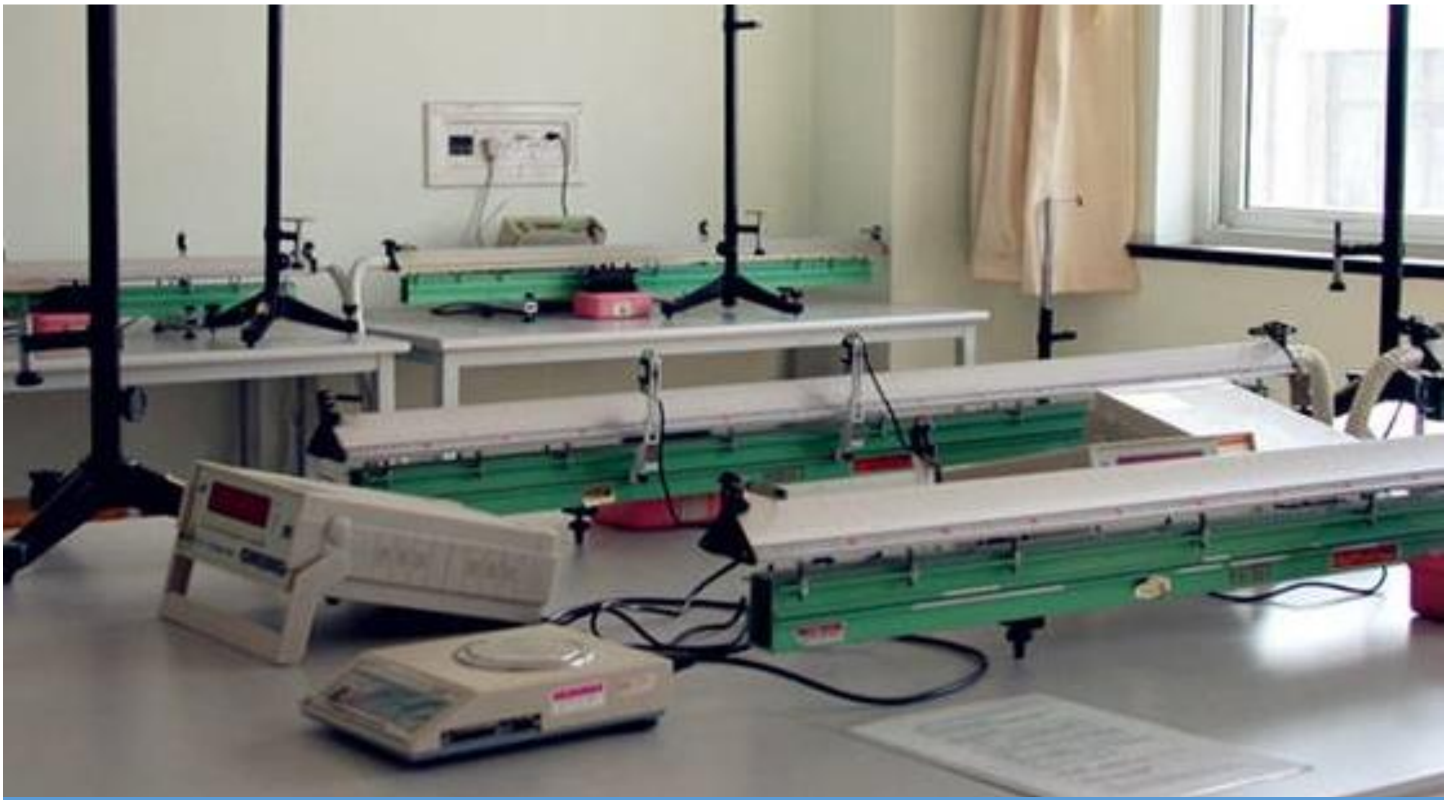

Simple Harmonic Motion

Oscillations in Mechanical Systems

Exercise 3 Report

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I. INTRODUCTION

This experiment will analyze the relationship between the oscillation period and the mass of the oscillator, check whether the oscillation period depends on the amplitude, and examine the relationship between the maximum speed and the amplitude.

According to Hooke's Law, within the elastic limit of deformation, the force F_x needed to be applied in order to stretch or compress a spring by the distance x is proportional to that distance:

$$F_x = kx \tag{1}$$

where k is the spring constant. In a spring-mass system (see Figure 1), where the origin (x = 0) of the coordinate system is set at the equilibrium position of the mass M, we have:

$$M\frac{d^2x}{dt^2} + (k_1 + k_2)x = 0 \tag{2}$$

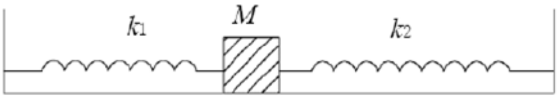

Figure 1: A Spring-Mass System

The general solution to Equation (2) is:

$$x(t) = Acos(\omega_0 t + \varphi_0) \tag{3}$$

where $\omega_0 = \sqrt{\frac{k_1+k_2}{M}}$ is the natural angular frequency of the oscillations, and φ_0 is the initial phase. The natural period of oscillation is:

$$T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{M}{k_1+k_2}} \tag{4}$$

Whenever the mass of the springs cannot be ignored,

$$\omega_0 = \sqrt{\frac{k_1+k_2}{M+m_0}} \tag{5}$$

where m_0 is the effective mass of the spring, which is 1/3 of the actual mass of the spring. In harmonic motions, the elastic potential energy of the system is $U = kx^2/2$ and the kinetic energy is $K = mv^2/2$. At the equilibrium position (x = 0), kinetic energy reaches its maximum K_{max}; at maximum displacement ($x = \pm A$), the potential energy reaches its maximum U_{max}.

In the absence of non-conservative forces, we have:

$$k = \frac{mv_{max}^2}{A^2} \tag{6}$$

II. MEASUREMENT PROCEDURE

A) MEASUREMENT SETUP

To measure the spring constant using the Jolly balance (see Figure 2), it is needed to place the small mirror C in the tube D and make three lines coincide: the line on the mirror, the line on the glass tube and its reflection in the mirror.

A: Sliding bar with metric scale;

H: Vernier for reading;

C: Small mirror with a horizontal line in the middle;

D: Fixed glass tube also with a horizontal line in the middle;

G: Knob for ascending and descending the sliding bar

S: Spring attached to top of the bar A

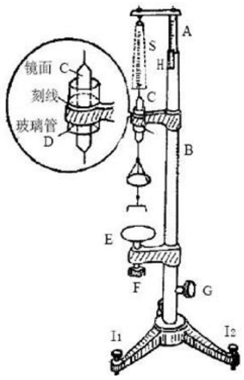

Figure 2: Jolly Balance

First, without adding any weight on the bottom end of the spring, adjust the knob *G* and make the three lines coincide; then read the scale L_1. Second, add mass *m* to the bottom of the spring, adjust knob *G* to make them into one line again and read the corresponding number on scale L_2. The spring constant is:

$$k = \frac{mg}{L_2 - L_1} \tag{6}$$

With a series of measurements, we may estimate the spring constant by linear fitting using the least squares method.

A photoelectric measuring system (see Figure 3) consists of two photoelectric gates and an electronic timer. Please note that for period measurements we use, we use the I-shape shutter. When measuring the speed of the object, we use a U-shaped shutter (see Figure 4), so that the light is blocked twice during a pass. The timer will then record the time interval Δt between the two generated signals. After the distance $\Delta x = \frac{1}{2}(x_{in} + x_{out})$ between the two arms of the U-shape shutter is measured, the speed of the object at the point of passing the gate is calculated as $v = \Delta x/\Delta t$.

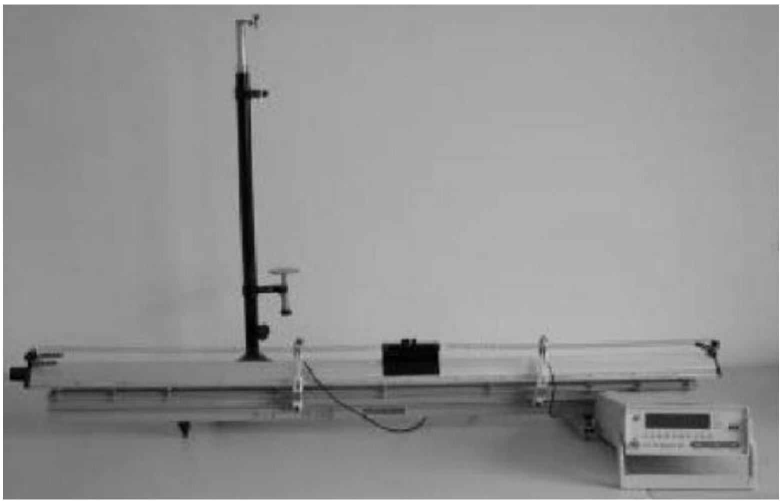

Figure 3: Measurement System Setup

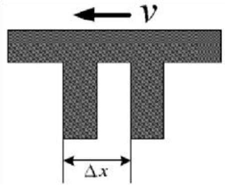

Figure 4: The U-shape Shutter

The precisions of the devices are listed in Table 1.

Measurements	*Precisions*
Jolly Balance (Length)	0.1 [mm]
Electronic Timer (“T” Mode)	0.1 [ms]
Air Track (Length)	0.1 [cm]
Electronic Timer (“S_2” Mode)	0.01 [ms]
Caliper	0.02 [mm]
Electronic Balance	0.01 [g]

Table 1: Precisions of Devices

B) EXPERIMENTAL PROCEDURES

I. Measure the Spring Constant.
   a. Adjust the Jolly balance to be vertical:
      Attach the spring and the mirror as shown in Figure 2. Add a 20 g preload and adjust knob I_1 and I_2 to make sure the mirror can move freely through the tube. Check whether the Jolly balance is parallel to the spring, and adjust knobs if necessary. Take a look at the balance from two orthogonal directions: from one direction, adjust the balance to be parallel to the spring; from the direction orthogonal to the previous one, check if the balance coincides with the spring.

   b. Adjust knob G and make the three lines in the tube coincide. Adjust the position of the tube to set the initial position L_0 within 5.0 ~ 10.0 cm.
   c. Record the reading L_0 on the scale, add mass m_1 and record L_1. Keep adding masses and take measurements for six different positions. The order of the masses should be recorded.
   d. Estimate the spring constant k_1 using the least squares method.
   e. Replace spring 1 with spring 2, repeat the measurements and calculate k_2.
   f. Remove the preload and repeat the measurement for springs 1 and 2 connected in series. Calculate k_3 and compare it with the theoretical value.

II. Find the Relation Between Oscillation Period T and Mass of Oscillator M.
   a. Adjust the air track so that it is horizontal.
      Make sure there is nothing placed on the air track, then turn on the air pump and make sure none of the holes on the air track are blocked. Place the object (cart) on the track without any initial velocity. Adjust the track until the object moves slowly back and forth in both directions.
   b. Measurements for horizontal air track.
      Attach springs to the sides of the cart, and set up the I-shape shutter. Make sure that the photoelectric gate is at the equilibrium position. Add weight m_1. Let the cart oscillate about the photoelectric gate. The amplitude of oscillations should be about 5 cm. Release the cart with a caliper or a ruler. Set the timer into the "T" mode. The timer in this mode will automatically record the time of 10 oscillation periods. Record the mass of the cart and the period. Add more weights and repeat measurements for 5 times. Analyze the relation between M and T by plotting a graph.
   c. Measurements for inclined air track.
      Control the inclination of the air track by placing plastic plates (3 pieces at a time) under the air track. Repeat the steps in (b) for two different inclinations (i.e., with 3 and 6 plates beneath the air track).
      Discuss the relation between M and T by graph plotting.

III. Find the Relation Between Oscillation Period T and Amplitude A.
   a. Keep the mass of the cart unchanged and change the amplitude (choose 6 different values). The recommended amplitude is about 5.0/ 10.0/ 15.0/ ... /30.0 cm.
   b. Linear-fit the data and comment on the relation between the oscillation period T and the amplitude A based on the correlation coefficient γ.

IV. Find the Relation Between the Maximum Speed and the Amplitude.
   a. Measure the outer distance x_{out} and the inner distance x_{in} of the U-shape shutter by a caliper. Calculate the distance $\Delta x = (x_{out} + x_{in})/2$.
   b. Change the shutter from I- to U-shape. Set the timer into the "S_2" mode. Let the cart oscillate. Record the second readings of the time interval Δt only if the two subsequent readings show the same digits to the left of the decimal point.
   c. Change the amplitude (choose 6 different values). The recommended amplitude is about 5.0/ 10.0/ 15.0/ ... /30.0 cm.

d. Measure the maximum speed v_{max} for different values of the amplitude *A*. Obtain the spring constant from Equation (6). Compare this result to that of the first part.

V. Mass measurement

a. Adjust the electronic balance every time before you use it. The level bubble should be in the center of the circle.
b. Add weights according to a fixed order. Weigh the cart with the I-shape shutter and with the U-shape shutter. Measure the mass of spring 1 and spring 2.
c. Record the data only after the circular symbol on the scales display disappears.

III. RESULTS & DATA PRESENTATION

A) SPRING CONSTANT

The spring constant measurement data are presented in Table 2.

Load Mass	Length [mm] ± 0.1 [mm]		
	Spring 1	*Spring 2*	*Spring Series*
0	88.0	96.2	85.0
4.78	107.5	115.9	124.8
9.5	127.8	135.1	165.7
14.29	147.8	156.2	204.9
19	168.7	174.7	245.2
23.8	187.5	195.2	285.6
28.52	208.0	217.0	325.2

Table 2: Spring Constant Measurement Data

By applying linear fit with Excel® 2016, the following length-mass graphs (see Figures 5 - 7) are got.

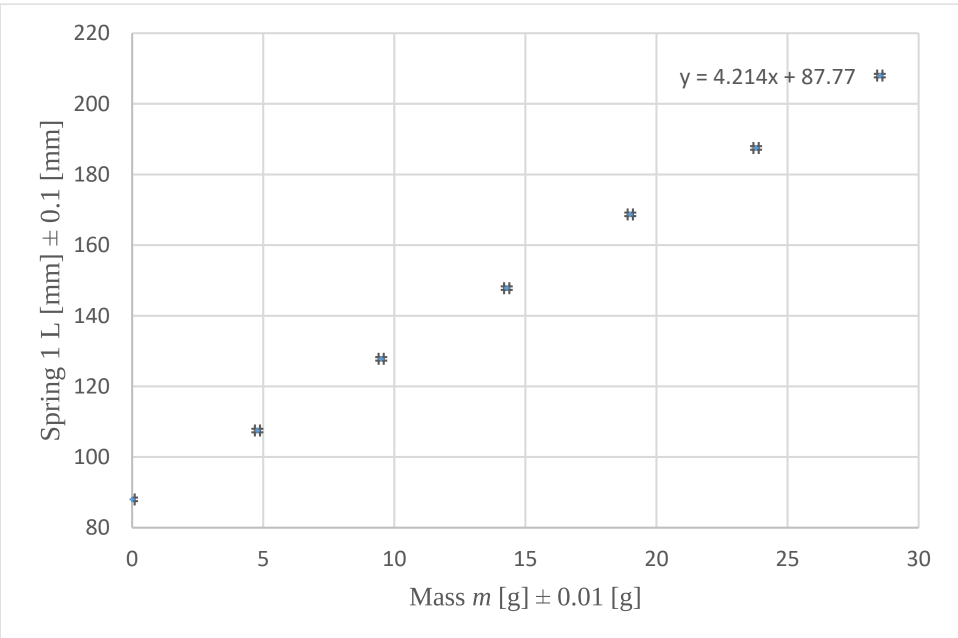

Figure 5: Spring 1

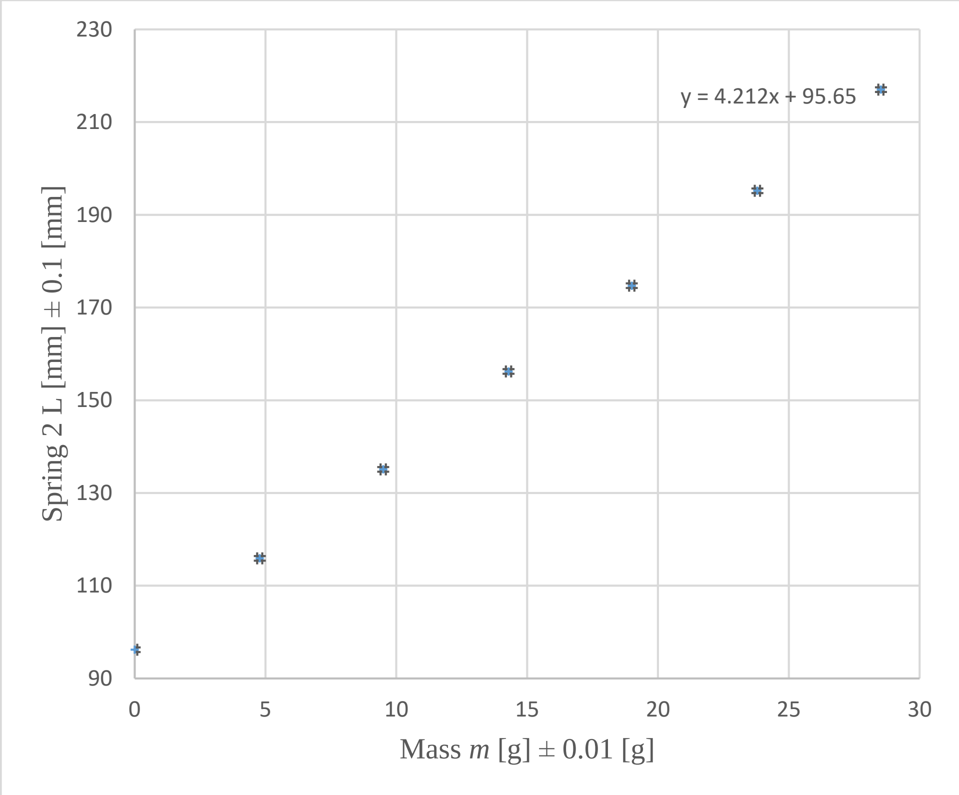

Figure 6: Spring 2

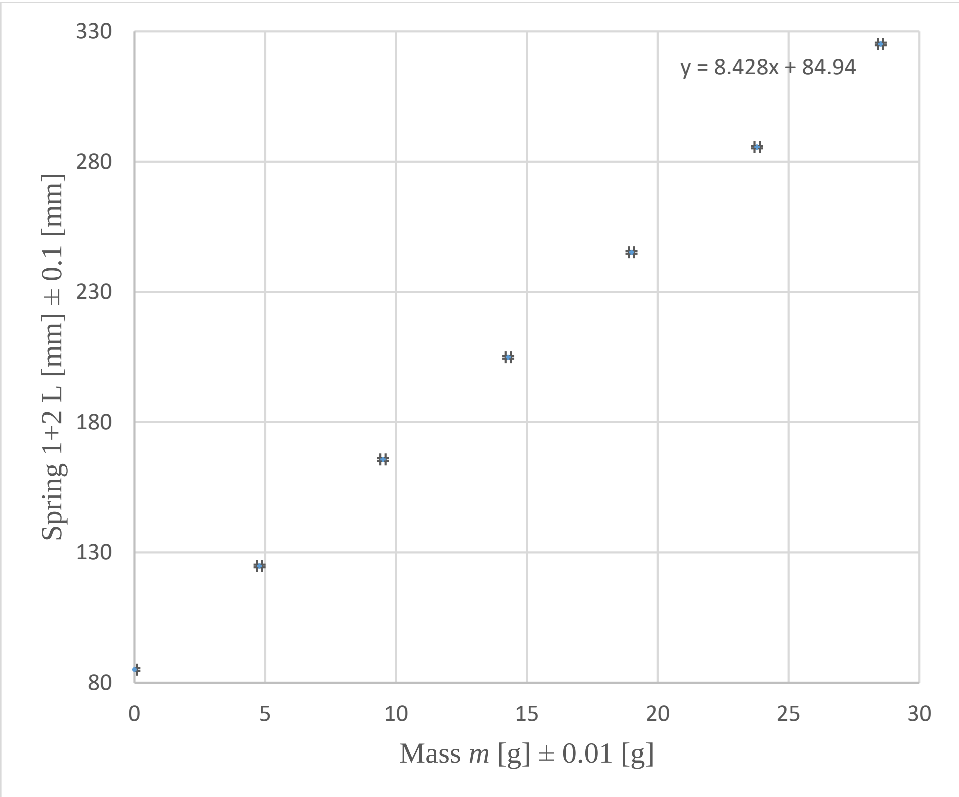

Figure 7: Spring 1+2

For Spring 1, the spring constant is:

$$k_1 = \frac{\Delta m}{\Delta L} g = \frac{9.794\ m/s^2}{4.214\ m/kg} = 2.324 \pm 0.019\ \ [N/kg]$$

Similarly, we get $k_2 = 2.325 \pm 0.019\ \ [N/kg]$, and $k_{1+2} = 1.162\ \pm 0.016[N/m]$. We thus can check that:

$$\frac{1}{\frac{1}{k_1}+\frac{1}{k_2}} = k_{1+2} = 1.162\ [N/m]$$

B) *T-M* RELATION

By applying linear fit with Excel® 2016, the following period-mass graphs (see Figures 8 - 10) are got (all the masses include the effective mass of the spring; the data on horizontal axis indicate the square roots of the masses, see Table 3).

Masses	m_1	m_2	m_3	m_4	m_5	m_6
Original Value [g] ± 0.01 [g]	190.87	195.59	200.38	205.09	209.89	214.61
Rooted Value [$\sqrt{g}$] ± 0.01 [$\sqrt{g}$]	13.816	13.985	14.156	14.321	14.488	14.650

Table 3: Square Roots of Masses

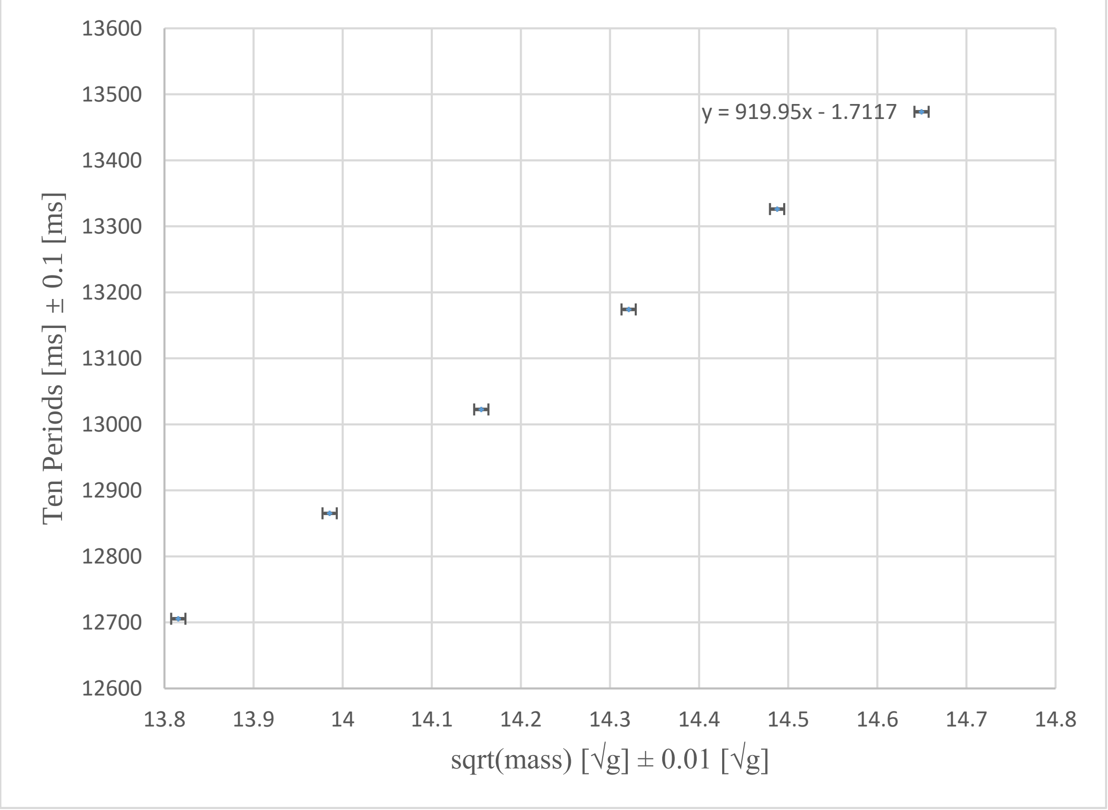

Figure 8: T-M Graph (Horizontal Air Track)

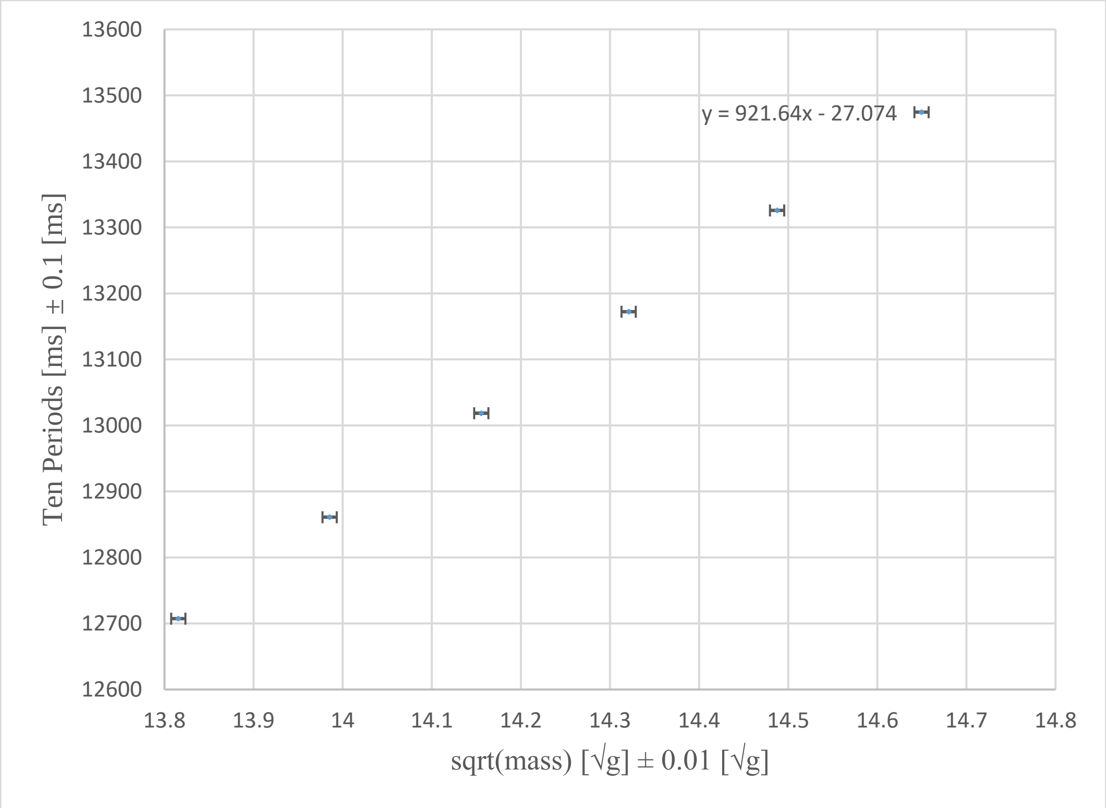

Figure 9: T-M Relation (Inclined Air Track 1)

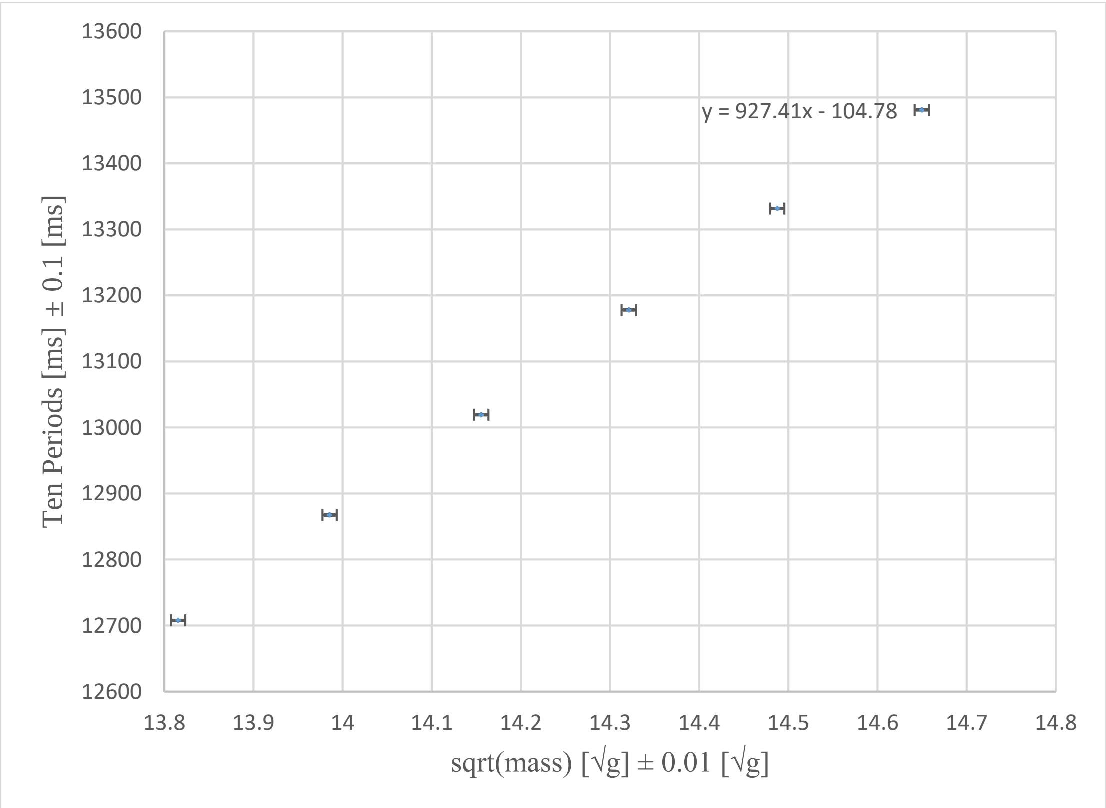

Figure 10: T-M Relation (Inclined Air Track 2)

It is crystal clear that $T \propto \sqrt{M}$, and that the inclination has no significant impact on the period. When plugging in $k_1 = 2.324\ N/m$ and $k_2 = 2.325\ N/m$, we can get:

$$\frac{\Delta T}{\Delta\sqrt{m}}\sqrt{k_1 + k_2} = \frac{920ms/\sqrt{g}}{10}\sqrt{(2.324 + 2.325)kg/s^2} = 6.28 \pm 0.02 \approx 2\pi$$

C) *T-A* RELATION

By applying linear fit with Excel® 2016, we can graph the *T-A* map as is shown in Figure 11.

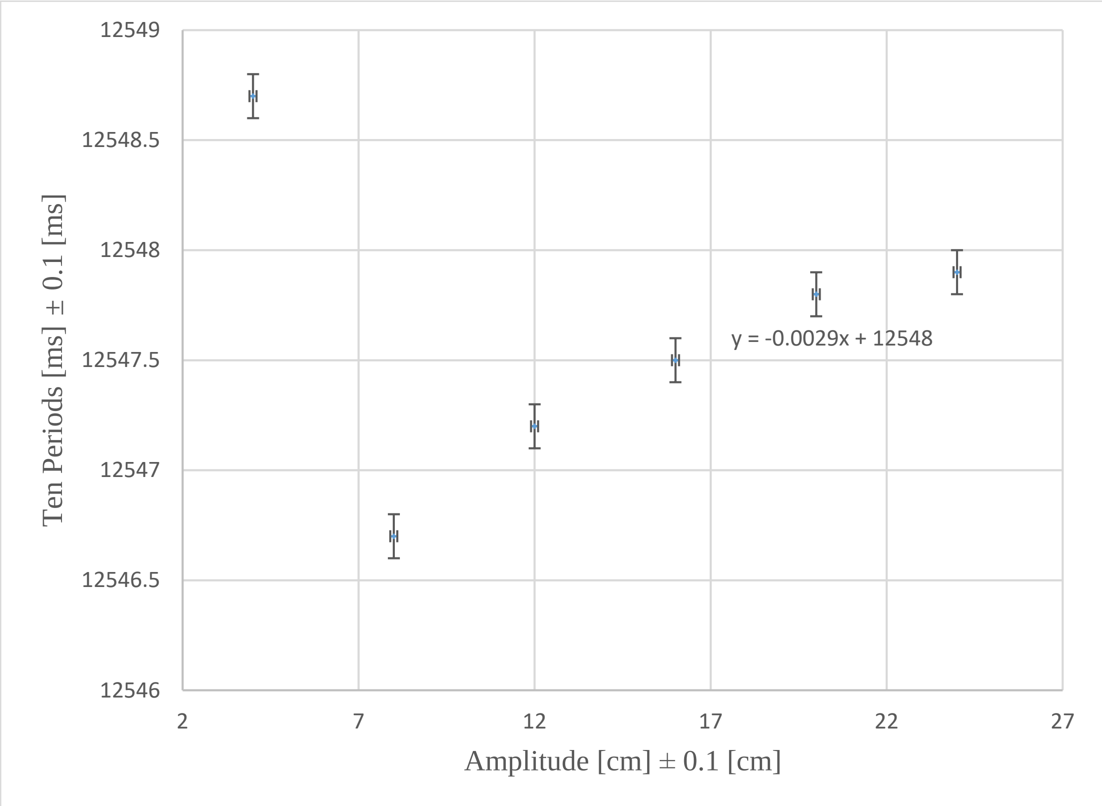

Figure 11: T-A Relation

It is clear that the period and the amplitude do not have any significant correlation.

D) $v_{max}^2 - A^2$ RELATION

Using the formula $\Delta x = (x_{out} + x_{in})/2$, the distance is calculated (see Table 4).

x_{in} [mm] ± 0.02 [mm]	x_{out} [mm] ± 0.02 [mm]	Δx [mm] ± 0.02 [mm]
5.10	15.00	10.05
5.04	15.12	10.08
5.06	15.08	10.07

Table 4: The Distance

For example, Δx_1 =(5.10+15.00)/2=10.05 [mm]. We can therefore get the average value:

$$\overline{\Delta x} = (10.05 + 10.08 + 10.07)/3 = 10.07 \text{ [mm]} \pm 0.04 \text{ [mm]}$$

Divide the distance by time to obtain the squared maximum instantaneous speed:

$$v_1^2 = \left(\frac{\overline{\Delta x}}{\Delta t_1}\right)^2 = \left(\frac{10.07mm}{53.89ms}\right)^2 = 0.03489\ [m^2/s^2]$$

Similarly, we can get the other speeds (see Table 5):

Δt [ms] ± 0.01[ms]	53.89	26.38	17.58	13.19	10.62	8.87
$v_{max}^2\ [m^2/s^2]$	0.03489	0.1456	0.3279	0.5825	0.8985	1.288
$A^2\ [m^2]$	0.0016	0.0064	0.0144	0.0256	0.0400	0.0576

Table 5: $\mathrm{v}_{\mathrm{max}}^2 - \mathrm{A}^2$ Relation

By applying linear fit with Excel® 2016, we can graph the $v_{max}^2 - A^2$ map as is shown in Figure 12.

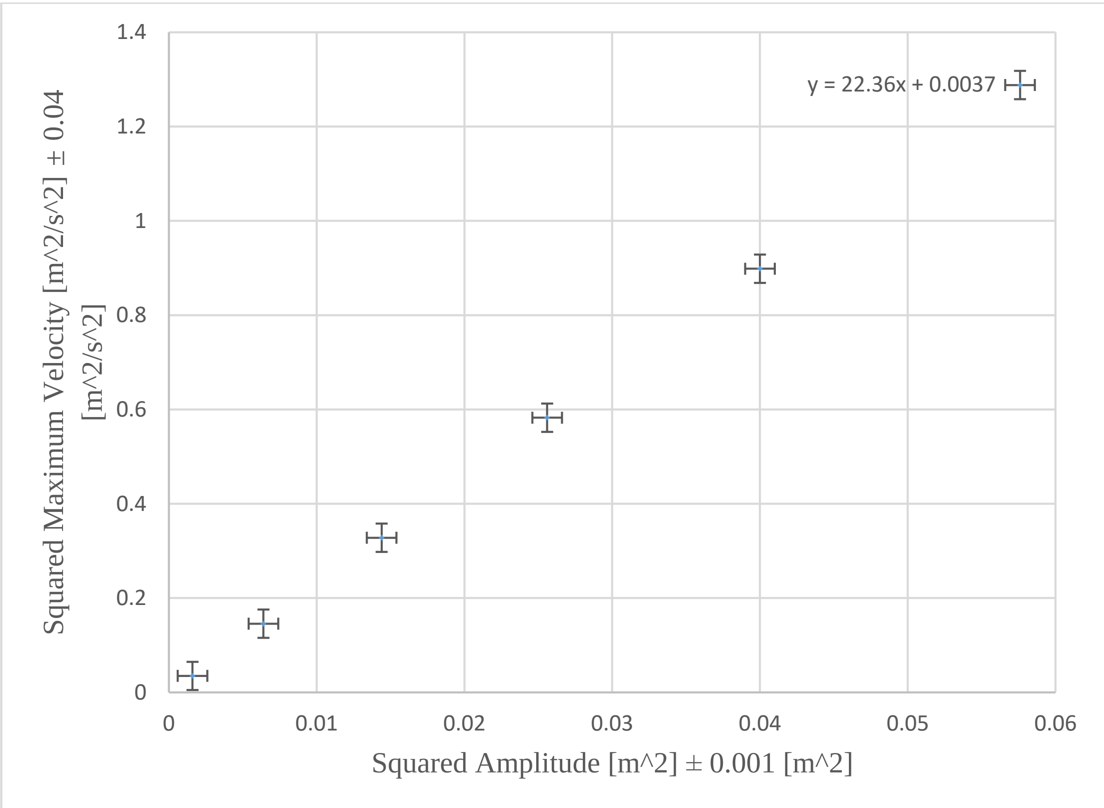

Figure 12: $\mathrm{v}_{\mathrm{max}}^2 - \mathrm{A}^2$ Relation

Hence, it is clear that $v_{max}^2 \propto A^2$, and we can check that:

$$\frac{\Delta v_{max}^2}{\Delta A^2} = 22.36\ [s^{-2}] \approx \frac{k_1+k_2}{M_{U-Shape}} = \frac{(2.324+2.325)kg/s^2}{197.58g} = 23.53\ [s^{-2}]$$

IV. UNCERTAINTY ANALYSIS & CALCULATIONS

In the 8 graphs above (see Figures 5-12), the uncertainties are given by Excel® 2016 Data Analysis Toolbox (see Table 6).

Figure	*Uncertainty on Horizontal Axis*	*Uncertainty on Vertical Axis*	*Standard Error of Slope*	*Uncertainty Result (u)*	*Relative Uncertainty (u_r)*
5	0.01 [g]	0.1 [mm]	0.0205	Spring Constant k_1 2.324±0.019 [N/m]	0.8%
6	0.01 [g]	0.1 [mm]	0.0199	Spring Constant k_2 2.325±0.019 [N/m]	0.8%
7	0.01 [g]	0.1 [mm]	0.0171	Spring Constant k_{1+2} 1.162±0.016 [N/m]	1.3%
8	0.01 [$\sqrt{g}$]	0.1 [ms]	2.77	$\frac{\Delta 10T}{\Delta\sqrt{m}}$=920±3[ms/$\sqrt{g}$]	0.3%
9	0.01 [$\sqrt{g}$]	0.1 [ms]	2.00	$\frac{\Delta 10T}{\Delta\sqrt{m}}$=922±2[ms/$\sqrt{g}$]	0.2%
10	0.01 [$\sqrt{g}$]	0.1 [ms]	3.35	$\frac{\Delta 10T}{\Delta\sqrt{m}}$=927±3[ms/$\sqrt{g}$]	0.3%
11	0.1 [ms]	0.1 [cm]	0.045	(Not Available) Irrelevant Variables	
12	0.001[m^2]	0.04[m^2/s^2]	0.094	$\frac{\Delta v_{max}^2}{\Delta A^2} = 22.36 \pm 0.09\ [s^{-2}]$	0.4%

The standard deviation of distance $\triangle x$ can be calculated as below:

$$STDEV_x = \sqrt{\frac{1}{n-1}\sum(x_i - \bar{x})} = 0.0153\ [mm] \tag{8-1}$$

Since n=3, $\frac{t_{0.95}}{\sqrt{n}} = 2.48$, the type-A uncertainty of the distance is:

$$\Delta_A = STDEV_x \cdot \frac{t_{0.95}}{\sqrt{n}} = 0.0379[mm] \tag{8-2}$$

Hence, the total uncertainty of the distance is:

$$\mathrm{u} = \sqrt{\Delta_A^2 + \Delta_B^2} = \sqrt{0.0379^2 + 0.02^2} = 0.04[mm] \tag{9}$$

and the relative uncertainty of the distance is:

$$u_r = \frac{u}{\bar{s}} \times 100\% = 0.4\% \tag{10}$$

V. CONCLUSIONS & DISCUSSIONS

The inaccuracies of this experiment come from: (1) the limitation of experimental equipment; (2) the difference between real spring and ideal spring; (3) the deformation when measuring the distance $\triangle x$; (4) the distance is not “zero,” so the instantaneous speed is not completely accurate; (5) the masses are placed by hand, so sweat may influence the mass; (6) slight friction (air drag) exerted on the cart and masses.

By comparison with theory, the results obtained are generally satisfying. As is shown in the "Results & Data Presentation" part, we validate the spring constant formula $\frac{1}{\frac{1}{k_1}+\frac{1}{k_2}} = k_{1+2}$; also validate the relations $T \propto \sqrt{M}$, and $v_{max}^2 \propto A^2$. It is found that period and amplitude are irrelevant.

To improve this experiment, I have the following suggestions:

- Use calibrated force sensor to detect the tension in the springs. This method can result in better precision on the spring constants. Also, the vibration of the spring will be greatly reduced, because the tension can now be changed continuously, so the waiting time can be saved.
- Instead of using a ruler (which may still have normal velocity due to the tremor on hand) to release the cart, design a set of equipment like electromagnet relay to make sure that the cart has zero initial velocity. Fix this equipment to the table, then it can also make sure that the amplitude will not change during the *T*-*M* relation experiment.
- To avoid air drag and friction, replace the air track with a vacuum tube with magnetic levitation, and replace the cart with maglev train, such that the train can float like Elon Musk's "Hyperloop."
- Use springs that are new, such that the possibility of fatigue and uneven *k* can be reduced.
- Make the "U-shape" aptly smaller to make the instantaneous speed more precise.
- Change the conditions of this experiment (such as the mass of the object) and redo the experiment for several cycles to validate the reliability of the relations we obtained and to avoid the contingency.