Monády

Monady sú použiteľný nástroj pre programátora poskytujúci:

- modularitu – skladať zložitejšie výpočty z jednoduchších (no side-effects),
- flexibilitu – výsledný kód je ľahšie adaptovateľný na zmeny,
- izoluje side-effect operácie (napr. IO) od čisto funkcionálneho zvyšku.

Štruktúra prednášok:

- Monády - prvý dotyk
  - Functor
  - Applicative
  - Monády – princípy a zákony
- Najbežnejšie monády
  - Maybe/Error monad
  - List monad
  - IO monad
  - State monad
  - Reader/Writer monad
  - Continuation monad
- Transformátory monád
- Monády v praxi

Monády – úvod

- Phil Wadler: https://homepages.inf.ed.ac.uk/wadler/papers/marktoberdorf/baastad.pdf
  Monads for Functional Programming In *Advanced Functional Programming*, Springer Verlag, LNCS 925, 1995,
- Jeff Newbern's: All About Monads https://www.cs.rit.edu/~swm/cs561/All_About_Monads.pdf
- A Gentle Introduction to Haskell,
  https://www.haskell.org/tutorial/monads.html
  https://wiki.haskell.org/All_About_Monads
- Sujit Kamthe: Understanding Functor and Monad With a Bag of Peanuts
  https://medium.com/beingprofessional/understanding-functor-and-monad-with-a-bag-of-peanuts-8fa702b3f69e
- Functors, Applicatives, And Monads In Pictures
  http://adit.io/posts/2013-04-17-functors,_applicatives,_and_monads_in_pictures.html
- Monads in Haskell and Category Theory
  https://www.diva-portal.org/smash/get/diva2:1369286/FULLTEXT01.pdf

Monads, Arrows, and Idioms

Philip Wadler, https://homepages.inf.ed.ac.uk/wadler/topics/monads.html

Články Phila Wadlera na stránke

- Monads for functional programming
- The essence of functional programming
- Comprehending monads
- The arrow calculus
- Monadic constraint programming
- Idioms are oblivious, arrows are meticulous, monads are promiscuous
- The marriage of effects and monads
- How to declare an imperative
- Imperative functional programming

What the hell are Monads?

Noel Winstanley,
https://web.archive.org/web/19991018214519/http://www.dcs.gla.ac.uk/~nww/Monad.html

Obsah:

- Maybe
- State
- The Monad Class
- Do notation
- Monadic IO
- Programming in the IO Monad

All About Monads

Jeff Newbern, https://www.cs.rit.edu/~swm/cs561/All_About_Monads.pdf

Obsah:

Part I - Understanding Monads

What is a monad? Meet the Monads. Doing it with class Monad support in Haskell

Part II - A Catalog of Standard Monads

Introduction. The Identity monad. The Maybe monad. The Error monad. The List monad. The IO monad. The State monad. The Reader monad. The Writer monad. The Continuation monad.

Part III - Monads in the Real

Combining monads the hard way. Monad transformers. Standard monad transformers. Anatomy of a monad transformer. More examples with monad transformers. Managing the transformer.

Když egyptský král Ptolemaios žádal slavného matematika Euklida o jednodušší cestu k pochopení matematiky (jako faraon se nechtěl obtěžovat těžkou prací studenta), Euklides mu nekompromisně odpověděl:
"V matematice neexistuje žádná královská cesta."

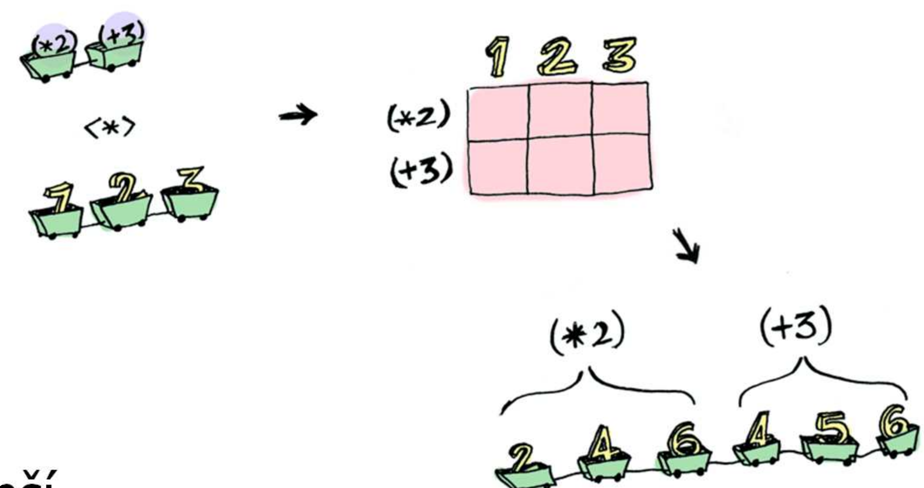
Roadmap

- Haskell má triedy, ale sú to vlastne konceptuálne interface (Java)
- Haskell má podtriedy, čo je konceptuálne dedenie na interface (Java)
- dedenie na interface ste určite v Jave videli, napr. na kolekciách

Relevantné triedy v Haskelli:

- Functor
- Applicatives
- Monad
- MonadPlus
- ...

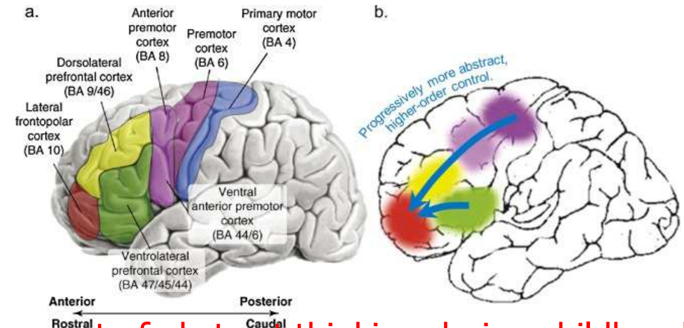
Takže monáda nie je najjednoduchší typ v tejto hierarchii



Alternatívny prístup:
Functors, Applicatives, And Monads In Pictures
http://adit.io/posts/2013-04-17-functors,_applicatives,_and_monads_in_pictures.html

Functor

prvotná idea



[Development of abstract thinking during childhood and adolescence: The role of rostrolateral prefrontal cortex]

```
double :: [Int] -> [Int]
double [] = []
double (x:xs) = (x+x):double xs
```

```
sqr :: [Int] -> [Int]
sqr [] = []
sqr (x:xs) = x*x: sqr xs
```

```
map :: (a->b) -> [a] -> [b]
map f [] = []
map f (x:xs) = f x : map f xs
```

```
map (*2)
```

map (\x->x+x)

```
map (^2)
```

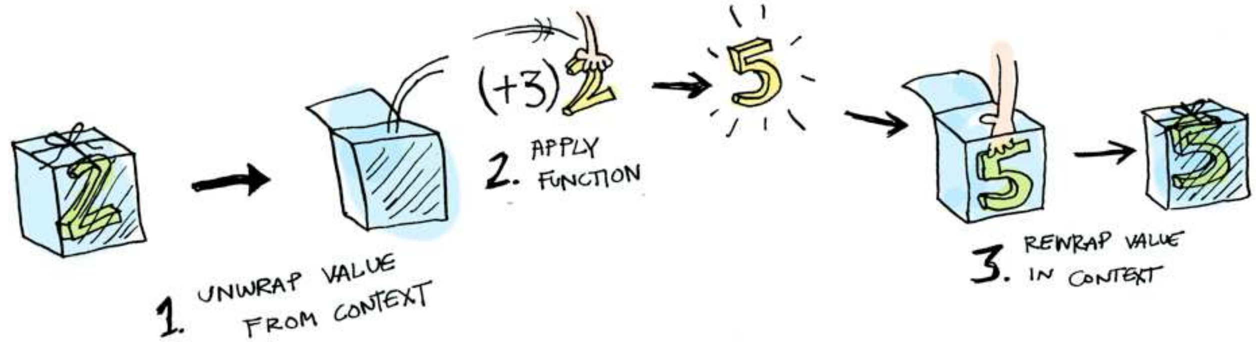
map (\x->x*x)

```
class Functor f where
  fmap :: (a->b) -> f a -> f b
```

fmap aplikuje funkciu f na hodnoty zabalené do typu, ktorý implementuje interface Functor

typ [ ] implementuje interface Functor tak, že fmap = map

Functor



http://adit.io/posts/2013-04-17-functors,_applicatives,_and_monads_in_pictures.html

Zoberme jednoduchšiu triedu, z modulu Data.Functor je definovaná takto:

-- každý typ t, ak implementuje Funtor t,

class Functor t where -- musí mať funkciu fmap s profilom

fmap :: (a -> b) -> t a -> t b -- haskell class je podobne java interface

a každá jej inštancia musí spĺňať <u>dve pravidlá</u> (to je <u>sémantika</u>, mimo syntaxe)

- fmap id = id -- identita
- fmap (p . q) = (fmap p) . (fmap q) -- kompozícia

Cvičenie1: Príklad inštancie pre **data M_1 a = Raise String | Return a**, overte, že platia obe sémantické pravidlá:

instance Functor M1 where

fmap **f** (Raise str) = Raise str

fmap **f** (Return x) = Return (**f** x)

Cvičenie

```
class Functor t where
  fmap :: (a -> b) -> t a -> t b
```

Definícia:

```
fmap  f  (Raise str)    =  Raise str
fmap  f  (Return x)     =  Return (f x)
```

Dokázať:

```
fmap id        = id
fmap (p . q)  = (fmap p) . (fmap q)
```

Cvičenie1 (pokrač.):

- fmap id =? id
  - fmap id (Raise str) = Raise str
  - fmap id (Return x) = Return (id x) = Return x

- fmap (p.q) =? (fmap p) . (fmap q)
  - Prípad Raise error:
  - L.S. = fmap (p.q) (Raise str) = Raise str
  - P.S. = ((fmap p) . (fmap q)) (Raise str) = (fmap p) ((fmap q) (Raise str)) = Raise str

  - Prípad Return hodnota:
  - L.S. = fmap (p.q) (Return x) = Return ((p.q) x) = (Return (p (q x)))
  - P.S. = ((fmap p) . (fmap q)) (Return x)

    = (fmap p) ((fmap q) (Return x))

    = (fmap p) (Return (q x)) = (Return (p (q x)))…. q.e.d.

Functor

Maybe, List

```
class Functor t where
  fmap :: (a -> b) -> t a -> t b

fmap id        = id
fmap (p . q)  = (fmap p) . (fmap q)
```

Cvičenie2: Definujte inštanciu triedy Functor pre typy:

data MyMaybe a = MyJust a | MyNothing deriving (Show)   -- alias Maybe a

data MyList a   = Null | Cons a (MyList a) deriving (Show)   -- alias [a]

… a pochopíte, ako je Functor definovaný pre štandardné typy Maybe a [].

```
> fmap (even) (Cons 1 (Cons 2 Null))                -- f : Int->Bool
Cons False (Cons True Null)
> fmap (\s -> s+s) (Cons 1 (Cons 2 Null))           -- f : Int->Int
Cons 2 (Cons 4 Null)
> fmap (show) (Cons 1 (Cons 2 Null))                -- f : Int->String
Cons "1" (Cons "2" Null)

> fmap ((\t -> t++t) . (show)) (Cons 1 (Cons 2 Null))    -- f : (String->String).(Int->String)
Cons "11" (Cons "22" Null)
> fmap (\t -> t++t)   (fmap (show) (Cons 1 (Cons 2 Null))) -- “overenie” vlastnosti kompozície
Cons "11" (Cons "22" Null)

> fmap id (Cons 1 (Cons 2 Null))                    --  overenie vlastnosti identity
Cons 1 (Cons 2 Null)
```

Functor

Maybe, List

```
class Functor t where
  fmap :: (a -> b) -> t a -> t b

fmap id        = id
fmap (p . q)   = (fmap p) . (fmap q)
```

Cvičenie2 (pokrač.): Definujte inštanciu triedy Functor pre typy:

```
data MyMaybe a = MyJust a | MyNothing deriving (Show)      -- alias Maybe a
data MyList a    = Null | Cons a (MyList a) deriving (Show)  -- alias [a]

instance Functor MyMaybe  where
      fmap f MyNothing    =  MyNothing
      fmap f (MyJust x)   =  MyJust (f x)

instance  Functor MyList  where
      fmap f Null = Null
      fmap f (Cons x xs) = Cons (f x) (fmap f xs)

instance  Functor []  where
      fmap = map
```

```
> fmap even [1,2,3]
[False,True,False]
> fmap (*2) [1,2,3]
[2,4,6]
> fmap (show) [1,2,3]
["1","2","3"]
> fmap (\x->x++x) $ fmap (show) [1,2,3]
["11","22","33"]
> fmap ((\x->x++x). show) [1,2,3]
["11","22","33"]
```

... stále ale chýba dôkaz platnosti dvoch vlastností ...

Functor – strom

```
class Functor t where
   fmap :: (a -> b) -> t a -> t b
fmap id         = id
fmap (p . q)  = (fmap p) . (fmap q)
```

Cvičenie3: Binárny strom (skoro ako tradičný LExp, ale parametrizovaný typ):

data LExp a = ID a | APP (LExp a) (LExp a) | ABS a (LExp a) deriving (Show)

instance Functor LExp where

```
    fmap f (ID x)                    = ID (f x)
    fmap f (APP left right)          = APP (fmap f left) (fmap f right)
    fmap f (ABS x body)              = ABS (f x) (fmap f body)
```

```
omega = ABS "x" (APP (ID "x") (ID "x"))
> fmap (\t -> t++t) omega
ABS "xx" (APP (ID "xx") (ID "xx"))
> fmap (\t -> (length t)) omega
ABS 1 (APP (ID 1) (ID 1))
```

Cvičenie4: Ľubovoľne n-árny strom (prezývaný RoseTree alias Rhododendron):

data RoseTree a = Rose a [RoseTree a]

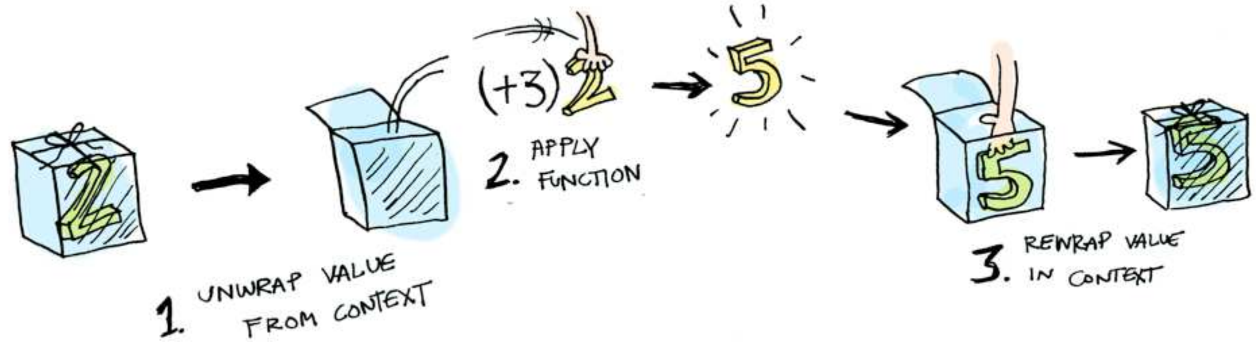
instance Functor RoseTree where

```
    fmap f (Rose a bs)        = Rose (f a) (map (fmap f) bs)
```

... opäť chýba dôkaz platnosti vlastností pre Cvičenie 3 aj 4 ...

Functor

zhrnutie



http://adit.io/posts/2013-04-17-functors,_applicatives,_and_monads_in_pictures.html

```
instance Functor [] where
  -- fmap :: (a->b) -> [a] -> [b]
   fmap = map
```

```
instance Functor Maybe where
   -- fmap :: (a->b) -> Maybe a -> Maybe b
   fmap _ Nothing = Nothing
   fmap g (Just x) = Just (g x)
```

```
instance Functor IO where
  -- fmap :: (a->b) -> IO a -> IO b
   fmap g mx = do { x<-mx; return (g x) }
```

```
infixl 4 <$>
<$>  = fmap
main :: IO ()
main = do  res <- words <$> getLine
           res <- fmap words getLine
           putStrLn $ show res
```

```
fmap f x … f <$> x
```

```
double :: Functor t => t Integer -> t Integer
double = fmap (*2)

sqr :: Functor t => t Integer -> t Integer
sqr = fmap (^2)
```

```
double (Just 7) = Just 14
double [1,2,3,4] = [2,4,6,8]

double (Branch (Leaf 7) (Leaf 9)) =
          Branch (Leaf 14) (Leaf 18)

double (Rose 3 [Rose 5 [], Rose 7 []]) =
          Rose 6 [Rose 10 [],Rose 14 []]
```

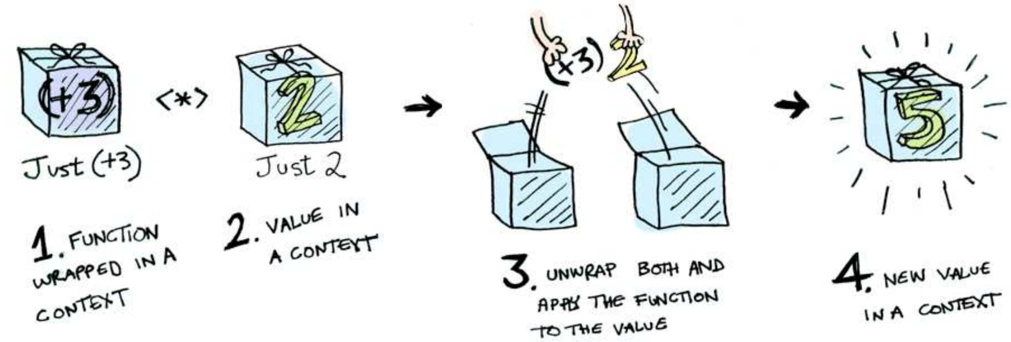
Applicative

prvotná idea

- Functor predstavuje abstrakciu aplikácie **unárnej funkcie** na každý prvok "Functor-like" dátovej štruktúry, nech je akákoľvek komplikovaná...
- Čo, ak by sme mali funkcie s veľa argumentami (nie len unárne):
  - fmap0 :: a -> f a
  - fmap1 :: (a->b) -> f a -> f b -- fmap
  - fmap2 :: (a->b->c) -> f a -> f b -> f c
  - fmap3 :: (a->b->c->d) -> f a -> f b -> f c -> f d -- ☹
- riešenie = **Currying** je transformácia funkcie s mnohými argumentami na unárnu, ktorá vráti inú funkciu,ktorá skonzumuje všetky ďalšie argumenty
  - **pure :: a -> f a**
  - **(<*>) :: f (a->b) -> f a -> f b** **infixl**

  Napr. nech g chce "tri argumenty"
  - pure g <*> x <*> y <*> z = ((pure g <*> x) <*> y) <*> z
- Hierarchia -- x :: f a, y :: f b, z :: f c
  - pmap0 g0 = pure g0 -- g0 je konštanta, lebo má 0-args.
  - fmap1 g1 x = pure g1 <*> x -- g1 :: a->b, pure g1 :: f (a->b)
  - fmap2 g2 x y = pure g2 <*> x <*> y -- g2 :: a->b->c, pure g2::f (a->b->c)
  - fmap3 g3 x y z = pure g3 <*> x <*> y <*> z -- g3 :: a->b->c->d,x::f a,y::f b,z::f c

Applicative



http://adit.io/posts/2013-04-17-functors,_applicatives,_and_monads_in_pictures.html

class **Functor f => Applicative f** where -- Applicative je podtrieda Functor

pure :: a -> f a

(<*>) :: f (a -> b) -> f a -> f b (infixl 4)

a každá jej inštancia musí spĺňať pravidlá (to je sémantika, mimo syntaxe)

- pure id <*> v = v -- identita
- pure (.) <*> u <*> v <*> w = u <*> (v <*> w) -- kompozícia
- pure f <*> pure x = pure (f x) -- homomorfizmus
- u <*> pure y = pure ($ y) <*> u = pure (\g->g y) <*> u -- výmena

infixr 9 .
(.) :: (b -> c) -> (a -> b) -> (a -> c)
(f . g) x = f (g x)

Príklad (pre M1):

Return id <*> Return 4 = Return 4

Return (.) <*> Return (+1) <*> Return (+2) <*> Return 4 = Return 7

Return (+1) <*> (Return (+2) <*> Return 4) = Return 7

Return (+4) <*> Return 3 = Return 7 -- pure f <*> pure x = pure (f x)

data M1 a = Raise String | Return a deriving(Show, Read, Eq)

Applicative

```
class Functor f => Applicative f where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b

pure id <*> v = v                                    -- identita
pure (.) <*> u <*> v <*> w = u <*> (v <*> w) -- kompozícia
pure f <*> pure x = pure (f x)                       -- homomorfizmus
u <*> pure y = pure ($ y) <*> u = pure (\g->g y) <*> u
```

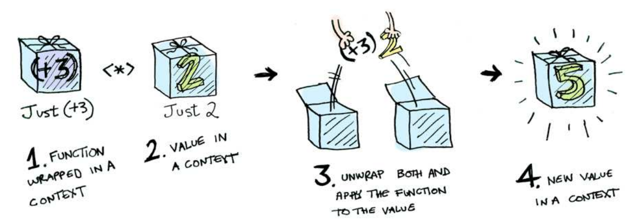
Cvičenie5: definujte inštanciu M1 pre triedu Applicative a overte 4 pravidlá:

```
instance Applicative M1 where
        pure a                  = Return a
        (Raise e)  <*> _        = Raise e       -- e:: String, Raise e::M1 a
        (Return f) <*> a        = fmap f a      -- f::a->b, Return f :: M1(a->b)
```

```
infixr 0 $
($) :: (a -> b) -> a -> b
f $ x = f x
```

Príklad:

1) Return id <*> Return 4 = Return 4



2) L.S. = Return (.) <*> Return (+1) <*> Return (+2) <*> Return 4 = Return 7
   P.S. = Return (+1) <*> (Return (+2) <*> Return 4) = Return 7

3) Return (+4) <*> Return 3 = Return 7 -- pure f <*> pure x = pure (f x)

4) Return (+2) <*> Return 7 = Return 9 = Return ($ 7) <*> Return (+2)

```
data M1 a     =    Raise String | Return a  deriving(Show, Read, Eq)
```

Applicative

```
class Functor f => Applicative f where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b

pure id <*> v = v                              -- identita
pure (.) <*> u <*> v <*> w = u <*> (v <*> w) -- kompozícia
pure f <*> pure x = pure (f x)                 -- homomorfizmus
u <*> pure y = pure ($ y) <*> u = pure (\g->g y) <*> u
```

Cvičenie5 (pokrač.): definujte inštanciu M1 pre Applicative a overte pravidlá:

```
instance Applicative M1 where
        pure a              = Return a
        (Raise e)  <*> _    = Raise e     -- e:: String, Raise e::M1 a
        (Return f) <*> a    = fmap f a    -- f::a->b, Return f :: M1(a->b)
```

```
infixr 0 $
($) :: (a -> b) -> a -> b
f $ x = f x
```

Dôkaz:

1) (Return id) <*> v = fmap id v = v   pravidlo identity pre Functors

3) pure f <*> pure x = (Return f) <*> (Return x) = fmap f (Return x) = Return (f x) = pure (f x)

2) (Return (.)) <*> (Return fu) <*> (Return fv) <*> (Return fw) =
   (Return (.) fu) <*> (Return fv) <*> (Return fw) =
   (Return ((.) fu) fv) <*> (Return fw) = (Return (fu . fv)) <*> (Return fw) =
   (Return ((fu . fv) fw)) = Return (fu (fv (fw)))

4) L.S. = (Return f) <*> (Return y) = fmap f (Return y) = (Return (f y))
   P.S. = (Return ($ y)) <*> (Return f) = fmap ($ y) (Return f) = Return (($ y) f) =
   Return ((\g->g y) f) = Return (f y)

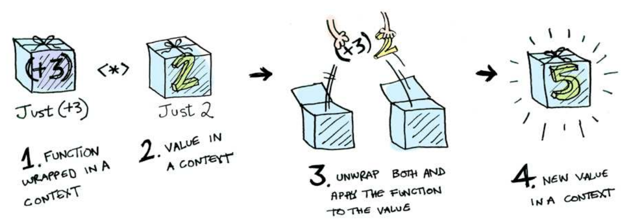
```
data M1 a     =   Raise String | Return a  deriving(Show, Read, Eq)
```

Applicative

```
class Functor f => Applicative f where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b

pure id <*> v = v                                  -- identita
pure (.) <*> u <*> v <*> w = u <*> (v <*> w)       -- kompozícia
pure f <*> pure x = pure (f x)                     -- homomorfizmus
u <*> pure y = pure ($ y) <*> u = pure (\g->g y) <*> u
```

Cvičenie5'': definujte inštanciu Maybe pre triedu Applicative a overte pravidlá:

```
instance Applicative Maybe where
    pure            :: a -> Maybe a
    pure            = Just
    pure x          = Just x
    (<*>)           :: Maybe (a->b) -> Maybe a -> Maybe b
    Nothing  <*> _  = Nothing
    (Just g) <*> a  = fmap g a
```



Príklad:

```
pure (*2) <*> Just 7                                = Just 14
pure (+) <*> Just 7 <*> Just 9                      = Just 16
pure (+) <*> Nothing <*> Just 9                     = Nothing
pure (\x y z->(x,y,z)) <*> Just 1 <*> Just 2 <*> Just 3   = Just (1,2,3)
```

```
data Maybe a    =    Just a | Nothing  deriving(Show, Read, Eq)
```

Applicative

```
class Functor f => Applicative f where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b

pure id <*> v = v                              -- identita
pure (.) <*> u <*> v <*> w = u <*> (v <*> w)   -- kompozícia
pure f <*> pure x = pure (f x)                 -- homomorfizmus
u <*> pure y = pure ($ y) <*> u = pure (\g->g y) <*> u
```

Cvičenie6: definujte inštanciu [] pre triedu Applicative a overte pravidlá:

```
instance Applicative [] where
    pure a        = [a]
    fs  <*> xs    = [ f x | f <- fs, x <- xs]
```

Príklad:

```
1)  pure (+1) <*> [1..5]                          = [2,3,4,5,6]
    pure (+) <*> [1,2] <*> [11,12]                = [12,13,13,14]
    pure (,) <*> [1,2] <*> [11,12]                = [(1,11),(1,12),(2,11),(2,12)]
2)  pure (.) <*> pure (+1) <*> pure (+3) <*> pure 9      = 13
    pure (.) <*> pure (+1) <*> pure (+3) <*> [9]         = [13]
    pure (.) <*> pure (+1) <*> pure (+3) <*> Just 9      = Just 13
3)  pure (+1) <*> pure 7                          = 8
4)  pure ($ 7) <*> pure (+1)                      = 8
```

type [a]   =   [] | a:[a]

Applicative

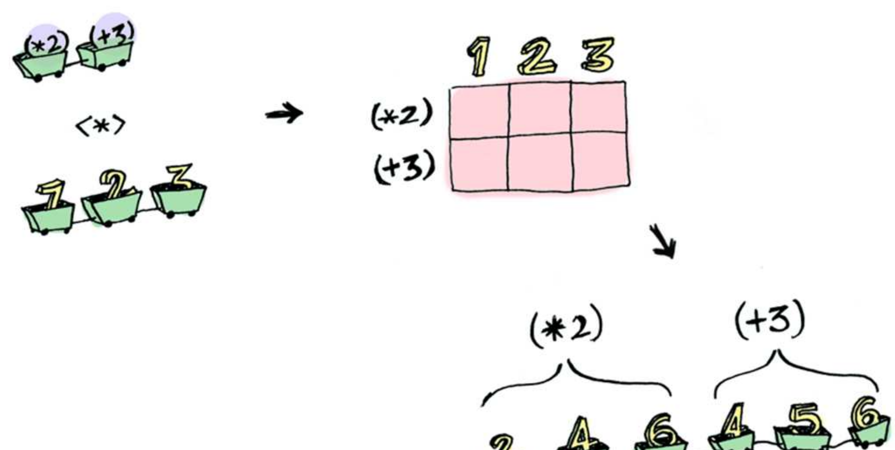
```
class Functor f => Applicative f where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b

pure id <*> v = v                               -- identita
pure (.) <*> u <*> v <*> w = u <*> (v <*> w)    -- kompozícia
pure f <*> pure x = pure (f x)                  -- homomorfizmus
u <*> pure y = pure ($ y) <*> u = pure (\g->g y) <*> u
```

Cvičenie6: definujte inštanciu [] pre Applicative, a overte pravidlá:

```
instance Applicative [] where
      pure a          = [a]
      fs  <*> xs      = [ f x | f <- fs, x <- xs]
```

Dôkaz:

1) (pure id) <*> v = [id] <*> v = v
2) [(.)] <*> [ui] <*> [vj] <*> [wk] =
   [(.)ui] <*> [vj] <*> [wk] =
   [ui . vj] <*> [wk] = [(ui . vj) wk] =
   [(ui (vj wk) | i <- .., j <-.., k <-..]
3) pure f <*> pure x = [f] <*> [x] = [f x]
4) [f1,… fn] <*> [y] = [f1 y, … fn y]



https://adit.io/posts/2013-04-17-functors,_applicatives,_and_monads_in_pictures.html

Príklady:

[(*2), (+3)] <*> [1,2,3] = [2,4,6,4,5,6]

(,) <$> [1,2,3] <*> [4,5,6] = [(1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6)]

(\x y z -> (x,y,z)) <$> [1,2] <*> [3,4] <*> [5,6] = [(1,3,5),(1,3,6),(1,4,5),(1,4,6),(2,3,5),(2,3,6),(2,4,5),(2,4,6)]

pure (,,) <*> [1,2] <*> [3,4] <*> [5,6] = [(1,3,5),(1,3,6),(1,4,5),(1,4,6),(2,3,5),(2,3,6),(2,4,5),(2,4,6)]

Kartézsky súčin

domáca úloha

```
module KSucin where
cart  :: [[t]] -> [[t]]
```

Príklad:

cart [[1,2], [3,4], [5]] = [[1,3,5],[1,4,5],[2,3,5],[2,4,5]]

cart [[1,2], [3,4], [5,6,7]] = [[1,3,5],[1,3,6],[1,3,7],[1,4,5],[1,4,6],[1,4,7],[2,3,5],[2,3,6],
[2,3,7],[2,4,5],[2,4,6],[2,4,7]]

cart [[1,2], [3,4], [5,6]] = [[1,3,5],[1,3,6],[1,4,5],[1,4,6],[2,3,5],[2,3,6],[2,4,5],[2,4,6]]
cart [[1,2], [3,4], []] = []
cart [["janka", "danka"], ["misko", "palko"]] = [["janka","misko"],["janka","palko"],["danka","misko"],
["danka","palko"]]

GHC.Base

- https://hackage.haskell.org/package/base-4.15.0.0/docs/GHC-Base.html

```
▸ Applicative []                                    #
```

Since: base-2.1

```
▾ Applicative Maybe                                 #
```

Since: base-2.1

Defined in GHC.Base

Methods

```
pure :: a -> Maybe a

(<*>) :: Maybe (a -> b) -> Maybe a -> Maybe b

liftA2 :: (a -> b -> c) -> Maybe a -> Maybe b -> Maybe c

(*>) :: Maybe a -> Maybe b -> Maybe b

(<*) :: Maybe a -> Maybe b -> Maybe a
```

```
▸ Applicative IO                                    #
```

Since: base-2.1

Monáda

(class Monad)

monáda **m** je typ implementujúci dve funkcie:

```
class Applicative m => Monad m where        -- interface predpisuje tieto funkcie
   return   :: a -> m a                     -- to bude pure z Applicatives
   >>=      :: m a -> (a -> m b) -> m b     -- náš `bind`
```

ktoré spĺňajú isté (sémantické) zákony:

neutrálnosť return:

- return c >>= (\x->g)   **=**   g[x/c]
- m >>= \x->return x   **=**   m

neutrálnosť asociativita:

- m1 >>= (****x->m2 >>= (****y->m3)) **=** (m1 >>= (****x->m2)) >>= (****y->m3)

inak zapísané:

```
return c  >>=  f          =    f c              -- ľavo neutrálny prvok
m  >>=  return            =    m                -- pravo neutrálny prvok
(m >>= f) >>= g           =    m >>= (\x-> f x >>= g)
                                                -- asocitativita >>=
```