Funkcie a funkcionály na ceste k Wholemeal (functional) programming

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I-18

http://dai.fmph.uniba.sk/courses/FPRO/

Čo je wholemeal (celozrnné)

Geraint Jones: Wholemeal programming means to **think big**:

- **work with an entire list**, rather than a sequence of elements
- develop **a solution space**, rather than an individual solution
- imagine **a graph**, rather than a single path.

first

- **solve a more general problem**,

then

- extract the interesting bits and pieces by transforming the general program into more specialized ones

Wholemeal programming je štýl rozmýšlania, programovania

... privedie vás k *šlachtickým* manierom vo funkcionálnom svete

Celozrnný programátor musí

poznať funkcie a najzákladnejšie funkcionály

- map/filter
  - map f xs = map f $[x_1, \ldots, x_n]$ = $[f\ x_1, \ldots, f\ x_n]$ = [f x | x <- xs]
  - filter f xs = filter p $[x_1, \ldots, x_n]$ = [x | x <- xs, p x]
- foldr/foldl
  - foldr f z $[x_1, \ldots, x_n] = (f\ x_1\ (f\ x_2\ \ldots\ (f\ x_n\ z)..))$
  - foldl f z $[x_1, \ldots, x_n] = (..((f\ z\ x_1)\ x_2)\ \ldots\ x_n)$
- scanr/scanl
  - scanr f z $[x_1, \ldots, x_n]$ = reverse $[z, (f\ x_n\ z), \ldots, (f\ x_2 \ldots (f\ x_n\ z)..), (f\ x_1\ (f\ x_2 \ldots (f\ x_n\ z)..))]$
  - scanl f z $[x_1, \ldots, x_n] = [z, (f\ z\ x_1), ((f\ z\ x_1)\ x_2), \ldots, (..((f\ z\ x_1)\ x_2)\ \ldots\ x_n)]$
  - scanr1 f $[x_1, \ldots, x_n]$ = reverse $[x_n, (f\ x_{n-1}\ x_n), \ldots, (f\ x_1\ (f\ x_2 \ldots (f\ x_{n-1}\ x_n)..))]$
  - scanl1 f $[x_1, \ldots, x_n] = [x_1, (f\ x_1\ x_2), ((f\ x_1\ x_2)\ x_3), \ldots, (..((f\ x_1\ x_2)\ x_3)\ \ldots\ x_n)]$
- iterate
  - iterate f x = $[x, (f\ x), ((f\ \ x)\ x), \ldots, f^n\ x, \ldots]$
- concat, … a t.ď.

Extrémny príklad celozrnného

2 vzájomne rekurzívne fcie

```
rozdelParneNeparne :: [Integer] -> ([Integer],[Integer])
rozdelParneNeparne [] = ([],[])
rozdelParneNeparne (x:xs) = (xp, x:xn) where (xp, xn) = rozdelNeparneParne xs
```

```
rozdelNeparneParne :: [Integer] -> ([Integer],[Integer])
rozdelNeparneParne [] = ([],[])
rozdelNeparneParne (x:xs) = (x:xp, xn) where  (xp, xn) = rozdelParneNeparne xs
```

```
rozdielSuctu :: [Integer] -> Integer
rozdielSuctu xs = sum parneMiesta - sum neparneMiesta
    where (parneMiesta, neparneMiesta) = rozdelParneNeparne xs
```

Celozrnné riešenie:

```
rozdielSuctu  = negate . foldr (-) 0
alebo len -foldr(-)0
```

RozdielSuctu.hs

Krok-po-kroku
(len pre tých, čo to nepochopili ešte)

- **Krok 1** - zbierame párne a nepárne prvky do zoznamov

```
rozdielSuctu'' xs  = (sum p) - (sum n)
        where (p,n) = foldr (\x -> \(a,b) -> (b,x:a)) ([],[]) xs
```

- **Krok 2** - prečo nepočítať súčet už hneď

```
rozdielSuctu''' xs  = p - n
        where (p,n) = foldr (\x -> \(a,b) -> (b,a+x)) (0,0) xs
```

- **Krok 3** – ušetrený where, zistíme, čo je uncurry

```
rozdielSuctu'''' xs  = uncurry (-) $ foldr (\x -> \(a,b) -> (b,a+x)) (0,0) xs
uncurry :: (a -> b -> c) -> (a, b) -> c
uncurry f (a,b) = f a b
```

- **Krok 4** – ušetrený explicitný argument

```
rozdielSuctu'''''   = uncurry (-) . foldr (\x -> \(a,b) -> (b,a+x)) (0,0)
```

RozdielSuctu.hs

Celozrnné krok-po-kroku

(a na jednoduchých príkladoch)

Čo robí táto funkcia ?

```
foo                 :: [Integer] -> Integer
foo []               = 0
foo (x:xs)  | odd x  = (3*x + 1) + foo xs
            | otherwise = foo xs
```

Sčíta 3x+1 pre každý prvok x vstupného zoznamu, ale len tie nepárne...

foo' xs = sum [3*x+1 | x <- xs, odd x] – toto je výrazný progres v čitateľnosti

```
foo''  xs  = sum (map (\x -> 3*x+1) ( filter odd xs)) -- to isté len s filter/map
foo''' xs  = sum $ map (\x -> 3*x+1) $ filter odd xs -- poznajúc operátor $
foo''''    = sum . map (\x -> 3*x+1) . filter odd      -- poznajúc kompozíciu .
foo'''''   = sum . map ((+1).(*3)) . filter odd        -- 2xpoznajúc kompozíciu
foo''''''  = foldr (+) 0 . map ((+1).(*3)) . filter odd -- extrémna verzia bez sum
```

Intro.hs

Celozrnné krok-po-kroku

(a na príkladoch)

Čo robí táto funkcia ?

```
goo                    :: [Integer] -> Integer
goo  []                = 1
goo  (x:xs)  | even x    = (x-2) * goo xs
             | otherwise = goo xs
```

Vynásobí všetky párne prvky vstupného zoznamu zmenšené o 2

goo' xs = product [x-2 | x <- xs, even x] -- výrazný progres v čitateľnosti

goo ' '   = product . map (subtract 2) . filter (even)

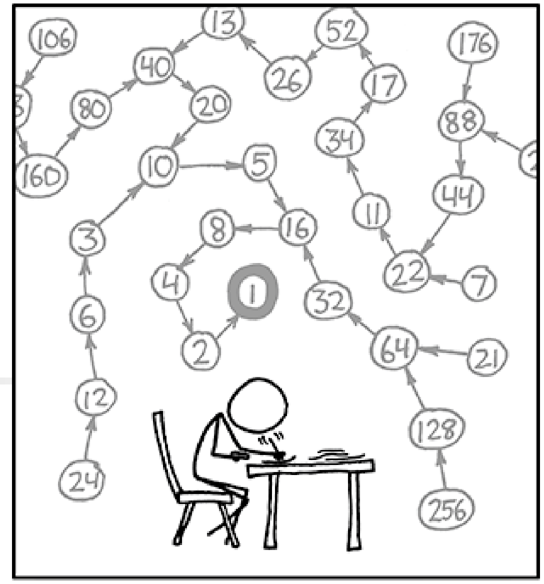
goo''' = foldl (*) 1 . map (subtract 2) . filter (even) -- extrémna verzia bez product

Intro.hs

Collatz

(a na príkladoch)

$$f(n) = \begin{cases} n/2 & \text{if } n \equiv 0 \pmod{2} \\ 3n+1 & \text{if } n \equiv 1 \pmod{2}. \end{cases}$$



THE COLLATZ CONJECTURE STATES THAT IF YOU PICK A NUMBER, AND IF IT'S EVEN DIVIDE IT BY TWO AND IF IT'S ODD MULTIPLY IT BY THREE AND ADD ONE, AND YOU REPEAT THIS PROCEDURE LONG ENOUGH, EVENTUALLY YOUR FRIENDS WILL STOP CALLING TO SEE IF YOU WANT TO HANG OUT.

Čo robí táto funkcia ?

```
collatz    :: Integer -> [Integer]
collatz 1  = []
collatz n | even n          = n : collatz (n `div` 2)
          | otherwise       = n : collatz (3 * n + 1)
```

To sú prvky tzv. Colatzovej postupnosti

```
collatz'= takeWhile (/=1) . iterate (\x -> if even x then x `div` 2 else 3 * x + 1)
```

iterate :: (a -> a) -> a -> [a]

iterate f x = [x, f x, f f x, f f f x, … , f^nx, …]

27, 82, **41**, 124, 62, **31**, 94, **47**, 142, **71**, 214, **107**, 322, **161**, 484, 242, **121**, 364, 182, **91**, 274, **137**, 412, 206, **103**, 310, **155**, 466, **233**, 700, 350, **175**, 526, **263**, 790, **395**, 1186, **593**, 1780, 890, **445**, 1336, 668, 334, **167**, 502, **251**, 754, **377**, 1132, 566, **283**, 850, **425**, 1276, 638, **319**, 958, **479**, 1438, **719**, 2158, **1079**, 3238, **1619**, 4858, **2429**, 7288, 3644, 1822, **911**, 2734, **1367**, 4102, **2051**, 6154, **3077**, 9232, 4616, 2308, 1154, **577**, 1732, 866, **433**, 1300, 650, **325**, 976, 488, 244, 122, **61**, 184, 92, 46, **23**, 70, **35**, 106, **53**, 160, 80, 40, 20, 10, **5**, 16, 8, 4, 2, **1**

Intro.hs

Celozrnné krok-po-kroku

(a na príkladoch)

$$f(n) = \begin{cases} n/2 & \text{if } n \equiv 0 \pmod{2} \\ 3n+1 & \text{if } n \equiv 1 \pmod{2}. \end{cases}$$

Intro.hs

Čo robí táto funkcia ?

```
moo       :: Integer -> Integer
moo 1     = 0
moo n     | even n          = n + moo (n `div` 2)
          | otherwise       = moo (3 * n + 1)
```

súčet párnych prvkov Collatzovej postupnosti, teda sum . filter (even) . hoo

```
2+(                                   -- nezapočítali sme dvojicu (1, s+2)
    snd $                             -- z poslednej dvojice zober druhú zložku
      last $                          -- zober poslednú dvojicu
        takeWhile ((/=1).fst) $       -- kým prvá zložka dvojice <> 1
          iterate (\(x,s) -> if even x then (x `div` 2,x+s) else (3 * x + 1,s) )
          (n,0)
    )
moo'' n = snd $ last $ takeWhile ((/=1).fst) $ -- z jemne zoprimalizované
        iterate (\(x,s) -> if even x then (x `div` 2,x+s) else (3 * x + 1,s) ) (n,2)
```

Cifry
(niektoré vaše riešenia)

```
module Cifry where
cifry 12345 == [1,2,3,4,5]
cifryR 12345 == [5,4,3,2,1]

cifry n = reverse (cifryR n)
cifryR 0 = []
cifryR n = (n `mod` 10):(cifryR (n `div` 10))
```

Intro.hs

Cifry

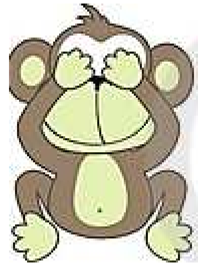
```
module Cifry where
cifry 12345 == [1,2,3,4,5]
cifryR 12345 == [5,4,3,2,1]

cifry      :: Integer -> [Integer]
cifry n    = map(`mod` 10) $ reverse $
                 takeWhile (> 0) $ iterate (`div`10) n
iterate (`div` 10) 12345 == [12345,1234,123,12,1,0,0,0,0,0,0,0,0,0...]
                            [1,12,123,1234,12345]
                            [1,  2,    3,     4,      5]
cifry' = map(`mod` 10) . reverse . takeWhile (> 0) . iterate (`div`10)
cifryR n = map(`mod` 10) $ takeWhile (> 0) $ iterate (`div`10) n
cifryR'   = map(`mod` 10) . takeWhile (> 0) . iterate (`div`10)
```

Intro.hs

Binárne číslo $\{1\}^{+}\{0\}^{*}$

- $\underbrace{111...111}_{m}\underbrace{00.....0000}_{n} = (2^m-1) * 2^n$



```
null $                                True
   dropWhile (==1) $                  []
      dropWhile(==0) $                [1, 1]
         map (`mod` 2) $              [0, 0, 1, 1]
            takeWhile (>0) $          [12, 6, 3, 1]
               iterate (`div` 2)      12 = [12,6,3,1,0,0,0...]
```

suma + i*cenaPiva = $(2^m-1) * 2^n$

suma \`mod\` cenaPiva = $((2^m-1) * 2^n)$ \`mod\` cenaPiva

suma \`mod\` cenaPiva = ((2^m-1) \`mod\` cenaPiva

*

2^n \`mod\` cenaPiva

) \`mod\` cenaPiva

mod 11	2^n	2^{m-1}	mod 11
1	1	0	0
2	2	1	1
4	4	3	3
8	8	7	7
5	16	15	4
10	32	31	9
9	64	63	8
7	128	127	6
3	256	255	2
6	512	511	5
1	1024	1023	0

Kombinácie s opakovaním

kso

```
repeat [] = [ [], [], [], [], [], ...           ::[[t]]
[[]] : repeat [] = [ [[]], [], [], [], [],... ::[[[t]]]
```

```
kso          :: [t] -> Int -> [[t]]
kso xs k  = (foldr f ([[]] : repeat []) xs) !! k
                 f x = scanl1 $ (++) . map (x :)
                 f x y = (scanl1 $ (++) . map (x :)) y
                 f x y = scanl1 ((++) . map (x :)) y
                 f x y = scanl1 (\acc -> \ws -> ((++) . map (x :)) acc ws) y
                 f x y = scanl1 (\acc -> \ws -> ((++)  (map (x :) acc) ws)) y
                 f x y = scanl1 (\acc -> \ws -> ((map (x :) acc) ++ ws)) y
                 f :: t -> [[[t]]] -> [[[t]]]
                 f x y = scanl1 g y
                      where g          :: [[t]] -> [[t]] -> [[t]]
                            g acc ws =(map (x :) acc) ++ ws
```

https://rosettacode.org/wiki/Combinations_with_repetitions#Haskell https://rosettacode.org/wiki/Combinations_with_repetitions#Haskell

Kombinácie s opakovaním

kso

```
repeat [] = [ [], [], [], [], [], ...              ::[[t]]
[[]] : repeat [] = [ [[]], [], [], [], [],...  ::[[[t]]]
```

```
kso          :: [t] -> Int -> [[t]]
kso xs k  = (foldr f ([[]] : repeat []) xs) !! k
                    f x y = scanl1 g y                          f :: t -> [[[t]]] -> [[[t]]]
                      where                                     g :: [[t]] -> [[t]] -> [[t]]
                        g acc ws =(map (x :) acc) ++ ws
```

f 4 ([[]] : repeat []) = [[[]], [[4]], [[4,4]], [[4,4,4]], [[4,4,4,4]], [[4,4,4,4,4]], [[4,4,4,4,4,4]],

f 3 $ f 4 ([[]] : repeat []) = [[[]], [[3],[4]], [[3,3],[3,4],[4,4]],[[3,3,3],[3,3,4],[3,4,4],[4,4,4]],

f 2 $ f 3 $ f 4 ([[]] : repeat []) = [[[]], [[2],[3],[4]],[[2,2],[2,3],[2,4],[3,3],[3,4],[4,4]],
[[2,2,2],[2,2,3],[2,2,4],[2,3,3],[2,3,4],[2,4,4],[3,3,3],[3,3,4],[3,4,4],[4,4,4]],…

f 1 $f 2 $ f 3 $ f 4 ([[]] : repeat []) = [[[]], [[1],[2],[3],[4]],
[[1,1],[1,2],[1,3],[1,4],[2,2],[2,3],[2,4],[3,3],[3,4],[4,4]],
[[1,1,1],[1,1,2],[1,1,3],[1,1,4],[1,2,2],[1,2,3],[1,2,4],[1,3,3],[1,3,4],[1,4,4],
[2,2,2],[2,2,3],[2,2,4],[2,3,3],[2,3,4],[2,4,4],[3,3,3],[3,3,4],[3,4,4],[4,4,4]],
[[1,1,1,1],[1,1,1,2],[1,1,1,3],[1,1,1,4],[1,1,2,2],[1,1,2,3],[1,1,2,4],[1,1,3,3],[1,1,3,4],[1,1,4,4],[1,2,2,2],[1,2,2,3],[1,2,2,4],[1,2,3,3],[1,2,3,4],[1,2,4,4],[1,3,3,3],[1,3,3,4],[1,3,4,4],[1,4,4,4],[2,2,2,2],[2,2,2,3],[2,2,2,4],[2,2,3,3],[2,2,3,4],[2,2,4,4],[2,3,3,3],[2,3,3,4],[2,3,4,4],[2,4,4,4],[3,3,3,3],[3,3,3,4],[3,3,4,4],[3,4,4,4],[4,4,4,4]]]

Maximálny súčet
súvislej podpostupnosti

0	1	2	3	4	5	6	7
-1	2	1	-3	2	3	-3	1

to

from	0	1	2	3	4	5	6	7
0	-1	1	2	-1	-1	2	-1	0
1	×	2	3	0	2	5	2	3
2	×	×	1	-2	0	3	0	1
3	×	×	×	-3	-1	2	-1	0
4	×	×	×	×	2	5	2	3
5	×	×	×	×	×	3	0	1
6	×	×	×	×	×	×	-3	-2
7	×	×	×	×	×	×	×	1

$x_{from} + \ldots + x_{to}$

Predošlý stĺpec
xs
nasledujúci stĺpec
map(+x) xs ++ [x]

nešikovné:
stĺpec otočíme
nasledujúci stĺpec
x: map(+x) xs

Stĺpce tejto tabuľky vyrábame postupne
[[-1], [1,2], [2,3,1], [-1,0,-2,-3], [-1,2,0,-1,2], [2,5,3,2,5,2],[-1,2,0,-1,2,0,-3]]

Intro.hs

Pamäťová zložitosť $O(n^2)$ či $O(n^3)$

Maximálny súčet

```
maxSucet' :: [Int] -> Int
maxSucet' [] = 0
maxSucet' xs =
    maximum (map (maximum) – maximum  trojuholníkovej matice
        (init (                                  -- posledný prvok - trojuholníková
            foldl (\xss -> \x -> (x:(map (+x) (head xss))): xss)   [[]]   xs)))

maxSucet'' xs = maximum $ map (maximum) $
        init $ foldl (\xss -> \x -> (x:(map (+x) (head xss))): xss) [[]] xs

maxSucet''' = maximum . map (maximum) .
        init . foldl (\xss  -> \x -> (x:(map (+x) (head xss))):xss) [[]]

maxSucet' [(-1), 2, 1, (-3), 2, 3, 1] == 6
maxSucet'' [(-1), 2, 1, (-3), 2, 3, 1] == 6
```

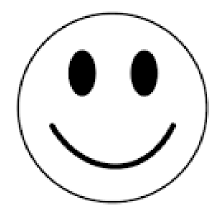
Intro.hs

Kadane Algo

	0	1	2	3	4	5	6	7
	-1	2	1	-3	2	3	-3	1
tempMax	0	2	3	0	2	5	2	3
globalMax	0	2	3	3	3	5	5	5

```
kadane   :: [Int] -> Int -> Int -> Int -- list -> tempMax -> globalMax -> max
kadane []           _       globalMax = globalMax
kadane (x:xs) tempMax globalMax = kadane xs newTempMax newGlobalMax
    where
          newTempMax = max (tempMax + x) 0
          newGlobalMax = max globalMax newTempMax

kadane' :: [Int] -> Int
kadane' (x:xs) = snd $ foldr pom (0,0) xs
   where pom x (tempMax, globalMax) =
         let newTempMax = max (tempMax + x) 0
           in (newTempMax, max globalMax newTempMax)
```



Pamäťová zložitosť O(n)

```
kadane [(-1), 2, 1, (-3), 2, 3, 1] 0 0 == 6
kadane' [(-1), 2, 1, (-3), 2, 3, 1] == 6
```

Kadane.hs

Maximálny súčet

```
maxSucet s = maxSucet' s [] 0 [] 0

maxSucet' :: [Int] -> [Int] -> Int -> [Int] -> Int -> (Int, [Int])
maxSucet' [] curMaxS curMaxSum _ _ = (curMaxSum, curMaxS)
maxSucet' (x:xs) curMaxS curMaxSum indexS indexSum
        | newIndexSum < 0  = maxSucet' xs curMaxS curMaxSum [] 0
        | otherwise        =
              maxSucet' xs newMaxS newMaxSum newIndexS newIndexSum
         where
          newIndexSum = indexSum + x
          newIndexS = indexS ++ [x]
          newMaxSum = max newIndexSum curMaxSum
          newMaxS =
                if newMaxSum == newIndexSum then newIndexS else curMaxS
```

Najčastejšie vyskytujúce slovo

Nájdi najčastejšie vyskytujúcie sa slovo v reťazci

```
-- rozdeľ na slová podľa oddelovača, viac pozri Data.List.Split
splitOneOf          :: String -> String -> [String]
splitWords          = filter(/= "") . splitOneOf " .,;!@#$%^&*()'"

chunks              :: [String] -> [[String]]
chunks []           = []
chunks xs@(w:_) = takeWhile (==w) xs: chunks (dropWhile (==w) xs)

type FreqTable      = [(Int,String)]
chunkLengths        :: [[String]] -> FreqTable
chunkLengths xs     = map (\chunk -> (length chunk, head chunk)) xs
```

"?: " splitWords hamlet
["There","was","this","king","

"?: " chunks ["a", "a", "a", "b", "b", "c"]
[["a","a","a"],["b","b"],["c"]]

"?: " chunkLengths $ chunks ["a", "a", "a", "b", "b", "c"]
[(3,"a"),(2,"b"),(1,"c")]

MostFrequent.hs

Najčastejšie vyskytujúce slovo

```
mostFrequent :: String -> String
mostFrequent ws =
   snd $ last $ sort $ chunkLengths $ chunks $ sort $ splitWords $ map toLower ws
```

"?: " sort [(3,"d"),(1,"b"), (2,"a")]
[(1,"b"),(2,"a"),(3,"d")]

```
-- funkcionálna verzia
mostFrequent' =
   snd .last . sort . chunkLengths . chunks . sort . splitWords . map toLower
```

"?: " mostFrequent' hamlet
"the"

```
-- zátvorková verzia pre rodených Lispistov
mostFrequent'' ws =
         snd (
           last (
             sort (
               chunkLengths (
                 chunks (
                   sort (
                     splitWords (
                       map toLower ws
           )))))))
```

Vstupný text:

```
hamlet = "There was this king sitting in his garden all alane " ++
  "When his brother in his ear poured a wee bit of henbane. " ++
  "He stole his brother's crown and his money and his widow. " ++
  "But the dead king walked and got his son and said Hey listen, kiddo! " ...
```

MostFrequent.hs

Kartézsky súčin

- fuj ☺ riešenie

```
cart xss = sequence xss
```

- tradičné, a priznajme, dobre čitateľné riešenie:

```
cp          :: [[t]] -> [[t]]
cp []       = [[]]
cp (xs:xss)   = [(x:ys) | x <- xs, ys <- cp xss]
```

- Marianové riešenie

pridáme jeden prvok do každej množiny cartTemp 1 [[4,5],[6,7]] == [[1,4,5],[1,6,7]]

```
-- verzia 1
cartTemp                 :: t -> [[t]] -> [[t]]
cartTemp element xss = foldr (\xs rekurzia -> (element:xs):rekurzia) [] xss
-- verzia 2
cartTemp element = foldr pom []  where
                             pom xs rek = (element:xs):rek
```

cartesianMarian.hs

riešenie – pokrač.

prvky jednej množiny kombinujeme s mnohými množinami

```
cartTemp2 [1, 2, 3] [[4, 5],[6,7],[8,9]] ==
    [[1,4,5],[1,6,7],[1,8,9],[2,4,5],[2,6,7],[2,8,9],[3,4,5],[3,6,7],[3,8,9]]
cartTemp2' xs yss = concat [ cartTemp x yss | x<-xs]

cartTemp2          :: [t] -> [[t]] -> [[t]]
cartTemp2 [] _     = []
                                         y++ (cartTemp x yss)
cartTemp2 xs yss = foldr (\x y -> (foldr (:) (cartTemp x yss) y)) [] xs
```

Kartézsky súčin množiny množín

```
--cart [ [1,2], [3,4], [5] ] = [[2,4,5],[2,3,5],[1,4,5],[1,3,5]]
cart      :: [[t]] -> [[t]]
cart xss  = foldr (\x y -> cartTemp2 x y) [[]] xss
```

cartesianMarian.hs

Kartézsky – transformácie

```
-- iniciálne riešenie
cp_1 []             = [[]]
cp_1 (xs:xss)  = [(x:ys) | x <- xs, ys <- cp_1 xss]

-- rozbité na vnútorný a vonkajší list-comprehension
cp_2 []          = [[]]
cp_2 (xs:xss)   = concat [ [(x:ys) | ys <- cp_2 xss ] | x <- xs]

-- vnútorný list=comprehension prepíšeme cez map
cp_3 []          = [[]]
cp_3 (xs:xss)   = concat [ map (x:) (cp_3 xss) | x <- xs]

-- zavedieme foldr
cp_4 xss   = foldr pom [[]] xss  where
             pom xs rek = concat [ map (x:) rek | x <- xs]
```

cartesianMarian.hs

Kartézsky – transformácie

```
-- odstránime concat
cp_5 xss   = foldr pom [[]] xss  where
              pom xs rek = foldr (\x -> \rek2 -> (map (x:) rek) ++ rek2) [] xs

-- slušnejšie prepísané
cp_6 xss   = foldr pom [[]] xss  where
              pom xs rek = foldr (pom2 rek) [] xs
              pom2 rek x rek2 = (map (x:) rek) ++ rek2

-- odstránime map
cp_7 xss   = foldr pom [[]] xss  where
              pom xs rek = foldr (pom2 rek) [] xs
              pom2 rek x rek2 = (foldr (pom3 x) [] rek) ++ rek2
              pom3 x y ys = (x:y):ys
```

cartesianStrachey.hs

Kartézsky – transformácie

```
-- odstránime append
cp_8 xss   = foldr pom [[]] xss  where
                pom xs rek = foldr (pom2 rek) [] xs
                pom2 rek x rek2 = foldr (:) rek2 (foldr (pom3 x) [] rek)
                pom3 x y ys = (x:y):ys

-- jediný problém, že to ide aj s tromi foldami
-- Strachey's functional pearl, forty years on
```

https://spivey.oriel.ox.ac.uk/mike/firstpearl.pdf

cartesianStrachey.hs