Write the following in English Sentences. Say whether they are true or false.

Exercise (2.7.1). $\forall x \in \mathbb{R}, x^2 > 0$

For all x in the real numbers, x^2 is greater than 0.

This is false, for example if we let $x = 0$, then $x > 0$.

Exercise (2.7.2). $\forall x \in \mathbb{R}, \exists n \in \mathbb{N}, x^n \geq 0$

For all x in the real numbers, there exists an n in the natural numbers such that $x^n \geq 0$.

This is true, if we let $n = 2$ then any x will be above or equal to zero.

Exercise (2.7.3). $\exists a \in \mathbb{R}, \forall x \in \mathbb{R}, ax = x$

There exists a real number a, such that for all real numbers x, $ax = x$.

This is true, if we let $a = 1$ then $ax = 1x = x$.

Exercise (2.7.4). $\forall X \in \mathcal{P}(\mathbb{N}), X \subseteq \mathbb{R}$

For all X in the powerset of the natural numbers, X is a subset of the Real Numbers.

This is a true statement because every subset of the natural numbers is a subset of the real numbers.

For the following, we're just staying whether its true or false.

Exercise (2.7.5). $\forall n \in \mathbb{N}, \exists X \in \mathcal{P}(\mathbb{N}), |X| = n$

True

Exercise (2.7.6). $\exists n \in \mathbb{N}, \forall X \in \mathcal{P}(\mathbb{N}),$

False

Exercise (2.7.7). $\forall X \subseteq \mathbb{N}, \exists n \in \mathbb{Z}, |X| = n$

False

Exercise (2.7.8). $\forall n \in \mathbb{Z}, \exists X \subseteq \mathbb{N}, |X| = n$

False

Exercise (2.7.9). $\forall n \in \mathbb{Z}, \exists m \in \mathbb{Z}, m = n + 5$

True

Exercise (2.7.10). $\exists m \in \mathbb{Z}, \forall n \in \mathbb{Z}, m = n + 5$

True

For these we are translating sentences into logic

Exercise (2.9.1). if f is a polynomial and its degree is greater than 2, then f' is not constant.

P: f is a polynomial

Q: f has a degree greater than 2

R: f' is constant

Translation: (P $\wedge$ Q) $\Rightarrow \neg$ R

Exercise (2.9.4). For every prime number p, there is another prime number q with $q > p$

let $\mathbb{P}$ be the set of prime numbers.

Translation: $\forall p \in \mathbb{P}, \exists q \in \mathbb{P}, q > p$

Exercise (2.9.6). For every positive number ε, there is a positive number M for which

$|f(x) - b| < \varepsilon$, whenever $x > M$

Exercise (2.9.7).