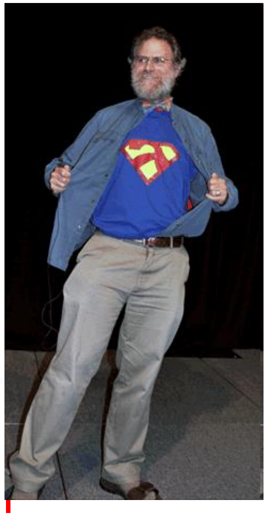
Monády – úvod



Phil Wadler: http://homepages.inf.ed.ac.uk/wadler/topics/monads.html

- Monads for Functional Programming In *Advanced Functional Programming* , Springer Verlag, LNCS 925, 1995,

- Noel Winstanley: What the hell are Monads?, 1999

http://web.cecs.pdx.edu/~antoy/Courses/TPFLP/lectures/MONADS/Noel/research/monads.html

- Jeff Newbern's: All About Monads
  https://www.cs.rit.edu/~swm/cs561/All_About_Monads.pdf

- Dan Bensen: A (hopefully) painless introduction to monads,
  http://www.prairienet.org/~dsb/monads.htm

Monady sú použiteľný nástroj pre programátora poskytujúci:

- modularitu – skladať zložitejšie výpočty z jednoduchších (no side-effects),
- flexibilitu – výsledný kód je ľahšie adaptovateľný na zmeny,
- izoluje side-effect operácie (napr. IO) od čisto funkcionálneho zvyšku.

```
sequence       :: Monad m => [m a] -> m [a]
sequence []     = return []
sequence (c:cs) = do { x <- c;
                       xs <- sequence cs;
                       return (x:xs) }
```

Maybe monad

Maybe je podobné Exception (Nothing~~Raise String, Just a ~~Return a)

```
data Maybe a = Nothing | Just a

instance Monad Maybe where
    return v          = Just v          -- vrát' hodnotu
    fail              = Nothing         -- vrát' neúspech

    Nothing  >>= f = Nothing            -- ak už nastal neúspech, trvá do konca
    (Just x)  >>= f  = f x              -- ak je zatial' úspech, závisí to na výpočte f
```

```
> sequence [Just "a", Just "b", Just "d"]
Just ["a","b","d"]
> sequence [Just "a", Just "b", Nothing, Just "d"]
Nothing
```

Maybe MonadPlus

```
data Maybe a = Nothing | Just a

class Monad m => MonadPlus m where – podtrieda, resp. podinterface
    mzero     :: m a                          -- ø
    mplus     :: m a -> m a -> m a            -- disjunkcia

instance MonadPlus Maybe where
    mzero              = Nothing              -- fail...
    Just x `mplus` y= Just x                  -- or
    Nothing `mplus` y = y

> Just "a" `mplus` Just "b"
Just "a"
> Just "a" `mplus` Nothing
Just "a"
> Nothing `mplus` Just "b"
Just "b"
```

Zákony monád a monádPlus

- vlastnosti return a >>=:

```
return x >>= f              = f x                  -- return ako identita zľava
p >>= return                = p                    -- retrun ako identita sprava
p >>= (\x -> (f x >>= g))= (p >>= (\x -> f x)) >>= g  -- "asociativita"
```

- vlastnosti zero a `plus`:

```
zero `plus` p               = p                    -- zero ako identita zľava
p `plus` zero               = p                    -- zero ako identita sprava
p `plus` (q `plus` r)       = (p `plus` q) `plus` r     -- asociativita
```

- vlastnosti zero `plus` a >>= :

```
zero >>= f                  = zero                 -- zero ako identita zľava
p >>= (\x->zero)            = zero                 -- zero ako identita sprava
(p `plus` q) >>= f          = (p >>= f) `plus` (q >>= f)     -- distribut.
```

List monad

- List monad použijeme, ak simulujeme nedeterministický výpočet

```
data List a = Null | Cons a (List a) deriving (Show)        -- alias [a]

instance Functor List where                    -- to je vlastne map
    fmap f Nil = Nil
    fmap f (Cons x xs) = Cons (f x) (fmap f xs)

instance Monad List where
    return x        = [x]                       :: a -> [a]
    m >>= f         = concatMap f m             :: [a] -> (a -> [b]) -> [b]
    concatMap       = concat . map f m
```

```
return :: a -> [a]
>>=    :: [a] -> (a -> [b]) -> [b]
```

List monad

```
type List a            = [a]

instance Functor List where
    fmap = map

instance Monad List where
    return v           = [x]
    [] >>= f           = []
    (x:xs) >>= f       = f x ++ (xs >>= f)          -- concatMap f (x:xs)

instance MonadPlus List where
    mzero                          = []
    [] `mplus` ys                  = ys
    (x:xs) `mplus` ys              = x : (xs `plus` ys)  -- mplus je klasický append
```

List monad - vlastnosti

Príklad, tzv. listMonad M a = List a = [a]

return x = [x] :: a -> [a]

m >>= f = concatMap f m :: [a] -> (a -> [b]) -> [b]

concatMap = concat . map f m

Cvičenie: overme platnosť zákonov:

- return c >>= (\x->g) **=** g[x/c]
  - [c] >>= (\x->g) = concatMap (\x->g) [c] = concat . map (\x->g) [c] = concat [g[x/c]] = g[x/c]
- m >>= \x->return x **=** m
  - $[c_1, \dots, c_n]$ >>= (\x->return x) = concatMap (\x->return x) $[c_1, \dots, c_n]$ = concat . map (\x->return x) $[c_1, \dots, c_n]$ = concat $[[c_1], \dots, [c_n]]$ = $[c_1, \dots, c_n]$
- m1 >>= (****x->m2 >>= (****y->m3)) **=** (m1 >>= (****x->m2)) >>= (****y->m3)
  - ($[c_1, \dots, c_n]$ >>= (\x->$[d_1, \dots, d_m]$)) >>= (\y->m3) =
    (concat $[[d_1[x/c_1], \dots, d_m[x/ c_1]], \dots [d_1[x/ c_n], \dots, d_m[x/ c_n]]]$) >>= (\y->m3) =
    ($[d_1[x/c_1], \dots, d_m[x/ c_1], \dots, d_1[x/ c_n], \dots, d_m[x/ c_n]]$) >>= (\y->m3) =
    ($[d_1[x/c_1], \dots, d_m[x/ c_1], \dots, d_1[x/ c_n], \dots, d_m[x/ c_n]]$) >>= (\y->$[e_1, \dots, e_k]$) = …
    $[e_i[y/d_j[x/c_l]]]$

Zákony monadPlus pre List

- vlastnosti zero a `plus`:

```
zero `plus` p              = p                       -- [] ++ p = p
p `plus` zero              = p                       -- p ++ [] = p
p `plus` (q `plus` r)      = (p `plus` q) `plus` r   -- asociativita ++
```

- vlastnosti zero `plus` a >>= :

```
zero >>= f                 = zero                    -- concat . map f [] = []
p >>= (\x->zero)           = zero                    -- concat . map (\x->[]) p = []
(p `plus` q) >>= f         = (p >>= f) `plus` (q >>= f)
                                                     -- concat . map f (p ++ q) =
                                                          concat . map f p
                                                          ++
                                                          concat . map f q
```

List monad vs. comprehension

```
squares lst = do      x <- lst
                      return (x * x)
-- vlastne znamená
squares lst =         lst >>= \x -> return (x * x)
-- po dosadení
squares lst = concat . map (\x -> [x * x]) lst
-- eta redukcia
squares = concat . map (\x -> [x * x])
-- a takto by sme to napísali bez všetkého
squares = map (\x -> x * x)

-- iný príklad: kartézsky súčin
cart xs ys  =   do  x <- xs
                    y <- ys
                    return (x,y)
```

Guard
(Control.Monad)

```
pythagoras =   [(x, y, z) |  z <- [1..],            -- pythagorejské trojuholníky
                             x <- [1..z],
                             y <- [x..z],
                             x*x+y*y == z*z]
pythagoras' =   do  z <- [1..]
                    x <- [1..z]
                    y <- [x..z]                                   -- zlé riešenie, prečo ?
                    if x*x+y*y == z*z then  return (x,y,z)  else  return ()

                   if x*x+y*y == z*z then return "hogo-fogo"    else []
                   return (x,y,z)                  resp. ["hogo-fogo"]

                   if x*x+y*y == z*z then return () else []
                   resp. guard (x*x+y*y == z*z)
                   return (x,y,z)
```

Guard
(Control.Monad)

```
guard :: (MonadPlus m) => Bool -> m ()
guard True   = return ()
guard False  = mzero
> guard (9 >5) >> return "hogo" :: [String]
["hogo"]
> guard (5 > 9) >> return "fogo" :: [String]
[]
```

Kartézsky súčin

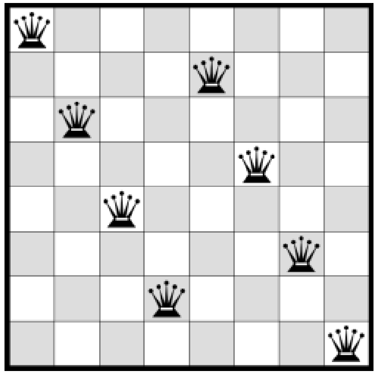
```
listComprehension xs ys = [(x,y) | x<-xs, y<-ys ]
guardedListComprehension xs ys =
                    [(x,y) | x<-xs, y<-ys, x<=y, x*y == 24 ]
```

```
> listComprehension [1,2,3] ['a','b','c']
[(1,'a'),(1,'b'),(1,'c'),(2,'a'),(2,'b'),(2,'c'),(3,'a'),(3,'b'),(3,'c')]

> guardedListComprehension [1..10] [1..10]
[(3,8),(4,6)]
```

```
monadComprehension xs ys = do { x<-xs; y<-ys; return (x,y) }
guardedMonadComprehension xs ys =
          do { x<-xs; y<-ys; guard (x<=y); guard (x*y==24); return (x,y) }
```

```
> monadComprehension [1,2,3] ['a','b','c']
[(1,'a'),(1,'b'),(1,'c'),(2,'a'),(2,'b'),(2,'c'),(3,'a'),(3,'b'),(3,'c')]
> guardedMonadComprehension [1..10] [1..10]
[(3,8),(4,6)]
```



Backtracking

```
check (i,j) (m,n)      = (i==m) || (j==n) || (j+i==n+m) || (j+m==i+n)

safe p n   = all (True==)  [not (check (i,j) (m+1,n)) | (i,j) <- zip [1..m] p]
                           where m=length p

-- backtrack
queens n = queens1 n n
queens1 n v   | n==0 = [[]]
              | otherwise = [y++[p] | y <- queens1 (n-1) v, p <- [1..v], safe y p]

mqueens n = mqueens1 n n
mqueens1 n v         | n==0 = return []
                     | otherwise =   do   y <- mqueens1 (n-1) v
                                          p <- [1..v]
                                          guard (safe y p)
                                          return (y++[p])
```

filterM

(Control.Monad)

```
filterM     :: (Monad m) => (a -> m Bool) -> [a] -> m [a]

> filterM (\x->[True, False]) [1,2,3]          -- potenčná množina, powerset
[[1,2,3],[1,2],[1,3],[1],[2,3],[2],[3],[]]

filterM          :: (Monad m) => (a -> m Bool) -> [a] -> m [a]
filterM _ []      =  return []
filterM p (x:xs) =  do
                          flg <- p x
                          ys  <- filterM p xs
                          return (if flg then x:ys else ys)
```

mapM, forM

(Control.Monad)

```
mapM    :: (Monad m) => (a -> m b) -> [a] -> m [b]
mapM f  = sequence . map f

forM     :: (Monad m) => [a] -> (a -> m b) -> m [b] -- len zámena args.
forM     = flip mapM
```

```
> mapM (\x->[x,11*x]) [1,2,3]
[[1,2,3],[1,2,33],[1,22,3],[1,22,33],[11,2,3],[11,2,33],[11,22,3],[11,22,33]]

> forM [1,2,3] (\x->[x,11*x])
[[1,2,3],[1,2,33],[1,22,3],[1,22,33],[11,2,3],[11,2,33],[11,22,3],[11,22,33]]

> mapM print [1,2,3]
1
2
3
[(),(),()]
```

```
> mapM_ print [1,2,3]
1
2
3
```

foldM

(Control.Monad)

```
foldM              :: (Monad m) => (a -> b -> m a) -> a -> [b] -> m a
foldM _ a []       = return a
foldM f a (x:xs)   = f a x >>= \y -> foldM f y xs
```

foldM f a_1 [x_1, …, x_n] =
    do {
        a_2 <- f a_1 x_1;
        a_3 <- f a_2 x_2;
        …
        a_n <- f a_{n-1} x_{n-1};
        return f a_n x_n }

```
> foldM (\y -> \x ->
        do { print (show x++"..."++ show y);
             return (x*y)})
     1 [1..10]
???
```

```
> foldM (\y -> \x ->  do print (show x++"..."++ show y); return (x*y))  1 [1..10]
???
```

```
newtype Either a b = Right a | Left b
instance (Error e) => Monad (Either e) where
    return x = Right x
    Right x >>= f = f x
    Left err >>= f = Left err
    fail msg = Left (strMsg msg)
```

Error monad

```
data Term = Con Int | Div Term Term deriving(Show, Read, Eq)

eval          :: Term -> Either String Int
eval(Con a)    = return a
eval(Div t u) = do
                    valT <- eval t
                    valU <- eval u
                    if valU == 0 then
                        fail "div by zero"
                    else
                        return (valT `div` valU)
> eval (Div (Con 1972) (Con 23))
Right 85
> eval (Div (Con 1972) (Con 0))
*** Exception: div by zero
```

Writer monad
(Control.Monad.Writer)

```
newtype Writer w a = Writer { runWriter :: (a, w) }
instance (Monoid w) => Monad (Writer w) where
      return x = Writer (x, mempty)
      (Writer (x,v)) >>= f =
         let (Writer (y, v')) = f x in Writer (y, v `mappend` v')
```

```
data Term = Con Int | Div Term Term deriving(Show, Read, Eq)

out :: Int -> Writer [String] Int
out x = writer (x, ["number: " ++ show x])

eval          :: Term -> Writer [String] Int
eval(Con a)   = out a
eval(Div t u) = do
                  valT <- eval t
                  valU <- eval u
                  out (valT `div` valU)
                  return (valT `div` valU)
```

```
> eval (Div (Con 1972) (Con 23))
WriterT (Identity (85,["number: 1972","number: 23","number: 85"]))
> runWriter  $ eval (Div (Con 1972) (Con 23))
(85,["number: 1972","number: 23","number: 85"])
```

Writer monad

(Control.Monad.Writer)

```
-- tell :: MonadWriter w m => w -> m ()

gcd' :: Int -> Int -> Writer [String] Int
gcd' a b  | b == 0 = do
                tell ["result " ++ show a]
                return a
          | otherwise = do
                tell [show a ++ " mod " ++ show b ++ " = " ++ show (a `mod` b)]
                gcd' b (a `mod` b)
```

```
> gcd' 18 12
WriterT (Identity (6,["18 mod 12 = 6","12 mod 6 = 0","result 6"]))

> runWriter (gcd' 2016 48)
(48,["2016 mod 48 = 0","result 48"])

> mapM putStrLn (snd $ runWriter (gcd' 2016 48))
2016 mod 48 = 0
result 48
[(),()]
```

State monad

(Control.Monad.State)

```
newtype State s a = State { runState :: (s -> (a,s)) }

instance Monad (State s) where
   return a          = State \s -> (a,s)
   (State x) >>= f = State \s ->
                              let (v,s') = x s in runState (f v) s',

class (Monad m) => MonadState s m | m -> s where
   get :: m s                              -- get vráti stav z monády
   put :: s -> m ()                        -- put prepíše stav v monáde

modify :: (MonadState s m) => (s -> s) -> m ()
modify f = do    s <- get
                 put (f s)
```

newtype

newtype State s a = State { runState :: (s -> (a,s)) }

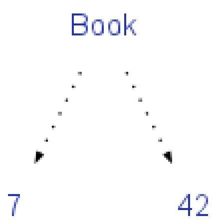
State s a má rovnakú reprezentáciu ako (s -> (a,s))

data State s a = State { runState :: (s -> (a,s)) }

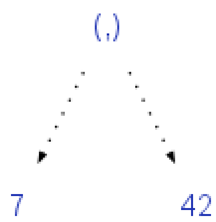
State s a je reprezentovaná krabicou State s pointrom na (s -> (a,s))

Príklad:

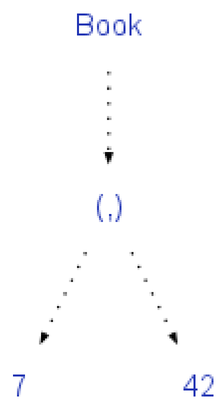
data Book = Book Int Int



newtype Book = Book (Int, Int)



data Book = Book (Int, Int)



http://stackoverflow.com/questions/5889696/difference-between-data-and-newtype-in-haskell

```
pop :: State Stack Int
pop = state(\(x:xs) -> (x,xs))

push :: Int -> State Stack ()
push a = state(\xs -> ((),a:xs))
```

State Stack

```
type Stack = [Int]

pushAll :: Int -> State Stack String
pushAll 0    = return ""
pushAll n    = do
             push n
             str <- pushAll (n-1)
             nn <- pop
             return (show nn ++ str)
```

```
> evalState (pushAll 10) []
"10987654321"
> execState (pushAll 10) []
[]
```

```
type Stack = [Int]

pushAll' :: Int -> State Stack String
pushAll' 0    = return ""
pushAll' n    = do
             stack <- get  -- push n
             put (n:stack)
             str <- pushAll (n-1)
             (nn:stack') <- get  -- nn <- pop
             put stack'
             return (show nn ++ str)
```

```
> evalState (pushAll' 10) []
"10987654321"
> execState (pushAll' 10) []
[]
```

Preorder so stavom

(Control.Monad.State)

```
data Tree a =      Nil |
                   Node a (Tree a) (Tree a)
                   deriving (Show, Eq)

preorder :: Tree a -> State [a] ()              -- stav a výstupná hodnota
preorder Nil                          = return ()
preorder (Node value left right)      =
                                      do {
                                          str<-get; -- get state=preorderlist
                                          put (value:str);  -- modify (value:)
                                          preorder left;
                                          preorder right;
                                          return () }
```

```
e :: Tree String
e = Node "c" (Node "a" Nil Nil) (Node "b" Nil Nil)

> execState (preorder e) []
["b","a","c"]

> evalState (preorder e) []
()
```

Prečíslovanie binárneho stromu

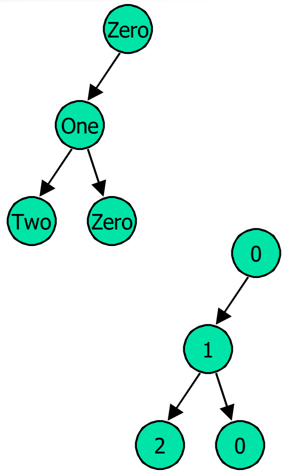
```
index :: Tree a -> State Int (Tree Int)          -- stav a výstupná hodnota
index Nil              = return Nil
index (Node value left right) =
                    do {
                              i <- get;
                              put (i+1);
                              ileft <- index left;
                              iright <- index right;
                              return (Node i ileft iright) }
```

```
> e'
Node "d" (Node "c" (Node "a" Nil Nil) (Node "b" Nil Nil)) (Node "c" (Node "a" Nil
    Nil) (Node "b" Nil Nil))

> evalState (index e') 0
Node 0 (Node 1 (Node 2 Nil Nil) (Node 3 Nil Nil)) (Node 4 (Node 5 Nil Nil) (Node 6
    Nil Nil))

> execState (index e') 0
7
```

Prečíslovanie stromu 2




```
type Table a = [a]

numberTree :: Eq a => Tree a -> State (Table a) (Tree Int)
numberTree Nil                   = return Nil
numberTree (Node x t1 t2)        =  do    num <- numberNode x
                                          nt1 <- numberTree t1
                                          nt2 <- numberTree t2
                                          return (Node num nt1 nt2)
   where
   numberNode :: Eq a => a -> State (Table a) Int
   numberNode x                  = do     table <- get
                                          (newTable, newPos) <- return (nNode x table)
                                          put newTable
                                          return newPos

   nNode::  (Eq a) => a -> Table a -> (Table a, Int)
   nNode x table                 = case (findIndexInList (== x) table) of
                                          Nothing -> (table ++ [x], length table)
                                          Just i  -> (table, i)
```

Prečíslovanie stromu 2

```
numTree :: (Eq a) => Tree a -> Tree Int
numTree t = evalState (numberTree t) []
```

```
> numTree ( Node "Zero"
                (Node "One" (Node "Two" Nil Nil)
                (Node "One" (Node "Zero" Nil Nil) Nil)) Nil)

Node 0 (Node 1 (Node 2 Nil Nil) (Node 1 (Node 0 Nil Nil) Nil)) Nil
```