Question.2-05

함수 $\vec{f}$가 $\vec{\theta}$에 대해 다음과 같이 주어졌을 때, Jacobian matrix $\dfrac{\partial \vec{f}(\vec{\theta})}{\partial \vec{\theta}}$를 구하시오.

$$\vec{\theta} = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \qquad \vec{f}(\vec{\theta}) = \begin{pmatrix} f_1(\vec{\theta}) \\ f_2(\vec{\theta}) \\ f_3(\vec{\theta}) \end{pmatrix} = \begin{pmatrix} (\theta_1)^3 - 2(\theta_2)^2 + 10 \\ ln(\theta_1) - sin(\theta_1)cos(\theta_2) \\ e^{\theta_1+10} - e^{2\theta_2} \end{pmatrix}$$

주어진 $\vec{\theta}, \vec{f}$에 대한 Jacobian은

$$\frac{\partial \vec{f}(\vec{\theta})}{\partial \vec{\theta}} = \begin{pmatrix} \frac{\partial f_1(\vec{\theta})}{\partial \theta_1} & \frac{\partial f_1(\vec{\theta})}{\partial \theta_2} \\ \frac{\partial f_2(\vec{\theta})}{\partial \theta_1} & \frac{\partial f_2(\vec{\theta})}{\partial \theta_2} \\ \frac{\partial f_3(\vec{\theta})}{\partial \theta_1} & \frac{\partial f_3(\vec{\theta})}{\partial \theta_2} \end{pmatrix} = \begin{pmatrix} \nabla_{\vec{\theta}} f_1(\vec{\theta}) \\ \nabla_{\vec{\theta}} f_2(\vec{\theta}) \\ \nabla_{\vec{\theta}} f_3(\vec{\theta}) \end{pmatrix}$$

이다. 이때 각각 $\nabla_{\vec{\theta}} f_1(\vec{\theta})$, $\nabla_{\vec{\theta}} f_2(\vec{\theta})$, $\nabla_{\vec{\theta}} f_3(\vec{\theta})$를 구하면 다음과 같다.

$$\frac{\partial f_1(\vec{\theta})}{\partial \theta_1} = \frac{\partial}{\partial \theta_1}\left[(\theta_1)^3 - 2(\theta_2)^2 + 10\right] = 3(\theta_1)^2$$

$$\frac{\partial f_1(\vec{\theta})}{\partial \theta_2} = \frac{\partial}{\partial \theta_2}\left[(\theta_1)^3 - 2(\theta_2)^2 + 10\right] = -4\theta_2$$

$$\longrightarrow \nabla_{\vec{\theta}} f_1(\vec{\theta}) = \left(\frac{\partial f_1(\vec{\theta})}{\partial \theta_1}, \frac{\partial f_1(\vec{\theta})}{\partial \theta_2}\right) = \left(3(\theta_1)^2, -4\theta_2\right)$$

$$\frac{\partial f_2(\vec{\theta})}{\partial \theta_1} = \frac{\partial}{\partial \theta_1}\left[ln(\theta_1) - sin(\theta_1)cos(\theta_2)\right] = \frac{1}{\theta_1} - cos(\theta_1)cos(\theta_2)$$

$$\frac{\partial f_2(\vec{\theta})}{\partial \theta_2} = \frac{\partial}{\partial \theta_2}\left[ln(\theta_1) - sin(\theta_1)cos(\theta_2)\right] = sin(\theta_1)sin(\theta_2)$$

$$\longrightarrow \nabla_{\vec{\theta}} f_2(\vec{\theta}) = \left(\frac{\partial f_2(\vec{\theta})}{\partial \theta_1}, \frac{\partial f_2(\vec{\theta})}{\partial \theta_2}\right) = \left(\frac{1}{\theta_1} - cos(\theta_1)cos(\theta_2), sin(\theta_1)sin(\theta_2)\right)$$

$$\frac{\partial f_3(\vec{\theta})}{\partial \theta_1} = \frac{\partial}{\partial \theta_1}\left[e^{(\theta_1+10)} - e^{2\theta_2}\right] = e^{\theta_1+10}$$

$$\frac{\partial f_3(\vec{\theta})}{\partial \theta_2} = \frac{\partial}{\partial \theta_2}\left[e^{(\theta_1+10)} - e^{2\theta_2}\right] = -2e^{2\theta_2}$$

$$\longrightarrow \nabla_{\vec{\theta}} f_3(\vec{\theta}) = \left(\frac{\partial f_3(\vec{\theta})}{\partial \theta_1}, \frac{\partial f_3(\vec{\theta})}{\partial \theta_2}\right) = \left(e^{\theta_1+10}, -2e^{2\theta_2}\right)$$

따라서 구하고자하는 Jacobian은 다음과 같다.

$$\frac{\partial \vec{f}(\vec{\theta})}{\partial \vec{\theta}} = \begin{pmatrix} \frac{\partial f_1(\vec{\theta})}{\partial \theta_1} & \frac{\partial f_1(\vec{\theta})}{\partial \theta_2} \\ \frac{\partial f_2(\vec{\theta})}{\partial \theta_1} & \frac{\partial f_2(\vec{\theta})}{\partial \theta_2} \\ \frac{\partial f_3(\vec{\theta})}{\partial \theta_1} & \frac{\partial f_3(\vec{\theta})}{\partial \theta_3} \end{pmatrix} = \begin{pmatrix} 3(\theta_1)^2 & -4\theta_2 \\ \frac{1}{\theta_1} - cos(\theta_1)cos(\theta_2) & sin(\theta_1)sin(\theta_2) \\ e^{\theta_1+10} & -2e^{2\theta_2} \end{pmatrix}$$