Math Notes

Solving Quadratic Equations

What is a quadratic equation?
- It is in the form $ax^2 + bx + c$ where a cannot be 0.
- There are two answers to the equation
- The highest power of the equation must be 2.

Solving quadratic equations
- The same meaning as getting the roots of the equation.
- You have to find the two values of x

Ways of solving quadratic equations
- Extracting common factors
- Cross Method
- Special rules
- General formula
- Completing the square
- Graphical Method (Will be covered in next chapter)

Method #1) Extracting common factors
- Can only be used when $c = 0$
- When $c = 0$, the equation is $ax^2 + bx = 0$
- After factorising, $ax^2 + bx = 0$ will result in $x(ax+b) = 0$
- To get 0, either or both factors must be 0.
  That means, $x = 0$ or $ax+b = 0$
- Example
  $3x^2 + 2x = 0$
  $x(3x+2) = 0$
  $x = 0$
  OR
  $3x+2 = 0$
  $3x = 2$
  $x = 2/3$

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Method #2) Cross Method

- Click on this hyperlink to review the cross method solution.
- After factorizing, you will get the equation in the form $(x - \alpha)(x - \beta)$ where α and β are the roots.
- Example

  $x^2 + 7x + 12 = 0$

  $(x+3)(x+4) = 0$

  $x+3 = 0$ OR $x+4 = 0$

  $x = -3$ OR $x = -4$

Method #3) Special Rules

- Can only be used if the equation is in either of these three forms:

  1) $d^2 + 2de + e^2$

  2) $d^2 - 2de + e^2$

  3) $d^2 - e^2$

  Note: the variable a and b in the original special rules are changed with d and e to avoid confusion.
- For #1 and #2, in the quadratic equation, $ax^2 + bx + c$, ax^2 must be in the form d^2, c must be in the form e^2 and bx must be in the form 2de.
- Example (#1)

  $x^2 + 2x + 1 = 0$

  $(x+1)^2 = 0$

  $x+1 = 0$

  $x = -1$
- Example (#2)

  $x^2 - 2x + 1 = 0$

  $(x-1)^2 = 0$

  $x-1 = 0$

  $x = 1$
- For #3, in the quadratic equation, ax^2 must be in the form d^2 and c must be in the form e^2
- Example

  $x^2 - 4 = 0$

  $x^2 - 2^2 = 0$

  $(x+2)(x-2) = 0$

  $x+2 = 0$ OR $x-2 = 0$

  $x = -2$ OR $x = 2$

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Method #4) General Formula

- The general formula is in the form of x = (-b±√(b^2 -4ac))/2a
- It can be obtained from changing the subject of the equation ax^2 + bx + c to x.
- Simply substitute the values of a, b and c into the general formula to obtain the value of x. Without the calculator, obtaining the value would be tedious.

Method #5) Completing the square

- The coefficient of x^2 in the expression ax^2 + bx + c must be 1.
- c = (b/2)^2
- Add and subtract (b/2)^2 at the same time to keep the expression the same.
- Complete the square on the LHS and move everything else to the RHS.
- Square root the LHS and RHS. Remember the plus-minus sign.
- If the value of x is square root of a negative value, leave it as it is, as you cannot square root a negative value.
- It uses special rule #1 and #1
- Example (Special rule #1)
  x^2 + 2x -8 = 0
  x^2 + 2x + (2/2)^2 - (2/2)^2 - 8 = 0
  (x + 2/2)^2 - (2/2)^2 - 8 = 0
  (x + 1)^2 = 9
  x + 1 = ±3
  x = -1 + 3 OR x = -1 - 3
  x = 2 OR x = -4
- Example (Special rule #2)
  x^2 - 2x - 8 = 0
  x^2 - 2x + (-2/2)^2 - (-2/2)^2 - 8 = 0
  (x - 2/2)^2 - (-2/2)^2 - 8 = 0
  (x - 1)^2 - 1 - 8 = 0
  (x - 1)^2 = 9
  x - 1 = ±3
  x = 1 + 3 OR x = 1 - 3
  x = 4 OR x = -2