Question.2-03

함수 f가 $\vec{\theta}$에 대해 다음과 같이 주어졌을 때, Jacobian matrix $\frac{\partial f(\vec{\theta})}{\partial \vec{\theta}}$를 구하시오.

1) $\vec{\theta} = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix}$ $f(\vec{\theta}) = (\theta_1)^3 - 2(\theta_2)^2 + \theta_1\theta_3$

2) $\vec{\theta} = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix}$ $f(\vec{\theta}) = ln^2(\theta_1) - 3e^{\theta_2\theta_3} + tan(\theta_3)$

1) 주어진 $f(\vec{\theta})$가 $\theta_1, \theta_2, \theta_3$에 대한 함수이므로 Jacobian을 구하기 위해선 $\frac{\partial f(\vec{\theta})}{\partial \theta_1}$, $\frac{\partial f(\vec{\theta})}{\partial \theta_2}$, $\frac{\partial f(\vec{\theta})}{\partial \theta_3}$를 각각 구해야 한다. 이를 계산해보면 다음과 같다.

$$\frac{\partial f(\vec{\theta})}{\partial \theta_1} = \frac{\partial}{\partial \theta_1}[(\theta_1)^3 - 2(\theta_2)^2 + \theta_1\theta_3]$$
$$= \frac{\partial}{\partial \theta_1}[(\theta_1)^3] - 2\frac{\partial}{\partial \theta_1}[(\theta_2)^2] + \frac{\partial}{\partial \theta_1}[\theta_1\theta_3]$$
$$= 3\theta_1^2 + \theta_3$$

$$\frac{\partial f(\vec{\theta})}{\partial \theta_2} = \frac{\partial}{\partial \theta_2}[(\theta_1)^3 - 2(\theta_2)^2 + \theta_1\theta_3]$$
$$= \frac{\partial}{\partial \theta_2}[(\theta_1)^3] - 2\frac{\partial}{\partial \theta_2}[(\theta_2)^2] + \frac{\partial}{\partial \theta_2}[\theta_1\theta_3]$$
$$= -4\theta_2$$

$$\frac{\partial f(\vec{\theta})}{\partial \theta_3} = \frac{\partial}{\partial \theta_3}[(\theta_1)^3 - 2(\theta_2)^2 + \theta_1\theta_3]$$
$$= \frac{\partial}{\partial \theta_3}[(\theta_1)^3] - 2\frac{\partial}{\partial \theta_3}[(\theta_2)^2] + \frac{\partial}{\partial \theta_3}[\theta_1\theta_3]$$
$$= \theta_1$$

따라서 Jacobian은 다음과 같다.

$$\frac{\partial f(\vec{\theta})}{\partial \vec{\theta}} = \left(\frac{\partial f(\vec{\theta})}{\partial \theta_1}, \frac{\partial f(\vec{\theta})}{\partial \theta_2}, \frac{\partial f(\vec{\theta})}{\partial \theta_3} \right)$$
$$= (3\theta_1^2 + \theta_3, -4\theta_2, \theta_1)$$

2) 이와 같은 방법으로 partial derivative를 구해보면

$$\frac{\partial f(\vec{\theta})}{\partial \theta_1} = \frac{\partial}{\partial \theta_1}[ln^2(\theta_1) - 3e^{\theta_2\theta_3} + tan(\theta_3)]$$
$$= \frac{\partial}{\partial \theta_1}[ln^2(\theta_1)] - 3\frac{\partial}{\partial \theta_1}[e^{\theta_2\theta_3}] + \frac{\partial}{\partial \theta_1}[tan(\theta_3)]$$
$$= 2ln(\theta_1) \cdot \frac{\partial}{\partial \theta_1}[ln(\theta_1)]$$
$$= \frac{2}{\theta_1}ln(\theta_1)$$

$$\frac{\partial f(\vec{\theta})}{\partial \theta_2} = \frac{\partial}{\partial \theta_2}\left[\ln^2(\theta_1) - 3e^{\theta_2\theta_3} + \tan(\theta_3)\right]$$

$$= \frac{\partial}{\partial \theta_2}\left[\ln^2(\theta_1)\right] - 3\frac{\partial}{\partial \theta_2}\left[e^{\theta_2\theta_3}\right] + \frac{\partial}{\partial \theta_2}\left[\tan(\theta_3)\right]$$

$$= -3e^{\theta_2\theta_3} \cdot \frac{\partial}{\partial \theta_2}\left[\theta_2\theta_3\right]$$

$$= -3\theta_3 e^{\theta_2\theta_3}$$

$$\frac{\partial f(\vec{\theta})}{\partial \theta_3} = \frac{\partial}{\partial \theta_3}\left[\ln^2(\theta_1) - 3e^{\theta_2\theta_3} + \tan(\theta_3)\right]$$

$$= \frac{\partial}{\partial \theta_3}\left[\ln^2(\theta_1)\right] - 3\frac{\partial}{\partial \theta_3}\left[e^{\theta_2\theta_3}\right] + \frac{\partial}{\partial \theta_3}\left[\tan(\theta_3)\right]$$

$$= -3e^{\theta_2\theta_3} \cdot \frac{\partial}{\partial \theta_3}\left[\theta_2\theta_3\right] + \sec^2(\theta_3)$$

$$= -3\theta_2 e^{\theta_2\theta_3} + \sec^2(\theta_3)$$

따라서 Jacobian은 다음과 같다.

$$\frac{\partial f(\vec{\theta})}{\partial \vec{\theta}} = \left(\frac{\partial f(\vec{\theta})}{\partial \theta_1}, \frac{\partial f(\vec{\theta})}{\partial \theta_2}, \frac{\partial f(\vec{\theta})}{\partial \theta_3}\right)$$

$$= \left(\frac{2}{\theta_1}\ln(\theta_1), -3\theta_3 e^{\theta_2\theta_3}, -3\theta_2 e^{\theta_2\theta_3} + \sec^2(\theta_3)\right)$$