I. OBJECTIVES

The objective of this lab is to:

- Study the principle of the Hall effect and its ap-plications by using a Hall probe.
- Verify that the Hall voltage is proportional to the magnetic field.
- Study the sensitivity of an integrated Hall probe by calculating the magnetic field at the center of a solenoid.
- Measure the magnetic field distribution along the axis of the solenoid and compare it with the corresponding theoretical curve.

II. THEORETICAL BACKGROUND

2.1 Hall Effect

In a magnetic field B, when the electric current I passes through the semiconductor sheet in the direction shown in Figure 1, an electric potential difference between the sides a and b of the sheet is generated. The corresponding electric field is perpendicular to both the direction of the current and the direction of the magnetic field. This effect is known as the Hall effect, and the electric potential difference is called the Hall voltage U_H.

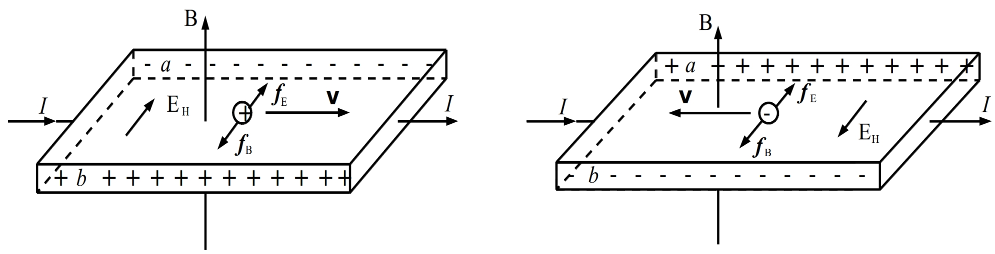

Figure 1. The principle of the Hall effect.

Microscopically, the Lorentz force F_B leads to the deflection of the moving charges, and their accumulation on one side of the sheet, which in turn increases the magnitude of the transverse electric field E_H (the Hall field). Due to this field, there is an electric force F_E acting upon the charges, and since F_B and F_E act in opposite directions, a balance is eventually reached and U_H stabilizes. The sign of U_H depends on the sign of the charge carriers. Therefore the type of charge carriers in semiconductors can be determined by analyzing the sign of U_H.

When the external magnetic field is not too strong, the Hall voltage is proportional to both the current and the magnitude of the magnetic field, and inversely proportional to the thickness of the sheet d

$$U_H = R_H \frac{IB}{d} = KIB \tag{1}$$

where R_H is the Hall coefficient and $K = R_H/d = K_H /I$, where K_H is the sensitivity of the Hall element.

2.2 Integrated Hall Probe

The magnitude of the magnetic field can be found by measuring the Hall voltage with a Hall probe when the sensitivity K_H and the current I are fixed. Since the Hall voltage is usually very small, it should be amplified before the measurement.

Silicon can be used to design both the Hall probe and the integrated circuits, so it is convenient to arrange the Hall probe and the electric circuits into a single device. Such a device is called an *integrated Hall probe*.

The integrated Hall probe SS495A consists of a Hall sensor, an amplifier, and a voltage compensator (Figure 2). The output voltage U can be read ignoring the residual voltage. The working voltage $U_S = 5$ V, and the output voltage U_0 is approximately 2.5 V when the magnetic field is zero. The relation between the output voltage U and the magnitude of the magnetic field is

$$B = \frac{U - U_0}{K_{\mathrm{H}}} \tag{2}$$

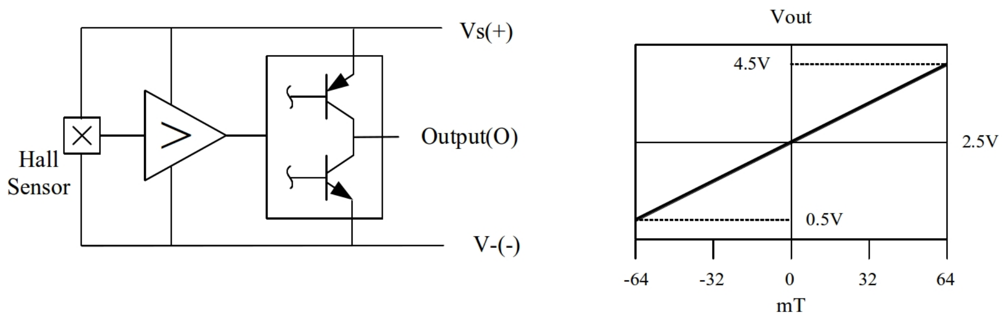

Figure 2. The integrated Hall probe SS495A (left). The relation between the output voltage U and the magnitude of the magnetic field B (right).

2.3 Magnetic Field Distribution Inside a Solenoid

The magnetic field distribution on the axis of a single layer solenoid is

$$B(x) = \mu_0 \frac{N}{L} I_{\mathrm{M}} \left\{ \frac{L + 2x}{2\left[D^2 + (L + 2x)^2\right]^{\frac{1}{2}}} + \frac{L - 2x}{2\left[D^2 + (L - 2x)^2\right]^{\frac{1}{2}}} \right\} = C(x) I_{\mathrm{M}}, \tag{3}$$

where N is the number of turns of the solenoid, L is its length, I_M is the current through the solenoid wire, and D is the solenoid's diameter. The magnetic permeability of vacuum is $\mu_0 = 4\pi \times 10^{-7}$ H/m.

The solenoid used in this exercise has ten layers, and the magnetic field B(x) for each layer can be calculated using Eq. (3). Then the net magnetic on the axis of the solenoid can be found by adding contributions due to all layers. The theoretical value of the magnetic field inside the solenoid with $I_M = 0.1$ A is given in Table 1.

x [cm]	B [mT]	x [cm]	B [mT]
±0.0	1.4366	±8.0	1.4057
±1.0	1.4363	±9.0	1.3856
±2.0	1.4356	±10.0	1.3478
±3.0	1.4343	±11.0	1.2685
±4.0	1.4323	±11.5	1.1963
±5.0	1.4292	±12.0	1.0863
±6.0	1.4245	±12.5	0.9261
±7.0	1.4173	±13.0	0.7233

Table 1. Theoretical value of the magnetic field inside the solenoid.

III. APPARATUS

The experimental setup shown in Figure 3 consists of an integrated Hall probe SS495A (see Figure 4) with K_H = 31.25 ± 1.25 V/T (at the working voltage 5 V) or K_H = 3.125 ± 0.125 mV/G, a solenoid, a power supply, a voltmeter, a DC voltage divider, and a set of connecting wires.

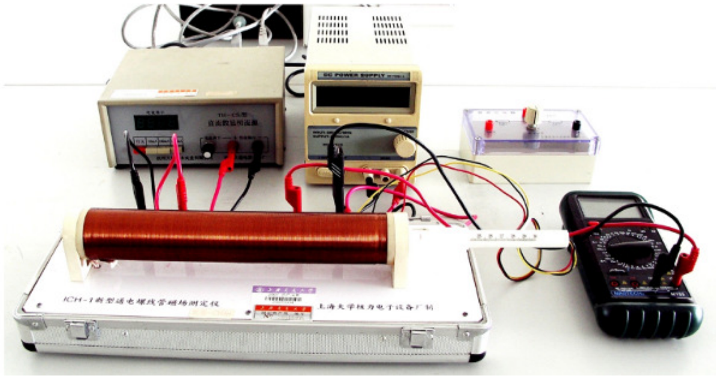

Figure 3. Measurement setup.

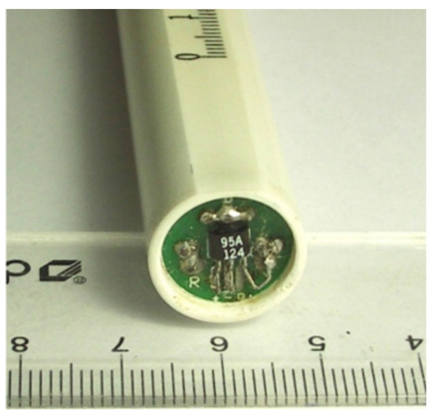

Figure 4. Integrated Hall probe SS495A.

The chart below shows the uncertainties of the devices (see Table 2).

Table 2: The uncertainty of the devices.

Devices	*Uncertainty*
Voltage Source U_s	0.5% [V]
Multi-meter (U_0)	$0.05\% + 6\times10^{-3or4}$ [V]
Multi-meter (U)	$0.05\% + 6\times10^{-3or4}$ [V]
Current Source I_m	2% [mA]
Position x	0.05 [cm]

IV. MEASUREMENT PROCEDURE

4.1 Relation Between Sensitivity K_H and Working Voltage U_S

1. Place the integrated Hall probe at the center of the solenoid. Set the working voltage at 5 V and measure the output voltage U_0 (I_M = 0) and U (I_M = 250 mA). Take the theoretical value of B(x = 0) from Table 1 and calculate the sensitivity of the probe K_H by using Eq. (2).
2. Measure K_H for different values of U_S (from 2.8 V to 10 V). Calculate K_H/U_S and plot the curve K_H/U_S vs. U_S.

4.2 Relation Between Output Voltage U and Magnetic Field B

1. With B=0, U_S=5 V, connect the 2.4 ~ 2.6 V output terminal of the DC voltage divider and the negative port of the voltmeter. Adjust the voltage until $U_0 = 0$.
2. Place the integrated Hall probe at the center of the solenoid and measure the output voltage U for different values of I_M ranging from 0 to 500 mA, with intervals of 50 mA.
3. Explain the relation between B(x = 0) and the Hall voltage U_H. Pay attention to the fact that the output voltage U is the amplified signal from U_H. The theoretical value of B(x = 0) can be found from Table 1.
4. Plot the curve U vs. B and find the sensitivity K_H by a linear fit (use a computer). Compare the value you obtained with the theoretical value in given in the Apparatus section and the value you have found in the first part.

4.3 Magnetic Field Distribution Inside the Solenoid

1. Measure the magnetic field distribution along the axis of the solenoid for I_M = 250 mA, record the output voltage U and the corresponding position x. Then find B = B(x). (Use the value of K_H found in the previous part of the experiment).
2. Use a computer to plot the theoretical and the experimental curve showing the magnetic field distribution inside the solenoid. Use dots for the data measured and a solid line for the theoretical curve. The origin of the plot should be at the center of the solenoid.

V. RESULTS & UNCERTAINTIES

5.1 Relation Between Sensitivity K_H and Working Voltage U_S

Based on the theoretical value, the sensitivity of the probe K_H is

$$K_H = \frac{U - U_0}{B} = \frac{2.616 - 2.500}{1.4366 \times 10^{-3} \times 2.5} = 32 \pm 2 \text{ [V/T]}$$

Below is the calculation for uncertainty

$$u_{K_H} = \sqrt{(\frac{\partial K_H}{\partial U} \cdot u_U)^2 + (\frac{\partial K_H}{\partial U_0} \cdot u_{U_0})^2} = 2 \text{ [V/T]}$$

$$u_{r,K_H} = \frac{u_{K_H}}{K_H} = 6\%$$

When we change the voltage U_s, we can still use the formulas above to calculate the sensitivities K_H and corresponding uncertainties. Below shows how to calculate the values of K_H/U_s and corresponding uncertainties

$$\frac{K_H}{U_s} = \frac{U - U_0}{B \cdot U_s} = \frac{5.248 - 5.024}{1.4366 \times 10^{-3} \times 2.5 \times 2.80} = 6.2 \pm 0.3 [\text{T}^{-1}]$$

$$u_{K_H/U_s} = \sqrt{(\frac{\partial K_H/U_s}{\partial U} \cdot u_U)^2 + (\frac{\partial K_H/U_s}{\partial U_0} \cdot u_{U_0})^2 + (\frac{\partial K_H/U_s}{\partial U_S} \cdot u_{U_S})^2} = 0.3 \text{ [T}^{-1}]$$

$$u_{r,K_H/U_s} = \frac{u_{K_H/U_s}}{K_H/U_s} = 5\%$$

The results with corresponding uncertainties are as followings (Table 3).

Table 3: Results for the first part.

#	K_H [V/T]	Uncertainty [V/T]	Uncertainty u_r	K_H/U_s [T^{-1}]	Uncertainty [T^{-1}]	Uncertainty u_r
1	13.7	0.5	4%	4.89	0.19	4%
2	18.0	0.6	3%	5.61	0.17	3%
3	21.2	0.6	3%	5.90	0.17	3%
4	24.4	0.6	2%	6.10	0.16	3%
(Data above with $6*10^{-4}$ V uncertainty for U/U_0)						
5	27	2	8%	6.1	0.5	8%
6	30	2	7%	6.3	0.5	7%
7	33	2	6%	6.3	0.4	7%
8	36	2	6%	6.4	0.4	7%
9	38	2	6%	6.4	0.4	6%
10	41	2	5%	6.4	0.4	6%
11	43	2	5%	6.4	0.4	6%
12	46	2	5%	6.4	0.3	5%
13	48	2	5%	6.3	0.3	5%
14	51	2	5%	6.4	0.3	5%
15	53	2	4%	6.3	0.3	5%
16	53	2	5%	6.0	0.3	5%
17	58	2	4%	6.3	0.3	5%
18	61	2	4%	6.4	0.3	5%
19	62	3	4%	6.2	0.3	5%

The uncertainty of U_S is given directly as 0.5%, so it very easy to get the following values (Table 4).

Table 3: The uncertainty for Us.

#	U_S [V]	Uncertainty [V]	Uncertainty u_r
1	2.800	0.014	0.5%
2	3.200	0.016	0.5%
3	3.600	0.018	0.5%
4	4.00	0.02	0.5%
5	4.40	0.02	0.5%
6	4.80	0.02	0.5%
11	6.80	0.03	0.5%
12	7.20	0.04	0.5%
13	7.60	0.04	0.5%
14	8.00	0.04	0.5%
15	8.40	0.04	0.5%
16	8.80	0.04	0.5%

7	5.20	0.03	0.5%	17	9.20	0.05	0.5%
8	5.60	0.03	0.5%	18	9.60	0.05	0.5%
9	6.00	0.03	0.5%	19	10.00	0.05	0.5%
10	6.40	0.03	0.5%				

Below is the scatter for K_H/U_s - U_s relation (Figure 5).

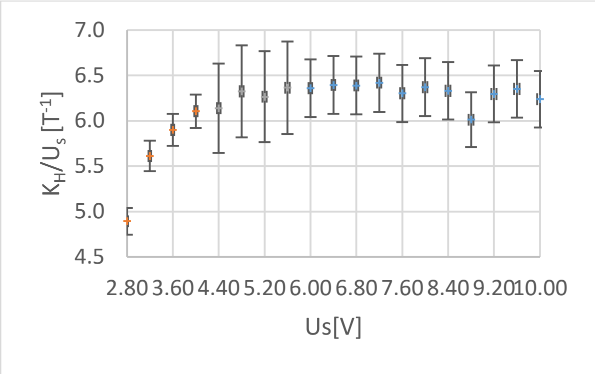

Figure 5: K_H/U_s - U_s relation.

5.2 Relation Between Output Voltage U and Magnetic Field B

Since B $\propto I_M$, calculate B can be calculated based on the values in Table 1. For example,

$$\mathrm{B} = \frac{I_M}{0.1\mathrm{A}} \cdot 1.4366\mathrm{mT} = \frac{500\mathrm{mA}}{0.1\mathrm{A}} \cdot 1.4366\mathrm{mT} = 7.18 \pm 0.14\ [\mathrm{mT}]$$

The uncertainty can be calculated as

$$u_B = \sqrt{\left(\frac{\partial B}{\partial I_M} \cdot I_M\right)^2} = 0.14[\mathrm{mT}]$$

$$u_{r,B} = \frac{u_B}{B} = 2\%$$

The U is measured directly. The uncertainty can be calculated as

$$u_U = 0.05\%\mathrm{U} + 0.6 = 0.7[\mathrm{mT}]$$

$$u_{r,U} = \frac{u_U}{U} = 0.3\%$$

Then the following data table (Table 5) can be got.

Table 5: Data for the second part.

#	B [mT]	Uncertainty [mV]	u_r	U [mV]	Uncertainty [mV]	u_r
1	0.000	0.000	2%	0.1	0.6	>100%
2	0.718	0.014	2%	23.6	0.6	3%
3	1.44	0.03	2%	46.2	0.6	1.3%
4	2.15	0.04	2%	68.8	0.6	0.9%
5	2.87	0.06	2%	91.5	0.6	0.7%
6	3.59	0.07	2%	114.2	0.7	0.6%
7	4.31	0.09	2%	129.6	0.7	0.5%
8	5.03	0.10	2%	159.4	0.7	0.4%
9	5.75	0.11	2%	181.8	0.7	0.4%
10	6.46	0.13	2%	204.4	0.7	0.3%
11	7.18	0.14	2%	226.7	0.7	0.3%

By applying linear-fit on Microsoft Excel 2016®, the following figure is got (Figure 6).

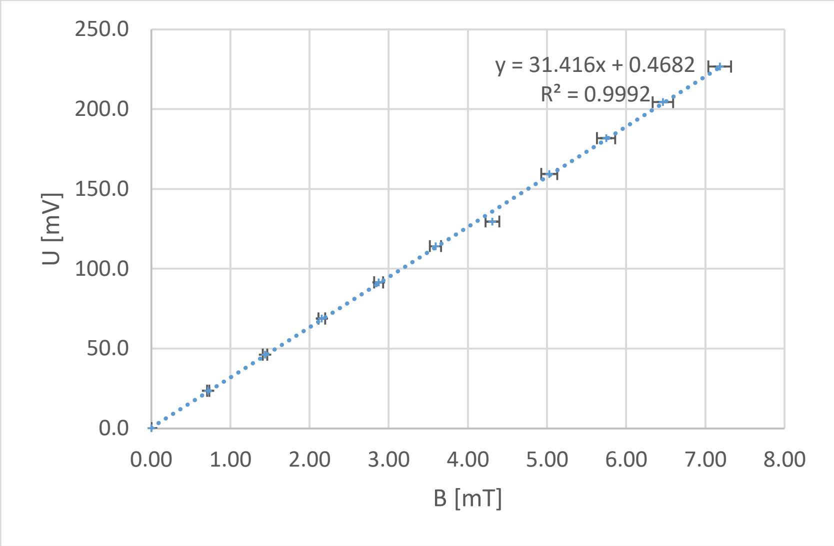

Figure 6: The linear fit for the second part.

It is clear that $U \propto B$, and since U is amplified value of U_H, i.e., $U_H \propto U$, it can be concluded that $U_H \propto B$. One explanation for this is that the higher B is, the larger the Lorentz force exerted on the charges is, then there will be more charges moved to the outer sides of the component, thereby creating a larger U_H.

The software gives the "Upper 95%" and "Lower 95%" values (32.084 and 30.749 respectively), so that we can calculate the uncertainty of the slope:

$$\text{Uncertainty} = \frac{Upper\ 95\% - Lower\ 95\%}{2} = 0.7$$

i.e. the slope (corresponding to K_H) in this experiment is 31.4 ± 0.7 V/T. It has been calculated in the first part that the theoretical value of K_H is 32 ± 2 V/T; while in the Apparatus part, the value is 31.25 ± 1.25 V/T - all of them are very close to each other. Therefore, the experimental result in this lab is quite satisfying.

5.3 Magnetic Field Distribution Inside the Solenoid

Since K_H is found as 31.4 V/T in the previous part, now B can be calculated as

$$\mathrm{B} = \frac{U}{K_H} = 1.87 \pm 0.06\ [\mathrm{mT}]$$

The uncertainty can be calculated as

$$u_B = \sqrt{\left(\frac{\partial B}{\partial K_H} \cdot u_{K_H}\right)^2 + \left(\frac{\partial B}{\partial U} \cdot u_U\right)^2} = 0.06\ [\mathrm{mT}]$$

The uncertainty of x is given directly, so the calculation part can be omitted. Then the following chart (Table 6) is got.

Table 6: Data for the third part.

#	x	Uncertainty		B	Uncertainty		#	x	Uncertainty		B	Uncertainty	
	[cm]	[cm]	u_r	[mT]	[mT]	u_r		[cm]	[V/T]	u_r	[mT]	[mT]	u_r
1	-13.00	0.05	0.4%	1.87	0.06	3%	21	1.00	0.05	5%	3.66	0.10	3%
2	-12.90	0.05	0.4%	1.97	0.06	3%	22	3.00	0.05	1.7%	3.66	0.10	3%
3	-12.80	0.05	0.4%	2.11	0.06	3%	23	5.00	0.05	1.0%	3.64	0.10	3%
4	-12.70	0.05	0.4%	2.17	0.07	3%	24	7.00	0.05	0.7%	3.61	0.10	3%

5	-12.60	0.05	0.4%	2.28	0.07	3%		25	8.00	0.05	0.6%	3.58	0.10	3%
6	-12.40	0.05	0.4%	2.49	0.07	3%		26	9.00	0.05	0.6%	3.54	0.10	3%
7	-12.20	0.05	0.4%	2.66	0.08	3%		27	10.00	0.05	0.5%	3.44	0.09	3%
8	-12.00	0.05	0.4%	2.79	0.08	3%		28	11.00	0.05	0.5%	3.24	0.09	3%
9	-11.80	0.05	0.4%	2.92	0.08	3%		29	11.20	0.05	0.4%	3.18	0.09	3%
10	-11.60	0.05	0.4%	3.02	0.08	3%		30	11.40	0.05	0.4%	3.11	0.09	3%
11	-11.40	0.05	0.4%	3.11	0.09	3%		31	11.60	0.05	0.4%	3.02	0.08	3%
12	-11.20	0.05	0.4%	3.17	0.09	3%		32	11.80	0.05	0.4%	2.91	0.08	3%
13	-11.00	0.05	0.5%	3.24	0.09	3%		33	12.00	0.05	0.4%	2.78	0.08	3%
14	-10.00	0.05	0.5%	3.44	0.09	3%		34	12.20	0.05	0.4%	2.65	0.08	3%
15	-9.00	0.05	0.6%	3.53	0.10	3%		35	12.40	0.05	0.4%	2.47	0.07	3%
16	-8.00	0.05	0.6%	3.58	0.10	3%		36	12.60	0.05	0.4%	2.30	0.07	3%
17	-7.00	0.05	0.7%	3.61	0.10	3%		37	12.70	0.05	0.4%	2.19	0.07	3%
18	-5.00	0.05	1.0%	3.64	0.10	3%		38	12.80	0.05	0.4%	2.09	0.06	3%
19	-3.00	0.05	1.7%	3.65	0.10	3%		39	12.90	0.05	0.4%	1.98	0.06	3%
20	-1.00	0.05	5%	3.65	0.10	3%		40	13.00	0.05	0.4%	1.89	0.06	3%

Meanwhile, the theoretical value of B can be found as

$$B = \frac{0.25\text{A}}{0.1\text{A}} \cdot 1.4366\text{mT} = 3.592\ [\text{mT}]$$

Then the following chart (Table 7) can be got.

Table 7: Data for the theoretical B's.

#	x [cm]	B [mT]		#	x [cm]	B [mT]		#	x [cm]	B [mT]
1	-13.0	1.808		12	-4.0	3.581		22	6.0	3.561
2	-12.5	2.315		13	-3.0	3.586		23	7.0	3.543
3	-12.0	2.716		14	-2.0	3.589		24	8.0	3.514
4	-11.5	2.991		15	-1.0	3.591		25	9.0	3.464
5	-11.0	3.171		16	0.0	3.592		26	10.0	3.370
6	-10.0	3.370		17	1.0	3.591		27	11.0	3.171
7	-9.0	3.464		18	2.0	3.589		28	11.5	2.991
8	-8.0	3.514		19	3.0	3.586		29	12.0	2.716
9	-7.0	3.543		20	4.0	3.581		30	12.5	2.315
10	-6.0	3.561		21	5.0	3.573		31	13.0	1.808
11	-5.0	3.573								

Then the following figure (Figure 7) is got.

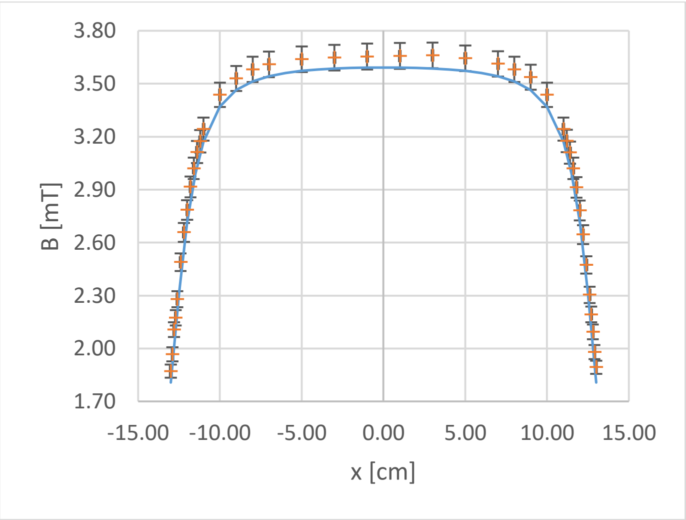

Figure 7: The plot for the third part

VI. CONCLUSIONS & DISCUSSIONS

In this lab, we studied the principle of the Hall effect and its applications by using a Hall probe, and we verified that the Hall voltage is proportional to the magnetic field; we also studied the sensitivity of an integrated Hall probe by calculating the magnetic field at the center of a solenoid; and we successfully measured the magnetic field distribution along the axis of the solenoid.

For the first part, it turns out that K_H/U_s should be a constant with respect to $U_{S.}$ However, we did not get very ideal values for the first four data points. For the second part, is is clear that the output voltage U and the magnetic field B have a linear relation. The experimental result is quite satisfying, with R^2 value of 0.9992, which is very close to the ideal value 1. For the third part, the experimental distribution is within the acceptable range to the theoretical curve, as can be seen in Figure 7.

The inaccuracies in this experiment may come from: i) the electromagnetic interference in the environment (including but not limited to mobile phones, personal computers, etc.); ii) the error due to the limited waiting time for the voltmeter to maintain at a very steady value; iii) the error due to naked-eye observation on the ruler; iv) the inaccuracy of the meters.

In addition, I have the following suggestions and ideas:

- To avoid quantum effect (which makes theory more complicated), we should use Hall probes made of metals such as Ag, Li, Rb, etc. We should not use Hall probes made of semiconductor or other divalent metal.
- To make this lab more interesting, we can further use this lab to calculate the *density of charge carriers* (denoted by n) in the conductor.

Denote the thickness of the conductor as b, the charge of carrier q, the average speed of charge carriers as u. Then $I = bdnqu$. In steady state, electro force is equal to Lorentz force: $qE = quB$.
It is easy to see that $U = R_H \frac{IB}{d} = Eb = uBb = B\frac{bdnqu}{dnq} = \frac{1}{nq} \cdot \frac{IB}{d}$.
Thus $R_H = \frac{1}{nq}$, i.e.

$$n = \frac{1}{R_H q}$$

For monovalent metals like Li, K, Na, etc, q is just the basic charge unit e.