Chapter 6

Exercise (2). Suppose $n \in \mathbb{Z}$. If n^2 is odd, then n is odd.

Proof: Suppose for the sake of contradiction that n^2 is odd and n is not odd, Then n is even, so $n = 2k$ for some $k \in \mathbb{Z}$. Therefore $n^2 = (2k)^2 = 4k^2 = 2(2k^2) = 2b$, where $b \in \mathbb{Z}$ by closure properties of the integers. So n^2 is even, this is a contradiction. So it must be the case that if n^2 is odd then n is odd. $\square$

Exercise (3). Prove that $\sqrt[3]{2}$ is irrational.

Proof: Suppose for the sake of contradiction that $\sqrt[3]{2}$ is not irrational. Then $\sqrt[3]{2}$ is a rational and is in the form $\sqrt[3]{2} = \frac{a}{b}$ where $a, b \in \mathbb{Z}$ and the ratio $\frac{a}{b}$ being in its most reduced form. Observe that cubing both sides $2 = (\frac{a}{b})^3 = \frac{a^3}{b^3}$ so that $2b^3 = a^3$. This implies that a is an even number which is contradictory to $\frac{a}{b}$ existing in its lowest forms. Thus $sqrt[3]2$ is irrational. $\square$

Exercise (4). Prove that $\sqrt{6}$ is irrational.

Proof: Write your answer here. $\square$

Exercise (8). Suppose $a, b, c \in \mathbb{Z}$. If $a^2 + b^2 = c^2$, then a or b is even.

Proof: Write your answer here. $\square$

Exercise (9). Suppose $a, b \in \mathbb{R}$. If a is rational and ab is irrational, then b is irrational.

Proof: Write your answer here. $\square$

Exercise (11). There exist no integers a and b for which $18a + 6b = 1$.

Proof: Write your answer here. $\square$

Exercise (12). For every positive $x \in \mathbb{Q}$, there is a positive $y \in \mathbb{Q}$ for which $y < x$.

Proof: Write your answer here. $\square$

Exercise (16). If a and b are positive real numbers, then $a + b \geq 2\sqrt{ab}$.

Proof: Write your answer here. $\square$

Exercise (19). The product of any five consecutive integers is divisible by 120. (For example, the product of 3, 4, 5, 6 and 7 is 2520, and $2520 = 120 \cdot 21$.)

Proof: Write your answer here. □

Chapter 7

Exercise (1). Suppose $x \in \mathbb{Z}$. Then x is even if and only if $3x + 5$ is odd.

Proof: Write your answer here. □

Exercise (4). Let a be an integer. Then $a^2 + 4a + 5$ is odd if an d only if a is even.

Proof: Write your answer here. □

Exercise (7). Suppose $x, y \in \mathbb{R}$. Then $(x + y)^2 = x^2 + y^2$ if and only if $x = 0$ or $y = 0$.

Proof: Write your answer here. □

Exercise (Reflection Problem). *Proof:* Write your answer here. □

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