

# Why Functional Programming Matters



John Hughes & Mary Sheeran

**QuviQ**  
...

**CHALMERS**

# Functional Programming à la 1940s

- Minimalist: who needs booleans?
- A boolean just *makes a choice!*

**true**    x   y = x

**false**   x   y = y

- We can *define* if-then-else!

**ifte**   bool   t   e =  
            bool   t   e

# Who needs integers?

- A (positive) integer just *counts loop iterations!*

**two**     $f\ x = f(f\ x)$

**one**     $f\ x = f\ x$

**zero**     $f\ x = x$

- To recover a “normal” integer...

```
*Church> two (+1) 0  
2
```

# Look, Ma, we can add!

- Addition by *sequencing* loops

**add** m n f x = m f (n f x)

m  
n

- Multiplication by *nesting* loops!

**mul** m n f x = m (n f) x

n m

\*Church> add one (mul two two) (+1) 0

# Factorial à la 1940

```
fact n =  
  ifte (iszzero n)  
    one  
    (mul n (fact (decr n)))
```

```
*Church> fact (add one (mul two two)) (+1) 0  
120
```

# A couple more auxiliaries

- Testing for zero

```
iszero n =
  n (\_ -> false) true
```

- Decrementing...

```
decr n =
  n (\m f x-> f (m incr zero))
    zero
    (\x->x)
    zero
```

Booleans, integers, (and other  
data structures) *can be entirely  
replaced by functions!*

*"Church encodings"*

Early versions of the Glasgow  
Haskell compiler actually  
implemented data-structures  
this way!



*Alonzo Church*

# Before you try this at home...

# Church.hs:27:35:

Occurs check: cannot construct the infinite type:  
 $t \sim t \rightarrow t \rightarrow t$

## Expected type:

## Actual type:

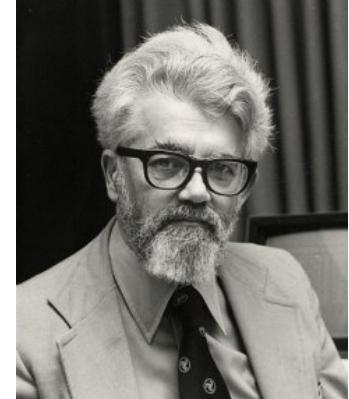
But wait, there's more...

Relevant bindings include

The type-checker needs a *little bit* of help

```
fact ::  
  (forall a. (a->a)->a->a) ->  
  (a->a) -> a -> a
```

# Factorial à la 1960



```
(LABEL FACT (LAMBDA (N)
  (COND ((ZEROP N) 1)
        (T (TIMES N (FACT (SUB1 N)))))))
```

## Higher-order functions!

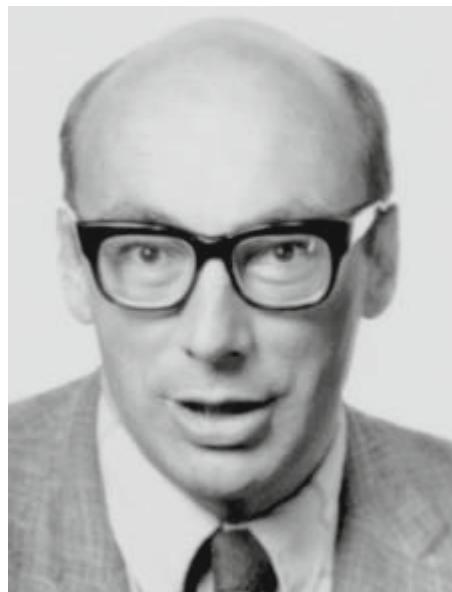
```
(MAPLIST FACT (QUOTE (1 2 3 4 5)))
(1 2 6 24 120)
```

# The Next 700 Programming Languages

P. J. Landin

*Univac Division of Sperry Rand Corp., New York, New York*

“... today ... 1,700 special programming languages used to ‘communicate’ in over 700 application areas.”—*Computer Software Issues*, an American Mathematical Association Prospectus, July 1965.



## Factorial in ISWIM

**fac(5)**

```
where rec fac(n) =  
      (n=1) → 1;  
                  n*fac(n-1)
```

# Laws

(MAPLIST F (REVERSE L))  $\equiv$  (REVERSE (MAPLIST F L))



What's the point of two  
different ways to do the  
same thing?

Wouldn't *two* facilities be  
better than one?

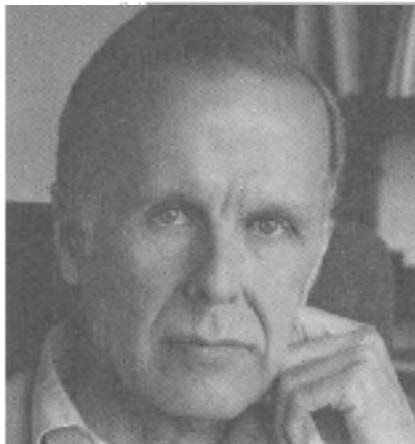
**Expressive power should  
be by design, rather  
than by accident!**

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# Can Programming Be Liberated from the von Neumann Style? A Functional Style and Its Algebra of Programs

John Backus  
IBM Research Laboratory, San Jose

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Turing award 1977

[Paper 1978](#)

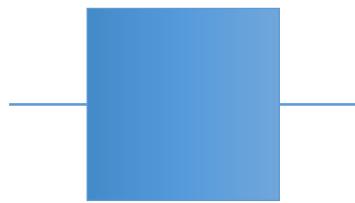
**Conventional programming  
languages are growing ever  
more enormous,  
but not stronger.**

**Inherent defects at the most basic level cause them to be both fat and weak:**

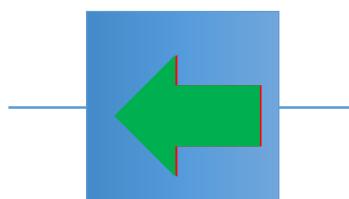
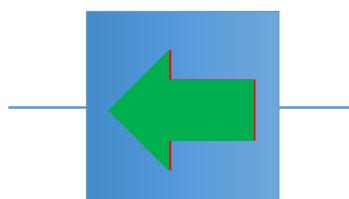
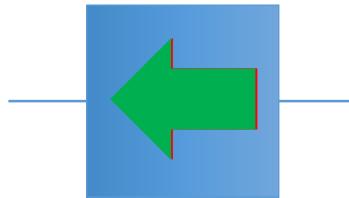
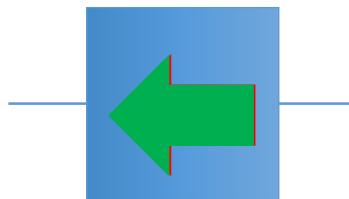
# Word-at-a-time



their inability to effectively use  
powerful combining forms  
for building new programs from  
existing ones



apply to all

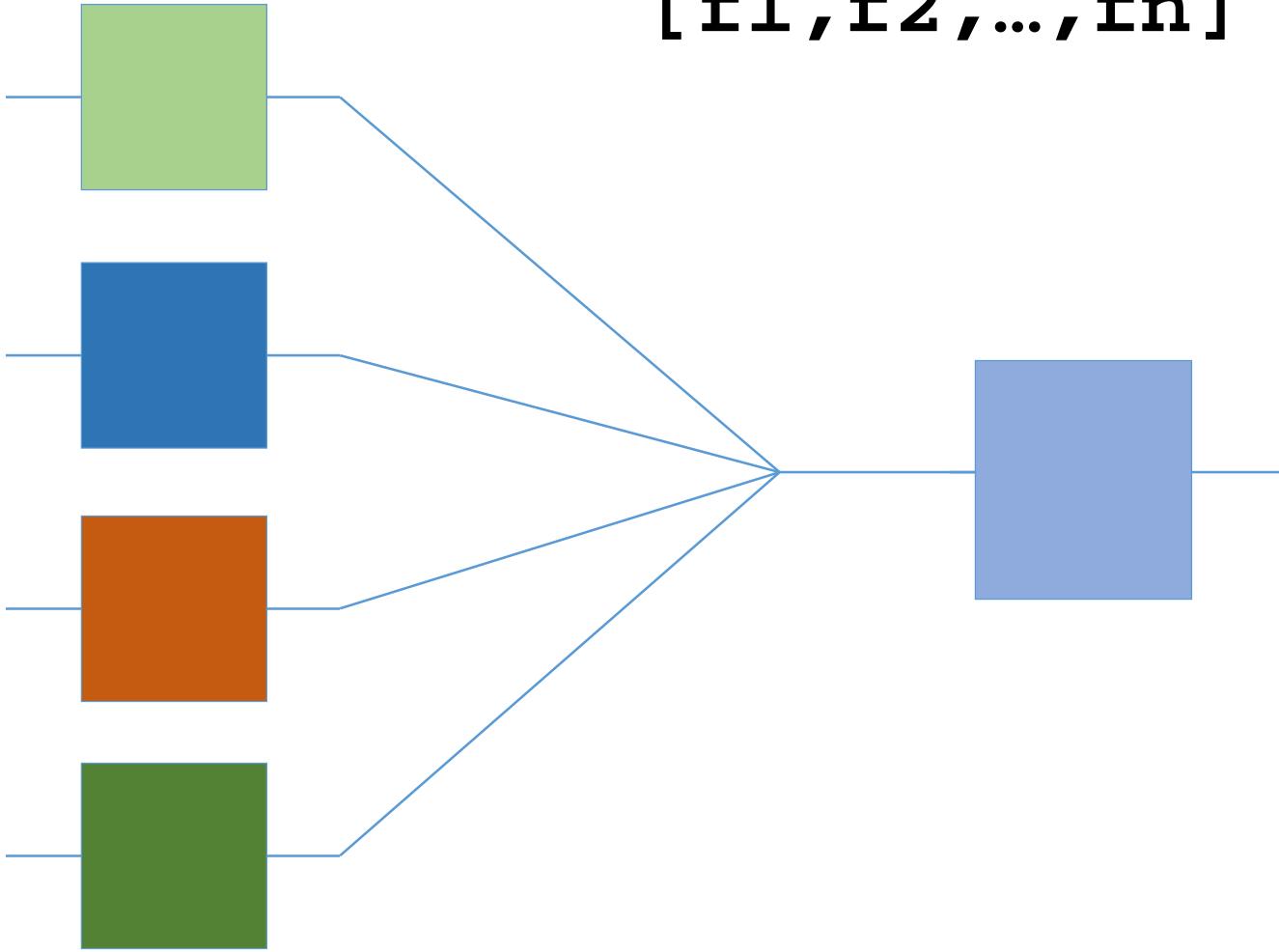


$\alpha f$

# construction



**their lack of useful  
mathematical properties for  
reasoning about programs**

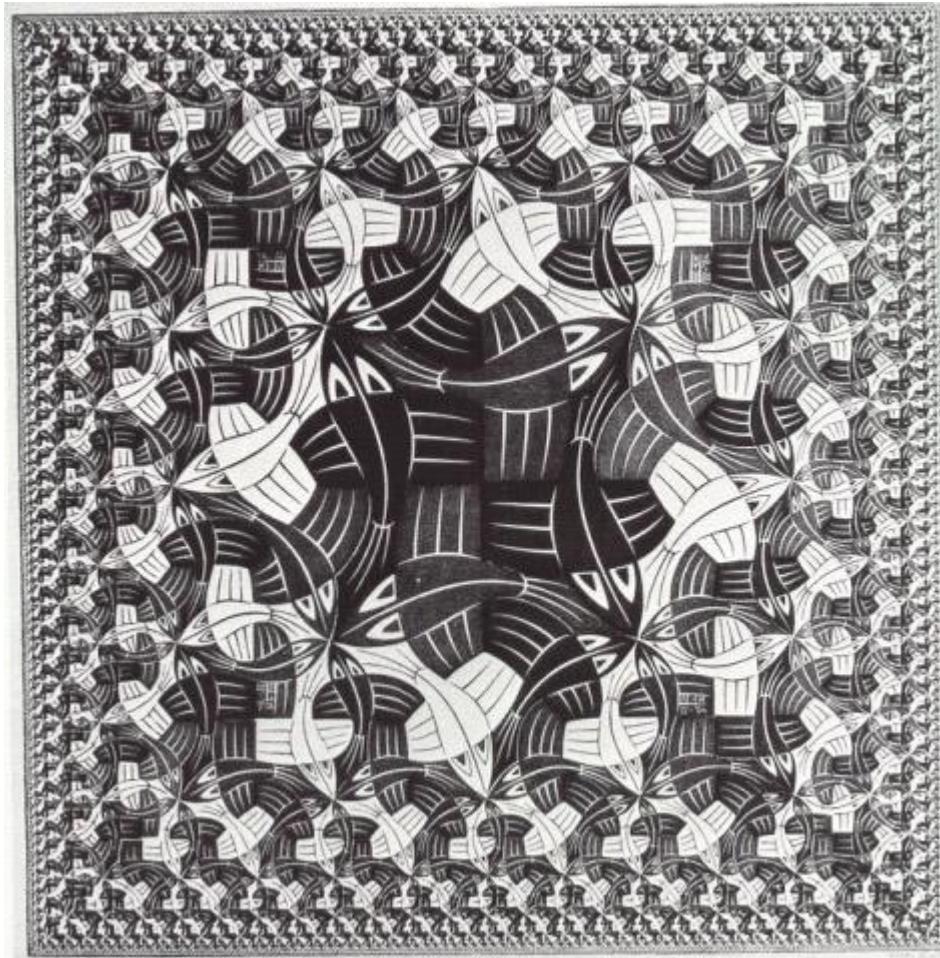
$$[f_1, f_2, \dots, f_n] \bullet g$$


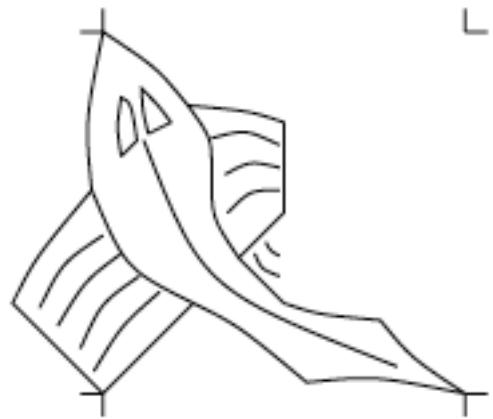
 $[ f_1, f_2, \dots, f_n ] \bullet g$  $[ f_1 \bullet g, f_2 \bullet g, \dots, f_n \bullet g ]$

```
c := 0;  
for i := 1 step 1 until n do  
  c := c + a[i] × b[i]
```

**Def IP = ( / + ) • (α ×) • Trans**

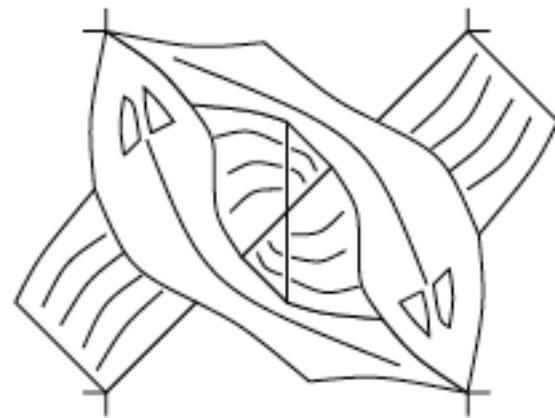
*Peter Henderson, Functional Geometry, 1982*



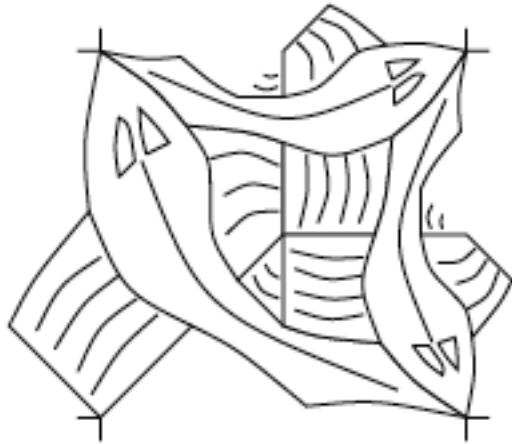


L

fish

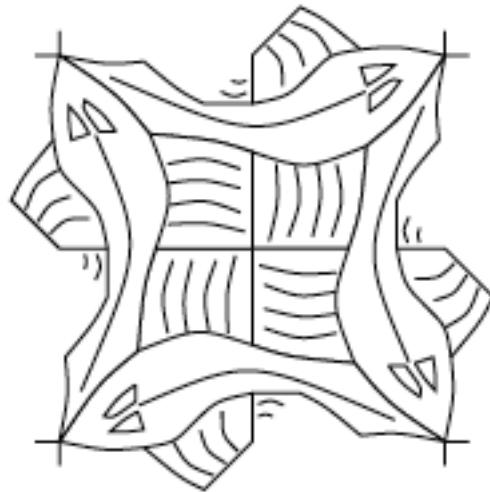


**over (fish, rot (rot (fish)))**



```
t = over (fish, over (fish2, fish3))
```

```
fish2 = flip (rot45 fish)
fish3 = rot (rot (rot (fish2)))
```



```
u = over (over (fish2, rot (fish2)),  
          over (rot (rot (fish2)),  
                 rot (rot (rot (fish2))))))
```

P	Q
R	S

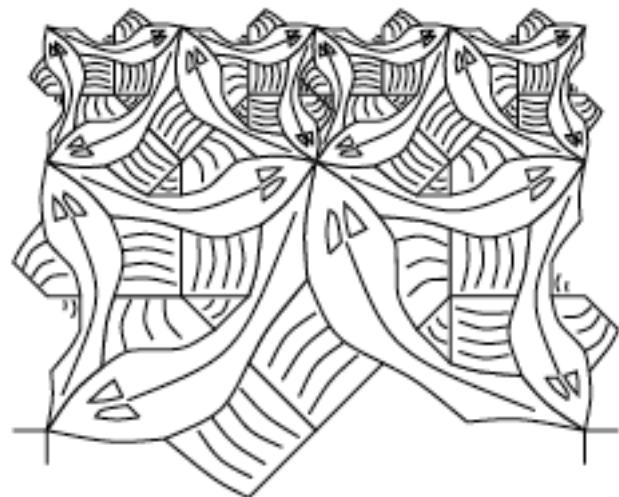
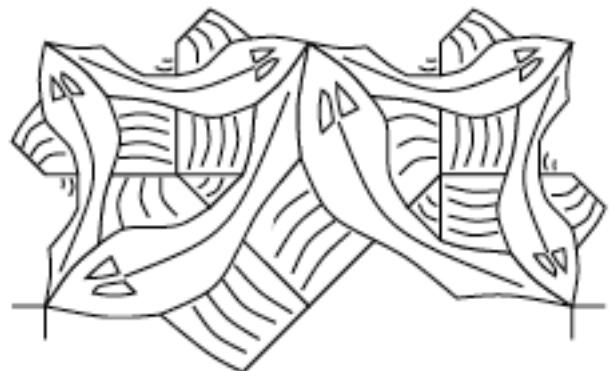
quartet

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「

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```
quartet(nil, nil,  
        rot(t), t)
```

**side1**

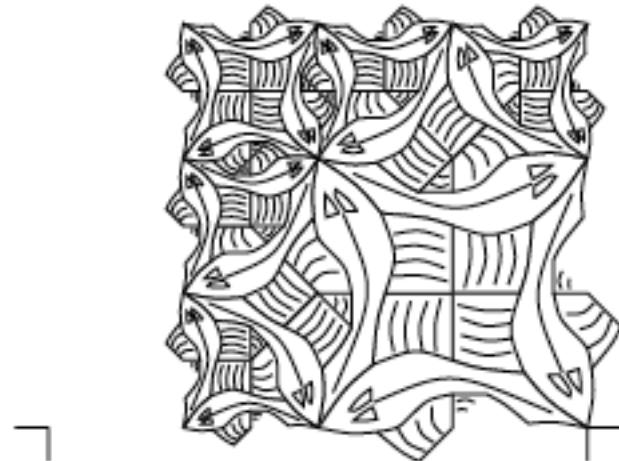
```
quartet(side1,side1,  
        rot(t), t )
```

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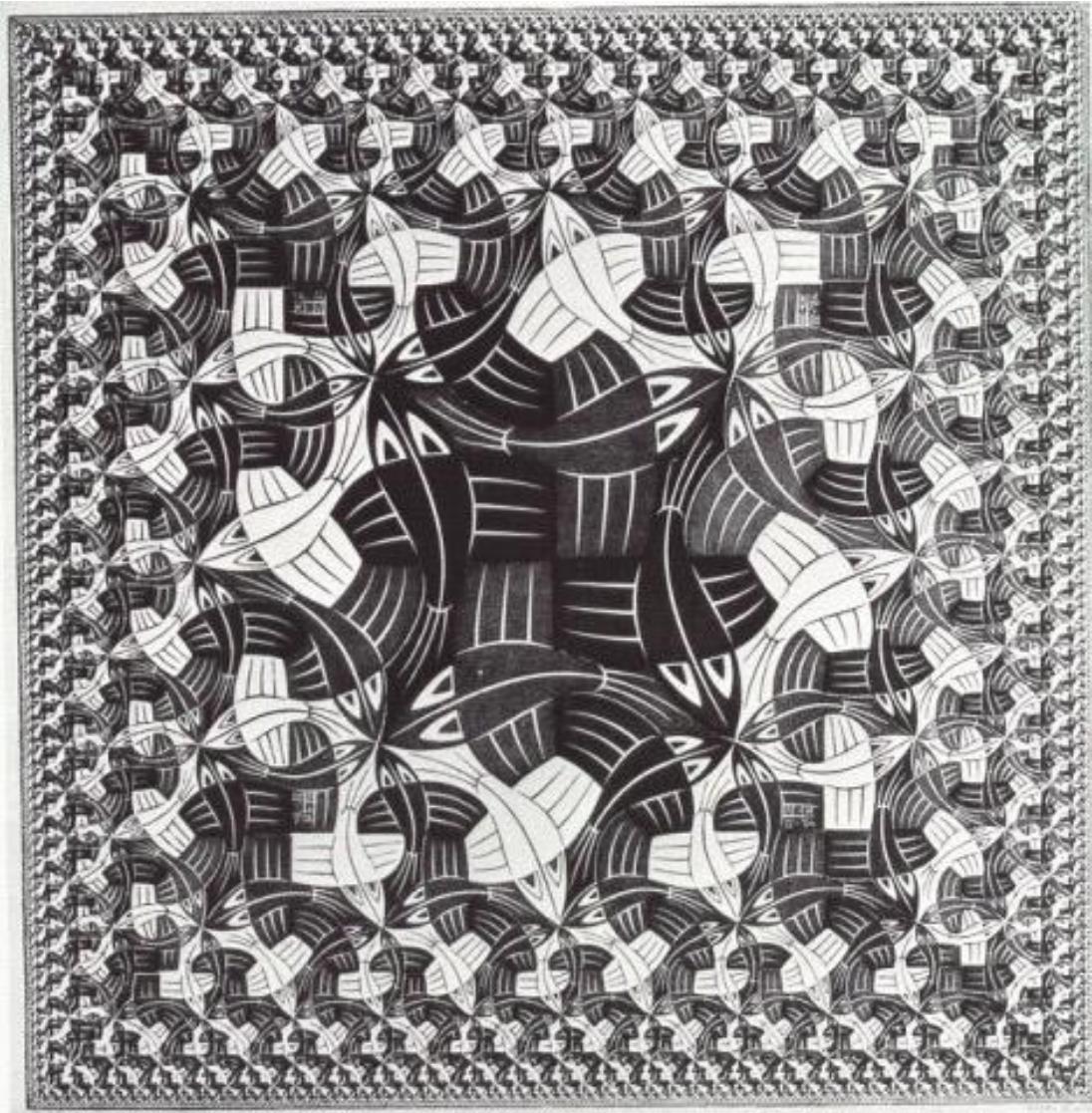
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quartet (nil,nil,nil,u)  
**corner1**

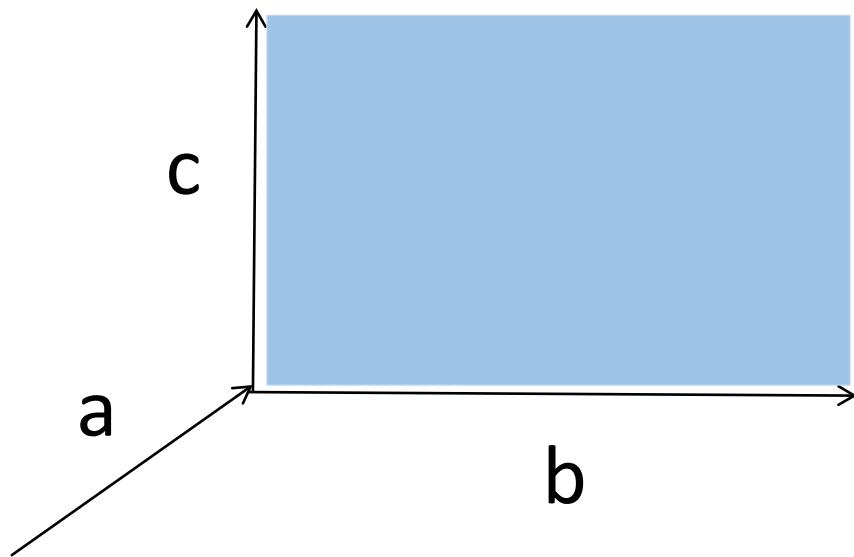
quartet(**corner1**,  
          **side1**,  
          **rot(side1)**,  
          )

```
squarelimit = nonet(
    corner,           side,
    rot(side),       u,
    rot(corner),     rot(rot(side)), rot(rot(corner)))
```

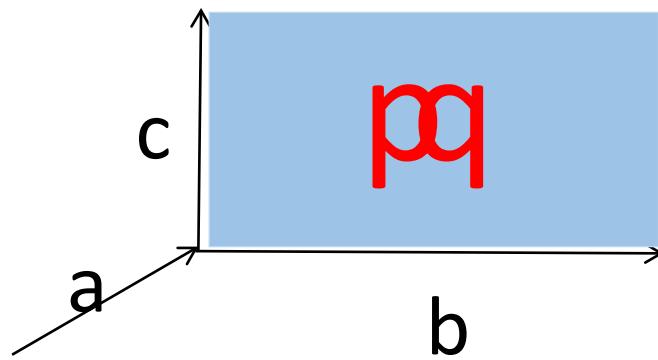


picture = function

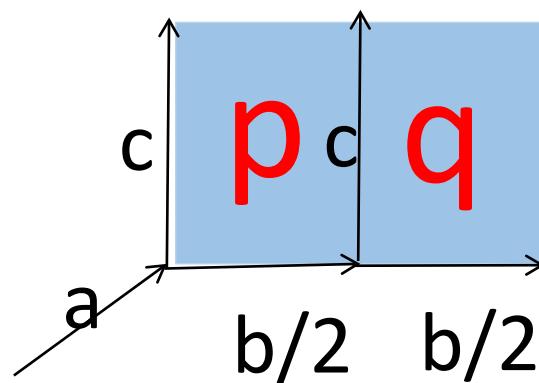
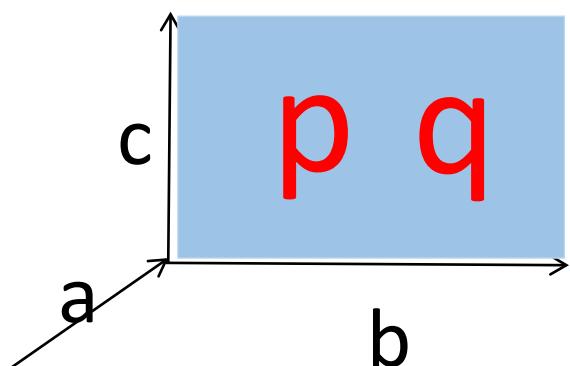
picture = function



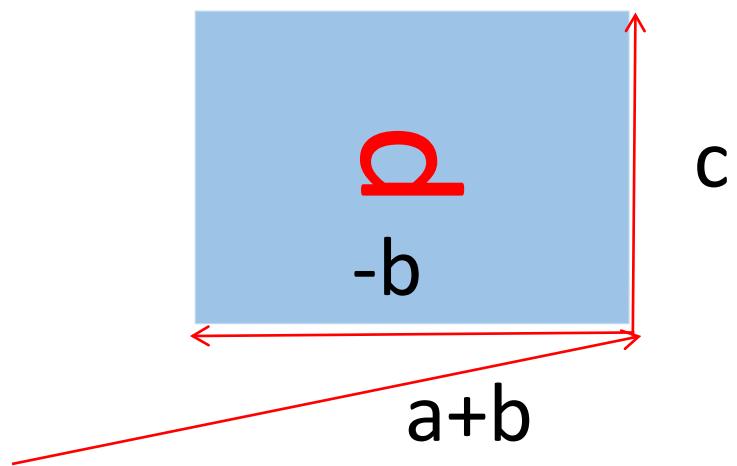
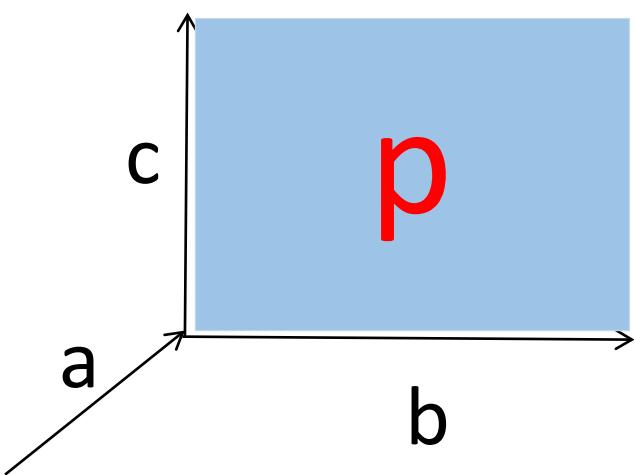
**over** (p,q) (a,b,c) =  
p(a,b,c) **U** q(a,b,c)



**beside** ( $p, q$ ) ( $a, b, c$ ) =  
 $p(a, b/2, c) \cup q(a+b/2, b/2, c)$



$$\text{rot}(p) (a, b, c) = p(a+b, c, -b)$$



# Laws

**rot(above(p,q))**

=

**beside(rot(p),rot(q))**



It seems there is a positive correlation between the simplicity of the rules and the quality of the algebra as a description tool.

Whole values

Combining forms

Algebra as litmus test

Functions as representations

# Haskell vs. Ada vs. C++ vs. Awk vs. ...

## An Experiment in Software Prototyping Productivity\*

Paul Hudak

Mark P. Jones

Yale University

Department of Computer Science

New Haven, CT 06518

{hudak-paul, jones-mark}@cs.yale.edu



July 4, 1994



Time 40.0:

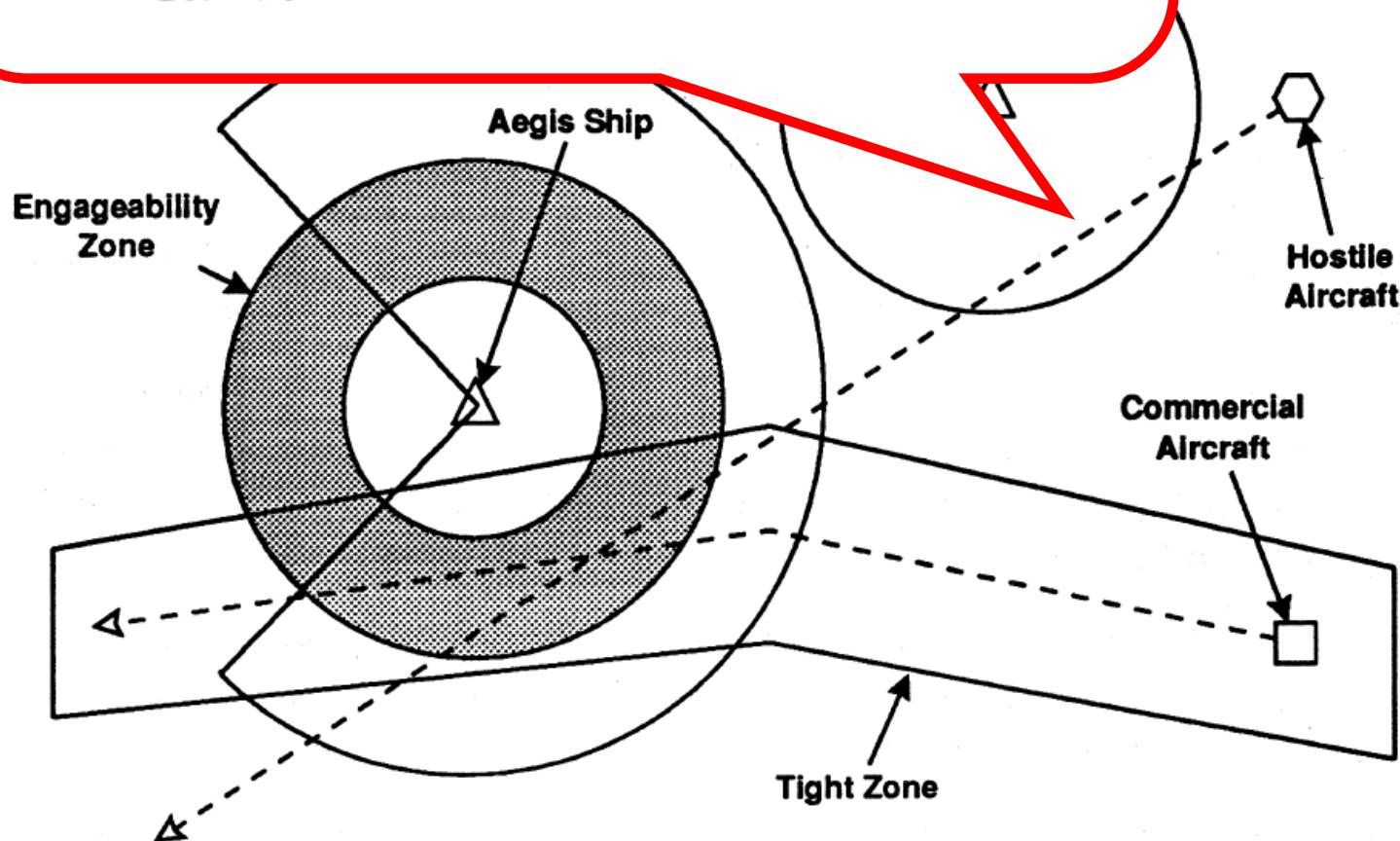
commercial aircraft: (100.0, 43.0)

-- In engageability zone

-- In tight zone

hostile craft: (210.0, 136.0)

-- In carrier slave doctrine



# Functions as Data

```
> type Region = Point -> Bool
```

Including 29 lines of  
inferable type  
signatures/synonyms

A student, given 8 days  
to learn Haskell, w/o  
knowledge of Yale group

Language	Lines of code	Lines of documentation	Development time (hours)
(1) Haskell	85	465	10
(2) Ada	767	7	23
(3) Ada9X	800	—	28
(4) C++	1105	130	—
(5) Awk/Nawk	250	150	—
(6) Rapide	157	0	54
(7) Griffin	251	0	34
(8) Proteus	293	79	26
(9) Relational Lisp	274	12	3
(10) Haskell	156	112	8

Figure 3: Summary of Prototype Software Development Metrics

# Reaction...

“too cute for its own good”

...higher-order functions just a trick, probably not useful in other contexts

# Lazy Evaluation (1976)



Henderson and Morris  
*A lazy evaluator*



Friedman and Wise  
*CONS should not evaluate  
its arguments*

# “The Whole Value” can be $\infty$ !

- The *infinite list* of natural numbers

```
[0, 1, 2, 3 ...]
```

*Consumer decides  
how many to  
compute*

- All the iterations of a function

```
iterate f x = [x, f x, f (f x), ...]
```

- A consumer for numerical methods

```
limit eps xs =
```

*<first element of xs within eps of its predecessor>*

# Some numerical algorithms

- Newton-Raphson square root

```
sqrt a = limit eps (iterate next 1.0)  
where next x = (x + a/x) / 2
```

- Derivatives

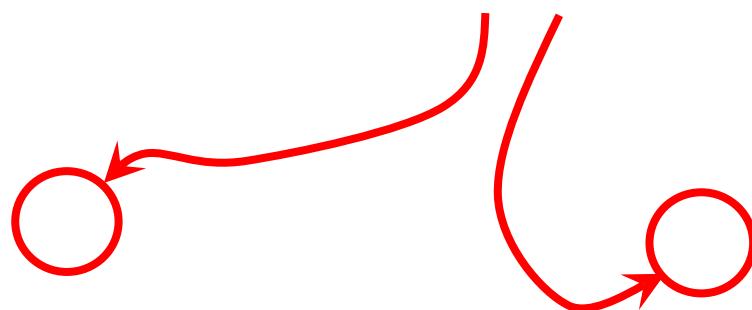
```
deriv f x =  
limit eps (map slope (iterate ((/ 2) 1.0))  
where slope h = (f (x+h) - f x) / h
```

*Same convergence check*

*Different approximation sequences*

# Speeding up convergence

The smaller  $h$  is, the better  
the approximation



Differentiation

Integration

The right  
answer

$$A + B * h^n$$

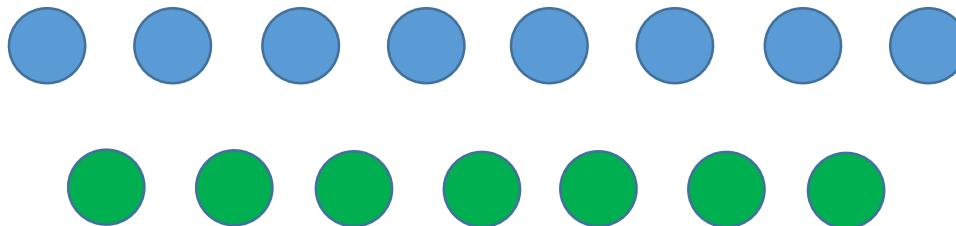
An error term

# Eliminating the error term

- Given:

$$\begin{aligned} & A + B * h^n \\ & A + B * (h/2)^n \end{aligned} \quad \left. \vphantom{\begin{array}{c} A + B * h^n \\ A + B * (h/2)^n \end{array}} \right\} \text{Two successive approximations}$$

- Solve for A and B!



**improve n xs converges faster than xs**

# Really fast derivative

```
deriv f x =  
  limit eps  
    ( improve 2  
      ( improve 1  
        ( map slope ( iterate ( / 2 ) 1.0 ) ) ) )  
  
The convergence check  
The improvements  
The approximations
```

Everything is programmed *separately* and easy to understand—thanks to “whole value programming”

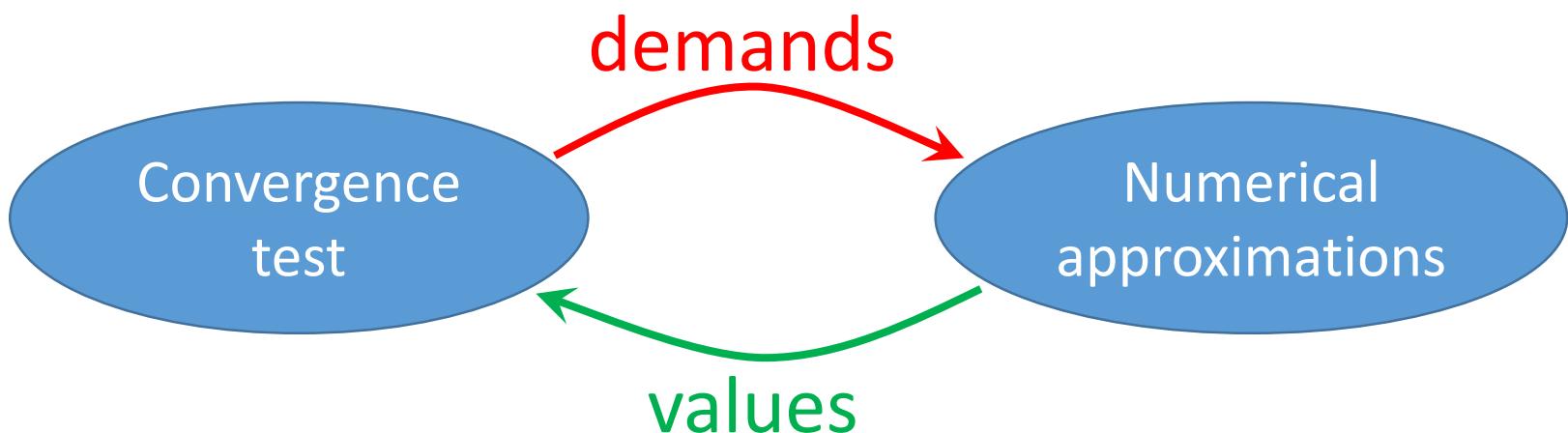
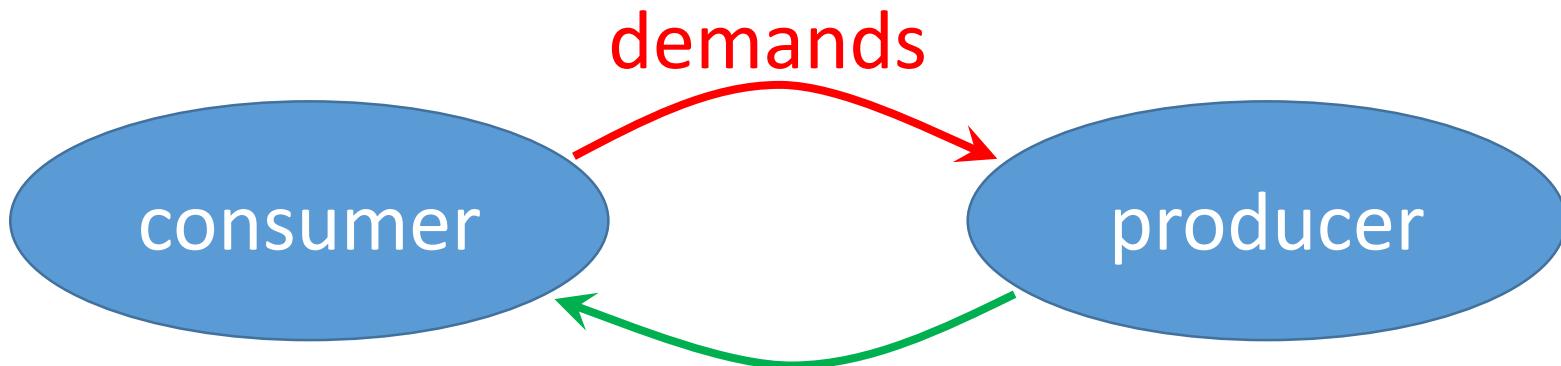
# Why Functional Programming Matters

John Hughes  
The University, Glasgow

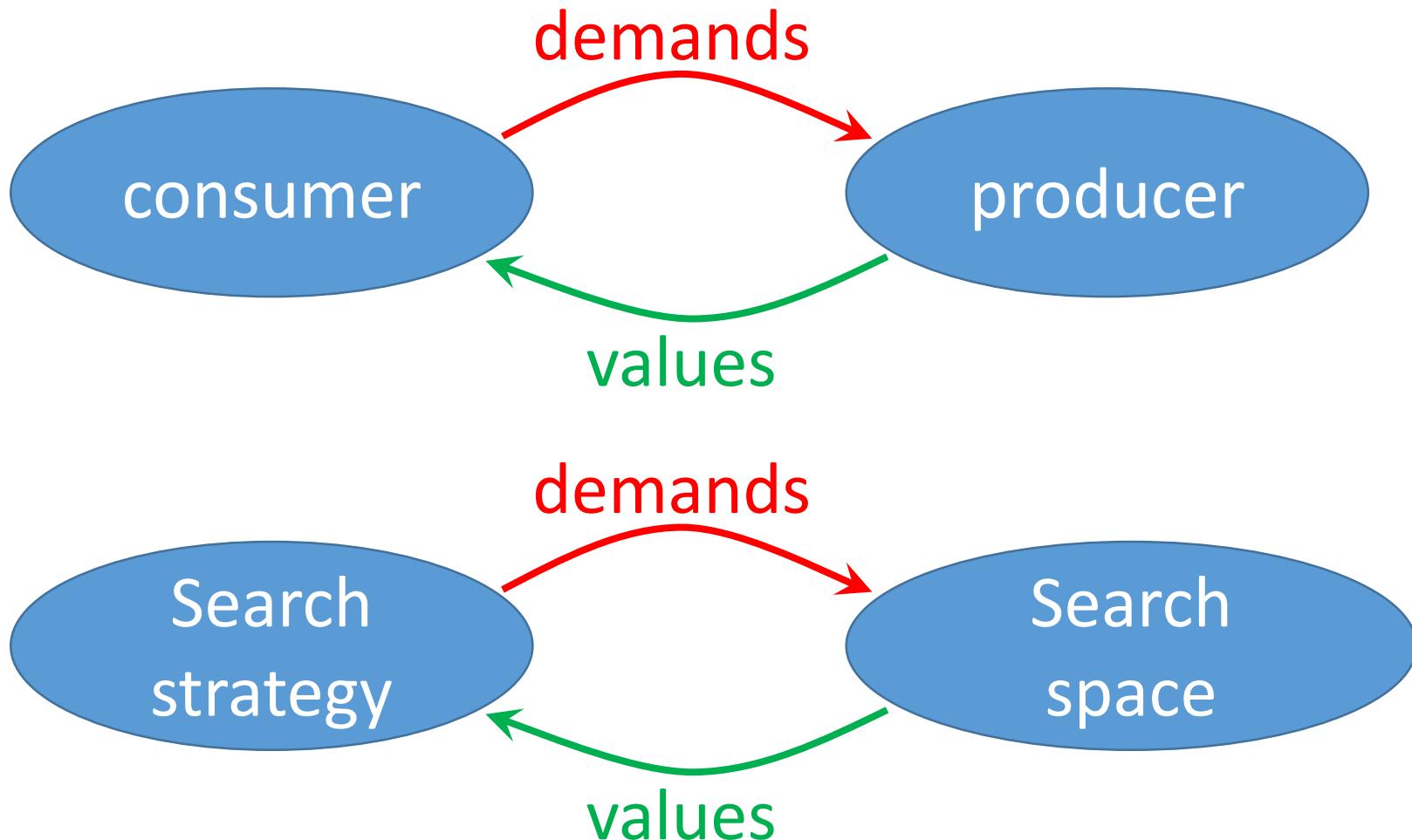


1990

# Lazy producer-consumer

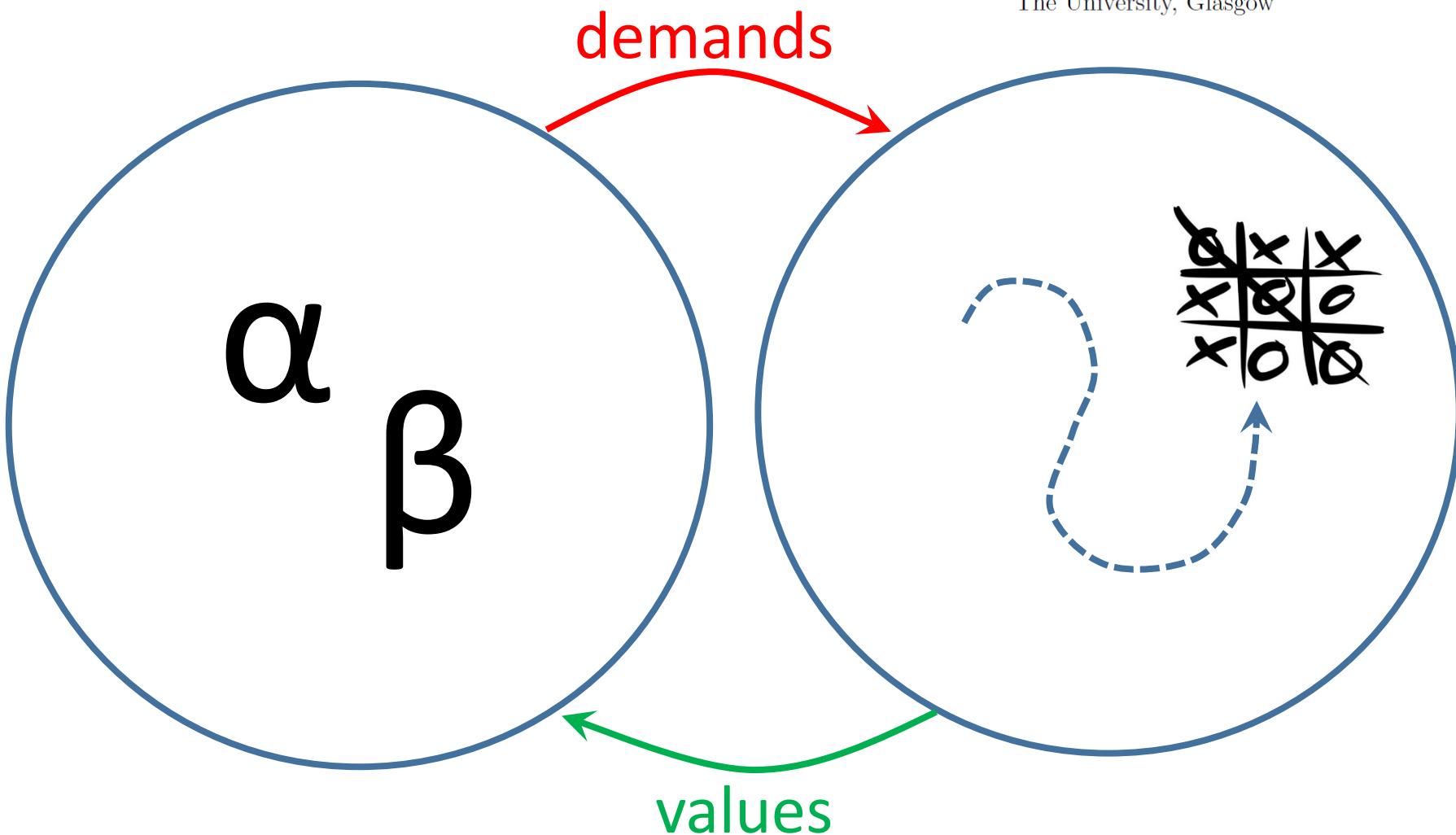


# Lazy producer-consumer

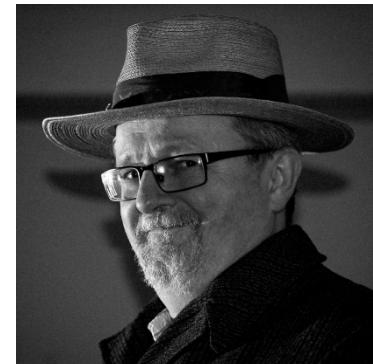


# Why Functional Programming Matters

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The University, Glasgow

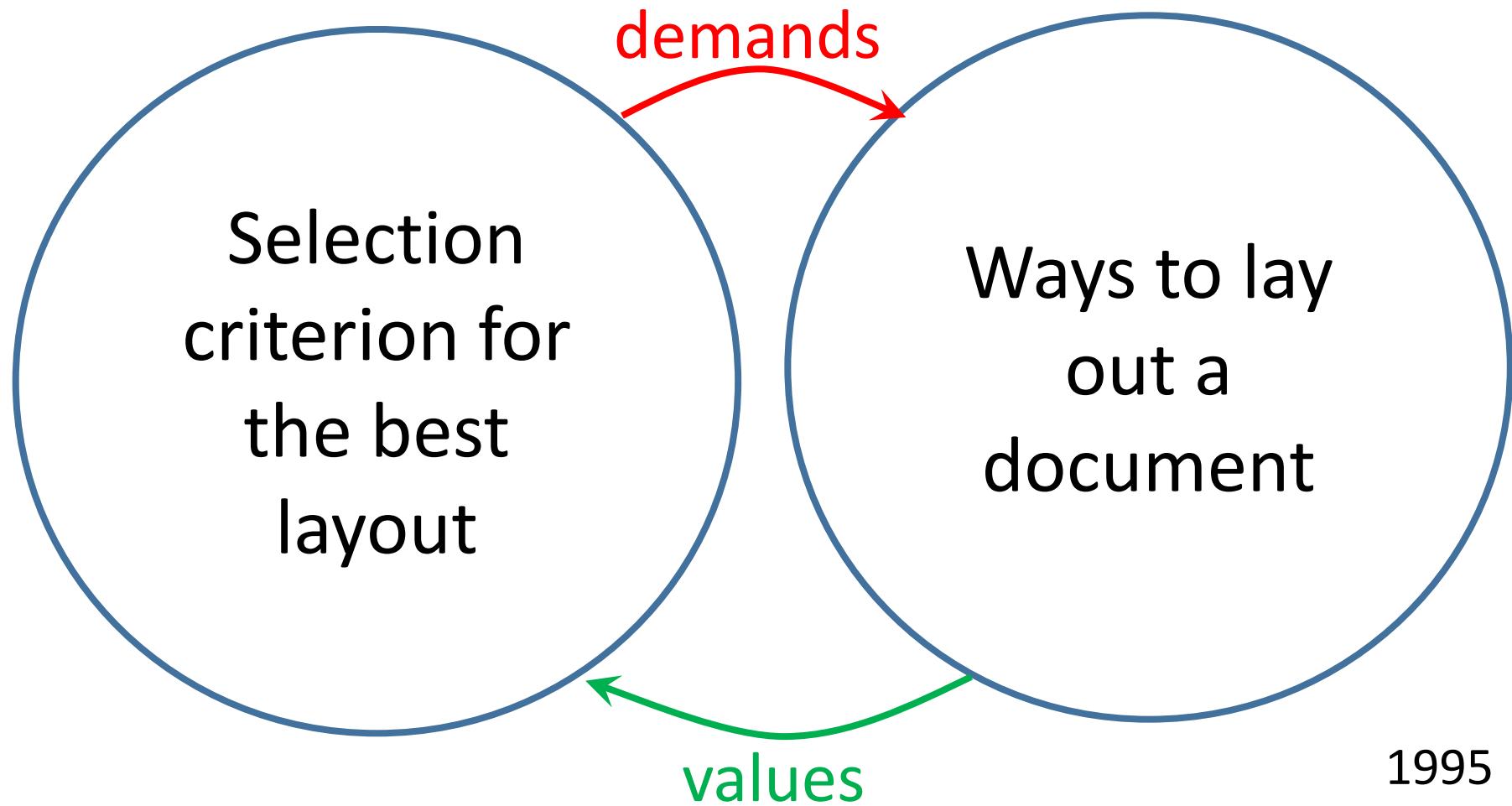


# The Design of a Pretty-printing Library



John Hughes

Chalmers Tekniska Högskola, Göteborg, Sweden.



# Improved on by...





# QuickCheck: A Lightweight Tool for Random Testing of Haskell Programs



Koen Claessen  
Chalmers University of Technology  
koen@cs.chalmers.se

John Hughes  
Chalmers University of Technology  
rjmh@cs.chalmers.se

```
prop_reverse() ->
?FORALL(Xs,list(int())),
    reverse(reverse(Xs)) == Xs).
```

```
3> eqc:quickcheck(qc:prop_reverse()).
```

```
.....  
.....
```

```
OK, passed 100 tests
true
```

2000



# QuickCheck: A Lightweight Tool for Random Testing of Haskell Programs



Koen Claessen  
Chalmers University of Technology  
koen@cs.chalmers.se

John Hughes  
Chalmers University of Technology  
rjmh@cs.chalmers.se

```
prop_wrong() ->
?FORALL(xs,list(int())),
reverse(xs) == xs).
```

```
4> eqc:quickcheck(qc:prop_wrong()).
Failed! After 1 tests.
[-36,-29,20,31,-47,-63,80,-7,93,-87,-29,33,64,58]
Shrinking xx.x.x...xx(4 times)
[0,1]←
false
```

*minimal counterexample*

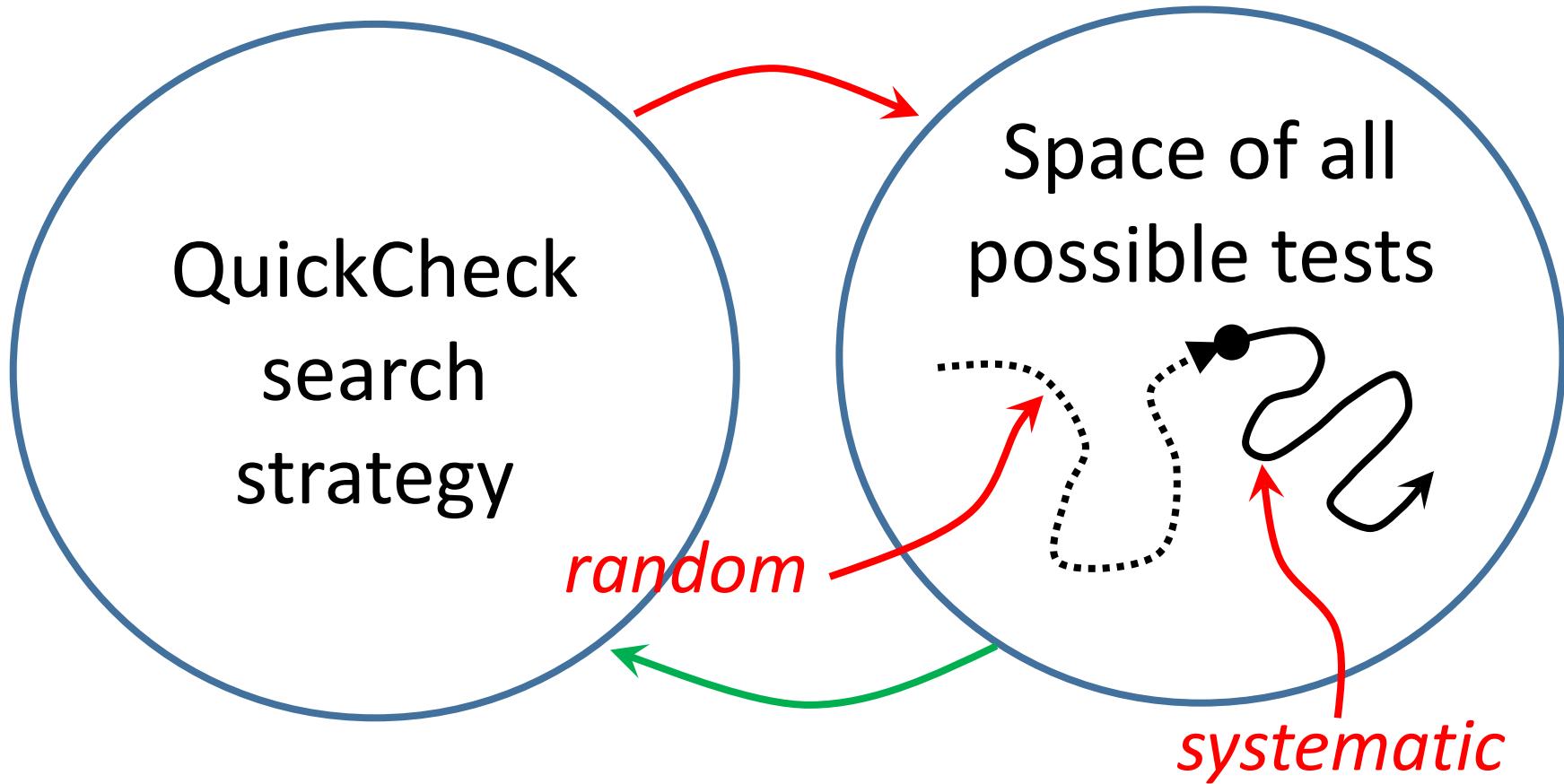


# QuickCheck: A Lightweight Tool for Random Testing of Haskell Programs



Koen Claessen  
Chalmers University of Technology  
[koen@cs.chalmers.se](mailto:koen@cs.chalmers.se)

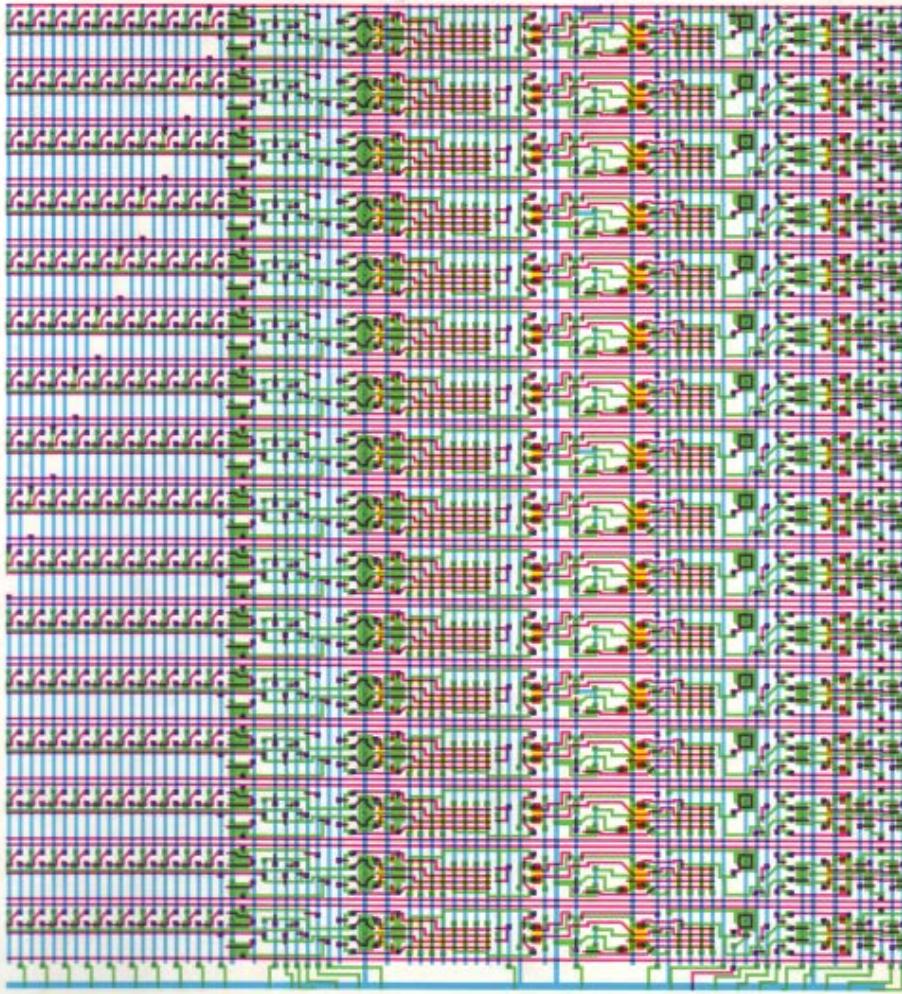
John Hughes  
Chalmers University of Technology  
[rjmh@cs.chalmers.se](mailto:rjmh@cs.chalmers.se)



INTRODUCTION TO

# VLSI SYSTEMS

CARVER MEAD • LYNN CONWAY



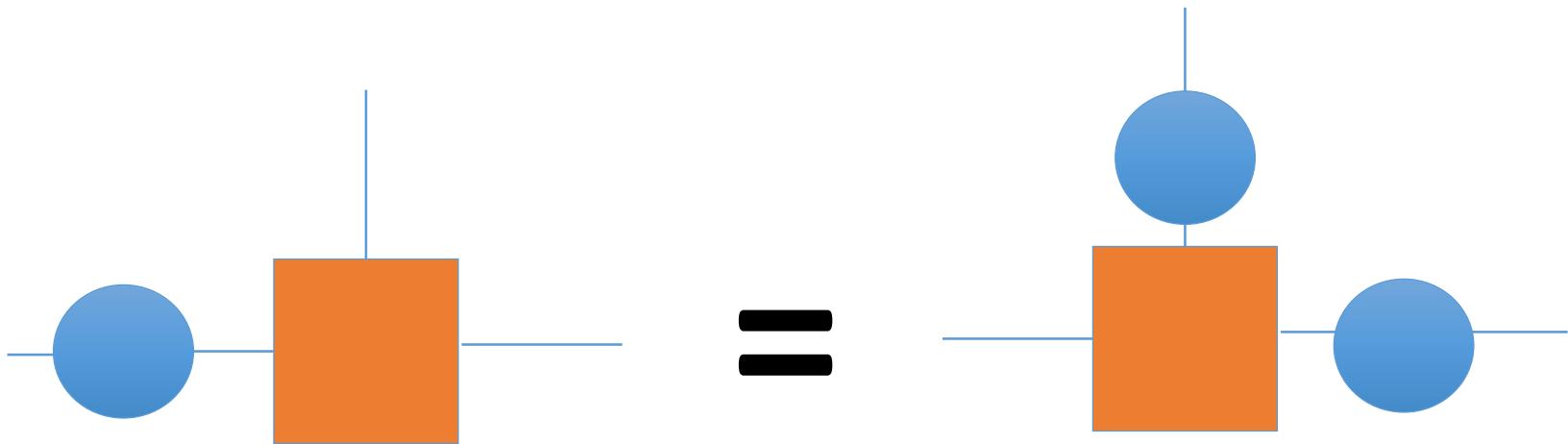
# muFP—Circuits as values

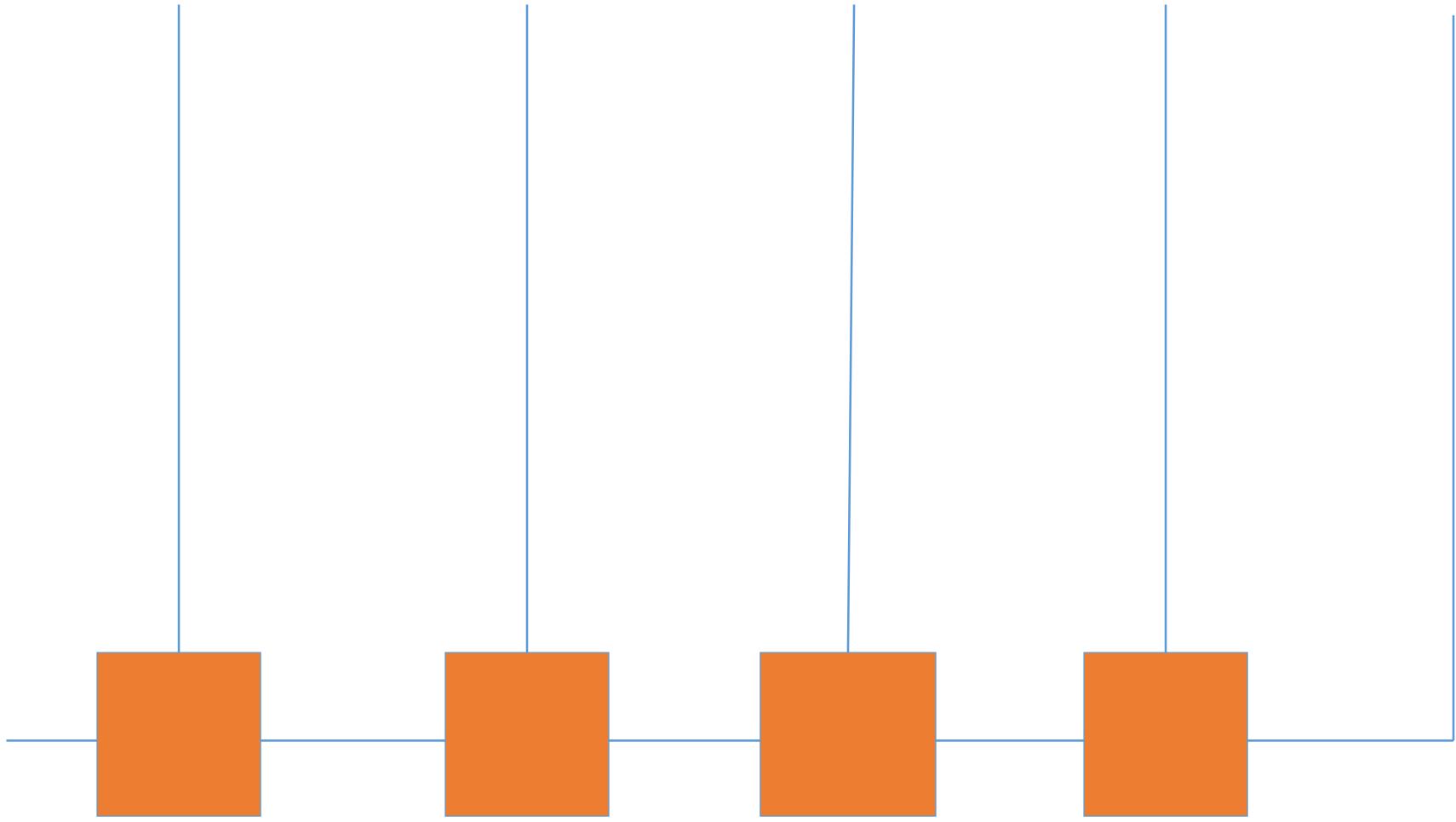


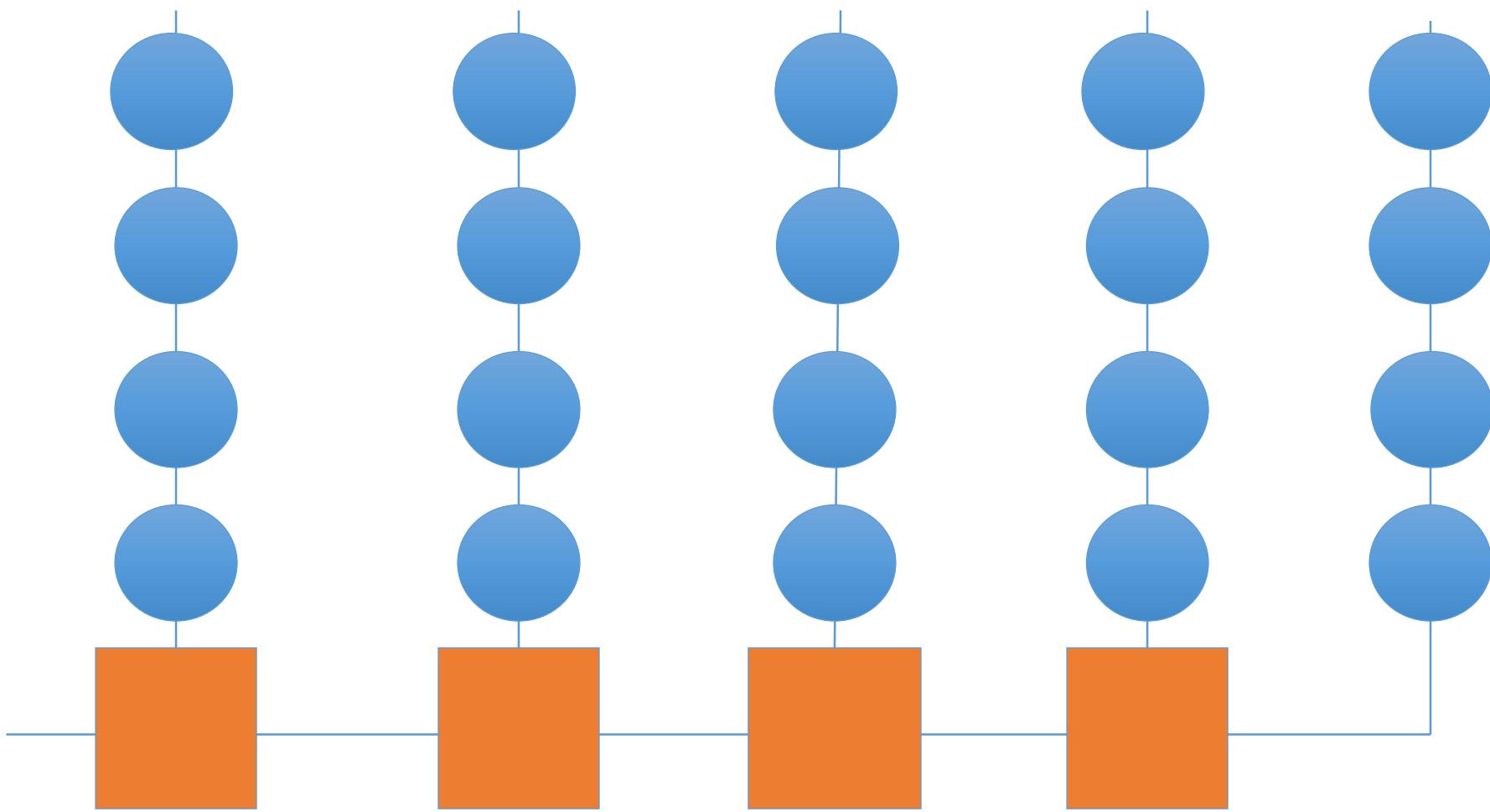
- Backus FP + unit delays
- Inherits many combining forms and laws
- Good for reasoning about alternative designs

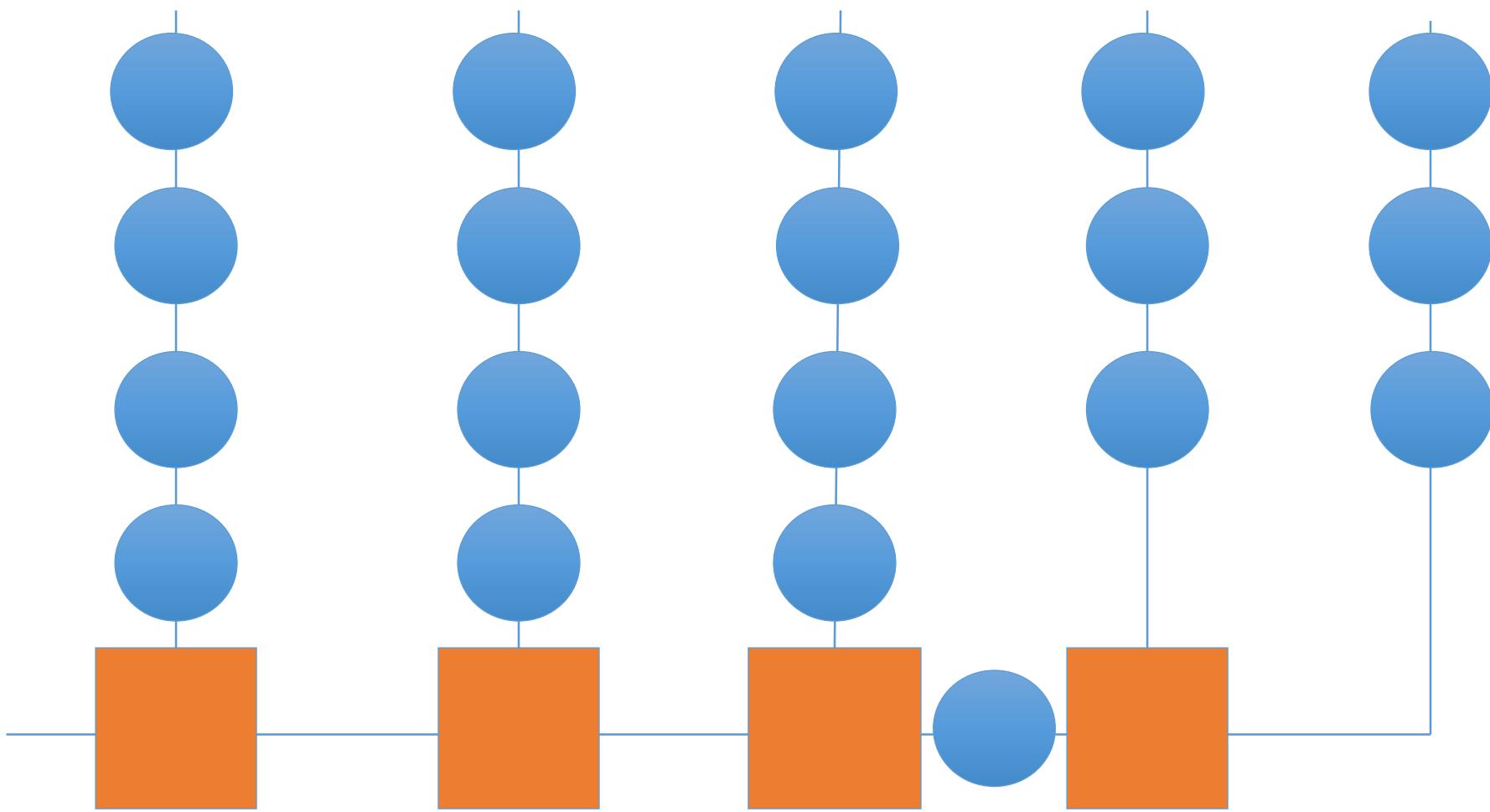


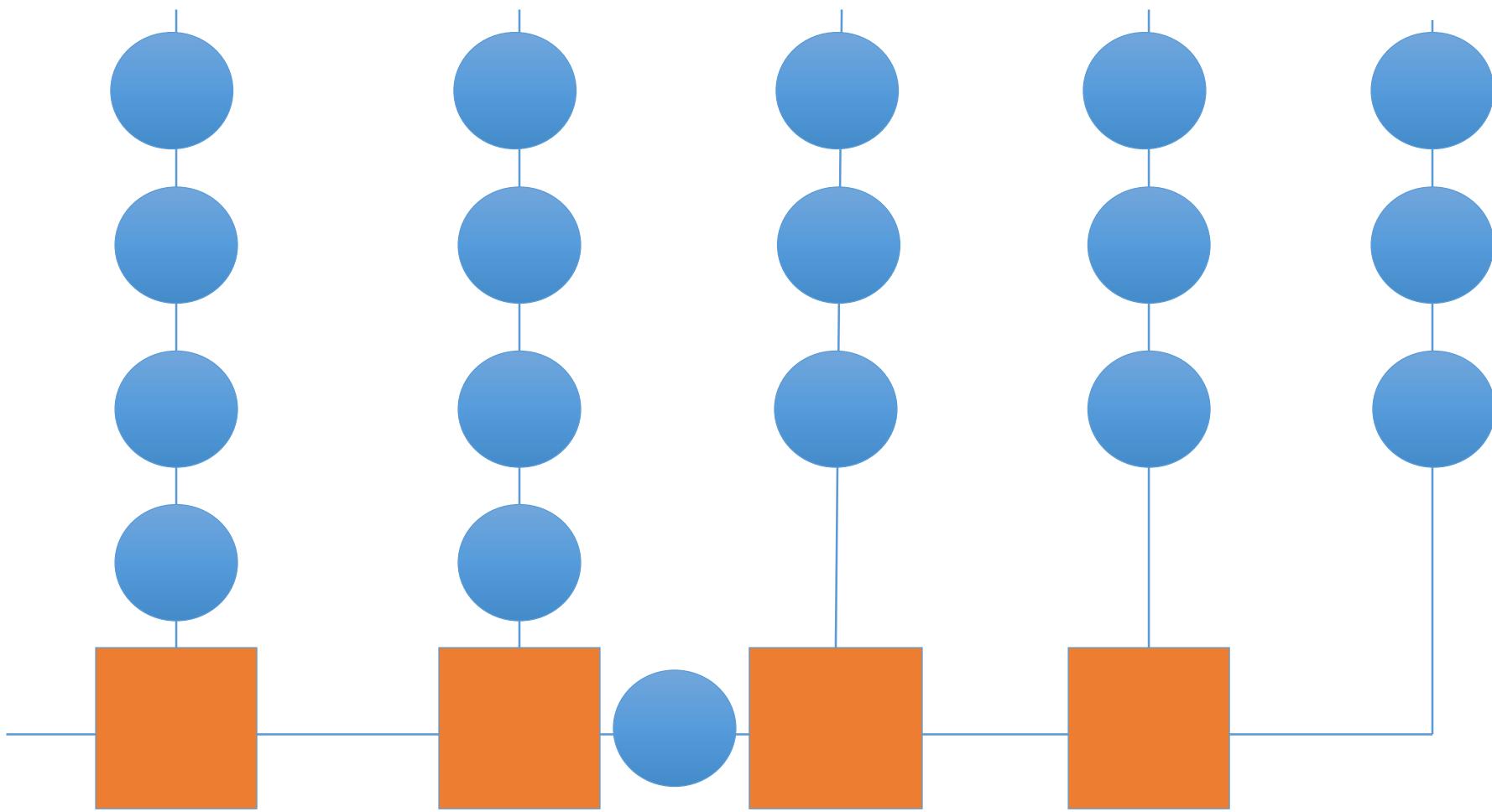
# Example law: “retiming”

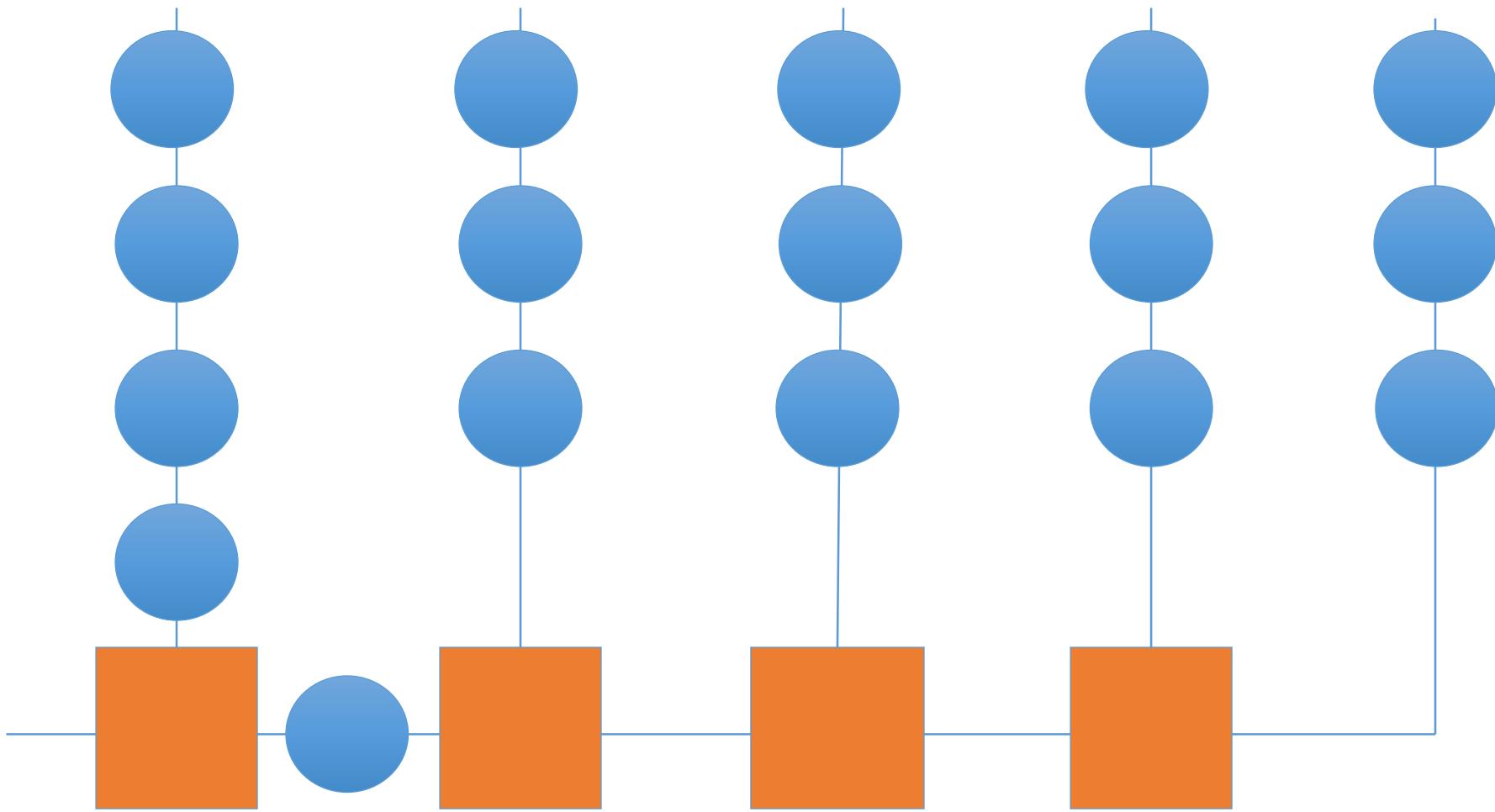


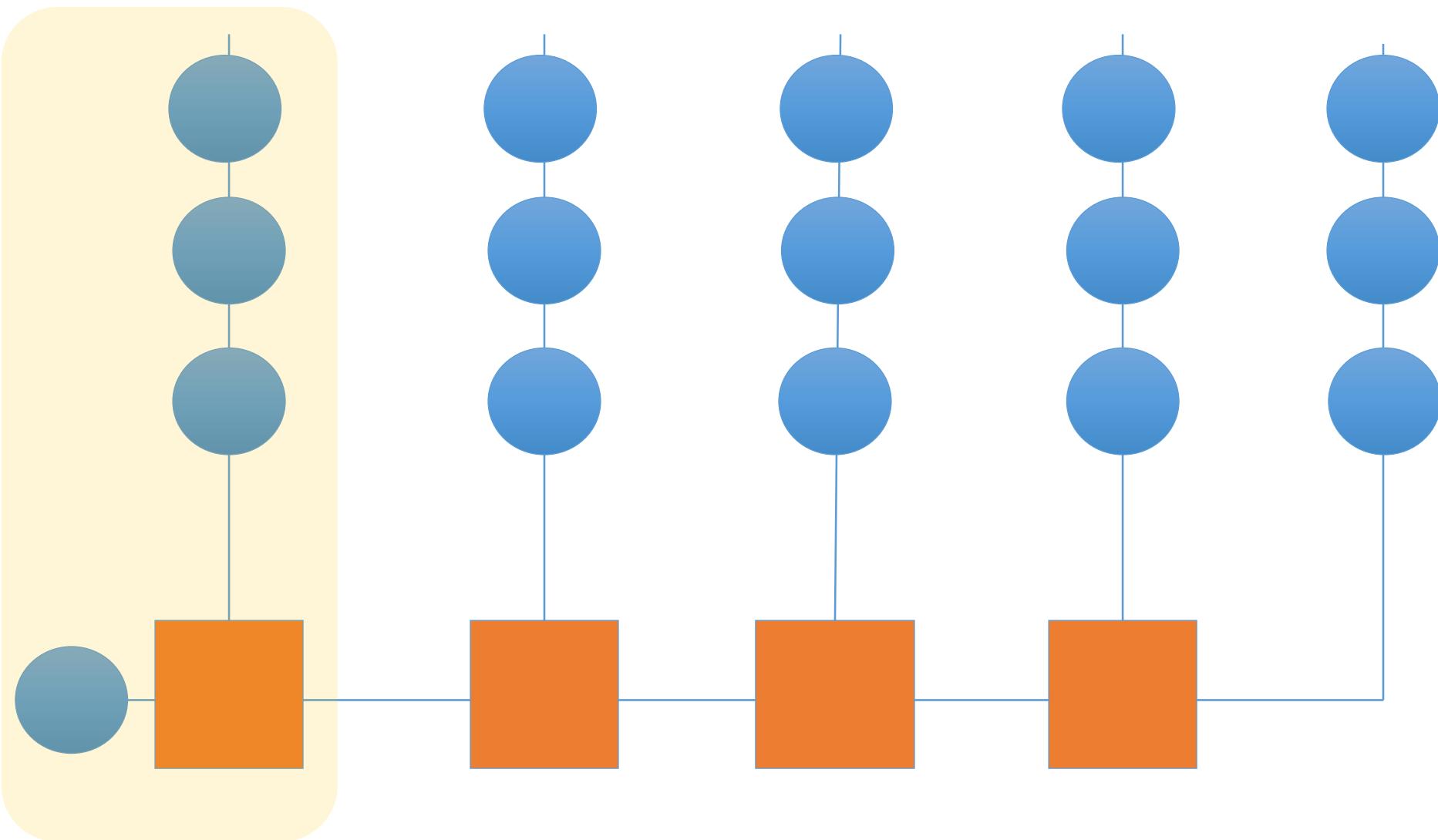


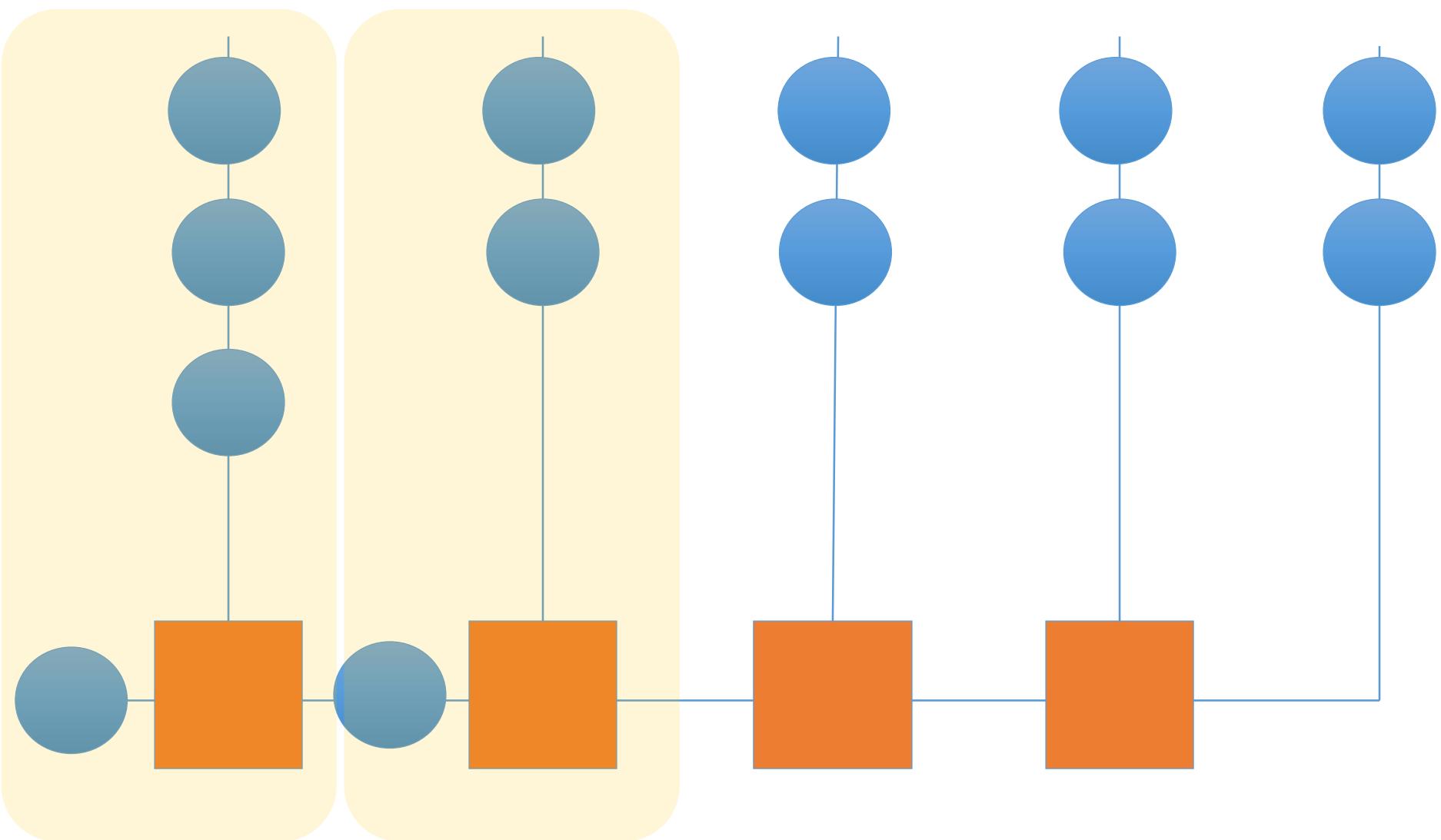


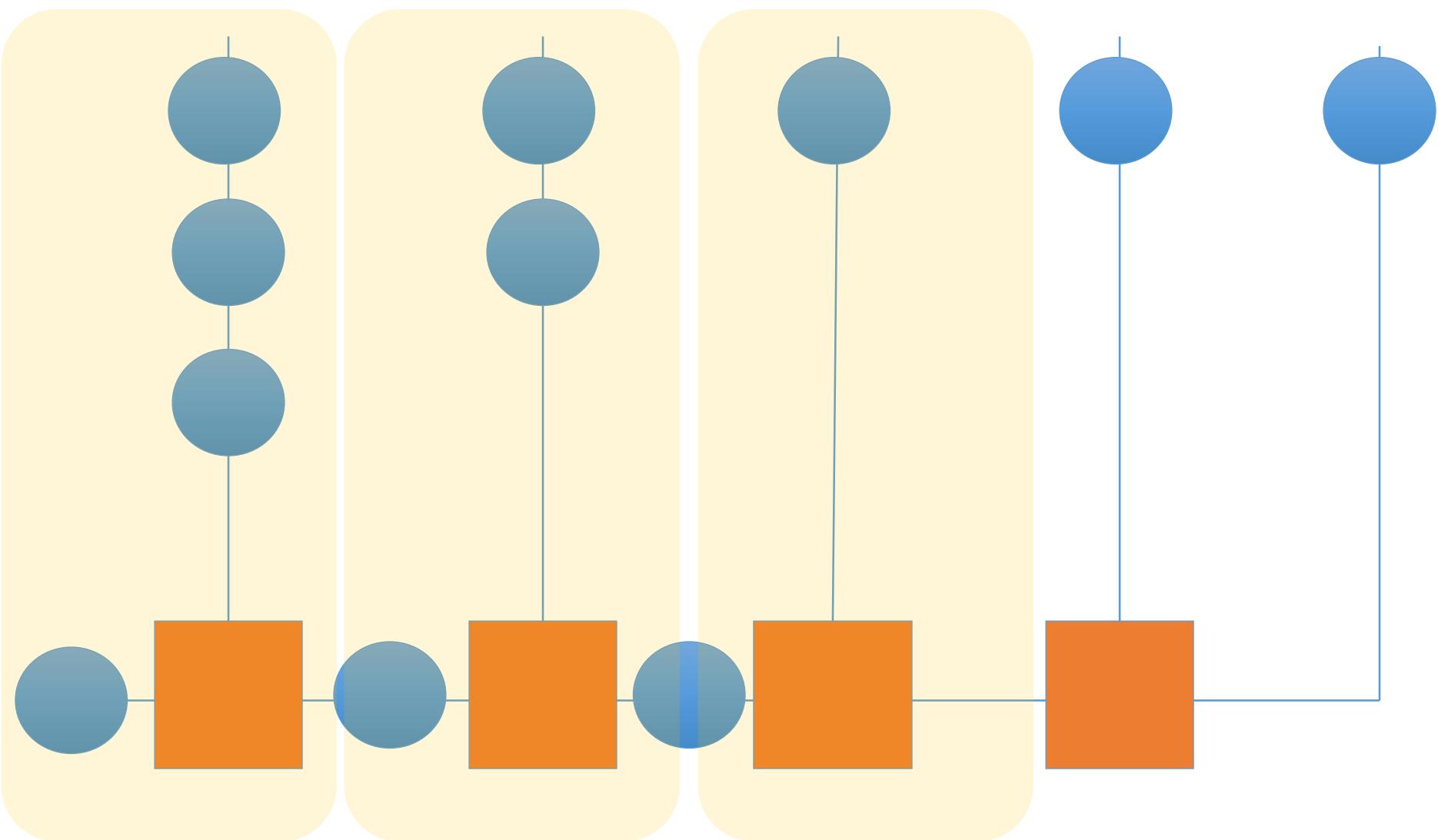


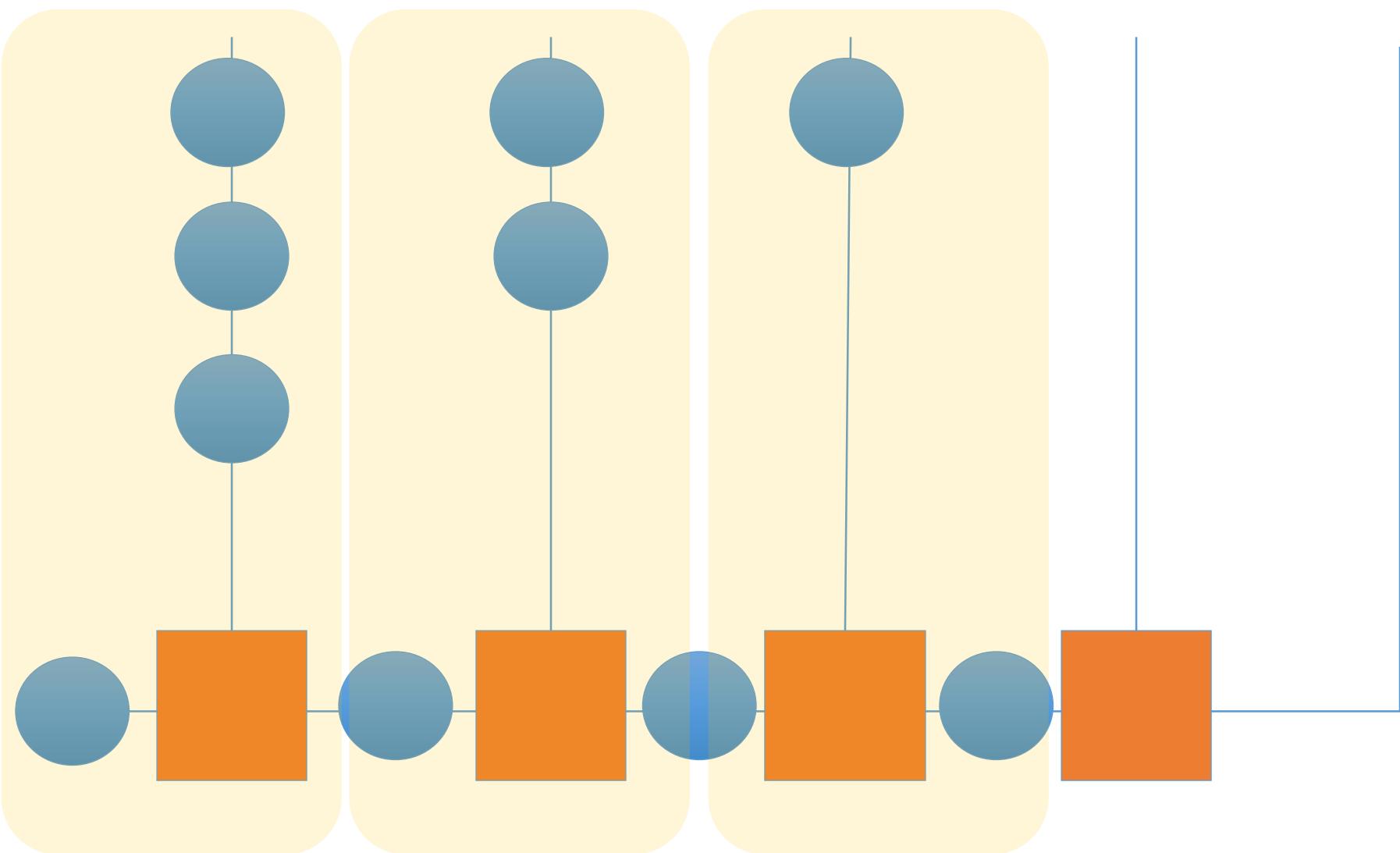












# Users!

# Plessey video motion estimation

“Using muFP, the array processing element was described in just one line of code and the complete array required four lines of muFP description. muFP enabled the effects of adding or moving data latches within the array to be assessed quickly.”

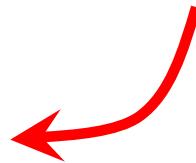
Bhandal et al, An array processor for video picture motion estimation,  
Systolic Array Processors, 1990, Prentice Hall

work with Plessey done by G. Jones and W. Luk

# Lava

*Semantics*

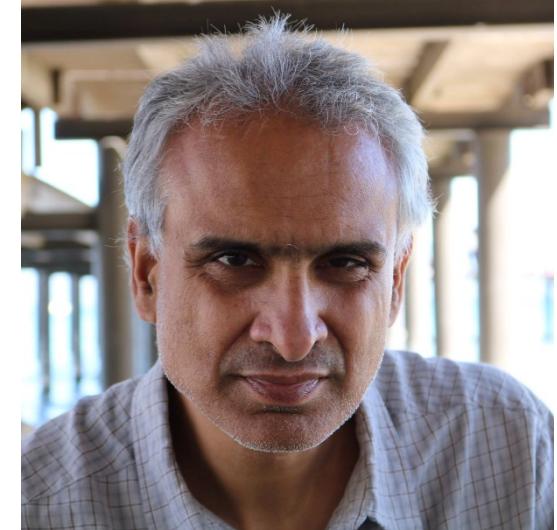
muFP



+

Functional  
Geometry

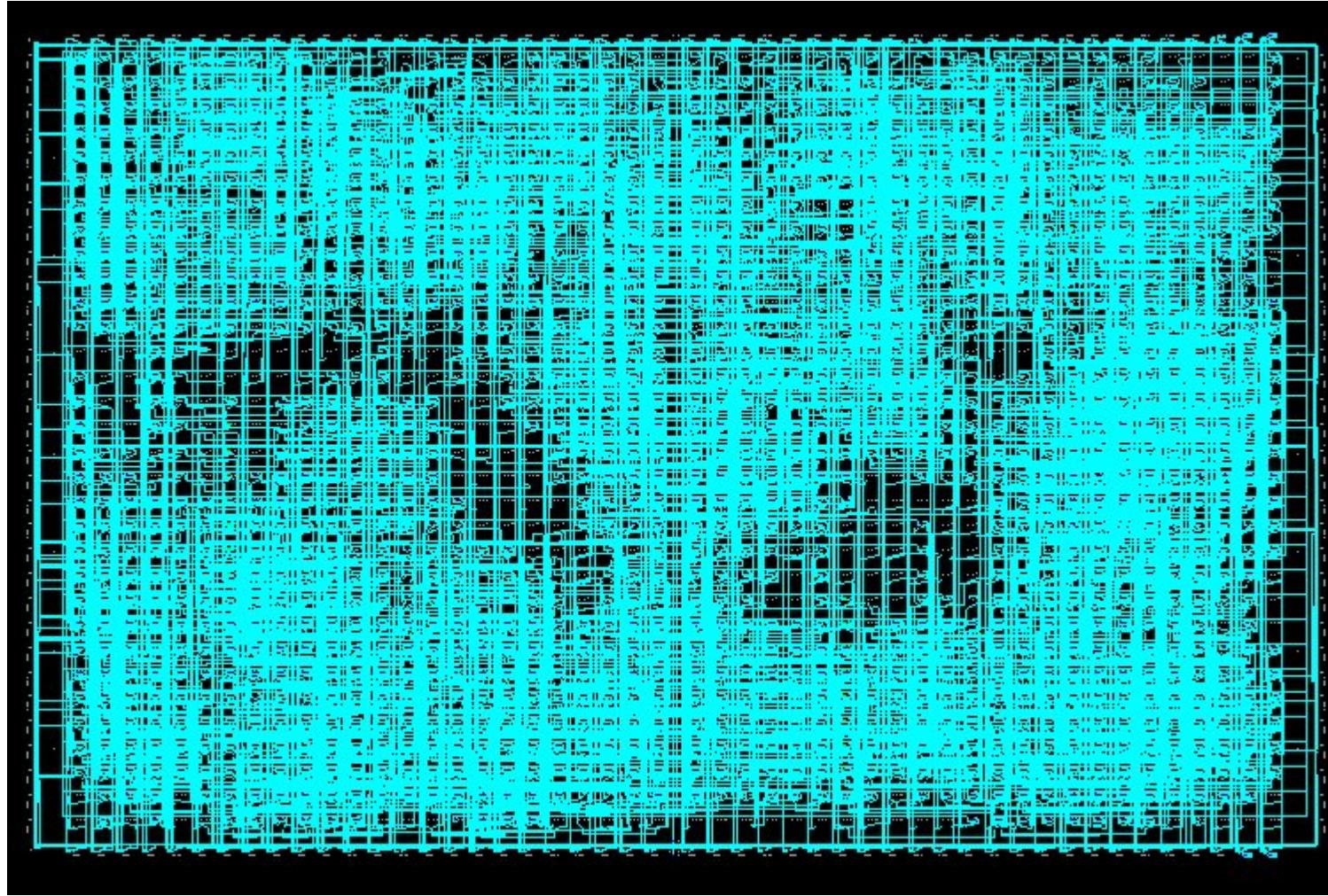
*Placement*



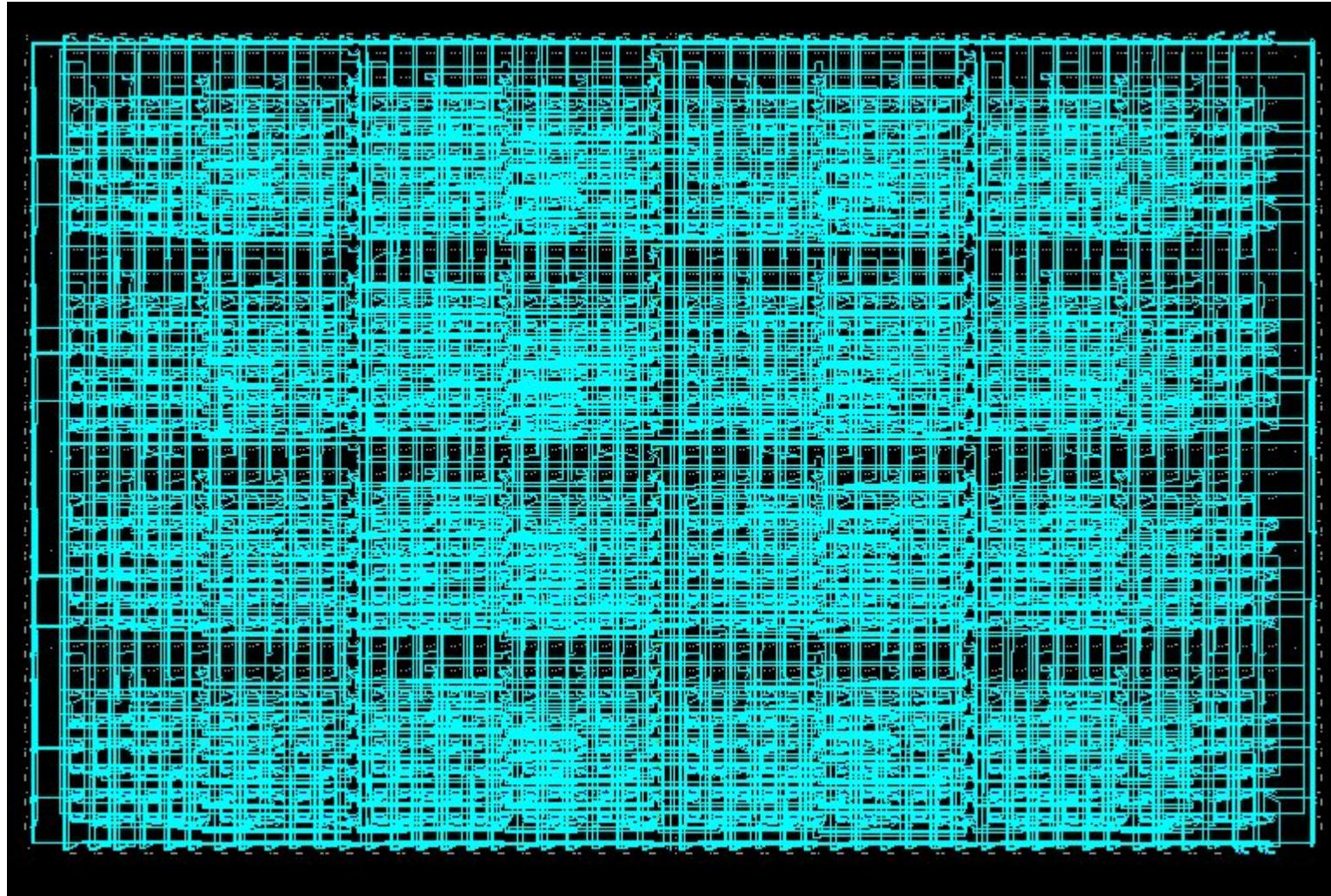
Satnam Singh  
Xilinx

→ FPGA layouts on Xilinx chips

# Four adder trees—no placement



# Four adder trees—Lava



Intel

$$4195835.0 - 3145727.0 * (4195835.0 / 3145727.0) = 0$$

Intel

$$4195835.0 - 3145727.0 * (4195835.0 / 3145727.0) = 0$$

Flawed Pentium

$$4195835.0 - 3145727.0 * (4195835.0 / 3145727.0) = 256$$

# \$475 million



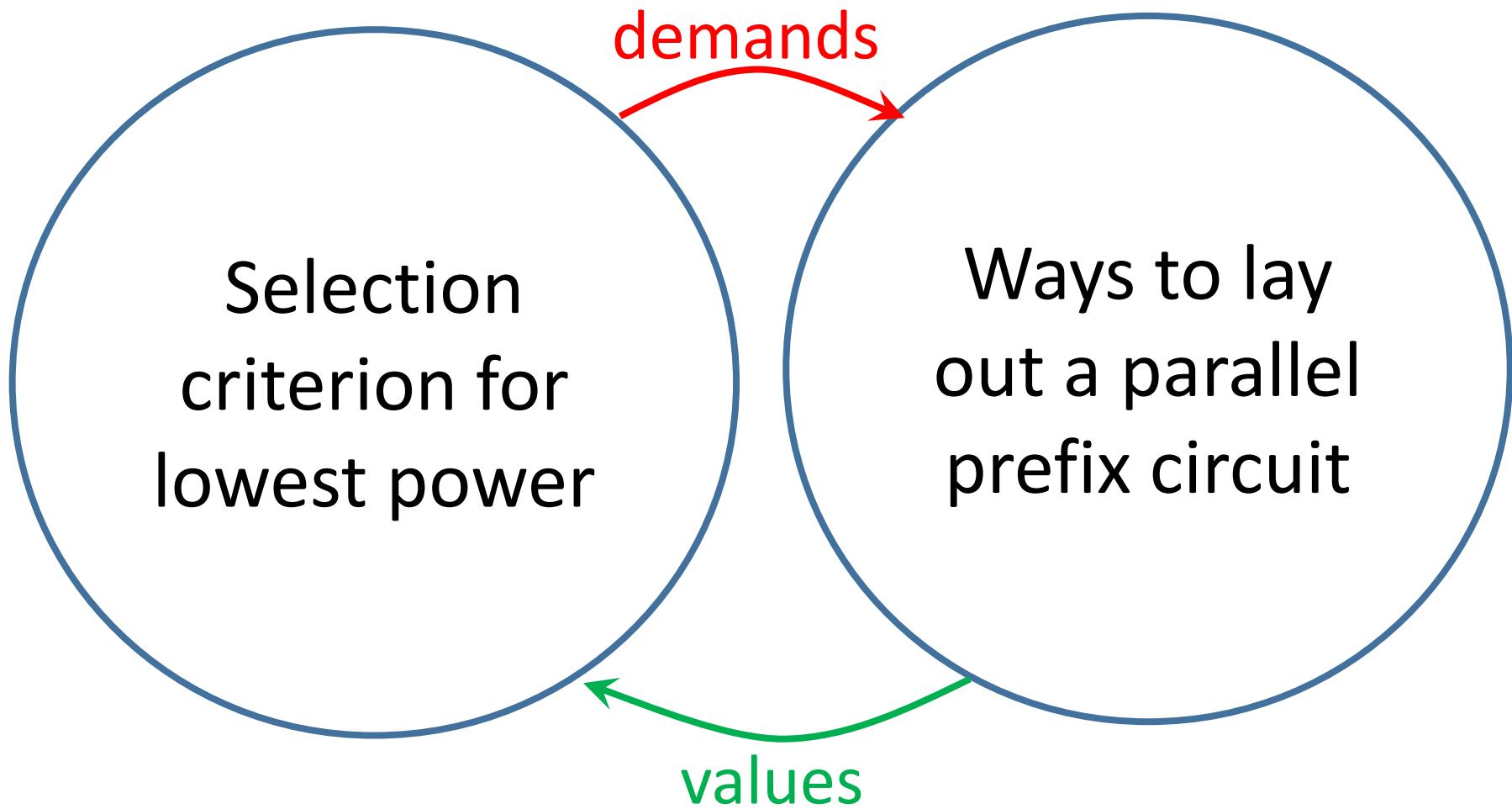
# fli

Lazy functional language,  
1000s users

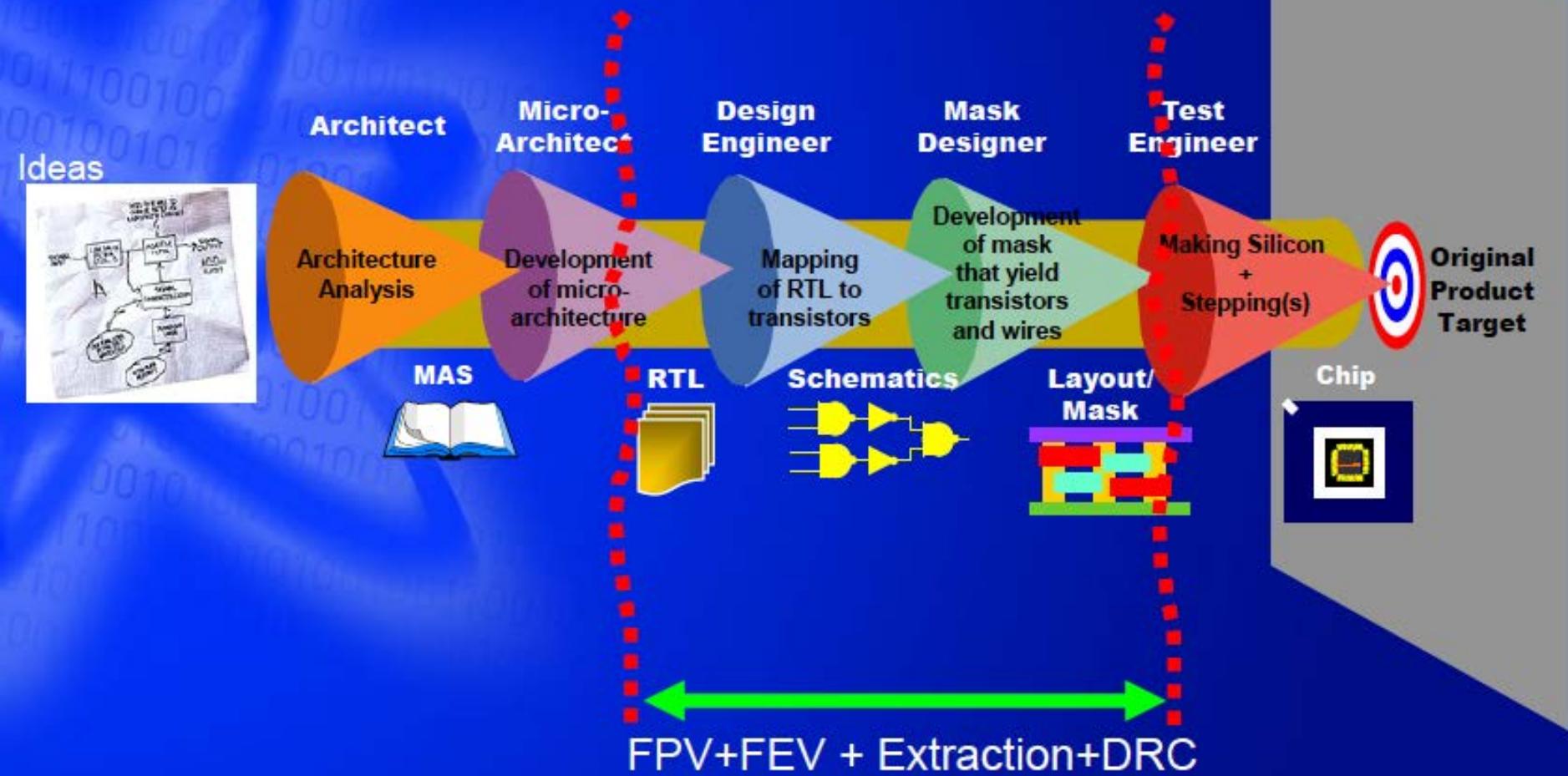


- Design
- High-level specification
- Scripting
- Implementation of formal verification tools  
and theorem provers
- Object language for theorem proving

Thanks to Carl Seger (formerly of Intel)

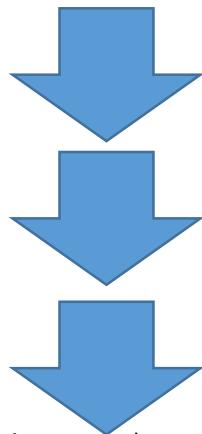


# Solid Formal Link with Good Return of the Investment

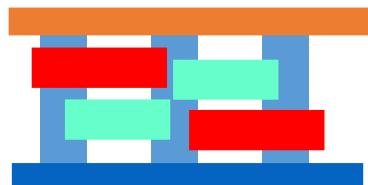


Thanks to Carl Seger (formerly of Intel)

# SPEC



Layout/  
Mask



# Bluespec—FP for hardware



Haskell-like language (architecture) +  
atomic transition rules (H/W modelling)

Frees designers to explore *better algorithms*,  
making major architectural change easy



Types, Functional Programming and  
Atomic Transactions in Hardware Design  
Rishiyur Nikhil LNCS 8000

# Bluecheck

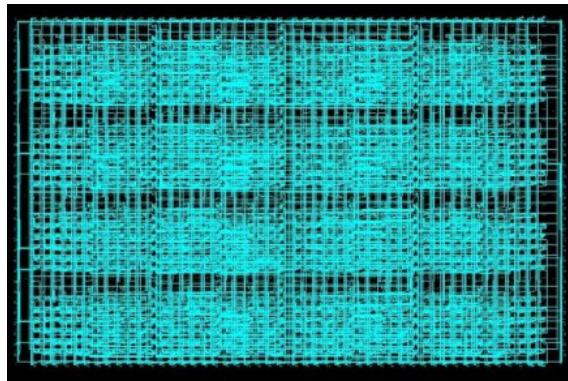
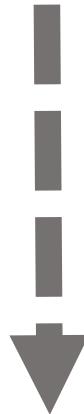
- QuickCheck in Bluespec!
- Generates and shrinks tests *on the FPGA!*

[A Generic Synthesisable Test Bench \(Naylor and Moore, Memocode 2015\)](#)

**two**  $f(x) = f(f(x))$

**one**  $f(x) = f(x)$

**zero**  $f(x) = x$



**Combining  
forms**

**whole  
values**

**Simple  
laws**

**Functions as  
representations**

