Haskell

A Purely Functional Language

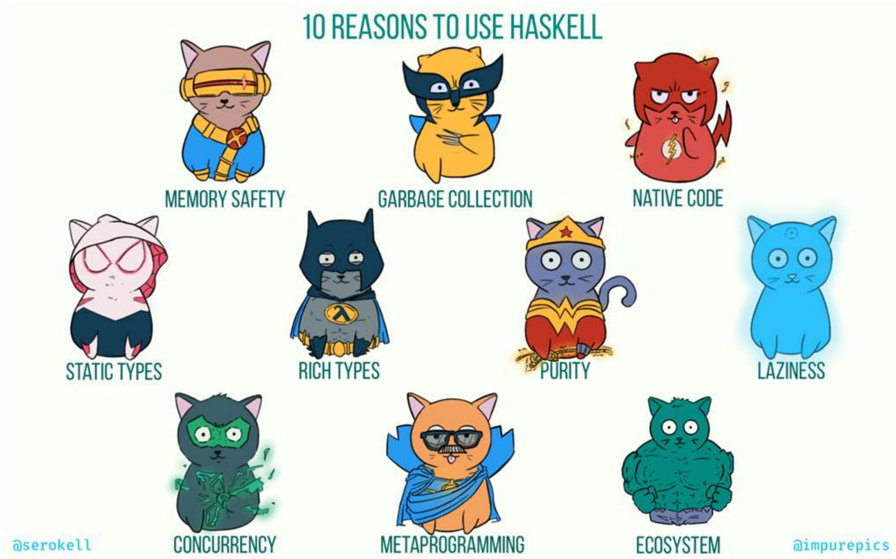
featuring static typing, higher-order functions, polymorphism, type classes and monadic effects

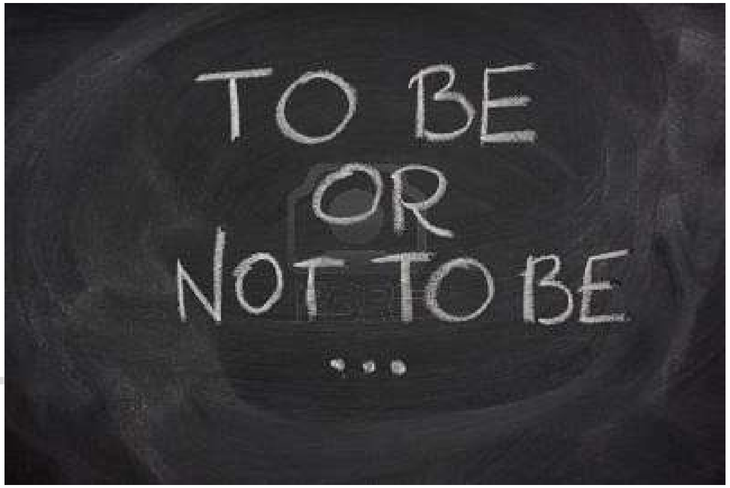
Funkcie a funkcionály

a ich

referečná transparentosť

Peter Borovanský





- 0x2B | ~0x2B
- 0x2B | !0x2B
- 0x2B || ~0x2B
- 0x2B || !0x2B

Všetko, čo by ste chceli vedieť o Haskelli, ale báli ste sa spýtať...

Funkcia

a čo s ňou

- **aplikovať** – na argument/y
  - Haskell: f 5, goo 17 21, fib (n-2) -- zátvorky píšeme kvôli zložitému argumentu, (fib n)-2
  - inde: f(5), goo(17,21) fib(n-2)

Celá pravda o Haskelli:

že **a b c d = (((a b) c) d)**...lebo operátor aplikácie funkcie na argument je *ľavo-asociatívny*, teda, ak zabudnem zátvorky, tak ich iniciatívne chápe grupované doľava

- **abstrahovať** / abstrakcia
- jazyk je funkcionálny, ak viete vytvoriť funkciu (λ-abstrakciu) počas behu program
  - Haskell:(+) = \x -> \y -> x+y
  - Python: `lambda x, y: x + y`
  - JS: (a, b) => a + b
  - Java: (a, b) -> a + b
  - Kotlin: a:Int, b:Int -> a + b
  - Swift: a, b in a+b
  - C, Pascal, … ☹
- syntax doesn't matter

Všetko, čo by ste chceli vedieť o Haskelli, ale báli ste sa spýtať...

Funkcia

a čo s ňou

- má typ
  - Python, JS, ...: ☹
  - **Haskell**:

    (+) :: Int -> Int -> Int,   čo **nie je** to isté ako :: (Int, Int) -> Int
  - **Curry**ing (+ 4) :: Int -> Int, (+ 4 7) :: Int = 11
  - Celá pravda o Haskelli:
  - **Int -> Int -> Int -> Char = Int -> (Int -> (Int -> Char))** ... lebo operátor funkčného typu **->** je *pravo asociatívny*, teda ak zabudnem zátvorky, tak ich chápe doprava. Explicitne, (Int->Int) -> (Int -> Int)
  - **Int -> Int -> String != (Int, Int) -> String**... lebo prvé je funkcia, ktorá vráti funkciu, ktorá vráti String. Vďaka *currying* ju volám takto: f 4 5, čo je (f 4) 5. Druhé je funkcia, ktorá čaká dvojicu. Musím ju volať takto: g (4,5), a vyzerám skôr Javista, a nie Haskellista
  - Príklad:

```
f :: Int -> (Int -> Int)
f a b = 10*a+b
f 5 7 = 57
f 5 :: Int -> Int
f 5 = \b -> 10*5+b
f 5 b = 10*5+b
f 5 y = 10*5+y
```

```
g :: (Int -> Int) -> Int
g h = h 7
g (+11) = 18
g (\x->x+11) = 18
g (^2) = 49
g (\x->x^2) = 49
g (*5) = 35
```

Všetko, čo by ste chceli vedieť o Haskelli, ale báli ste sa spýtať...

Funkcia

a čo s ňou

- komponovať – f . g - z matematiky $(f \circ g)(x) = f(g(x))$
- je asociatívna, nie je komutatívna, identita id = \x->x je neutrálny prvok

Haskell:

```
(.) :: (b->c) -> (a->b) -> (a->c)
(.) = \f -> \g -> \x -> f (g x)
f . g = \x -> f (g x)
(f . g) x = f (g x)

composeMany :: [a->a] -> (a->a)
composeMany [] = id
composeMany (f:fs) = f . composeMany fs

-- ak poznáte reduce.py ≈ foldr
composeMany xs = foldr (.) id xs
```

Bezargumentový snobizmus

```
composeMany = foldr (.) id
```

pramení tiež z matematiky, lebo matematik tiež radšej napíše f = g miesto **∀x: f(x) = g(x)**

Python, JS, ...:

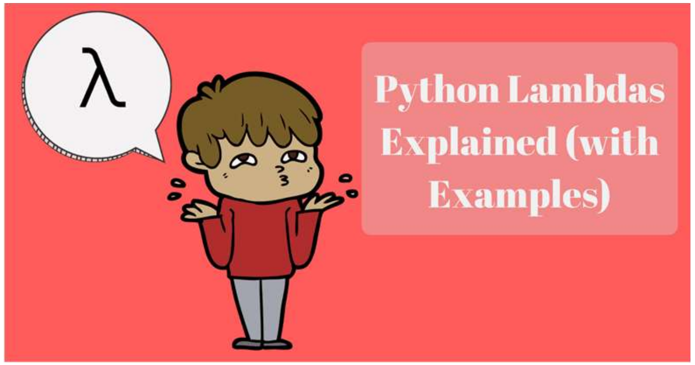
```
def compose(f, g):
    return lambda x: f(g(x))


def composeMany(*fs):
    return reduce(compose, fs)

print(composeMany(
        lambda x: x+1,
        lambda x: x+2,
        lambda x: x*3
        )(10))
```

Python Kvíz

pre aplikovancov




```
print(map(lambda x: x*x, [1,2,3,4,5]))
print(list(map(lambda x: x*x, [1,2,3,4,5])))
print(list(filter(lambda y:y>10,map(lambda x: x*x, [1,2,3,4,5]))))
```

<map object at 0x037

[1, 4, 9, 16, 25]

[16, 25]

```
from functools import reduce
print(reduce((lambda x, y: x * y), [1, 2, 3, 4]))
```

24

```
print(reduce((lambda x, y: x + y), [1, 2, 3, 4]))
print(reduce((lambda x, y: x - y), [1, 2, 3, 4]))
```

10

-8

```
def compose(f, g):
        return lambda x: f(g(x))
print(compose( lambda x: x+1, lambda x: x*3 )(10))
```

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```
def composeMany(*fs):
        return reduce(compose, fs)
print(composeMany(lambda x:x+1, lambda x:x+2, lambda x:x*3)(10))
```

33

lambdas.hs

All Things Pythonic

Guido van Rossum: The fate of reduce() in Python 3000, (r.2005)

- Python aquired lambdas, reduce(), filter() and map() thanks a Lisp hacker
- despite of the PR value, I think these features should be cut from Python 3
- **Update: lambda, filter and map will stay (the latter two with small changes, returning iterators instead of lists). Only reduce will be removed from the 3.0 standard library. You can import it from functools.**

https://www.artima.com/weblogs/viewpost.jsp?thread=98196

Lambda & kompozícia

prvočíselná etuda

6 spôsobov ako naprogramovať prvočíselný test, ilustrácia možností jazyka

```
-- upravené riešenie s HaskellThonu, delitele len po sqrt
jePrvocislo n = null (delitele n)
sqrti n        = floor (sqrt $ fromIntegral n)
delitele n     = [d | d <- [2..sqrti n], n `mod` d == 0]

-- explicitne viditeľné lambda abstrakcie
jePrvocislo' =  \n -> null (delitele' n)
sqrti'       =  \n -> floor (sqrt $ fromIntegral n)
delitele'    =  \n -> [d | d <- [2..sqrti' n], n `mod` d == 0]

-- 'funkcionálny' snobizmus, zamlčané explicitné argumenty, poučívanie kompozície fcií
jePrvocislo'' =  null . delitele''
sqrti''       = floor . sqrt . fromIntegral
delitele''    = \n -> [d | d <- [2..sqrti'' n], n `mod` d == 0]
```

primes.hs

Clojure

prvočíselná etuda

```
-- closure: sqrt a delitele sú vnorené funkcie, ktoré vidia do prostredia materskej fcie
jePrvocislo''' n = null delitele where
                   sqrtn = floor (sqrt $ fromIntegral n)
                   delitele = [d | d <- [2..sqrtn], n `mod` d == 0]

-- let … in … vždy nahradí where, ale naopak to neplatí
jePrvocislo'''' n = let sqrtn = floor (sqrt $ fromIntegral n)
                        delitele = [d | d <- [2..sqrtn], n `mod` d == 0]
                    in null delitele

-- konečne riešenie správne aj matematicky … ☺
jePrvocislo''''' n = n>1 && null delitele where
                     sqrtn = floor (sqrt $ fromIntegral n)
                     delitele = [d | d <- [2..sqrtn], n `mod` d == 0]
```

```
main = do putStrLn $ show $ filter jePrvocislo [1..100]
          putStrLn $ (show . filter jePrvocislo'') [1..100]
          putStrLn $ (show . filter jePrvocislo''') [1..100]
          putStrLn $ (show . filter jePrvocislo'''') [1..100]
          putStrLn $ (show . filter jePrvocislo''''') [1..100]
```

```
*Main> main
[1,2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,89,97]
[1,2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,89,97]
[1,2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,89,97]
[1,2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,89,97]
[2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,89,97]
```

primes.hs

Všetko, čo by ste chceli vediet o Haskelli, ale báli ste sa spýtať

Otázka z interview

a ako na ňu

Haskell:

```
dvojica a b = pair
    where pair f = f a b
dvojica :: s->t->((s->t->v)->v)
-- inak
dvojica a b = \f -> f a b
dvojica a b f = f a b
prvy (dvojica a b) = a
druhy (dvojica a b) = b
prvy p = p (\ a-> \b -> a)
druhy p = p (\ a-> \b -> b)
print $ prvy (dvojica 4 5)
print $ druhy (dvojica 4 5)
prvy p = p true
   where true a b = a
druhy p = p false
    where false a b = b
```

Python, JS, ...:

```
def dvojica(a, b):
    def pair(f):
        return f(a, b)
    return pair
# inak
def dvojica(a,b):
    return lambda f: f(a,b)

def head(p):
    return p(lambda a,b :a)

def tail(p):
    return p(lambda a,b:b)

print(head(dvojica(4,5)))
print(tail(dvojica(4,5)))
```

pair.hs, pair.py

Často sa *obrábabie* zoznamov prezentuje ako funkcionálne programovanie, pritom je to len nevyhnutný úvod k lepšiemu...

Zoznamová zoznamka

Haskell:

xs = [1,2,3,4,5] [1..5]

length xs

xs!!i

neexistuje-immutable list

head xs

tail xs

last xs

init xs

take n xs

drop n xs

take m (drop n xs)

xs++xs

reverse xs

Python:

[1,2,3,4,5]		
len xs		
xs[i]	.. indexy 0..length xs-1	
xs[i]=...		
xs[0]		1
xs[1:]		[2,3,4,5]
xs[len(xs)-1]		5
xs[:len(xs)-1]		[1,2,3,4]
xs[:n]		
xs[n:]		
xs[n:n+m]		
xs+xs		[1,2,3,4,5,1,2,3,4,5]
xs.reverse()		returns void

import Data.List

http://hackage.haskell.org/package/base-4.12.0.0/docs/Data-List.html

base-4.12.0.0: Basic libraries | Quick Jump

Data.List

Copyright	(c) The University of Glasg
License	BSD-style (see the file libra
Maintainer	libraries@haskell.org
Stability	stable
Portability	portable
Safe Haskell	Trustworthy
Language	Haskell2010

Operations on lists.

Basic functions

Contents

`(++) :: [a] -> [a] -> [a]` | infixr 5 | # Source

Append two lists, i.e.,

```
[x1, ..., xm] ++ [y1, ..., yn] == [x1, ..., xm, y1, ..., yn]
[x1, ..., xm] ++ [y1, ...] == [x1, ..., xm, y1, ...]
```

If the first list is not finite, the result is the first list.

`head :: [a] -> a` # Source

Extract the first element of a list, which must be non-empty.

`last :: [a] -> a` # Source

Extract the last element of a list, which must be finite and non-empty.

`tail :: [a] -> [a]` # Source

Extract the elements after the head of a list, which must be non-empty.

`init :: [a] -> [a]` # Source

Return all the elements of a list except the last one. The list must be non-empty.

Možno vás to prekvapí

- ale zoznam – list – [a]

- **je immutable/nemenný asi ako String v Jave**
- raz, keď ho vytvoríte, nikdy ho už nezmeníte, len môžete vytvoriť nový, trochu iný
- inak povedané
  indexovanie xs!!i existuje, ale niečo ako nahradenie xs[i] = v neexistuje

Chce to objaviť inú dátovú štruktúru, a sú: Data.Array, Data.Set, ...
- ale tie sú tiež immutable/nemenné
- …lebo filozófia

- prvý pocit mutable-dátovej štruktúry poskytnú až monády, state monad

- **je homogénny v type**

- v zozname typu [a] sú len hodnoty typu a, a žiadne iné
- žiaden Any, Object, mother-of-all-types ... neexistuje, našťastie

List-comprehension

Každý poriadny kurz FP začína funkcionálmi map a filter:

...ale my sme trénovali list-comprehension:

```
[ f x | x <- xs, p x]        [ f(x) for x in xs if p(x)]
```

map, filter sú deriváti list-comprehension

```
map          ::   (a -> b) -> [a] -> [b]
map f xs     =    [ f x | x <- xs]

filter       ::   (a -> Bool) -> [a] -> [a]
filter p xs  =    [ x | x <- xs, p x]
```

Všetko, čo ste chceli zmeniť, a nikdy sa vám to nepodarilo

- zoznam ("pole") xs vieme indexovať indexami i <- [0..length xs-1]

```
        xs!!i                        -- getter
```

- neexistuje setter xs[i] = value

```
set  ::  [t] -> Int -> t -> [t]
set xs i value | i < 0            = xs    -- out of range
               | i >= length xs   = xs    -- out of range
               | otherwise        = (if i == 0 then value else y):set ys (i-1) value
                                    where (y:ys) = xs
               | otherwise        = let (y:ys) = xs in
                                    (if i == 0 then value else y):set ys (i-1) value
set''  ::  [t] -> Int -> t -> [t]
set'' xs i value | i < 0            = xs    -- out of range
                 | i >= length xs   = xs    -- out of range
                 | otherwise        = [xs!!j | j <- [0.. i-1] ] ++ [value] ++
                                      [xs!!j | j <- [i+1..length xs-1] ]
```

zoznam.hs

Zoznamová rekurzia

```
-- vyber prvých n prvkov zo zoznamu
take                :: Int -> [a] -> [a]
take 0 _            = []
take _ []           = []
take n (x:xs)       = x : (take (n-1) xs)

-- dĺžka zoznamu
length              :: [a] -> Int
length  []          = 0
length  (x:xs)      = 1 + length xs
```

```
"?: " take 5 [1,3..100]
[1,3,5,7,9]
"?: " length (take 5 [1,3..100])
5
"?: " length $ take 5 [1,3..100]
5
```

Hypotéza (pre ľubovoľné n a xs) platí:

- length (take n xs) = n
- length $ take n xs = n   -- dolárová notácia
- (length . take n) xs = n   -- kompozícia funkcií z matematike

Dôkaz - length (take n xs) = n
(matematická indukcia)

Indukcia (vzhľadom na dĺžku/štruktúru xs):

- **xs = []**

  length (take n []) = 0

  0 = 0

  č.b.t.d.

- **xs = (y:ys)**

  length (take n (y:ys)) = n

  length (y:take (n-1) ys) = n

  1 + <u>length (take (n-1) ys)</u> = n

  indukčný predpoklad, |ys| < |xs|

  1 + <u>(n-1)</u> = n

č.b.t.d.

```
Definície z predošlej strany:
take             :: Int -> [a] -> [a]
take 0 _         = []
take _ []        = []
take n (x:xs) = x : take (n-1) xs
```

```
length           :: [a] -> Int
length  []        = 0
length  (x:xs) = 1 + length xs
```

QuickCheck

Elegantný nástroj na testovanie (!!! nie dôkaz !!!) hypotéz

"?: " import Test.QuickCheck

"?: " quickCheck (\(xs,n) -> length (take n xs) == n)

*** Failed! Falsifiable (after 2 tests and 1 shrink):

"?: " verboseCheck (\(xs,n) -> length (take n xs) == n)

Passed:

([],0)

Passed:

([()],1)

Failed:

([],-1)

*** Failed! Failed:

Neplatí to pre n záporne, lebo napr. take (-3) [1..100] = [],

resp. naša definícia nepokrýva prípad n < 0

!!! ALE MY SME TO AJ TAK “DOKÁZALI”... !!! ☹

QuickCheck.hs

QuickCheck

Podmienka: miesto písania

if n >= 0 then length (take n s) == n **else True**

Napíšeme pre-condition pomocou **==>**

"?: " verboseCheck (\(xs,n) -> **n>=0 ==>** length (take n xs) == n)

Passed:

([],0)

Failed:

([()],2)

Neplatí to pre ak length xs < n

"?: " quickCheck (\(xs,n) -> n>=0 && length xs >= n ==>
length (take n xs) == n)

*** Gave up! Passed only 35 tests.

Tvrdenie sme **overili** na niekoľkých prípadoch, ale to **nie je dôkaz**.

V dôkaze môžeme urobiť chybu (ako na slajde 2), QuickCheck slúži ako nástoj na hľadanie/odhaľovanie kontrapríkladov, kedy naše tvrdenie neplatí.

QuickCheck.hs

QuickCheck

Don't write tests!

Generate them from properties

- miesto písania unit testov, quickcheck vám ich (nejaké) vygeneruje
- vy potom nepíšete testy, ale vlastnosti vašich programov.

O niečom podobnom dávno snívali/dúfali Hoare, Dijkstra, ...

- s rozdielom, že vlastnosti programov chceli dokázať,
- miesto hľadania kontrapríkladu.

Quickcheck:

- generuje náhodné vstupné hodnoty, pre základné aj definované typy
  - Int, Bool, ...
  - [Int], String, …
  - Int->Int, Int->Bool
- ak nájde kontrapríklad (už vieme, že to neplatí), snaží sa ho zminimalizovat/zjednodušiť, napr: length (take n xs)==n neplatí pre length (take 21 [5,-192,3981,-291,2220,-192,22,12,-192,-1]) == 21

QuickCheck

Don't write tests!

Generate them from properties

autori: Koen Claessen, John Hughes

Príklad Parretovho pravidla 20:80 - za 20% energie chytíte 80% problémov

Príklad (viac tu):

Collatz (viac tu) je funkcia f(n)= if n `mod`2 == 0 then n/2 else 3n+1.

```
f      :: Integer -> Integer
f n    | even(n) = n `div` 2
       | odd(n)  = 3*n + 1
collatz   :: Integer -> Bool
collatz   1 = True
collatz   n = collatz (f n)
```

```
"?: " quickCheck (\n -> n > 0 ==> collatz(n))
+++ OK, passed 100 tests.
"?: " quickCheckWith stdArgs{ maxSuccess = 100000 }
                   (\n -> n > 0 ==> collatz(n))
+++ OK, passed 100000 tests.
```

Paul Erdős: "Mathematics may not be ready for such problems.“ offered $500 for its solution.

Kritérium delieteľnosti 11

- rodné číslo 786115 3333 (ženské, *15.nov1978)
- 7861153333 `mod` 11 == 0
- 11 | 7861153333 iff 11 | 7+6+1+3+3 –(8+1+5+3+3) = 0

- naše rodné čísla sú delitelné 11, ľahká kontrola

- čísla kariet majú tiež kontrolu, Luhnnov algo, DÚ1

- čo bankové účty
- 7000155733 / 8180 – soc.poisťovňa
- cifry násobíme váhami 6,3,7,9,10,5,8,4,2,1, sčítame, výsledok delitelný 11
- 11 | 7*6+0*3+0*7+0*9+1*10+5*5+5*8+7*4+3*2+3*1
- (sum $ zipWith (*) [7,0,0,0,1,5,5,7,3,3] [6,3,7,9,10,5,8,4,2,1]) `mod` 11
- (sum $ zipWith (*) [2,7,0,1,1,3,2,4,4,3] [6,3,7,9,10,5,8,4,2,1]) `mod` 11

Kvíz - platí/neplatí ?

(neseriózny prístup ale intuíciu treba tiež trénovať)

- length [m..n] == n-m+1

  "?: " quickCheck ((\(n,m) -> length [m..n] == n-m+1))

  *** Failed! Falsifiable (after 3 tests and 1 shrink):

  "?: " quickCheck ((\(n,m) -> **m <= n ==>** length [m..n] == n-m+1))

  +++ OK, passed 100 tests.

- length (xs ++ ys) == length xs + length ys

  "?: " quickCheck((\xs->\ys->(length (xs++ys)==length xs + length ys)))

  +++ OK, passed 100 tests.

- length (reverse xs) == length xs

  quickCheck((\xs -> (length (reverse xs) == length xs)))

  +++ OK, passed 100 tests.

- (xs, ys) == unzip (zip xs ys)

  quickCheck((\xs -> \ys -> ((xs, ys) == unzip (zip xs ys))))

  *** Failed! Falsifiable (after 3 tests and 1 shrink):

  quickCheck((\xs -> \ys -> (length xs == length ys ==>

  (xs, ys) == unzip (zip xs ys))))

QuickCheck.hs

Počet cifier ešte raz

funkcionálny štýl

```
pocetCifier :: Integer -> Int
pocetCifier n = length $ show n
                    pocetCifier = length . Show

pocetCifier' :: Integer -> Int
pocetCifier' n =  fromIntegral $ ceiling $ (logBase 10 (fromIntegral n))
                    pocetCifier' =  fromIntegral . ceiling . (logBase 10) . fromIntegral

pocetCifier'' :: Integer -> Int
pocetCifier'' n =  length $ takeWhile (/=0) $ iterate (`div` 10) n
                    pocetCifier'' =  length . takeWhile (/=0) . iterate (`div` 10)

hypoteza1 = quickCheck(\n -> (n > 0) ==> pocetCifier n == pocetCifier'' n)
hypoteza2 = quickCheck(\n -> (n > 0) ==> pocetCifier n == pocetCifier' n)
hypoteza2' = quickCheck(\n -> (n > 1) ==> pocetCifier n == pocetCifier' n)
hypoteza2'' = quickCheck(\n -> (n > 10) ==> pocetCifier n == pocetCifier' n)
-- platí/neplatí ?
```

cifry.hs

Funkcia/predikát argumentom

- zober zo zoznamu tie prvky, ktoré spĺňajú podmienku (test)
  Booleovská podmienka príde ako argument funkcie a má typ (a -> Bool):

```
filter          :: (a -> Bool) -> [a] -> [a]
filter p xs     = [ x | x <- xs, p x ]
```

> **> filter even [1..10]**
> **[2,4,6,8,10]**

alternatívna definícia:

```
filter p []      = []
filter p (x:xs)  = if p x then x:(filter p xs) else filter p xs
```

vlastnosti (zväčša úplne zrejmé ?):

- filter True xs = xs ... [x | x <- xs, True] = [x | x <- xs] = xs
- filter False xs = [] ... [x | x <- xs, False] = [ ]
- filter p1 (filter p2 xs) = filter (p1 && p2) xs
- (filter p1 xs) ++ (filter p2 xs) = filter (p1 || p2) xs

```
filter p []        = []
filter p (x:xs)    = if p x then x:(filter p xs) else filter p xs
```

Dôkaz

filter p1 (filter p2 xs) = filter (p1 && p2) xs

Indukcia vzhľadom na parameter xs

- []

  L.S. = filter p1 (filter p2 []) = filter p1 [] = [] = filter (p1 && p2) [] = P.S.

- (x:xs)

  L.S. = filter p1 (filter p2 (x:xs)) = ... definícia

  filter p1 (if p2 x then x:(filter p2 xs) else filter p2 xs) = ... filter dnu cez if

  if p2 x then filter p1 (x:(filter p2 xs)) else filter p1 (filter p2 xs) = ... indukcia

  if p2 x then filter p1 (x:(filter p2 xs)) else filter (p1 && p2) xs = ... definícia

  if p2 x then

          if p1 x then x:(filter p1 (filter p2 xs)) else filter p1 (filter p2 xs)

  else filter (p1 && p2) xs = ... 2 x indukcia

  if p2 x then

          if p1 x then x:(filter (p1 && p2) xs) else filter (p1 && p2) xs

  else filter (p1 && p2) xs =

QuickCheck.hs

```
filter p []         = []
filter p (x:xs)     = if p x then x:(filter p xs) else filter p xs
```

Dôkaz

filter p1 (filter p2 xs) = filter (p1 && p2) xs

<u>if p2 x then</u>

<u>if p1 x then</u> x:(filter (p1 && p2) xs) else filter (p1 && p2) xs

else filter (p1 && p2) xs = ... požívame vlastnosť if-then-else

if A then	if A && B then C
if B then C	else D
else D	
else D	

if (p1 && p2) x then x:(filter (p1 && p2) xs) else filter (p1 && p2) xs = ... def.

filter (p1 && p2) (x:xs) = P.S.

č.b.t.d.

QuickCheck a funkcie

Funkcie sú hodnoty ako každé iné
Ako vie QuickCheck pracovať s funkciami ?

- je skladanie funkcií komutatívne ?

```
"?: " import Text.Show.Functions
"?: " quickCheck(
    (\x -> \f -> \g -> (f.g) x == (g.f) x)::Int->(Int->Int)->(Int->Int)->Bool)
*** Failed! Falsifiable (after 2 tests):
```

- je skladanie funkcií asociatívne ?

```
"?: " quickCheck(
    (\x -> \f -> \g -> \h -> (f.(g.h)) x == ((f.g).h) x)
            ::Int->(Int->Int)->(Int->Int)->(Int->Int)->Bool)
+++ OK, passed 100 tests.
```

Opäť to NIE je DÔKAZ, len 100 pokusov.

QuickCheck.hs

QuickCheck a predikáty

Predikát je len funkcia s výsledným typom Bool

- filter p1 (filter p2 xs) = filter (p1 && p2) xs

?: " quickCheck ((\xs -> \p1 -> \p2 ->
filter p1 (filter p2 xs) == filter (p1 && p2) xs)
:: [Int] -> (Int->Bool) -> (Int->Bool) -> Bool)

<interactive>:113:91: Couldn't match expected type 'Bool' ---

NEPLATÍ LEBO ANI TYPY NESEDIA, && je definovaný na Bool, a nie na funkciách Int->Bool

- filter p1 (filter p2 xs) = filter (\x-> p1 x && p2 x) xs

  +++ OK, passed 100 tests.

Opäť to NIE je DÔKAZ (ten už bol), len 100 pokusov.

- (filter p1 xs) ++ (filter p2 xs) = filter (\x -> p1 x || p2 x) xs

"?: " quickCheck ((\xs -> \p1 -> \p2 ->
(filter p1 xs) ++ (filter p2 xs) == filter (\x -> p1 x || p2 x) xs)
:: [Int] -> (Int->Bool) -> (Int->Bool) -> Bool)

*** Failed! Falsifiable (after 3 tests):
[0] <function> <function>

QuickCheck.hs

Rekapitulácia

videli sme tzv. **Property Based Testing** pomocou **QuickCheck**:

- najznámejšie dva funkcionály: map, filter – ktoré poznáte aj z Pythonu
- quickCheck náhodne generujúci testy/kontrapríklady pre typy
  - základné typy: Int, Bool, String…
  - zoznamy: [Int], [t]
  - funkcie: Int->Int, a->b, …
- množstvo 'ekvivalentných' tvrdení, niektoré boli neekvivalentné...

Property Based Testing (PBT):

- rôzne implementácie QuickCheck v jazykoch:
  - Scala (Scala Check), F# (FsCheck), Clojure (test.check), Python (Hypothesis)
- musí implementovať:
  - generovanie dát pre základné typy, parametrické typy, funkčné typy, ...
  - generovanie dát pre používateľom definované typy
  - zjednodušovanie kontrapríkladu (shrinking)

https://en.wikipedia.org/wiki/QuickCheck

Funkcia argumentom map

- funktor, ktorý aplikuje funkciu (1.argument) na všetky prvy zoznamu

```
map             :: (a->b) -> [a] -> [b]
map f []        = []
map f (x:xs)    = f x : map f xs
map f xs        = [ f x | x <- xs ]
```

- Príklady:

```
map (+1) [1,2,3,4,5]                    = [2,3,4,5,6]
map odd  [1,2,3,4,5]                    = [True,False,True,False,True]
and (map odd  [1,2,3,4,5])              = False

map head [ [1,0,0], [2,1,0], [3,0,1] ]  = [1, 2, 3]
map tail [ [1,0,0], [2,1,0], [3,0,1] ]  = [ [0,0], [1,0], [0,1] ]
map (0:) [[1],[2],[3]]                  = [[0,1],[0,2],[0,3]]
```

Vlastnosti map

- map id xs = xs ☑ map id = id
- map (f.g) xs = map f (map g xs) ☑ map f . map g = map (f.g)
- ~~head (map f xs) = f (head xs)~~ ☑ ~~head . map f = f . head~~
- ~~tail (map f xs) = map f (tail xs)~~ ☑ ~~tail . map f = map f . tail~~
- map f (xs++ys) = map f xs++map f ys ☑
- length (map f xs) = length xs ☑ length . map f = length
- map f (reverse xs) = reverse (map f xs) ☑ map f.reverse=reverse.map f
- ~~sort (map f xs) = map f (sort xs)~~ ☑ ~~sort . map f = map f . sort~~
- map f (concat xss) = concat (map (map f) xss) ☑

map f . concat = concat . map (map f)

```
concat              :: [[a]] -> [a]
concat []           = []
concat (xs:xss)     = xs ++ concat xss
concat [[1], [2,3], [4,5,6],[]] = [1,2,3,4,5,6]
```

Vlastnosti map, filter

Na zamyslenie:

- filter p (map f xs) = ??? (filter (p.f) xs)
- filter p (map f xs) = map f (filter (p.f) xs)
- filter p . map f = map f . filter (p.f)

Dôkaz:

```
filter p (map f xs)    = filter p [ f x | x<-xs]
                       = [y | y <- [ f x | x<-xs], p y]
                       = [f x |  x<-xs, p (f x)]
                       = map f [x |  x<-xs, p (f x)]
                       = map f (filter (p.f))
```

Quíz - prémia

nájdite pravdivé a zdôvodnite

- map f . take n = take n . map f

- map f . filter p = map fst . filter snd . map (fork (f,p))

  where fork :: (a->b, a->c) -> a -> (b,c)

  fork (f,g) x = (f x, g x)

- filter (p . g) = map (inverzna_g) . filter p . map g

  ak inverzna_g . g = id

- reverse . concat = concat . reverse . map reverse

- filter p . concat = concat . map (filter p)

QuickSort s QuickCheck
(na cvičeniach)

```
import Test.QuickCheck
import Data.List (sort)
qsort :: Ord a => [a] -> [a]                    -- Ord a – vieme triediť len porovnateľné typy
qsort []      = []                              -- analógia interface Comparable<a>
qsort (p:xs) = qsort (filter  (< p) xs) ++ [p] ++ qsort (filter (>= p) xs)

quickCheck( \xs -> length (qsort xs) == length xs)
quickCheck((\xs -> length (qsort xs) == length xs)::[Int]->Bool)
quickCheck((\xs -> qsort xs == sort xs)::[Int]->Bool)
quickCheck((\xs -> qsort(qsort xs) == qsort xs)::[Int]->Bool)

isSorted :: Ord a => [a] -> Bool
isSorted xs = sort xs == xs
isSorted' :: Ord a => [a] -> Bool
isSorted' [] = True
isSorted' xs = and $ zipWith (<=) (init xs) (tail xs)
quickCheck((\xs -> isSorted (qsort xs))::[Int]->Bool)
quickCheck((\xs -> isSorted' (qsort xs))::[Int]->Bool)
```

QuickSort.hs

Kombinatorika
(podobné nájdete v Prémii QC & Kombinatorika)

```
module Kombinatorika where
import Test.QuickCheck
import Data.List

fact n = product [1..n]
comb n k = (fact n) `div` ((fact k) * (fact (n-k)))

-- permutácie
perms :: [t] -> [[t]]
perms [] = [[]]
perms (x:xs) = [ insertInto x i ys | ys <- perms xs, i <- [0..length ys] ]
                    where insertInto x i xs = (take i xs) ++ (x:drop i xs)
qchPERM = quickCheck(\n -> (n > 0 && n < 10) ==> length (perms [1..n]) == fact n)
```

kbo ::	[t] -> Int -> [[t]]	?	n!
kso ::	[t] -> Int -> [[t]]		(n nad k)
vbo :: (Eq t) =>	[t] -> Int -> [[t]]		((n+k-1) nad k)
vso ::	[t] -> Int -> [[t]]		n.(n-1). … .(n-k+1)

Kombinatorika.hs

- Introduction to QuickCheck
  https://wiki.haskell.org/Introduction_to_QuickCheck2
- Introduction to QuickCheck by example: Number theory and Okasaki's red-black trees
  http://matt.might.net/articles/quick-quickcheck/
- K.Claessen, J.Hughes:QuickCheck:A Lightweight Tool for Random Testingof Haskell Programs
  https://www.eecs.northwestern.edu/~robby/courses/395-495-2009-fall/quick.pdf
- A QuickCheck Tutorial: Generators
  https://www.stackbuilders.com/news/a-quickcheck-tutorial-generators

Definované typy

Ak definujeme vlastnú dátovú štruktúru, ako využiť quickCheck ?

data BVS t = Nil | Node (BVS t) t (BVS t) deriving(Show, Eq)

- dva konštruktory **Nil** a **Node** _ _ _
- deriving popisuje patričnosť do triedy class - (resp. implements interface)
  - Show – automaticky vygenerovaná funkcia show :: BVS t ->String
  - Eq – automaticky vygenerované funkcie ==,/= :: BVS t -> BVS t -> Bool

Ako definovať funkciu, ktorá vracia náhodný strom, napr. BVS Int ?

Existuje nejaká náhodná funkcia, napr. nextInt :: Int ?

Nie je to v rozpore s Referenčnou transparentnosťou ?

Java a Reflexivita
(malá odbočka)

Skúsme si tú istú otázku preformulovať v Jave, ktorú poznáme

- Napíšte funkciu, ktorá vytvorí náhodnú inštanciu ľubovoľnej triedy
  Object gener(String className)

- Nechceme mať náhodný generátor pre každú triedu, lebo pre nami definované triedy by sme ho museli písať sami…

- Reflexivita (Java Reflection Model), od slajdu 11
- https://github.com/Programovanie4/Prednasky/blob/master/13/13_java.pdf
- java primitívne typy (int, char, double, …), String…
- polia (int[], …)
- triedy s default konštruktorom (Stvorec(), …)
- triedy s konštruktorom s parametrami – rekurzívne pre každý parameter konštruktora, potom zavolanie konštruktora s náhodnými parametrami
- generické triedy

QuickCheck – Generátor

(pre základné typy)

- trieda Arbitrary t definuje generátor Gen t pre hodnoty typu t:

```
class Arbitrary a where
    arbitrary :: Gen t
```

a volá sa pomocou funkcie generate :: Gen t -> IO t

Pre preddefinované typy to už niekto zadefinoval:

```
"?: " (generate arbitrary) :: IO Int          23, 45, 12, 49,  12, …
"?: " generate arbitrary :: IO Char           't'"w', '\199', …
"?: " generate arbitrary :: IO (Char, Int)    ('6',0), ('<',-7)
"?: " generate arbitrary :: IO [Int]          [-29,-17,10], [-10,9]
"?: " generate arbitrary :: IO Double         -5.5026813
"?: " generate arbitrary :: IO Bool           True, False, False
"?: " do { fst <- generate arbitrary::IO Int;
           snd <- generate arbitrary::IO Char;
           return (fst, snd) }                (-6, 'r'), (15, 'a'), …
```

QuickCheck – Generátor
(pre funkčné typy)

```
"?: " generate arbitrary :: IO (Int->Int)                          <function>
"?: " do {f<-generate arbitrary :: IO (Integer->Integer); return (f 7)}    9, 11

"?: " do {
        f<-generate arbitrary :: IO (Integer->Integer);
        g<-generate arbitrary :: IO (Integer->Integer);
        x<-generate arbitrary :: IO Integer;
        return (((f.g) x) == ((g.f) x)) }               False, False, False, True

"?: " do {
        f<-generate arbitrary :: IO (Integer->Integer);
        g<-generate arbitrary :: IO (Integer->Integer);
        h<-generate arbitrary :: IO (Integer->Integer);
        x<-generate arbitrary :: IO Integer;
        return ((((f.g).h) x) == (((f.g).h) x)) }        True, True, True, True
```

Generatory.hs

Generátory
(pre definované typy)

```
kocka :: Gen Int
kocka = choose(1,6)                    -- "?: " generate kocka
                                       -- "?: " generate (choose(1,10))

yesno :: Gen Bool
yesno = choose(True, False)            -- "?: " generate yesno
                                  -- "?: " generate (choose(True, False))

data  Minca = Hlava | Panna deriving (Show)
instance Arbitrary Minca where
    arbitrary = oneof [return Hlava, return Panna]
                              "?: " generate (arbitrary::Gen Minca)
                              "?: " (generate arbitrary)::IO Minca

falosnaMinca :: Gen Minca
falosnaMinca = frequency [(1,return Hlava), (2,return Panna)]
                                       -- "?: " generate falosnaMinca
```

Pre nami definované typy XXX musíme definovať inštanciu triedy Arbitrary XXX

Generatory.hs

Generátory - zoznam

```
arbitraryListMax8Len :: Arbitrary a => Gen [a]    -- náhodný zoznam len <= 8
arbitraryListMax8Len =
    do {
      k <- choose (0, 8)::(Gen Int);
      sequence [ arbitrary | _ <- [1..k] ] }
```

```
"?: " generate (arbitraryListMax8Len::Gen [Int])
[-21,12,17,16,4,-20]
```

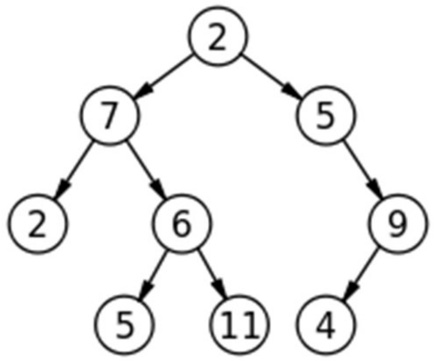
```
arbitraryList :: Arbitrary a => Gen [a]
arbitraryList =
  mysized ( \n -> do {
                       k <- choose (0, n) ;
                       sequence [ arbitrary | _ <- [1..k] ] }
              )
mysized :: (Int -> Gen a) -> Gen a
mysized f = f 50
```

```
"?: " generate (arbitraryList::Gen [Int])
[-9,7,14,24,18,28,-4,0,22,12,-14]
```

```
"?: " generate
          (mysized (\n -> choose(n,n)))
50
```

Generatory.hs

Generátory - strom




```
data Tree t = Leaf t | Node (Tree t) t (Tree t)
      deriving (Show, Ord, Eq)

instance Arbitrary a => Arbitrary (Tree a) where
  arbitrary = frequency
              [
                (1, liftM Leaf arbitrary )
              , (1, liftM3 Node arbitrary arbitrary arbitrary)
              ]
```

"?: " generate (arbitrary :: Gen (Tree Int))
Leaf (-18)

```
strom :: Gen (Tree Int)
strom = frequency [
                (1, liftM Leaf arbitrary )
              , (10, liftM3 Node arbitrary arbitrary arbitrary)
              ]
```

"?: " generate strom
Node (Node (Leaf (-2)) 3 (Leaf (-6))) 23 (Leaf 22)

Generatory.hs

BVS – binárny vyhľadávací

```
data BVS t = Nil | Node (BVS t) t (BVS t)  deriving(Show, Ord, Eq)

-- je binárny vyhľadávací strom
isBVS                 :: (Ord t) => BVS t -> Bool          -- t vieme porovnávať <

-- nájdi v binárnom vyhľadávacom strome
find                  :: (Ord t) => t -> (BVS t) -> Bool -- analógia Comparable<t>
find _ Nil                                  = False
find x (Node left value right)              | x == value = True
                                            | x < value   = find x right
                                            | x > value   = find x left

flat                           :: BVS t -> [t]
flat Nil                       = []
flat (Node left value right)  = flat left ++ [value] ++ flat right
```

Tree.hs

BVS - isBVS

Príšerne neefektívne riešenie, prepíšte lepšie:

```
isBVS    :: (Ord t) => BVS t -> Bool
isBVS Nil = True
isBVS (Node left value right) =
      (all (<value) (flat left))
      &&
      (all (>value) (flat right))
      &&
      isBVS left
      &&
      isBVS right
```

Tree.hs

BVS - testy

```
qch1 = verbose((\x -> \tree -> find x tree)::Int->(BVS Int)->Bool)
qch2 = quickCheck((\x -> \tree -> ((find x tree) == (elem x (flat tree))))
                                                    ::Int->BVS Int->Bool)
{--
"?: " qch2
*** Failed! Falsifiable (after 3 tests):
1 ; Node Nil (-2) (Node Nil 1 Nil)
--}
qch3 = quickCheck((\x -> \tree -> (isBVS tree) ==>
            ((find x tree) == (elem x (flat tree)))):Int->BVS Int->Property)
{--
*** Failed! Falsifiable (after 2 tests):
0 ; Node (Node Nil (-1) (Node Nil 0 Nil)) 1 Nil
--}
```

KDE je chyba v definícii BVS ??

Tree.hs

BVS – tajnička

Don't write tests!

Generate them
from properties

```
find  :: (Ord t) => t -> (BVS t) -> Bool
find _ Nil = False
find x (Node left value right)  | x == value = True
                                | x < value = find x right
                                | x > value = find x left

                                | x < value = find x left
                                | x > value = find x right
```

Haskell – foldr

```
foldr            :: (a -> b -> b) -> b -> [a] -> b

foldr f z []      = z
foldr f z (x:xs) = f x (foldr f z xs)

a : b : c : [] -> f a (f b (f c z))
```

```
     :                    f
    / \                  / \
   a   :    foldr f z   a   f
      / \   -------->      / \
     b   :                b   f
        / \                  / \
       c  []                c   z
```

```
-- g je vnorená lokálna funkcia

foldr :: (a -> b -> b) -> b -> [a] -> b
foldr f z = g
  where   g []     = z
          g (x:xs) = f x (g xs)
```

```
Main> foldr (+) 0 [1..100]
5050
```

```
Main> foldr (\x y->10*y+x) 0 [1,2,3,4]
4321
```

Haskell – foldl

```
foldl           :: (a -> b -> a) -> a -> [b] -> a

foldl f z []      =  z
foldl f z (x:xs) =  foldl f (f z x) xs

a : b : c : [] -> f (f (f z a) b) c
```

```
  :                     f
 / \                   / \
a   :    foldl f z    f   c
   / \   ------>     / \
  b   :             f   b
     / \           / \
    c  []         z   a
```

```
Main> foldl (+) 0 [1..100]
5050

Main> foldl (\x y->10*x+y) 0 [1,2,3,4]
1234
```

Vypočítajte

- foldr max (-999) [1,2,3,4]
  foldl max (-999) [1,2,3,4]

- foldr (_ -> \y ->(y+1)) 0 [3,2,1,2,4]
  foldl (\x -> _ ->(x+1)) 0 [3,2,1,2,4]

- foldr (-) 0 [1..100] =

(1-(2-(3-(4-...-(100-0))))) = 1-2 + 3-4 + 5-6 + ... + (99-100) = -50

- foldl (-) 0 [1..100] =

  (...(((0-1)-2)-3) ... - 100) = -5050

Kvíz

```
    :                      f
   / \                    / \
  a   :    foldl f z     f   c
     / \    ------>     / \
    b   :              f   b
       / \            / \
      c  []          z   a
```

foldr (:) [] xs =   xs

foldr (:) ys xs =   xs++ys

foldr ? ? xs = reverse xs

foldr ((:) . h) [] = ???

```
    :                      f
   / \                    / \
  a   :    foldr f z     a   f
     / \   -------->        / \
    b   :                  b   f
       / \                    / \
      c  []                  c   z
```

Pre tých, čo zvládli kvíz, odmena !

kliknite si podľa vašej politickej orientácie

http://foldl.com/

http://foldr.com/

Funkcia je hodnotou

- [a->a] je zoznam funkcií typu a->a
  napríklad: [(+1),(+2),(*3)] je [\x->x+1,\x->x+2,\x->x*3]

- čo je foldr (.) id [(+1),(+2),(*3)] ?? [a->a]
  akého je typu
  foldr (.) id [(+1),(+2),(*3)] 100 303
  foldl (.) id [(+1),(+2),(*3)] 100 ???

lebo skladanie fcií je asociatívne:

- ((f . g) . h) x = (f . g) (h x) = f (g (h x)) = f ((g . h) x) = (f . (g . h)) x

- funkcie nevieme porovnávať, napr. head [(+1),(+2),(*3)] == id

- funkcie vieme permutovať, length $ permutations [(+1),(+2),(*3),(^2)]

fold.hs

Maximálna permutácia funkcií

- zoznam funkcií aplikujeme na zoznam argumentov

```
apply          :: [a -> b] -> [a] -> [b]
apply fs args = [ f a | f <- fs, a <- args]
```

```
apply [(+1),(+2),(*3)] [100, 200]
[101,201,102,202,300,600]
```

Dokážte/vyvraťte: map f . apply fs = apply (map (f.) fs)

- čo počíta tento výraz

```
maximum $
    apply
      (map (foldr (.) id) (permutations [(+1),(^2),(*3),(+2),(/3)]))
      [100]
```

31827

- ((+1).(+2).(*3).(^2).(/3)) 100

3336.333333333334

- ((/3).(^2).(*3).(+2).(+1)) 100

31827.0

fold.hs

take pomocou foldr/foldl

Výsledkom foldr ?f? ?z? xs je funkcia, do ktorej keď dosadíme n, vráti take n:

… preto aj ?z? musí byť funkcia, do ktorej keď dosadíme n, vráti take n []:

```
take'        :: Int -> [a] -> [a]
take' n xs  =  (foldr pomfcia (\_ -> []) xs) n where
                    pomfcia x h = \n -> if n == 0 then []
                                               else x:(h (n-1))
                    alebo
                    pomfcia x h n =  if n == 0 then [] else x:(h (n-1))
                    alebo
take''' n xs = foldr (\a -> \h -> \n -> case n of
                                          0 -> []
                                          n -> a:(h (n-1)) )
                      (\_ -> [])
                    xs
                    n
```

foldoviny.hs

Zákon fúzie – pre foldr

Fussion Law:

Nech g1, g2 sú binárne funkcie, z1, z2 konštanty
Ak pre funkciu f platí :

f z1 = z2 && f (g1 a b) = g2 a (f b)

potom platí

f . (foldr g1 z1 xs) = foldr g2 z2 xs

Príklad použitia Fussion Law:

$$(n*).\ \underbrace{\text{foldr } (+)\ 0}_{\text{sum}} = \text{foldr } ((+).(n*))\ 0$$

Dôkaz (pomocou Fussion Law): overíme predpoklady
čo je čo ?!:

f = (n*), z1 = z2 = 0, g1 = (+), g2 = (+). (n*)

treba overiť:

- (n*) 0 = 0 ☑
- Ľ.S.=(n*) (a+b) = (n*a + n*b) = (+).(n*) a ((n*) b) = P.S. ☑

Vlastnosti

Acid Rain (fold/build/deforestation theorem)

$$\underbrace{\underbrace{\text{foldr f z}}_{\text{[x]->u}}\ .\ \underbrace{\text{g (:) []}}_{\text{t->[x]}}}_{\text{t->u}} = \text{g f z}$$

Intuícia: Keď máme vytvoriť zoznam pomocou funkcie g zo zoznamových konštruktorov (:) [], na ktorý následne pustíme foldr, ktorý nahradí (:) za f a [] za z, namiesto toho môžeme konštruovať priamo výsledný zoznam pomocou g f z.

Otypujme si to (aspoň):

Ak z :: u, potom f :: x->u->u, foldr f z :: [x]->u.

Ľavá strana: ([x]->u).(t->[x]) výsledkom je typ t->u

Pravá strana: g :: (x -> u -> u) -> u -> (t ->u) foldr f z . g (:) [] = g f z

length . map _ = length

map :: (a -> b) -> [a] -> [b]
map h = foldr ((:) . h) []   -- (:).h a as = (:)(h a as) = h a: as
        = (\x -> \y -> foldr (x . h) y) (:) []
g

length   :: [a] -> Int
length   = foldr (\ _ -> \n -> n+1) 0
f   z

length   .   map h   = ... length

Ľ.S. = (foldr (\ _ -> \n -> n+1) 0) . (foldr ((:) . h) []) =
= podľa Acid Rain theorem (f = (\ _ -> \n -> n+1), z = 0, ale čo je g ? …
g x y = (foldr (x . h) y)
g f z = (foldr (f . h) z) = foldr ((\ _ ->\n -> n+1) . h) 0 =
foldr ((_ ->\n -> n+1)) 0 = length = P.S.
lebo (tento krok pomalšie):
((\ _->\n ->n+1) . h) x y = (\ _->\n->n+1) (h x) y = (\n->n+1) y = y+1

Iný príklad acid rain

```
g     :: (Integer ->u -> u) -> u -> Integer -> u
g h w 0 = w
g h w n = h n (g h w (n-1))
```

```
"?: " ((foldr (*) 1) . (g (:) [])) 100
93326215443944152681699238856266700490715968264381621468592963895217599993229915608941463976156518286253697920827223758251185210916864000000000000000000000000
"?: " g (*) 1 100
93326215443944152681699238856266700490715968264381621468592963895217599993229915608941463976156518286253697920827223758251185210916864000000000000000000000000
```

```
g'     :: Integer -> [t]
g' 0 = []
g' n =  n : (g' (n-1))

g''     :: Integer -> Integer
g'' 0 = 1
g'' n = n * (g'' (n-1))
```

acid_rain.hs

foldr a foldl pre pokročilejších

definujte foldl pomocou foldr, alebo naopak:

```
   :                   f
  / \                 / \
 a   :   foldr f z   a   f
    / \  --------->     / \
   b   :               b   f
      / \                 / \
     c  []               c   z
```

myfoldl f z xs = foldr (\x -> \y -> (f y x)) z (myReverse xs)
myfoldr f z xs = foldl (\x -> \y -> (f y x)) z (myReverse xs)

(flip f)

```
   :                   f
  / \                 / \
 a   :   foldl f z   f   c
    / \  ------>    / \
   b   :           f   b
      / \         / \
     c  []       z   a
```

- odstránime myReverse
  myReverse xs = foldr (\x -> \y -> (y ++ [x])) [] xs
  myfoldl' f z xs = foldr (\x -> \y -> (f y x)) z
  (foldr (\x -> \y -> (y ++ [x])) [] xs)

- odstránime ++
  xs ++ ys = foldr (:) ys xs
  myfoldl'' f z xs = foldr (\x -> \y -> (f y x)) z
  (foldr (\x -> \y -> (foldr (:) [x] y)) [] xs)

hmmm..., teoreticky (možno) zaujímavé, prakticky nepoužiteľné ...

foldr a foldl posledný krát

Zamyslime sa, ako z foldr urobíme foldl:

```
    :                      f
   / \                    / \
  a   :   foldr f z      a   f
     / \  -------->         / \
    b   :                  b   f
       / \                    / \
      c  []                  c   z
```

induktívne predpokladajme, že rekurzívne volanie foldr nám vráti výsledok, t.j. hodnotu y, ktorá zodpovedá foldl:

- y = myfoldl f [b,c] = \z -> f (f z b) c

```
    :                        f
   / \                      / \
  a   :   foldl f z        f   c
     / \  ------>         / \
    b   :                f   b
       / \              / \
      c  []            z   a
```

nech x je ďalší prvok zoznamu, t.j.

- x = a

ako musí vyzerať funkcia ?, ktorou fold-r-ujeme, aby sme dostali myfoldl f [a,b,c] = \z' -> f (f (f z' a) b) c = ? x y

- ? = (\x y z' -> y (f z' x))

dosaďme:

- (\z' -> (\z -> f (f z b) c) (f z' a)) =
- (\z' -> f (f (f z' a) b) c) =
- \z' -> f (f (f z' a) b) c

Pre tých, čo neveria, fakt posledný krát

? = (\x y z' -> y (f z' x))

- myfoldl''' f xs z = foldr (\x y z -> y (f z x)) id xs z
- myfoldl''' f [] = id
- myfoldl''' f [c] = (\x y z -> y (f z x)) c id = \z -> f z c
- myfoldl''' f [b,c] = (\x y z -> y (f z x)) b (\w -> f w c) =
  \z -> (\w -> f w c) (f z b) =
  \z -> f (f z b) c
- myfoldl''' f [a,b,c] = (\x y z -> y (f z x)) a (\w -> f (f w b) c) =
  \z -> (\w -> f (f w b) c) (f z a) =
  \z -> f (f (f z a) b) c
- myfoldl '''' f z xs = foldr (\x y z -> y (f x z)) id xs z

... doma skúste foldr pomocou foldl ...