Math Notes

Algebraic Fractions and Manipulation

Special Rules -
1) $(a + b)^2 = a^2 + 2ab + b^2$
2) $(a - b)^2 = a^2 - 2ab + b^2$
3) $(a + b)(a - b) = a^2 - b^2$

The above can be proven by expansion.
WARNING: $(ab)^2 = (a^2)(b^2)$ and not $a^2 + 2ab + b^2$

RECAP

What is an Algebraic Equation?
- An algebraic equation has an equal sign and have two sides, left hand side (LHS) and a right hand side (RHS). The LHS and the RHS have two different variables making it and equation.
- An example would be the equation of a line, $y = mx + c$

What is an Algebraic Expression?
- An algebraic expression has no equal sign meaning it does not equate to anything. However, you can factorize or expand it.
- An example would be the quadratic expression, $ax^2 + bx + c$

Terms
- A variable is an algebraic term that has a value that may vary.
  Examples:
  x^2 and x (Most commonly used)
- Coefficient is the factor in front of a variable.
  Examples:
  Coefficient of $9x^2$ is 9.
  Coefficient of $3x$ is 3.
- Constant is a value that remains constant
  Examples:
  10 and 15

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What is expansion?

- In layman terms, expansion is to remove the brackets by multiplying the factors together.
- Examples,
  $a(b + c) = ab + ac$
  $(a + b)(c + d) = ac + ad + bc + bd$

What is factorization?

- It is the opposite of expansion, forming brackets from equations or expressions.
- Factorization can be done with three methods.

Method #1) Extraction of Common Factors

Expression: $ax + bx$

- To factorize the above expression, one must evaluate and extract common factors.
- In the above case, the common factor of both terms is x.
- $ax + bx = x(a + b)$

Method #2) Grouping

Expression 1: $3a + 3b + ca + bc$

- To factorize the above expression, one must use method #1 before grouping.
- After extracting, group the terms with the same factors.
- Using method #1,
  $3a + 3b + ca + bc = a(3 + c) + b(3 + c)$
- After extracting, group the terms.
  $a(3 + c) + b(3 + c) = (a + b)(3 + c)$

Expression 2: $3a - 3b + ac - bc$

- Using method #1,
  $3a - 3b - ac + bc = 3(a - b) + c(b - a)$
- To obtain the same factor for the above factor, add a negative.
  $3(a - b) + c(b - a) = 3(a - b) - c(a - b)$
- As you can see, you can reverse the order of the subtraction with this method.
  Eg. $(a - b) = -(b - a)$
- Using method #2,
  $3(a - b) - c(a - b) = (3 - c)(a - b)$

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Method #3) Cross Method

Expression: $10x^2 - 7x - 12$

- It is used to factorize quadratic equation.
- Firstly, draw a cross
- Secondly, place the values
- Thirdly, input the values accordingly.
- At the top left section of the cross, values are multiplied downwards to form the values in the bottom left section respectively.
- At the top right section of the cross, values are added downwards to form the value in the bottom right section.
- To obtain the values at the top right section, values in the top left section are multiplied diagonally.

What are algebraic fractions?

- Algebraic fractions are fractions using a variable in the numerator or denominator, such as 1/x. Because division by 0 is impossible, variables in the denominator have certain restriction. For example, in 1/x, x can never be 0, or in 4/abc, neither a, b nor c can be 0.
- Similar to normal fractions, you can simplify fractions by canceling common factors from top and bottom.
- Both numerators and denominators are bracketed, but not shown, so take note when working with the fractions.

Multiplication of Algebraic Fractions

- Similar to normal fractions, when multiplying algebraic fractions, you multiply the numerator and denominator of one fraction with another fraction.
- Example

  (a/c) * (b/d) = a*b/c*d = ab/cd

  (ab/ac) * (c/b) = (b/c) * (c/b) = 1

Division of Algebraic Fractions

- Similar to normal fractions, when dividing algebraic fractions, you take reciprocal for the second fraction and multiply.
- Example

  (a/c) / (b/d) = (a/c) * (d/b) = ad/bc

  Note. b, c and d cannot be 0.

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Simplification of Algebraic Fractions

- You can only cancel when there are common factors.
- The following examples can be simplified
  yx/2x = y/2
  (y^2)(x)/2y = xy/2
- The following examples cannot be simplified
  x/x+y
  y/y+9

Addition of Algebraic Fractions

- To add two algebraic fractions, the common denominator must be found first. The easiest way of obtaining common denominator is multiplying both fraction's denominator with each other.
- The sum is the addition of the numerators over the common denominator.
- An example would be adding (2/(x+1)) to (3/x)
  (2/(x+1)) + (3/x) = (2x/(x^2 + x)) + ((3x + 3)/(x^2 + x)) = (5x + 3)/(x^2 + x)

Subtraction of Algebraic Fractions

- To subtract two algebraic fractions, similar to addition, the common denominator must be found.
- The difference is the subtraction of the numerators over the common denominator.
- An example would be subtracting (3/x) from (2/(x+1))
  (2/(x+1)) - (3/x) = (2x/(x^2 + x)) - ((3x + 3)/(x^2 + x)) = -(x+3)/(x^2 + x)

What is a formula?

- A formula is an equation with only one variable on the LHS and everything else on the RHS.
- The variable on the LHS is the subject of the formula.
- Examples
  In the equation of a straight line, y = mx + c, y is the subject.

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Change of Subject

- Changing of subject is to put the variable on the LHS and everything else on the RHS.
- Example #1) Change the subject of the equation $p = v + t$ to t.
  $p = v + t$
  $t = p - v$
- Example #2) Change the subject of the equation $y^2 = x^2 + 2xy$ to x.
  $10 = x^2 + 2y$
  $x^2 = 10 - 2y$
  $x = \pm\sqrt{(10 - 2y)}$
- Note: when square rooting, make sure a $\pm$ sign is present, as a square may be obtained from squaring a negative and positive value .