Section 4.5

Exercise (4.58). Use Table 4, Appendix 3, to find the following probabilities for a standard normal random variable Z:

(a) $P(0 \leq Z \leq 1.2)$

Solution: $.5000 - .1151 = .3849$ □

(b) $P(-.9 \leq Z \leq 0)$

Solution: $.5000 - .1841 = .3159$ □

(c) $P(.3 \leq Z \leq 1.56)$

Solution: $.3821 - .0594 = .3227$ □

(d) $P(-.2 \leq Z \leq .2)$

Solution: $2 * (.5000 - .4207) = .1586$ □

(e) $P(-1.56 \leq Z \leq -.2)$

Solution: $(.5000 - .0594) + (.5000 - .4207) = .5199$ □

Exercise (4.59). If Z is a standard normal random variable, find the value z_0 such that

(a) $P(Z > z_0) = .5$.

Solution: $z_0 = 0$ □

(b) $P(Z < z_0) = .8643$.

Solution: $z_0 = 1.1$ □

(c) $P(-z_0 < Z < z_0) = .90$.

Solution:

$$\begin{aligned}
.90 &= P(Z < z_0) - 1 + P(Z < z_0) \\
&= 2P(Z < z_0) - 1 \\
1.90 &= 2p(Z < z_0) \\
0.95 &= p(Z < z_0) \\
z_0 &= 1.645
\end{aligned}$$

□

(d) $P(-z_0 < Z < z_0) = .99$.

Solution: $z_0 = 2.576$ □

Exercise (4.61). What is the median of a normally distributed random variable with mean μ and standard deviation σ?

Solution: μ is the median □

Exercise (4.63). A company that manufactures and bottles apple juice uses a machine that automatically fills 16-ounce bottles. There is some variation, however, in the amounts of liquid dispensed into the bottles that are filled. The amount dispensed has been observed to be approximately normally distributed with mean 16 ounces and standard deviation 1 ounce.

(a) Use Table 4, Appendix 3, to determine the proportion of bottles that will have more than 17 ounces dispensed into them.

Solution:

$$z = \frac{\gamma - \mu}{\sigma}$$
$$z = \frac{17 - 16}{1} = 1$$
$$p(Z > 1) = .1587$$

□

Exercise (4.64). The weekly amount of money spent on maintenance and repairs by a company was observed, over a long period of time, to be approximately normally distributed with mean \$400 and standard deviation \$20. If \$450 is budgeted for next week, what is the probability that the actual costs will exceed the budgeted amount?

(a) Answer the question, using Table 4, Appendix 3.

Solution:

$$z = \frac{\gamma - \mu}{\sigma}$$
$$z = \frac{450 - 400}{20} = 2.5$$
$$P(Z > 2.5) = .0062$$

□

Exercise (4.65). In Exercise 4.64, how much should be budgeted for weekly repairs and maintenance to provide that the probability the budgeted amount will be exceeded in a given week is only .1?

Solution: Choose $z = 1.28$ from table ($P = .1003$), let n denote how much we should budget

$$z = 1.28 = \frac{n - 400}{20}$$

$$25.6 = n - 400$$

$$425.6 = n$$

□

Exercise (4.66). A machining operation produces bearings with diameters that are normally distributed with mean 3.0005 inches and standard deviation .0010 inch. Specifications require the bearing diameters to lie in the interval $3.000 \pm .0020$ inches. Those outside the interval are considered scrap and must be remachined. With the existing machine setting, what fraction of total production will be scrap?

(a) Answer the question, using Table 4, Appendix 3.

Solution:

$$\begin{aligned}
(z_0 + z_1) &= \frac{\gamma_0 - \mu}{\sigma} + \frac{\gamma_1 - \mu}{\sigma} \\
&= \frac{3.0020 - 3.0005}{.0010} + \frac{2.9980 - 3.0005}{.0010} \\
&\to P(Z > 1.5) + P(Z < -2.5) \\
&= .0668 + .0062 \\
&= .0730
\end{aligned}$$

□

Exercise (4.68). The grade point averages (GPAs) of a large population of college students are approximately normally distributed with mean 2.4 and standard deviation .8. What fraction of the students will possess a GPA in excess of 3.0?

(a) Answer the question, using Table 4, Appendix 3.

Solution:

$$z = \frac{\gamma - \mu}{\sigma} = \frac{3.0 - 2.4}{.8} = 0.75$$
$$P(Z > .75) = .2266$$

□

Exercise (4.69). Refer to Exercise 4.68. If students possessing a GPA less than 1.9 are dropped from college, what percentage of the students will be dropped?

Solution:

$$z = \frac{\gamma - \mu}{\sigma} = \frac{1.9 - 2.4}{.8} = -0.625$$
$$P(Z < -0.625) = (.2676 + .2643)/2 = 0.26595$$

□

Exercise (4.70). Refer to Exercise 4.68. Suppose that three students are randomly selected from the student body. What is the probability that all three will possess a GPA in excess of 3.0?

Solution: $.2266^3 = 0.01163536$ □

Exercise (4.73). The width of bolts of fabric is normally distributed with mean 950 mm (millimeters) and standard deviation 10 mm.

(a) What is the probability that a randomly chosen bolt has a width of between 947 and 958 mm?

Solution:

$$z_0 = \frac{\gamma_0 - \mu}{\sigma} = \frac{947 - 950}{10} = -0.3$$
$$z_1 = \frac{\gamma_1 - \mu}{\sigma} = \frac{958 - 950}{10} = 0.8$$
$$P(-0.3 \leq Z \leq 0.8) = 1 - (.3821 + .2119) = 0.406$$

□

(b) What is the appropriate value for C such that a randomly chosen bolt has a width less than C with probability .8531?

Solution:

$$P(z_0 \leq Z) = (1 - .8531) = .1469$$
$$\rightarrow z_0 = 1.5$$
$$\rightarrow \frac{\gamma - 950}{10} = 1.05$$
$$\gamma = 10.5 + 950 = 960.5 = C$$

□

Exercise (4.77). The SAT and ACT college entrance exams are taken by thousands of students each year. The mathematics portions of each of these exams produce scores that are approximately normally distributed. In recent years, SAT mathematics exam scores have averaged 480 with standard deviation 100. The average and standard deviation for ACT mathematics scores are 18 and 6, respectively.

(a) An engineering school sets 550 as the minimum SAT math score for new students. What percentage of students will score below 550 in a typical year?

Solution:

$$z = \frac{\gamma - \mu}{\sigma} = \frac{550 - 480}{100} = 0.7$$
$$P(Z < 0.7) = 1 - .2420 = 0.758$$

□

(b) What score should the engineering school set as a comparable standard on the ACT math test?

Solution:

$$z = \frac{\gamma - 18}{6} = 0.7$$
$$\gamma - 18 = 4.2$$
$$\gamma = 22.2$$

□

Section 4.6

Exercise (4.89). The operator of a pumping station has observed that demand for water during early afternoon hours has an approximately exponential distribution with mean 100 cfs (cubic feet per second).

(a) Find the probability that the demand will exceed 200 cfs during the early afternoon on a randomly selected day.

Solution:

$$\int_{\alpha}^{\infty} \frac{1}{\beta} e^{-\gamma/\beta} dy = -e^{-\alpha/\beta}\Big]_{\alpha}^{\infty} = 1 - e^{-\alpha/\beta} = 1 - e^{-200/100} = 0.8646647$$

□

(b) What water-pumping capacity should the station maintain during early afternoons so that the probability that demand will exceed capacity on a randomly selected day is only .01?

Solution:

$$\begin{aligned} 1 - e^{-n/100} &= .01 \\ -e^{-n/100} &= -.99 \\ e^{-n/100} &= .99 \\ -n/100 &= ln(.99) \\ n &= -100ln(.99) \\ n &= 1.005034 \end{aligned}$$

□

Exercise (4.91). If Y has an exponential distribution and $P(Y > 2) = .0821$, what is

(a) $\beta = E(Y)$?

Solution:

$$f(x) = e^{-2/x} = .0821$$

$$-2/x = ln(0.0821)$$

$$-2/ln(0.0821) = x$$

$$.8000585 = x$$

□

(b) $P(Y \leq 1.7)$?

Solution: $1 - e^{-1.7/.800585} = 0.8803814$ □

Exercise (4.93). Historical evidence indicates that times between fatal accidents on scheduled American domestic passenger flights have an approximately exponential distribution. Assume that the mean time between accidents is 44 days.

(a) If one of the accidents occurred on July 1 of a randomly selected year in the study period, what is the probability that another accident occurred that same month?

Solution: $P(Y \leq 31) = \int_0^{31} \frac{1}{44} e^{-y/44} dy = e^{0/44} - e^{-31/44} = .5057$ □

(b) What is the variance of the times between accidents?

Solution: $44 = \frac{1}{\lambda}$ so $\lambda = 1/44$ so $V[X] = 1/((1/44)^2) = 1936$ □

Exercise (4.94). One-hour carbon monoxide concentrations in air samples from a large city have an approximately exponential distribution with mean 3.6 ppm (parts per million).

(a) Find the probability that the carbon monoxide concentration exceeds 9 ppm during a randomly selected one-hour period.

Solution: $P(Y > 9) = \int_9^{\infty} \frac{1}{3.6} e^{-y/3.6} dy = 0 - e^{-9/3.6} = 0.082085$ □

(b) A traffic-control strategy reduced the mean to 2.5 ppm. Now find the probability that the concentration exceeds 9 ppm.

Solution: $P(Y > 9) = \int_9^{\infty} \frac{1}{2.5} e^{-y/2.5} dy = 0 - e^{-9/2.5} = 0.02732372$

□

Exercise (4.97). A manufacturing plant uses a specific bulk product. The amount of product used in one day can be modeled by an exponential distribution with $\beta = 4$ (measurements in tons). Find the probability that the plant will use more than 4 tons on a given day.

Solution: □

Exercise (4.104). The lifetime (in hours) Y of an electronic component is a random variable with density function given by

$$f(y) = \begin{cases} \frac{1}{100}e^{-y/100}, & y > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

Three of these components operate independently in a piece of equipment. The equipment fails if at least two of the components fail. Find the probability that the equipment will operate for at least 200 hours without failure.

Solution: $\square$

Exercise (4.105). Four-week summer rainfall totals in a section of the Midwest United States have approximately a gamma distribution with $\alpha = 1.6$ and $\beta = 2.0$.

(a) Find the mean and variance of the four-week rainfall totals.

Solution: $\square$

(b) Applet Exercise What is the probability that the four-week rainfall total exceeds 4 inches?

Solution: $\square$

Exercise (4.109). The weekly amount of downtime Y (in hours) for an industrial machine has approximately a gamma distribution with $\alpha = 3$ and $\beta = 2$. The loss L (in dollars) to the industrial operation as a result of this downtime is given by $L = 30Y + 2Y^2$. Find the expected value and variance of L.

Solution: $\square$

Section 4.7

Exercise (4.123). The relative humidity Y, when measured at a location, has a probability density function given by

$$f(y) = \begin{cases} ky^3(1-y)^2, & 0 \le y \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find the value of k that makes $f(y)$ a density function.

Solution: $\square$

Exercise (4.124). The percentage of impurities per batch in a chemical product is a random variable Y with density function

$$f(y) = \begin{cases} 12y^2(1-y), & 0 \le y \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

A batch with more than 40% impurities cannot be sold.

(a) Integrate the density directly to determine the probability that a randomly selected batch cannot be sold because of excessive impurities.

Solution: □

Exercise (4.126). The weekly repair cost Y for a machine has a probability density function given by

$$f(y) = \begin{cases} 3(1-y)^2, & 0 < y < 1, \\ 0, & \text{elsewhere,} \end{cases}$$

with measurements in hundreds of dollars. How much money should be budgeted each week for repair costs so that the actual cost will exceed the budgeted amount only 10% of the time?

Solution: □

Exercise (4.129). During an eight-hour shift, the proportion of time Y that a sheet-metal stamping machine is down for maintenance or repairs has a beta distribution with $\alpha = 1$ and $\beta = 2$. That is,

$$f(y) = \begin{cases} 2(1-y), & 0 \le y \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

The cost (in hundreds of dollars) of this downtime, due to lost production and cost of maintenance and repair, is given by $C = 10 + 20Y + 4Y^2$. Find the mean and variance of C.

Solution: □

Exercise (4.131). Errors in measuring the time of arrival of a wave front from an acoustic source sometimes have an approximate beta distribution. Suppose that these errors, measured in microseconds, have approximately a beta distribution with $\alpha = 1$ and $\beta = 2$.

(a) What is the probability that the measurement error in a randomly selected instance is less than .5 μs?

Solution: □

(b) Give the mean and standard deviation of the measurement errors.

Solution: □

Exercise (4.133). The proportion of time per day that all checkout counters in a supermarket are busy is a random variable Y with a density function given by

$$f(y) = \begin{cases} cy^2(1-y)^4, & 0 \le y \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find the value of c that makes $f(y)$ a probability density function.

Solution: □

(b) Find $E(Y)$. (Use what you have learned about the beta-type distribution. Compare your answers to those obtained in Exercise 4.28.)

Solution: $\square$

(c) Calculate the standard deviation of Y.

Solution: $\square$

Section 4.9

Exercise (4.140). Identify the distributions of the random variables with the following moment-generating functions:

(a) $m(t) = (1 - 4t)^{-2}$.

Solution: $\square$

(b) $m(t) = 1/(1 - 3.2t)$.

Solution: $\square$

(c) $m(t) = e^{-5t+6t^2}$.

Solution: $\square$

Exercise (4.141). If $\theta_1 < \theta_2$, derive the moment-generating function of a random variable that has a uniform distribution on the interval (θ_1, θ_2).

Solution: $\square$

Exercise (4.145). A random variable Y has the density function

$$f(y) = \begin{cases} e^y, & y < 0, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find $E(e^{3Y/2})$.

Solution: $\square$

(b) Find the moment-generating function for Y.

Solution: $\square$

(c) Find $V(Y)$.

Solution: $\square$