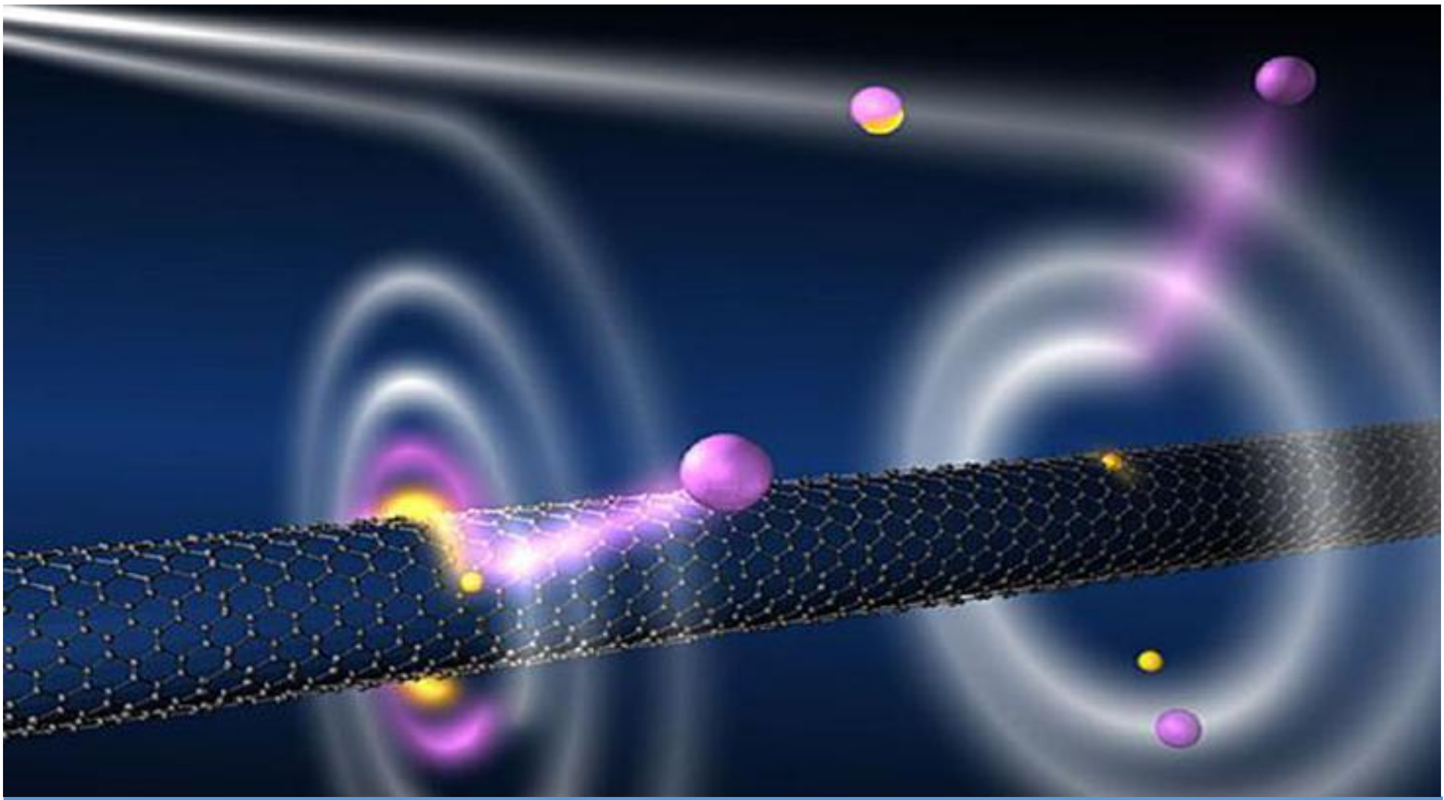

Measurements of the Moment of Inertia

Exercise 1 Report by Fenglei Gu

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I. INTRODUCTION

The main purpose of this experiment is to get familiar with the constant-torque method for measuring the moment of inertia of a rigid body.

Moment of inertia of a rigid body about an axis is a quantitative attribute that defines the body's resistance (inertia) to a change of angular velocity in rotation about that axis. In physics, we have the Second Law of Dynamics for Rotational Motion:

$$\tau_z = I\beta_z \tag{1}$$

where τ_z is the component of the torque about the axis of rotation, I is the moment of inertia about this axis, β_z is the angular acceleration component. The moment of inertia is an additive quantity, i.e. $I_{AB} = I_A + I_B$, where A and B are two rigid bodies about the same axis, AB is the combined rigid body composed of A and B. If the moment of inertia of a rigid body with mass m about an axis through the body's center of mass is I_0, then for any axis parallel to that axis, then we have Parallel Axis Theorem (Steiner's Theorem):

$$I = I_0 + md^2 \tag{2}$$

where d is the distance between the axes.

II. MEASUREMENT PROCEDURE

A) MEASUREMENT SETUP

Figure 1. *Measurement Setup*. SJTU, 2016.

The measurement setup is shown in Figure 1. suppose that the empty turntable is initially rotating and its moment of inertia with respect to the rotation axis is I_1. Since the bearings of the turntable are not frictionless, there will be a non-zero frictional torque M_μ causing the turntable to decelerate with angular acceleration β_1, so that the second law of dynamics for rotational motion of the empty turntable reads:

$$M_\mu = -I_1\beta_1 \tag{3}$$

Below the turntable, there is a conical pulley of radius R. Attached to the other end of the string passing through a disk pulley, there is a weight with mass m. If the mass moves downwards with constant acceleration a, the tension in the string T is constant and $T = m\,(g - a)$. If the turntable rotates with angular acceleration β_2, then $a = R\beta_2$. The torque the string exerts on the pulley (and hence the turntable) is then $TR = m\,(g - R\beta_2)R$. Taking into account the frictional torque, the equation of motion for the turntable reads:

$$m(g - R\beta_2)R - M_\mu = I_1\beta_2 \tag{4}$$

Eliminating M_μ from Equations (3) and (4), we find:

$$I_1 = \frac{mR(g-R\beta_2)}{\beta_2-\beta_1} \tag{5}$$

Similarly, if a rigid body with an unknown moment of inertia is placed on the turntable:

$$I_2 = \frac{mR(g-R\beta_4)}{\beta_4-\beta_3} \tag{6}$$

where β_3 is the magnitude of angular deceleration of the turntable with the body, and β_4 is its angular acceleration, when the mass m is released and moves downwards. Considering that the moment of inertia is an additive quantity, the moment of inertia of the rigid object placed on the turntable, with respect to the axis of rotation, may be found as the difference:

$$I_3 = I_2 - I_1 \tag{7}$$

At the edge of the turntable two shielding pins are fixed. An integrated counter-type electronic timer is used to measure the consecutive number k and the time t of the photo-gate signal. If (k,t) is a set of the measurement data, the corresponding angular position is:

$$\theta = k\pi = \omega_0 t + \frac{1}{2}\beta t^2 \tag{8}$$

where ω_0 is the initial angular speed.

B) EXPERIMENTAL PROCEDURE

1. Measure the mass of the weight, the hoop, the disk, and the cylinder, as well as the radius of the cone pulley and the cylinder. Calculate the moment of inertia of the hoop and the disk analytically. Use a reliable source to find the local value of the acceleration due to gravity in Minhang Heath.
2. Turn the electronic timer on and switch it to mode 1-2 (single gate, multiple pulses).
3. Place the instrument close to the edge of the desk and stretch the disk pulley arm outside, so that the weight can move downwards unobstructed.
4. Level the turntable with the bubble level.
5. Make the turntable rotating and press the start button on the timer. After at least 8 signals are recorded, stop the turntable and record the data in your data sheet.
6. Attach the weight to one end of the string. Place the string on the disk pulley, thread through the hole in the arm, and wind the string around the third ring of the cone pulley. Adjust the arm holder so that the string goes through the center of the hole.
7. Release the weight and start the timer. Stop the turntable when the weight hits the floor. Write down the recorded data.
8. The angular acceleration can be found by plotting $\theta = k\pi$ against t and performing a quadratic fit using data processing software. (The magnitude of the angular acceleration is equal to the coefficient next to t^2 multiplied by 2. The uncertainty of the angular acceleration can be read directly from the fitting result.) The moment of inertia of the empty turntable is found by using the formulae in Section 3 and the data from step 5 and 7. Repeat steps 5 ~ 7 with a rigid object placed on the turntable. Equation (7) is used to find the moment of inertia of the rigid object.

Each student should measure the moment of inertia of the empty turntable, the hoop, the disk and the cylinder, as well as verify the parallel-axis theorem by placing two cylinders off the axis while keeping their center of mass on the axis in order to keep the rotation steady.

The distances between the holes and the center of the turntable are ca. 45, 60, 75, 90, 105 mm, respectively. The timer's resolution is 0.0001 s, and the error is 0.004%.

III. RESULTS & DATA PRESENTATION

A) ANGULAR ACCELERATION

Using Microsoft Office Excel 2016, the decelerations and accelerations are calculated (see Table 1), while graphs are made accordingly (see Figures 2 - 6).

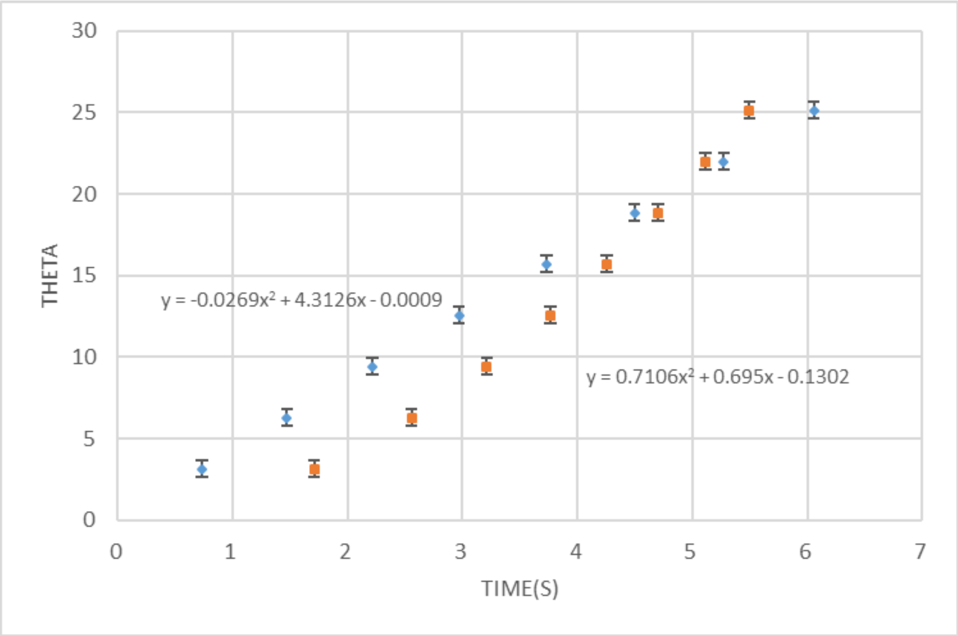

Figure 2: *Empty Turntable*

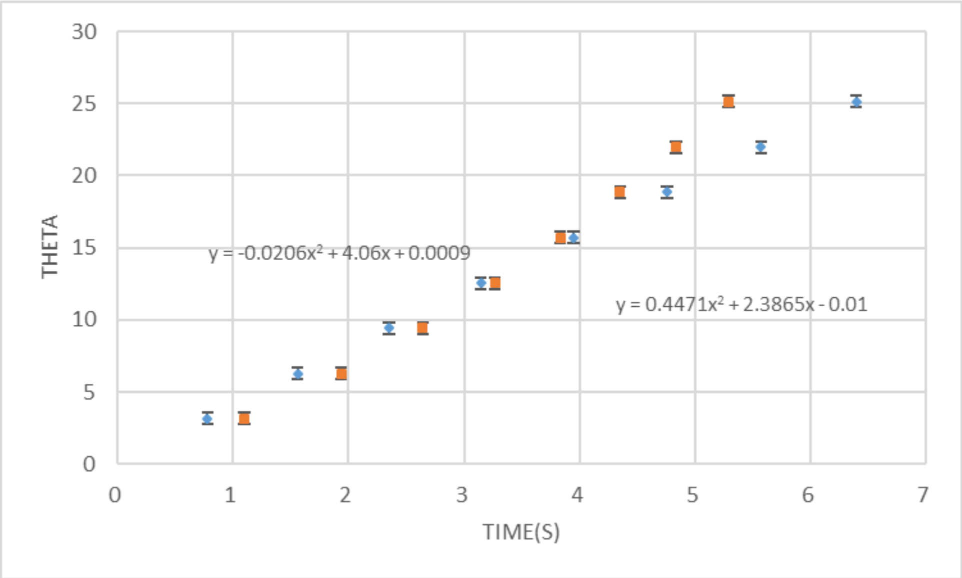

Figure 3: *With Disk*

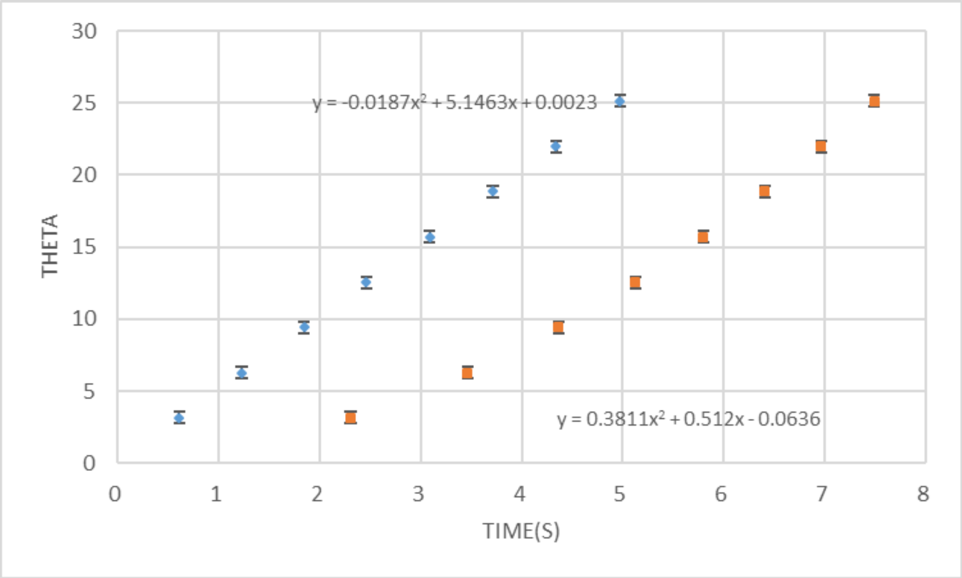

Figure 4: *With Hoop*

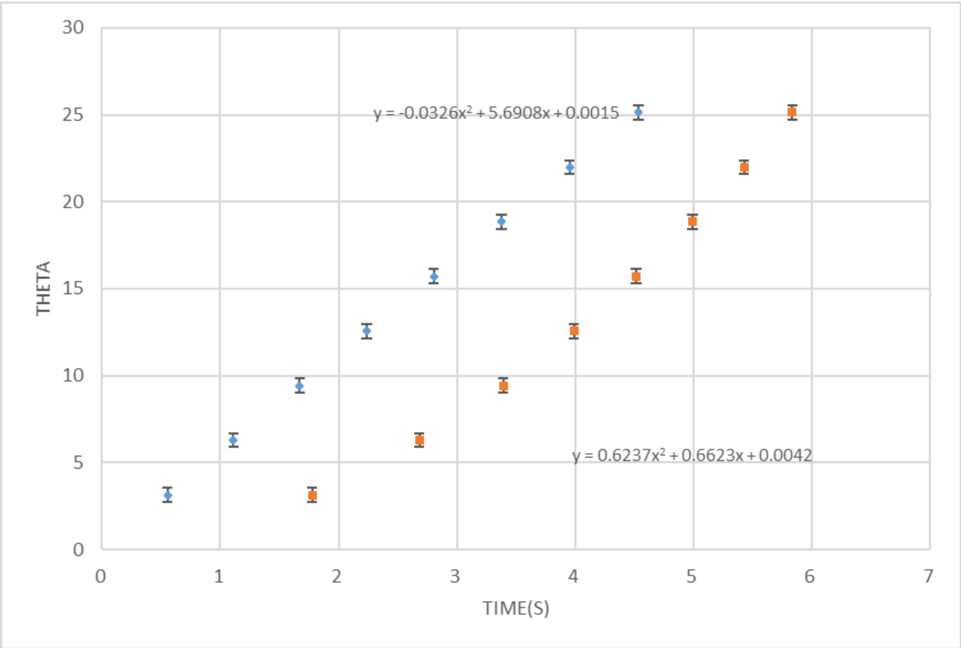

Figure 5: *Cylinders in Holes 1 & 2*

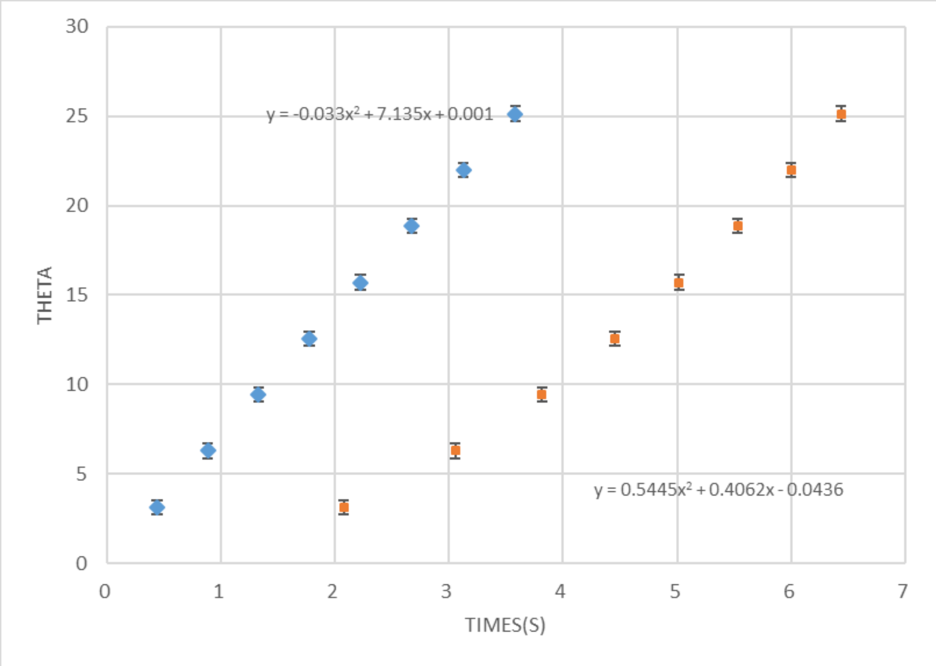

Figure 6: *Cylinders in Holes 3 & 4*

	Decelerations [rad·s^{-2}]	Accelerations [rad·s^{-2}]
Empty	-0.0269	0.7106
With Disk	-0.0206	0.4471
With Hoop	-0.0187	0.3811
Cylinders in Holes 1 & 2	-0.0326	0.6237
Cylinders in Holes 3 & 4	-0.0303	0.5445

Table 1: *Decelerations & Accelerations*

B) CALIPER RESULTS

The average values are got as followings (see Table 2).

	Values [mm] ± 0.02 [mm]
Disk Φ	239.87
Hoop Φ	225.08
Cylinder A Φ	30.01
Cylinder B Φ	30.00
Cone pulley Φ	50.05
Hole ① d	45.16
Hole ② d	45.06
Hole ③ d	75.50
Hole ④ d	75.40

Table 2: *Caliper Results*

Calculation method: (taking disk diameter as an example)

$$\text{Disk } \emptyset = \frac{239.86 + 239.84 + 239.88 + 239.90}{4} = 239.87 \text{ [mm]}$$

C) INERTIA VALUES

Taking "Cylinders in Holes 1 & 2" as an example.

$$I_1 = \frac{mR(g - R\beta_2)}{\beta_2 - \beta_1} = \frac{40.0\times10^{-2}\times50.05\times10^{-3}\times(9.794 - 50.05\times10^{-3}\times0.7106)}{0.7106 + 0.0269} = 0.0265 \text{ [kg·m}^2\text{]}$$

$$I_2 = \frac{mR(g - R\beta_4)}{\beta_4 - \beta_3} = \frac{40.0\times10^{-2}\times50.05\times10^{-3}\times(9.794 - 50.05\times10^{-3}\times0.6237)}{0.6237 + 0.0326} = 0.0298 \text{ [kg·m}^2\text{]}$$

$$I_3 = I_2 - I_1 = 0.0033 \text{ [kg·m}^2\text{]} \approx 2m_{Cylinder}d^2$$

It is easy to find that other cases (disk, hoop, etc.) obey the same rule as well (see Table 3).

	I_3 [kg·m^2]
With Disk	0.015
With Hoop	0.022
Cylinders in Holes 1 & 2	0.0033
Cylinders in Holes 3 & 4	0.0085

Table 3: *Values of Inertia*

V. CONCLUSIONS & DISCUSSION

In this experiment, the main sources of inaccuracy are: (1) error when measuring the diameters; (2) levelling; (3) frictions. By comparison with the theoretical data, the value of the measured inertia is basically in accordance with expectation, and can thus validate the theory.

To further improve this experiment, I suggest that the experiment be done in vacuum environment (cylinders, etc. can be placed with a robot arm) such that the air drag will not affect the data.

IV. UNCERTAINTY ANALYSIS & CALCULATIONS

WS-1 Caliper Measurements

The type-B uncertainty for all caliper measurements is $u_B =$ 0.02 [mm].

The type-A uncertainty for the disk diameter ϕ is

$$u_{\phi,A} = \frac{t_{0.95}}{\sqrt{4}} \sqrt{\frac{1}{4-1} \sum_{i=1}^{4} (\phi_i - \overline{\phi})^2} = \underline{\ 1.59\ } \times \underline{\ 0.12\ } = \underline{\ 0.2\ } \ [\text{ mm }].$$

The (total) uncertainty for the disk diameter

$$\boxed{u_\phi} = \sqrt{u_{\phi,A}^2 + u_{\phi,B}^2} = \sqrt{\underline{\ 0.2^2+0.02^2\ }} = \boxed{\underline{\ 0.2\ } \ [\text{ mm }]}.$$

Similarly, uncertainties of other calliper measurements can be found. The results of calculations are presented in Table WS-1.

Measurement	*type-A uncertainty u_A* [mm]	*uncertainty u* [mm]
hoop outer diameter ϕ_1	0.08	0.08
hoop inner diameter ϕ_2	0.02	0.03
cylinder A diameter ϕ_{ca}	0.02	0.03
cylinder B diameter ϕ_{cb}	0.02	0.03
cone pulley diameter ϕ_{cp}	0.04	0.04

Table WS-1: Uncertainty of calliper measurements.

The distance d from the center of holes to the axis of rotation is found as a result of two single measurements of distance $d = \frac{1}{2}(d_1 + d_2)$, with $u_{d_1} =$ 0.02 [mm] and $u_{d_2} =$ 0.02 [mm], so using the uncertainty propagation formula, we have

$$\boxed{u_d} = \sqrt{\left(\frac{\partial d}{\partial d_1}\right)^2 u_{d_1}^2 + \left(\frac{\partial d}{\partial d_2}\right)^2 u_{d_2}^2} = \sqrt{\frac{1}{4}u_{d_1}^2 + \frac{1}{4}u_{d_2}^2} = \boxed{0.014 \text{ [mm]}}.$$

WS-2 Mass Measurements

All the mass measurements, except for the mass of the hoop, are single measurements, so the uncertainty is estimated as the resolution of the electronic scales $\boxed{u_m = 0.1 \text{ [g]}}$.

For the mass of the hoop, the result is found by subtracting two masses $m_{\text{hoop}} = m_1 - m_2$, with $u_{m_1} = u_{m_2} = u_m$ hence from the uncertainty propagation formula

$$\boxed{u_{m,\text{hoop}}} = \sqrt{\left(\frac{\partial m_{\text{hoop}}}{\partial m_1}\right)^2 u_{m_1}^2 + \left(\frac{\partial m_{\text{hoop}}}{\partial m_2}\right)^2 u_{m_2}^2} = \sqrt{2u_m^2}$$
$$= \boxed{0.14 \text{ [g]}}.$$

WS-3 Theoretically Calculated Moment of Inertia

For the disk,

$$\boxed{u_{I_{\text{disk,theo}}}} = \sqrt{\left(\frac{\partial I}{\partial m}\right)^2 u_m^2 + \left(\frac{\partial I}{\partial \phi}\right)^2 u_\phi^2} = \sqrt{\left(\frac{1}{8}\phi^2\right)^2 u_m^2 + \left(\frac{1}{4}m\phi\right)^2 u_\phi^2}$$
$$= \sqrt{(1/8*479.7^2)^2*0.1^2+(1/4*488.3*479.7)*0.02^2}$$
$$= \boxed{3*10^2 \text{ [g}\cdot\text{mm}^2\text{]}}.$$

Similarly, for the hoop,

$$\boxed{u_{I_{\text{hoop,theo}}}} = \sqrt{\left(\frac{\partial I}{\partial m}\right)^2 u^2_{m,\text{hoop}} + \left(\frac{\partial I}{\partial \phi_1}\right)^2 u^2_{\phi_1} + \left(\frac{\partial I}{\partial \phi_2}\right)^2 u^2_{\phi_2}}$$

$$= \sqrt{\left(\frac{1}{8}(\phi_1^2 + \phi_2^2)\right)^2 u^2_{m,\text{hoop}} + \left(\frac{1}{4}m\phi_1\right)^2 u^2_{\phi_1} + \left(\frac{1}{4}m\phi_2\right)^2 u^2_{\phi_2}}$$

$$= \boxed{\underline{\quad 4*10^2 \quad} \ [\mathrm{g{\cdot}mm^2}]}.$$

For cylinder A with respect to the axis of symmetry,

$$u_{I_{\text{A,symmetry,theo}}} = \sqrt{\left(\frac{\partial I}{\partial m}\right)^2 u^2_m + \left(\frac{\partial I}{\partial \phi_{ca}}\right)^2 u^2_{\phi_{ca}}} = \sqrt{\left(\frac{1}{8}\phi_{ca}^2\right)^2 u^2_m + \left(\frac{1}{4}m\phi_{ca}\right)^2 u^2_{\phi_{ca}}}$$

$$= \boxed{\underline{\quad 8*10^2 \quad} \ [\mathrm{g{\cdot}mm^2}]}.$$

For cylinder B with respect to the axis of symmetry,

$$u_{I_{\text{B,symmetry,theo}}} = \sqrt{\left(\frac{\partial I}{\partial m}\right)^2 u^2_m + \left(\frac{\partial I}{\partial \phi_{cb}}\right)^2 u^2_{\phi_{cb}}} = \sqrt{\left(\frac{1}{8}\phi_{cb}^2\right)^2 u^2_m + \left(\frac{1}{4}m\phi_{cb}\right)^2 u^2_{\phi_{cb}}}$$

$$= \boxed{\underline{\quad 2*10^2 \quad} \ [\mathrm{g{\cdot}mm^2}]}.$$

Recall that from the parallel axis theorem, $I_{\text{theo}} = I_{\text{c.m.,theo}} + md^2$, where $I_{\text{c.m.,theo}}$ is the moment of inertia with respect to an axis through the center of mass (here the axis of symmetry, so that $I_{\text{c.m.,theo}} = I_{\text{symmetry,theo}}$). Hence, the corresponding uncertainty

$$u_{I_{\text{theo}}} = \sqrt{\left(\frac{\partial I}{\partial I_{\text{symmetry,theo}}}\right)^2 u^2_{I_{\text{symmetry,theo}}} + \left(\frac{\partial I}{\partial m}\right)^2 u^2_m + \left(\frac{\partial I}{\partial d}\right)^2 u^2_d}$$

$$= \sqrt{(1)^2 (u_{I_{\text{symmetry,theo}}})^2 + (d^2)^2 u^2_m + (2md)^2 u^2_d}.$$

In particular, for cylinder A in hole ①

$$\boxed{u_{I_{\text{A in ①, theo}}}} = \sqrt{(1)^2(u_{I_{\text{A,symmetry,theo}}})^2 + (d_1^2)^2 u_{m_A}^2 + (2m_A d_1)^2 u_{d_1}^2}$$

$$= \boxed{\underline{\quad 9*10^2 \quad} \; [\underline{\text{g·mm}^2}]}.$$

For cylinder B in hole ②,

$$\boxed{u_{I_{\text{B in ②,theo}}}} = \sqrt{(1)^2(u_{I_{\text{B,symmetry,theo}}})^2 + (d_2^2)^2 u_{m_B}^2 + (2m_B d_2)^2 u_{d_2}^2}$$

$$= \boxed{\underline{\quad 5*10^2 \quad} \; [\underline{\text{g·mm}^2}]}.$$

For cylinder A in hole ③,

$$\boxed{u_{I_{\text{A in ③,theo}}}} = \sqrt{(1)^2(u_{I_{\text{A,symmetry,theo}}})^2 + (d_3^2)^2 u_{m_A}^2 + (2m_A d_3)^2 u_{d_3}^2}$$

$$= \boxed{\underline{\quad 1.0*10^3 \quad} \; [\underline{\text{g·mm}^2}]}.$$

For cylinder B in hole ④,

$$\boxed{u_{I_{\text{B in ④,theo}}}} = \sqrt{(1)^2(u_{I_{\text{B,symmetry,theo}}})^2 + (d_4^2)^2 u_{m_B}^2 + (2m_B d_4)^2 u_{d_4}^2}$$

$$= \boxed{\underline{\quad 7*10^2 \quad} \; [\underline{\text{g·mm}^2}]}.$$

The remaining uncertainties are those generated by summation and subtraction among moments of inertia, so the uncertainty of these operations $I_2 = I_1 \pm I_3$ is $u_{I_2} = \sqrt{u_{I_1}^2 + u_{I_3}^2}$. In particular.

$$\boxed{u_{I_{\text{A in ①, B in ②,theo}}} = \underline{\quad 1.0*10^3 \quad} \; [\underline{\text{g·mm}^2}]},$$

$$\boxed{u_{I_{\text{A in ③, B in ④,theo}}} = \underline{\quad 1.0*10^3 \quad} \; [\underline{\text{g·mm}^2}]}.$$

WS-4 Time Measurements

For a single measurement of t, its type-B uncertainty is $u_t = \Delta_{t,B} = \Delta_{\text{relative}} + \Delta_{\text{dev}}$ with $\boxed{\Delta_{\text{dev}} = \underline{0.0001} \; [\underline{\;\text{s}\;}]}$. Uncertainties of individual measurements are given in Table WS-2.

		Deceleration					Acceleration			
Empty turntable	k	1	2	3	4	k	1	2	3	4
	u_t [s]	$1.3*10^4$	$1.6*10^4$	$1.9*10^4$	$2*10^4$	u_t [s]	$1.7*10^4$	$2*10^4$	$2*10^4$	$2*10^4$
	k	5	6	7	8	k	5	6	7	8
	u_t [s]	$3*10^4$	$3*10^4$	$3*10^4$	$3*10^4$	u_t [s]	$3*10^4$	$3*10^4$	$3*10^4$	$3*10^4$
		Deceleration					Acceleration			
With disk	k	1	2	3	4	k	1	2	3	4
	u_t [s]	$1.3*10^4$	$1.6*10^4$	$1.9*10^4$	$2*10^4$	u_t [s]	$1.4*10^4$	$1.8*10^4$	$2*10^4$	$3*10^4$
	k	5	6	7	8	k	5	6	7	8
	u_t [s]	$2*10^4$	$3*10^4$	$3*10^4$	$3*10^4$	u_t [s]	$3*10^4$	$3*10^4$	$3*10^4$	$3*10^4$
		Deceleration					Acceleration			
With hoop	k	1	2	3	4	k	1	2	3	4
	u_t [s]	$1.2*10^4$	$1.5*10^4$	$1.8*10^4$	$2*10^4$	u_t [s]	$1.9*10^4$	$2*10^4$	$3*10^4$	$3*10^4$
	k	5	6	7	8	k	5	6	7	8
	u_t [s]	$2*10^4$	$3*10^4$	$3*10^4$	$3*10^4$	u_t [s]	$3*10^4$	$3*10^4$	$4*10^4$	$4*10^4$
		Deceleration					Acceleration			
With cylinder A in hole ①, B in ②	k	1	2	3	4	k	1	2	3	4
	u_t [s]	$1.2*10^4$	$1.4*10^4$	$1.6*10^4$	$1.9*10^4$	u_t [s]	$1.7*10^4$	$2*10^4$	$2*10^4$	$3*10^4$
	k	5	6	7	8	k	5	6	7	8
	u_t [s]	$2*10^4$	$3*10^4$	$3*10^4$	$3*10^4$	u_t [s]	$3*10^4$	$3*10^4$	$3*10^4$	$3*10^4$
		Deceleration					Acceleration			
With cylinder A in hole ③, B in ④	k	1	2	3	4	k	1	2	3	4
	u_t [s]	$1.2*10^4$	$1.4*10^4$	$1.5*10^4$	$1.7*10^4$	u_t [s]	$1.8*10^4$	$2*10^4$	$2*10^4$	$2*10^4$
	k	5	6	7	8	k	5	6	7	8
	u_t [s]	$2*10^4$	$2*10^4$	$3*10^4$	$3*10^4$	u_t [s]	$3*10^4$	$3*10^4$	$3*10^4$	$4*10^4$

Table WS-2: Uncertainty of time measurements.

WS-5 Angular Acceleration

The angular acceleration/deceleration is equal to twice the value of the coefficient next to t^2, found in a quadratic fit procedure to the measurement data.

$$u_\beta = 2 \times t_{0.95} \times SD$$

where SD is the standard deviation of the coefficient, and $t_{0.95}$ = 2.57 has been found for $n-2$, with the number of measurements used in the fitting procedure n = 8. The fit was performed in Microsoft Office Excel [1] and the results are presented in Table WS-3.

[1] Please give the name of software that you used to perform the fit. Also, please print out the summary tables generated by the program, and attach them to this worksheet (combine tables together to save paper). That sounds hypocritical. The best way is to grade this report on an electronic version.

	SD [rad/s²]	$Deceleration\ u_{\beta_d}$ [rad/s²]	SD [rad/s²]	$Acceleration\ u_{\beta_a}$ [rad/s²]
Empty turntable	4.30*10^{-4}	2.037*10^{-3}	3.30*10^{-4}	1.540*10^{-3}
With disk	7.02*10^{-5}	3.651*10^{-4}	1.77*10^{-4}	9.342*10^{-3}
With hoop	2.17*10^{-4}	1.173*10^{-4}	2.46*10^{-5}	1.227*10^{-4}
A in ①, B in ②	1.67*10^{-4}	8.022*10^{-4}	2.02*10^{-4}	1.135*10^{-3}
A in ③, B in ④	1.89*10^{-4}	9.897*10^{-3}	3.32*10^{-4}	1.593*10^{-3}

Table WS-3: Uncertainty of the angular deceleration/acceleration.

WS-6 Experimentally Measured Moment of Inertia

The moment of inertia in this experiment is found from formulas (5), (6), (7) given in the manual, by measuring the angular acceleration, the mass of the weight, and the diameter of the cone pulley. To find the corresponding uncertainty of the moment of inertia, several applications of the uncertainty propagation formula are required.

The uncertainty of the moment of inertia of the combined object (turntable + rigid body) is found first as

$$u_I = \sqrt{\left(\frac{\partial I}{\partial m}u_m\right)^2 + \left(\frac{\partial I}{\partial \phi_{cp}}u_{\phi_{cp}}\right)^2 + \left(\frac{\partial I}{\partial \beta_a}u_{\beta_a}\right)^2 + \left(\frac{\partial I}{\partial \beta_d}u_{\beta_d}\right)^2},$$

with

$$\begin{aligned}
\frac{\partial I}{\partial m} &= \frac{(\phi_{cp}/2)(g-(\phi_{cp}/2)\beta_a)}{\beta_a-\beta_d},\\
\frac{\partial I}{\partial \phi_{cp}} &= \frac{m(g-\beta_a\phi_{cp})}{2(\beta_a-\beta_d)},\\
\frac{\partial I}{\partial \beta_a} &= \frac{m\phi_{cp}(\beta_d\phi_{cp}-2g)}{4(\beta_d-\beta_a)^2},\\
\frac{\partial I}{\partial \beta_d} &= -\frac{m\phi_{cp}(\beta_a\phi_{cp}-2g)}{4(\beta_a-\beta_d)^2}.
\end{aligned}$$

For instance, the uncertainty for the experimentally measured moment of inertia for the empty turntable is found as

$u^2_{I_{\text{turntable}}}$ = ____

____

____

$= \underline{\quad 0.11 \quad}$ [$kg^2 \cdot m^4$].

Hence

$$\boxed{u_{I_{\text{turntable}}} = \underline{\quad 0.3 \quad} \; [kg \cdot m^2]}.$$

In the same way, the uncertainty of the moments of inertia of other combined objects are found. The results are listed in Table WS-4.

Object	$u_{I_{\text{object}}}$ [$kg \cdot m^2$]
Turntable + disk	0.6
Turntable + hoop	0.13
Turntable + A in hole ①, B in ②	0.2
Turntable + A in hole ③, B in ④	0.4

Table WS-4: Uncertainty of the moment of inertia (combined objects).

The uncertainty of the moment of inertia of a rigid body (without the turntable) is found as $I_{\text{body}} = I - I_{\text{turntable}}$ is calculated as

$$u_{I_{\text{body}}} = \sqrt{\left(\frac{\partial I_{\text{body}}}{\partial I} u_I\right)^2 + \left(\frac{\partial I}{\partial I_{\text{turntable}}} u_{I_{\text{turntable}}}\right)^2} = \sqrt{u_I^2 + u_{I_{\text{turntable}}}^2},$$

and the results are presented in Table WS-5.

Rigid body	$u_{I_{\text{body}}}$ [$kg \cdot m^2$]
Disk	0.8
Hoop	0.8
A in hole ①, B in ②	0.8
A in hole ③, B in ④	0.8

Table WS-5: Uncertainty of the moment of inertia (rigid bodies).

For instance, the uncertainty for the moment of the disk is

$\boxed{u_{\text{I,disk}}} = \underline{\quad \text{sqrt}(u_I^2 + u_{I_{\text{disk}}}^2) \quad}$

$= \boxed{\underline{\quad 0.3 \quad} \; [kg \cdot m^2]}$