## Section 12.2

*Exercise* (18). Prove that the function $f : \mathbb{N} \to \mathbb{Z}$ defined as $f(n) = \frac{(-1)^n(2n-1)+1}{4}$ is bijective.

*Proof.* Write your answer here. $\square$

## Section 12.3

*Exercise* (1). Prove that if six integers are chosen at random, then at least two of them will have the same remainder when divided by 5.

*Proof.* Write your answer here. $\square$

*Exercise* (4). Consider a square whose side-length is one unit. Select any five points from inside this square. Prove that at least two of these points are within $\frac{\sqrt{2}}{2}$ units of each other.

*Remark: It may help your argument to include suitable drawings.*

*Proof.* Write your answer here. $\square$

## Section 12.4

*Exercise* (2). Suppose $A = \{1, 2, 3, 4\}, B = \{0, 1, 2\}, C = \{1, 2, 3\}$. Let $f : A \to B$ be $f = \{(1, 0), (2, 1), (3, 2), (4, 0)\}$, and $g : B \to C$ be $g = \{(0, 1), (1, 1), (2, 3)\}$. Find $g \circ f$.

*Solution.* Write your answer here. $\square$

*Exercise* (4). Suppose $A = \{a, b, c\}$. Let $f : A \to A$ be the function $f = \{(a, c), (b, c), (c, c)\}$, and let $g : A \to A$ be the function $g = \{(a, a), (b, b), (c, a)\}$. Find $g \circ f$ and $f \circ g$.

*Solution.* Write your answer here. $\square$

*Exercise* (6). Consider the functions $f, g : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = \frac{1}{x^2+1}$ and $g(x) = 3x + 2$. Find the formulas for $g \circ f$ and $f \circ g$.

*Solution.* Write your answer here. □

*Exercise* (7). Consider the functions $f, g : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ defined as $f(m, n) = (3m - 4n, 2m + n)$ and $g(m, n) = 5m + n, m)$. Find the formulas for $g \circ f$ and $f \circ g$.

*Solution.* Write your answer here. □

## Reflection

*Exercise* (Reflection Problem).

*Answers.*

**How long did it take you to complete each problem?:**

**What was easy?:**

**What was challenging? What made it challenging?:**

**What did you learn from comparing your answers to those in the book?:**

□

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