I. THEORETICAL BACKGROUND

1.1 Polarization of Light

The electric field vector E, which in the context of electromagnetic waves correspond-ing to the visible part of the spectrum is sometimes referred to as the light vector describes a time-dependent, propagating electric field. In the plane perpendicular to the propaga-tion direction of a light wave, the light vector may have different directions along which its magnitude oscillates. The light, for which the light vector maintains a certain oscilla-tion direction, is called linearly polarized and the axis defining the direction is called the polarization axis (see Figure 1).

The light with the light vector direction rotating about the propagation direction, so that its endpoint traces a circle, is called circularly polarized light. If the vector traces an ellipse, the light said to be elliptically polarized (see Figure 2).

Light emitted from ordinary light sources (natural light) is unpolarized. However it can be regarded as a statistical equal-weight mixture of linearly polarized waves with equal amplitudes. There the light may be also partially polarized, which means it can be regarded as a combination of a polarized and the natural (unpolarized) light. The direction

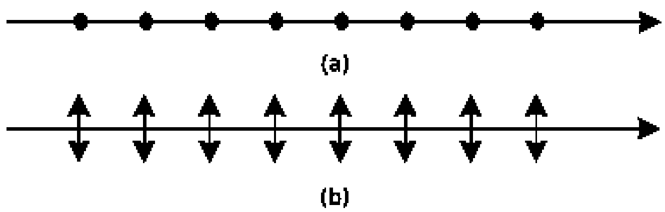

Figure 1. (a) Linearly polarized light with the polarization axis perpendicular to the page plane. (b) Linearly polarized light with the polarization axis parallel to the page plane.

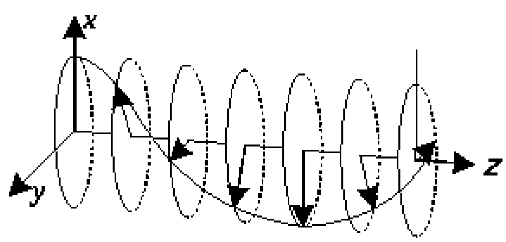

Figure 2. Elliptically polarized light propagating in the z direction. The light is polarized in the xy plane.

corresponding to the maximum amplitude of the light vector of such partially polarized light is the oscillation direction of the polarized component.

1.2 Polarizer

A device commonly used to produce polarized light is a polaroid (also called a polarizer). It polarizes the light using the principle of dichroism: a selective absorption mechanism tends to allow the light polarized in a certain direction (direction of the crys-tal alignment) to pass through the material, while the light polarized in all other directions is absorbed. This turns the incident natural light into linearly polarized. A polarization device can not only change incident natural light to polarized light (it then acts as a polarizer), but may also be used to detect and analyze linearly polarized, natural, and partially polarized light (it is then called an analyzer).

1.3 Malus' law

A visible effect in the light coming out of a polarization device is a change of the light brightness.

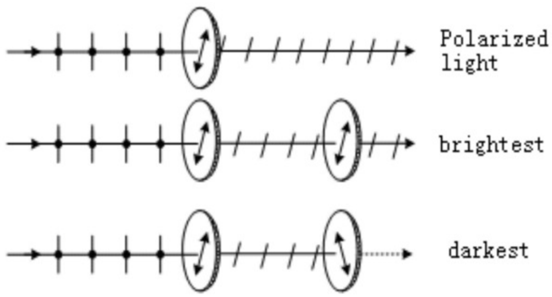

Figure 3. Change in the brightness of the light depends on the mutual orientation of the polarizer and the analyzer.

Suppose that we have two polarizers arranged so that their planes are parallel — the left one plays the role of a polarizer, the other one is an analyzer (see Figure 3). Let the angle between their transmission directions (polarization axes) be θ. The light is incident normally on the polarizer and then continues to the analizer. The intensity of the linearly polarized light leaving the analyzer is

$$I_{\text{light}} = I_{\text{light},0} \cos^2 \theta, \tag{1}$$

where $I_{\text{light},0}$ is the intensity of the linearly polarized light incident on the analyzer. Equation (1), named after Étienne-Louis Malus as the Malus' law, was derived in 1809.

Obviously, for a single polarizer, if polarized light is incident on it, then the transmitted light intensity will change periodically when rotating the polarizer. If the incident light is partially or elliptically polarized, the minimum intensity will not be zero as there will be always some component of the light polarized in the transmission direction. The incident light must be natural or circularly polarized if the intensity does not change at all. Hence, by using a polarizer, one can distinguish linearly polarized light from the natural and circularly polarized light.

1.4 Generation of Elliptically and Circularly Polarized Light; Half-wave and Quarter-wave Plates

Suppose that linearly polarized light is incident normally on a crystal plate whose surface is parallel to its optical axis, and the angle between the polarizing axis and the optical axis of the plate is α. Then the linearly polarized light is resolved into two waves: an e-wave with the oscillation direction parallel to the optical axis of the plate (extraordinary axis) and an o-wave whose oscillation direction is perpendicular to the optical axis (ordinary axis). They propagate in the same direction, but with different speeds. The resulting optical path difference over the thickness d of the plate is

$$\Delta = (n_{\text{e}} - n_{\text{o}})d,$$

and, consequently, the phase difference

$$\delta = \frac{2\pi}{\lambda}(n_{\text{e}} - n_{\text{o}})d,$$

where λ is the wavelength, n_e is the refractive index for the extraordinary axis, and n_o is the refractive index for the ordinary axis. In a so-called positive crystal $\delta > 0$, whereas in a negative one $\delta < 0$.

As shown in Figure 4, when the light propagates through the crystal plate, the two components of the light vector are

$$E_x = A_{\text{o}} \cos \omega t$$

$$E_y = A_{\text{e}} \cos(\omega t + \delta)$$

where $A_e = A \cos \alpha$, $A_o = A \sin \alpha$. Eliminating time from the above equations one obtains

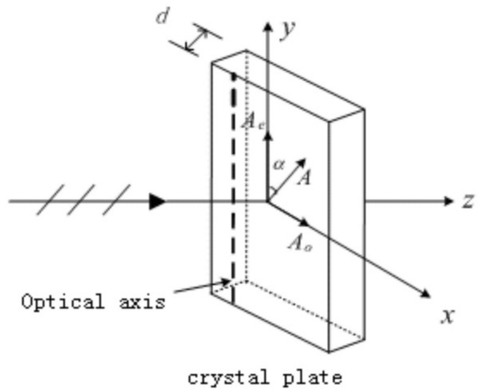

Figure 4. Linearly polarized light passing through a waveplate.

$$\frac{E_x^2}{A_\mathrm{o}^2} + \frac{E_y^2}{A_\mathrm{e}^2} - 2\frac{E_x E_y}{A_\mathrm{o} A_\mathrm{e}} \cos\delta = \sin^2\delta, \tag{2}$$

which is the equation of an ellipse.

When the thickness of the plate changes, the optical path difference changes as well. Some cases of particular interest, are discussed below:

► If Δ = kλ, where k = 0, 1, 2, . . ., the phase difference δ = 0, and Eq. (2) reduces to

$$E_y = \frac{A_\mathrm{e}}{A_\mathrm{o}} E_x,$$

which is a linear equation. Hence the transmitted light is linearly polarized with the oscillation direction remaining unchanged. A wave-plate that satisfies this condition is called a full-wave plate. The light goes through a full-wave plate without changing its polarization state.

► If Δ = (2k + 1)λ/2, where k = 0, 1, 2, . . ., the phase difference δ = π, and Eq. (2) becomes

$$E_y = -\frac{A_\mathrm{e}}{A_\mathrm{o}} E_x.$$

The transmitted light is also linearly polarized with the polarization axis rotated by the angle of 2α. A waveplate that satisfies the condition is called 1/2-wave plate or half-wave plate. When a polarized light passes through a half-wave plate, its polarization axis gets rotated by an angle 2α. If α = π/4, then the polarization axis of the transmitted light is perpendicular to that of the incident light.

► If Δ = (2k + 1)λ/4, where k = 0, 1, 2, . . ., the phase difference δ = ±π/2, then Eq. (2)becomes

$$\frac{E_x^2}{A_\mathrm{o}^2} \pm \frac{E_y^2}{A_\mathrm{e}^2} = 1.$$

The transmitted light is elliptically polarized with a waveplate that satisfies the above condition is called a 1/4-wave plate or a quarter-wave-plate and is an important optical element in many polarization experiments.

If $A_e = A_o = A$, then $E_x^2 + E_y^2 = A^2$, and the transmitted light is circularly polarized. Since the amplitudes of the o-wave and the e-wave are both functions of α, the polarization state after passing through a 1/4-wave plate will vary, depending on the angle:

► if α = 0, the transmitted light is linearly polarized with the polarization axis parallel to the optical axis of the 1/4–wave plate;

► if $\alpha = \pi/2$, the transmitted light is linearly polarized with the polarization axis perpendicular to the optical axis of the 1/4–wave plate;

► if $\alpha = \pi/4$, the transmitted light is circularly polarized;

► otherwise, the transmitted light is elliptically polarized.

II. APPARATUS

The measurement setup consists of: a semiconductor laser, a tungsten iodine lamp, a silicon photo-cell, a UT51 digital universal meter, as well as two polarizers, 1/2-wave and 1/4–wave plates (the uncertainty of the the angle is 2^{o}) and a lens with a glass sheet. The elements are placed on an optical bench.

The uncertainties of the angles on the polarizers (analyzers) and wave-plates are 2°. The uncertainty of the current meter (on 20μA-scale) is 0.001 μA.

III. MEASUREMENT PROCEDURES

3.1 Apparatus Adjustment

1. Fix the laser at one of the ends of the bench, place the lens and the glass sheet in front of it. Make sure that the light passes through the center of the lens.
2. Adjust the distance between the lens and the laser to the focal length of the lens.
3. Move the glass sheet along the bench. If the size of the light spot on the glass varies significantly, repeat Step 2.
4. Remove the glass sheet, and place the photo-cell on the bench. Set the digital universal meter in the appropriate mode and range.

3.2 Demonstration of Malus' Law

1. Assemble the measurement setup as shown in Figure 5. Make sure that the laser ray passes through the polarizer to generate linearly polarized light before continuing to the analyzer and the silicon photo-cell.
2. Rotate the analyzer for 360° and observe a change in the light intensity to find the maximum electric current I_0.
3. Set the angle of analyzer to 90° and adjust the angle of the polarizer until the electric current measured by the multimeter reaches its minimum. At this point, the polarizing axes of the polarizer and the analyzer are perpendicular to each other.
4. Rotate the analyzer from 90° to 0° and record the magnitude of the current I every 5°. Record the values in a table and plot the graph I/I_0 vs. $\cos^2\theta$. Perform linear fitting and compare the data with the theoretical result.

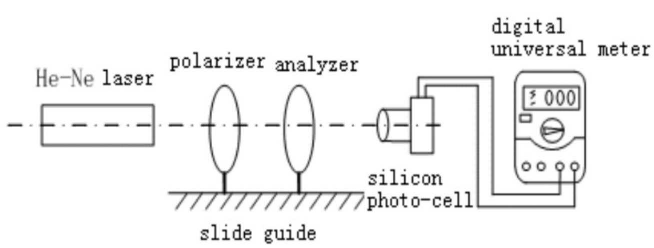

Figure 5. Experimental setup for the demonstration of Malus' law.

3.3 Linearly Polarized Light and the Half-wave Plate

1. Set up the equipment on the optical bench as shown in Figure 6. A is the analyzer and P is the polarizer. Set the polarizing axes of A and P perpendicular to each other before placing the 1/2-wave plate in the apparatus; extinction of the light can be observed on screen.
2. After inserting the 1/2-wave plate, rotate it to make the light extinction appear again and set this position as the initial position.

3. Rotate the 1/2-wave plate for $\alpha = 10^{\circ}$ from the initial position and the light ex-tinction will be broken. Then rotate A to make the light extinction appear again, record the angle of rotation $\Delta\theta$ in a table.
4. Rotate the 1/2-wave plate for 10° from the previous position (now $\alpha = 20^{\circ}$) and repeat Step 3. Repeat this step (increase α) for 8 times. Plot the graph $\Delta\theta$ vs. θ.
5. Analyze the data and answer the following questions in your lab report:
   (a) How many times can the light extinction be observed when the 1/2-wave plate rotates for 360◦?
   (b) How many times can the light extinction be observed when the analyzer rotates for 360◦?
   (c) Explain the polarization state of linearly polarized light after passing through the half-wave-plate.

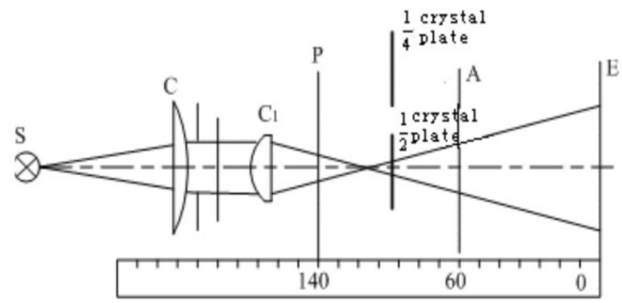

Figure 6. Experimental setup for the 1/2-wave plate.

3.4 Circularly and Elliptically Polarized Light and the 1/4-wave Plate

1. Set up the equipment on the optical bench as shown in Figure 6. A is the analyzer and P is the polarizer. Set the polarizing axes of A and P perpendicular to each other before placing the 1/4-wave plate in the apparatus; extinction of the light can be observed on screen. At this point the angle $\theta = 90^{\circ}$.
2. After inserting the 1/4-wave plate, rotate it to make the light extinction appear again and set this position as the initial position. At this point $\alpha = 0^{\circ}$. Rotate the 1/4-wave plate and observe the change in the light intensity.
3. Rotate the analyzer for 360° and record the light intensity (which is indicated by the current I) for every 10°. Record the data in a table.
4. Rotate the 1/4-wave plate for 20°, repeat Step 3.
5. Rotate the 1/4-wave plate for 45°, repeat Step 3.
6. Rotate the 1/4-wave plate for 70°. Then rotate the analyzer and record its position and the magnitude of the current when the light intensity reaches a maximum.
7. Use a computer to plot the relation between the rotation angle of the analyzer and the light amplitude in polar coordinates. Normalize the amplitude by its maximum value. Mark the position recorded in Step 6 and compare it with the data recorded in Step 4. Pay attention to the fact that the light intensity is found indirectly by measuring the electric current, and the intensity is proportional to the amplitude squared. The current indicates the intensity, not the amplitude.
8. Compare the result of Step 5 with that for the circular polarization. Plot a linear fit to the data when the angle is 45°.

IV. RESULTS & UNCERTAINTIES

4.1 Demonstration of Malus' Law

The maximum electric current I_0 is found to be 7.455±0.001 μA. Then we can calculate the I/I_0 values as:

$$I/I_0 = \frac{7.450}{7.455} = 0.99933 \pm 0.00019$$

Below is the calculation for uncertainty

$$u_{I/I_0} = \sqrt{\left(\frac{\partial I/I_0}{\partial I} \cdot u_I\right)^2 + \left(\frac{\partial I/I_0}{\partial I_0} \cdot u_{I_0}\right)^2} = 0.00019$$

$$u_{r,I/I_0} = \frac{u_{I/I_0}}{I/I_0} = 0.019\%$$

The results with corresponding uncertainties are as followings (Table 1).

Table 1: Results for the first part.

θ [°]	$\cos^2(\theta)$	I [μA]	Uncertainty		I/I0	Uncertainty	
90	0.00	0.004	0.001	25%	0.00054	0.00013	25%
85	0.01	0.066	0.001	1.5%	0.00885	0.00013	2%
80	0.03	0.244	0.001	0.4%	0.03273	0.00013	0.4%
75	0.07	0.523	0.001	0.19%	0.07015	0.00013	0.2%
70	0.12	0.897	0.001	0.11%	0.12032	0.00014	0.11%
65	0.18	1.398	0.001	0.07%	0.18753	0.00014	0.07%
60	0.25	1.936	0.001	0.05%	0.25969	0.00014	0.05%
55	0.33	2.504	0.001	0.04%	0.33588	0.00014	0.04%
50	0.41	3.139	0.001	0.03%	0.42106	0.00015	0.03%
45	0.50	3.779	0.001	0.03%	0.50691	0.00015	0.03%
40	0.59	4.443	0.001	0.02%	0.59598	0.00016	0.03%
35	0.67	5.080	0.001	0.02%	0.68142	0.00016	0.02%
30	0.75	5.670	0.001	0.018%	0.76056	0.00017	0.02%
25	0.82	6.184	0.001	0.016%	0.82951	0.00017	0.02%
20	0.88	6.650	0.001	0.015%	0.89202	0.00018	0.02%
15	0.93	6.968	0.001	0.014%	0.93467	0.00018	0.02%
10	0.97	7.241	0.001	0.014%	0.97129	0.00019	0.019%
5	0.99	7.420	0.001	0.013%	0.99531	0.00019	0.019%
0	1.00	7.450	0.001	0.013%	0.99933	0.00019	0.019%

By applying linear fitting on Excel, the following figure (see Figure 7 on next page) is got.

The software gives R square value of the slope as 0.9999. It also gives the "Upper 95%" and "Lower 95%" values (0.995558 and 1.005932 respectively), so that we can calculate the uncertainty of the slope:

$$\text{Uncertainty} = \frac{Upper\ 95\% - Lower\ 95\%}{2} = 0.005$$

i.e. the slope in this experiment is 1.001 ± 0.005. It can be seen that the result perfectly fits with theoretical expectation, as the slope is very close to 1, and the R^2 value is as high as 0.9999.

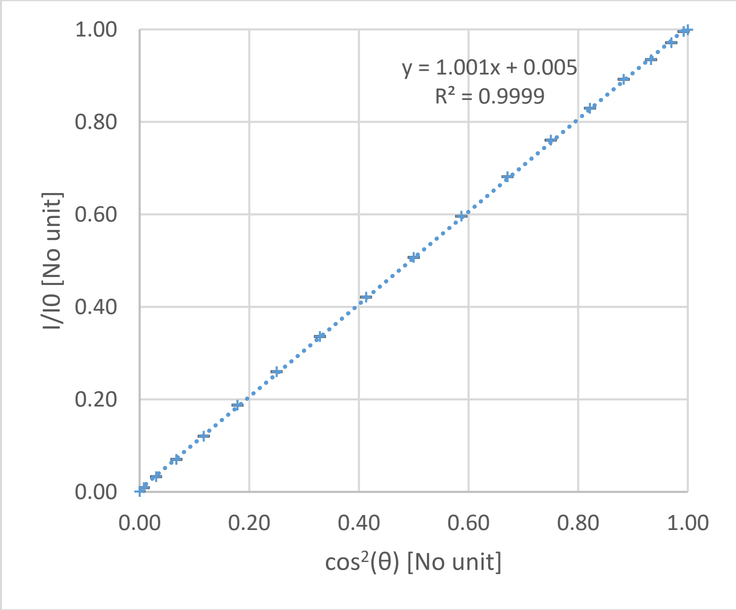

Figure 7: The plot for the first part.

4.2 Linearly Polarized Light and the Half-wave Plate

The angles are measured directly and corresponding uncertainties are given directly, thus the following data table (Table 2) can be got immediately.

Table 2: Data for the second part.

θ[°]	△θ[°]	Uncertainty	
0	0	2	N/A
10	22	2	9%
20	42	2	5%
30	62	2	3%
40	82	2	2%
50	102	2	2%
60	122	2	1.6%
70	142	2	1.4%
80	162	2	1.2%
90	183	2	1.1%

Then the following figure is got (Figure 8).

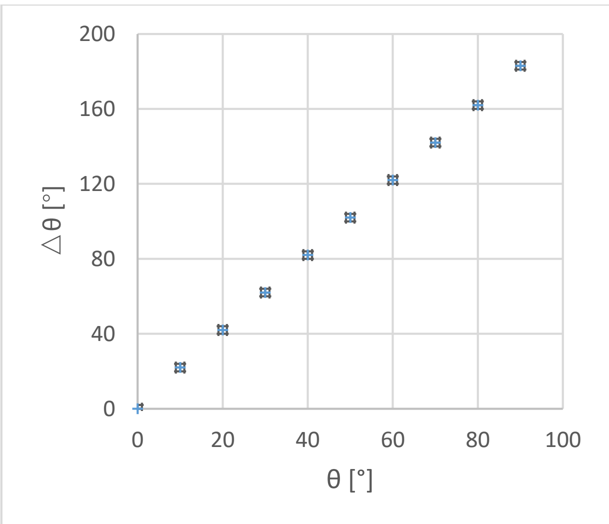

Figure 8: The plot for the second part.

The software gives slope as 2.016, which is very close to the theoretical value 2, with a small deviation of 0.8%. The software also gives the R^2 value as 0.9999, which is very close to the ideal 1. Meanwhile, it gives the “Upper 95%” and “Lower 95%” values (2.0016 and 2.0310 respectively), so that we can calculate the uncertainty of the slope:

$$\text{Uncertainty} = \frac{Upper\ 95\% - Lower\ 95\%}{2} = 0.015$$

i.e. the slope in this experiment is 2.016 ± 0.015. Therefore, the experimental result in this lab is quite satisfying.

The answers to the questions in the Lab Manual are as followings:

a) Four times of light extinction can be observed when the 1/2-wave plate rotates for 360◦.
b) Two times of light extinction can be observed when the analyzer rotates for 360◦.
c) After passing through the half-wave-plate, the linearly polarized light is still linearly polarized.

4.3 Circularly and Elliptically Polarized Light and the 1/4-wave Plate

We can calculate the $\sqrt{I/I_0}$ values as:

$$\sqrt{I/I_0} = \frac{4.730}{4.755} = 0.9974 \pm 0.0002$$

Below is the calculation for uncertainty

$$u_{\sqrt{I/I_0}} = \sqrt{\left(\frac{\partial\sqrt{I/I_0}}{\partial I}\cdot u_I\right)^2 + \left(\frac{\partial\sqrt{I/I_0}}{\partial I_0}\cdot u_{I_0}\right)^2} = 0.0002$$

$$u_{r,\sqrt{I/I_0}} = \frac{u_{\sqrt{I/I_0}}}{\sqrt{I/I_0}} = 0.02\%$$

The results with corresponding uncertainties are as followings (Table 3).

Table 3: Results for the first part.

θ [°]	$\sqrt{I/I_0}$ (0°)	Uncertainty			$\sqrt{I/I_0}$ (20°)	Uncertainty			$\sqrt{I/I_0}$ (45°)	Uncertainty	
0	0.9974	0.0002	0.02%		0.9585	0.0002	0.02%		0.9326	0.0004	0.04%
10	0.9768	0.0002	0.02%		0.9927	0.0002	0.02%		0.9507	0.0004	0.04%
20	0.9253	0.0002	0.02%		0.9993	0.0002	0.02%		0.9692	0.0004	0.04%
30	0.8431	0.0002	0.03%		0.9791	0.0002	0.02%		0.9849	0.0004	0.04%
40	0.7393	0.0002	0.03%		0.9317	0.0002	0.02%		0.9952	0.0004	0.04%
50	0.6124	0.0002	0.04%		0.8624	0.0002	0.03%		1.0000	0.0004	0.04%
60	0.4710	0.0003	0.06%		0.7644	0.0002	0.03%		0.9972	0.0004	0.04%
70	0.3049	0.0004	0.12%		0.6563	0.0002	0.04%		0.9892	0.0004	0.04%
80	0.1413	0.0008	0.5%		0.5399	0.0003	0.05%		0.9760	0.0004	0.04%
90	0.0580	0.0018	3%		0.4178	0.0003	0.08%		0.9594	0.0004	0.04%
100	0.2141	0.0005	0.2%		0.3294	0.0004	0.12%		0.9407	0.0004	0.04%
110	0.3779	0.0003	0.08%		0.3065	0.0004	0.13%		0.9220	0.0004	0.04%
120	0.5379	0.0003	0.05%		0.3686	0.0004	0.10%		0.9052	0.0004	0.04%
130	0.6769	0.0002	0.03%		0.4775	0.0003	0.06%		0.8946	0.0004	0.04%
140	0.7948	0.0002	0.03%		0.5948	0.0003	0.04%		0.8908	0.0004	0.04%
150	0.8885	0.0002	0.02%		0.7118	0.0002	0.03%		0.8929	0.0004	0.04%
160	0.9546	0.0002	0.02%		0.8183	0.0002	0.03%		0.9026	0.0004	0.04%
170	0.9920	0.0002	0.02%		0.8989	0.0002	0.03%		0.9176	0.0004	0.04%
180	0.9997	0.0002	0.02%		0.9589	0.0002	0.02%		0.9352	0.0004	0.04%
190	0.9756	0.0002	0.02%		0.9933	0.0002	0.02%		0.9521	0.0004	0.04%
200	0.9216	0.0002	0.02%		1.0000	0.0002	0.02%		0.9694	0.0004	0.04%
210	0.8430	0.0002	0.03%		0.9798	0.0002	0.02%		0.9836	0.0004	0.04%

220	0.7406	0.0002	0.03%		0.9316	0.0002	0.02%		0.9924	0.0004	0.04%
230	0.6054	0.0002	0.04%		0.8589	0.0002	0.03%		0.9954	0.0004	0.04%
240	0.4634	0.0003	0.06%		0.7650	0.0002	0.03%		0.9931	0.0004	0.04%
250	0.2968	0.0004	0.13%		0.6559	0.0002	0.04%		0.9836	0.0004	0.04%
260	0.1337	0.0008	0.6%		0.5348	0.0003	0.05%		0.9711	0.0004	0.04%
270	0.0632	0.0017	3%		0.4134	0.0003	0.08%		0.9548	0.0004	0.04%
280	0.2190	0.0005	0.2%		0.3263	0.0004	0.12%		0.9362	0.0004	0.04%
290	0.3831	0.0003	0.08%		0.3062	0.0004	0.13%		0.9170	0.0004	0.04%
300	0.5397	0.0003	0.05%		0.3670	0.0004	0.10%		0.9042	0.0004	0.04%
310	0.6699	0.0002	0.03%		0.4758	0.0003	0.06%		0.8937	0.0004	0.04%
320	0.7909	0.0002	0.03%		0.5975	0.0003	0.04%		0.8896	0.0004	0.04%
330	0.8832	0.0002	0.02%		0.7114	0.0002	0.03%		0.8921	0.0004	0.04%
340	0.9488	0.0002	0.02%		0.8186	0.0002	0.03%		0.9011	0.0004	0.04%
350	0.9880	0.0002	0.02%		0.8973	0.0002	0.03%		0.9158	0.0004	0.04%

Then we can get the following plots (Figures 9 – 11).

By applying linear fit for the 45° data, we see that the slope is -0.00014, which is very close to the theoretical value 0. The software also gives the R^2 value as 0.1508, which is very close to the ideal 0 (totally irrelevant). Meanwhile, it gives the “Upper 95%” and “Lower 95%” values (-0.00026 and -2.5e-05 respectively), so that we can calculate the uncertainty of the slope:

$$\text{Uncertainty} = \frac{Upper\ 95\% - Lower\ 95\%}{2} = 0.00012$$

i.e. the slope in this experiment is -0.00014 ± 0.00012. Therefore, the experimental result in this lab is quite satisfying.

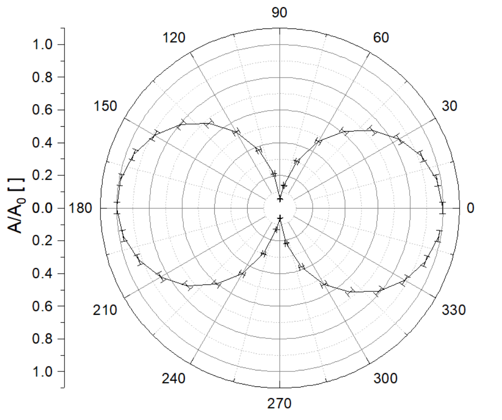

Figure 9: Plot for 0 °

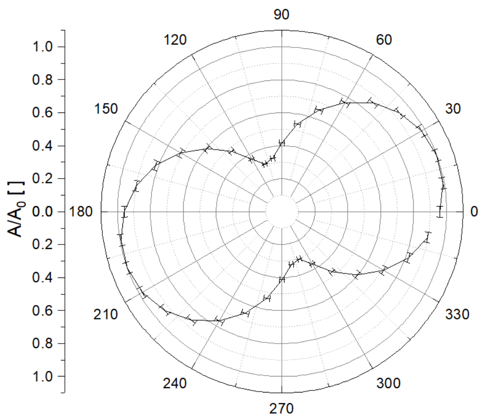

Figure 10: Plot for 20 °

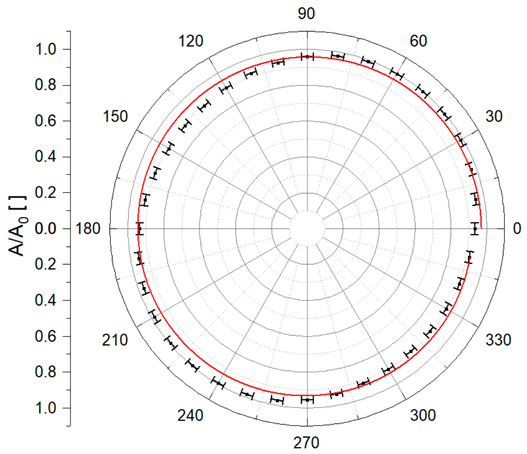

Figure 11: Plot for 45 °

When the 1/4-wave plate is rotated for 70◦, the light intensity reaches a maximum (with the magnitude of current 4.107±0.001 μA), if the analyzer is rotated for 355±2°. By comparison with the results got when the 1/4-wave plate is rotated for 20◦, it can be derived that: i) the absolute values of the maximum light intensity (current) are theoretically the same; ii) the rotation angles when the maximum intensity is reached should sum up to $k\pi$, where k is an integer.

V. CONCLUSIONS & DISCUSSIONS

In this lab, we understood some properties of light, in particular, we verified Malus' law and studied the polarization phenomenon, as well as the way half- and quarter-wave plates work in optical systems. We also investigated in the generation and detection of elliptically and circularly polarized light.

For the first part, it turns out that the Malus' Law is correct, thanks to the high R square value (0.9999, very close to the ideal 1). For the second part, it is clear that when a polarized light passes through a half-wave plate, its polarization axis gets rotated by an angle 2α, if the angle between the polarizing axis and the optical axis of the plate is α. For the third part, the experimental curve is generally in accordance with the theoretical expectation, as can be seen in Figures 9 – 11.

The inaccuracies in this experiment may come from: i) the diffused light in the environment (especially the light coming from the torches); ii) the error due to the limited waiting time for the meter to maintain at a very steady value; iii) the error due to naked-eye observation for the focal length adjustment; iv) the inaccuracy of the meters.

In addition, I have the following suggestions and ideas:

- We can use some opaque covers to prevent the sensor from receiving the unwanted diffuse-reflected light, so that the data can be more accurate.
- Assume that some naughty guys erased the "2" "4" signs on the components so that we cannot distinguish polarizer, half-wave plate, quarter-wave plate directly from the appearance. We can then distinguish them by lending an Na-light source from a nearby chemistry lab or biochemistry lab. How to do that? Well, we use a piece of glass to observe the reflected Na-light through these three mixed-up components. In

this process, we should rotate these components while changing the direction of the reflected light. If, during each round of rotation process, two times of light extinction can be observed, then this component must be a polarizer. At this time, the incident angle of the Na-light is the Brewster's angle, while the reflected light is a linearly polarized light with its vibration plane perpendicular to the incident plane. Now that we have a piece of polarizer and a polarized light with known vibration direction, we can make the left two wave-plates rotate and check the change of intensity of the emergent light. If, there is always two times of light extinction for each round of rotation, regardless of the orientation, then the component must be a half-wave plate. This is because the emergent light of a half-wave plate is still linearly polarized, While the emergent light of a quarter-wave plate, in most cases, will be elliptically polarized.