Measurement of the Speed of Sound

VP160 Physics Lab Exercise 4 Report

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I. INTRODUCTION

Sound is a mechanical wave that propagates through a compressible medium. Sound with the frequency higher than 20000 Hz is called *ultrasound*. In this experiment an ultrasonic wave is chosen as the signal source, because its wavelength is short enough to measure the speed of sound precisely. The phase speed v, the frequency f and the length λ of a wave are related by the formula

$$v = \lambda f \tag{1}$$

For motion with constant speed v along a straight line, we have

$$v = \frac{L}{t} \tag{2}$$

where L is the distance traveled over time t. Below are three methods for measuring the speed of sound.

- **Resonance Method**
  Two piezoelectric transducers S_1 and S_2 (see Figure 2) are parallel placed a distance L from each other, receiving as well as reflecting the sound waves. If

$$L = \mathrm{n}\frac{\lambda}{2} \tag{3}$$

  where n is an integer, then standing waves will form, and maximum output power will be observed in the oscillography (see Figure 1).

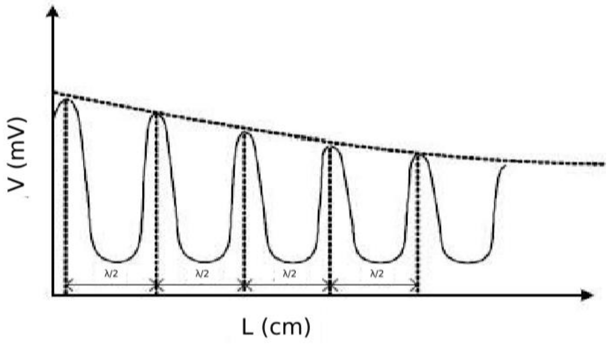

Figure 1: Relationship between the signal voltage and the distance between the transducers. SJTU, 2018.

  After the position corresponding to each maximum is measured, it is easy to find the wavelength. Since the frequency f is displayed directly on the signal generator, it is easy to calculate the speed of sound by Equation (1).

- **Phase-comparison method**

  If the phase of the wave at two points on the wave propagation direction is equal, then the distance between these points L has to be a multiple of the wavelength, *i.e.*

$$L = n\lambda \tag{4}$$

  where n is an integer. Lissajous figures are used to identify the values of L. When the two superimposed harmonic motions have identical frequency $\omega_x = \omega_y$ and phase difference $|\varphi_x - \varphi_y| = n\pi$, where n is an integer, the Lissajous figure will show as a straight line. For other values of the phase difference the figures will have an elliptical shape.

- **Time-difference method**

When an ultrasonic pulse signal emitted by S_1 arrives at S_2, it is received and returned back to the processor. By contrasting the original signal with the received one, one can measure the time needed for the sound to travel from S_1 to S_2 over a distance of L. When the values of L and t are known, the phase speed of sound can be found from Equation (2).

II. MEASUREMENT SETUP

The device set-up is shown as the figure below (see Figure 2).

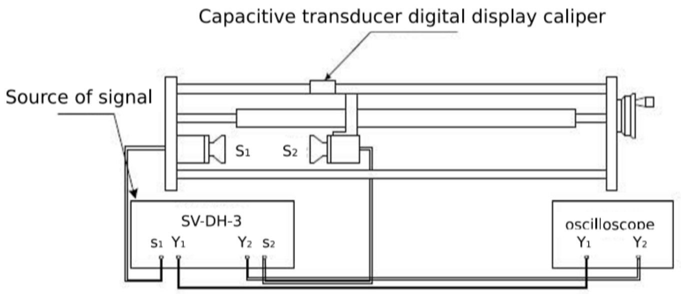

Figure 2: Experimental setup, consisting of a signal source, two piezoelectric transducers S_1 and S_2, and oscilloscope. SJTU, 2018.

The chart below shows the uncertainties of the devices (see Table 1).

Table 1: The uncertainty of the devices.

Devices	*Uncertainty*
Source of signal	0.001 [kHz]
Caliper	0.001 [mm]
Cursor function (oscilloscope)	0.2 [μs]
Thermometer	1 [°C]

III. EXPERIMENTAL PROCUDURES

A) RESONANCE METHOD

1. The initial distance between S_1 and S_2 is set at around 1 cm. Then the signal source and the oscilloscope are turned on.
2. The following options are then set on the panel of the signal source.
   (1) The *Continuous Wave* is chosen for *Method*.
   (2) Adjust *Signal Strength* until a 10 V peak voltage is observed on the oscilloscope.

(3) The *Signal Frequency* is set at 35.659 kHz with uncertainty of 0.001 kHz, such that the peak-to-peak voltage reaches its maximum. The number is recorded on the data sheet.

3. Increase L gradually by moving S_2, and observe the output voltage of S_2 on the oscilloscope. The position of S_2 as L_2 when the output voltage reaches a maximum is recorded on the data sheet.
4. Repeat step 3 to collect 12 values of L_2 and calculate v. All data are recorded on the data sheet.

B) PHASE-COMPARISON METHOD

1. The Lissajous figures are used to observe the phase difference between the transmitted and the received signals. The S_2 is moved such that we can record the position when the Lissajous figure becomes a straight line with the same slope.
2. Step 1 is repeated, in order to collect 12 sets of data. All data are recorded in the data sheet.

C) TIME-DIFFERENCE METHOD (LIQUID)

1. Switch to the device with water as the medium.
2. The mode *Pulse Wave* is chosen for *Method* on the panel of the signal source.
3. Adjust the frequency to 100 Hz and the width to 500 μs.
4. The cursor function of the oscilloscope is used to measure the time and the distance between the starting points of neighboring periods. All 12 pairs of data are recorded on the data sheet and then v_{water} is calculated.

IV. RESULTS & DATA PRESENTATION

All the data collected are shown as below (see Tables 2 & 3).

Table 2: Data for the Resonance Method and the Phase-comparison Method.

n	$L_{\text{Resonance}}$ [mm]±0.001[mm]		$L_{\text{Phase-comparison}}$ [mm]±0.001[mm]
0	118.530		118.089
1	123.700		127.802
2	127.870		137.585
3	133.420		148.449
4	138.341		158.256
5	143.319		167.249
6	148.284		178.132
7	153.180		186.732
8	158.229		196.607
9	163.020		206.400
10	167.895		216.205
11	172.910		226.790

Table 3: Data for the Time-difference Method.

n	t [ms]±0.2[μs]	$L_{\text{Time-difference}}$ [mm]±0.001[mm]
0	76.0	100.559
1	82.4	113.800
2	86.8	120.000
3	90.8	125.000
4	94.0	130.000
5	97.2	135.000
6	100.4	140.000
7	103.6	145.000
8	106.8	150.000
9	112.8	158.950
10	118.4	167.400
11	126.0	178.700

By applying linear-fit on Microsoft Excel 2016®, the following figures are got (see Figures 3 - 5). Tables of fitting parameters are shown in the Part 5 (Uncertainty and Calculations) of this report.

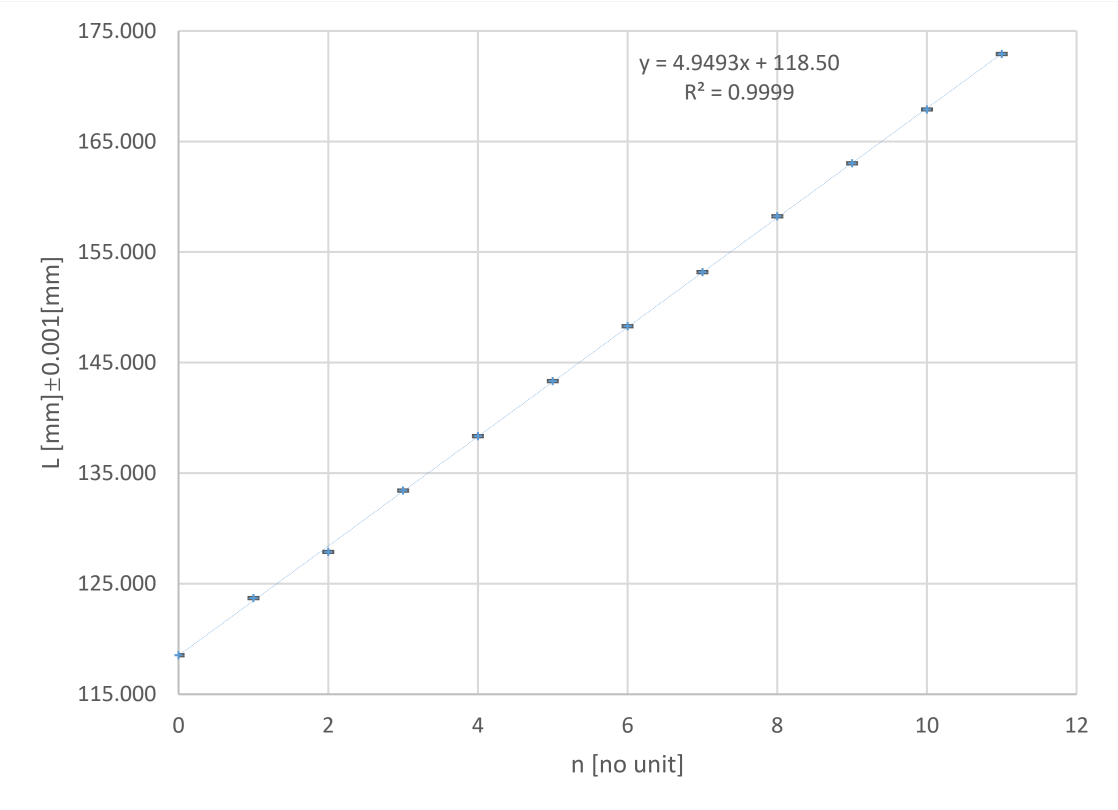

Figure 3: Linear-fit for the Resonance Method.

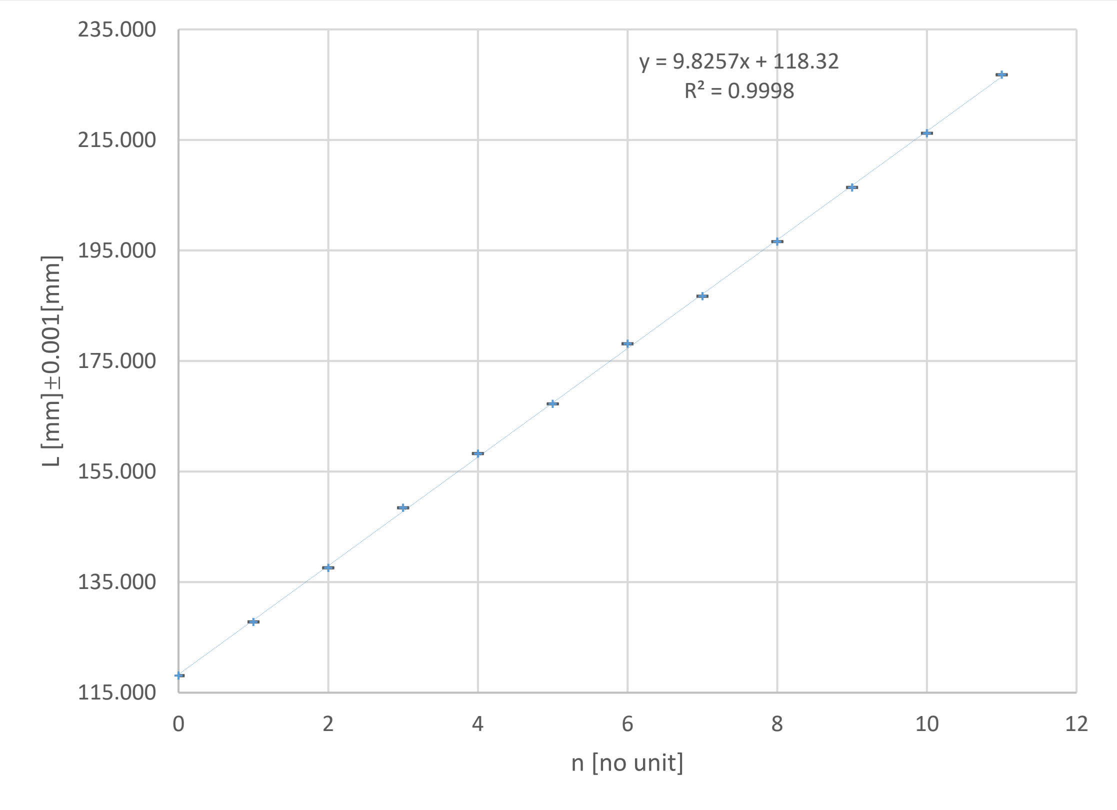

Figure 4: Linear-fit for the Phase-comparison Method.

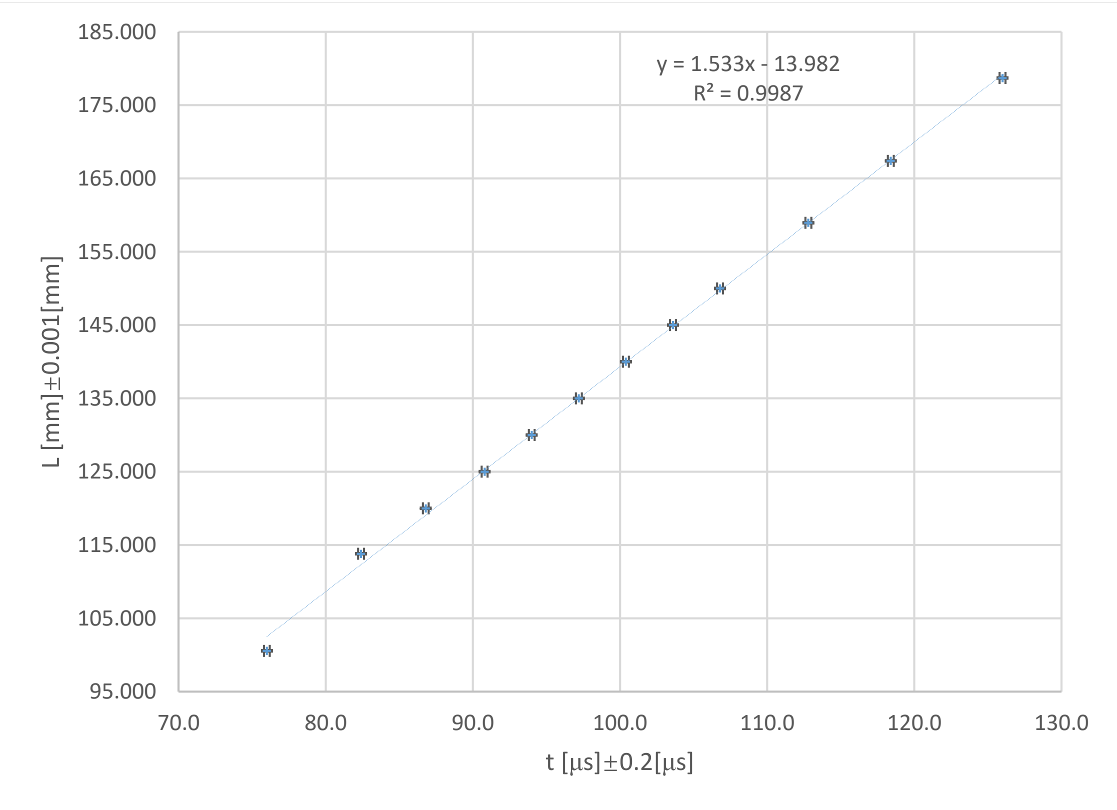

Figure 5: Linear-fit for the Time-difference Method.

Based on the slopes of these linear-fits, the following results can be found.

- **Resonance Method**

  Due to Equation (3), $\frac{\mathrm{d}L}{\mathrm{d}n} = \frac{\lambda}{2}$. Hence, $\lambda = 9.90 \pm 0.07$[mm]. Thus,

  $$v = \lambda f = 353 \pm 2[\mathrm{m/s}]$$

  In this case, the relative uncertainty is very small (0.6%), which corresponds to an R^2 value (0.9999) very close to 1.

- **Phase-comparison Method**

  Due to Equation (4), $\frac{\mathrm{d}L}{\mathrm{d}n} = \lambda$. Hence, $\lambda = 9.83 \pm 0.09$[mm]. Thus,

  $$v = \lambda f = 351 \pm 3[\mathrm{m/s}]$$

  In this case, the relative uncertainty is small (0.9%), which corresponds to an R^2 value (0.9998) close to 1.

- **Time-difference Method (Liquid)**

  Due to Equation (2),

  $$v = \frac{dL}{dt} = 1.53 \pm 0.04[\mathrm{km/s}]$$

  In this case, the relative uncertainty is not so small (3%), which corresponds to an R^2 value (0.9987) not so close to 1.

All these results are got under room temperature (25℃).

V. UNCERTAINTY ANALYSIS & CALCULATIONS

Below is the table of fitting parameters[1] given by Microsoft Excel 2016® (see Table 4).

Table 4: Table of fitting parameters.

Method	*Given directly by the software*				*Calculated*
	R^2	Lower 95%	Upper 95%	Coefficient	Uncertainty of slope
Resonance	0.9999	9.824744	9.972627	9.8986853	0.04
Phase-comparison	0.9998	9.731204	9.920152	9.8256783	0.09
Time-difference	0.9987	1.494425	1.571497	1.53296083	0.04

[1] Some other parameters, such as standard error, multiple R, adjusted R square, SS, MS, F, significance F, and P-value, etc., are also given by the software. To save space for this report, they are not included.

Furthermore, the uncertainties of the final results (see Table 5) in the Resonance Method as well as the Phase-comparison Method should be calculated based on the uncertainty of the slope (the uncertainty of the final results in the Time-difference Method is just the uncertainty of slope).

Table 5:The uncertainties of the final results.

Methods	*Speed*	*Uncertainty*	*Relative Uncertainty*
Resonance	353[m/s]	2[m/s]	0.6%
Phase-comparison	351[m/s]	3[m/s]	0.9%
Time-difference	1.53[km/s]	0.04[km/s]	3%

Take the resonance method as an example for calculation:

$$u_\lambda = \frac{upper\ bound - lower\ bound}{2} = 0.04[mm]$$

$$u_v = \sqrt{(\frac{\partial v}{\partial \lambda} \cdot u_\lambda)^2 + (\frac{\partial v}{\partial f} \cdot u_f)^2} = 2\ [m/s]$$

$$u_{rv} = \frac{u_v}{v} = 0.6\%$$

where the uncertainty of frequency u_f is given by the device as 0.001[kHz].

VI. CONCLUSIONS & DISCUSSIONS

In this experiment, the inaccuracies may come from: i) the noise waves; ii) the error due to the naked-eye observation on the screen of the oscilloscope; iii) the inaccuracy of the output frequency of the source of signal; iv) the sensitivity of the oscilloscope; v) the difference of natural frequency of S_1 and S_2, which may influence the resonance method.

By comparison with the literature data[2] (346.13 [m/s] & 1.4966 [km/s]), the results in this experiment (see Part 4 of this report) are relatively high. A probable reason for the higher result of sound speed in air is that the human bodies as well as the oscilloscope heat the air up, such that the speed measured increases. With regard to the speed in liquid, the higher

[2] See: Anshen, Qi, et al. *General Physics Tutorials: Mechanics*. Higher Education Press, 2012. page 344.

result may come from the impurities in water – in fact, the more concentrated the liquid is, the faster sound can propagate.

To improve this experiment, I have the following suggestions:

- For the resonance method, start from a place close to the knot;
- For the resonance method, use something like a single chip microcomputer to record the maximum value, instead of naked-eye observation;
- Avoid moving the transducer back and forth;
- Make full use of the measurement range, i.e. collect more than 12 sets of data;