Homework 4

Christopher Munoz

2.112 Three radar sets, operating independently, are set to detect any aircraft flying through a certain area. Each set has a probability of .02 of failing to detect a plane in its area. If an aircraft enters the area, what is the probability that it

a goes undetected?

$$0.02 * 0.02 * 0.02 = 0.000008$$

b is detected by all three radar sets?

$$0.98 * 0.98 * .0.98 = 0.941192$$

2.139 Refer to Exercise 2.112. Let the random variable Y represent the number of radar sets that detect a particular aircraft. Compute the probabilities associated with each value of Y.

$$\text{Let } P(Y = K) = \binom{3}{k} * (0.98)^k * (0.02)^{3-k}$$
$$P(Y = 0) = \text{No radars detect aircraft} = (0.02)^2 = 0.000008$$
$$P(Y = 1) = \text{One radars detects aircraft} = (0.98)^1 * (0.02)^2 = 0.00392$$
$$P(Y = 2) = \text{Two radars detect aircraft}(0.98)^2 * (0.02)^1 = 0.019208$$
$$P(Y = 3) = \text{Three radars detect aircraft} = 0.941192$$

3.1 When the health department tested private wells in a county for two impurities commonly found in drinking water, it found that 20% of the wells had neither impurity, 40% had impurity A, and 50% had impurity B. (Obviously, some had both impurities.) If a well is randomly chosen from those in the county, find the probability distribution for Y , the number of impurities found in the well.

$$\begin{aligned} P(Y = 0) &= \text{No Impurities} = .20 \\ P(Y = 1) &= \text{One Impurity} = .70 \\ P(Y = 2) &= \text{Both Impurities} = .10 \end{aligned}$$

3.2 You and a friend play a game where you each toss a balanced coin. If the upper faces on the coins are both tails, you win \$1; if the faces are both heads, you win \$2; if the coins do not match (one shows a head, the other a tail), you lose \$1 (win (−\$1)). Give the probability distribution for your winnings, Y , on a single play of this game.

$$\begin{aligned} S &= (HH, TT, HT, TH) \\ P(Y = 1) &= \text{Both coins heads} = .50 * .50 = .25 \\ P(Y = 2) &= \text{Both coins tails} = .50 * .50 = .25 \\ P(Y = -1) &= \text{Coins don't match} = .50 \end{aligned}$$

3.3 A group of four components is known to contain two defectives. An inspector tests the components one at a time until the two defectives are located. Once she locates the two defectives, she stops testing, but the second defective is tested to ensure accuracy. Let Y denote the number of the test on which the second defective is found. Find the probability distribution for Y .

$$\begin{aligned} P(Y = 2) &= \text{Second defective found in case 2} = \frac{1}{6} \\ P(Y = 3) &= \text{Second defective found in case 3} = \frac{1}{3} \\ P(Y = 4) &= \text{Second defective found in case 4} = \frac{1}{2} \end{aligned}$$

3.5 A problem in a test given to small children asks them to match each of three pictures of animals to the word identifying that animal. If a child assigns the three words at random to the three pictures, find the probability distribution for Y , the number of correct matches.

$$P(Y=0) = \text{No animals correct} = \frac{2}{6}$$
$$P(Y=1) = \text{One animal correct} = \frac{3}{6}$$
$$P(Y=2) = \text{All animals correct} = \frac{1}{6}$$

3.8 A single cell can either die, with probability .1, or split into two cells, with probability .9, producing a new generation of cells. Each cell in the new generation dies or splits into two cells independently with the same probabilities as the initial cell. Find the probability distribution for the number of cells in the next generation.

$$P(Y=0) = \text{Terminus at } 0 = 0.109$$
$$P(Y=2) = \text{Terminus at } 2 = 0.162$$
$$P(Y=4) = \text{Terminus at } 4 = 0.729$$

3.11 Persons entering a blood bank are such that 1 in 3 have type O+ blood and 1 in 15 have type O− blood. Consider three randomly selected donors for the blood bank. Let X denote the number of donors with type O+ blood and Y denote the number with type O− blood. Find the probability distributions for X and Y . Also find the probability distribution for X + Y , the number of donors who have type O blood.

For X (number with O+ blood): This follows binomial distribution with n=3, p=1/3

$$P(X=0) = \binom{3}{0}(1/3)^0(2/3)^3 = \frac{8}{27}$$
$$P(X=1) = \binom{3}{1}(1/3)^1(2/3)^2 = \frac{12}{27}$$
$$P(X=2) = \binom{3}{2}(1/3)^2(2/3)^1 = \frac{6}{27}$$
$$P(X=3) = \binom{3}{3}(1/3)^3(2/3)^0 = \frac{1}{27}$$

For Y (number with O- blood): This follows binomial distribution with n=3, p=1/15

$$P(Y=0) = \binom{3}{0}(1/15)^0(14/15)^3 = \frac{2744}{3375}$$
$$P(Y=1) = \binom{3}{1}(1/15)^1(14/15)^2 = \frac{588}{3375}$$
$$P(Y=2) = \binom{3}{2}(1/15)^2(14/15)^1 = \frac{42}{3375}$$
$$P(Y=3) = \binom{3}{3}(1/15)^3(14/15)^0 = \frac{1}{3375}$$

For X + Y (total with type O blood): This follows binomial distribution with n=3, p=1/3+1/15=6/15=2/5

$$P(X+Y=0) = \binom{3}{0}(2/5)^0(3/5)^3 = \frac{27}{125}$$
$$P(X+Y=1) = \binom{3}{1}(2/5)^1(3/5)^2 = \frac{54}{125}$$
$$P(X+Y=2) = \binom{3}{2}(2/5)^2(3/5)^1 = \frac{36}{125}$$
$$P(X+Y=3) = \binom{3}{3}(2/5)^3(3/5)^0 = \frac{8}{125}$$

3.12 Let Y be a random variable with p(y) given in the accompanying table. Find E(Y), E(1/Y), E(Y 2 − 1), and V (Y).

y	1	2	3	4
$p(y)$	.4	.3	.2	.1

$$E(Y) = 1(.4) + 2(.3) + 3(.2) + 4(.1) = 2.0$$
$$E(1/Y) = \frac{1}{1}(.4) + \frac{1}{2}(.3) + \frac{1}{3}(.2) + \frac{1}{4}(.1) = 0.658$$
$$E(Y^2) = 1^2(.4) + 2^2(.3) + 3^2(.2) + 4^2(.1) = 5.0$$
$$E(Y^2 - 1) = E(Y^2) - 1 = 5.0 - 1 = 4.0$$
$$V(Y) = E(Y^2) - [E(Y)]^2 = 5.0 - 2.0^2 = 1.0$$

3.13 Refer to the coin-tossing game in Exercise 3.2. Calculate the mean and variance of Y , your winnings on a single play of the game. Note that E(Y) > 0. How much should you pay to play this game if your net winnings, the difference between the payoff and cost of playing, are to have mean 0?

$$\begin{aligned} E(Y) &= (-1)(.5) + (1)(.25) + (2)(.25) = 0.25 \\ E(Y^2) &= (-1)^2(.5) + (1)^2(.25) + (2)^2(.25) = 1.75 \\ V(Y) &= E(Y^2) - [E(Y)]^2 = 1.75 - 0.25^2 = 1.6875 \\ &\text{To have mean 0: pay \$0.25 per game} \end{aligned}$$

3.14 The maximum patent life for a new drug is 17 years. Subtracting the length of time required by the FDA for testing and approval of the drug provides the actual patent life for the drug—that is, the length of time that the company has to recover research and development costs and to make a profit. The distribution of the lengths of actual patent lives for new drugs is given below:

Years, y	3	4	5	6	7	8	9	10	11	12	13
$p(y)$	.03	.05	.07	.10	.14	.20	.18	.12	.07	.03	.01

a Find the mean patent life for a new drug.

$$\begin{aligned} E(Y) &= 3(.03) + 4(.05) + 5(.07) + 6(.10) + 7(.14) \\ &+8(.20) + 9(.18) + 10(.12) + 11(.07) + 12(.03) + 13(.01) = 7.95 \end{aligned}$$

b Find the standard deviation of Y = the length of life of a randomly selected new drug.

$$\begin{aligned} E(Y^2) &= 9(.03) + 16(.05) + 25(.07) + 36(.10) + 49(.14) \\ &+64(.20) + 81(.18) + 100(.12) + 121(.07) + 144(.03)+ \\ &169(.01) = 68.87 \end{aligned}$$

$$V(Y) = 68.87 - 7.95^2 = 5.6475$$

$$\sigma = \sqrt{5.6475} = 2.376$$

c What is the probability that the value of Y falls in the interval $\pm$ 2 ?

$$P(\mu \pm 2\sigma) = P(3.198 \leq Y \leq 12.702) = 0.97$$

3.15 Who is the king of late night TV? An Internet survey estimates that, when given a choice between David Letterman and Jay Leno, 52% of the population prefers to watch Jay Leno. Three late night TV watchers are randomly selected and asked which of the two talk show hosts they prefer.

a Find the probability distribution for Y , the number of viewers in the sample who prefer Leno.

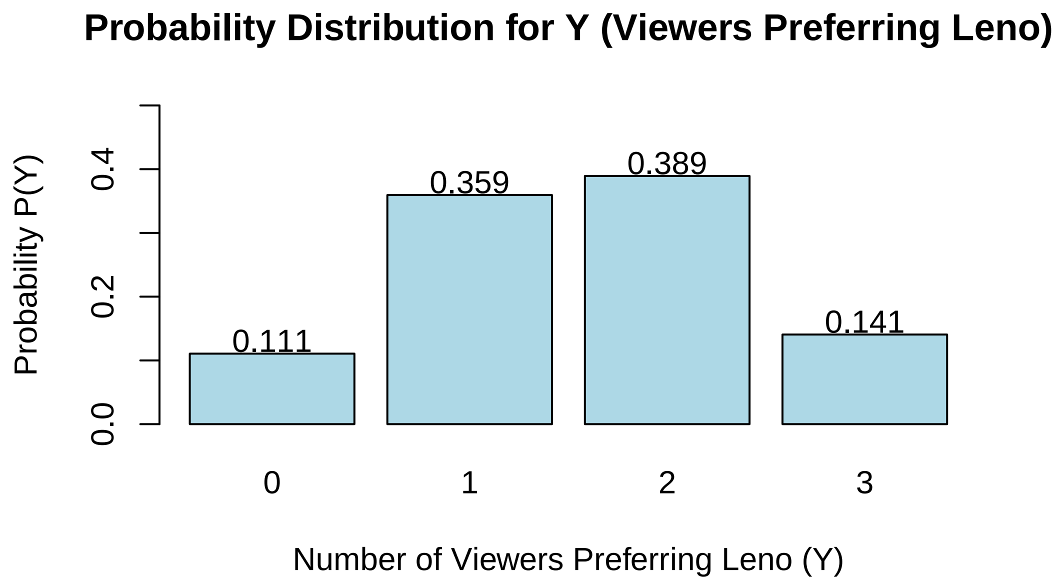
$$P(Y = 0) = \binom{3}{0}(0.52)^0(0.48)^3 = 0.110592$$
$$P(Y = 1) = \binom{3}{1}(0.52)^1(0.48)^2 = 0.359424$$
$$P(Y = 2) = \binom{3}{2}(0.52)^2(0.48)^1 = 0.389376$$
$$P(Y = 3) = \binom{3}{3}(0.52)^3(0.48)^0 = 0.140608$$

b Construct a probability histogram for p(y).

```r
y_values <- c(0, 1, 2, 3)
probabilities <- c(0.110592, 0.359424, 0.389376, 0.140608)

barplot(probabilities,
        names.arg = y_values,
        main = "Probability Distribution for Y (Viewers Preferring Leno)",
        xlab = "Number of Viewers Preferring Leno (Y)",
        ylab = "Probability P(Y)",
        col = "lightblue",
        border = "black",
        ylim = c(0, 0.5))

text(x = seq(0.7, 4.3, by = 1.2),
     y = probabilities + 0.02,
     labels = round(probabilities, 3))
```




```
#His viewers are surprisingly normal wink wink nudge nudge.
```

c What is the probability that exactly one of the three viewers prefers Leno?

$$P(Y = 1) = 0.359424$$

d What are the mean and standard deviation for Y ?

$$E(Y) = np = 3(0.52) = 1.56$$
$$\sigma = \sqrt{np(1-p)} = \sqrt{3(0.52)(0.48)} = 0.866$$

e What is the probability that the number of viewers favoring Leno falls within 2 standard deviations of the mean?

$$P(\mu \pm 2\sigma) = P(0 \leq Y \leq 3) = 1.0$$

3.18 Refer to Exercise 3.8. What is the mean number of cells in the second generation?

$$E(Y) = 0(0.109) + 2(0.162) + 4(0.729) = 3.24$$

3.19 An insurance company issues a one-year \$1000 policy insuring against an occurrence A that historically happens to 2 out of every 100 owners of the policy. Administrative fees are \$15 per policy and are not part of the company's "profit." How much should the company charge for the policy if it requires that the expected profit per policy be \$50? [Hint: If C is the premium for the policy, the company's "profit" is C −15 if A does not occur and C −15−1000 if A does occur.]

$$
\begin{aligned}
\text{Let } C &= \text{premium charged} \\
\text{Profit} &= C - 15 \text{ if no occurrence (prob} = 0.98) \\
\text{Profit} &= C - 15 - 1000 \text{ if occurrence (prob} = 0.02) \\
E(\text{Profit}) &= (C - 15)(0.98) + (C - 1015)(0.02) = 50 \\
C - 15 - 20.3 &= 50 \\
C &= 85.30
\end{aligned}
$$

3.23 In a gambling game a person draws a single card from an ordinary 52-card playing deck. A person is paid \$15 for drawing a jack or a queen and \$5 for drawing a king or an ace. A person who draws any other card pays \$4. If a person plays this game, what is the expected gain?

$$
\begin{aligned}
P(\text{Jack or Queen}) &= \frac{8}{52}, \text{Win } \$15 \\
P(\text{King or Ace}) &= \frac{8}{52}, \text{Win } \$5 \\
P(\text{Other}) &= \frac{36}{52}, \text{Lose } \$4 \\
E(\text{Gain}) = 15(\frac{8}{52}) + 5(\frac{8}{52}) + (-4)(\frac{36}{52}) &= -\$0.38
\end{aligned}
$$

3.25 Two construction contracts are to be randomly assigned to one or more of three firms: I, II, and III. Any firm may receive both contracts. If each contract will yield a profit of \$90,000 for the firm, find the expected profit for firm I. If firms I and II are actually owned by the same individual, what is the owner's expected total profit?

$$
\begin{aligned}
P(\text{Firm I gets 0 contracts}) &= \frac{4}{9} \\
P(\text{Firm I gets 1 contract}) &= \frac{4}{9} \\
P(\text{Firm I gets 2 contracts}) &= \frac{1}{9} \\
E(\text{Profit for Firm I}) = 0(\frac{4}{9}) + 90000(\frac{4}{9}) + 180000(\frac{1}{9}) &= \$60000 \\
E(\text{Profit for Firms I and II combined}) &= \$120000
\end{aligned}
$$