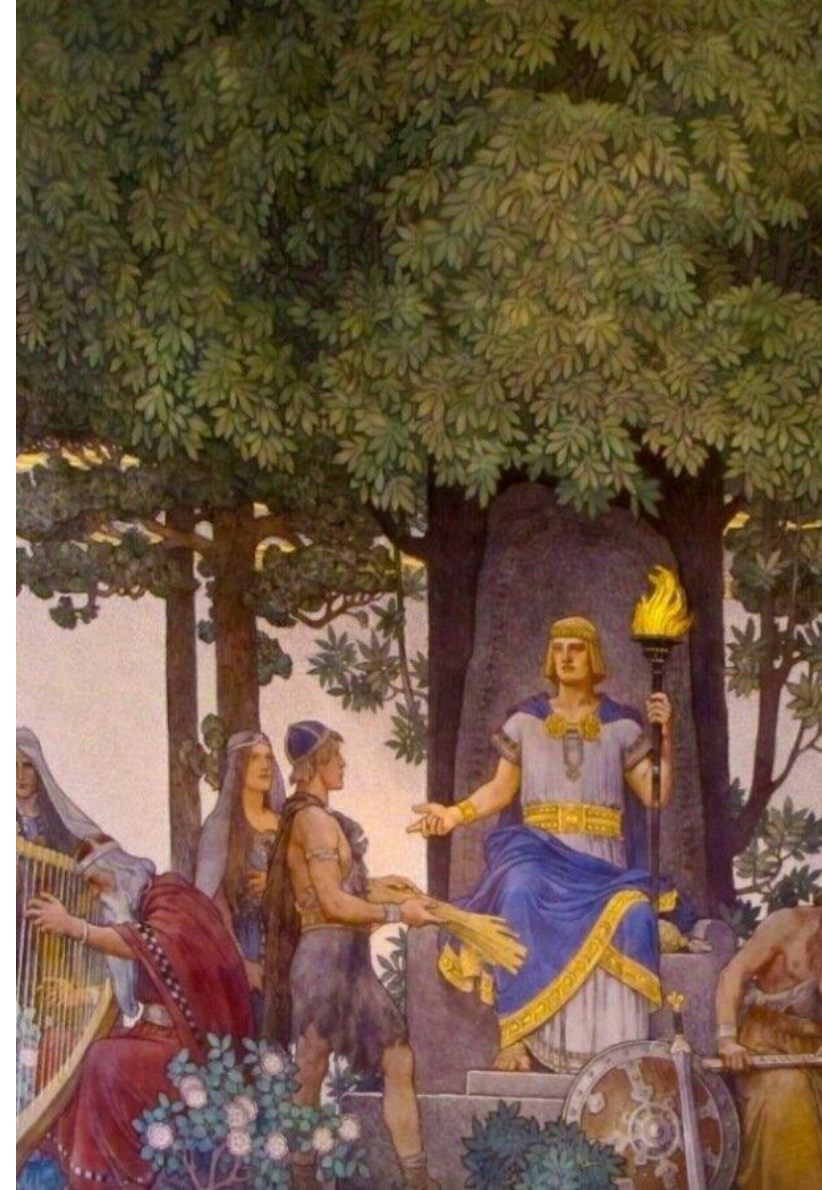


AFFINITY FUNCTIONS AND CENTRALITY MEASURES IN SOCIAL NETWORK ANALYSIS

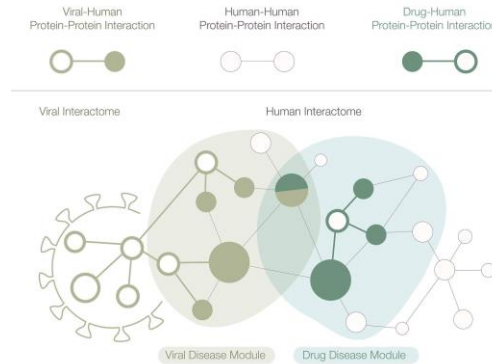
21, September, 2021

J. Fumanal Idocin, O. Córdón, A. Alonso Betanzos, H. Bustince, M. Minárová

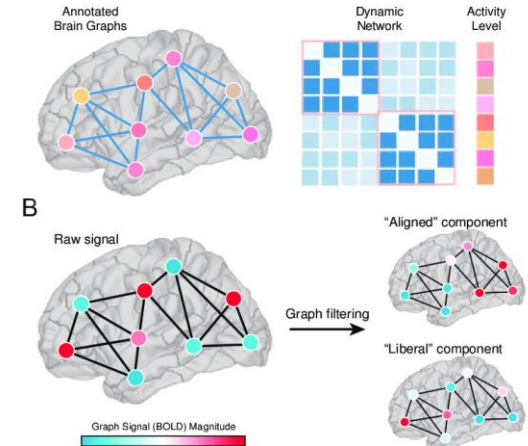
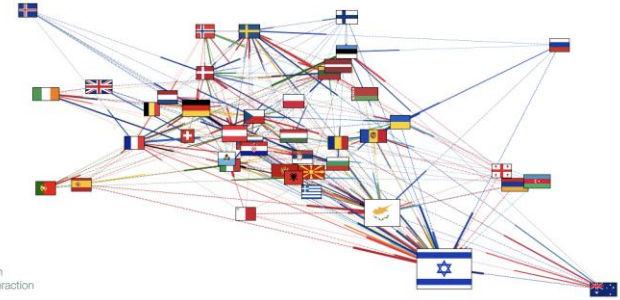


SOCIAL NETWORK ANALYSIS

- Extract **patterns** and knowledge from **relationships** in social networks.
- Typical problems: **detect clusters, leaders, propagate information, process mining...**

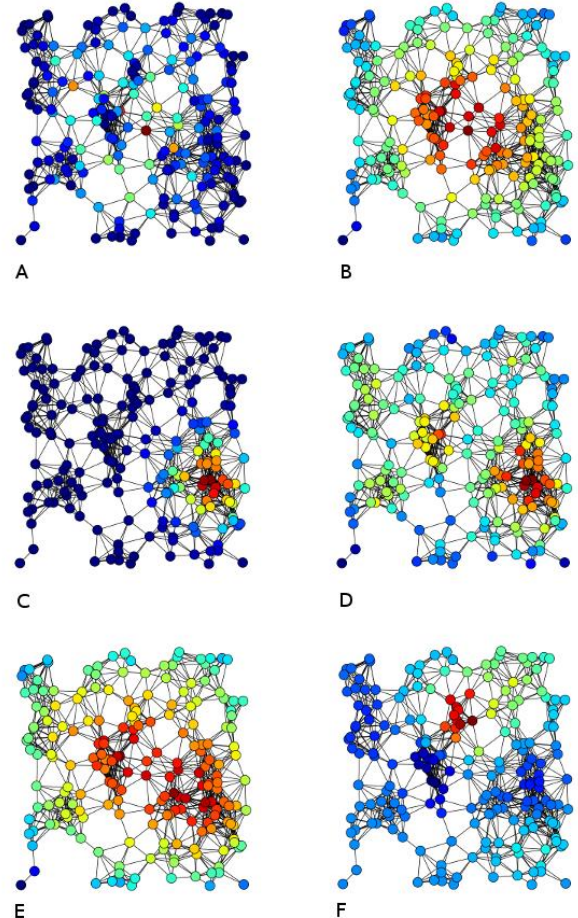


Eurovision 2018 Final Votes



CENTRALITY MEASURES

- **Centrality measures** are metrics to ponder the importance of each actor in the network.
- Some prototypical examples:
 - **Degree centrality**: Node degree.
 - **Betweenness centrality**: Number of times actor X is in the shortest path between two other actors.
 - **Closeness centrality**: Average of the shortest path between X and the rest of the actors in the network.



AFFINITY FUNCTIONS

- Functions that characterize a **pairwise relationship** between actors x & y , according to some **social criteria**:

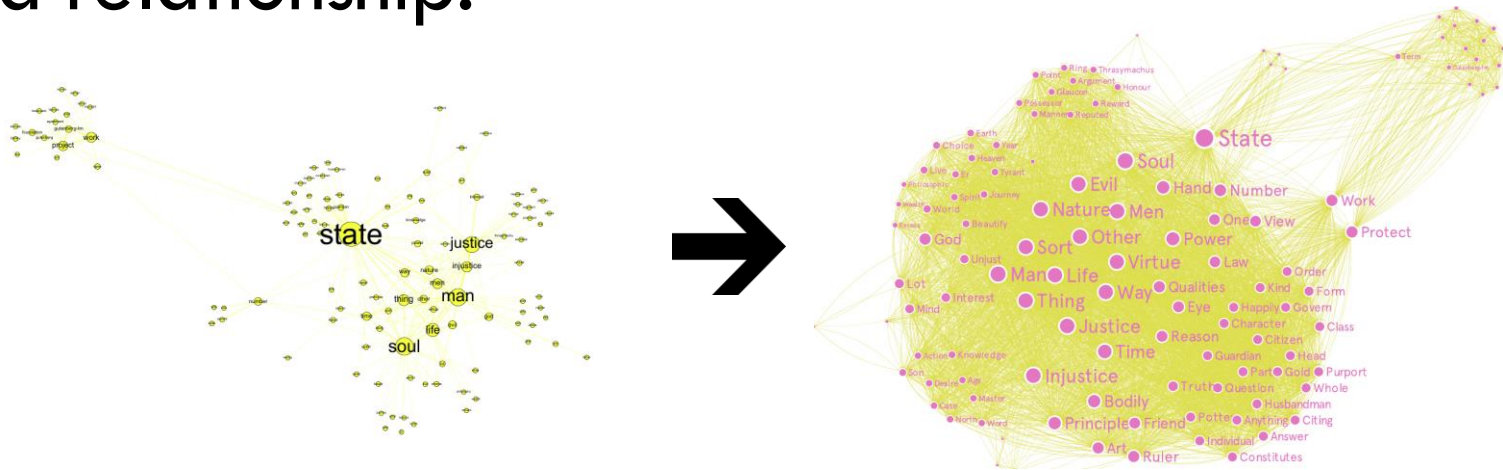
$$F_c(x, y) \rightarrow [0,1]$$

- 0: no affinity at all
- 1: complete affinity

(Similar to fuzzy sets, but the idea of **memberships** is changed by the idea of **affinity**)

AFFINITY FUNCTIONS

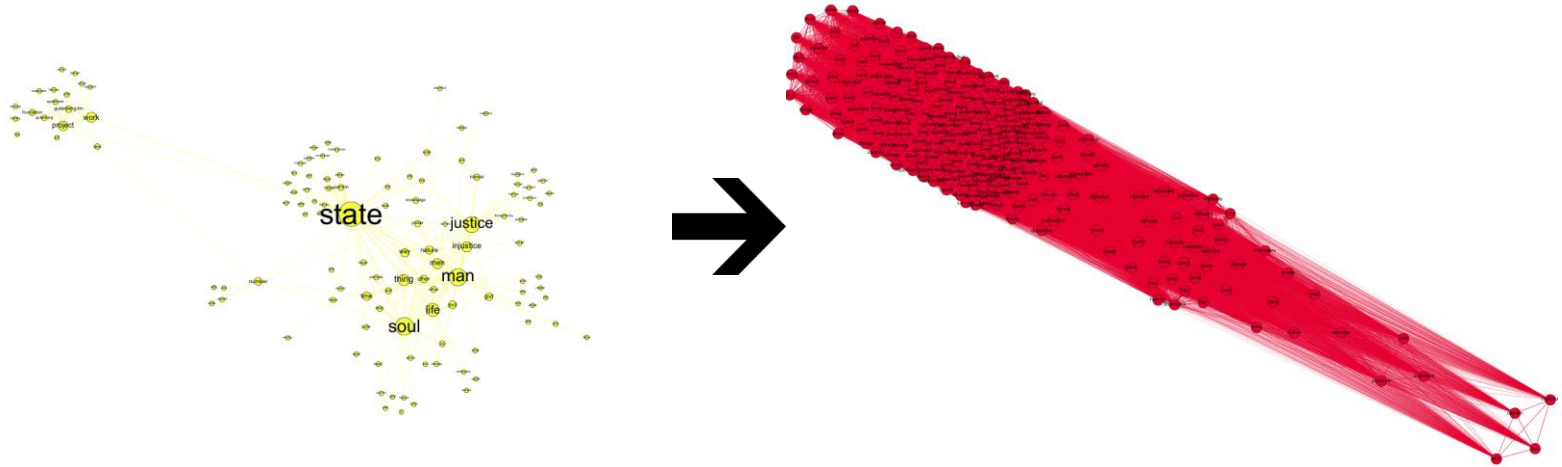
- **Best Common Friend** Affinity: importance of the best shared relationship.



Fumanal-Idocin, J., Alonso-Betanzos, A., Cordón, O., Bustince, H., & Minárová, M. (2020). Community detection and social network analysis based on the Italian wars of the 15th century. *Future Generation Computer Systems*, 113, 25-40.

AFFINITY FUNCTIONS

- **Machiavelli** Affinity: similarity in the social structures around both actors.



Fumanal-Idocin, J., Alonso-Betanzos, A., Cordón, O., Bustince, H., & Minárová, M. (2020). Community detection and social network analysis based on the Italian wars of the 15th century. *Future Generation Computer Systems*, 113, 25-40.

AFFINITY FUNCTIONS

- Some **affinity** functions dramatically **increase** the **density** of the graph.
 - In those cases, the idea of **shortest path** or **degree** do not **work**.
 - However, we can be interested in **both expressing** the social trait of the **affinity** function and ponder it with a **centrality** measure.



SEMANTIC VALUE

Centrality measure for
affinity functions

PROPOSAL

Centrality measure + Affinity functions

PROPOSAL

Centrality measure + Affinity functions

Semantic Value

SEMANTIC VALUE

- **Semantic Value:** ✕
 - Ponder each actor based on:
 - What the actor gives to the network (Intrinsic value) ⌋
 - What the actor receives from the network (Extrinsic value) ⌋

SEMANTIC VALUE

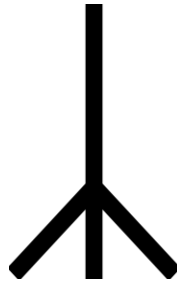
- The Semantic Value is the **natural union of both:**

$$\mathbb{X}(x) = \cup(\mathbb{A}(x), \mathbb{Y}(x))$$

SEMANTIC VALUE

Unique/intrinsic value of Actor X:

What the actor gives to the network (Application dependant)

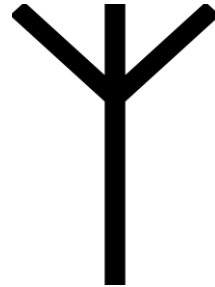


(Runic letter “calc”, meaning “Chalice”)

SEMANTIC VALUE

Extrinsic value of Actor X: the interaction with others

What the actor receives from the network (Has an **actual formula**)



(Runic letter “Eolh”, meaning “Elk”)

SEMANTIC VALUE

Extrinsic value: the interaction with others

$$\Psi(x) = \bigcup_{i=1}^a \{F_C(X_i, x) \bowtie (X_i) - \bigcup_{j \in J} \{\cap(F_C(X_i, x) \bowtie (X_i), F_C(X_j, x) \bowtie (X_j))\}\}$$

SEMANTIC VALUE

Extrinsic value: the interaction with others

$$\Psi(x) = \bigcup_{i=1}^a \{F_C(X_i, x) \bowtie (X_i) - \bigcup_{j \in J} \{\cap(F_C(X_i, x) \bowtie (X_i), F_C(X_j, x) \bowtie (X_j))\}\}$$

Affinity functions



SEMANTIC VALUE

Extrinsic value: the interaction with others

$$\Psi(x) = \bigcup_{i=1}^a \{F_C(X_i, x) \bowtie (X_i) - \bigcup_{j \in J} \{\cap(F_C(X_i, x) \bowtie (X_i), F_C(X_j, x) \bowtie (X_j))\}\}$$

Affinity functions



IMPORTANT: Circular dependencies

SEMANTIC VALUE

- Approximating the original formula: remove the recursivity by **simplifying** the **Semantic** Value for the **Intrinsic** value.
- Determining the **intrinsic** value?
 - **Application dependant**: for our case, the number of times each actor appeared in the original text.

SEMANTIC VALUE

- We can convert the **Semantic Value** into a **Centrality Measure**.
- **Compatible** con **Affinity** functions! (or at least some of them)

$\bowtie \rightarrow S$

$\Join \rightarrow I$

$\Upsilon \rightarrow E$

$\cup \rightarrow +$

$\cap \rightarrow *$

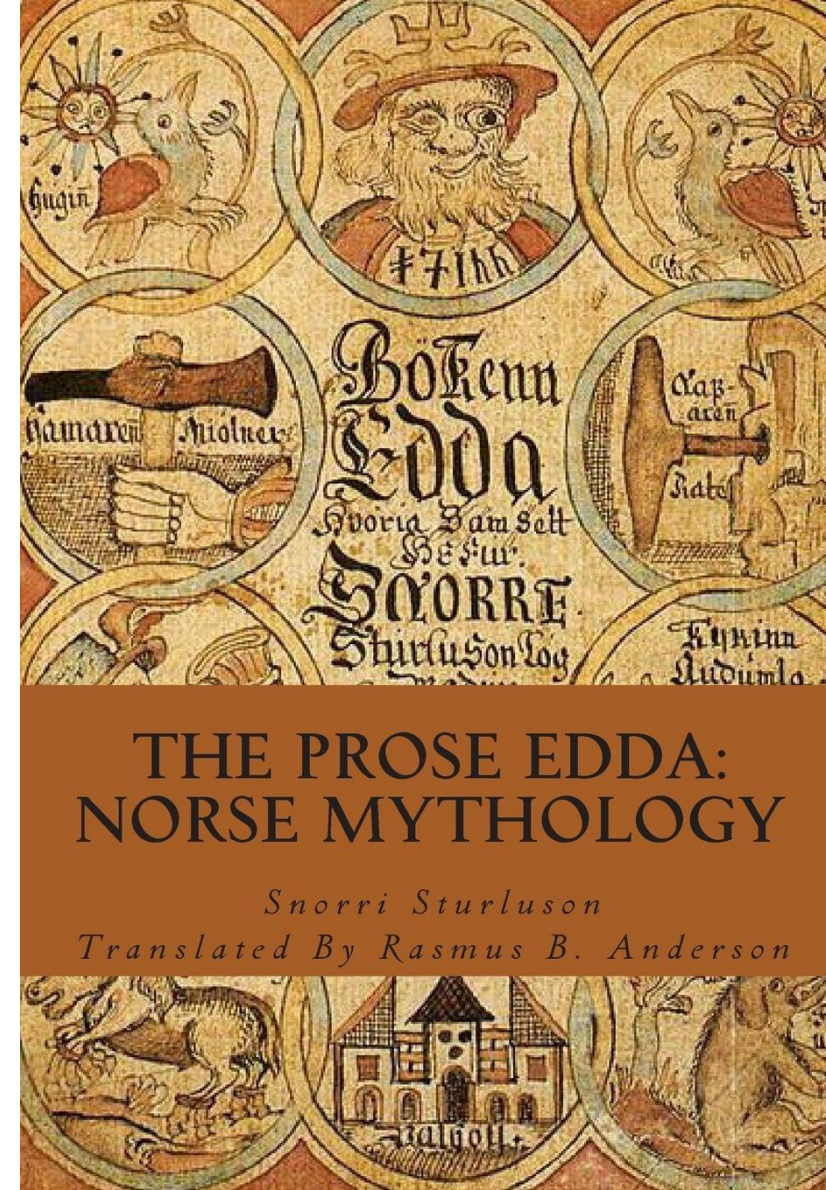


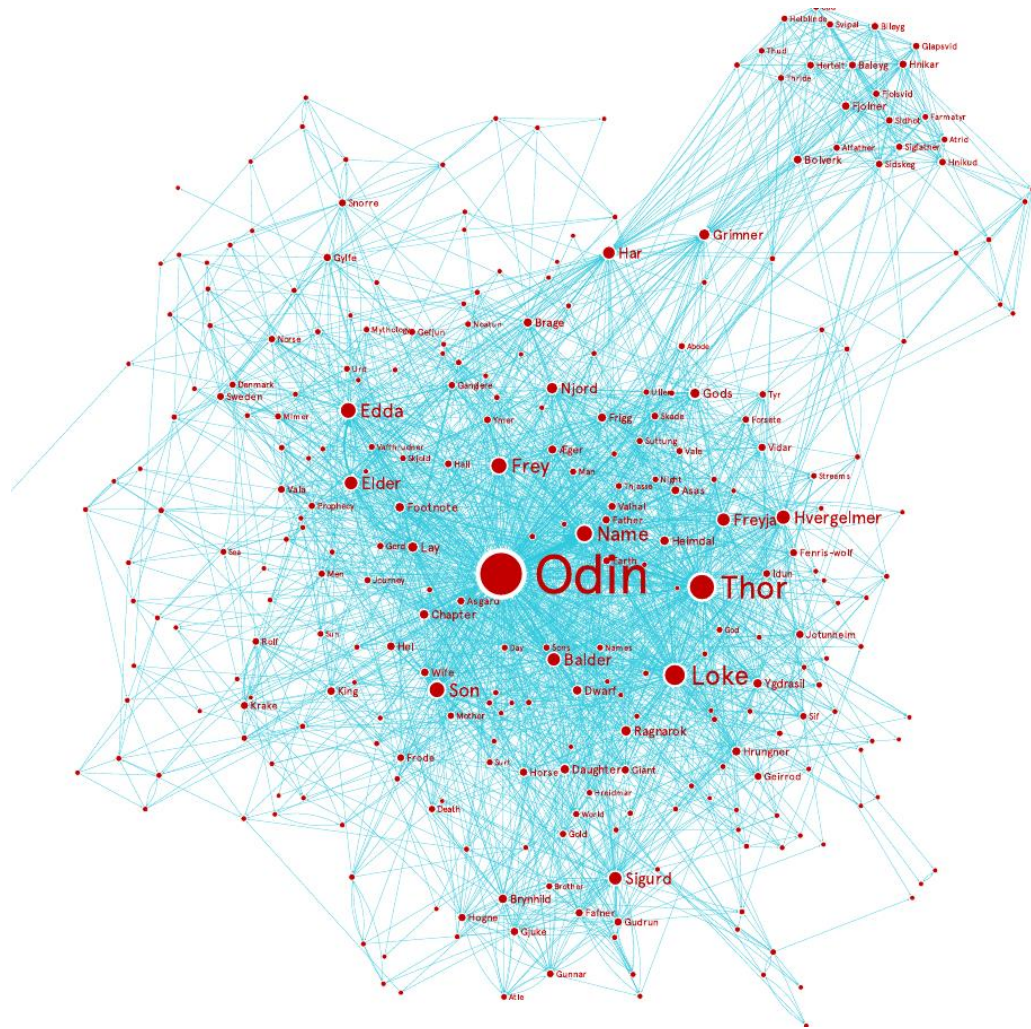
EXPERIMENTAL RESULTS

Viking gods and Social
network analysis

OUR TESTS: *THE YOUNGER EDDA*

- The Prose Edda or the Younger Edda by Snorri Sturluson (1179-1241) is a medieval Icelandic compilation of **mythical** texts.
- The original stories contain material from traditional sources, reaching the Viking Age.
- Stories of: Odin, Thor, Loki, Ragnarök, and so on...





The Younger Edda

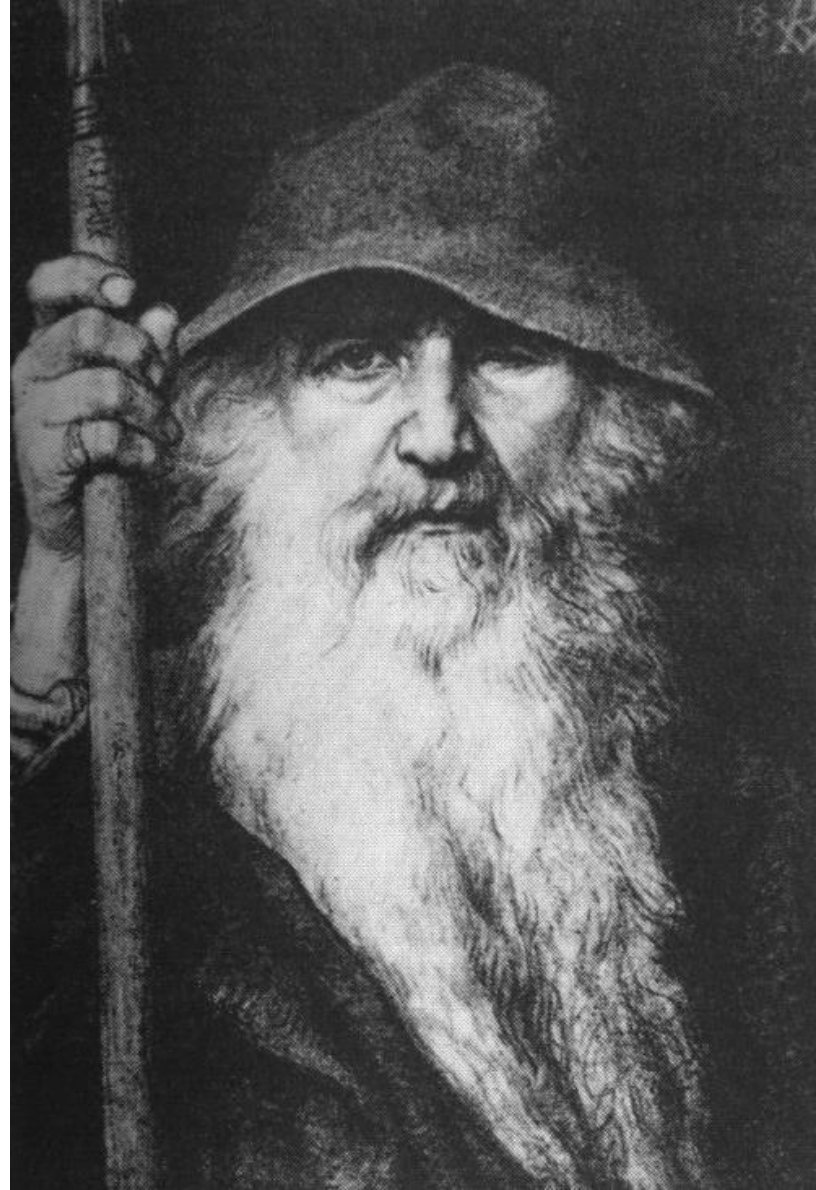
OUR TESTS: *THE YOUNGER EDDA*

	<i>S</i>	<i>E</i>	Freq. (<i>I</i>)	Degree	Betweenness	Closeness	Eigencentality
Odin	215.00	106.00	109	1113	0.47	0.75	0.37
Thor	176.63	44.63	132	508	0.14	0.61	0.26
Loki	100.58	34.58	66	291	0.06	0.56	0.22
King	53.63	13.63	40	79	0.02	0.49	0.07
Frey	51.01	20.01	31	167	0.02	0.53	0.17
Har	50.24	16.24	34	116	0.03	0.52	0.09
Sigurd	45.02	19.02	26	178	0.02	0.52	0.13
Balder	43.69	14.69	29	151	0.02	0.52	0.14
Freyja	28.78	10.78	18	154	0.01	0.51	0.16
Norse	24.06	4.06	20	37	0.00	0.46	0.04

OUR TESTS: *THE YOUNGER EDDA*

Odin is the top for all metrics.

- Expected: he has many names, many attributes...



OUR TESTS: *THE YOUNGER EDDA*

Top 3 are all gods:

Odin, Thor, Loki.

(Top 10, 6 of them)



OUR TESTS: *THE YOUNGER EDDA*

NOT many abstract concepts:

Anthropocentrism!

- NOT expected! We expected abstract ideas to receive more information from the rest of the network than individual actors.



CONCLUSIONS

- Affinity functions require special considerations to work with centrality measures.
- Semantic value: intrinsic + extrinsic
- Semantic value as centrality measure DOES work with affinity functions
 - Properties and patterns related to the original material seem to appear more clearly.

REFERENCES

- About **Myth Networks** and **Semantic Value**: Fumanal-Idocin, J., Cordón, O., Dimuro, G., Minárová, M., and Bustince, H. (2021). The Concept of Semantic Value in Social Network Analysis: an Application to Comparative Mythology, *arXiv preprint*.
- **Affinity functions**: Fumanal-Idocin, J., Alonso-Betanzos, A., Cordón, O., Bustince, H., & Minárová, M. (2020). Community detection and social network analysis based on the Italian wars of the 15th century. *Future Generation Computer Systems*, 113, 25-40.



Check out the code! https://github.com/Fuminides/noumenon_project