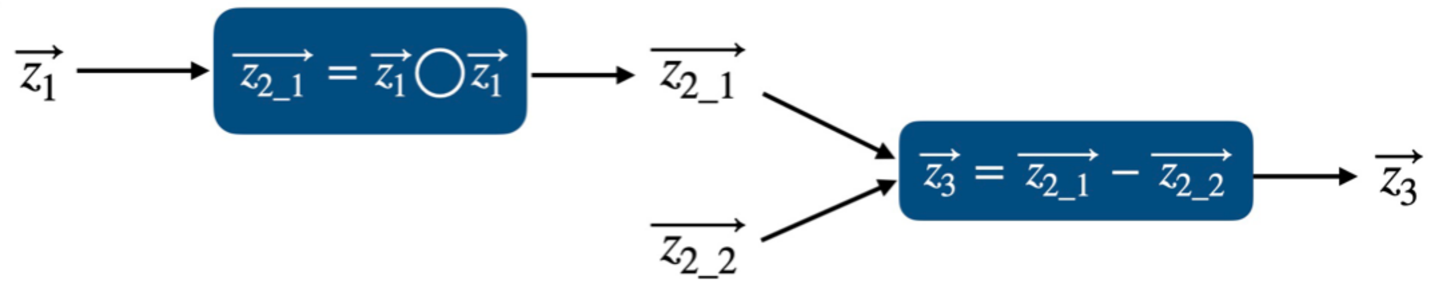
Question.2-09

다음 연산에서 $\dfrac{\partial \vec{z_3}}{\partial \vec{z_1}}$를 구하시오.



주어진 상황에서 $\dfrac{\partial \vec{z}_3}{\partial \vec{z}_1}$을 구하기 위해선 $\dfrac{\partial \vec{z}_3}{\partial \vec{z}_{2_1}}$와 $\dfrac{\partial \vec{z}_{2_1}}{\partial \vec{z}_1}$을 구해야한다. 이를 구해보면 다음과 같다.

$$\frac{\partial \vec{z}_{2\_1}}{\partial \vec{z}_1} = \begin{pmatrix} \frac{\partial z_{2\_1}^{(1)}}{\partial z_1^{(1)}} & \frac{\partial z_{2\_1}^{(1)}}{\partial z_1^{(2)}} & \cdots & \frac{\partial z_{2\_1}^{(1)}}{\partial z_1^{(n)}} \\ \frac{\partial z_{2\_1}^{(2)}}{\partial z_1^{(1)}} & \frac{\partial z_{2\_1}^{(2)}}{\partial z_1^{(2)}} & \cdots & \frac{\partial z_{2\_1}^{(2)}}{\partial z_1^{(n)}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial z_{2\_1}^{(n)}}{\partial z_1^{(1)}} & \frac{\partial z_{2\_1}^{(n)}}{\partial z_1^{(2)}} & \cdots & \frac{\partial z_{2\_1}^{(n)}}{\partial z_1^{(n)}} \end{pmatrix} = \begin{pmatrix} 2z_1^{(1)} & 0 & \cdots & 0 \\ 0 & 2z_1^{(2)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 2z_1^{(n)} \end{pmatrix}$$

$$\frac{\partial \vec{z}_3}{\partial \vec{z}_{2\_1}} = \begin{pmatrix} \frac{\partial z_3^{(1)}}{\partial z_{2\_1}^{(1)}} & \frac{\partial z_3^{(1)}}{\partial z_{2\_1}^{(2)}} & \cdots & \frac{\partial z_3^{(1)}}{\partial z_{2\_1}^{(n)}} \\ \frac{\partial z_3^{(2)}}{\partial z_{2\_1}^{(1)}} & \frac{\partial z_3^{(2)}}{\partial z_{2\_1}^{(2)}} & \cdots & \frac{\partial z_3^{(2)}}{\partial z_{2\_1}^{(n)}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial z_3^{(n)}}{\partial z_{2\_1}^{(1)}} & \frac{\partial z_3^{(n)}}{\partial z_{2\_1}^{(2)}} & \cdots & \frac{\partial z_3^{(n)}}{\partial z_{2\_1}^{(n)}} \end{pmatrix} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

위의 Jacobian들과 Vector chain rule을 이용해 $\dfrac{\partial \vec{z}_3}{\partial \vec{z}_1}$을 구하면 다음과 같다.

$$\frac{\partial \vec{z}_3}{\partial \vec{z}_1} = \frac{\partial \vec{z}_3}{\partial \vec{z}_{2\_1}} \cdot \frac{\partial \vec{z}_{2\_1}}{\partial \vec{z}_1} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} \begin{pmatrix} 2z_1^{(1)} & 0 & \cdots & 0 \\ 0 & 2z_1^{(2)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 2z_1^{(n)} \end{pmatrix} = \begin{pmatrix} 2z_1^{(1)} & 0 & \cdots & 0 \\ 0 & 2z_1^{(2)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 2z_1^{(n)} \end{pmatrix}$$