Chapter 4

Exercise (2). If x is an odd integer, then x^3 is odd.

Proof. Write your answer here. □

Exercise (4). Suppose $x, y \in \mathbb{Z}$. If x and y are odd, then xy is odd.

Proof. Write your answer here. □

Exercise (6). Suppose $a, b, c \in \mathbb{Z}$. If $a \mid b$ and $a \mid c$, then $a \mid (b + c)$.

Proof. Write your answer here. □

Exercise (11). Suppose $a, b, c, d \in \mathbb{Z}$. If $a \mid b$ and $c \mid d$, then $ac \mid bd$.

Proof. Write your answer here. □

Exercise (12). If $x \in \mathbb{R}$ and $0 < x < 4$, then $\frac{4}{x(4-x)} \geq 1$.

Proof. Write your answer here. □

Exercise (14). If $n \in \mathbb{Z}$, then $5n^2 + 3n + 7$ is odd. (Try cases.)

Proof. Write your answer here. □

Exercise (16). If two integers have the same parity, then their sum is even. (Try cases.)

Proof. Write your answer here. □

Exercise (18). Suppose x and y are positive real numbers. If $x < y$, then $x^2 < y^2$.

Proof. Write your answer here. □

Exercise (20). If a is an integer and $a^2 \mid a$, then $a \in \{-1, 0, 1\}$.

Proof. Write your answer here. □

Exercise (26). Every odd integer is a difference of two squares.

Proof. Write your answer here. □

Exercise (28). Let $a, b, c \in \mathbb{Z}$. Suppose a and b are not both zero, and $c \neq 0$. Prove that $c \gcd(a, b) \leq gcd(ca, cb)$.

Proof. Write your answer here. □

Exercise (Reflection Problem). Answer the following questions:

Proof.

- How long did it take you to complete each problem?
  Write your answer here.
- What was easy?
  Write your answer here.
- What was challenging? What made it challenging?
  Write your answer here.
- Compare your answers to the odd numbered exercises to those in the back of the textbook. What did you learn from this comparison?
  Write your answer here.

□