

CHAPTER 4

Exercise (14). If $n \in \mathbb{Z}$, then $5n^2 + 3n + 7$ is odd. (Try cases.)

Proof. Write your answer here.

□

Exercise (16). If two integers have the same parity, then their sum is even. (Try cases.)

Proof. Write your answer here.

□

Exercise (18). Suppose x and y are positive real numbers. If $x < y$, then $x^2 < y^2$.

Proof. Write your answer here.

□

Exercise (20). If a is an integer and $a^2 \mid a$, then $a \in \{-1, 0, 1\}$.

Proof. Write your answer here.

□

Exercise (26). Every odd integer is a difference of two squares.

Proof. Write your answer here.

□

Exercise (28). Let $a, b, c \in \mathbb{Z}$. Suppose a and b are not both zero, and $c \neq 0$. Prove that $c \gcd(a, b) \leq \gcd(ca, cb)$.

Proof. Write your answer here.

□

CHAPTER 5

Exercise (4). Suppose $a, b, c \in \mathbb{Z}$. If a does not divide bc , then a does not divide b .

Proof. Write your answer here.

□

Exercise (5). Suppose $x \in \mathbb{R}$. If $x^2 + 5x < -$ then $x < 0$.

Proof. Write your answer here.

□

Exercise (6). Suppose $x \in \mathbb{R}$. If $x^3 - x > 0$ then $x > -1$.

Proof. Write your answer here.

□

Exercise (7). Suppose $a, b \in \mathbb{Z}$. If both ab and $a + b$ are even, then both a and b are even.

Proof. Write your answer here.

□

Exercise (9). Suppose $n \in \mathbb{Z}$. If $3 \nmid n^2$, then $3 \nmid n$.

Proof. Write your answer here.

□

Exercise (10). Suppose $x, y, z \in \mathbb{Z}$ and $x \neq 0$. If $x \nmid yz$, then $x \nmid y$ and $x \nmid z$.

Proof. Write your answer here.

□

Exercise (16). Suppose $x, y \in \mathbb{Z}$. If $x + y$ is even, then x and y have the same parity.

Proof. Write your answer here.

□

Exercise (18). If $a, b \in \mathbb{Z}$, then $(a + b)^3 \equiv a^3 + b^3 \pmod{3}$.

Proof. Write your answer here.

□

Exercise (19). Let $a, b, c \in \mathbb{Z}$ and $n \in \mathbb{N}$. If $a \equiv b \pmod{n}$ and $a \equiv c \pmod{n}$, then $c \equiv b \pmod{n}$.

Proof. Write your answer here.

□

Exercise (22). Let $a \in \mathbb{Z}, n \in \mathbb{N}$. If a has remainder r when divided by n , then $a \equiv r \pmod{n}$.

Proof. Write your answer here.

□

Exercise (24). If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$, then $ac \equiv bd \pmod{n}$.

Proof. Write your answer here.

□

Exercise (25). Let $n \in \mathbb{N}$. If $2^n - 1$ is prime, then n is prime.

Proof. Write your answer here.

□

Exercise (32). If $a \equiv b \pmod{n}$, then a and b have the same remainder when divided by n .

Proof. Write your answer here.

□