Problem 2.1 We covert the following linear program to standard form:

$$\begin{aligned}
\text{maximize} \quad & z = 3x_1 + 5x_2 - 4x_3 && (1)\\
\text{subject to} \quad & 7x_1 - 2x_2 - 3x_3 \geq 4 && (2)\\
& -2x_1 + 4x_2 + 8x_3 = -3 && (3)\\
& 5x_1 - 3x_2 - 2x_3 \leq 9 && (4)\\
& x_1 \geq 1, x_2 \leq 7, x_3 \geq 0 && (5)
\end{aligned}$$

We convert the problem into a minimization problem and multiplying (1) by -1, adding slack to (2), multiplying (3) by -1 adding surplus to (4), and turning our general constraints into equalities in 5, We will be using the notation in the book. Our linear program in standard form ends up being:

$$\begin{aligned}
\text{minimize} \quad & \hat{z} = -3x_1 - 5x_2 + 4x_3 && (1)\\
\text{subject to} \quad & 7x_1 - 2x_2 - 3x_3 - e_4 = 4 && (2)\\
& 2x_1 - 4x_2 - 8x_3 = 3 && (3)\\
& 5x_1 - 3x_2 - 2x_3 + s_5 = 9 && (4)\\
& x_1 - e_6 = 1 && (5)\\
& x_2 + s_7 = 7 && (6)\\
& x_1, x_2, x_3, e_4, s_5, e_6, s_7 \geq 0 && (7)
\end{aligned}$$

Or in our matrix-vector form:

$$\begin{aligned}
\text{minimize} \quad & z = c^T x\\
\text{subject to} \quad & Ax = b\\
& x \geq 0
\end{aligned}$$

Where $A \in \mathbb{R}^{mxn}$, $b \in \mathbb{R}^n$, $x \in \mathbb{R}^n$ such that:

$$c = \begin{bmatrix} -3 & -5 & 4 & 0 & 0 & 0 & 0 \end{bmatrix}^T \qquad A = \begin{bmatrix} 7 & -2 & -3 & -1 & 0 & 0 & 0\\ 2 & -4 & -8 & 0 & 0 & 0 & 0\\ 5 & -3 & -2 & 0 & 1 & 0 & 0\\ 1 & 0 & 0 & 0 & 0 & -1 & 0\\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 & x_2 & x_3 & e_4 & s_5 & e_6 & s_7 \end{bmatrix}^T \qquad b = \begin{bmatrix} 4 & 3 & 9 & 1 & 7 \end{bmatrix}^T$$

Problem 2.2 We convert the flowing linear program to standard form:

$$\begin{aligned}
\text{minimize} \quad & z = x_1 - 5x_2 - 7x_3 && (1)\\
\text{subject to} \quad & 5x_1 - 2x_2 + 6x_3 \geq 5 && (2)\\
& 3x_1 + 4x_2 - 9x_3 = 3 && (3)\\
& 7x_1 + 3x_2 + 5x_3 \leq 9 && (4)\\
& x_1 \geq -2, x_2, x_3 \text{ free} && (5)
\end{aligned}$$

For this problem we start with substituting for the free variables first so we can inject those values into our function, we substitute $x_2 = x_2' - x_2''$ and $x_3 = x_3' - x_3''$, we also add excess to (2) and surplus to (4) and substitute our general constraints at (5) which then becomes:

$$\begin{aligned}
\text{minimize} \quad & z = x_1 - 5x_2' + 5x_2'' - 7x_3' + 7x_3'' && (1)\\
\text{subject to} \quad & 5x_1 - 2x_2' + 2x_2'' + 6x_3' - 6x_3'' - e_4 = 5 && (2)\\
& 3x_1 + 4x_2' - 4x_2'' - 9x_3' + 9x_3'' = 3 && (3)\\
& 7x_1 + 3x_2' - 3x_2'' + 5x_3' - 5x_3'' + s_5 = 9 && (4)\\
& -x_1 - s_6 = 2 && (5)\\
& x_1, x_2', x_2'', x_3', x_3'', e_4, s_5, s_6, \geq 0 && (6)
\end{aligned}$$

Or in our matrix-vector form:

$$\begin{aligned}
\text{minimize} \quad & z = c^T x\\
\text{subject to} \quad & Ax = b\\
& x \geq 0
\end{aligned}$$

Where $A \in \mathbb{R}^{mxn}$, $b \in \mathbb{R}^n$, $x, e, s \in \mathbb{R}^n$ such that:

$$c = \begin{bmatrix} 1 & -5 & 5 & -7 & 7 & 0 & 0 & 0 \end{bmatrix}^T \quad A = \begin{bmatrix} 5 & -2 & 2 & 6 & -6 & -1 & 0 & 0 \\ 2 & 4 & -4 & -9 & 9 & 0 & 0 & 0 \\ 7 & 3 & -3 & 5 & -5 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 & x_2' & x_2'' & x_3' & x_3'' & e_4 & s_5 & s_6 \end{bmatrix}^T \qquad b = \begin{bmatrix} 5 & 3 & 9 & 2 \end{bmatrix}$$