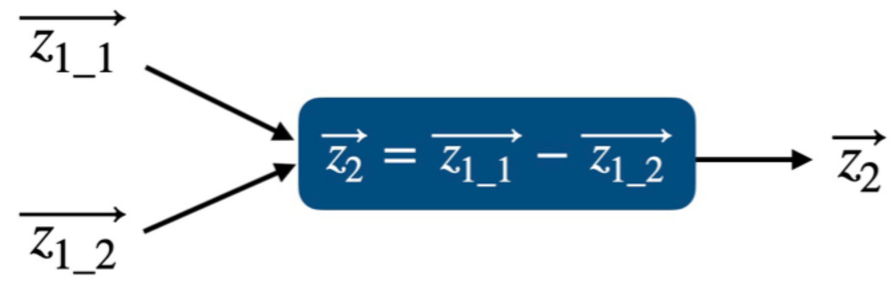
Question.2-07

다음 연산에서 $\dfrac{\partial \vec{z_2}}{\partial \vec{z_{1\_1}}}, \dfrac{\partial \vec{z_2}}{\partial \vec{z_{1\_2}}}$, 를 각각 구하시오.



주어진 연산에서

$$\vec{z}_{1\_1} = \begin{pmatrix} z_{1\_1}^{(1)} \\ z_{1\_1}^{(2)} \\ \vdots \\ z_{1\_1}^{(n)} \end{pmatrix}, \quad \vec{z}_{1\_2} = \begin{pmatrix} z_{1\_2}^{(1)} \\ z_{1\_2}^{(2)} \\ \vdots \\ z_{1\_2}^{(n)} \end{pmatrix}, \quad \vec{z}_2 = \begin{pmatrix} z_2^{(1)} \\ z_2^{(2)} \\ \vdots \\ z_2^{(n)} \end{pmatrix}$$

라 하면, $\vec{z}_2 = \vec{z}_{1\_1} + \vec{z}_{1\_2}$ 연산은

$$\begin{pmatrix} z_2^{(1)} \\ z_2^{(2)} \\ \vdots \\ z_2^{(n)} \end{pmatrix} = \begin{pmatrix} z_{1\_1}^{(1)} - z_{1\_2}^{(1)} \\ z_{1\_1}^{(2)} - z_{1\_2}^{(2)} \\ \vdots \\ z_{1\_1}^{(n)} - z_{1\_2}^{(n)} \end{pmatrix}$$

이 된다. 따라서 Jacobian matrices $\dfrac{\partial \vec{z}_2}{\partial \vec{z}_{1\_1}}, \dfrac{\partial \vec{z}_2}{\partial \vec{z}_{1\_2}}$ 는 다음과 같다.

$$\frac{\partial \vec{z}_2}{\partial \vec{z}_{1\_1}} = \begin{pmatrix} \frac{\partial z_2^{(1)}}{\partial z_{1\_1}^{(1)}} & \frac{\partial z_2^{(1)}}{\partial z_{1\_1}^{(2)}} & \cdots & \frac{\partial z_2^{(1)}}{\partial z_{1\_1}^{(n)}} \\ \frac{\partial z_2^{(2)}}{\partial z_{1\_1}^{(1)}} & \frac{\partial z_2^{(2)}}{\partial z_{1\_1}^{(2)}} & \cdots & \frac{\partial z_2^{(2)}}{\partial z_{1\_1}^{(n)}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial z_2^{(n)}}{\partial z_{1\_1}^{(1)}} & \frac{\partial z_2^{(n)}}{\partial z_{1\_1}^{(2)}} & \cdots & \frac{\partial z_2^{(n)}}{\partial z_{1\_1}^{(n)}} \end{pmatrix} = \begin{pmatrix} \frac{\partial}{\partial z_{1\_1}^{(1)}}[z_{1\_1}^{(1)} - z_{1\_2}^{(1)}] & \frac{\partial}{\partial z_{1\_1}^{(2)}}[z_{1\_1}^{(1)} - z_{1\_2}^{(1)}] & \cdots & \frac{\partial}{\partial z_{1\_1}^{(n)}}[z_{1\_1}^{(1)} - z_{1\_2}^{(1)}] \\ \frac{\partial}{\partial z_{1\_1}^{(1)}}[z_{1\_1}^{(2)} - z_{1\_2}^{(2)}] & \frac{\partial}{\partial z_{1\_1}^{(2)}}[z_{1\_1}^{(2)} - z_{1\_2}^{(2)}] & \cdots & \frac{\partial}{\partial z_{1\_1}^{(n)}}[z_{1\_1}^{(2)} - z_{1\_2}^{(2)}] \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial}{\partial z_{1\_1}^{(1)}}[z_{1\_1}^{(n)} - z_{1\_2}^{(n)}] & \frac{\partial}{\partial z_{1\_1}^{(2)}}[z_{1\_1}^{(n)} - z_{1\_2}^{(n)}] & \cdots & \frac{\partial}{\partial z_{1\_1}^{(n)}}[z_{1\_1}^{(n)} - z_{1\_2}^{(n)}] \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

$$\frac{\partial \vec{z}_2}{\partial \vec{z}_{1\_2}} = \begin{pmatrix} \frac{\partial z_2^{(1)}}{\partial z_{1\_2}^{(1)}} & \frac{\partial z_2^{(1)}}{\partial z_{1\_2}^{(2)}} & \cdots & \frac{\partial z_2^{(1)}}{\partial z_{1\_2}^{(n)}} \\ \frac{\partial z_2^{(2)}}{\partial z_{1\_2}^{(1)}} & \frac{\partial z_2^{(2)}}{\partial z_{1\_2}^{(2)}} & \cdots & \frac{\partial z_2^{(2)}}{\partial z_{1\_2}^{(n)}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial z_2^{(n)}}{\partial z_{1\_2}^{(1)}} & \frac{\partial z_2^{(n)}}{\partial z_{1\_2}^{(2)}} & \cdots & \frac{\partial z_2^{(n)}}{\partial z_{1\_2}^{(n)}} \end{pmatrix} = \begin{pmatrix} \frac{\partial}{\partial z_{1\_2}^{(1)}}[z_{1\_1}^{(1)} - z_{1\_2}^{(1)}] & \frac{\partial}{\partial z_{1\_2}^{(2)}}[z_{1\_1}^{(1)} - z_{1\_2}^{(1)}] & \cdots & \frac{\partial}{\partial z_{1\_2}^{(n)}}[z_{1\_1}^{(1)} - z_{1\_2}^{(1)}] \\ \frac{\partial}{\partial z_{1\_2}^{(1)}}[z_{1\_1}^{(2)} - z_{1\_2}^{(2)}] & \frac{\partial}{\partial z_{1\_2}^{(2)}}[z_{1\_1}^{(2)} - z_{1\_2}^{(2)}] & \cdots & \frac{\partial}{\partial z_{1\_2}^{(n)}}[z_{1\_1}^{(2)} - z_{1\_2}^{(2)}] \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial}{\partial z_{1\_2}^{(1)}}[z_{1\_1}^{(n)} - z_{1\_2}^{(n)}] & \frac{\partial}{\partial z_{1\_2}^{(2)}}[z_{1\_1}^{(n)} - z_{1\_2}^{(n)}] & \cdots & \frac{\partial}{\partial z_{1\_2}^{(n)}}[z_{1\_1}^{(n)} - z_{1\_2}^{(n)}] \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 & \cdots & 0 \\ 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -1 \end{pmatrix}$$