We will begin by building theorems from the ground up from basic rules

Theorem 0.1. *Convergence: For $A_n \to L$ means: For all $\epsilon > 0$, there exists N such that for all $n \geq N$, $\mid a_n - L \mid < \epsilon$.*

Theorem 0.2. *Triangle inequality*

$$\begin{aligned} \textit{Triangle inequality :} &\quad \mid a+b \mid \leq \mid a \mid + \mid b \mid \\ \textit{Reverse triangle :} &\quad ||a| - |b|| \leq |a-b| \\ \textit{Product bound :} &\quad |ab| = |a||b| \end{aligned}$$

We will now prove our first theorem

Theorem 0.3. *(Uniqueness of Limits) If $a_n \to L$ and $a_n \to M$ then $L = M$.*

Proof. Let $\epsilon > 0$ be arbitrary. Since $a_n \to L$ there exists an N_1 such that for all $n \geq N : |a_n - L| < \frac{\epsilon}{2}$.

Likewise since $a_n \to M$, there exists N_2 such that for all $n \geq N_2 : |a_n - M| < \frac{\epsilon}{2}$.

Let $N = \max\{N_1, N_2\}$. For $n \geq N$:

$$|L - M| = |L - a_n + a_n - M| \leq |a_n - L| + |a_n + M| < \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon$$

Since this holds for arbitrary $\epsilon > 0$, we must have $|L - M| = 0$, so L = M.

□