Section 3.8

Exercise (3.121). Let Y denote a random variable that has a Poisson distribution with mean $\lambda = 2$. Find:

(a) $P(Y = 4)$

Solution: Poisson distribution function is

$$P(Y = y) = \frac{e^{-\lambda}\lambda^y}{y!}$$

$$P(Y = 4) = \frac{e^{-2}(-2)^4}{4!} = 0.09022352$$

$\square$

(b) $P(Y \geq 4)$

Solution:

$$\begin{aligned} P(Y \geq 4) &= 1 - [P(Y = 3) + P(Y = 2) + P(Y = 1) + P(Y = 0)] \\ &= 1 - [\frac{e^{-2}(-2)^3}{3!} + \frac{e^{-2}(-2)^2}{2!} + \frac{e^{-2}(-2)^1}{1!} + \frac{e^{-2}(-2)^0}{0!}] \\ &= 1 - [0.180447 + 0.2706706 + 0.2706706 + 0.1353353] \\ &= 0.1428765 \end{aligned}$$

(table gives me 1-0.857 = 0.143 but wanted to do it full at least once) $\square$

(c) $P(Y < 4)$

Solution:

$$P(Y < 4) = P(Y \leq 3) = 0.857$$

(table) $\square$

(d) $P(Y \geq 4 \mid Y \geq 2)$

Solution:

$$P(Y \geq 4 \mid Y \geq 2) = \frac{P(Y \geq 4) \cap P(Y \geq 2)}{P(Y \geq 2)} = \frac{P(Y \geq 4)}{P(Y \geq 2)} = \frac{0.143}{1 - 0.406} = 0.241$$

(table and prev prob) $\square$

Exercise (3.122). Customers arrive at a checkout counter in a department store according to a Poisson distribution at an average of seven per hour. During a given hour, what are the probabilities that:

(a) no more than three customers arrive?

Solution:

$$P(Y \leq 3) = 0.082$$

(table) $\square$

(b) at least two customers arrive?

Solution:

$$P(Y \geq 2) = 1 - P(Y \leq 1) = 1 - 0.007 = 0.993$$

$\square$

(c) exactly five customers arrive?

Solution:

$$P(Y = 5) = \frac{e^{-7}7^5}{5!} = 0.1277167$$

$\square$

Exercise (3.125). Refer to Exercise 3.122. If it takes approximately ten minutes to serve each customer, find the mean and variance of the total service time for customers arriving during a 1-hour period. (Assume that a sufficient number of servers are available so that no customer must wait for service.) Is it likely that the total service time will exceed 2.5 hours?

Solution: Note that 10 minutes = 1/6 of an hour, $E(Y) = \lambda$ and $V(Y) = \lambda$ for Poisson.

$$E(\frac{1}{6}Y) = \frac{1}{6}E(Y) = \frac{1}{6} * 7 = 1.166667$$
$$V(\frac{1}{6}Y) = (\frac{1}{6})^2 V(Y) = \frac{1}{36} * 7 = 0.1944444$$

Not likely to exceed 2.5 hours of total service time since the variance is already so small, the standard deviation would be smaller $\square$

Exercise (3.127). The number of typing errors made by a typist has a Poisson distribution with an average of four errors per page. If more than four errors appear on a given page, the typist must retype the whole page. What is the probability that a randomly selected page does not need to be retyped?

Solution: $\lambda = 4$ and $Y \leq 4$ for this one.

$$P(Y \leq 4) = 0.629$$

(table) □

Exercise (3.131). The number of knots in a particular type of wood has a Poisson distribution with an average of 1.5 knots in 10 cubic feet of the wood. Find the probability that a 10-cubic-foot block of the wood has at most 1 knot.

Solution: $\lambda = 1.5$ and $Y \leq 1$.

$$P(Y \leq 1) = 0.558$$

(table really has everything doesn't it) □

Exercise (3.134). Consider a binomial experiment for $n = 20$, $p = .05$. Use Table 1, Appendix 3, to calculate the binomial probabilities for $Y = 0, 1, 2, 3,$ and 4. Calculate the same probabilities by using the Poisson approximation with $\lambda = np$. Compare.

Solution: Binomial Distribution for $P(Y \leq 4)$ for $n = 20$ and $p = 0.5$ is 0.997 according to back table. For our Poisson Distribution $\lambda = 20 * 0.05 = 1$, we refer to the table in the back and calculate it.

$$\begin{aligned} P(Y \leq 4) &= 0.996 \text{ (table)} \\ P(Y \leq 4) &= P(Y = 0) + P(Y = 1) + P(Y = 2) + P(Y = 3) + P(Y = 4) \\ &= e^{-1}[\frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!}] \\ &= 0.9963402 \text{ (full calculation)} \end{aligned}$$

Depending on who you are, they are pretty close. □

Exercise (3.135). A salesperson has found that the probability of a sale on a single contact is approximately .03. If the salesperson contacts 100 prospects, what is the approximate probability of making at least one sale?

Solution: We use Poisson Approximation

$$P(Y \geq 1) = 1 - P(Y = 0) = 1 - \frac{e^{-3}3^0}{0!} = e^{-3} = 0.9502129$$

□

Exercise (3.139). In the daily production of a certain kind of rope, the number of defects per foot Y is assumed to have a Poisson distribution with mean $\lambda = 2$. The profit per foot when the rope is sold is given by X, where $X = 50 - 2Y - Y^2$. Find the expected profit per foot.

Solution:

$$\begin{aligned} E(X) &= E[50 - 2Y - Y^2] \\ &= E(50) - 2E(Y) - E(Y^2) \\ &= 50 - 2(2) - [V(Y) + [E(Y)]^2] \\ &= 50 - 4 - (2 + 4) \\ &= 40 \end{aligned}$$

□

Exercise (3.141). A food manufacturer uses an extruder (a machine that produces bite-size cookies and snack food) that yields revenue for the firm at a rate of \$200 per hour when in operation. However, the extruder breaks down an average of two times every day it operates. If Y denotes the number of breakdowns per day, the daily revenue generated by the machine is $R = 1600 - 50Y^2$. Find the expected daily revenue for the extruder.

Solution:

$$\begin{aligned} E(R) &= E[1600 - 50Y^2] \\ &= E[1600] - 50E[Y^2] \\ &= 1600 - 50[V(Y) + E(Y)^2] \\ &= 1600 - 50[2 + 4] \\ &= 1300 \end{aligned}$$

□

Section 3.9

Exercise (3.145). If Y has a binomial distribution with n trials and probability of success p, show that the moment-generating function for Y is

$$m(t) = (pe^t + q)^n, \text{ where } q = 1 - p.$$

Solution: □

Exercise (3.146). Differentiate the moment-generating function in Exercise 3.145 to find $E(Y)$ and $E(Y^2)$. Then find $V(Y)$.

Solution: □

Exercise (3.147). If Y has a geometric distribution with probability of success p, show that the moment-generating function for Y is

$$m(t) = \frac{pe^t}{1 - qe^t}, \text{ where } q = 1 - p.$$

Solution: □

Exercise (3.149). Refer to Exercise 3.145. Use the uniqueness of moment-generating functions to give the distribution of a random variable with moment-generating function $m(t) = (0.6e^t + 0.4)^3$.

Solution: □

Exercise (3.151). Refer to Exercise 3.145. If Y has moment-generating function $m(t) = (0.7e^t + 0.3)^{10}$, what is $P(Y \leq 5)$?

Solution: □

Exercise (3.153). Find the distributions of the random variables that have each of the following moment-generating functions:

(a) $m(t) = [(1/3)e^t + (2/3)]^5$

Solution: □

(b) $m(t) = \frac{e^t}{2 - e^t}$

Solution: □

(c) $m(t) = e^{2(e^t - 1)}$

Solution: □

Exercise (3.155). Let $m(t) = \frac{1}{6}e^t + \frac{2}{6}e^{2t} + \frac{3}{6}e^{3t}$. Find the following:

(a) $E(Y)$

Solution: □

(b) $V(Y)$

Solution: □

(c) The distribution of Y

Solution: □

3.11

Exercise (3.167). Let Y be a random variable with mean 11 and variance 9. Using Tchebysheff's theorem, find:

(a) a lower bound for $P(6 < Y < 16)$

Solution: □

(b) the value of C such that $P(|Y - 11| \geq C) \leq 0.09$

Solution: □

Exercise (3.168). Would you rather take a multiple-choice test or a full-recall test? If you have absolutely no knowledge of the test material, you will score zero on a full-recall test. However, if you are given 5 choices for each multiple-choice question, you have at least one chance in five of guessing each correct answer! Suppose that a multiple-choice exam contains 100 questions, each with 5 possible answers, and you guess the answer to each of the questions.

(a) What is the expected value of the number Y of questions that will be correctly answered?

Solution: □

(b) Find the standard deviation of Y.

Solution: □

(c) Calculate the intervals $\mu \pm 2\sigma$ and $\mu \pm 3\sigma$.

Solution: □

(d) If the results of the exam are curved so that 50 correct answers is a passing score, are you likely to receive a passing score? Explain.

Solution: □

Exercise (3.171). For a certain type of soil the number of wireworms per cubic foot has a mean of 100. Assuming a Poisson distribution of wireworms, give an interval that will include at least 5/9 of the sample values of wireworm counts obtained from a large number of 1-cubic-foot samples.

Solution: □

4.2

Exercise (4.1). Let Y be a random variable with $p(y)$ given in the table below.

y	1	2	3	4
$p(y)$	.4	.3	.2	.1

(a) Give the distribution function, $F(y)$. Be sure to specify the value of $F(y)$ for all y, $-\infty < y < \infty$.

Solution: □

(b) Sketch the distribution function given in part (a).

Solution: □

Exercise (4.3). A Bernoulli random variable is one that assumes only two values, 0 and 1 with $p(1) = p$ and $p(0) = 1 - p \equiv q$.

(a) Sketch the corresponding distribution function.

Solution: □

(b) Show that this distribution function has the properties given in Theorem 4.1.

Solution: □

Exercise (4.5). Suppose that Y is a random variable that takes on only integer values $1, 2, \ldots$ and has distribution function $F(y)$. Show that the probability function $p(y) = P(Y = y)$ is given by

$$p(y) = \begin{cases} F(1), & y = 1, \\ F(y) - F(y-1), & y = 2, 3, \ldots \end{cases}$$

Solution: □

Exercise (4.7). Let Y be a binomial random variable with $n = 10$ and $p = 0.2$.

(a) Use Table 1, Appendix 3, to obtain $P(2 < Y < 5)$ and $P(2 \leq Y < 5)$. Are the probabilities that Y falls in the intervals $(2, 5)$ and $[2, 5)$ equal? Why or why not?

Solution: □

(b) Use Table 1, Appendix 3, to obtain $P(2 < Y \leq 5)$ and $P(2 \leq Y \leq 5)$. Are these two probabilities equal? Why or why not?

Solution: □

(c) Earlier in this section, we argued that if Y is continuous and $a < b$, then $P(a < Y < b) = P(a \le Y < b)$. Does the result in part (a) contradict this claim? Why?

Solution: □

Exercise (4.9). A random variable Y has the following distribution function:

$$F(y) = P(Y \le y) = \begin{cases} 0, & \text{for } y < 2, \\ 1/8, & \text{for } 2 \le y < 2.5, \\ 3/16, & \text{for } 2.5 \le y < 4, \\ 1/2, & \text{for } 4 \le y < 5.5, \\ 5/8, & \text{for } 5.5 \le y < 6, \\ 11/16, & \text{for } 6 \le y < 7, \\ 1, & \text{for } y \ge 7. \end{cases}$$

(a) Is Y a continuous or discrete random variable? Why?

Solution: □

(b) What values of Y are assigned positive probabilities?

Solution: □

(c) Find the probability function for Y.

Solution: □

(d) What is the median, $\phi_{.5}$, of Y?

Solution: □

Exercise (4.11). Suppose that Y possesses the density function

$$f(y) = \begin{cases} cy, & 0 \le y \le 2, \\ 0, & \text{elsewhere}. \end{cases}$$

(a) Find the value of c that makes $f(y)$ a probability density function.

Solution: □

(b) Find $F(y)$.

Solution: □

(c) Graph $f(y)$ and $F(y)$.

Solution: □

(d) Use $F(y)$ to find $P(1 \leq Y \leq 2)$.

Solution: □

(e) Use $f(y)$ and geometry to find $P(1 \leq Y \leq 2)$.

Solution: □

Exercise (4.13). A supplier of kerosene has a 150-gallon tank that is filled at the beginning of each week. His weekly demand shows a relative frequency behavior that increases steadily up to 100 gallons and then levels off between 100 and 150 gallons. If Y denotes weekly demand in hundreds of gallons, the relative frequency of demand can be modeled by

$$f(y) = \begin{cases} y, & 0 \leq y \leq 1, \\ 1, & 1 < y \leq 1.5, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find $F(y)$.

Solution: □

(b) Find $P(0 \leq Y \leq 0.5)$.

Solution: □

(c) Find $P(0.5 \leq Y \leq 1.2)$.

Solution: □

Exercise (4.17). The length of time required by students to complete a one-hour exam is a random variable with a density function given by

$$f(y) = \begin{cases} cy^2 + y, & 0 \leq y \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find c.

Solution: □

(b) Find $F(y)$.

Solution: □

(c) Graph $f(y)$ and $F(y)$.

Solution: □

(d) Use $F(y)$ in part (b) to find $F(-1)$, $F(0)$, and $F(1)$.

Solution: □

(e) Find the probability that a randomly selected student will finish in less than half an hour.

Solution: □

(f) Given that a particular student needs at least 15 minutes to complete the exam, find the probability that she will require at least 30 minutes to finish.

Solution: □

Exercise (4.19). Let the distribution function of a random variable Y be

$$F(y) = \begin{cases} 0, & y \leq 0, \\ \frac{y}{8}, & 0 < y < 2, \\ \frac{y^2}{16}, & 2 \leq y < 4, \\ 1, & y \geq 4. \end{cases}$$

(a) Find the density function of Y.

Solution: □

(b) Find $P(1 \leq Y \leq 3)$.

Solution: □

(c) Find $P(Y \geq 1.5)$.

Solution: □

(d) Find $P(Y \geq 1 \mid Y \leq 3)$.

Solution: □