

Topic 04 - Tutorial Sheet-04

Lambda Calculus

Exercise 1:

Keeping in mind alpha equivalence, choose an answer that is equivalent to the listed lambda term.

1. $\lambda xy.xz$

- (a) $\lambda xz.xz$
- (b) $\lambda mn.mz$
- (c) $\lambda z(\lambda.x.xz)$

2. $\lambda xy.xxy$

- (a) $\lambda mn.mnp$
- (b) $\lambda x.(\lambda y.xy)$
- (c) $\lambda a(\lambda b.aab)$

3. $\lambda xyz.zx$

- (a) $\lambda x.(\lambda y.(\lambda z))$
- (b) $\lambda tos.st$
- (c) $\lambda mnp.mn$

Exercise 2:

Which (two or more) of the following are equivalent?

1. $\text{mth } x \ y \ z = x * y * z$
2. $\text{mth } x \ y = \lambda z \rightarrow x * y * z$
3. $\text{mth } x = \lambda y \rightarrow \lambda z \rightarrow x * y * z$
4. $\text{mth} = \lambda x \rightarrow \lambda y \rightarrow \lambda z \rightarrow x * y * z$

Exercise 3:

The type of **mth** (above) is

`mth :: Num a => a -> a -> a -> a`

Write down the type of

`mth 3`

Exercise 4:

Rewrite, using Haskell and evaluate the following:

1. $(\lambda x.x)2$
2. $(\lambda x.(x * 2))4$
3. $(\lambda x.(\lambda y.x * y))3\ 4$
4. $(\lambda x.\lambda y.(if\ x < y\ then\ -1\ else\ if\ x == y\ then\ 0\ else\ 1))\ 3\ 4$
 (**Note:** Use of if inside the lambda expression.)

Exercise 5:

Rewrite the f function in the *where* clause using anonymous lambda syntax

```
addOneIfOdd n = case odd n of
  True  -> f n
  False -> n
  where f n = n + 1
```

Exercise 6:

Rewrite the following to use anonymous lambda syntax

```
addFive x y = (if x > y then x else y) + 5
```

Exercise 7:

Write a lambda version of the following functions:

1. **abs**: which takes an Integer and returns the non-negative value.
 e.g. $abs\ -1 = 1$, $abs\ 4 = 4$.
2. **mymax**: which takes two numbers and returns the larger of the two
3. **mymin**: which takes two numbers and returns the smaller of the two