

# Question.2-04

함수  $\vec{f}$ 가  $\theta$ 에 대해 다음과 같이 주어졌을 때, Jacobian matrix  $\frac{\partial \vec{f}(\theta)}{\partial \theta}$ 를 구하시오.

$$\vec{f}(\theta) = \begin{pmatrix} f_1(\theta) \\ f_2(\theta) \\ f_3(\theta) \end{pmatrix} = \begin{pmatrix} \theta^3 - 2\theta^2 + 10 \\ \ln(\theta) - \sin(\theta)\cos(\theta) \\ e^{\theta+10} - e^{2\theta} \end{pmatrix}$$

주어진 vector function  $\vec{f}$ 의 원소인  $f_1(\theta)$ ,  $f_2(\theta)$ ,  $f_3(\theta)$ 들에 대한 각각의 derivative를 구하면 다음과 같다.

$$\begin{aligned} \frac{\partial f_1(\theta)}{\partial \theta} &= \frac{\partial}{\partial \theta} [\theta^3 - 2\theta^2 + 10] \\ &= \frac{\partial}{\partial \theta} [\theta^3] - 2 \frac{\partial}{\partial \theta} [\theta^2] + \frac{\partial}{\partial \theta} [10] \\ &= 3\theta^2 - 4\theta \end{aligned}$$

$$\begin{aligned} \frac{\partial f_2(\theta)}{\partial \theta} &= \frac{\partial}{\partial \theta} [\ln(\theta) - \sin\theta \cos\theta] \\ &= \frac{\partial}{\partial \theta} [\ln(\theta)] - \frac{\partial}{\partial \theta} [\sin\theta \cos\theta] \quad \text{product rule } (fg)' = f'g + fg' \\ &= \frac{1}{\theta} - \left[ \left( \frac{\partial}{\partial \theta} [\sin\theta] \right) \cos\theta + \sin\theta \left( \frac{\partial}{\partial \theta} [\cos\theta] \right) \right] \\ &= \frac{1}{\theta} - (\cos^2\theta - \sin^2\theta) \quad \text{--- } \sin^2\theta + \cos^2\theta = 1 \\ &= \frac{1}{\theta} + 2\sin^2\theta - 1 \end{aligned}$$

$$\begin{aligned} \frac{\partial f_3(\theta)}{\partial \theta} &= \frac{\partial}{\partial \theta} [e^{\theta+10} - e^{2\theta}] \\ &= \frac{\partial}{\partial \theta} [e^{\theta+10}] - \frac{\partial}{\partial \theta} [e^{2\theta}] \\ &= e^{\theta+10} \frac{\partial}{\partial \theta} [\theta+10] - e^{2\theta} \frac{\partial}{\partial \theta} [2\theta] \\ &= e^{\theta+10} - 2e^{2\theta} \end{aligned}$$

따라서 Jacobian  $\frac{\partial \vec{f}(\theta)}{\partial \theta}$ 는 다음과 같다.

$$\frac{\partial \vec{f}(\theta)}{\partial \theta} = \begin{pmatrix} \frac{\partial f_1(\theta)}{\partial \theta} \\ \frac{\partial f_2(\theta)}{\partial \theta} \\ \frac{\partial f_3(\theta)}{\partial \theta} \end{pmatrix} = \begin{pmatrix} 3\theta^2 - 4\theta \\ \frac{1}{\theta} + 2\sin^2\theta - 1 \\ e^{\theta+10} - 2e^{2\theta} \end{pmatrix}$$