Measurement of Fluid Viscosity

Exercise 2 Report by Fenglei Gu

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I. INTRODUCTION

For a spherical object with radius R moving at speed v in an infinite volume of a liquid, the magnitude of the drag force is usually modeled as:

$$F_1 = 6\pi\eta vR \tag{1}$$

When a spherical object falls vertically downwards in a fluid, it is being acted upon by the following three forces: the viscous force $\mathbf{F}_1$ and the buoyancy force $\mathbf{F}_2$ both act upwards, and the weight of the object $\mathbf{F}_3$ is directed downwards. The magnitude of the buoyancy force is:

$$F_2 = \frac{4}{3}\pi R^3 \rho_1 g \tag{2-1}$$

where ρ_1 is the density of the fluid and g is the acceleration due to gravity. The weight of the object is:

$$F_3 = \frac{4}{3}\pi R^3 \rho_2 g \tag{2-2}$$

with ρ_2 being the density of the object. At equilibrium (terminal status):

$$F_3 = F_1 + F_2 \tag{2-3}$$

such that

$$\eta = \frac{2}{9} g R^2 \frac{\rho_2 - \rho_1}{v_t} \tag{3}$$

where v_t is the terminal speed. Equation (3) is equivalent to:

$$\eta = \frac{2}{9} g R^2 \frac{(\rho_2 - \rho_1)t}{s} \tag{4}$$

where s is the distance traveled in time t after reaching the terminal speed. Since the volume of the fluid used in the measurement is not infinite, Equation (1) should be modified as:

$$F_1 = 6\pi\eta vR\left(1 + 2.4\frac{R}{R_c}\right) \tag{1'}$$

where R_c is the radius of the container. Consequently, Equation (4) reads:

$$\eta = \frac{2(\rho_2 - \rho_1)t}{9s\left(1 + 2.4\frac{R}{R_c}\right)} g R^2 \tag{5}$$

Since the length L of the container is limited, there may be further corrections introduced, depending on the ratio on R_c/L.

II. MEASUREMENT PROCEDURE

A) MEASUREMENT SETUP

Figure 1 shows a Stokes' viscosity measurement device filled with castor oil in which motion of small metal balls will be observed.

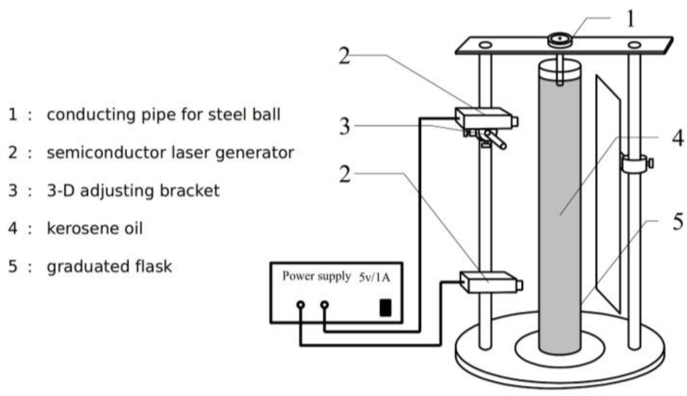

Figure 1: *Stokes' viscosity measurement apparatus*. SJTU, 2016.

Other measurement devices and respective precisions are listed in Table 1.

Devices	*Precisions*
Ruler	1 [mm]
Micrometer	0.001 [mm]
Caliper	0.01 [mm]
Densimeter	0.0001 [g/cm^3]
Electronic scale	0.001 [g]
Stopwatch	0.01 [s]
Thermometer	1 [℃]

Table 1: Other Devices and Respective Precisions.

B) EXPERIMENTAL PROCEDURES

1. Adjust Stokes' viscosity measurement device.
   a. Adjust the knobs beneath the base to make the plumb aiming at the center of the base.
   b. Turn on the two lasers, adjust the beams so that they are parallel and aim at the plumb line.
   c. Remove the plumb and place the graduated flask with castor oil at the center of the base.
   d. Place the guiding pipe on the top of the viscosity measurement device.
   e. Put a metal ball into the pipe and check whether the ball can block the laser beams. If not, repeat Step a.
2. Measure the terminal velocity of a falling ball.

a. Measure the vertical distance s between the two laser beams at least three times.

b. Put a metal ball into the guiding pipe. Start the stopwatch when the ball passes through the first beam, and stop it when it passes through the second one. Record the time t and repeat the procedure for at least 6 times.

3. Measurement of the ball density ρ_2.

a. Use electronic scales to measure the mass of 40 metal balls. Calculate the average to find the mass of a single ball.

b. Use a micrometer to measure the diameter of the metal balls. Repeat for ten times and calculate the average value.

c. Calculate the ball density ρ_2.

4. Final Calculations

a. Measure the density ρ_1 of the castor oil by using the provided densimeter (one measurement).

b. Use a caliper to measure the inner diameter D of the graduated flask for six times.

c. Read the ambient temperature from the thermometer placed in the lab.

d. Calculate the value of viscosity coefficient η using Equation (5).

III. RESULTS & DATA PRESENTATION

The data collected in this lab is shown below. Table 2 shows the distance measurement data. The average is calculated as:

$$\bar{S} = \frac{1}{n}\sum S_i = \frac{1}{3}(155.7 + 156.2 + 156.5) = 156.1\ [mm] \tag{6}$$

Measurements	*Distance S* [mm] ± 0.10 [mm]
1	155.7
2	156.2
3	156.5
Average	156.1

Table 2: Distance Measurement Data

Similarly, the average of time (see Table 3), diameter of the ball (see Table 4), diameter of the flask (see Table 5) is calculated as followings.

Measurements	*Time t* [s] ± 0.01 [s]
1	7.19
2	7.34
3	7.21
4	7.02
5	7.13
6	7.23

Average	7.19

Table 3: Time Measurement Data

Measurements	*Diameter d* [mm] ± 0.004 [mm]
1	2.000
2	1.985
3	1.995
4	2.005
5	2.001
6	2.001
7	2.003
8	1.999
9	1.998
10	2.005
Average	1.999

Table 4: Measurement Data for Balls

Measurements	*Diameter D* [mm] ± 0.02 [mm]
1	62.00
2	62.06
3	61.68
4	61.60
5	62.04
6	62.06
Average	61.91

Table 5: Measurement Data for the Flask

The density of balls can thus be calculated:

$$\rho_2 = \frac{m}{40\times\frac{1}{6}\pi d^3} = \frac{1.310\times10^{-3}}{40\times\frac{1}{6}\pi\times(1.999\times10^{-3})^3} = 7.830\times10^3\ [kg/m^3] \quad (7)$$

By uncertainty analysis (see Table 7), the following results are got (see Table 6):

Measurements	*Results*
S	(156.1±1.0) mm
t	(7.19±0.11) s
d	(1.999±0.006) mm
D	(61.9±0.2) mm

Table 6: Results of Measurements

Hence the viscosity can be estimated using Equation 5:

$$\eta = \frac{2(\rho_2-\rho_1)tgR^2}{9s(1+2.4\frac{R}{R_c})} = \frac{2\times(7.830-0.9560)\times10^3\times7.19\times9.794\times(1.999\times10^{-3})^2}{9\times0.1561\times(1+2.4\times\frac{1.999}{61.9})} = 2.53\ [kg\cdot m^{-1}\cdot s^{-1}]$$

IV. UNCERTAINTY ANALYSIS & CALCULATIONS

The standard deviation of distance S can be calculated as below:

$$STDEV_S = \sqrt{\frac{1}{n-1}\sum(S_i - \bar{S})} = 0.4041\ [mm] \tag{8-1}$$

Since n=3, $\frac{t_{0.95}}{\sqrt{n}} = 2.48$, the type-A uncertainty of the distance is:

$$\Delta_A = STDEV_S \cdot \frac{t_{0.95}}{\sqrt{n}} = 1.002\ [mm] \tag{8-2}$$

Hence, the total uncertainty of the distance is:

$$\mathrm{u} = \sqrt{\Delta_A^2 + \Delta_B^2} = \sqrt{1.002^2 + 0.10^2} = 1.0[mm] \tag{9}$$

and the relative uncertainty of the distance is:

$$u_r = \frac{u}{\bar{S}} \times 100\% = 0.6\% \tag{10}$$

Similarly, the uncertainties of other measurements can be calculated (see Table 6).

Measurements	Δ_A	Δ_B	u	u_r
S	1.002 [mm]	0.10 [mm]	1.0 [mm]	0.6%
t	0.112 [s]	0.01 [s]	0.11 [s]	1.5%
d	4.194×10^{-3} [mm]	4×10^{-3} [mm]	6×10^{-3} [mm]	0.3%
D	0.2197 [mm]	0.02 [mm]	0.2 [mm]	0.3%
ρ_1	/	1×10^{-4} [g/cm^3]	1×10^{-4} [g/cm^3]	0.01%
m	/	1×10^{-3} [g]	1×10^{-3} [g]	0.08%
T	/	0.5 K	0.5 K	0.2%

Table 7: Uncertainties

V. CONCLUSIONS & DISCUSSIONS

By comparison with the theoretical value[1] of castor oil's viscosity 0.985 $[kg \cdot m^{-1} \cdot s^{-1}]$ at 298.15 K and 100kPa, the experimental result 2.53 $[kg \cdot m^{-1} \cdot s^{-1}]$ is relatively high. There are two main reasons. First, the room temperature and atmosphere pressure in the lab is different from that of the reference book. Secondly, castor oil is mixture rather than pure substance, so the viscosity may vary.

[1] See Lide, D. R., ed. (2005). *CRC Handbook of Chemistry and Physics* (86th ed.). Boca Raton (FL): CRC Press.

With regard to the sources of inaccuracies, there may remain some tiny air bubbles attached on the balls, such that the buoyancy force $\mathbf{F}_2$ increases a lot. In addition, the time t is measured by human-controlled stopwatch, which is not very accurate.

Nevertheless, all data collected and results calculated in this experiment are within expectation, considering the limited equipment used in this experiment.

To improve this experiment, I have the following suggestions:

- the time t should be measured by a laser photoelectric timer, which measures the time when the laser light bunch is blocked by the ball;
- between different measurements (t_1, t_2, etc.) of time, wait for enough time before proceeding on to next measurement, such that the vortex and turbulent flows produced by the previous ball can be reduced;
- provide a thermostatic apparatus to the flask, such that the potential temperature change happened during the experiment can be avoided;
- taking the Reynolds number (R_e) into account, i.e., make secondary amendment by redoing the calculations replacing the Equation (1') with

$$F_1 = 6\pi\eta v R\left(1 + \frac{3}{16}R_e - \frac{19}{1080}R_e^2\right)\left(1 + 2.4\frac{R}{R_c}\right) \tag{11}$$

where $R_e = \frac{\rho_1 s R_c}{t\eta}$ is calculated by the data got in the first round of calculations.