Problem 4.2:
In this problem we explore descent directions for the linear function $f(x) = x_1 - 2x_2 + 3x_3$ and determine whether our solutions depend on the value of x. Let us consider the gradient of our linear function:

$$\nabla f(x) = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$$

We let d be the dscent direction

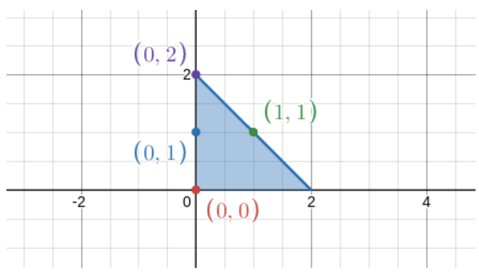
Problem 4.3: Consider the following problem:

$$\begin{aligned} \text{minimize} \quad & f(x) = -x_1 - x_2 \\ \text{subject to} \quad & x_1 + x_2 \leq 2 \\ & x_1, x_2 \geq 0 \end{aligned}$$

We will do the following:

{i} Determine the feasible directions at $x = (0,0)^T, (0,1)^T, (1,1)^T$ and $(0,2)^T$

{ii} Determine whether there exist feasible descent direction at these points, and hence determine which (if any) of the point can be local minimizers.



{i} Solution, below we let p be a set of vectors.

$$\begin{aligned} \bar{x} = (0,0)^T : \quad & \{p \in \mathbb{R}^2 | p_1 \geq 0, p_2 \geq 0\} \\ \bar{x} = (0,1)^T : \quad & \{p \in \mathbb{R}^2 | p_1 \geq 0, p_1 \leq 1\} \\ \bar{x} = (1,1)^T : \quad & \{p \in \mathbb{R}^2 | p_1 + p_2 \leq 0\} \\ \bar{x} = (0,2)^T : \quad & \{p \in \mathbb{R}^2 | p_1 + p_2 \leq 2\}, \text{ where } \{p_2 \leq 0\} \end{aligned}$$

{ii}
For the point $(0,0)^T$, every direction $p_1 \geq 0$ and $p_2 \geq 0$ is a feasible descent direction. Its not a local minimizer.
For the point $(0,1)^T$, there are feasible directions if $0 \leq p_1 \leq 1$ and p_2, not a local minimizer.
For point $(1,1)$, there are no feasible directions, this is actually the global minimizer
For point $(0,2)$ there are no feasible directions