Problem 1. There is no rational number whose square is 2.

Proof. Assume, for contradiction, that there exist integers p and q satisfying

$$\frac{p}{q} = \sqrt{2},$$

where p/q is a rational number in lowest terms. By squaring, this is the same as $\frac{p^2}{a^2} = 2$, and by clearing denominators it is the same as

$$p^2 = 2q^2.$$

Thus p^2 is divisible by 2, an even number. This implies that p is also divisible by 2 and can be expressed in the form p=2k for some $k \in \mathbb{Z}$. If we sbustitute the p in $p^2=2q^2$ for 2k, we get

$$(2k)^2 = 4(k^2) = 2q^2$$

Further reducing this gives us

$$2(k^2) = q^2$$

This is a contradiction as the result implies that q^2 is also even and thus q is even. Therefore p and q are both even and are irreducible.

Problem 2. (a) The negation of "For all real numbers satisfying a < b, there exists $n \in \mathbb{N}$ such that a + (1/n) < b" is

- (b) The negation of "There exists a real number x > 0 such that x < 1/n for all $n \in \mathbb{N}$ " is
- (b) The negation of "Between every two distinct real numbers there is a rational number" is

Problem 3. Suppose a and b are real numbers. Then

(a)
$$|a - b| \le |a| + |b|$$

$$(b) ||a-b|| \leq |a-b|$$

Proof.

Problem 4. Give an example of each, or state that it is impossible. (a) $f : \mathbb{N} \to \mathbb{N}$ that is one-to-one but not onto.

- (b) $f: \mathbb{N} \to \mathbb{N}$ that is onto but not one-to-one.
- (d) $f: \mathbb{N} \to \mathbb{Z}$ that is one-to-one and onto.

Problem 5. There exists an infinite collection of sets A_1, A_2, A_3, \ldots with the properties that every A_i has an infinite number of elements, and $A_i \cap A_j = \emptyset$ for all $i \neq j$, and $\bigcup_{i=1}^{\infty} A_i = \mathbb{N}$.