Problem 22. *If $f : A \to B$ has an inverse function then f is onto and f is one-to-one.*

Proof. □

Problem 23. *A real number $x \in \mathbb{R}$ is called* algebraic *if there exists $a_0, a_1, \dots, a_{n-1}, a_n \in \mathbb{Z}$, not all zero, so that*

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 = 0$$

That is, a real number is algebraic if it is a root of a polynomial equation with integer coefficients.

(a) *The numbers $\sqrt{2}$, $\sqrt[3]{2}$, and $\sqrt{3} + \sqrt{2}$ are algebraic.*

Proof. □

(b) *For fixed $n \in \mathbb{N}$, let A_n be the set of algebraic numbers which are roots of polynomials, with integer coefficients, of degree n. Then A_n is countable.*

Proof. □

(c) *The set of all algebraic numbers is countable.*

Proof. □

Problem 24. *There is an onto function $f : (0, 1) \to S$ where $S = \{(x, y) : 0 < x, y < 1\}$ is the unit square in the plane $\mathbb{R}^2$.*

Proof. □

Problem 25. (a) $\lim_{n\to\infty} \frac{2n+1}{5n+3} = \frac{2}{5}$

Proof. Let $\epsilon > 0$. □

(b) $\lim_{n\to\infty} \frac{2n^2}{n^3+1} = 0$

Proof. Let $\epsilon > 0$. □

(c) $\lim_{n\to\infty} \frac{\sin(n)}{\sqrt{n}} = o$

Proof. Let $\epsilon > 0$. □

Problem 26. *(a) A sequence with an infinite number of ones that does not converge to one.*

(b) A sequence with an infinite number of ones that converges to a limit not equal to one.

Problem 27. *Let (x_n) be a sequence that converges to x. Suppose $p(x)$ is a polynomial. Then*

$$\lim_{n\to\infty} p(x_n) = p(x).$$

Proof. $\square$

Problem 28. *Consider three sequences (x_n), (y_n), and (z_n) for which $x_n \leq y_n \leq z_n$ for each n. If $x_n \to \ell$ and $z_n \to \ell$ then $y_n \to \ell$.*

Proof. $\square$