.75, p < .001	z = -8.01, p < .001
	△knn9_sd	-0.03	0.047	0.97	[0.89, 1.07]	-0.60	.55	9867.50	6599.00	6630.43	χ2 = 29.92, p < .001	.0053	z = -0.01, p = .99	z = -269.00, p < .001
	△knn9_skewness	-0.05	0.047	0.95	[0.87, 1.05]	-1.01	.31	9866.80	6598.60	6630.00	χ2 = 30.35, p < .001	.0054	z = -32.69, p < .001	z = -4256.42, p < .001
	△mst_avg	0.16	0.048	1.17	[1.07, 1.29]	3.32	< .001	9856.80	6591.10	6622.53	χ2 = 37.82, p < .001	.0068	z = -2.26, p = .02	z = -16.17, p < .001
	△mst_sd	0.01	0.048	1.01	[0.92, 1.11]	0.21	.84	9867.80	6599.20	6630.59	χ2 = 29.76, p < .001	.0053	z = 0.36, p = .72	z = -1045.10, p < .001
	△mst_skewness	-0.03	0.047	0.97	[0.88, 1.06]	-0.67	.51	9867.40	6599.00	6630.39	χ2 = 29.96, p < .001	.0054	z = -13.70, p < .001	z = -5885.40, p < .001
	△dist_line_avg_inverse	0.02	0.049	1.02	[0.92, 1.12]	0.33	.74	9867.70	6599.10	6630.51	χ2 = 29.84, p < .001	.0053	z = 0.64, p = .52	z = -441.90, p < .001
	△dist_line_sd_inverse	-0.01	0.047	0.99	[0.90, 1.08]	-0.24	.81	9867.80	6599.20	6630.59	χ2 = 29.76, p < .001	.0053	z = -0.07, p = .95	z = -2294.66, p < .001
	△dist_line_skewness_inverse	0.04	0.047	1.04	[0.95, 1.14]	0.83	.41	9867.20	6598.80	6630.23	χ2 = 30.12, p < .001	.0054	z = -12.96, p < .001	z = -3087.97, p < .001

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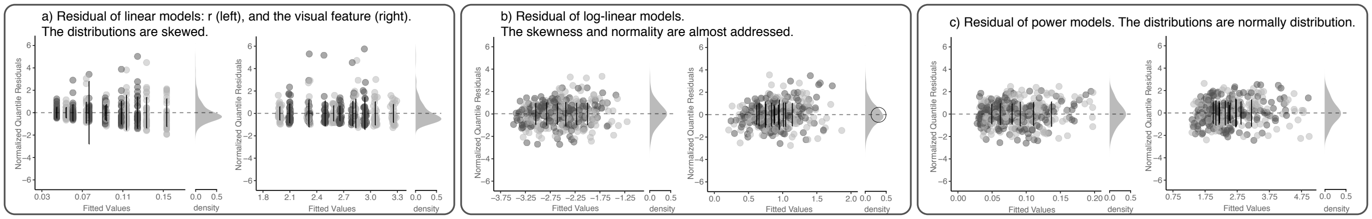

Fig. E1. The residual plots of the linear, log-linear, and power models, following Kay and Heer's presenting style. Note that the residuals of the linear models are highly skewed, while the log-linear and power models mitigate the skewed residuals.

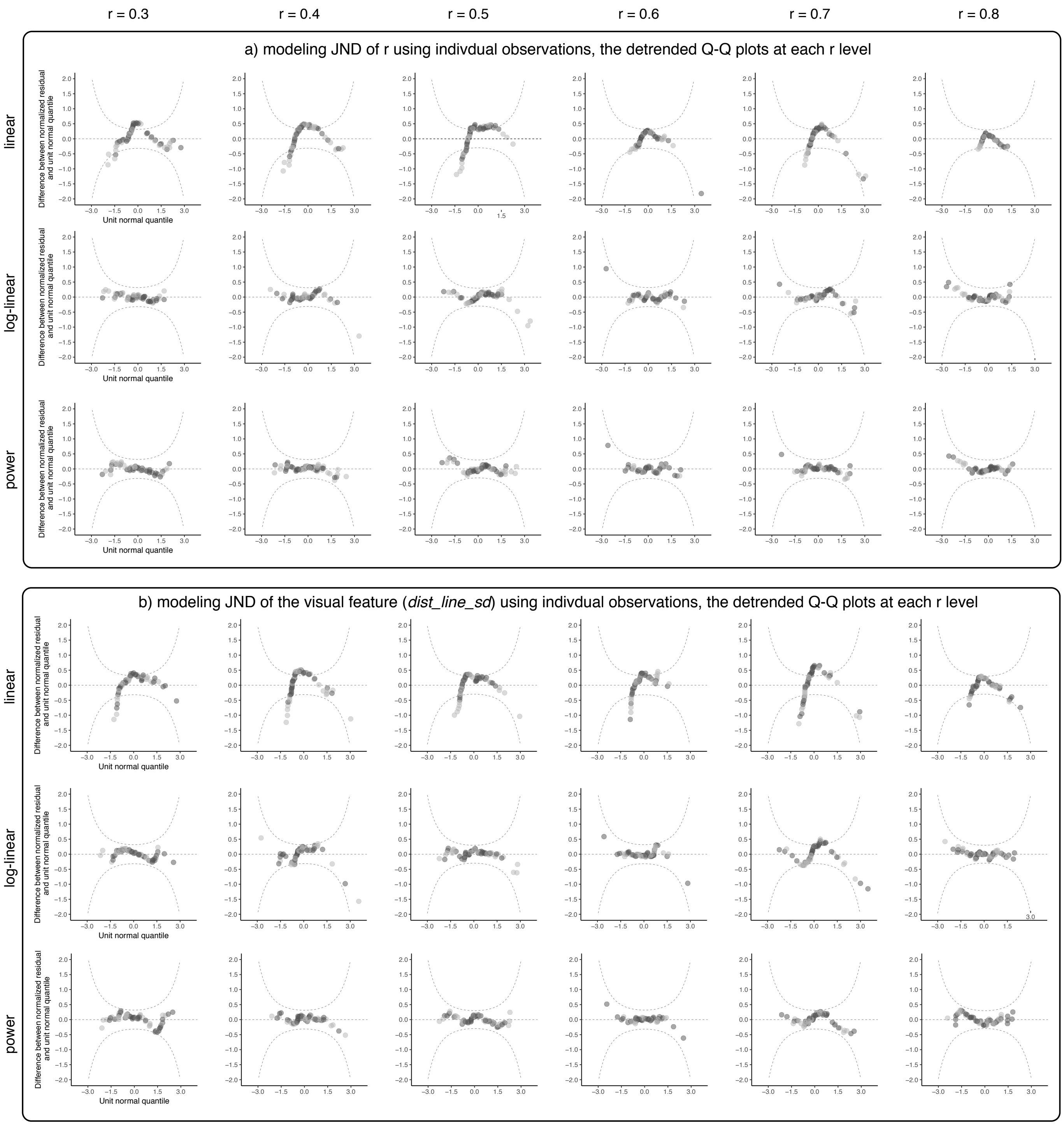

Fig. E2. The detrended Q-Q plots for the linear, log-linear, and power models at each r level. Similar to the detrended Q-Q plots based on all r levels in the main body of our paper, the residuals of the linear model are highly skewed and largely deviate from normality. The residuals of the log-linear model are largely corrected, but the power model tackles more observations and persists better normality.

TABLE G1
The results of other features using linear model based on mean observations

Feature: dist_line_sd

Model	Correlation Coefficients	Coefficients				R2	AIC
		β0	p	β1	p		
f_1: $JNDr \sim r$	-.9778	0.1860	< .001	-0.1791	< .001	.9561	-79.7720
f_2: $JNDv \sim v$	.9011	0.7975	.0101	0.0708	< .001	.8119	-3.0762
f_3: $JNDv \sim JNDr$	.9950	1.4717	< .001	10.8154	< .001	.9900	-40.7950
f_4: $v \sim r$	-.9962	38.0295	< .001	-27.4850	< .001	.9924	18.6965
f_1': *Substitution*	-	0.1864	-	-0.1798	-	-	-

Feature: ellipse_area

Model	Correlation Coefficients	Coefficients				R2	AIC
		β0	p	β1	p		
f_1: $JNDr \sim r$	-.9778	0.1860	< .001	-0.1791	< .001	.9561	-79.7720
f_2: $JNDv \sim v$	-.7450	6748.1167	< .001	-0.0516	.0038	.5551	184.4261
f_3: $JNDv \sim JNDr$	-.9750	5007.0600	< .001	-14080.3600	< .001	.9507	150.9729
f_4: $v \sim r$	-.9810	85006.0000	< .001	-49761.0000	< .001	.9624	218.2764
f_1': *Substitution*	-	0.1878	-	-0.1823	-	-	-

Feature: ellipse_minor

Model	Correlation Coefficients	Coefficients				R2	AIC
		β0	p	β1	p		
f_1: $JNDr \sim r$	-.9778	0.1860	< .001	-0.1791	< .001	.9561	-79.7720
f_2: $JNDv \sim v$	.9114	4.0874	.0762	0.0774	< .001	.8306	48.7316
f_3: $JNDv \sim JNDr$	.9950	10.6781	< .001	99.8083	< .001	.9901	12.4099
f_4: $v \sim r$	-.9960	326.0880	< .001	-232.6390	< .001	.9921	70.4632
f_1': *Substitution*	-	0.1867	-	-0.1803	-	-	-

Feature: conf_bounding_box_perp

Model	Correlation Coefficients	Coefficients				R2	AIC
		β0	p	β1	p		
f_1: $JNDr \sim r$	-.9778	0.1860	< .001	-0.1791	< .001	.9561	-79.7720
f_2: $JNDv \sim v$	.9174	1.9590	.2470	0.0832	< .001	.8416	41.9412
f_3: $JNDv \sim JNDr$	.9953	7.4785	< .001	78.3206	< .001	.9906	5.9266
f_4: $v \sim r$	-.9961	241.8870	< .001	-169.4050	< .001	.9922	62.6403
f_1': *Substitution*	-	0.1865	-	-0.1800	-	-	-

TABLE G2
The results of other features using linear model based on individual observations

Feature: dist_line_sd

Model	Coefficients								R2	AIC	Normality of residuals	Skewness	Kurtosis	Homoscedasticity
	β0	p	β1	p	β2	p	β3	p						
f_1: $JNDr \sim r$	0.1927	< .001	-0.1792	< .001	0.0202	.0167	-0.0195	.1809	.3076	-1213.7160	p < .001	2.1075	7.4827	p < .001
f_2: $JNDv \sim v$	0.8672	.0046	0.0761	< .001	-0.2949	.3314	0.0155	.2345	.0876	1228.8330	p < .001	2.3604	8.1541	p = .4596
f_3: $JNDv \sim JNDr$	1.5412	< .001	11.3588	< .001	-	-	-	-	.9648	-937.7772	p < .001	-0.9502	0.5097	p < .001
f_4: $v \sim r$	38.0921	< .001	-27.6028	< .001	-	-	-	-	.9915	455.0915	p < .001	-0.3038	-1.2431	p = .0011
f_1': *Substitution*	0.1959	-	-0.1849	-	-	-	-	-	-	-	-	-	-	-

Feature: ellipse_area

Model	Coefficients								R2	AIC	Normality of residuals	Skewness	Kurtosis	Homoscedasticity
	β0	p	β1	p	β2	p	β3	p						
f_1: $JNDr \sim r$	0.1927	< .001	-0.1792	< .001	0.0202	.0167	-0.0195	.1809	.3076	-1213.7160	p < .001	2.1075	7.4827	p < .001
f_2: $JNDv \sim v$	7677.6290	< .001	-0.0614	< .001	219.2846	.7900	-0.0109	.4450	.0772	6986.6580	p < .001	3.0652	14.9829	p = .4058
f_3: $JNDv \sim JNDr$	5974.7100	< .001	-19516.6183	< .001	-	-	-	-	.8592	5294.8880	p < .001	0.1027	-1.0428	p < .001
f_4: $v \sim r$	85259.7000	< .001	-50239.7000	< .001	-	-	-	-	.9629	6731.6310	p < .001	-0.3803	-1.5405	p = .3916
f_1': *Substitution*	0.1810	-	-0.1581	-	-	-	-	-	-	-	-	-	-	-

Feature: ellipse_minor

Model	Coefficients								R2	AIC	Normality of residuals	Skewness	Kurtosis	Homoscedasticity
	β0	p	β1	p	β2	p	β3	p						
f_1: $JNDr \sim r$	0.1927	< .001	-0.1792	< .001	0.0202	.0167	-0.0195	.1809	.3076	-1213.7160	p < .001	2.1075	7.4827	p < .001
f_2: $JNDv \sim v$	7.2060	.0101	0.0698	< .001	-2.0215	.4686	0.0126	.3656	.0660	2898.8070	p < .001	2.2677	7.5192	p = .5416
f_3: $JNDv \sim JNDr$	12.7507	< .001	87.8829	< .001	-	-	-	-	.9687	571.4967	p < .001	-0.9013	0.4779	p < .001
f_4: $v \sim r$	326.6220	< .001	-233.6570	< .001	-	-	-	-	.9921	2051.1070	p < .001	-0.3771	-1.5406	p = .3299
f_1': *Substitution*	0.1962	-	-0.1855	-	-	-	-	-	-	-	-	-	-	-

Feature: conf_bounding_box_perp

Model	Coefficients								R2	AIC	Normality of residuals	Skewness	Kurtosis	Homoscedasticity
	β0	p	β1	p	β2	p	β3	p						
f_1: $JNDr \sim r$	0.1927	< .001	-0.1792	< .001	0.0202	.0167	-0.0195	.1809	.3076	-1213.7160	p < .001	2.1075	7.4827	p < .001
f_2: $JNDv \sim v$	4.7497	.0185	0.0751	< .001	-1.7710	.3781	0.0137	.3046	.0808	2627.3450	p < .001	2.4186	8.3107	p = .7131
f_3: $JNDv \sim JNDr$	9.5077	< .001	68.0967	< .001	-	-	-	-	.9546	524.4871	p < .001	-0.7942	0.1198	p < .001
f_4: $v \sim r$	242.2637	< .001	-170.1396	< .001	-	-	-	-	.9916	1830.8990	p < .001	-0.2900	-1.3412	p = .0010
f_1': *Substitution*	0.1974	-	-0.1877	-	-	-	-	-	-	-	-	-	-	-

TABLE G3
The results of other features using log-linear model based on individual observations

Feature: dist_line_sd

Method	Coefficients								R2	AIC	Normality of residuals	Skewness	Kurtosis	Homoscedasticity
	β0	p	β1	p	β2	p	β3	p						
f_1: $log(JNDr) \sim r$	-1.4137	< .001	-2.0152	< .001	0.1365	.0021	-0.0815	.2837	.7941	-1724.3940	p = .0586	0.1170	0.5619	p = .9961
f_2: $log(JNDv) \sim v$	0.1903	< .001	0.0297	< .001	-0.1074	.0516	0.0059	.0130	.7104	748.3163	p < .001	0.4009	0.8595	p = .2798
f_3: $log(JNDv) \sim log(JNDr)$	1.8886	< .001	0.4038	< .001	-	-	-	-	.9917	-1460.4100	p < .001	-0.8475	-0.2552	p < .001
f_4: $v \sim r$	38.0921	< .001	-27.6028	< .001	-	-	-	-	.9915	455.0915	p < .001	-0.3038	-1.2431	p = .0011
f_1': *Substitution*	-1.4064	-	-2.0285	-	-	-	-	-	-	-	-	-	-	-

Feature: ellipse_area

Method	Coefficients								R2	AIC	Normality of residuals	Skewness	Kurtosis	Homoscedasticity
	β0	p	β1	p	β2	p	β3	p						
f_1: $log(JNDr) \sim r$	-1.4137	< .001	-2.0152	< .001	0.1365	.0021	-0.0815	.2837	.7941	-1724.3940	p = .0586	0.1170	0.5619	p = .9961
f_2: $log(JNDv) \sim v$	9.1210	< .001	-1.58E-05	< .001	0.1647	.0776	-4.1620E-06	.0098	.6464	6494.4460	p = .0213	0.1694	0.6152	p = .8166
f_3: $log(JNDv) \sim log(JNDr)$	7.1421	< .001	-0.4234	< .001	-	-	-	-	.9701	-938.3142	p < .001	0.4384	-0.5639	p < .001
f_4: $v \sim r$	85259.7000	< .001	-50239.7000	< .001	-	-	-	-	.9629	6731.6310	p < .001	-0.3803	-1.5405	p = .3916
f_1': *Substitution*	-1.4903	-	-1.8761	-	-	-	-	-	-	-	-	-	-	-

Feature: ellipse_minor

Method	Coefficients								R2	AIC	Normality of residuals	Skewness	Kurtosis	Homoscedasticity
	β0	p	β1	p	β2	p	β3	p						
f_1: $log(JNDr) \sim r$	-1.4137	< .001	-2.0152	< .001	0.1365	.0021	-0.0815	.2837	.7941	-1724.3940	p = .0586	0.1170	0.5619	p = .9961
f_2: $log(JNDv) \sim v$	2.2770	< .001	0.0033	< .001	-0.0848	.1877	5.7730E-04	.0715	.6906	2420.2810	p = .1207	0.1261	0.5229	p = .9869
f_3: $log(JNDv) \sim log(JNDr)$	3.9043	< .001	0.3837	< .001	-	-	-	-	.9954	-1625.1810	p < .001	-0.9555	-0.2695	p < .001
f_4: $v \sim r$	326.6220	< .001	-233.6570	< .001	-	-	-	-	.9921	2051.1070	p < .001	-0.3771	-1.5406	p = .3299
f_1': *Substitution*	-1.4068	-	-2.0276	-	-	-	-	-	-	-	-	-	-	-

Feature: conf_bounding_box_perp

Method	Coefficients								R2	AIC	Normality of residuals	Skewness	Kurtosis	Homoscedasticity
	β0	p	β1	p	β2	p	β3	p						
f_1: $log(JNDr) \sim r$	-1.4137	< .001	-2.0152	< .001	0.1365	.0021	-0.0815	.2837	.7941	-1724.3940	p = .0586	0.1170	0.5619	p = .9961
f_2: $log(JNDv) \sim v$	1.9512	< .001	0.0049	< .001	-0.0935	.1277	7.8710E-04	.0533	.6862	2160.0370	p = .0027	0.2769	0.8093	p = .7385
f_3: $log(JNDv) \sim log(JNDr)$	3.6867	< .001	0.4014	< .001	-	-	-	-	.9927	-1518.8680	p < .001	-0.8492	0.0545	p < .001
f_4: $v \sim r$	242.2637	< .001	-170.1396	< .001	-	-	-	-	.9916	1830.8990	p < .001	-0.2900	-1.3412	p = .0010
f_1': *Substitution*	-1.3881	-	-2.0617	-	-	-	-	-	-	-	-	-	-	-

TABLE G4
The results of other features using power model based on individual observations

Feature: dist_line_sd

Method	BCT	Coefficients								R2	AIC	Normality of residuals	Skewness	Kurtosis	Homoscedasticity
	ω	β0	p	β1	p	β2	p	β3	p						
f_1: $JND_r^{\omega} \sim r$	0.26	0.6629	< .001	-0.2646	< .001	0.0186	< .001	-0.0155	.0615	.8099	-1742.0840	p = .2866	-0.0298	-0.2223	p = .3326
f_2: $JND_v^{\omega} \sim v$	-0.17	0.9465	< .001	-0.0037	< .001	0.0187	< .001	-0.0010	< .001	.7387	703.7296	p = .0702	0.1061	-0.2850	p = .0607
f_3: $JND_v^{\omega} \sim JND_r^{\omega}$	-	1.0722	< .001	-0.3985	< .001	-	-	-	-	.9907	-4124.9540	p < .001	0.7579	-0.9203	p < .001
f_4: $v \sim r$	-	38.0921	< .001	-27.6028	< .001	-	-	-	-	.9915	455.0915	p < .001	-0.3038	-1.2431	p = .0011
f_1': *Substitution*	-	0.6672	-	-0.2549	-	-	-	-	-	-	-	-	-	-	-

Feature: ellipse_area

Method	BCT	Coefficients								R2	AIC	Normality of residuals	Skewness	Kurtosis	Homoscedasticity
	ω	β0	p	β1	p	β2	p	β3	p						
f_1: $JND_r^{\omega} \sim r$	0.26	0.6629	< .001	-0.2646	< .001	0.0186	< .001	-0.0155	.0615	.8099	-1742.0840	p = .2866	-0.0298	-0.2223	p = .3326
f_2: $JND_v^{\omega} \sim v$	-1.15	1.2720E-05	.0383	1.1550E-09	< .001	-1.3520E-05	.0277	3.7680E-10	< .001	.6506	6468.1120	p = .7608	0.0752	-0.0470	p = .5524
f_3: $JND_v^{\omega} \sim JND_r^{\omega}$	-	-4.0360E-05	< .001	2.3360E-04	< .001	-	-	-	-	.9342	-7787.4650	p < .001	-0.1377	-1.1536	p < .001
f_4: $v \sim r$	-	85259.7000	< .001	-50239.7000	< .001	-	-	-	-	.9629	6731.6310	p < .001	-0.3803	-1.5405	p = .3916
f_1': *Substitution*	-	0.6488	-	-0.2484	-	-	-	-	-	-	-	-	-	-	-

Feature: ellipse_minor

Method	BCT	Coefficients								R2	AIC	Normality of residuals	Skewness	Kurtosis	Homoscedasticity
	ω	β0	p	β1	p	β2	p	β3	p						
f_1: $JND_r^{\omega} \sim r$	0.26	0.6629	< .001	-0.2646	< .001	0.0186	< .001	-0.0155	.0615	.8099	-1742.0840	p = .2866	-0.0298	-0.2223	p = .3326
f_2: $JND_v^{\omega} \sim v$	-0.07	0.8502	< .001	-1.8260E-04	< .001	0.0053	.0869	-3.3300E-05	.0311	.6917	2409.8420	p = .3780	0.0217	-0.1942	p = .8543
f_3: $JND_v^{\omega} \sim JND_r^{\omega}$	-	0.8984	< .001	-0.1609	< .001	-	-	-	-	.9961	-3862.0720	p < .001	0.9281	-0.2506	p < .001
f_4: $v \sim r$	-	326.6220	< .001	-233.6570	< .001	-	-	-	-	.9921	2051.1070	p < .001	-0.3771	-1.5406	p = .3299
f_1': *Substitution*	-	0.6700	-	-0.2652	-	-	-	-	-	-	-	-	-	-	-

Feature: conf_bounding_box_perp

Method	BCT	Coefficients								R2	AIC	Normality of residuals	Skewness	Kurtosis	Homoscedasticity
	ω	β0	p	β1	p	β2	p	β3	p						
f_1: $JND_r^{\omega} \sim r$	0.26	0.6629	< .001	-0.2646	< .001	0.0186	< .001	-0.0155	.0615	.8099	-1742.0840	p = .2866	-0.0298	-0.2223	p = .3326
f_2: $JND_v^{\omega} \sim v$	-0.08	0.8719	< .001	-2.8100E-04	< .001	0.0068	.0173	-5.1470E-05	.0067	.6893	2140.4610	p = .3756	-0.0733	-0.1999	p = .2421
f_3: $JND_v^{\omega} \sim JND_r^{\omega}$	-	0.9232	< .001	-0.1779	< .001	-	-	-	-	.9914	-3633.9180	p < .001	0.7288	-0.0486	p < .001
f_4: $v \sim r$	-	242.2637	< .001	-170.1396	< .001	-	-	-	-	.9916	1830.8990	p < .001	-0.2900	-1.3412	p = .0010
f_1': *Substitution*	-	0.6710	-	-0.2688	-	-	-	-	-	-	-	-	-	-	-