Exercises: Determine the domain. For each of the folowing quantified statements taken randomly from mathematics textbooks, **determine a plausible domain.** Some of the domains are implicit.

Exercise (1). (Well-ordering principle) Every nonempty subset of $\mathbb{N}$ has a smallest element.

The plausible domain is $\{X \subseteq \mathbb{N}\,|X| \leq 1\}$

Exercise (2). Every finite connected graph G has a spanning tree.

The set of all finite connected graphs.

Exercise (3). A tree with n vertices has exactly $n-1$ edges.

The set of all trees.

Exercise (4). Every finite acyclic graph has at least one sink and at least one source.

The set of all finite acyclic graphs

Exercise (5). If u and v are different vertices of a digraph G, and if there is a path in G from u to v, then there is an acyclic path u to v.

The domain is the set of vertices in digraph G.

More translation practice: the implicit domain is $\mathbb{Z}$. Each of the following statements is implicitely quantified: that is, each is a "for all" statement, with domain $\mathbb{Z}$. For each of the following:
(i) translate into symbols
(ii) Write the negation of the statement in words
(iii) Write the contraposition of the original statement in words
(iv) Which are true?

Exercise (1). If a divides b, then ac divides bc for any c.

(i) $\forall a, b, \in \mathbb{Z}, a|b \Rightarrow \forall c \in \mathbb{Z}, ac|bc$
(ii) For all integers a and b, a divides b and there exists an integer c such that ac does not divide bc.
(iii) For all integers a and b, there exists an integer c such that if ac divides bc, then a does not divide b.
(iv) The original statement and the contrapositive are true.

Exercise (2). If ac divides bc, then a divides b.

(i) $\forall a, b, c \in \mathbb{Z}, ac|bc \Rightarrow a|b$
(ii) If ac divides bc, then a does not divide b
(iii) If a does not divide b, then ac does not divide bc.
(iv) The original statement and the contrapositive are false, negation is also false.

Exercise (3). If a divides b and a divides $b+2$ then $a = 1$ or $a = 2$.

(i) $\forall a, b \in \mathbb{Z}, (a|b \wedge a|(b+2)) \Rightarrow (a = 1 \vee a = 2)$
(ii) If a divides b and a divides $b+2$, then a is not equal to 1 or 2.
(iii) If a is not equal to 1 or 2, then a does not divide neither b and $b+2$.
(iv) The original statement is and contrapositive are true, negation is false.

Exercise (4). if xy is even, then x is even or y is even.

(i) $\forall x, y \in \mathbb{Z}, (2|xy) \Rightarrow (2|x \vee 2|y)$
(ii) If xy is even, then x and y are odd.
(iii) If x and y are odd, then xy is odd.
(iv) The original and the contraposition are true, negation false.

Exercise (5). The sum of two odd integers is odd

(i) $\forall x, y \in \mathbb{Z}, x + y = 2k + 1, \forall k \in \mathbb{Z}$
(ii) The sum of two odd integers is even.
(iii) If two integers are odd, then their sum is odd.
(iv) The original statement is false, the negation is true, the contrapositive is false.

Exercise (6). If a and b are odd, then $a^2 + b^2$ is not divisible by 4

(i) $\forall k, m \in \mathbb{Z}, (a = 2k + 1 \wedge b = 2m + 1) \Rightarrow (a^2 + b^2 \nmid 4)$
(ii) If a and b are odd, then $a^2 + b^2$ is divisible by 4.
(iii) If $a^2 + b^2$ is divisible by 4, then a or b are even numbers.
(iv) The original statement is true as well as the contrapositive, the negation is false.