Math Notes

Similarity and Congruency

Congruency

- Two figures are said to be congruent if they have the same shape & length, regardless of orientation. They are identical to each other in both shape and sizes, in that corresponding sides and angles are the same.
- If two figures are congruent, when one figure is slid, turned or flipped, it covers the the other figure perfectly.
- It was taught to us that congruency symbol was **"≡"** but it is not.
- For safety, write it in words
  Eg. ABC is congruent to A'B'C'

Polygons

- Two sides of the same length are called corresponding sides
- Similarly, two corners with same angles are called corresponding angles.
- Matching vertices are called corresponding vertices.
- When stating a congruence relationship, corresponding vertices must be named in the correct order.
- Example
  Quadrilateral ABCD is congruent to Quadrilateral A'B'C'D'
  Quadrilateral BCDA is congruent to Quadrilateral B'C'D'A'
  Quadrilateral ABCD is NOT congruent to Quadrilateral B'C'D'A'

Tests for congruency of triangles

- SSS
- SAS
- ASA/AAS
- RHS

Test #1) SSS (Side-side-side)

- All three sides of one triangle must be equal to the corresponding sides of the other triangle for them to be congruent to one another.

Test #2) SAS (Side-angle-side)

- Two sides must be equal to the corresponding sides of the other triangle
- The included angle must be equal to the corresponding angle of the other triangle

Math Notes

Test #3) ASA/AAS (Angle-side-angle/angle-angle-side)
- Two angles must be equal to the corresponding angles of the other triangle.
- With two angles, the last angle can be obtained.
- A side must be equal to the corresponding side of the other triangle.

Test #4) RHS (Right-angle Hypotenuse Side)
- The hypotenuse of one right-angled triangle must be equal to the other right-angled triangle's hypotenuse.
- The other side of the right-angled triangle must be equal to the other right-angled triangle's side

Invalid tests for congruency
- AAA (Angle-angle-angle)
  Changing the scale of one triangle keeps the interior angle the same but not the sides. Therefore it is not a test for congruency.
- SSA/ASS (Side-side-angle/Angle-side-side)
  Draw Triangle ABC such that angle BAC = 30 degree, AB = 7cm and BC = 4cm. There are two possible positions for vertex C and therefore it cannot be used as a test for congruency.

Similarity
- Two polygons are similar if all corresponding angles of are equal and the ratios of the corresponding sides are equal.

Test for similarity of triangles
- AA
- SSS (Proportional)
- SAS (Proportional)

Test #1) AA
- If two angles are equal to the corresponding angles of the other triangle, all angles can be determined.
- When all angles are equal to the corresponding angles, they are similar as the sides can increase, proportional to its corresponding sides.

Math Notes

Test #2) SSS (Proportional)

- If the ratios of the corresponding sides are equal, then one triangle is an enlargement of the other.
- The proportion of the corresponding sides is known as the enlargement factor.

Test #3) SAS (Proportional)

- When 2 corresponding sides are proportional and the included angle is equal to the corresponding angle of the other triangle, the shape is fixed.
- However, the scale is not fixed.

Notes

- Remember to state reasoning of equal angles
- Examples
  abc = ade (corr. angles, bc // de)
  bce = dcg (vert. opp. angles)
- Remember to state common angles
- Example
  Angle ABC is common
- Remember to state the test of congruency
- Examples
  ABC is congruent to A'B'C' (SAS)

Area of similar figures

- When two figures are similar, the ratio of their areas is equal to the square of the ratio of any two corresponding lengths of the two figures.
- If A1 and A2 denote the areas of similar figures, and L1 and L2 denote their corresponding lengths, we have
  (A1/A2) = (L1/L2)^2
- Example
  A square of side 2cm and a square of side 6cm
  Ratio of corresponding sides = 2/6
  Ratio of areas = (2/6)^2 = 4/36 = 1/9

Math Notes

Volume of similar figures

- When two solids are similar, the ratio of their volumes is equal to the cube of the ratio of any two corresponding lengths of the two solid.
- Instead of squaring the ratio of two corresponding lengths, cube it.
- If V1 and V2 denote the volumes of similar solids and L1 and L2 denote their corresponding lengths, we have
  (V1/V2) = (L1/L2)^3
- Example
  A cube of side 2 cm and a cube of side 6cm
  Ratio of corresponding length = 2/6
  Ratio of volumes = (2/6)^3 = 8/216 = 1/27