

2.4 Exact equations

2.4.1 problem1

Consider the differential equation: $(\sin(y) - y \sin(x))dx + (\cos(x) + x \cos(y) - 3y)dy = 0$, we solve via the method of exact equations:

$$\begin{aligned} M(x, y) &= (\sin(y) - y \sin(x)) & \frac{\delta M}{\delta y} &= \cos(y) - \sin(x) \\ N(x, y) &= (\cos(x) + x \cos(y) - 3y) & \frac{\delta N}{\delta y} &= \cos(y) - \sin(x) \end{aligned}$$

Now we solve:

$$f(x, y) = \int (\sin(y) - y \sin(x))dx = x \sin(y) + y \cos(x) + g(y) \quad (1)$$

$$\frac{\delta f}{\delta y} = x \cos(y) + \cos(x) + g'(y) = \cos(x) + x \cos(y) - 3y \quad \rightarrow g'(y) = -3y \quad (2)$$

$$\int g'(y)dy = \int -3ydy \quad (3)$$

$$g(y) = -\frac{3}{2}y^2 \quad (4)$$

Thus our solution is:

$$\boxed{x \sin(y) + y \cos(x) - \frac{3}{2}y^2 = C} \quad (5)$$