Available online at www.sciencedirect.com
SCIENCE DIRECT
International Journal of Fatigue 27 (2005) 847–852
International Journal of Fatigue
www.elsevier.com/locate/ijfatigue

The influence of grain size variation on metal fatigue

Jenny Andersson*

Department of Mathematical Statistics, Chalmers University of Technology, SE-412 96 Göteborg, Sweden

Received 21 July 2004; received in revised form 13 October 2004; accepted 18 November 2004

Abstract

The aim of the present study is to investigate the influence of the variation of metal grain sizes on fatigue lives. The grain structure is simulated from a Poisson–Voronoi model and the short crack growth model of Navarro and de los Rios is applied. The resulting fatigue life decreased with increasing component size, probably reflecting the fact that with increasing number of grains there is a larger probability of finding a large grain where the crack starts. The standard deviation of the logarithm of the lives was in the order of 0.2–0.4, i.e. the variation in grain size explains only part of the observed variance in real fatigue data.

Keywords: Short crack; Grain structure; Voronoi tessellation

1. Introduction

Supposedly identical components made of metal often show substantial differences in fatigue lives. The differences are apparent even during controlled tests with identical stress levels. Miller writes in [3] that the scatter in fatigue data needs to be put in a perspective by for example detailed studies of the effect of material structure on early crack growth. One model of early (short) crack growth has been developed by Navarro and de los Rios in [5–10]. The purpose of this study is to investigate the effect of grain size variation on fatigue life. Since the main part of the fatigue life is explained by the crack initiation, the model of Navarro–de los Rios will be used, as in [12], but modified to handle grains of varying sizes. A stochastic grain structure will be obtained by simulation. Similar ideas have been used by Ahmadi and Zenner [1] in a study of the growth of microcracks under the influence of cyclic loading. They compared simulations of cracks in a two-dimensional hexagonal lattice with experiments and the distribution of cracks was claimed to be in quantitatively good agreement between simulations and experiments. The main differences in the ideas from the present paper is the deterministic grain structure and our focus on scatter. A stochastic grain structure, a Voronoi tessellation, is used by Meyer, Brückner-Foit and Möslang [2] but focusing more on the crack patterns when several cracks are allowed to grow. Here also the results were found to be in good agreement with experiments.

The grain model is introduced in Section 2.1 and the Navarro–de los Rios model with modifications is described in Section 2.2 along with some computational details. The results are presented in Section 3 and analysed in Section 4.

2. Model

2.1. Grain structure

In the proposed model the metal grain structure is a Voronoi tessellation in two or three dimensions of points generated from a Poisson process (see Fig. 1). The reason for using a Voronoi tessellation can be argued as follows. If, in the crystallisation process of a one phase metal, all grains begin to grow simultaneously and at the same rate the resulting grain structure would be a Voronoi tessellation. The tessellation could be modified by allowing the grains to begin their growth at different times and by using a different point process with more or less clustering of the points.

A tessellation partitions an Euclidean space ($\mathbb{R}^n$) into sets, (C_i), with non-overlapping interior, that is $\mathbb{R}^n = \cup_i C_i$. Let $\{p_i\}$ be a set of points. Each point p_i in this set, from now

* Tel.: +41 772 82 94; fax: +31 772 35 08.
E-mail address: jennya@math.chalmers.se.

0142-1123/$ - see front matter © 2005 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijfatigue.2004.11.007

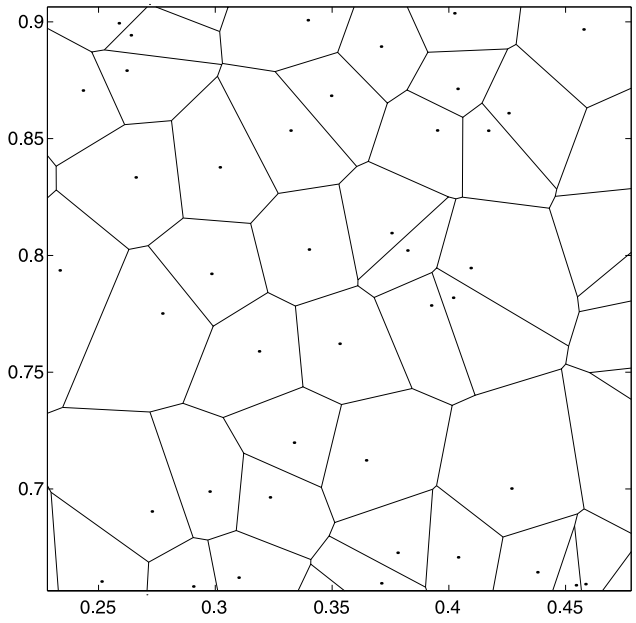

Fig. 1. A Voronoi tessellation of points generated from a Poisson process.

on called nuclei, generates a cell (or grain) C_i. One grain C_i consists of all points in $\mathbb{R}^n$ which has p_i as their nearest nuclei

$$C_i = \{x \in \mathbb{R}^n : ||p_i - x|| \leq ||p_j - x||, \forall p_j\}, \quad (1)$$

where $\|\cdot\|$ is the Euclidean distance. If the set of points $\{p_i\}$ is locally finite (any finite region contains a finite number of points) the C_is are called a Voronoi tessellation and C_i a Voronoi cell. A general reference on the properties of Voronoi tessellations is [11] and a more mathematical one is [4].

The realisations of Voronoi tessellations were accomplished using MATLAB version 6.5.1. First points were generated according to a Poisson process in two dimensions. Secondly, the function 'voronoin' were used on these points giving the Voronoi tessellation. To handle the effect of edges, points were generated in a slightly larger area than the one needed.

The metal simulated here is assumed to have a face-centred cubic (FCC) atomic structure and one phase (homogeneous in terms of chemical composition). In the model each grain is given a random (uniformly distributed) slip plane direction which determines the directions for the other slip planes (Fig. 2).

2.2. Crack growth model

The crack growth model is adapted from Navarro–de los Rios model for short crack growth under uniaxial loading [5–10]. The crack is modelled on the surface of the metal and consequently the three dimensional structure is disregarded. Since the point here is to use a grain structure with grains of different sizes, the Navarro–de los Rios

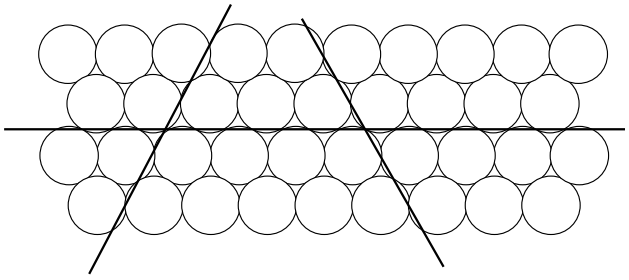

Fig. 2. Slip planes in a closed packed metal seen in two dimensions.

model, which is described for grains of equal size (as in [12]), has to be modified to the current situation. In short, the Navarro–de los Rios model considers the plastic slip produced ahead of a crack to be represented by a continuous distribution of dislocations. It is assumed that when slip is initiated in a grain the entire grain undergoes slip and is only blocked by the grain boundary, i.e. the front of the plastic zone coincides with the grain boundary. Slip is initiated in the next grain when the stress ahead of the plastic zone is enough to move new dislocations. This stress only depends on the position of the crack tip relative to the grain boundary.

The crack is initiated in the centre of a large grain with a slip plane close to the plane of maximum shear stress, that is the angle between the slip plane and the load direction is close to 45°. In making a decision in which grain to start, a compromise is made between size and direction of slip planes. If l is the length of the grain along a slip plain going through the centre of the grain and θ is the angle between the slip plane and the plane of maximum shear stress, a new length is calculated by $l_c = l \cos 2\theta$ (this is repeated for the three slip planes through the centre of the grain). This calculation reflects the fact that the sheer is zero both perpendicular and parallel to the main load direction. The grain selected for the crack to start in is the one with maximal l_c. The crack is supposed to grow along a slip plane at all times.

The crack growth rate is determined by

$$\frac{da}{dN} = f\phi, \quad (2)$$

where a is half the surface crack length, N the number of load cycles, f represents the fraction of dislocations ahead of the crack that participates in the crack growth process and depends on the applied stress and the material and ϕ is the plastic displacement of the crack-tip given by

$$\phi = \frac{2(1-\nu)\sqrt{1-n^2}}{\mu n}\sigma a, \quad (3)$$

where σ is the applied load, μ the shear modulus and ν Poisson's ratio. Here, $n = a/c$ is a dimensionless parameter, c the length of half the crack and half the plastic zone (see Fig. 3).

The slip band is blocked by the grain boundary and the crack will grow at a decreasing rate as it approaches

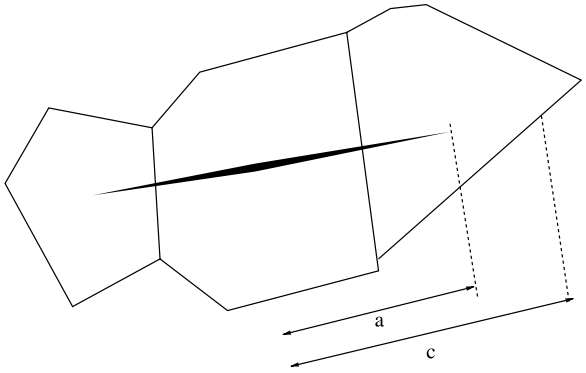

Fig. 3. Illustration of the parameters c and a.

the boundary until slip can be transfered to the next grain. This happens at a critical value of n equal to

$$n_c^i = \cos\left(\frac{\pi}{2}\frac{\sigma - \sigma_{Li}}{\sigma_{comp}}\right), \tag{4}$$

where σ_{comp} is the resistance to plastic deformation of the crack tip. Consecutive grains are numbered $i=1,2,3,\ldots$. When σ is smaller than σ_{Li} the stress is not enough to overcome the boundary and the crack stops. The minimum stress required for slip propagation is given by

$$\sigma_{Li} = \sigma_{FL}\frac{m_i}{m_1}\sqrt{\frac{\bar{d}_i}{2c_i}} \tag{5}$$

where c_i is the length of half the crack plus half the plastic zone when the crack grows in grain i, $\bar{d}_i$ is the mean of the length the crack has grown in each grain, σ_{FL} is the fatigue stress and

$$\frac{m_i}{m_1} = 1 + 2.07\left(\frac{2}{\pi}\arctan(0.522(i-1)2)\right)^{1.86}. \tag{6}$$

is the ratio of grain orientations.

When a new slip band is initiated in the next grain and the plastic zone is supposed to span the entire new grain, and therefore, n decreases to

$$n_s^{i+1} = \frac{c_i}{c_{i+1}}n_c^i, \tag{7}$$

which is a rescaling of the old value of n by the new value of c. According to the model, the crack will grow along that slip plane in the new grain that is closest to the plane of maximum shear stress (the angle between this plane and the loading direction is 45°), regardless of which direction the slip plane takes in the third dimension under the surface.

The growth rate Eq. (2) can be integrated over a grain (or over parts of a grain) to give the number of cycles spent in that grain

$$\Delta N_i = \frac{\mu}{f(1-\nu)2\sigma}(\arcsin n_c^i - \arcsin n_s^i). \tag{8}$$

Table 1
Parameter values for commercially pure aluminium

Parameter	Value
μ	25.0 GPa
σ_{comp}	50.0 MPa
σ_{FL}	42.5 MPa
ν	0.33

The total number of cycles is then obtained by summing over all grains.

In the Navarro–de los Rios model all the grains are assumed to be equal in size and because of the symmetry in that case it is enough to do calculations on half the crack. Here, however, the crack may not grow at the same rate at both directions after the first grain. Practically, this is solved by considering the two growth directions separately. As an approximation of the total cracklength after, say, N cycles, we can calculate how long the crack is in both directions separately, by using Eq. (8). The total cracklength after N cycles is then obtained by adding the cracklengths in the two directions. The growth rate at N cycles is approximated by adding the growth rates for the two directions at N cycles.

The values of the parameters used in the calculations are the same as in [12] for commercially pure aluminium. These are shown in Table 1. They used $f=6.16\times10^{-5}$ $(2(\sigma-\sigma_{FL}))^{2.696}$.

3. Results

Simulations were made of two-dimensional Voronoi tessellations where the number of nuclei were taken from a Poisson distribution with expectation (denoted λA) 2000, 4000 and 9000, which corresponds to looking at

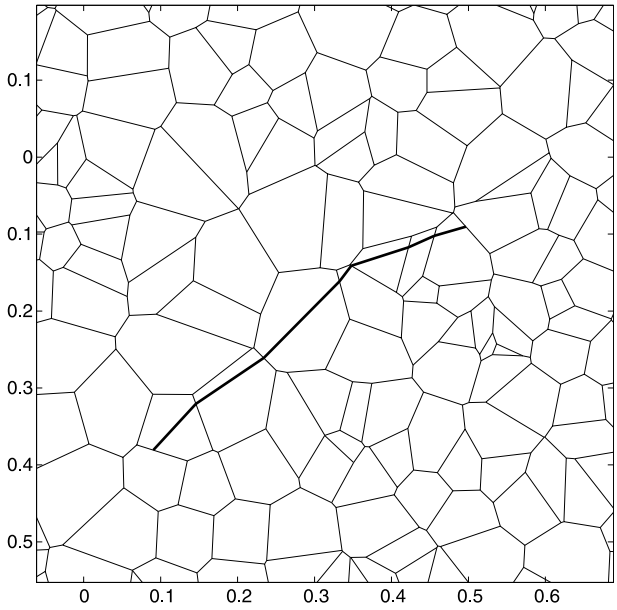

Fig. 4. A simulated crack and grain structure.

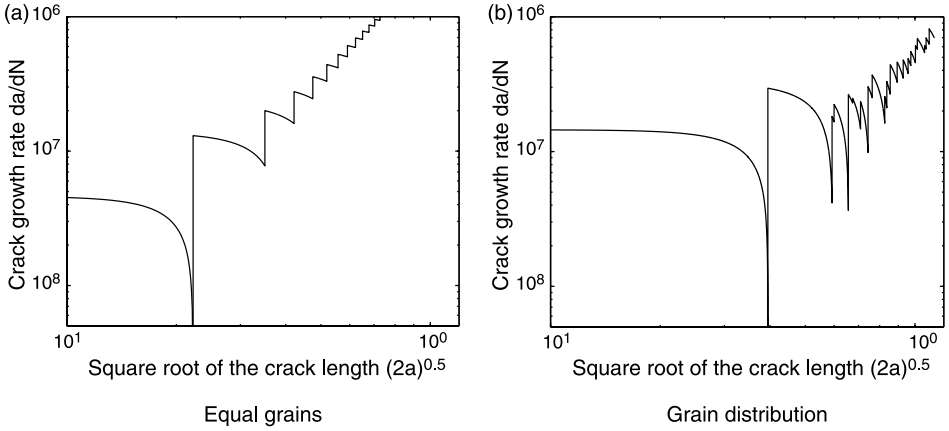

Fig. 5. Logarithmic crack growth rate plots for the original Navarro–de los Rios model to the left and for the Voronoi tessellation model to the right.

components of increasing size. More specifically, squares of sides 2, 4 and 6 giving the area A to be 4, 16 and 36, respectively, were used with an intensity, λ, of points per unit area as 250. The unit of the area is not important since the grain size is only included implicitly in the Navarro–de los Rios models, i.e. in the material constants. These choices gave a mean intercept length of the grains in the tessellations as approximately 0.050 calculated from the simulations. The crack path was determined as described in Section 2.2 and the crack was allowed to grow to a maximum (it could stop before, if $\sigma < \sigma_{\mathrm{Li}}$ in Eq. (4)) length of 10 times the mean grain size. The crack growth rate as a function of crack length and the number of cycles to failure were calculated, the latter resulting in a Wöhler curve for the short crack growth. For each value of the expectation the simulation was repeated 1000 times.

Fig. 4 shows an example of a simulated crack, not showing the entire simulated square, and Fig. 5 the crack growth rate as a function of crack length for this crack. In the latter figure, there is also the corresponding plot for a grain structure with equal grain sizes. The growth rate decreases when the crack get close to a boundary, then increases sharply as the crack resumes its growth in the next grain. Fig. 6 shows a Wöhler curve for the initial crack growth for expectation in the Poisson distribution equal to 9000. As a comparison the results from using a model without grain size variation is plotted in the same figure. A regression was made on the lives for $\Delta\sigma$ ranging from 94 to 100 MPa to $N = a(\Delta\sigma)^b$ with the values of the coefficients in Table 2 as the result, i.e. the life decreases with λA or equivalently component size.

The observations at $N = 10^8$ are of cracks that have stopped before they were 10 times the mean grain size long. The variation conditional on finite fatigue life of the number of cycles to failure first increases with the applied load and the decreases (plot in Fig. 7). Fig. 8 shows the percentage of cracks that stopped, i.e. the fatigue life is infinite.

It is often claimed that the intercept lengths of the grains in metals with unimodal distribution of grains is approximately lognormal. In Fig. 9 is a quantile plot of the logarithm of the intercept lengths in a simulation. The points are supposed to follow a straight line if the lognormal distribution is appropriate.

4. Discussion

The discussion following is purely qualitative because of lack of real data. There is no evaluation of the Voronoi model as a grain structure apart form the comparison of

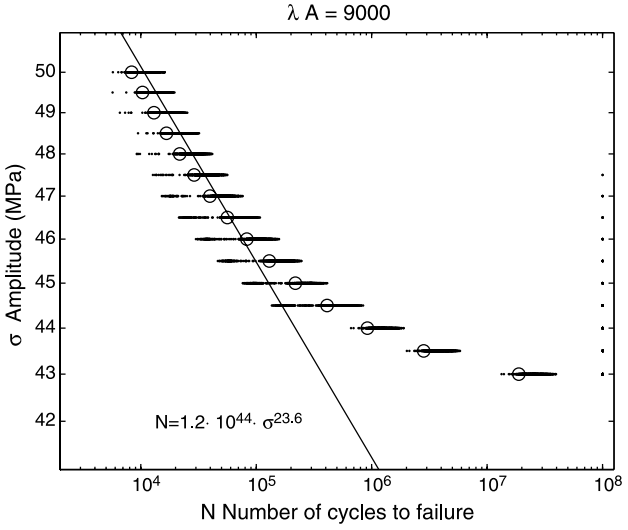

Fig. 6. Wöhler curve for the initial crack growth. The unfilled rings corresponds to life lengths calculated from the model with equal grain sizes.

Table 2
Coefficients in $N = a(\Delta\sigma)^b$, when $\Delta\sigma$ is in MPa

Expectation (λA)	$a \times 10^{44}$	b
1000	1.91	−23.7
4000	1.75	−23.7
9000	1.24	−23.6

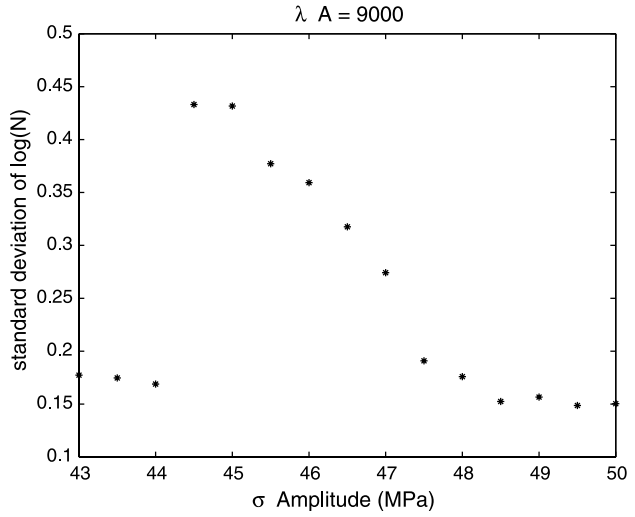

Fig. 7. The standard deviation conditional on finite fatigue life of the number of cycles in Fig. 6 as a function of the load.

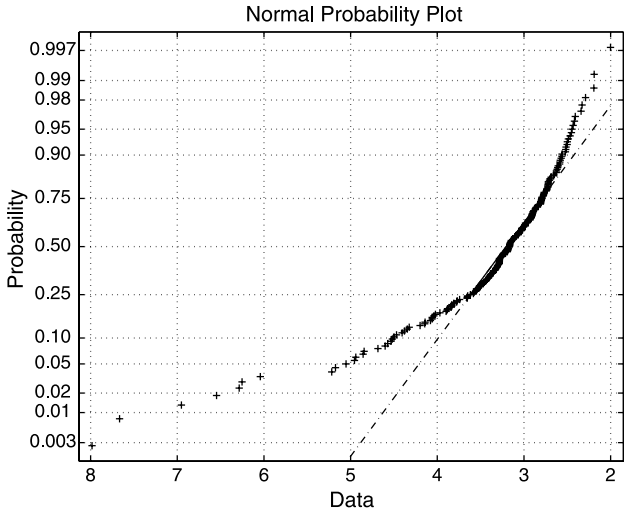

Fig. 9. A normal quantile plot of the logarithm of 110 intercept lengths in a two-dimensional Voronoi tessellation.

the intercepts with the lognormal distribution. The agreement is not very good since the observations are not on a straight line. In this context, however, this is not the crucial thing to compare but rather the distribution of large grains which is more important for the crack growth. Probably the common knowledge of the intercept lengths being lognormal is not always accurate so the best way to do a good comparison would be to use a real material.

The calculated fatigue lives should be seen as an example of what is possible to do with this modelling approach. The crack growth model of Navarro–de los Rios is used only in the form of grains of equal sizes as a first approximation. For future simulations the results for unequal grain sizes in Vallellano, Navarro and Domínguez [13] can be used instead. The reason for not using it here was the extra complexity in programming. Our belief is that the approximation does not change the results qualitatively and other approximations made have probably a larger impact.

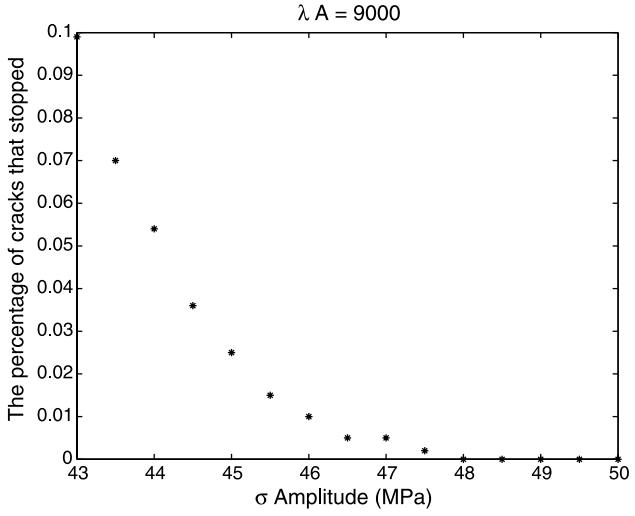

Fig. 8. The percentage of cracks that stopped in Fig. 6, i.e. the number of observations at $N=10^8$ cycles.

Comparing the simulations to a computation with equal grain sizes show that the crack growth rate curve is more irregular. The advantage of using a grain structure with varying grain sizes as opposed to one with equal grains is that the crack can stop and that it is possible to calculate the variance of the fatigue lives.

There are many simulations with infinite life even for higher loads which is not observed in real data. The explanation is that if a crack stops in a real material there may be a crack that can continue somewhere else in the structure. If a crack stops here there is no other crack that starts at another location. In principle it is possible to simulate that situation, however, then a decision have to be made when to stop creating new cracks.

The standard deviation of the lives conditional on finite life first increases with the load and then decreases as expected from observations. The increase in the beginning is due to the censored data which really have large fatigue lives and hence would increase the standard deviation if they were accounted for.

The fatigue life decreased with increasing number of grains, probably reflecting the fact that with increasing number of grains there is a larger probability of finding a large grain (where the crack is assumed to start). The standard deviation of the logarithm of the lives conditional on finite life is in the order of 0.2–0.4 depending on the load.

Grain size variation gives rise to a longer fatigue life compared with a structure with equal grains. One possible explanation is that even if the crack starts in a large grain and grows fast there, the next grains is probably smaller than in a structure of equally sized grains, and therefore, it grows slower in the second grain. The fatigue life decreases, as

expected with component size, i.e. the size of the area simulated. However, only a part of fatigue life variation observed in doing experiments is explained by the varying grain size according to the simulations.

Acknowledgements

I am grateful to my supervisors Jacques de Maré and Thomas Svensson for starting me on this project and helpful discussions.

References

[1] Ahmadi A, Zenner H. Simulation of microcrack propagation for different load sequences. In: Blom AF, editor. Fatigue 2002: Proceedings of the eight international fatigue congress, vol. 3; 2002. p. 1997–2004.

[2] Meyer S, Brückner-Foit A, Möslang A. A stochastic simulation model for microcrack initiation in martensitic steel. Comput Mater Sci 2003; 26:102–10.

[3] Miller KJ. A historical perspective of the important parameters of metal fatigue; and problems for the next century. In: Wu XR, Wang ZG, editors. Fatigue '99: Proceedings of the seventh international fatigue congress, vol. 1–4. Beijing: China Higher Education Press; 1999. p. 15–39.

[4] Mller J. Lecture notes on random Voronoi tessellations. Berlin: Springer; 1994.

[5] Navarro A, de los Rios ER. A model for short fatigue crack propagation with an interpretation of the short–long transition. Fatigue Fract Eng Mater Struct 1987;10:169–86.

[6] Navarro A, de los Rios ER. A microstructurally-short fatigue crack growth equation. Fatigue Fract Eng Mater Struct 1988;11:383–96.

[7] Navarro A, de los Rios ER. Short and long fatigue crack growth: a unified model. Philos Mag A 1988;57:15–36.

[8] Navarro A, de los Rios ER. An alternative model of the blocking of dislocations at grain boundaries. Philos Mag A 1988;57:37–42.

[9] Navarro A, de los Rios ER. Compact solution for a multizone BCS crack model with bounded or unbounded end conditions. Philos Mag A 1988;57:43–50.

[10] Navarro A, de los Rios ER. Considerations of grain orientation and work hardening on short-fatigue-crack modelling. Philos Mag A 1990;61:435–49.

[11] Okabe A, Boots B, Sugihara K. Spatial tessellations: concepts and applications of Voronoi diagrams. New York: Wiley; 1992.

[12] Turnbull A, de los Rios ER. Predicting fatigue life in commercially pure aluminium using a short crack growth model. Fatigue Fract Eng Mater Struct 1995;18:1469–81.

[13] Vallellano C, Navarro A, Domínguez J. Compact formulation for modelling cracks in infinite solids using distributed dislocations. Philos Mag A 2002;82:81–92.