Section 3.5

Exercise (3.92). Ten percent of the engines manufactured on an assembly line are defective. If engines are randomly selected one at a time and tested, what is the probability that the first nondefective engine will be found on the second trial?

Solution: We have Y-Geom(.10) where we want the second success so

$$P(Y = 2) = (1 - .90)^{2-1}(.90) = 0.09$$

□

Exercise (3.93). Refer to Exercise 3.92. What is the probability that the third nondefective engine will be found

a on the fifth trial?

Solution:

$$P(Y = 5) = \binom{5-1}{3-1}(.90)^3(1 - .90)^{5-3} = 0.04374$$

□

b on or before the fifth trial?

Solution:

$$\begin{aligned}
P(Y \leq 5) &= P(Y = 3) + P(Y = 4) + P(Y = 5) \\
&= \binom{3-1}{3-1}(.90)^3(1 - .90)^{3-3} + \binom{4-1}{3-1}(.90)^3(1 - .90)^{4-3} + 0.04374 \\
&= 0.729 + 0.2187 + 0.04374 \\
&= 0.99144
\end{aligned}$$

□

Exercise (3.97). A geological study indicates that an exploratory oil well should strike oil with probability .2.

a What is the probability that the first strike comes on the third well drilled?

Solution:

$$P(Y = 3) = (1 - .2)^{3-1}(.2) = 0.128$$

□

b What is the probability that the third strike comes on the seventh well drilled?

Solution: Negative Binomial Distribution with $r = 3, p = .2, k = 7$.

$$P(Y = k) = \binom{k-1}{r-1} p^r (1-p)^{k-r}$$
$$P(Y = 7) = \binom{7-1}{3-1} (.2)^3 (.8)^4$$
$$= 15(0.008)(0.4096)$$
$$= 0.049152$$

□

c What assumptions did you make to obtain the answers to parts (a) and (b)?

Solution: That the trials were independent. □

d Find the mean and variance of the number of wells that must be drilled if the company wants to set up three producing wells.

Solution:

$$E(Y) = \frac{r}{p} = \frac{3}{.2} = 15$$
$$V(Y) = \frac{r(1-p)}{p^2} = \frac{3(.8)}{.2^2} = 6$$

□

Section 3.6

Exercise (3.102). An urn contains ten marbles, of which five are green, two are blue, and three are red. Three marbles are to be drawn from the urn, one at a time without replacement. What is the probability that all three marbles drawn will be green?

Solution: Our variables for Hypergeo: $N = 10, K = 5, n = 3, k = 3$

$$\begin{aligned} P(X = k) &= \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}} \\ &= \frac{\binom{5}{3}\binom{10-5}{3-3}}{\binom{10}{3}} \\ &= \frac{\binom{5}{3}\binom{5}{0}}{\binom{10}{3}} \\ &= 0.08333333 \end{aligned}$$

$\square$

Exercise (3.104). Twenty identical looking packets of white power are such that 15 contain cocaine and 5 do not. Four packets were randomly selected, and the contents were tested and found to contain cocaine. Two additional packets were selected from the remainder and sold by undercover police officers to a single buyer. What is the probability that the 6 packets randomly selected are such that the first 4 all contain cocaine and the 2 sold to the buyer do not?

Solution: Event 1(first four contain cocaine) $X_1 :: N_1 = 20, K_1 = 15, n_1 = 4, k_1 = 4$

Event 2(Choose 2 no cocaine) $X_2 :: N_2 = 16, K_2 = 11, n_2 = 2, k_1 = 0$

$$P(X_1 = 4) = \frac{\binom{15}{4}\binom{5}{0}}{\binom{20}{4}} = .2817$$

$$P(X_2 = 0) = \frac{\binom{11}{0}\binom{5}{2}}{\binom{16}{2}} = .0833$$

$$P(X_1 \cap X_2) = .2817 * .0833 = 0.0235$$

$\square$

Exercise (3.107). A group of six software packages available to solve a linear programming problem has been ranked from 1 to 6 (best to worst). An engineering firm, unaware of the rankings, randomly se- lected and then purchased two of the packages. Let Y denote the number of packages purchased by the firm that are ranked 3, 4, 5, or 6. Give the probability distribution for Y.

Solution: Our variables for hypergeometric distribution $N = 6, K = 4, n = 2$ so our probability distribution is

$$P(Y = k) = \frac{\binom{4}{k}\binom{6-4}{2-k}}{\binom{6}{2}}$$

□

Exercise (3.113). A jury of 6 persons was selected from a group of 20 potential jurors, of whom 8 were African American and 12 were white. The jury was supposedly randomly selected, but it contained only 1 African American member. Do you have any reason to doubt the randomness of the selection?

Solution: Our variables for Hypergeometric distribution is $N = 20, K = 8, n = 6, k = \{1, 0\}$.

$$\begin{aligned} P(K \leq 1) &= P(k = 0) + P(k = 1) \\ &= \frac{\binom{8}{3}\binom{12}{5}}{\binom{20}{6}} + \frac{\binom{8}{0}\binom{12}{6}}{\binom{20}{6}} \\ &= .187 \end{aligned}$$

□

That probability seems pretty reasonable to me.

Exercise (3.117). In an assembly-line production of industrial robots, gearbox assemblies can be installed in one minute each if holes have been properly drilled in the boxes and in ten minutes if the holes must be redrilled. Twenty gearboxes are in stock, 2 with improperly drilled holes. Five gearboxes must be selected from the 20 that are available for installation in the next five robots.

a Find the probability that all 5 gearboxes will fit properly.

Solution: Hypergeometric distribution with variables $N = 20, K = 18, n = 5, k = 5$.

$$P(X = k) = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$$

$$P(X = 5) = \frac{\binom{18}{5}\binom{2}{0}}{\binom{20}{5}}$$

$$= 0.5526316$$

□

b Find the mean, variance, and standard deviation of the time it takes to install these 5 gearboxes.

Solution: Installation time is $T = 5 + 9X$

$$E(T) = E(5 + 9X) = 5 + 9E(X) = 5 + 9(0.5) = 9.5 \text{ minutes}$$

$$\text{Var}(T) = \text{Var}(5 + 9X) = 9^2 \cdot \text{Var}(X) = 81(0.3553) = 28.78$$

$$\text{SD}(T) = \sqrt{\text{Var}(T)} = \sqrt{28.78} = 5.36 \text{ minutes}$$

□

Below is an exercise I did by accident and I didn't feel like omitting it.

Exercise (3.96). The telephone lines serving an airline reservation office are all busy about 60% of the time.

a If you are calling this office, what is the probability that you will complete your call on the first try? The second try? The third try?

Solution: Since this is a geometric series, we use our formula $P(Y = k) = (1-p)^{(k-1)} * p$ for

$p = .6$ and k to represents our trials.

$$P(Y = 1) = .4$$

$$P(Y = 2) = .6 * .4 = .24$$

$$P(Y = 3) = (.6)^2 * .4 = .144$$

□

b If you and a friend must both complete calls to this office, what is the probability that a total of four tries will be necessary for both of you to get through?

Solution: We use a Negative Binomial Distribution for r-th success where $p = .4, r = 2$

$$
\begin{aligned}
P(Y = k) &= \binom{k-1}{r-1} p^r (1-p)^{k-r} \\
&= \binom{4-1}{2-1} .4^2 (1 - .4)^{4-2} \\
&= 3 * (.16)(0.36) \\
&= 0.1728
\end{aligned}
$$

□