Probability HW 3

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2.71 If two events, A and B, are such that P(A) = .5, P(B) = .3, and $P(A \cap B) = .1$, find the following:
a P(A|B)

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.1}{.3} = \frac{1}{3}.$$

b P(B|A)

$$P(B|A) = \frac{(B \cap A)}{P(A)} = \frac{(A \cap B)}{P(A)} = \frac{.1}{.5} = \frac{1}{5}.$$

c $P(A|A \cup B)$

$$P(A|A \cup B) = \frac{P(A \cap (A \cup B))}{P(A \cup B)} = \frac{5}{7}.$$

d $P(A|A \cap B)$

$$P(A|A \cap B) = \frac{P(A \cap (A \cap B)}{P(A \cap B)} = \frac{P(A \cap B)}{P(A \cap B)} = 1.$$

e $P(A \cap Bf|A \cup B)$

$$P(A \cap B|A \cup B) = \frac{P((A \cap B) \cap (A \cup B))}{P(A \cup B)} = \frac{P(A \cap B)}{P(A \cup B)} = \frac{.1}{.7} = \frac{1}{7}.$$

2.73 Gregor Mendel was a monk who, in 1865, suggested a theory of inheritance based on the science of genetics. He identified heterozygous individuals for flower color that had two alleles (one r = recessive white color allele and one R = dominant red color allele). When these individuals were mated, 3/4 of the offspring were observed to have red flowers, and 1/4 had white flowers. The following table summarizes this mating; each parent gives one of its alleles to form the gene of the offspring.

	Parent 2	
Parent 1	r	R
r	rr	rR
R	Rr	RR

We assume that each parent is equally likely to give either of the two alleles and that, if either one or two of the alleles in a pair is dominant (R), the offspring will have red flowers. What is the probability that an offspring has
a at least one dominant allele?

$$\frac{3}{4} = 0.75.$$

b at least one recessive allele?

$$\frac{3}{4} = 0.75.$$

c one recessive allele, given that the offspring has red flowers?

$$P(r|R) = \frac{P(r \cap R)}{P(R)} = \frac{.50}{.75} = \frac{2}{3} = 0.6666667$$

2.74 One hundred adults were interviewed in a telephone survey. Of interest was their opinions regarding the loan burdens of college students and whether the respondent had a child currently in college. Their responses are summarized in the table below:

	Loan Burden			
Child in College	Too High (A)	About Right (B)	Too Little (C)	*Total*
Yes (D)	.20	.09	.01	.30
No (E)	.41	.21	.08	.70
Total	.61	.30	.09	1.00

Which of the following are independent events?
a A and D

$$P(A \cap D) = .20 - \text{Dependent}$$

b B and D

$$P(B \cap D) = 0.9 - \text{Independent}$$

c C and D

$$P(C \cap D) = 0.1 - \text{Dependent}$$

2.77 A study of the posttreatment behavior of a large number of drug abusers suggests that the likelihood of conviction within a two-year period after treatment may depend upon the offenders education. The proportions of the total number of cases falling in four education–conviction categories are shown in the following table:

	Status within 2 Years after Treatment		
Education	Convicted	Not Convicted	*Total*
10 years or more	.10	.30	.40
9 years or less	.27	.33	.60
Total	.37	.63	1.00

Suppose that a single offender is selected from the treatment program. Define the events: A: The offender has 10 or more years of education.
B: The offender is convicted within two years after completion of treatment. Find the following:
a $P(A)$

$$P(A) = .40$$

b $P(B)$

$$P(B) = .37$$

c $P(A \cap B)$

$$P(A \cap B) = .10$$

d $P(A \cup B)$

$$P(A \cup B) = .40 + .37 - 10 = .67$$

e $P(\overline{A})$

$$P(\overline{A}) = .60$$

f $P(\overline{A \cup B})$

$$P(\overline{A \cup B}) = .90$$

g $P(\overline{A \cap B})$

$$P(\overline{A \cap B}) = .63$$

h $P(A|B)$

$$P(A|B) = .27$$

i $P(B|A)$

$$P(B|A) = .25$$

2.79 Suppose that A and B are mutually exclusive events, with P(A) > 0 and P(B) < 1. Are A and B independent? Prove your answer

Suppose A and B are mutually exclusive events, by definition of mutually exclusive this means that $(A \cap B) = 0$. Note that by the axiom of probability $P(B) \geq 0$. Thus $P(A)P(B) > 0$ and $P(A)$ and $P(B)$ are not independent. (Note that although $P(B) = 0$ is possible, this is a degenerate case so we ignore it as the event not occurring at all makes the problem stupid.)

2.87 Suppose that A and B are two events such that P(A) + P(B) > 1.
a What is the smallest possible value for P(A B)?

$$P(A) + P(B) - 1$$

b What is the largest possible value for P(A   B)?

$$P(A) \text{ if } P(A) < P(B) \text{ and } P(B) \text{ if } P(B) < P(A).$$

2.91 Can A an B be mutually exclusive if P(A) = .4 and P(B) = .7? If P(A) = .4 and P(B) = .3? Why

$$\text{No because } P(A) + P(B) > 1$$

9.92 A policy requiring all hospital employees to take lie detector tests may reduce losses due to theft, but some employees regard such tests as a violation of their rights. Past experience indicates that lie detectors have accuracy rates that vary from 92% to 99%. To gain some insight into the risks that employees face when taking a lie detector test, suppose that the probability is .05 that a lie detector concludes that a person is lying who, in fact, is telling the truth and suppose that any pair of tests are independent. What is the probability that a machine will conclude that
a each of three employees is lying when all are telling the truth?

$$0.05^3 = 0.000125$$

b at least one of the three employees is lying when all are telling the truth?

$$1 - .95^3 = 0.142625$$

2.94 A smoke detector system uses two devices, A and B. If smoke is present, the probability that it will be detected by device A is .95; by device B, .90; and by both devices, .88.
a If smoke is present, find the probability that the smoke will be detected by either device A or B or both devices.

Let A = Device A detects smoke.

Let B = Device B detects smoke.

$$P(A \cup B) = .95 + .90 - .88 = .97$$

b Find the probability that the smoke will be undetected.

$$P(\overline{A \cup B}) = 1 - .97 = 0.03$$

2.111 An advertising agency notices that approximately 1 in 50 potential buyers of a product sees a given magazine ad, and 1 in 5 sees a corresponding ad on television. One in 100 sees both. One in 3 actually purchases the product after seeing the ad, 1 in 10 without seeing it. What is the probability that a randomly selected potential customer will purchase the product?

Let S_M: sees magazine ad,
S_T: sees TV ad,

$P(S_M) = \frac{1}{50} = 0.02$,
$P(S_T) = \frac{1}{5} = 0.2$,
$P(S_M \cap S_T) = \frac{1}{100} = 0.01$,
$P(\text{Purchase}|S_M \cup S_T) = \frac{1}{3}$,
$P(\text{Purchase}|\overline{S_M \cup S_T}) = \frac{1}{10}$.

$$P(S_M \cup S_T) = 0.02 + 0.2 - 0.01 = 0.21$$

$$P(\overline{S_M \cup S_T}) = 1 - 0.21 = 0.79$$

$$P(\text{Purchase}) = \frac{1}{3} \cdot 0.21 + \frac{1}{10} \cdot 0.79 = 0.149$$

2.114 A lie detector will show a positive reading (indicate a lie) 10% of the time when a person is telling the truth and 95% of the time when the person is lying. Suppose two people are suspects in a one-person crime and (for certain) one is guilty and will lie. Assume further that the lie detector operates independently for the truthful person and the liar. What is the probability that the detector

a shows positive reading for both suspects?

Let P: positive reading, T: truthful ($P(P|T) = 0.1$), L: lying ($P(P|L) = 0.95$).

$$P(P_1 \cap P_2) = 0.1 \cdot 0.95 = 0.095$$

b shows a positive reading for the guilty suspect and a negative reading for the innocent suspect?

$$P(P|L) \cdot P(\overline{P}|T) = 0.95 \cdot 0.9 = 0.855$$

c is completely wrong—that is, that it gives a positive reading for the innocent suspect and a negative reading for the guilty?

$$P(P|T) \cdot P(\overline{P}|L) = 0.1 \cdot 0.05 = 0.005$$

d gives a positive reading for either or both of the two suspects?

$$P(P_1 \cup P_2) = 0.1 + 0.95 - 0.095 = 0.955$$

2.116 A communications network has a built-in safeguard system against failures. In this system if line I fails, it is bypassed and line II is used. If line II also fails, it is bypassed and line III is used. The probability of failure of any one of these three lines is .01, and the failures of these lines are independent events. What is the probability that this system of three lines does not completely fail?

$$1 - (0.01)^3 = 0.999999$$

2.118 An accident victim will die unless in the next 10 minutes he receives some type A, Rh-positive blood, which can be supplied by a single donor. The hospital requires 2 minutes to type a prospective donor's blood and 2 minutes to complete the transfer of blood. Many untyped donors are available, and 40% of them have type A, Rh-positive blood. What is the probability that the accident victim will be saved if only one blood-typing kit is available? Assume that the typing kit is reusable but can process only one donor at a time.

$$P(\text{saved}) = 0.4 + 0.6 \cdot 0.4 = 0.64$$

*2.119 Suppose that two balanced dice are tossed repeatedly and the sum of the two uppermost faces is determined on each toss. What is the probability that we obtain
a a sum of 3 before we obtain a sum of 7?

$$P(3 \text{ before } 7) = \frac{\frac{1}{18}}{\frac{1}{18} + \frac{1}{6}} = \frac{1}{4}$$

b a sum of 4 before we obtain a sum of 7?

$$P(4 \text{ before } 7) = \frac{\frac{1}{12}}{\frac{1}{12} + \frac{1}{6}} = \frac{1}{3}$$

2.125 A diagnostic test for a disease is such that it (correctly) detects the disease in 90% of the individuals who actually have the disease. Also, if a person does not have the disease, the test will report that he or she does not have it with probability .9. Only 1% of the population has the disease in question. If a person is chosen at random from the population and the diagnostic test indicates that she has the disease, what is the conditional probability that she does, in fact, have the disease? Are you surprised by the answer? Would you call this diagnostic test reliable?

Let D = Has disease
Let T = Test Positive

$$P(D|T) = \frac{P(T|D)P(D)}{P(T|D)P(D) + P(T|\overline{D})P(\overline{D})} = \frac{0.9 \cdot 0.01}{0.9 \cdot 0.01 + 0.1 \cdot 0.99} = \frac{0.009}{0.009 + 0.099} = \frac{1}{12} = \frac{0.009}{0.108} = 0.0833$$

2.129 Males and females are observed to react differently to a given set of circumstances. It has been observed that 70% of the females react positively to these circumstances, whereas only 40% of males react positively. A group of 20 people, 15 female and 5 male, was subjected to these circumstances, and the subjects were asked to describe their reactions on a written questionnaire. A response picked at random from the 20 was negative. What is the probability that it was that of a male?

Let M = Male
Let F = Female
Let N = Negative Reaction

$$P(M|N) = \frac{P(N|M)P(M)}{P(N|M)P(M) + P(N|F)P(F)} = \frac{0.6 \cdot 0.25}{0.6 \cdot 0.25 + 0.3 \cdot 0.75} = \frac{0.15}{0.15 + 0.225} = \frac{0.15}{0.375} = 0.4$$

2.133 A student answers a multiple-choice examination question that offers four possible answers. Suppose the probability that the student knows the answer to the question is .8 and the prob- ability that the student will guess is .2. Assume that if the student guesses, the probability of selecting the correct answer is .25. If the student correctly answers a question, what is the probability that the student really knew the correct answer?

Let K = Knows answer
Let G = Guesses
Let C = Correct Answers

$$P(K|C) = \frac{P(C|K)P(K)}{P(C|K)P(K) + P(C|G)P(G)} = \frac{1 \cdot 0.8}{1 \cdot 0.8 + 0.25 \cdot 0.2} = \frac{0.8}{0.8 + 0.05} = \frac{0.8}{0.85} = 0.9412$$