Section 12.1

Exercise (1). Suppose $A = \{0, 1, 2, 3, 4\}, B = \{2, 3, 4, 5\}$ and $f = \{(0, 3), (1, 3), (2, 4), (3, 2), (4, 2)\}$. State the domain and range of f. Find $f(2)$ and $f(1)$.

Solution: Write your answer here. □

Exercise (3). There are four different functions $f : \{a, b\} \to \{0, 1\}$. List them. Diagrams suffice.

Solution: Write your answer here. □

Exercise (7). Consider the set $f = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : 3x + y = 4\}$. Is this a function from $\mathbb{Z}$to $\mathbb{Z}$? Explain.

Solution: Write your answer here. □

Exercise (9). Consider the set $f = \{(x, x^2) : x \in \mathbb{R}\}$. Is this a function from $\mathbb{R}$to $\mathbb{R}$? Explain.

Solution: Write your answer here. □

Exercise (12). Is the set $\theta = \{((x, y), (3y, 2x, x + y)) : x, y \in \mathbb{R}\}$ a function? If so, what is the domain and range? What can be said about the codomain?

Solution: Write your answer here. □

Section 12.2

Exercise (1). Let $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c\}$. Give and example of a function $f : A \to B$ that is neither injective nor surjective.

Solution: Write your answer here. □

Exercise (5). A function $f : \mathbb{Z} \to \mathbb{Z}$ is defined as $f(n) = 2n+1$. Verify whether this function is injective and whether it is surjective.

If it is either of these things, this requires a proof. If it is not one of these things, that requires a well presented counterexample.

Solution: Write your answer here. □

Exercise (7). A function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is defined as $f(m, n) = 2n - 4m$. Verify whether this function is injective and whether it is surjective.

Solution: Write your answer here. □

Exercise (8). A function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ is defined as $f(m, n) = (m + n, 2m + n)$. Verify whether this function is injective and whether it is surjective.

Solution: Write your answer here. □

Exercise (10). Prove the function $f : \mathbb{R} - \{1\} \to \mathbb{R} - \{1\}$ defined by $f(x) = \left(\frac{x+1}{x-1}\right)^3$ is bijective.

Proof: Write your answer here. □

Exercise (14). Consider the function $\theta : \mathcal{P}(\mathbb{Z}) \to \mathcal{P}(\mathbb{Z})$ defined as $\theta(X) = \overline{X}$. Is θ injective? Is it surjective? Bijective? Explain.

Solution: Write your answer here. □

Section 12.3

Exercise (1). Prove that if six integers are chosen at random, then at least two of them will have the same remainder when divided by 5.

Proof: Write your answer here. □

Exercise (4). Consider a square whose side-length is one unit. Select any five points from inside this square. Prove that at least two of these points are within $\frac{\sqrt{2}}{2}$ units of each other.

Remark: It may help your argument to include suitable drawings.

Proof: Write your answer here. □

Section 12.4

Exercise (2). Suppose $A = \{1,2,3,4\}, B = \{0,1,2\}, C = \{1,2,3\}$. Let $f : A \to B$ be $f = \{(1,0),(2,1),(3,2),(4,0)\}$, and $g : B \to C$ be $g = \{(0,1),(1,1),(2,3)\}$. Find $g \circ f$.

Solution: Write your answer here. □

Exercise (4). Suppose $A = \{a,b,c\}$. Let $f : A \to A$ be the function $f = \{(a,c),(b,c),(c,c)\}$, and let $g : A \to A$ be the function $g = \{(a,a),(b,b),(c,a)\}$. Find $g \circ f$ and $f \circ g$.

Solution: Write your answer here. □

Exercise (6). Consider the functions $f, g : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = \frac{1}{x^2+1}$ and $g(x) = 3x + 2$. Find the formulas for $g \circ f$ and $f \circ g$.

Solution: Write your answer here. □

Exercise (7). Consider the functions $f, g : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ defined as $f(m,n) = (3m - 4n, 2m + n)$ and $g(m,n) = 5m + n, m)$. Find the formulas for $g \circ f$ and $f \circ g$.

Solution: Write your answer here. □

Exercise (Reflection Problem).

Answers:

How long did it take you to complete each problem?:

What was easy?:

What was challenging? What made it challenging?:

What did you learn from comparing your answers to those in the book?:

$\square$

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