4.2.1

We are given the differential equation $y''+2y'+y=0$ with the solution $y_1 = xe^{-x}$ we will construct the second solution using the formula provided: $y_2 = y_1 \int \frac{e^{-\int p(x)dx}}{(y_1)^2}dx$.

$$y_2 = y_1 \int \frac{e^{-\int p(x)dx}}{(y_1)^2}dx \tag{1}$$

$$= xe^{-x} \int \frac{e^{-\int 2dx}}{(xe^{-x})^2}dx \qquad p(x) = 2 \tag{2}$$

$$= xe^{-x} \int \frac{e^{-2x}}{(xe^{-x})^2}dx \tag{3}$$

$$= xe^{-x} \int \frac{e^{-2x}}{(x^2e^{-2x})}dx \tag{4}$$

$$= xe^{-x} \int \frac{1}{x^2}dx \tag{5}$$

$$= xe^{-x} * (-\frac{1}{x}) \tag{6}$$

$$= -e^{-x} \tag{7}$$

Our second solution is

$$\boxed{y_2 = C_2e^{-x}}$$

4.2.2