

#### Monády – úvod



Phil Wadler: <a href="http://homepages.inf.ed.ac.uk/wadler/topics/monads.html">http://homepages.inf.ed.ac.uk/wadler/topics/monads.html</a>

- Monads for Functional Programming In Advanced Functional Programming, Springer Verlag, LNCS 925, 1995,
- Noel Winstanley: What the hell are Monads?, 1999
  <a href="http://web.cecs.pdx.edu/~antoy/Courses/TPFLP/lectures/MONADS/Noel/research/monads.html">http://web.cecs.pdx.edu/~antoy/Courses/TPFLP/lectures/MONADS/Noel/research/monads.html</a>
- Jeff Newbern's: All About Monads
   https://www.cs.rit.edu/~swm/cs561/All About Monads.pdf
- Dan Bensen: A (hopefully) painless introduction to monads, <u>http://www.prairienet.org/~dsb/monads.htm</u>

Monady sú použiteľný nástroj pre programátora poskytujúci:

- modularitu skladať zložitejšie výpočty z jednoduchších (no side-effects),
- flexibilitu výsledný kód je ľahšie adaptovateľný na zmeny,
- izoluje side-effect operácie (napr. IO) od čisto funkcionálneho zvyšku.

#### Maybe Monad

Maybe je podobné Exception (Nothing~~Raise String, Just a ~~Return a)

```
data Maybe a = Nothing | Just a
```

```
instance Monad Maybe where
```

```
return v = Just v
fail = Nothing
```

Nothing >>=  $\_=$  Nothing (Just x) >>= f = f x

- -- vráť hodnotu
  - -- vráť neúspech
  - -- ret'azenie, bind, ...
  - -- ak už nastal neúspech, trvá do konca
  - -- ak je zatiaľ úspech, závisí to na výpočte f

```
return v = Just v

fail = Nothing

Nothing >>= = Nothing

(Just x) >>= f = f x
```

#### Príklady na Maybe Monad

- konštaty: Just 1, Nothing :: Maybe Int, Just "b" :: Maybe String
- základné operácie:

```
(Just 1) >>= (\x -> Nothing) = Nothing
(Just 1) >>= (\x -> Just (1+x)) = Just 2
Nothing >>= (\x -> Just (1+x)) = Nothing
```

do-notácia, monad-comprehension:

```
do { x<-(Just 1); Nothing } :: Maybe t = Nothing do { x<-(Just 1); return (1+x) } :: Maybe Int = Just 2 do { x<-(return 1); return (1+x) } :: (Monad m, Num b) => m b do { x<-(return 1); return (1+x) } = 2 do { x<-(return 1); return (1+x) } :: Maybe Int = Just 2 do { x<-(return 1); return (1+x) } :: [Int] = [2] do { x<-Nothing; return (1+x) } :: Maybe t
```

### 4

#### Maybe sequence

sequence [] = return []

sequence :: Monad m => [m a] -> m [a]

#### Monadic Prelude

```
class Monad m where
                                             -- definition:(>>=), return
  return :: a -> m a
  (>>=) :: m a -> (a -> m b) -> m b
  (>>) :: m \ a -> m \ b -> m \ b -- zahodíme výsledok prvej monády
  p >> q = p >> = \ -> q
-- ak nezáleží na výsledkoch
sequence_ :: (Monad m) => [m a] -> m()
sequence_ = foldr (>>) (return ())
sequence_ (c:cs) = do { _ <- c; _ <- sequence cs; return () }</pre>
sequence_ [m_1, m_2, ... m_n] = m_1 >> = \setminus_- >
                                                           do \{ m_1;
                           m_2 >>= \setminus ->
                                                                m_2;
                           m_n >> = \setminus ->
                           return ()
                                                                m_n;
                                                                return ()
```

#### Maybe MonadPlus

data Maybe a = Nothing | Just a

```
class Monad m => MonadPlus m where – podtrieda, resp. podinterface
  mzero :: m a
                                       -- disjunkcia
  mplus :: m a -> m a
instance MonadPlus Maybe where
                       = Nothing -- fail...
  mzero
  Just x `mplus` y = Just x
                                    -- or
  Nothing 'mplus' y = y
Just "a" `mplus` Just "b"
                                              = Just "a" !!!
Just "a" `mplus` Nothing
                                              = Just "a"
Nothing `mplus` Just "b"
                                              = Just "b"
```

#### Zákony monád a monády Plus

vlastnosti return a >>=:

```
return x >>= f = f x -- return ako identita zl'ava
p >>= return = p -- retrun ako identita sprava
p >>= (x -> (f x >>= g))= (p >>= (x -> f x)) >>= g -- "asociativita"
```

vlastnosti zero a `plus`:

```
zero `plus` p = p -- zero ako identita zl'ava `plus'

p `plus` zero = p -- zero ako identita sprava `plus'

p `plus` (q `plus` r) = (p `plus` q) `plus` r -- asociativita
```

vlastnosti zero `plus` a >>= :

```
zero >>= f = zero -- zero ako identita zl'ava `>>=' 
p >>= (\x->zero) = zero -- zero ako identita sprava `>>=' 
(p `plus` q) >>= f = (p >>= f) `plus` (q >>= f) -- distributivita
```



#### Sheep family

```
adam = Sheep "Adam" Nothing Nothing
eve = Sheep "Eve" Nothing Nothing
uranus = Sheep "Uranus" Nothing Nothing
gaea = Sheep "Gaea" Nothing Nothing
kronos = Sheep "Kronos" (Just gaea) (Just uranus)
holly = Sheep "Holly" (Just eve) (Just adam)
roger = Sheep "Roger" (Just eve) (Just kronos)
molly = Sheep "Molly" (Just holly) (Just roger)
dolly = Sheep "Dolly" (Just molly) Nothing
```

-- a sheep has its name, and maybe mother and maybe father
data Sheep = Sheep {name :: String, mother :: Maybe Sheep, father :: Maybe Sheep}
deriving (Eq)

```
starý otec z matkinej strany
```

```
maternalGrandfather :: Sheep -> Maybe Sheep
maternalGrandfather' o = if mother o == Nothing then
                                                                       -- klasicky:
                                                                       -- o :: Sheep
                              Nothing
                                                             -- mother o :: Maybe Sheep
                          else
                              father (fromJust (mother o))
                                                             -- fromJust ... :: Sheep
maternalGrandfather s = do{ m <- mother s ;
                                                                       -- monadicky:
                              father m }
                                                                       -- m :: Sheep
matky otca otec
                                                        maternalGrandfather dolly = Just "Roger"
mothersPaternalGrandfather :: Sheep -> Maybe Sheep
mothersPaternalGrandfather s = do { m <- mother s ;
                                     qf <- father m ;
                                                                      -- m, qf :: Sheep
                                     father qf } mothersPaternalGrandfather dolly = Just "Kronos"
```

#### List monad

- List monad použijeme, ak simulujeme nedeterministický výpočet
  - ...parsery boli toho príkladom

```
data List a = Null | Cons a (List a) deriving (Show) -- alias [a]
instance Functor List where -- to je vlastne map
  fmap f Nil = Nil
  fmap f (Cons x xs) = Cons (f x) (fmap f xs)

instance Monad List where
  return x = [x] :: a -> [a]
  m >>= f = concatMap f m :: [a] -> (a -> [b]) -> [b]
  concatMap = concat . map f m
  m >>= f = concat (map f m)
```

```
return :: a -> [a]
>>= :: [a] -> (a -> [b]) -> [b]
```



#### List Monad a MonadPlus

```
type List a = [a]
```

instance Functor List where fmap = map

instance Monad List where

```
return v = [x]

[] >>= f = []

(x:xs) >>= f = f x ++ (xs >>= f) -- concatMap f (x:xs)
```

instance MonadPlus List where

```
mzero = []
[] `mplus` ys = ys
(x:xs) `mplus` ys = x : (xs `plus` ys) -- mplus je klasický append
```

#### List monad - vlastnosti

 $[e_i[y/d_i[x/c_i]]]$ 

```
Príklad, tzv. listMonad M a = List a = [a]
                       :: a -> [a]
return x = [x]
m >>= f = concatMap f m :: [a] -> (a -> [b]) -> [b]
concatMap = concat \cdot map f m
Cvičenie: overme platnosť zákonov:
return c >>= (\x->g)
                               = q[x/c]
    • [c] >>= (\x->g) = concatMap (\x->g) [c] = concat . map (\x->g) [c] =
       concat \lceil q[x/c] \rceil = q[x/c]
m >>= \x->return x
    concat . map (x->return x) [c_1, ..., c_n] = concat [[c_1], ..., [c_n]] = [c_1, ..., c_n]
• m1 >>= (\x->m2 >>= (\y->m3)) = (m1 >>= (\x->m2)) >>= (\y->m3)
    • ([c_1, ..., c_n] >> = (\x->[d_1, ..., d_m])) >> = (\y->m3) =
       (\text{concat} [ [d_1[x/c_1], ..., d_m[x/c_1]], ... [d_1[x/c_n], ..., d_m[x/c_n]] ] ) >>= (\y->m3) =
       ( [d_1[x/c_1], ..., d_m[x/c_1], ..., d_1[x/c_n], ..., d_m[x/c_n] ] ) >>= (y->m3) =
       ([d_1[x/c_1], ..., d_m[x/c_1], ..., d_1[x/c_n], ..., d_m[x/c_n]]) >>= (y->[e_1, ..., e_k]) = ...
```

#### Zákony monadPlus pre List

```
vlastnosti zero a `plus`:
                         -- preložené do ľudštiny:
zero 'plus' p
                     = p -- [] ++ p = p
p `plus` zero
           = p 	 -- p ++ [] = p
p `plus` (q `plus` r) = (p `plus` q) `plus` r -- asociativita ++
vlastnosti zero `plus` a >>= :
                     = zero -- concat . map f [] = []
zero >>= f
p >>= (x->zero) = zero -- concat . map (x->[]) p = []
(p 'plus' q) >>= f = (p >>= f) 'plus' (q >>= f)
                                    -- concat . map f(p ++ q) =
                                           concat . map f p
                                           ++
                                           concat . map f q
```

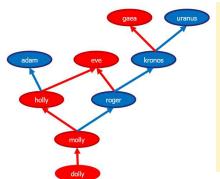
#### List monad vs. comprehension

```
squares lst = do \{x < - lst\}
                    return (x * x) }
                                                             -- vlastne znamená
squares lst =
             lst >>= \x -> return (x * x)
                                                             -- po dosadení >>=, return
squares lst = concat . map (\x -> [x * x]) lst
                                                             -- eta redukcia
squares = concat . map (x \rightarrow [x * x])
                                                             -- takto by sme to napísali
squares = map (\x -> x * x)
                                                   -- iný príklad: kartézsky súčin
cart xs ys = do x < -xs
                  y <- ys
```

return (x,y)



# Sheep family



```
adam = Sheep "Adam" Nothing Nothing
eve = Sheep "Eve" Nothing Nothing
uranus = Sheep "Uranus" Nothing Nothing
gaea = Sheep "Gaea" Nothing Nothing
kronos = Sheep "Kronos" (Just gaea) (Just uranus)
holly = Sheep "Holly" (Just eve) (Just adam)
roger = Sheep "Roger" (Just eve) (Just kronos)
molly = Sheep "Molly" (Just holly) (Just roger)
dolly = Sheep "Dolly" (Just molly) Nothing
```

```
data Sheep = Sheep { name::String, mother::Maybe Sheep, father::Maybe Sheep}
parents :: Sheep -> [Maybe Sheep]
parents x = [father x, mother x]
                                                  parents dolly = [Nothing, Just "Molly"]
parents :: Sheep -> Maybe [Sheep]
parents x = sequence [father x, mother x]
                                                               parents dolly = Nothing
                                                               parents roger = Just ["Kronos","Eve"]
parents' :: Sheep -> [Sheep]
parents' x = (if father x == Nothing then [] else [ fromJust (father x) ]) ++
             (if mother x == Nothing then [] else [ fromJust (mother x) ])
                                                               parents' dolly = ["Molly"]
parents" :: Sheep -> Maybe [Sheep]
                                                               parents' roger = ["Kronos", "Eve"]
parents" x = do \{ o < -father x; return [o] \} `mplus` (do m < -mother x; return [m])
                                                           parents" dolly = Just ["Molly"]
parents'" :: Sheep -> Maybe [Sheep]
                                                           parents" roger = Just ["Kronos"]
parents'' x = do \{ o < -father x; m < -mother x; return ([o] `mplus` [m]) \}
          parents'' dolly = Nothing
                                                           parents" roger = Just ["Kronos", "Eve"]
```

#### Sheep family

adam = Sheep "Adam" Nothing Nothing
eve = Sheep "Eve" Nothing Nothing
uranus = Sheep "Uranus" Nothing Nothing
gaea = Sheep "Gaea" Nothing Nothing
kronos = Sheep "Kronos" (Just gaea) (Just uranus)
holly = Sheep "Holly" (Just eve) (Just adam)
roger = Sheep "Roger" (Just eve) (Just kronos)
molly = Sheep "Molly" (Just holly) (Just roger)
dolly = Sheep "Dolly" (Just molly) Nothing

```
1) Definujte predkov po ženskej línii, teda k_mother 1 je mama, k_mother 2 je babka, k_mother 3 je prababka, ...
k_mother :: Int -> Sheep -> Maybe Sheep k_mother 0 dolly = Just "Dolly"
k_mother 1 dolly = Just "Molly"
k_mother 2 dolly = Just "Holly"
k_mother 3 dolly = Just "Eve"
k_mother 4 dolly = Nothing
```

```
uranus
                                  gaea
adam
                                         kronos
    holly
                            roger
                molly
```

2) Definujte všetkých predkov k-tej úrovne, opakovať v zozname sa môžu:

```
k_predecesors :: Int -> Sheep -> [Sheep]
```

k\_predecesors 1 dolly = ["Molly"]

k\_predecesors 2 dolly = ["Roger","Holly"]

k\_predecesors 3 dolly = ["Kronos","Eve","Adam","Eve"]

k\_predecesors 4 dolly = ["Uranus","Gaea"]

k\_predecesors 5 dolly = []

### Guard (Control.Monad)

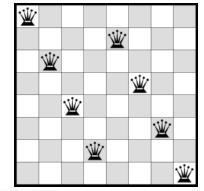
```
pythagoras = [(x, y, z) | z < -[1..], -- pythagorejské trojuholníky
                         x < -[1..z],
                         y < -[x..z],
                         x*x+y*y == z*z
pythagoras' = do z < -[1..]
                  x < - [1..z]
                  y < -[x..z]
                                                   -- zlé riešenie, prečo?
                  if x*x+y*y == z*z then return (x,y,z) else return ()
pythagoras" ... if x*x+y*y == z*z then return "hogo-fogo" else []
                 return (x,y,z)
                               resp. ["hogo-fogo"]
pythagoras''' ... if x*x+y*y == z*z then return () else []
                 resp. guard (x*x+y*y == z*z)
                 return (x,y,z)
```



```
guard :: (MonadPlus m) => Bool -> m ()
guard True = return ()
guard False = mzero
> guard (9 >5) >> return "hogo" :: [String]
["hogo"]
> guard (5 > 9) >> return "fogo" :: [String]
[]
```

Kartézsky súčin

```
listComprehension xs ys = [(x,y) | x<-xs, y<-ys]
listComprehension [1,2,3] ['a','b','c'] = [(1,'a'),(1,'b'),(1,'c'),(2,'a'),(2,'b'),(2,'c'),(3,'a'),(3,'b'),(3,'c')]
guardedListComprehension xs ys = [(x,y) | x<-xs, y<-ys, x<=y, x*y == 24]
guardedListComprehension [1..10] [1..10]
                                                     = [(3,8),(4,6)]
monadComprehension xs ys = do { x<-xs; y<-ys; return (x,y) }
monadComprehension [1,2,3] ['a','b','c'] = [(1,'a'),(1,'b'),(1,'c'),(2,'a'),(2,'b'),(2,'c'),(3,'a'),(3,'b'),(3,'c')]
guardedMonadComprehension xs ys =
         do { x<-xs; y<-ys; guard (x<=y); guard (x*y==24); return (x,y) }
quarded Monad Comprehension [1..10] [1..10] = [(3,8),(4,6)]
```



#### Backtracking

```
-- konsistencia riešenia (teraz nepodstatné):
check (i,j) (m,n) = (i==m) || (j==n) || (j+i==n+m) || (j+m==i+n)
safe p n = all (True==) \lceil not (check (i,j) (m+1,n)) \mid (i,j) < -zip \lceil 1...m \rceil p \rceil
                           where m=length p
-- backtrack
queens size = queens1 size where
 queens1 n | n==0 = [[]]
             | otherwise = [cr++[row] | cr<-queens1 (n-1), row<-[1..size], safe cr row]
length $ queens 10 = 724 (3.16 secs, 1,897,245,320 bytes)
mqueens size = mqueens1 size where
  mqueens1 n | n==0 = return []
               otherwise = do cr <- mqueens1 (n-1)
                                  row <- [1..size]
                                  quard (safe cr row)
                                  return (cr + + [row])
```





zdroj: http://learnyouahaskell.com/a-fistful-of-monads#the-list-monad

```
type KnightPos = (Int,Int)
-- jeden krok koňa na šachovnici
moveKnight :: KnightPos -> [KnightPos]
moveKnight (c,r) = do (c',r') <- [(c+2,r-1),(c+2,r+1),(c-2,r-1),(c-2,r+1),(c+1,r-2),(c+1,r+2),(c-1,r-2),(c-1,r+2)]
guard (c' `elem` [1..8] && r' `elem` [1..8]) -- stále na ploche return (c',r')
```

#### -- kam sa dostane kôň na k krokov

```
length $ ink 7(0,0) = 45016
length $ nub $ ink 7(0,0) = 32
```

#### filterM (Control.Monad)

```
filterM :: (Monad m) => (a -> m Bool) -> [a] -> m [a]

> filterM (\x->[True, False]) [1,2,3] -- potenčná množina, powerset
[[1,2,3],[1,2],[1,3],[1],[2,3],[2],[3],[]]

filterM :: (Monad m) => (a -> m Bool) -> [a] -> m [a]
filterM _ [] = return []
filterM p (x:xs) = do

flg <- p x
ys <- filterM p xs
return (if flg then x:ys else ys)
```

## 4

#### mapM, forM

(Control.Monad)

```
mapM :: (Monad m) => (a -> m b) -> [a] -> m [b]
mapM f = sequence . map f
         :: (Monad m) => [a] -> (a -> m b) -> m [b] -- len zámena args.
forM
         = flip mapM
forM
> mapM (\x->[x,11*x]) [1,2,3]
[[1,2,3],[1,2,33],[1,22,3],[1,22,33],[11,2,3],[11,2,33],[11,22,3],[11,22,33]]
> forM [1,2,3] (\x->[x,11*x])
[[1,2,3],[1,2,33],[1,22,3],[1,22,33],[11,2,3],[11,2,33],[11,22,3],[11,22,33]]
> mapM print [1,2,3]
                                        > mapM_ print [1,2,3]
                                        23
3
[(),(),()]
```

#### foldM

(Control.Monad)

```
:: (Monad m) => (a -> b -> m a) -> a -> [b] -> m a
foldM
foldM _ a [] = return a
foldM f a (x:xs) = f a x >> = y -> foldM f y xs
                                 > foldM (\y -> \x ->
foldM f a_1 [x_1, ..., x_n] =
                                      do { print (show x++"..."++ show y);
   do {
                                           return (x*y)})
         a_2 < -f a_1 x_1;
                                     1 [1..10]
         a_3 < -f a_2 x_2;
                                 ???
         a_n < -f a_{n-1} x_{n-1};
         return f a_n x_n }
```

> foldM (\y -> \x -> do print (show x++"..."++ show y); return (x\*y)) 1 [1..10] ???

# newtype Either a b = Right a | Left b instance (Error e) => Monad (Either e) where return x = Right x Right x >>= f = f x Left err >>= f = Left err fail msg = Left (strMsg msg)

#### **Error monad**

data Term = Con Int | Div Term Term deriving(Show, Read, Eq)

```
:: Term -> Either String Int
eval
eval(Con a) = return a
eval(Div t u) = do
                  valT <- eval t
                  valU <- eval u
                  if valU == 0 then
                     fail "div by zero"
                  else
                     return (valT `div` valU)
> eval (Div (Con 1972) (Con 23))
Right 85
> eval (Div (Con 1972) (Con 0))
*** Exception: div by zero
```

# newtype Writer w a = Writer { runWriter :: (a, w) } instance (Monoid w) => Monad (Writer w) where return x = Writer (x, mempty) (Writer (x,v)) >>= f = let (Writer (y, v')) = f x in Writer (y, v `mappend` v')

#### Writer monad

(Control.Monad.Writer)

```
data Term = Con Int | Div Term Term deriving(Show, Read, Eq)
out :: Int -> Writer [String] Int
out x = writer(x, ["number: " ++ show x])
           :: Term -> Writer [String] Int
eval
eval(Con a) = out a
eval(Div t u) = do
                   valT <- eval t
                   valU <- eval u
                   out (valT `div` valU)
                   return (valT `div` valU)
> eval (Div (Con 1972) (Con 23))
WriterT (Identity (85,["number: 1972","number: 23","number: 85"]))
> runWriter $ eval (Div (Con 1972) (Con 23))
(85,["number: 1972","number: 23","number: 85"])
```

## 4

#### Writer monad

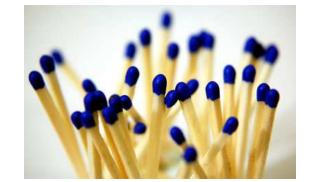
(Control.Monad.Writer)

```
-- tell :: MonadWriter w m => w -> m ()
gcd' :: Int -> Int -> Writer [String] Int
gcd' \ a \ b \ | \ b == 0 = do
                tell ["result " ++ show a]
                return a
         otherwise = do
                tell [show a ++ " mod " ++ show b ++ " = " ++ show (a `mod` b)]
                gcd' b (a `mod` b)
> gcd' 18 12
> runWriter (gcd' 2016 48)
(48,["2016 \mod 48 = 0","result 48"])
> mapM putStrLn (snd $ runWriter (qcd' 2016 48))
2016 \mod 48 = 0
result 48
[(),()]
```





```
type Kopa = Int
finished :: Kopa -> Bool
                                      -- kedy hra končí
finished = (== 0)
valid :: Kopa -> Int -> Bool -- korektný ťah
valid kopa beriem = (kopa >= beriem) && beriem < 4 && beriem > 0
getDigit :: String -> IO Int
getDigit prompt = do putStr prompt
                    x <- getChar
                     if isDigit x
                       then return (digitToInt x)
                     else
                       getDigit ""
```



### IO Monad

(dvaja hráči)

```
play2 :: Kopa -> Bool -> IO ()
play2 kopa hrac =
  do putStrLn ("kopa:" ++ (show kopa))
     if finished kopa then
       putStrLn ("Hrac " ++ (show (not $ hrac)) ++ " vyhral!")
     else
       do putStrLn ("Ide hrac " ++ (show hrac))
           beriem <- getDigit "kolko beries : "
           if valid kopa beriem then
             play2 (kopa - beriem) (not $ hrac)
           else
             do putStrLn "zly tah"
                 play2 kopa hrac
```

nim2 :: IO () nim2 = play2 (nextInt 10 20) True

-- generujeme náhodnú kopu 10..19

#### IO Monad

(jeden hráč proti kompu)



```
play1 :: Kopa -> IO ()
                                                  strategia :: Int -> Int
play1 kopa =
                                                  strategia kopa
  do putStrLn ("kopa:" ++ (show kopa))
                                                     | kopa `mod` 4 == 0 = kopa-1
     if finished kopa then
                                                     otherwise = kopa - (kopa `mod` 4)
       putStrLn "prehral si :("
     else
                                                  nim1 :: IO ()
       do beriem <- getDigit "kolko beries : "
                                                  nim1 = play1 (nextInt 10 20)
           if valid kopa beriem then
             let kopa' = kopa - beriem in
                if finished kopa' then
                  putStrLn "vyhral si :)"
                else
                  do putStrLn ("ja beriem:" ++ (show (kopa' - (strategia kopa'))))
                      play1 (strategia kopa')
           else
              do putStrLn "zly tah"
                 play1 kopa
```

#### State monad

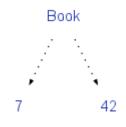
(Control.Monad.State)

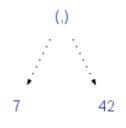
```
newtype State s a = State { runState :: (s -> (a,s)) }
instance Monad (State s) where
                  = State \s -> (a,s)
 return a
  (State x) >> = f = State \s ->
                          let (v,s') = x s in runState (f v) s',
class (Monad m) => MonadState s m \mid m -> s where
                                   -- get vráti stav z monády
 get :: m s
                                   -- put prepíše stav v monáde
 put :: s -> m ()
modify :: (MonadState s m) => (s -> s) -> m ()
modify f = do s < -get
                 put (f s)
```

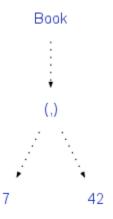
### Čo je newtype vs. data vs. type

```
newtype State s a = State { runState :: (s \rightarrow (a,s)) }
State s a má rovnakú reprezentáciu ako (s \rightarrow (a,s)), ale nie je to type State s a = s \rightarrow (a,s)
```

```
data State s a = State { runState :: (s -> (a,s)) }
State s a je reprezentovaná krabicou State s pointrom na (s -> (a,s))
Príklad:
```







```
pop :: State Stack Int
pop = state(\(x:xs) -> (x,xs))

push :: Int -> State Stack ()
push a = state(\(xs -> ((),a:xs)))
```

#### State Stack

```
Stav výsledok
                                       type Stack = [Int]
type Stack = [Int]
                                        pushAll' :: Int -> State Stack String
pushAll :: Int -> State Stack String
                                        pushAll' 0 = return ""
pushAll 0 = return ""
                                        pushAll' n = do
pushAll n = do {
                                                   stack <- get -- push n
           push n;
                                                   put (n:stack)
           str <- pushAll (n-1);
                                                   str <- pushAll (n-1)
           nn <- pop; ——
                                                   (nn:stack') <- get -- nn <- pop
           return (show nn ++ str)}
                                                   put stack'
                                                   return (show nn ++ str)
evalState vráti výslednú hodnotu
> evalState (pushAll 10) []
                                        > evalState (pushAll' 10) []
"10987654321"
                                        "10987654321"
execState vráti výsledný stav
                                        > execState (pushAll' 10) []
> execState (pushAll 10) []
```

#### Preorder so stavom

(Control.Monad.State)

```
data Tree a =
                       Nil I
                       Node a (Tree a) (Tree a)
                       deriving (Show, Eq)
    preorder :: Tree a -> State [a] ()
                                                     -- stav a výstupná hodnota
    preorder Nil
                                           = return ()
    preorder (Node value left right)
                                           do {
                                                     str<-get; -- get state=preorderlist</pre>
                                                     put (value:str); -- modify (value:)
                                                     preorder left;
e :: Tree String
                                                     preorder right;
e = Node "c" (Node "a" Nil Nil) (Node "b" Nil Nil)
                                                     return () }
> execState (preorder e) []
["b","a","c"]
> evalState (preorder e) []
()
```

### 4

#### Prečíslovanie binárneho stromu

```
index :: Tree a -> State Int (Tree Int) -- stav a výstupná hodnota
index Nil
                    = return Nil
index (Node value left right) =
                    do {
                              i <- get;
                              put (i+1);
                              ileft <- index left;</pre>
                              iright <- index right;</pre>
                              return (Node i ileft iright) }
> e'
Node "d" (Node "c" (Node "a" Nil Nil) (Node "b" Nil Nil)) (Node "c" (Node "a" Nil
   Nil) (Node "b" Nil Nil))
> evalState (index e') 0
Node 0 (Node 1 (Node 2 Nil Nil) (Node 3 Nil Nil)) (Node 4 (Node 5 Nil Nil) (Node 6
   Nil Nil))
> execState (index e') 0
```

#### Prečíslovanie stromu 2

```
type Table a = [a]
numberTree :: Eq a => Tree a -> State (Table a) (Tree Int)
numberTree Nil
                            = return Nil
numberTree (Node x t1 t2)
                               = do num <- numberNode x
                                      nt1 <- numberTree t1
                                       nt2 <- numberTree t2
                                       return (Node num nt1 nt2)
   where
   numberNode :: Eq a => a -> State (Table a) Int
   numberNode x
                             = do
                                      table <- get
                                       (newTable, newPos) <- return (nNode x table)
                                       put newTable
                                       return newPos
   nNode:: (Eq a) => a -> Table a -> (Table a, Int)
   nNode x table
                             = case (findIndexInList (== x) table) of
                                       Nothing -> (table ++ [x], length table)
                                       Just i -> (table, i)
```

#### Prečíslovanie stromu 2

```
numTree :: (Eq a) => Tree a -> Tree Int
numTree t = evalState (numberTree t) []
```

```
> numTree ( Node "Zero"

(Node "One" (Node "Two" Nil Nil)

(Node "One" (Node "Zero" Nil Nil) Nil)
```

Node 0 (Node 1 (Node 2 Nil Nil) (Node 1 (Node 0 Nil Nil) Nil)) Nil