

Monady sú použiteľný nástroj pre programátora poskytujúci:

- modularitu skladať zložitejšie výpočty z jednoduchších (no side-effects),
- flexibilitu výsledný kód je ľahšie adaptovateľný na zmeny,
- izoluje side-effect operácie (napr. IO) od čisto funkcionálneho zvyšku.

Štruktúra prednášok:

- Monády prvý dotyk
 - Functor
 - Applicative
 - Monády princípy a zákony
- Najbežnejšie monády
 - Maybe/Error monad
 - List monad
 - IO monad
 - State monad
 - Reader/Writer monad
 - Continuation monad
- Transformátory monád
- Monády v praxi



Control.Monad

```
sequence :: (Monad m) => [m a] -> m [a]
mapM
         :: (Monad m) => (a -> m b) -> [a] -> m [b]
         :: (Monad m) => [a] -> (a -> m b) -> m [b]
forM
mapM f as = sequence (map f as)
forM = flip mapM
zipWithM :: (Monad m) => (a -> b -> m c) -> [a] -> [b] -> m [c]
zipWithM f xs ys = sequence (zipWith f xs ys)
replicateM :: (Monad m) => Int -> m a -> m [a]
replicateM n x = sequence (replicate n x)
filterM :: (Monad m) => (a -> m Bool) -> [a] -> m [a]
foldM
       :: (Monad m) => (a -> b -> m a) -> a -> [b] -> m a
guard :: (MonadPlus m) => Bool -> m ()
guard True = return ()
guard False = zero
```

```
sequence_ :: (Monad m) => [m a] -> m ()
mapM_{-} :: (Monad m) => (a -> m b) -> [a] -> m ()
forM
          :: (Monad m) => [a] -> (a -> m b) -> m ()
mapM_ f as = sequence_ (map f as)
forM = flip mapM
zipWithM_{-} :: (Monad m) => (a -> b -> m c) -> [a] -> [b] -> m ()
zipWithM f xs ys = sequence (zipWith f xs ys)
replicateM_ :: (Monad m) => Int -> m a -> m ()
replicateM n x = sequence (replicate n x)
foldM_{-}:: (Monad m) => (a -> b -> m a) -> a -> [b] -> m ()
```

1

mapM, forM

mapM f = sequence . map f

(Control.Monad)

mapM

```
:: (Monad m) = [a] - (a - m b) - m [b] - len zámena args.
forM
           = flip mapM
forM
> mapM (\x->[x,11*x]) [1,2,3]
[[1,2,3],[1,2,33],[1,22,3],[11,22,33],[11,2,33],[11,22,33],[11,22,33]]
> mapM (\x -> [True, False]) [1,2,3] [[True,True,True],[True,False],[True,False,True],[True,False],
 [False,True,True],[False,True,False],[False,False,True],[False,False,False]]
> forM [1,2,3] (\x->[x,11*x]) [[1,2,3],[1,22,3],[1,22,33],[11,2,33],[11,22,3],[11,22,33]]
> mapM print [1,2,3]
                                    > mapM print [1,2,3]
                                                               mapM_ (putStrLn.show)
                                                               [1,2,3]
[(),(),()]
```

:: (Monad m) => (a -> m b) -> [a] -> m [b]

filterM (Control.Monad)

```
filterM :: (Monad m) => (a -> m Bool) -> [a] -> m [a]
```

```
> filterM (\x->[True, False]) [1,2,3]
[[1,2,3],[1,2],[1,3],[1],[2,3],[2],[3],[]]
```

-- potenčná množina, powerset

```
filterM :: (Monad m) => (a -> m Bool) -> [a] -> m [a] filterM \_ [] = return [] filterM p (x:xs) = do flg <- p x ys <- filterM p xs return (if flg then x:ys else ys)
```



foldM

(Control.Monad)

```
foldM
                   :: (Monad m) => (a -> b -> m a) -> a -> [b] -> m a
foldM _ a [] = return a
foldM f a (x:xs) = f a x >> = \y -> foldM f y xs
                                           foldM f a (x:xs) = do y < -f a x
foldM f a_1 [x_1, ..., x_n] =
                                                                  foldM f y xs
   do {
         a_2 < -f a_1 x_1;
         a_3 < -f a_2 x_2;
                                   > foldM (\y -> \x ->
         a_n < -f a_{n-1} x_{n-1};
                                             return (x*y)})
         return f a_n x_n }
```

```
do { print (show x++"..."++ show y);
   1 [1..10]
???
```

```
> foldM (\y -> \x -> do print (show x++"..."++ show y); return (x+y)) 0 [1..10]
???
```

newtype Either a b = Right a | Left b instance (Error e) => Monad (Either e) where return x = Right x Right x >>= f = f x Left err >>= f = Left err fail msg = Left (strMsg msg)

Error monad

data Term = Con Int | Div Term Term deriving(Show, Read, Eq)

```
eval :: Term -> Either String Int
eval(Con a) = return a
eval(Div t u) = do
                  valT <- eval t
                  valU <- eval u
                  if valU == 0 then
                     fail "div by zero"
                                             -- throwError "div by zero"
                  else
                     return (valT `div` valU)
> eval (Div (Con 1972) (Con 23))
Right 85
> eval (Div (Con 1972) (Con 0))
*** Exception: div by zero
```



Reader monad

(Control.Monad.Reader)

```
main :: IO ()
main = do params <- loadParams
          let result = func1 params
          print result
data Params = Params { p1 :: String, p2 :: String, p3 :: String }
                                                                deriving (Show)
loadParams :: IO Params
loadParams = do p1 <- lookupEnv "JAVA_HOME"</pre>
                 p2 <- lookupEnv "OS"
                 p3 <- lookupEnv "HOMEDRIVE"
                 return $ Params (fromMaybe "no java" p1)
                                 (fromMaybe "unknown" p2)
                                 (fromMaybe "no drive" p3)
func1 :: Params -> String
func1 params = "Result: " ++ (show (func2 params))
func2 :: Params -> Int
func2 params = 2 + floor (func3 params)
func3:: Params -> Float
func3 params = (fromIntegral $ length $ p1 params ++ p2 params ++ p3 params)*3.14
```

Reader monad

(Control.Monad.Reader)

Reader monáda sa používa, ak máme **nemenné** prostredie, ktoré zdieľa viac výpočtov

```
newtype Reader r a = Reader (r -> a) -- r je typ čítaného prostredia, a je typ výsledku
```

```
data Reader r a = Reader { runReader :: (r -> a) }
```

class Monad m => MonadReader r m | m -> r where

func :: Reader Params a

runReader func params :: a

-- runReader :: Reader r a -> r -> a -- runReader func :: r -> a

-- získa prostredie

ask :: Params

func :: Reader Params String

func = do params <- **ask**

...



Reader monad

(Control.Monad.Reader)

```
main' :: IO ()
main' = do params <- loadParams
           let result = runReader func1' params
           putStrLn result
func1':: Reader Params String
func1' = do params <- ask
            result <- func2'
            return $ "Result: " ++ (show result)
func2':: Reader Params Int.
func2' = do params <- ask
            result <- func3'
            return $ 2+floor(result)
func3':: Reader Params Float
func3' = do params <- ask
```

loadParams :: IO Params

params :: Params

func1':: Reader Params String runReader:: Reader r a -> r -> a

result :: String

ask :: m r, Reader Params String

params :: Params

params <- **ask**let result = (fromIntegral \$ length \$ p1 params++p2 params++p3 params)*3.14

return result

Reader.hs

Writer monad

(Control.Monad.Writer)

```
newtype Writer w a = Writer { runWriter :: (a, w) }
instance (Monoid w) => Monad (Writer w) where
return x = Writer (x, mempty)
(Writer (x,v)) >>= f =
let (Writer (y, v')) = f x
in Writer (y, v `mappend` v')
```

Writer monáda sa používa,ak máme výpočet produkujúci stream dát,ktoré akumulujeme

```
data Term = Con Int | Div Term Term deriving(Show, Read, Eq)
line
        :: Term -> Int -> String
line t a = "eval (" ++ show t ++ ") \leq " ++ show a ++ "\n"
eval
          :: Term -> Writer String Int
eval x@(Con a) =
      do tell (line x a)
         return a
eval x@(Div t u) =
      do valT <- eval t
          valU <- eval u
          tell (line x (valT `div` valU))
          return (valT `div` valU)
```

```
eval :: Term -> Writer String Int
eval x@(Con a) = writer (a, line x a)

eval x@(Div t u) =
   do valT <- eval t
   valU <- eval u
   let result = (valT `div` valU)
   writer (result, (line x result))
```



(Control.Monad.Writer)

newtype Writer w a = Writer { runWriter :: (a, w) }
instance (Monoid w) => Monad (Writer w) where
 return x = Writer (x, mempty)
 (Writer (x,v)) >>= f =
 let (Writer (y, v')) = f x
 in Writer (y, v `mappend` v')

```
Writer String Int
Writer w a
w = typ akumulátora
a = typ výsledku
-- vráti dvojicu, hodnotu a akumulátor
runWriter :: Writer w a -> (a,w)
-- vráti len akumulátor
execWriter :: :: Writer w a -> w
-- pripíše hodnotu do akumulátora, žiaden výsledok
tell :: w -> m ()
-- pripíše hodnotu do akumulátora, vráti výsledok
writer :: (a,w) -> m a
out :: Int -> Writer [String] Int
out x = writer(x, ["number: " ++ show x])
mult :: Writer [String] Int
mult = do \{a \leftarrow out 3; b \leftarrow out 5; return (a*b) \}
```

```
t::Term
t = (Div (Div (Con 1972) (Con 2)) (Con
23))
> eval t
WriterT (Identity (42,
"eval (Con 1972) <=1972\neval (Con 2) <=2\neval (Div (Con
1972) (Con 2)) <=986\neval (Con 23) <=23\neval (Div (Div
(Con 1972) (Con 2)) (Con 23)) <=42\n")
> runWriter (eval t)
(42,"eval (Con 1972) <= 1972 \setminus (Con 2) <= 2 \setminus (Div
(Con 1972) (Con 2)) <=986\neval (Con 23) <=23\neval (Div
(Div (Con 1972) (Con 2)) (Con 23)) <=42\n")
> execWriter (eval t)
"eval (Con 1972) <=1972\neval (Con 2) <=2\neval (Div (Con
1972) (Con 2)) <=986\neval (Con 23) <=23\neval (Div (Div
(Con 1972) (Con 2)) (Con 23)) <=42\n"
> putStr $ execWriter (eval t)
eval (Con 1972) <=1972
eval (Con 2) <=2
eval (Div (Con 1972) (Con 2)) <=986
eval (Con 23) <=23
eval (Div (Div (Con 1972) (Con 2)) (Con 23)) <=42
> runWriter mult
(15,["number: 3","number: 5"])
> execWriter mult
["number: 3","number: 5"]
> mapM_ putStrLn $ execWriter (mult)
number: 3
number: 5
                                             Writer.hs
```



Writer monad

(Control.Monad.Writer)

40 mod 24 = 16 64 mod 40 = 24 2024 mod 64 = 40

```
gcd':: Int -> Int -> Writer [String] Int
gcd'ab \mid b == 0 = do
                    tell ["result " ++ show a]
                    return a
          otherwise = do
                    tell [show a ++ " mod " ++ show b ++ " = " ++ show (a `mod` b)]
                    gcd' b (a `mod` b)
gcd' :: Int -> Int -> Writer [String] Int
gcd' a b \mid b == 0 = writer (a, ["result" ++ show a])
       | otherwise = do let modulo = (a `mod` b)
                       result <- gcd' b modulo
                       writer (result, [show a ++ " mod " ++ show b ++ " = " ++ show modulo])
> mapM putStrLn (execWriter $ gcd' 2024 64)
result 8
16 \mod 8 = 0
24 \mod 16 = 8
```



Euclid's Game

hra pre dvoch hráčov začínajú s dvomi prirodzenými číslami na tabuli Jediné pravidlo:

odčítajte menšie od väčšieho a napíšte na tabuľu, ale také, aké tam nie je Ten kto napíše posledné číslo vyhráva, prehráva ten, čo už nevie ťahať

Aká je víťazná stratégia?

```
13 6
7
1
5
2
11
9
4
3
8
10
12 modrý vyhral
```

```
18 12
6 modrý vyhral
```

```
16 6
10
4
2
14
12
8 červený vyhral
```

```
21 9
12
3
6
18
15 modrý vyhral
```

```
23 8
15
7
1
14
22
21
20
19
18
17
16
13
12
11
10
9
6
5
4
3
2 modrý vyhral
```

State monad

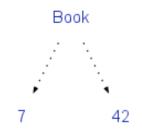
(Control.Monad.State)

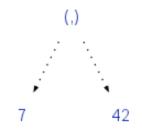
```
newtype State s a = State { runState :: (s -> (a,s)) }
instance Monad (State s) where
                  = State \s -> (a,s)
 return a
 (State x) >> = f = State \s ->
                          let (v,s') = x s in runState (f v) s'
class (Monad m) => MonadState s m | m -> s where
                                  -- get vráti stav z monády
 get :: m s
 put :: s -> m ()
                                  -- put prepíše stav v monáde
modify :: (MonadState s m) => (s -> s) -> m ()
modify f = do s < -get
                 put (f s)
```

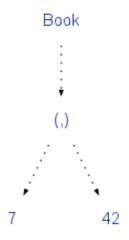
Čo je newtype vs. data vs. type

newtype State s a = State { runState :: (s -> (a,s)) } State s a má rovnakú reprezentáciu ako (s -> (a,s)), ale nie je to **type** State s a = s -> (a,s)

data State s a = State { runState :: (s -> (a,s)) }
State s a je reprezentovaná krabicou State s pointrom na (s -> (a,s))
Príklad:







```
newtype State s a = State { runState :: (s -> (a,s)) }
```



State s a (basics-1)

newtype State $s = State \{ runState :: (s -> (a,s)) \}$

```
runState :: State s a -> (s -> (a, s)) -- vráti funkciu state monády
```

evalState :: State s a -> s -> a -- vráti výsledok state monády pre stav s

 execState :: State s a -> s -> s -- vráti výsledný stav state monády pre vstupný stav s

```
:t runState ((return "hello") :: State Int String)
runState ((return "hello") :: State Int String) :: Int -> (String, Int)
runState ((return "hello") :: State Int String) 77 = ("hello",77)
evalState ((return "hello") :: State Int String) 77 = "hello"
execState ((return "hello") :: State Int String) 77 = 77
```

```
newtype State s a = State { runState :: (s -> (a,s)) }
```

State s a

(basics-2)

```
return :: a -> State s a
                              -- monáda s výsledkom x::a, stavom s
return x s = (x,s)
                                                 -- return x = \slash s \rightarrow (x,s)
get :: State s s
                             -- stav state monády je jej výsledkom
get s = (s,s)
                                                  -- qet = \s -> (s,s)
runState get 1 = (1,1)
                            -- prepíše stav monády x, výsledok je nezaujímavý
put :: s -> State s ()
put x s = ((),x)
                                                 -- put x = \s -> ((),x)
runState (put 5) 1 = ((),5)
runState (do { put 5; return 'X' }) 1 = ('X',5)
modify :: (s \rightarrow s) \rightarrow State s()
modify f = do \{ x < - get; put (f x) \}
runState (modify (+3)) 1 = ((),4)
runState (do { modify (+3); return "hello"}) 1 = ("hello",4)
```

```
newtype State s = State \{ runState :: (s -> (a,s)) \}
```

State s a (basics-3)

let increment = do { x < - get; put (x+1); return x } in runState increment 77 = (77,78)

```
gets :: (s -> b) -> State s b -- aplikuje funkciu na stav monády
gets f = do \{ x < - get; return (f x) \}
runState (gets (+1)) 77 = (78,77)
```

evalState (gets (+1)) 77 = 78

-- vráti výsledok state monády pre vstupný stav s, po aplikovaný funkcie

execState (gets (+1)) 77 = 77 -- vráti výsledný stav state monády pre vstupný stav s, a ten sa nezmenil

runState (modify (+1)) 77 = ((),78)

Eval s vlastnou State Monad

(bolo minule)

```
data Term = Con Int | Div Term Term deriving(Show, Read, Eq)
type State = Int
data SM a = SM (State-> (a, State))
instance Functor SM where ...
instance Applicative SM where ...
instance Monad SM where ...
incState :: SM ()
incState = SM (x \rightarrow ((),x+1))
evalSM' :: Term -> SM Int
                                                             > goSM' t
evalSM'(Con a) = return a
evalSM'(Div t u) = do valT < -evalSM' t
                      valU<-evalSM' u
                      incState
                      return(valT `div` valU)
              :: Term -> State
goSM
               = let SM p = evalSM t, (result, state) = p 0 in state
goSM t
```

Eval

```
data Term = Con Int | Div Term Term deriving(Show, Read, Eq)
type Stav = Int

evalSM :: Term -> State Stav Int
evalSM (Con a) = return a
evalSM (Div t u) = do valT<-evalSM t
valU<-evalSM u
modify (+1)

runState
(42,2)
> execState
2
> evalState
42
```

return(valT `div` valU)

```
> runState (evalSM t) 0
(42,2)
> execState (evalSM t) 0
2
> evalState (evalSM t) 0
42
```

```
pop :: State Stack Int
pop = state(\(x:xs) -> (x,xs))

push :: Int -> State Stack ()
push a = state(\xs -> ((),a:xs))
```

State Stack

```
type Stack = [Int]
type Stack = [Int]
                       stav
                              výsledok
                                            pushAll' :: Int -> State Stack String
pushAll :: Int -> State Stack String
                                            pushAll' 0 = return ""
pushAll 0 = return ""
                                            pushAll' n = do
pushAll n = do {
                                                        stack <- get -- push n
           push n;
                                                        put (n:stack)
           str <- pushAll (n-1);
                                                        str <- pushAll (n-1)
           nn <- pop; -
                                                        (nn:stack') <- get -- nn <- pop
           return (show nn ++ str)}
                                                        put stack'
evalState vráti výslednú hodnotu
                                                        return (show nn ++ str)
> evalState (pushAll 10) []
"10987654321"
                                            > evalState (pushAll' 10) []
execState vráti výsledný stav
                                            "10987654321"
> execState (pushAll 10) []
                                            > execState (pushAll' 10) []
```

Parkovací automat

Parkovaci automat sa zapína/vypína na signal '.', na začiatku je vypnutý. Keď je zapnutý, tak počíta počet automobilov, ktoré vošli '(' a počet automobilov, ktoré odišli ,)'. Stav automatu je ich rozdiel, výsledok výpočtu je počet automobilov, ktoré na parkovisku zostali, počas období, keď bol zapnutý

main = mapM_ print [evalState (loop input) (False, 0) |

Dobré zátvorky ???

```
type Stav = (Bool, Int, Int) -- .., počet (, počet [
                                                                         "() -> True"
                                                                         "(()) -> True"
loop :: String -> State Stav Bool
                                                                         "(()()) -> True"
loop [] = do (ok, parents, brackets) <- get
                                                                         '(()()(())) -> True"
                return $ ok && parents == 0 && brackets == 0
                                                                         ")( -> False"
                                                                         "([)] -> True"
loop (x:xs) = do (ok,parents, brackets) <- get
                                                                         "[[]] -> True"
                                                                         "][ -> False"
                        case x of
                                                                         "()[] -> True"
                           '(' -> put (ok,parents+1, brackets)
                                -> put (ok,parents, brackets+1)
                           ')' | parents > 0 -> put (ok,parents-1, brackets)
                           ')' | parents <= 0 -> put (False,0,0)
                                -> put $ if (brackets>0) then (ok,parents, brackets-1)
                                                           else (False,0,0)
              loop xs
main = mapM_ print [ evalState (loop input) (True, 0, 0) | input <- [
```

"()", "(())", "(()())", "(()()(()))", ")(", "([)]", "[[]]", "][", "()[]"]]

> main

Preorder so stavom

(Control.Monad.State)

```
data Tree a =
                       Nil
                       Node a (Tree a) (Tree a) deriving (Show, Eq)
                                 stav
   preorder :: Tree a -> State [a] ()
                                                    -- stav a výstupná hodnota
   preorder Nil
                                           = return ()
   preorder (Node value left right)
                                          do {
                                                              str :: [a]
                                                    str<-get; -- get state=preorderlist</pre>
e :: Tree String
                                                    put (value:str); -- modify (value:)
e = Node "c" (Node "a" Nil Nil) (Node "b" Nil Nil)
                                                    preorder left;
                                                    preorder right;
> execState (preorder e) [] -- stav
                                                    return () }
["b","a","c"]
> evalState (preorder e) [] -- výsledok
()
```

stav výsledok

Prečíslovanie binárneho stromu

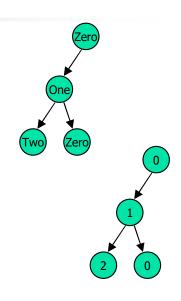
```
> e'
Node "d" (Node "c" (Node "a" Nil Nil) (Node "b" Nil Nil)) (Node "c" (Node
"a" Nil Nil) (Node "b" Nil Nil))
> evalState (reindex e') 0
Node 0 (Node 1 (Node 2 Nil Nil) (Node 3 Nil Nil)) (Node 4 (Node 5 Nil Nil)
(Node 6 Nil Nil))
> execState (reindex e') 0
7
```

stav výsledok

Prečíslovanie stromu 2

```
type Table a = [a]
```

```
numberTree :: Eq a => Tree a -> State (Table a) (Tree Int)
numberTree Nil = return Nil
numberTree (Node x t1 t2) = do num <- numberNode x
nt1 <- numberTree t1
nt2 <- numberTree t2
return (Node num nt1 nt2)
```



where

```
addNode:: (Eq a) => a -> Table a -> (Table a, Int)
addNode x table = case (findIndexInList (== x) table) of
Nothing -> (table ++ [x], length table)
Just i -> (table, i)
```

tree.hs

Prečíslovanie stromu 2

```
numTree :: (Eq a) => Tree a -> Tree Int
numTree t = evalState (numberTree t) []
```

Piškvorky

```
size = 11
data PiskyState = PiskyState { playground::[[Tile]], onTurn::Bool, generator::StdGen }
data Tile = Empty | X | O deriving (Eq., Show)
-- player True/False
nextPlayer :: Bool->Bool
nextPlayer x = not x
-- player's sign
sign :: Bool -> Tile
sign True = X
sign False = 0
-- no Empty on board
finish :: State PiskyState Bool
finish = do pstate <- get
            return $ any (==Empty) ( concat (playground pstate))
```

Piškvorky

```
move :: State PiskyState (Int,Int)
move = do pstate <- get
             let free = [(i,j) | i < -[0..size-1], j < -[0..size-1], (playground pstate)!!i!!j == Empty]
             let gen = generator pstate
             let (r, gen') = randomR (0, length free - 1) gen
             put $ pstate { generator = gen' }
             return $ free !! r
update :: (Int,Int) -> State PiskyState ()
update (row,col) = do pstate <- get
                         let player = onTurn pstate
                         let s = sign player
                         let newPG = [[ if i == row && j == col then s else (playground pstate)!!i!!j
                                                  | j<-[0..size-1]] | i<-[0..size-1]]
                         put $ pstate { onTurn = nextPlayer player, playground=newPG }
oneTurn :: State PiskyState Bool
```

Piskvorky.hs

oneTurn = do { (row, col) <- move; update (row, col) ; finish }

Piškvorky

```
pinit :: StdGen -> PiskyState
pinit gen = PiskyState (take size (repeat (take size (repeat Empty))))
               True
               gen
main:: IO()
main = do g <- getStdGen
       let istate = pinit g
       putStr (show $ execState (sequence $ take 100 $ repeat oneTurn) istate)
> main
.OXXOX.XXXX
0000.X0.00X
XX000000X00
0XX00000.0X
X.0X.X0.X00
000XXX0X0XX
0X000.X..XX
X.XOOXOX.XO
XOXX.OXX.OO
.xxo.x.xxoo
.xxxox..xox
```