

Haskell

A Purely Functional Language

featuring static typing, higher-order functions,
polymorphism, type classes and monadic effects

Funkcie a funkcionály

referečná transparentosť

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I-18

<http://dai.fmph.uniba.sk/courses/FPRO/>





Zoznamová rekurzia

```
-- vyber prvých n prvkov zo zoznamu
take      :: Int -> [a] -> [a]
take 0 _   = []
take _ []  = []
take n (x:xs) = x : (take (n-1) xs)
```

```
-- dĺžka zoznamu
length    :: [a] -> Int
length [] = 0
length (x:xs) = 1 + length xs
```

Hypotéza (pre ľubovoľné n a xs) platí:

- $\text{length (take } n \text{ xs)} = n$
- $\text{length } \$ \text{ take } n \text{ xs} = n$
- $(\text{length} . \text{take } n) \text{ xs} = n$

-- dolárová notácia

-- kompozícia funkcií z matematike .

```
"?: " take 5 [1,3..100]
[1,3,5,7,9]
"?: " length (take 5 [1,3..100])
5
"?: " length $ take 5 [1,3..100]
5
```



Dôkaz - $\text{length} (\text{take } n \text{ } xs) = n$

(matematická indukcia)

Indukcia (vzhľadom na dĺžku/štruktúru xs):

- **$xs = []$**

$$\text{length} (\text{take } n []) = 0$$

$$0 = 0$$

č.b.t.d.

- **$xs = (y:ys)$**

$$\text{length} (\text{take } n (y:ys)) = n$$

$$\text{length} (y:\text{take } (n-1) ys) = n$$

$$1 + \text{length} (\text{take } (n-1) ys) = n$$

indukčný predpoklad, $|ys| < |xs|$

$$1 + (n-1) = n$$

č.b.t.d.

Definície z predošlej strany:

$\text{take} \quad \quad \quad :: \text{Int} \rightarrow [a] \rightarrow [a]$

$\text{take } 0 \quad \quad = []$

$\text{take } _ [] = []$

$\text{take } n (x:xs) = x : \text{take } (n-1) xs$

$\text{length} \quad \quad :: [a] \rightarrow \text{Int}$

$\text{length} [] = 0$

$\text{length } (x:xs) = 1 + \text{length } xs$



QuickCheck

Elegantný nástroj na testovanie (!!! nie dôkaz !!!) hypotéz

```
"?: " import Test.QuickCheck
```

```
"?: " quickCheck (\(xs,n) -> length (take n xs) == n)
```

```
*** Failed! Falsifiable (after 2 tests and 1 shrink):
```

```
"?: " verboseCheck (\(xs,n) -> length (take n xs) == n)
```

```
Passed:
```

```
([],0)
```

```
Passed:
```

```
([()],1)
```

```
Failed:
```

```
([],-1)
```

```
*** Failed! Failed:
```

Neplatí to pre n záporne, lebo napr. `take (-3) [1..100] = []`,

resp. naša definícia nepokrýva prípad $n < 0$

!!! ALE MY SME TO AJ TAK "DOKÁZALI"... !!!





QuickCheck

Podmienka: miesto písania

if n >= 0 then length (take n s) == n **else True**

Napíšeme pre-condition pomocou ==>

"?: " verboseCheck (\(xs,n) -> **n>=0** ==> length (take n xs) == n)

Passed:

([],0)

Failed:

([()],2)

Neplatí to pre ak length xs < n ☹️

"?: " quickCheck (\(xs,n) -> **n>=0 && length xs >= n** ==>

length (take n xs) == n)

*** Gave up! Passed only 35 tests.



Tvrdenie sme **overili** na niekoľkých prípadoch, ale to **nie je dôkaz**.

V dôkaze môžeme urobiť chybu (ako na slajde 2), QuickCheck slúži ako nástroj na hľadanie/odhaľovanie kontrapríkladov, kedy naše tvrdenie neplatí.

Don't write tests!

Generate them
from properties



QuickCheck

- miesto písania unit testov, quickcheck vám ich (nejaké) vygeneruje
- vy potom nepíšete testy, ale vlastnosti vašich programov.

O niečom podobnom dávno snívali/dúfali Hoare, Dijkstra, ...

- s rozdielom, že vlastnosti programov chceli dokázať,
- miesto hľadania kontrapríkladu.

Quickcheck:

- generuje náhodné vstupné hodnoty, pre základné aj definované typy
 - Int, Bool, ...
 - [Int], String, ...
 - Int->Int, Int->Bool
- ak nájde kontrapríklad (už vieme, že to neplatí), snaží sa ho zminimalizovať/zjednodušiť, napr: `length (take n xs) == n` neplatí pre `length (take 21 [5,-192,3981,-291,2220,-192,22,12,-192,-1]) == 21`

Don't write tests!

Generate them
from properties



QuickCheck

autori: [Koen Claessen](#), [John Hughes](#)

Príklad Parretovho pravidla 20:80 - za 20% energie chytíte 80% problémov

Príklad (viac [tu](#)):

Collatz (viac [tu](#)) je funkcia $f(n) = \text{if } n \bmod 2 == 0 \text{ then } n/2 \text{ else } 3n+1$.

```
f      :: Integer -> Integer
f n    | even(n) = n `div` 2
      | odd(n)  = 3*n + 1
collatz :: Integer -> Bool
collatz 1 = True
collatz n = collatz (f n)
```

```
"?: " quickCheck (\n -> n > 0 ==> collatz(n))
+++ OK, passed 100 tests.
"?: " quickCheckWith stdArgs{ maxSuccess = 100000 }
      (\n -> n > 0 ==> collatz(n))
+++ OK, passed 100000 tests.
```

[Paul Erdős](#): "Mathematics may not be ready for such problems." offered \$500 for its solution.

Kvíz - platí/neplatí ?

(neseriózny prístup ale intuíciu treba tiež trénovať)

- `length [m..n] == n-m+1` 😞
"?: " `quickCheck ((\ (n,m) -> length [m..n] == n-m+1))`
*** Failed! Falsifiable (after 3 tests and 1 shrink):
"?: " `quickCheck ((\ (n,m) -> m <= n ==> length [m..n] == n-m+1))` 😊
+++ OK, passed 100 tests.
- `length (xs ++ ys) == length xs + length ys` 😊
"?: " `quickCheck((\xs->\ys->(length (xs++ys)==length xs + length ys)))`
+++ OK, passed 100 tests.
- `length (reverse xs) == length xs` 😊
`quickCheck((\xs -> (length (reverse xs) == length xs)))`
+++ OK, passed 100 tests.
- `(xs, ys) == unzip (zip xs ys)` 😞
`quickCheck((\xs -> \ys -> ((xs, ys) == unzip (zip xs ys))))`
*** Failed! Falsifiable (after 3 tests and 1 shrink):
`quickCheck((\xs -> \ys -> (length xs == length ys ==>`
`(xs, ys) == unzip (zip xs ys))))` 😊



Funkcia/predikát argumentom

- zober zo zoznamu tie prvky, ktoré spĺňajú podmienku (test)
Booleovská podmienka príde ako argument funkcie a má typ $(a \rightarrow \text{Bool})$:

`filter` $:: (a \rightarrow \text{Bool}) \rightarrow [a] \rightarrow [a]$

`filter p xs` $= [x \mid x \leftarrow xs, p\ x]$

alternatívna definícia:

`filter p []` $= []$

`filter p (x:xs)` $= \text{if } p\ x \text{ then } x:(\text{filter } p\ xs) \text{ else } \text{filter } p\ xs$

**> filter even [1..10]
[2,4,6,8,10]**

vlastnosti (zväčša úplne zrejmé ?):

- `filter True xs` $= xs$... $[x \mid x \leftarrow xs, \text{True}] = [x \mid x \leftarrow xs] = xs$
- `filter False xs` $= []$... $[x \mid x \leftarrow xs, \text{False}] = []$
- `filter p1 (filter p2 xs)` $= \text{filter } (p1 \ \&\& \ p2) \ xs$
- `(filter p1 xs) ++ (filter p2 xs)` $= \text{filter } (p1 \ || \ p2) \ xs$

$$\begin{aligned}\text{filter } p \ [] &= [] \\ \text{filter } p \ (x:xs) &= \text{if } p \ x \text{ then } x:(\text{filter } p \ xs) \text{ else } \text{filter } p \ xs\end{aligned}$$

Dôkaz

$\text{filter } p1 \ (\text{filter } p2 \ xs) = \text{filter } (p1 \ \&\& \ p2) \ xs$

Indukcia vzhľadom na parameter xs

- []
L.S. = $\text{filter } p1 \ (\text{filter } p2 \ []) = \text{filter } p1 \ [] = [] = \text{filter } (p1 \ \&\& \ p2) \ [] = \text{P.S.}$
- (x:xs)
L.S. = $\text{filter } p1 \ (\text{filter } p2 \ (x:xs)) = \dots \text{definícia}$
 $\text{filter } p1 \ (\text{if } p2 \ x \text{ then } x:(\text{filter } p2 \ xs) \text{ else } \text{filter } p2 \ xs) = \dots \text{filter dnu cez if}$
 $\text{if } p2 \ x \text{ then } \text{filter } p1 \ (x:(\text{filter } p2 \ xs)) \text{ else } \text{filter } p1 \ (\text{filter } p2 \ xs) = \dots \text{indukcia}$
 $\text{if } p2 \ x \text{ then } \text{filter } p1 \ (x:(\text{filter } p2 \ xs)) \text{ else } \text{filter } (p1 \ \&\& \ p2) \ xs = \dots \text{definícia}$
 $\text{if } p2 \ x \text{ then}$
 $\quad \text{if } p1 \ x \text{ then } x:(\text{filter } p1 \ (\text{filter } p2 \ xs)) \text{ else } \text{filter } p1 \ (\text{filter } p2 \ xs)$
 $\text{else } \text{filter } (p1 \ \&\& \ p2) \ xs = \dots \text{2 x indukcia}$
 $\text{if } p2 \ x \text{ then}$
 $\quad \text{if } p1 \ x \text{ then } x:(\text{filter } (p1 \ \&\& \ p2) \ xs) \text{ else } \text{filter } (p1 \ \&\& \ p2) \ xs$
 $\text{else } \text{filter } (p1 \ \&\& \ p2) \ xs =$

$\text{filter } p [] = []$
 $\text{filter } p (x:xs) = \text{if } p \ x \text{ then } x:(\text{filter } p \ xs) \text{ else } \text{filter } p \ xs$

Dôkaz

$\text{filter } p1 (\text{filter } p2 \ xs) = \text{filter } (p1 \ \&\& \ p2) \ xs$

if $p2 \ x$ then

if $p1 \ x$ then $x:(\text{filter } (p1 \ \&\& \ p2) \ xs)$ else $\text{filter } (p1 \ \&\& \ p2) \ xs$

else $\text{filter } (p1 \ \&\& \ p2) \ xs = \dots$ **požívame vlastnosť if-then-else**

if A then

if $A \ \&\& \ B$ then C

if B then C

else D

else D

else D

if $(p1 \ \&\& \ p2) \ x$ then $x:(\text{filter } (p1 \ \&\& \ p2) \ xs)$ else $\text{filter } (p1 \ \&\& \ p2) \ xs = \dots$ **def.**

$\text{filter } (p1 \ \&\& \ p2) \ (x:xs) = \text{P.S.}$

č.b.t.d.



QuickCheck a funkcie

Funkcie sú hodnoty ako každé iné
Ako vie QuickCheck pracovať s funkciami ?

- je skladanie funkcií komutatívne ?

```
"?: " import Text.Show.Functions
```



```
"?: " quickCheck(
```

```
  (\x -> \f -> \g -> (f.g) x == (g.f) x)::Int->(Int->Int)->(Int->Int)->Bool)
```

```
*** Failed! Falsifiable (after 2 tests):
```

- je skladanie funkcií asociatívne ?

```
"?: " quickCheck(
```

```
  (\x -> \f -> \g -> \h -> (f.(g.h)) x == ((f.g).h) x)
  ::Int->(Int->Int)->(Int->Int)->(Int->Int)->Bool)
```



```
+++ OK, passed 100 tests.
```

Opäť to NIE je DÔKAZ, len 100 pokusov.

QuickCheck a predikáty

Predikát je len funkcia s výsledným typom Bool

- `filter p1 (filter p2 xs) = filter (p1 && p2) xs`



?: " quickCheck (\xs -> \p1 -> \p2 ->

filter p1 (filter p2 xs) == filter (p1 && p2) xs)

:: [Int] -> (Int->Bool) -> (Int->Bool) -> Bool)

<interactive>:113:91: Couldn't match expected type 'Bool' ---

NEPLATÍ LEBO ANI TYPY NESEDIA, && je definovaný na Bool, a nie na funkciách Int->Bool

- `filter p1 (filter p2 xs) = filter (\x-> p1 x && p2 x) xs`



+++ OK, passed 100 tests.

Opäť to NIE je DÔKAZ (ten už bol), len 100 pokusov.

- `(filter p1 xs) ++ (filter p2 xs) = filter (\x -> p1 x || p2 x) xs`

"?: " quickCheck (\xs -> \p1 -> \p2 ->

(filter p1 xs) ++ (filter p2 xs) == filter (\x -> p1 x || p2 x) xs)

:: [Int] -> (Int->Bool) -> (Int->Bool) -> Bool)

*** Failed! Falsifiable (after 3 tests):

[0] <function> <function>

Funkcia argumentom

map

- funktor, ktorý aplikuje funkciu (1.argument) na všetky prvky zoznamu

`map` $:: (a \rightarrow b) \rightarrow [a] \rightarrow [b]$

`map f []` $= []$

`map f (x:xs)` $= f\ x : \text{map } f\ xs$

`map f xs` $= [f\ x \mid x \leftarrow xs]$

- Príklady:

`map (+1) [1,2,3,4,5]` $= [2,3,4,5,6]$

`map odd [1,2,3,4,5]` $= [\text{True}, \text{False}, \text{True}, \text{False}, \text{True}]$

`and (map odd [1,2,3,4,5])` $= \text{False}$

`map head [[1,0,0], [2,1,0], [3,0,1]]` $= [1, 2, 3]$

`map tail [[1,0,0], [2,1,0], [3,0,1]]` $= [[0,0], [1,0], [0,1]]$

`map (0:) [[1],[2],[3]]` $= [[0,1],[0,2],[0,3]]$



Vlastnosti map

- $\text{map id xs} = \text{xs}$ ☒ $\text{map id} = \text{id}$
- $\text{map (f.g) xs} = \text{map f (map g xs)}$ ☒ $\text{map f} . \text{map g} = \text{map (f.g)}$
- ~~■ $\text{head (map f xs)} = \text{f (head xs)}$ ☒ ~~$\text{head} . \text{map f} = \text{f} . \text{head}$~~~~
- ~~■ $\text{tail (map f xs)} = \text{map f (tail xs)}$ ☒ ~~$\text{tail} . \text{map f} = \text{map f} . \text{tail}$~~~~
- $\text{map f (xs ++ ys)} = \text{map f xs} ++ \text{map f ys}$ ☒
- $\text{length (map f xs)} = \text{length xs}$ ☒ $\text{length} . \text{map f} = \text{length}$
- $\text{map f (reverse xs)} = \text{reverse (map f xs)}$ ☒ $\text{map f} . \text{reverse} = \text{reverse} . \text{map f}$
- ~~■ $\text{sort (map f xs)} = \text{map f (sort xs)}$ ☒ ~~$\text{sort} . \text{map f} = \text{map f} . \text{sort}$~~~~
- $\text{map f (concat xss)} = \text{concat (map (map f) xss)}$ ☒

$\text{map f} . \text{concat} = \text{concat} . \text{map (map f)}$

$\text{concat} :: [[a]] \rightarrow [a]$

$\text{concat []} = []$



$\text{concat (xs:xss)} = \text{xs} ++ \text{concat xss}$

$\text{concat} [[1], [2,3], [4,5,6], []] = [1,2,3,4,5,6]$



Vlastnosti map, filter

Na zamyslenie:

- `filter p (map f xs)` = `???` `(filter (p.f) xs)` 
- `filter p (map f xs)` = `map f (filter (p.f) xs)` 
- `filter p . map f` = `map f . filter (p.f)`

Dôkaz:

`filter p (map f xs)`
= `filter p [f x | x<-xs]`
= `[y | y <- [f x | x<-xs], p y]`
= `[f x | x<-xs, p (f x)]`
= `map f [x | x<-xs, p (f x)]`
= `map f (filter (p.f))`



Quíz - prémia

nájdite pravdivé a zdôvodnite

- $\text{map } f . \text{take } n = \text{take } n . \text{map } f$
- $\text{map } f . \text{filter } p = \text{map } \text{fst} . \text{filter } \text{snd} . \text{map } (\text{fork } (f,p))$
where $\text{fork} :: (a \rightarrow b, a \rightarrow c) \rightarrow a \rightarrow (b,c)$
 $\text{fork } (f,g) x = (f x, g x)$
- $\text{filter } (p . g) = \text{map } (\text{inverzna_g}) . \text{filter } p . \text{map } g$
ak $\text{inverzna_g} . g = \text{id}$
- $\text{reverse} . \text{concat} = \text{concat} . \text{reverse} . \text{map } \text{reverse}$
- $\text{filter } p . \text{concat} = \text{concat} . \text{map } (\text{filter } p)$



QuickSort s QuickCheck

(podobné bolo na cvičeniach)

```
import Test.QuickCheck
```

```
import Data.List (sort)
```

```
qsort :: Ord a => [a] -> [a]
```

-- Ord a – vieme triediť len porovnateľné typy

```
qsort [] = []
```

-- analógia interface Comparable<a>

```
qsort (p:xs) = qsort (filter (< p) xs) ++ [p] ++ qsort (filter (>= p) xs)
```

```
quickCheck(\xs -> length (qsort xs) == length xs)
```

```
quickCheck((\xs -> length (qsort xs) == length xs)::[Int]->Bool)
```

```
quickCheck((\xs -> qsort xs == sort xs)::[Int]->Bool)
```

```
quickCheck((\xs -> qsort(qsort xs) == qsort xs)::[Int]->Bool)
```

```
isSorted :: Ord a => [a] -> Bool
```

```
isSorted xs = sort xs == xs
```

```
isSorted' :: Ord a => [a] -> Bool
```

```
isSorted' [] = True
```

```
isSorted' xs = and $ zipWith (<=) (init xs) (tail xs)
```

```
quickCheck((\xs -> isSorted (qsort xs))::[Int]->Bool)
```

```
quickCheck((\xs -> isSorted' (qsort xs))::[Int]->Bool)
```



Kombinatorika

(podobné nájdete v Prémii QC & Kombinatorika)

```
module Kombinatorika where
import Test.QuickCheck
import Data.List
```

```
fact n = product [1..n]
```

```
comb n k = (fact n) `div` ((fact k) * (fact (n-k)))
```

```
-- permutácie
```

```
perms :: [t] -> [[t]]
```

```
perms [] = [[]]
```

```
perms (x:xs) = [ insertInto x i ys | ys <- perms xs, i <- [0..length ys] ]
```

```
    where insertInto x i xs = (take i xs) ++ (x:drop i xs)
```

```
qchPERM = quickCheck(\n -> (n > 0 && n < 10) ==> length (perms [1..n]) == fact n)
```

```
kbo :: [t] -> Int -> [[t]]
```

```
kso :: [t] -> Int -> [[t]]
```

```
vbo :: (Eq t) => [t] -> Int -> [[t]]
```

```
vso :: [t] -> Int -> [[t]]
```

?

$n!$

$(n \text{ nad } k)$

$((n+k-1) \text{ nad } k)$

$n.(n-1). \dots .(n-k+1)$

Deliteľnosť 11

(bolo na cvičeniach)

- SK67 8360 5207 0042 0002 6991
- 6783605207004200026991 = 11 * 616691382454927275181
- Rodné číslo (.cz, .sk) je deliteľné 11

oneStep :: Integer -> Integer

oneStep = \n -> abs \$

~~uncurry (-) \$ foldr (\c > \ (sp,sn) -> (c+sn, sp)) (0,0) \$~~
map (`mod` 10) \$ takeWhile (>0) \$ iterate (`div` 10) n

JaroF:

foldr (-) 0 \$

allSteps :: Integer -> Bool

allSteps = \n -> 0 == (head \$ dropWhile (>9) \$ iterate oneStep n)

qch1 = quickCheck(\n -> (n>0) ==> allSteps n == (n `mod` 11 == 0))

***Eleven**> qch1

+++ OK, passed 100 tests.

Rule for Divisibility by 11

10,813?

10,813

1+8+3= 12

0+1= 1

12 - 1 = 11

11 ÷ 11





BiLandia

(Hejného metóda)

```
pocetMoznosti 0 = 0 -- Martina
pocetMoznosti 1 = 1
pocetMoznosti n | n `mod` 2 == 0 = pocetMoznosti (n `div` 2) + pocetMoznosti (n-1)
                | otherwise = pocetMoznosti (n-1)
```

```
pocetMoznosti' 0 = 1 -- Jarka
pocetMoznosti' 1 = 1
pocetMoznosti' x | x `mod` 2 == 1 = pocetMoznosti' (x-1)
                | otherwise = (pocetMoznosti' (x-2)) + (pocetMoznosti' (x `div` 2))
```

```
qch = quickCheck(\n -> (0<=n && n <= 1000) ==> pocetMoznosti n == pocetMoznosti' n) -- failed: (
```

```
qch1 = quickCheck(\n -> (0<n && n <= 1000) ==> pocetMoznosti n == pocetMoznosti' n) -- passed
```

```
pocetMoznosti'' 0 = 1 -- Samo
```

```
pocetMoznosti'' n = sum (map pocetMoznosti'' [0..(div n 2)])
```

```
pocetMoznosti''' 0 = 1 -- and The Winner is: Jakub
```

```
pocetMoznosti''' n = sum [pocetMoznosti''' x | x <- [0..n `div` 2]]
```

```
qch2 = quickCheck(\n -> (0<n && n <= 1000) ==> pocetMoznosti n == pocetMoznosti'' n) -- passed
```



- Introduction to QuickCheck
[https://wiki.haskell.org/Introduction to QuickCheck2](https://wiki.haskell.org/Introduction_to_QuickCheck2)
- Introduction to QuickCheck by example: Number theory and Okasaki's red-black trees
<http://matt.might.net/articles/quick-quickcheck/>
- K.Claessen, J.Hughes:QuickCheck:A Lightweight Tool for Random Testing of Haskell Programs
<https://www.eecs.northwestern.edu/~robby/courses/395-495-2009-fall/quick.pdf>
- A QuickCheck Tutorial: Generators
<https://www.stackbuilders.com/news/a-quickcheck-tutorial-generators>



Rekapitulácia

videli sme tzv. **Property Based Testing** pomocou **QuickCheck**:

- najznámejšie dva funkcionály: map, filter – ktoré poznáte aj z Pythonu
- quickCheck náhodne generujúci testy/kontrapríklady pre typy
 - základné typy: Int, Bool, String...
 - zoznamy: [Int], [t]
 - funkcie: Int->Int, a->b, ...
- množstvo 'ekvivalentných' tvrdení, niektoré boli neekvivalentné...

Property Based Testing (PBT):

- rôzne implementácie QuickCheck v jazykoch:
 - Scala (Scala Check), F# (FsCheck), Clojure (test.check), Python (Hypothesis)
- musí implementovať:
 - generovanie dát pre základné typy, parametrické typy, funkčné typy, ...
 - generovanie dát pre používateľom definované typy
 - zjednodušovanie kontrapríkladu (shrinking)



Definované typy

Ak definujeme vlastnú dátovú štruktúru, ako využiť quickCheck ?

```
data BVS t = Nil | Node (BVS t) t (BVS t) deriving(Show, Eq)
```

- dva konštruktory **Nil** a **Node** _ _ _
- deriving popisuje patričnosť do triedy class - (resp. implements interface)
 - Show – automaticky vygenerovaná funkcia show :: BVS t ->String
 - Eq – automaticky vygenerované funkcie ==,/= :: BVS t -> BVS t -> Bool

Ako definovať funkciu, ktorá vracia náhodný strom, napr. BVS Int ?

Existuje nejaká náhodná funkcia, napr. nextInt :: Int ?

Nie je to v rozpore s Referenčnou transparentnosťou ?

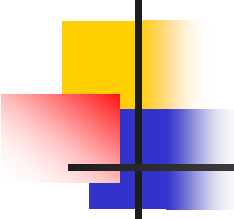


Java a Reflexivita

(malá odbočka)

Skúsme si tú istú otázku preformulovať v Jave, ktorú poznáme

- Napíšte funkciu, ktorá vytvorí náhodnú inštanciu ľubovoľnej triedy
Object gener(String className)
- Nechceme mať náhodný generátor pre každú triedu, lebo pre nami definované triedy by sme ho museli písať sami...
- Reflexivita (Java Reflection Model), od slajdu 11
- https://github.com/Programovanie4/Prednasky/blob/master/13/13_java.pdf
- java primitívne typy (int, char, double, ...), String...
- polia (int[], ...)
- triedy s default konštruktorom (Stvorec(), ...)
- triedy s konštruktorom s parametrami – rekurzívne pre každý parameter konšuktora, potom zavolanie konšuktora s náhodnými parametrami
- generické triedy



QuickCheck – Generátor

(pre základné typy)

- trieda Arbitrary t definuje generátor Gen t pre hodnoty typu t:
class Arbitrary a where
 arbitrary :: Gen t
a volá sa pomocou funkcie generate :: Gen t -> IO t

Pre preddefinované typy to už niekto zdefinoval:

"?: " (generate arbitrary) :: IO Int	23, 45, 12, 49, 12, ...
"?: " generate arbitrary :: IO Char	't','w', '\199', ...
"?: " generate arbitrary :: IO (Char, Int)	('6',0), ('<','-7)
"?: " generate arbitrary :: IO [Int]	[-29,-17,10], [-10,9]
"?: " generate arbitrary :: IO Double	-5.5026813
"?: " generate arbitrary :: IO Bool	True, False, False
"?: " do { fst <- generate arbitrary::IO Int; snd <- generate arbitrary::IO Char; return (fst, snd) }	(-6, 'r'), (15, 'a'), ...



QuickCheck – Generátor

(pre funkčné typy)

```
"?: " generate arbitrary :: IO (Int->Int) <function>
```

```
"?: " do {f<-generate arbitrary :: IO (Integer->Integer); return (f 7)} 9, 11
```

```
"?: " do {  
    f<-generate arbitrary :: IO (Integer->Integer);  
    g<-generate arbitrary :: IO (Integer->Integer);  
    x<-generate arbitrary :: IO Integer;  
    return (((f.g) x) == ((g.f) x)) } False, False, False, True
```

```
"?: " do {  
    f<-generate arbitrary :: IO (Integer->Integer);  
    g<-generate arbitrary :: IO (Integer->Integer);  
    h<-generate arbitrary :: IO (Integer->Integer);  
    x<-generate arbitrary :: IO Integer;  
    return (((((f.g).h) x) == (((f.g).h) x)) } True, True, True, True
```



Generátory

(pre definované typy)

```
kocka :: Gen Int
```

```
kocka = choose(1,6)
```

```
-- "?: " generate kocka
```

```
-- "?: " generate (choose(1,10))
```

```
yesno :: Gen Bool
```

```
yesno = choose(True, False)
```

```
-- "?: " generate yesno
```

```
-- "?: " generate (choose(True, False))
```

```
data Minca = Hlava | Panna deriving (Show)
```

```
instance Arbitrary Minca where
```

```
    arbitrary = oneof [return Hlava, return Panna]
```

Pre nami definované typy
XXX musíme definovať
inštanciu triedy Arbitrary XXX

```
"?: " generate (arbitrary::Gen Minca)
```

```
"?: " (generate arbitrary)::IO Minca
```

```
falosnaMinca :: Gen Minca
```

```
falosnaMinca = frequency [(1,return Hlava), (2,return Panna)]
```

```
-- "?: " generate falosnaMinca
```



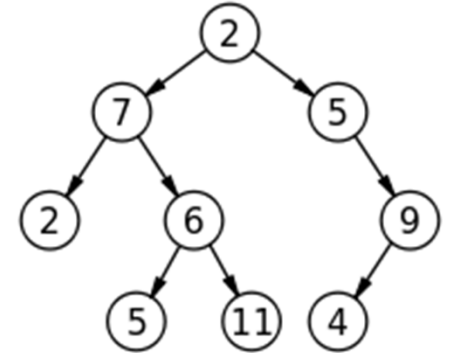
Generátory - zoznam

```
arbitraryListMax8Len :: Arbitrary a => Gen [a]    -- náhodný zoznam len <= 8
arbitraryListMax8Len =                          "?: " generate (arbitraryListMax8Len::Gen [Int])
  do {                                           [-21,12,17,16,4,-20]
    k <- choose (0, 8)::(Gen Int);
    sequence [ arbitrary | _ <- [1..k] ] }
```

```
arbitraryList :: Arbitrary a => Gen [a]
arbitraryList =                                "?: " generate (arbitraryList::Gen [Int])
  mysized ( \n -> do {                          [-9,7,14,24,18,28,-4,0,22,12,-14]
    k <- choose (0, n) ;
    sequence [ arbitrary | _ <- [1..k] ] }
  )
```

```
mysized :: (Int -> Gen a) -> Gen a              "?: " generate
mysized f = f 50                               (mysized (\n -> choose(n,n)))
50
```

Generátory - strom



```
data Tree t = Leaf t | Node (Tree t) t (Tree t)
    deriving (Show, Ord, Eq)
```

```
instance Arbitrary a => Arbitrary (Tree a) where
    arbitrary = frequency
```

```
    [
        (1, liftM Leaf arbitrary )           "?: " generate (arbitrary :: Gen (Tree Int))
        , (1, liftM3 Node arbitrary arbitrary arbitrary)
    ]
```

```
strom :: Gen (Tree Int)
strom = frequency [
    (1, liftM Leaf arbitrary )           "?: " generate strom
    , (10, liftM3 Node arbitrary arbitrary arbitrary)
    ]
Node (Node (Leaf (-2)) 3 (Leaf (-6))) 23 (Leaf 22)
```



BVS – binárny vyhľadávací

```
data BVS t = Nil | Node (BVS t) t (BVS t) deriving(Show, Ord, Eq)
```

```
-- je binárny vyhľadávací strom
```

```
isBVS          :: (Ord t) => BVS t -> Bool          -- t vieme porovnávať <
```

```
-- nájsť v binárnom vyhľadávacom strome
```

```
find           :: (Ord t) => t -> (BVS t) -> Bool -- analógia Comparable<t>
```

```
find _ Nil     = False
```

```
find x (Node left value right) | x == value = True  
                                | x < value  = find x right  
                                | x > value  = find x left
```

```
flat           :: BVS t -> [t]          -- opäť riešenie Jara/Turbo
```

```
flat Nil       = []
```

```
flat (Node left value right) = flat left ++ [value] ++ flat right
```



BVS - isBVS

Príšerne neefektívne riešenie, prepíšte lepšie:

```
isBVS :: (Ord t) => BVS t -> Bool
```

```
isBVS Nil = True
```

```
isBVS (Node left value right) =
```

```
    (all (<value) (flat left))
```

```
    &&
```

```
    (all (>value) (flat right))
```

```
    &&
```

```
    isBVS left
```

```
    &&
```

```
    isBVS right
```




BVS - testy

```
qch1 = verbose((\x -> \tree -> find x tree)::Int->(BVS Int)->Bool)
qch2 = quickCheck((\x -> \tree -> ((find x tree) == (elem x (flat tree))))
                ::Int->BVS Int->Bool)
```

```
{--
```

```
"?: " qch2
```

```
*** Failed! Falsifiable (after 3 tests):
```

```
1 ; Node Nil (-2) (Node Nil 1 Nil)
```

```
--}
```

```
qch3 = quickCheck((\x -> \tree -> (isBVS tree) ==>
                ((find x tree) == (elem x (flat tree))))::Int->BVS Int->Property)
```

```
{--
```

```
*** Failed! Falsifiable (after 2 tests):
```

```
0 ; Node (Node Nil (-1) (Node Nil 0 Nil)) 1 Nil
```

```
--}
```

KDE je chyba v definícii BVS ??

Don't write tests!

Generate them
from properties



BVS – tajnička

```
find :: (Ord t) => t -> (BVS t) -> Bool
```

```
find _ Nil = False
```

```
find x (Node left value right) | x == value = True
```

```
    | x < value = find x right
```

```
    | x > value = find x left
```

```
    | x < value = find x left
```

```
    | x > value = find x right
```

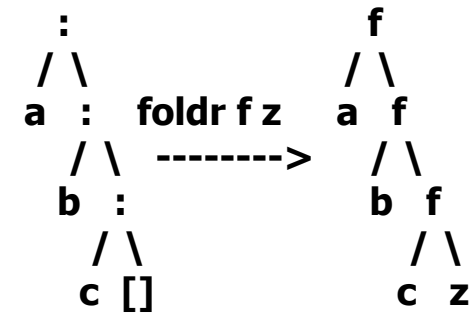
Haskell – foldr

`foldr` $:: (a \rightarrow b \rightarrow b) \rightarrow b \rightarrow [a] \rightarrow b$

`foldr f z []` = `z`

`foldr f z (x:xs)` = `f x (foldr f z xs)`

`a : b : c : []` $\rightarrow f\ a\ (f\ b\ (f\ c\ z))$



```
Main> foldr (+) 0 [1..100]
5050
```

```
Main> foldr (\x y->10*y+x) 0 [1,2,3,4]
4321
```

-- `g` je vnorená lokálna funkcia

```
foldr :: (a -> b -> b) -> b -> [a] -> b
foldr f z = g
  where g []      = z
        g (x:xs) = f x (g xs)
```



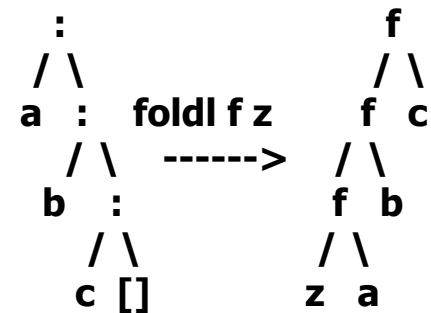
Haskell – foldl

`foldl` $:: (a \rightarrow b \rightarrow a) \rightarrow a \rightarrow [b] \rightarrow a$

`foldl f z []` = `z`

`foldl f z (x:xs)` = `foldl f (f z x) xs`

`a : b : c : []` \rightarrow `f (f (f z a) b) c`



```
Main> foldl (+) 0 [1..100]
5050
```

```
Main> foldl (\x y->10*x+y) 0 [1,2,3,4]
1234
```



Vypočítajte

- `foldr max (-999) [1,2,3,4]`
`foldl max (-999) [1,2,3,4]`
- `foldr (_ -> \y ->(y+1)) 0 [3,2,1,2,4]`
`foldl (\x -> _ ->(x+1)) 0 [3,2,1,2,4]`

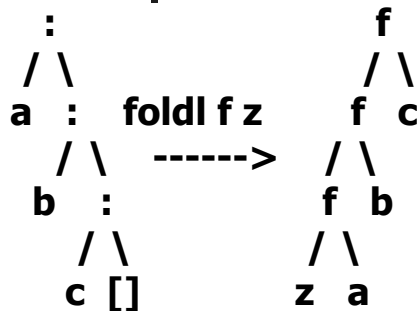
- `foldr (-) 0 [1..100] =`

$$(1-(2-(3-(4-\dots-(100-0)))))) = 1-2 + 3-4 + 5-6 + \dots + (99-100) = -50$$

- `foldl (-) 0 [1..100] =`

$$(\dots(((0-1)-2)-3) \dots - 100) = -5050$$

Kvíz

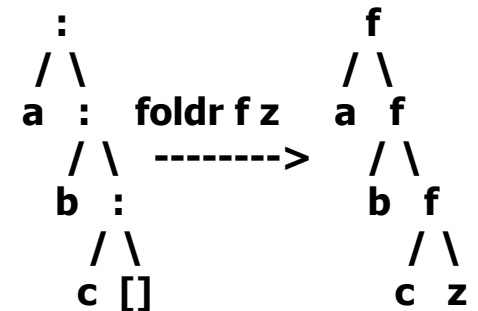


$$\text{foldr } (:) [] \text{ xs} = \text{xs}$$

$$\text{foldr } (:) \text{ ys xs} = \text{xs} ++ \text{ys}$$

$$\text{foldr } ? ? \text{ xs} = \text{reverse xs}$$

$$\text{foldr } ((:) . h) [] = ???$$



<http://foldl.com/>



Pre tých, čo zvládli kvíz, odmena !
kliknite si podľa vašej politickej
orientácie

<http://foldr.com/>





Funkcia je hodnotou

- $[a \rightarrow a]$ je zoznam funkcií typu $a \rightarrow a$
napríklad: $[(+1), (+2), (*3)]$ je $[\backslash x \rightarrow x+1, \backslash x \rightarrow x+2, \backslash x \rightarrow x*3]$
- čo je foldr $(.)$ id $[(+1), (+2), (*3)]$??
akého je typu $[a \rightarrow a]$
foldr $(.)$ id $[(+1), (+2), (*3)]$ 100 303
foldl $(.)$ id $[(+1), (+2), (*3)]$ 100 ???

lebo skladanie fcií je asociatívne:

- $((f . g) . h) x = (f . g) (h x) = f (g (h x)) = f ((g . h) x) = (f . (g . h)) x$
- funkcie nevieme porovnávať, napr. $\text{head } [(+1), (+2), (*3)] == \text{id}$
- funkcie vieme permutovať, $\text{length } \$ \text{permutations } [(+1), (+2), (*3), (^2)]$



Maximálna permutácia funkcií

- zoznam funkcií aplikujeme na zoznam argumentov

```
apply      :: [a -> b] -> [a] -> [b]
apply fs args = [ f a | f <- fs, a <- args]
```

```
apply [(+1),(+2),(*3)] [100, 200]
[101,201,102,202,300,600]
```

Dokážte/vyvráťte: `map f . apply fs = apply (map (f.) fs)`

- čo počíta tento výraz

```
maximum $
```

```
  apply
```

```
    (map (foldr (.) id) (permutations [(+1),(^2),(*3),(+2),(/3)]))
    [100]
```

```
31827
```

- `((+1).(+2).(*3).(^2).(/3)) 100`

```
3336.333333333334
```

- `((/3).(^2).(*3).(+2).(+1)) 100`

```
31827.0
```




take pomocou foldr/foldl

Výsledkom foldr `?f? ?z?` xs je funkcia, do ktorej keď dosadíme n, vráti take n:
... preto aj `?z?` musí byť funkcia, do ktorej keď dosadíme n, vráti take n []:

`take' :: Int -> [a] -> [a]`

`take' n xs = (foldr pomfcia (_ -> []) xs) n where`

`pomfcia x h = \n -> if n == 0 then []
 else x:(h (n-1))`

`alebo`

`pomfcia x h n = if n == 0 then [] else x:(h (n-1))`

`alebo`

`take''' n xs = foldr (\a -> \h -> \n -> case n of
 0 -> []
 n -> a:(h (n-1)))`

`(_ -> [])`

`xs`

`n`



Zákon fúzie – pre foldr

Fussion Law:

Nech g_1, g_2 sú binárne funkcie, z_1, z_2 konštanty

Ak pre funkciu f platí :

$$f\ z_1 = z_2 \ \&\& \ f\ (g_1\ a\ b) = g_2\ a\ (f\ b)$$

potom platí

$$f . (\text{foldr}\ g_1\ z_1\ xs) = \text{foldr}\ g_2\ z_2\ xs$$

Príklad použitia Fussion Law:

$$(n^*). \underbrace{\text{foldr}\ (+)\ 0}_{\text{sum}} = \text{foldr}\ ((+)\cdot(n^*))\ 0$$

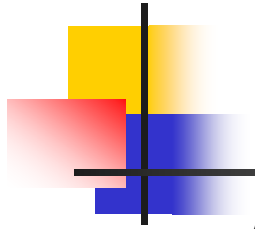
Dôkaz (pomocou Fussion Law): overíme predpoklady

čo je čo ?!:

$$f = (n^*),\ z_1 = z_2 = 0,\ g_1 = (+),\ g_2 = (+) \cdot (n^*)$$

treba overiť:

- $(n^*)\ 0 = 0$ ☒
- $L.S. = (n^*)\ (a+b) = (n^*a + n^*b) = (+) \cdot (n^*)\ a\ ((n^*)\ b) = P.S.$ ☒



Vlastnosti



Acid Rain (fold/build/deforestation theorem)

$$\underbrace{\text{foldr } f \ z}_{[x] \rightarrow u} \cdot \underbrace{g \ (\ :) \ []}_{t \rightarrow [x]} = g \ f \ z$$

$t \rightarrow u$

Intuícia: Keď máme vytvoriť zoznam pomocou funkcie g zo zoznamových konštruktorov $(:) []$, na ktorý následne pustíme foldr , ktorý nahradí $(:)$ za f a $[]$ za z , namiesto toho môžeme konštruovať priamo výsledný zoznam pomocou $g \ f \ z$.

Otypujme si to (aspoň):

Ak $z :: u$, potom $f :: x \rightarrow u \rightarrow u$, $\text{foldr } f \ z :: [x] \rightarrow u$.

Ľavá strana: $([x] \rightarrow u) \cdot (t \rightarrow [x])$ výsledkom je typ $t \rightarrow u$

Pravá strana: $g :: (x \rightarrow u \rightarrow u) \rightarrow u \rightarrow (t \rightarrow u)$

$$\text{foldr } f \ z \ . \ g \ (:) \ [] = g \ f \ z$$

length . map _ = length

map :: (a -> b) -> [a] -> [b]

map h = foldr ((:) . h) []

-- (:) . h a as = (:) (h a as) = h a : as

= $\underbrace{(\backslash x \rightarrow \backslash y \rightarrow \text{foldr } (x . h) \ y)}_g (:) \ []$

length :: [a] -> Int

length = foldr $\underbrace{(\backslash _ \rightarrow \backslash n \rightarrow n+1)}_f \underbrace{0}_z$

length . map h = length

L.S. = $\underbrace{(\text{foldr } (\backslash _ \rightarrow \backslash n \rightarrow n+1) \ 0)} \ . \ \underbrace{(\text{foldr } ((:) . h) \ [])}$ =

= podľa Acid Rain theorem ($f = (\backslash _ \rightarrow \backslash n \rightarrow n+1)$, $z = 0$, ale čo je g ? ...

$g \ x \ y = (\text{foldr } (x . h) \ y)$

$g \ f \ z = (\text{foldr } (f . h) \ z) = \text{foldr } ((\backslash _ \rightarrow \backslash n \rightarrow n+1) . h) \ 0 =$

$\text{foldr } ((\backslash _ \rightarrow \backslash n \rightarrow n+1)) \ 0 = \text{length} = \text{P.S.}$

lebo (tento krok pomalšie):

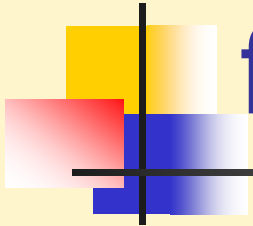
$((\backslash _ \rightarrow \backslash n \rightarrow n+1) . h) \ x \ y = (\backslash _ \rightarrow \backslash n \rightarrow n+1) \ (h \ x) \ y = (\backslash n \rightarrow n+1) \ y = y+1$

$$g \ h \ w \ n = h \ n \ (g \ h \ w \ (n-1))$$

9332621544394415268169923885626670049071596826438162146859296389521759999322991560894146397615651828625369792082722375825118521091686400000000000000000000

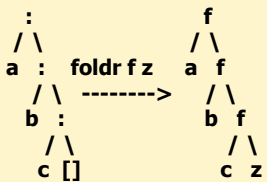
9332621544394415268169923885626670049071596826438162146859296389521759999322991560894146397615651828625369792082722375825118521091686400000000000000000000

$$g' n = n : (g' (n-1))$$
$$g''_n = n * (g''_{n-1})$$

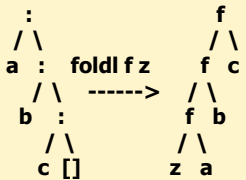


foldr a foldl pre pokročilejších

definujte foldl pomocou foldr, alebo naopak:



$\text{myfoldl } f \ z \ xs = \text{foldr } (\backslash x \rightarrow \backslash y \rightarrow (f \ y \ x)) \ z \ (\text{myReverse } xs)$
 $\text{myfoldr } f \ z \ xs = \text{foldl } (\backslash x \rightarrow \backslash y \rightarrow (f \ y \ x)) \ z \ (\text{myReverse } xs)$



■ odstránime myReverse

$\text{myReverse } xs = \text{foldr } (\backslash x \rightarrow \backslash y \rightarrow (y ++ [x])) \ [] \ xs$

$\text{myfoldl}' \ f \ z \ xs = \text{foldr } (\backslash x \rightarrow \backslash y \rightarrow (f \ y \ x)) \ z$
 $(\text{foldr } (\backslash x \rightarrow \backslash y \rightarrow (y ++ [x])) \ [] \ xs)$

■ odstránime ++

$xs ++ ys = \text{foldr } (:) \ ys \ xs$

$\text{myfoldl}'' \ f \ z \ xs = \text{foldr } (\backslash x \rightarrow \backslash y \rightarrow (f \ y \ x)) \ z$
 $(\text{foldr } (\backslash x \rightarrow \backslash y \rightarrow (\text{foldr } (:) \ [x] \ y)) \ [] \ xs)$

hmmm..., teoreticky (možno) zaujímavé, prakticky nepoužiteľné ...

foldr a foldl posledný krát

Zamyslime sa, ako z foldr urobíme foldl:

induktívne predpokladajme, že rekurzívne volanie foldr nám vráti výsledok, t.j. hodnotu y , ktorá zodpovedá foldl:

- $y = \text{myfoldl } f \text{ [b,c]} = \lambda z \rightarrow f (f z b) c$

nech x je ďalší prvok zoznamu, t.j.

- $x = a$

ako musí vyzerat' funkcia $?$, ktorou fold-r-ujeme, aby sme dostali $\text{myfoldl } f \text{ [a,b,c]} = \lambda z' \rightarrow f (f (f z' a) b) c = ? \ x \ y$

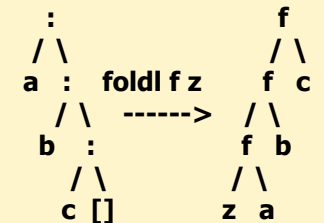
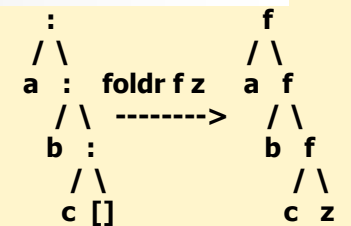
- $? = (\lambda x \ y \ z' \rightarrow y (f z' x))$

dosad'me:

- $(\lambda z' \rightarrow (\lambda z \rightarrow f (f z b) c) (f z' a)) =$

- $(\lambda z' \rightarrow f (f (f z' a) b) c) =$

- $\lambda z' \rightarrow f (f (f z' a) b) c$



Pre tých, čo neveria, fakt posledný krát

$$? = (\backslash x y z' \rightarrow y (f z' x))$$

$\text{myfoldl}''' f \text{ xs } z = \text{foldr} (\backslash x y z \rightarrow y (f z x)) \text{ id } \text{ xs } z$

- $\text{myfoldl}''' f [] = \text{id}$
- $\text{myfoldl}''' f [c] = (\backslash x y z \rightarrow y (f z x)) c \text{ id} = \backslash z \rightarrow f z c$
- $\text{myfoldl}''' f [b,c] = (\backslash x y z \rightarrow y (f z x)) b (\backslash w \rightarrow f w c) =$
 $\backslash z \rightarrow (\backslash w \rightarrow f w c) (f z b) =$
 $\backslash z \rightarrow f (f z b) c$
- $\text{myfoldl}''' f [a,b,c] = (\backslash x y z \rightarrow y (f z x)) a (\backslash w \rightarrow f (f w b) c) =$
 $\backslash z \rightarrow (\backslash w \rightarrow f (f w b) c) (f z a) =$
 $\backslash z \rightarrow f (f (f z a) b) c$
- $\text{myfoldl}''' f z \text{ xs} = \text{foldr} (\backslash x y z \rightarrow y (f x z)) \text{ id } \text{ xs } z$

... doma skúste foldr pomocou foldl ...