Question.2-08

다음 연산에서 $\frac{\partial \vec{z_2}}{\partial \vec{z_1}}$를 구하시오.

$$\vec{z_1} \longrightarrow \boxed{\vec{z_2} = \vec{z_1} \bigcirc \vec{z_1}} \longrightarrow \vec{z_2}$$

이때 ◯은 Hadamard product을 의미하며,

$\vec{z_1} \bigcirc \vec{z_1} = \begin{pmatrix} z_1^{(1)} * z_1^{(1)} \\ z_1^{(2)} * z_1^{(2)} \\ \vdots \\ z_1^{(n)} * z_1^{(n)} \end{pmatrix}$ 로 계산된다.

주어진 연산에서

$$\vec{z_1} = \begin{pmatrix} z_1^{(1)} \\ z_1^{(2)} \\ \vdots \\ z_1^{(n)} \end{pmatrix}, \quad \vec{z_2} = \begin{pmatrix} z_2^{(1)} \\ z_2^{(2)} \\ \vdots \\ z_2^{(n)} \end{pmatrix}$$

라 하면, $\vec{z_2} = \vec{z_1} \bigcirc \vec{z_1}$ 연산은

$$\begin{pmatrix} z_2^{(1)} \\ z_2^{(2)} \\ \vdots \\ z_2^{(n)} \end{pmatrix} = \begin{pmatrix} z_1^{(1)} \times z_1^{(1)} \\ z_1^{(2)} \times z_1^{(2)} \\ \vdots \\ z_1^{(n)} \times z_1^{(n)} \end{pmatrix} = \begin{pmatrix} (z_1^{(1)})^2 \\ (z_1^{(2)})^2 \\ \vdots \\ (z_1^{(n)})^2 \end{pmatrix}$$

이 된다. 따라서 Jacobian matrix $\frac{\partial \vec{z_2}}{\partial \vec{z_1}}$는 다음과 같다.

$$\frac{\partial \vec{z_2}}{\partial \vec{z_1}} = \begin{pmatrix} \frac{\partial z_2^{(1)}}{\partial z_1^{(1)}} & \frac{\partial z_2^{(1)}}{\partial z_1^{(2)}} & \cdots & \frac{\partial z_2^{(1)}}{\partial z_1^{(n)}} \\ \frac{\partial z_2^{(2)}}{\partial z_1^{(1)}} & \frac{\partial z_2^{(2)}}{\partial z_1^{(2)}} & \cdots & \frac{\partial z_2^{(2)}}{\partial z_1^{(n)}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial z_2^{(n)}}{\partial z_1^{(1)}} & \frac{\partial z_2^{(n)}}{\partial z_1^{(2)}} & \cdots & \frac{\partial z_2^{(n)}}{\partial z_1^{(n)}} \end{pmatrix} = \begin{pmatrix} \frac{\partial}{\partial z_1^{(1)}}[(z_1^{(1)})^2] & \frac{\partial}{\partial z_1^{(2)}}[(z_1^{(1)})^2] & \cdots & \frac{\partial}{\partial z_1^{(n)}}[(z_1^{(1)})^2] \\ \frac{\partial}{\partial z_1^{(1)}}[(z_1^{(2)})^2] & \frac{\partial}{\partial z_1^{(2)}}[(z_1^{(2)})^2] & \cdots & \frac{\partial}{\partial z_1^{(n)}}[(z_1^{(2)})^2] \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial}{\partial z_1^{(1)}}[(z_1^{(n)})^2] & \frac{\partial}{\partial z_1^{(2)}}[(z_1^{(n)})^2] & \cdots & \frac{\partial}{\partial z_1^{(n)}}[(z_1^{(n)})^2] \end{pmatrix}$$

$$= \begin{pmatrix} 2z_1^{(1)} & 0 & \cdots & 0 \\ 0 & 2z_1^{(2)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 2z_1^{(n)} \end{pmatrix}$$