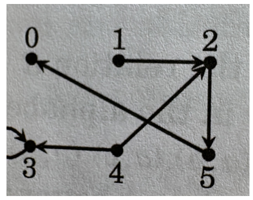
Section 11.1

Exercise (5). Here is a digram for a relation R on a set A. Write the sets A and R.



Solution:

$$A = 0, 1, 2, 3, 4, 5$$

$$R = (1, 2), (2, 5), (4, 2), (4, 3), (3, 3), (5, 0)$$

□

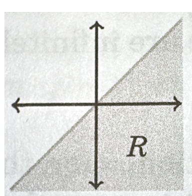
Exercise (6). Congruence modulo 5 is a relation on the set $A = \mathbb{Z}$. In this relation xRy $x \equiv y \pmod 5$. Write out the set R in set-builder notation.

Solution:

$$R = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid x \equiv y\}$$

□

In the following exercises, subsets R of $\mathbb{R}^2$ or $\mathbb{Z}^2$ are indicated by gray shading. In each case, R is a familiar relation on $\mathbb{R}$ or $\mathbb{Z}$. State it.

Exercise (12). 

Proof: R is the relation: $\geq$, in particular this is $x \geq y$. □

Section 11.2

Note: When a property does not hold, it suffices to describe **a** *counterexample.*

Exercise (2). Consider the relation $R = \{(a, b), (a, c), (c, c), (b, b), (c, b), (b, c)\}$ on the set $A = \{a, b, c\}$. Is R reflexive? Symmetric? Transitive? If a property does not hold, say why.

Solution: R is reflexive, □

:x

Exercise (5). Consider the relation $R = \{(0,0), (\sqrt{2}, 0), (0, \sqrt[2]{}), (\sqrt{2}, \sqrt{2})$ on $\mathbb{R}$. Is R reflexive? Symmetric? Transitive? If a property does not hold, say why.

Solution: Write your answer here. $\square$

Exercise (6). Consider the relation $R = \{(x, x) : x \in \mathbb{Z}\}$ on $\mathbb{Z}$. Is R reflexive? Symmetric? Transitive? If a property does not hold, say why.

Solution: Write your answer here. $\square$

Exercise (8). Define a relation on $\mathbb{Z}$as xRy if $|x - y| < 1$. Is R reflexive? Symmetric? Transitive? If a property does not hold, say why. What familiar relation is this?

Solution: Write your answer here. $\square$

Exercise (12). Prove that the relation $|$ (divides) on the set $\mathbb{Z}$is reflexive and transitive. (Use example 11.8 as a guide if you are unsure how to proceed.)

Solution: Write your answer here. $\square$

Exercise (14). Suppose R is a symmetric and transitive relation on a set A, and there is an element $a \in A$ for which aRx for every $x \in A$. Prove that R is reflexive.

Proof: Write your answer here. $\square$

Exercise (15). Prove or disprove: If a relation is symmetric and transitive, then it is also reflexive.

Solution: Write your answer here. $\square$

Section 11.3

Exercise (3). Let $A = \{a, b, c, d, e\}$. Suppose R is an equivalence relation on A. Suppose R has three equivalence classes. Also aRd and bRc. Write out R as a set.

Solution: Write your answer here. $\square$

Exercise (5). There are two different equivalence relations on the set $A = \{a, b\}$. Describe them. Diagrams will suffice.

Solution: Write your answer here. $\square$

Exercise (8). Define a relation R on $\mathbb{Z}$as xRy iff $x^2 + y^2$ is even. Prove R is an equivalence relation. Describe its equivalence classes.

Proof: Write your answer here. $\square$

Solution: Write your answer here. □

Section 11.4

Exercise (1). List all the partitions of the set $A = \{a, b\}$. Compare your answer to the answer to Exercise 5 of Section 11.3.

Solution: Write your answer here. □

Exercise (4). Suppose P is a partition of a set A. Define a relation R on A by declaring xRy if and only if $x, y \in X$ for some $X \in P$. Prove that R is an equivalence relation on A. Then prove that P is the set of equivalence classes of R.

Proof: Write your answer here. □

Exercise (Reflection Problem).

- How long did it take you to complete each problem?

  Answer: □

- What was easy?

  Answer: □

- What was challenging? What made it challenging?

  Answer: □

- Compare your answers to the odd numbered exercises to those in the back of the textbook. What did you learn from this comparison?

  Answer: □

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