Problem 2.2:
Theorem: Let Z be an n x r null-space matrix for the matrix A. If Y is any invertible r x r matrix, $\hat{Z} = ZY$ is also a null-space matrix for A.

Proof: Given that Z is an n x r null-space matrix for matrix A, we know that $AZ = 0$, we also know that Y is an invertible matrix. We need to show that $A\hat{Z} = 0$ or $AZY = 0$ for $\hat{Z} = ZY$ to be a null-space matrix for A. Consider the following:

$$\begin{aligned} A\hat{Z} &= A(ZY) \\ A(ZY) &= (AZ)Y \\ A(ZY) &= 0Y \\ AZY &= 0 \text{ or } A\hat{Z} = 0 \end{aligned}$$

Thus proving $\hat{Z} = ZY$ is also a null-space matrix for A.