I. THEORETICAL BACKGROUND

The goal of this exercise is to:

- study the shape of the magnetic hysteresis loop and the magnetization curve;
- understand how to use these characteristics to discuss properties of ferromagnetic materials;
- quantitatively study the concepts of the coercive field strength, the residual magnetic field and the magnetic susceptibility;
- visualize the magnetization curve and the magnetic hysteresis loop on the oscilloscope;
- find the Curie temperature of a ferromagnetic material.

1.1 Magnetization

The relationship between the magnetic field B, the magnetization M, and the auxiliary magnetic field H is:

$$B = \mu_0(H + M) = (\chi_m + 1)\,\mu_0 H = \mu_r\mu_0 H = \mu H,$$

where $\mu_0 = 4\pi \cdot 10^{-7}$ H/m is the magnetic permeability of vacuum, χ_m is the magnetic susceptibility of the material, the dimensionless quantity μ_r is called the relative magnetic permeability of the material, and $\mu = \mu_r\mu_0$ is the material's (ab-solute) magnetic permeability. For a paramagnetic material, $\chi_m > 0$ and μ_r is slightly greater than 1. On the other hand, for a diamagnetic material, $\chi_m < 0$ with the absolute value between 10^{-4} and 10^{-5} and μ_r is slightly less than 1. For a ferromagnetic material, $\chi_m \gg 1$, so that $\mu_r \gg 1$.

If the temperature is increased above a certain value, a ferromagnet will turn into a paramagnet, that is a material with randomly oriented magnetic dipole moments. This critical value of the temperature is called the *Curie temperature*, and on the graph μ vs. T it is the temperature corresponding to the point where the slope of the tangent line is maximum.

1.2 Magnetic Hysteresis

In addition to high magnetic permeability, ferro-magnets have another important property, which is the magnetic hysteresis. When a ferromagnet is being magnetized, the magnetic field B depends not only on the current value of the auxiliary magnetic field H, but also on the previous state of the material, as shown in Figure 1. The curve OA, where the magnetic field B grows as the auxiliary magnetic field H increases, describes the process of magnetizing of an (initially demagnetized) ferromagnet. This curve is called the magnetization curve.

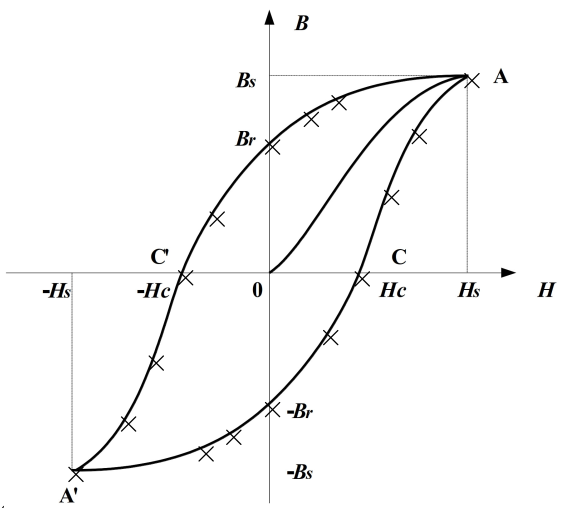

Figure 1. The magnetization curve and the magnetic hysteresis loop of a ferromagnetic material. The crosses indicate the points to be recorded to describe the loop.

When the auxiliary magnetic field is increased to a certain value HS, the magnetic field B hardly increases and reaches a saturation state. If then the auxiliary magnetic field is being decreased, the magnetic field B is not decreasing along the original path, but rather choosing another path $AC'A'$. Furthermore, when H increases from the value $-H_S$, B will reach A along the curve $A'CA$, and finally form a closed curve. When H = 0, we have $|B| = B_r$, where B_r is called the remnant magnetic field (yielding the corresponding remnant magnetization). In order to make B = 0, an auxiliary magnetic field has to be applied in the reverse direction. When the auxiliary magnetic field reaches the value $H = -H_C$, where H_C is called the *coercive field strength*, the material is demagnetized and the magnetic field B, and hence the magnetization, is zero.

1.3 Visualization on the Oscilloscope

Figure 2 presents a diagram of a circuit that is used to visualize the magnetization curve and the magnetic hysteresis loop on the oscilloscope. In this exercise they are studied for a ferromagnet sample that has an average magnetic path of L and the number of the turns in the coil is N_1. If the electric current through the coil is i_1, then according to Ampère's law

$$HL = N_1 i_1.$$

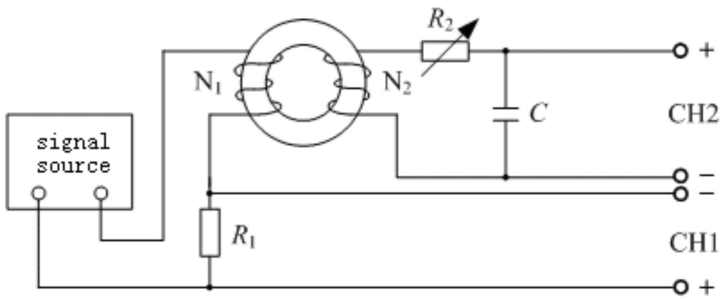

Figure 2. The electric circuit for visualization of the magnetization curve and the magnetic hysteresis loop on the oscilloscope. $R_1 = 10\ \Omega$ and R_2 is a variable resistor with maximum resistance of 2.2 kΩ.

In this case the input voltage for the deflection plate of the x axis channel of the oscilloscope is

$$U_{R_1} = R_1 i_1 = \frac{R_1 L}{N_1} H,$$

where R_1, L, and N_1 are constant. Hence, the input voltage for the x axis channel is proportional to the auxiliary magnetic field H.

If the cross-sectional area of the sample is S, then according to the law of electromagnetic induction, the induced electromotive force in a secondary coil with the number of turns N_2, is

$$\mathcal{E}_2 = -N_2 S \frac{\mathrm{d}B}{\mathrm{dt}}. \tag{1}$$

Assuming that the number of turns N_2 in the secondary coil is small, the electromotive force due to self-induction can be ignored. Moreover, if the values of the resistance R_2 and the capacitance C are chosen so that $R_2 \gg 1/\omega C$, then

$$E_2 = R_2 i_2. \tag{2}$$

Substituting $i_2 = dq/dt = C dU_C/dt$ into Eq. (2) and combining with Eq. (1) one obtains

$$U_C = -\frac{N_2 S B}{R_2 C}, \tag{3}$$

which shows that the input voltage U_C for the y axis channel is proportional to the magnetic field B.

1.4 Measurement of the Curie Temperature with an AC Bridge

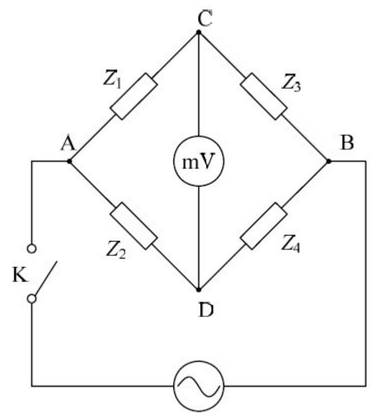

Figure 3. Four–side impedance bridge.

In an AC bridge (Figure 3), the four sides of the bridge are combinations of resistive, capacitive, and inductive elements combined in series or in parallel. The idea is to adjust the parameters of the bridge in order to eliminate a potential difference between the points C and D. When the bridge reaches this equilibrium state, one should have

$$\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}.$$

To make this equality hold, the moduli and the phases of both sides should be equal, i.e.

$$\frac{|Z_1|}{|Z_2|} = \frac{|Z_3|}{|Z_4|} \qquad \text{and} \qquad \varphi_1 + \varphi_4 = \varphi_2 + \varphi_3,$$

respectively. Then the bridge is balanced, i.e. the potential difference between the points C and D is equal to zero.

Figure 4 shows a RL AC bridge used in this experiment. The input current source is supplied by a signal generator, allowing to chose a suitable (high) output frequency. The elements Z_1 and Z_2 are resistors, whereas Z_3 and Z_4 are inductors with a finite resistance r_1 and r_2, respectively. The impedances of these four elements are

$$Z_1 = R_1, \qquad Z_2 = R_2, \qquad Z_3 = r_1 + j\omega L_1, \qquad Z_4 = r_2 + j\omega L_2,$$

with j being the imaginary unit ($j^2 = -1$) and ω — the angular frequency of the signal provided by the voltage source. When the bridge is in balance

$$r_2 = \frac{R_2}{R_1} r_1, \qquad L_2 = \frac{R_2}{R_1} L_1.$$

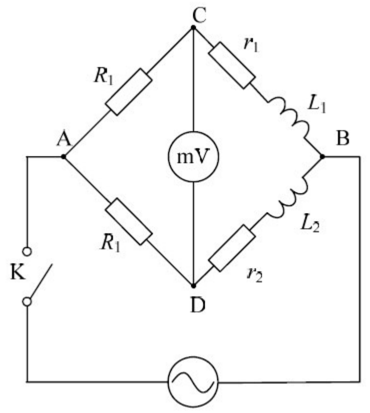

Figure 4. AC bridge.

When the temperature is increased to the critical value, the ferromagnet undergoes a transition to the paramagnetic state, the potential difference between C and D will suddenly becomes nearly zero again and at this very point the balance is recovered. This particular transition point allows one to estimate the Curie temperature, i.e. the temperature of the ferromagnetic-paramagnetic phase transition, by observing the voltage-temperature curve.

II. Apparatus

The apparatus consists of the following main elements: a signal gen-erator, an oscilloscope, a digital multimeter, a platinum resistance thermometer, a glass vacuum-tube heating pipe, a voltage/current source, a wiring block, a $4.418 \cdot 10^{-6}$ F capacitor, a 5.95 Ω resistor, a 2.2 kΩ variable resistor.

The uncertainty of L is $1 \cdot 10^{-2}$ m, of S is $1 \cdot 10^{-7}$ m^2, of C is $1 \cdot 10^{-9}$ F, of R_1 is 0.01 Ω, of R_2 is 0.1 Ω, of U_R (horizontal axis of the oscilloscope) is 1 mV, of U_C (vertical axis of the oscilloscope) is 0.4 mV.

The parameters of other elements are: $L = 3.61 \cdot 10^{-2}$ m, $S = 1.25 \cdot 10^{-5}$ m^2, $C = 4.418 \cdot 10^{-6}$ F, and $N_1 = N_2 = 100$.

III. Procedures

3.1 Magnetic Hysteresis Loop Measurement

1. Assemble the circuit according to Figure 2 with the signal source disconnected.
2. Turn on the signal source and adjust the frequency to 1 kHz or above. We must not short the signal source, or set the frequency below 1 kHz (otherwise the equipment will be damaged).
3. Connect the signal source to the circuit and observe the U_{R_1} vs. U_C loop under different conditions:

   (1) with different signal frequencies (1 ~ 2 kHz) and amplitudes (1 ~ 5 V),

   (2) with different values of the resistance R_2.
4. Obtain the saturated hysteresis loop for a certain frequency (1 ~ 2 kHz) and suitable value of the resistance R_2 (about 1 kΩ); shift the picture of the loop to the center of the oscilloscope screen, adjust its size and show the visualization to the instructor.
5. Trace the whole loop with no less than 16 points (see Figure 1 for the location of the points).
6. Turn off the signal source, disconnect the circuit and measure the resistances.

7. Plot the magnetic hysteresis loop (H–B loop), and find $\pm B_r$, $\pm H_c$, $\pm B_s$,$\pm H_s$.

3.2 Measurement of the Curie Temperature with an AC Bridge

This part is cancelled.

IV. CALCULATIONS & RESULTS

We set our signal source to 1000 Hz and 6.700 Vpp, and the variable resistor is set to 1594.8 Ω (other parameters can be found in Part 2 of this report). Then we get the curve below (Figure 5) on the oscilloscope.

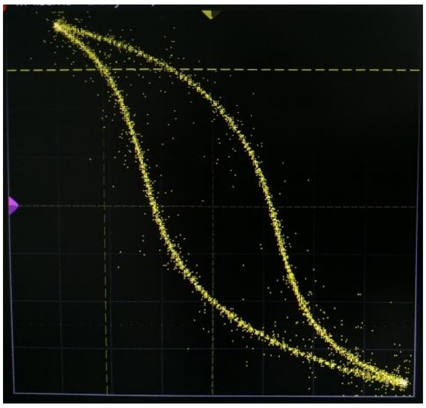

Figure 5. The curve on the oscilloscope. The horizontal axis is U_R, the vertical axis is U_C.

Based on the collected data, we can plot the U_C - U_R graph as below (Figure 6).

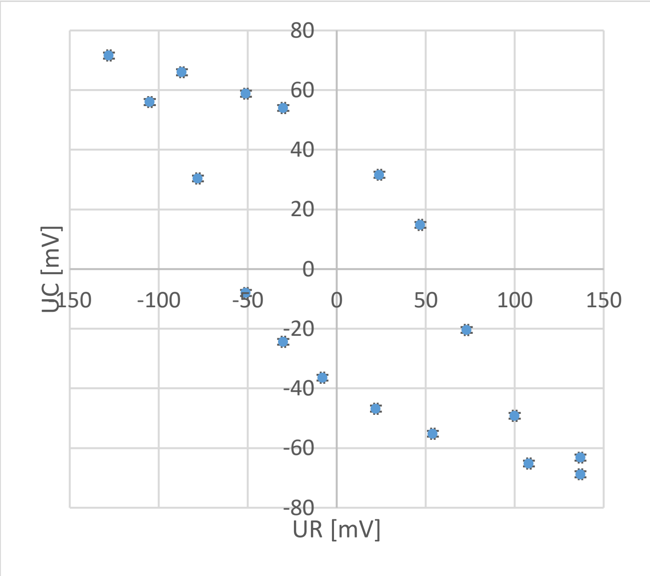

Figure 6. The U_C - U_R relation.

We can then calculate the corresponding values of B and H. Below is a set of sample calculation. (The uncertainty analysis can be found in Part 5 of this report.)

$$H = \frac{N_1}{R_1 L} U_R = \frac{100}{5.95 \cdot 3.61 \cdot 10^{-2}} \cdot 0.137 = 63.8 \pm 0.3[\mathrm{A/m}]$$

$$B = -\frac{R_2 C}{N_2 S} U_C = -\frac{1594.8 \cdot 4.418 \cdot 10^{-6}}{100 \cdot 1.25 \cdot 10^{-5}} \cdot (-68.8) = 388 \pm 3\ [\mathrm{mT}]$$

The values of all the data points are as followings (Table 1).

Table 1. Data for H and B.

H [A/m]	B [mT]		H [A/m]	B [mT]		H [A/m]	B [mT]
-59.6	-404		33.99	114.99		10.24	264
-40.5	-372		46.6	277		-3.724	205.2
-23.74	-331		63.8	356		-13.97	137.5
-13.97	-304		63.8	388		-23.74	45.09
11.17	-178.1		50.3	368		-36.31	-171.4
21.88	-83.4		25.14	311		-48.9	-316

Then we can get the following plot (Figure 7).

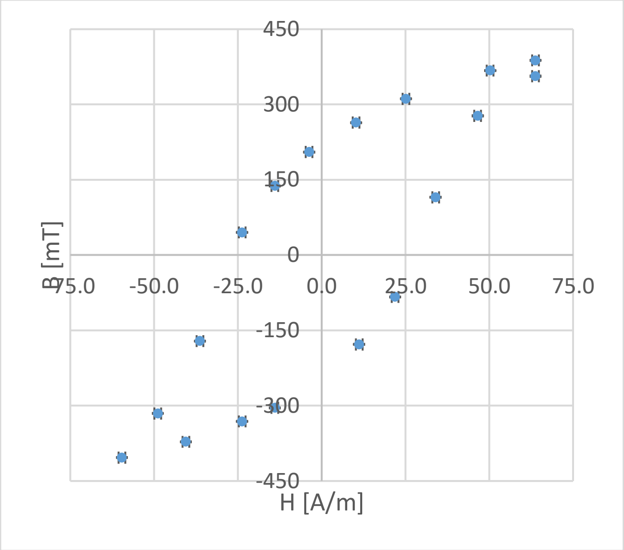

Figure 7. The B-H relation.

We also found the following values: $|B_r|$ = 220 [mT], $|H_c|$ = 27 [A/m], $|B_s|$ = 405 [mT], $|H_s|$ = 65 [A/m].

V. UNCERTAINTY ANALYSIS

The uncertainties of U_C and U_R are already given by the device (see Part 2 of this report), so we will focus on the uncertainties of H and B here. Below is a sample calculation.

$$u_H = \sqrt{(\frac{\partial H}{\partial R_1} \cdot u_{R_1})^2 + (\frac{\partial H}{\partial L} \cdot u_L)^2 + (\frac{\partial H}{\partial U_R} \cdot u_{U_R})^2} = 0.3 \text{ [A/m]}$$

$$u_{r,H} = \frac{u_H}{H} = 0.5\%$$

$$u_B = \sqrt{(\frac{\partial B}{\partial R_2} \cdot u_{R_2})^2 + (\frac{\partial B}{\partial C} \cdot u_C)^2 + (\frac{\partial B}{\partial S} \cdot u_S)^2 + (\frac{\partial B}{\partial U_C} \cdot u_{U_C})^2} = 3 \text{ [mT]}$$

$$u_{r,B} = \frac{u_B}{B} = 0.8\%$$

The uncertainties of other sets of data can be found in Table 2 (each cell correspond to the cell in Table 1).

Table 2. Uncertainties for H and B.

u_H [A/m]	u_B [mT]		u_H [A/m]	u_B [mT]		u_H [A/m]	u_B [mT]
0.3	3		0.16	0.9		0.05	2
0.2	3		0.2	2		0.017	1.6
0.11	3		0.3	3		0.07	1.1
0.07	2		0.3	3		0.11	0.3
0.06	1.4		0.3	3		0.16	1.3
0.10	0.6		0.12	2		0.2	2

Table 2. Relative uncertainties for H and B.

ur_H	ur_B		ur_H	ur_B		ur_H	u_B
0.5%	0.7%		0.5%	0.8%		0.5%	0.8%
0.5%	0.8%		0.4%	0.7%		0.5%	0.8%
0.5%	0.9%		0.5%	0.8%		0.5%	0.8%
0.5%	0.7%		0.5%	0.8%		0.5%	0.7%
0.5%	0.8%		0.6%	0.8%		0.4%	0.8%
0.5%	0.7%		0.5%	0.6%		0.4%	0.6%

VI. CONCLUSIONS & DISCUSSIONS

In this experiment, the inaccuracies may come from: i) the noises (dots on the oscilloscope); ii) the error due to the naked-eye observation on the screen of the oscilloscope; iii) the inner resistance of the circuit; iv) the sensitivity of the oscilloscope; v) the inaccurate number of the capacitance denoted on the capacitor.

By comparison with an ideal curve[1] of hysteresis loop, the result in this experiment is quite satisfying. But still, there are some interfering pixels (noises), which is acceptable in a real circuit.

In addition, I have the following suggestions and ideas:

- When we find that the B-H loop is not centro-symmetry to the original point, then there might be remnant magnetism, so we can gradually decrease the voltage, and then redo the experiment;
- If we find the B-H loop of a new material is very flat, then we can use that kind of material to make permanent magnets; adversely, we can use that kind of material to make voltage transformers.

[1] See Hu Youqiu, *Electromagnetics and Electrodynamics*, Science Press, 2016, page 153.