

Question.2-06

$\vec{\theta}, \vec{f}$ 가 다음과 같이 주어지고,

$$\vec{\theta} = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix} \quad \vec{f}(\vec{\theta}) = \begin{pmatrix} f_1(\vec{\theta}) \\ f_2(\vec{\theta}) \\ f_3(\vec{\theta}) \end{pmatrix}$$

$\vec{\theta}$ 에 대한 함수 \vec{f} 는 다음과 같을 때,

$$f_i(\vec{\theta}) = \sin(\theta_i)$$

Jacobian matrix $\frac{\partial \vec{f}(\vec{\theta})}{\partial \vec{\theta}}$ 를 구하시오.

3개의 vector function \vec{f} 를 정리하면

$$\vec{f}(\vec{\theta}) = \begin{pmatrix} f_1(\vec{\theta}) \\ f_2(\vec{\theta}) \\ f_3(\vec{\theta}) \end{pmatrix} = \begin{pmatrix} \sin(\theta_1) \\ \sin(\theta_2) \\ \sin(\theta_3) \end{pmatrix}$$

가 된다. 이때 Jacobian $\frac{\partial \vec{f}(\vec{\theta})}{\partial \vec{\theta}}$ 를 구하면 다음과 같다.

$$\begin{aligned} \frac{\partial \vec{f}(\vec{\theta})}{\partial \vec{\theta}} &= \begin{pmatrix} \frac{\partial f_1(\vec{\theta})}{\partial \theta_1} & \frac{\partial f_1(\vec{\theta})}{\partial \theta_2} & \frac{\partial f_1(\vec{\theta})}{\partial \theta_3} \\ \frac{\partial f_2(\vec{\theta})}{\partial \theta_1} & \frac{\partial f_2(\vec{\theta})}{\partial \theta_2} & \frac{\partial f_2(\vec{\theta})}{\partial \theta_3} \\ \frac{\partial f_3(\vec{\theta})}{\partial \theta_1} & \frac{\partial f_3(\vec{\theta})}{\partial \theta_2} & \frac{\partial f_3(\vec{\theta})}{\partial \theta_3} \end{pmatrix} = \begin{pmatrix} \frac{\partial}{\partial \theta_1} [\sin(\theta_1)] & \frac{\partial}{\partial \theta_2} [\sin(\theta_1)] & \frac{\partial}{\partial \theta_3} [\sin(\theta_1)] \\ \frac{\partial}{\partial \theta_1} [\sin(\theta_2)] & \frac{\partial}{\partial \theta_2} [\sin(\theta_2)] & \frac{\partial}{\partial \theta_3} [\sin(\theta_2)] \\ \frac{\partial}{\partial \theta_1} [\sin(\theta_3)] & \frac{\partial}{\partial \theta_2} [\sin(\theta_3)] & \frac{\partial}{\partial \theta_3} [\sin(\theta_3)] \end{pmatrix} \\ &= \begin{pmatrix} \cos(\theta_1) & 0 & 0 \\ 0 & \cos(\theta_2) & 0 \\ 0 & 0 & \cos(\theta_3) \end{pmatrix} \end{aligned}$$