

Problem 2.1 We covert the following linear program to standard form:

$$\text{maximize } z = 3x_1 + 5x_2 - 4x_3 \quad (1)$$

$$\text{subject to } 7x_1 - 2x_2 - 3x_3 \geq 4 \quad (2)$$

$$-2x_1 + 4x_2 + 8x_3 = -3 \quad (3)$$

$$5x_1 - 3x_2 - 2x_3 \leq 9 \quad (4)$$

$$x_1 \geq 1, x_2 \leq 7, x_3 \geq 0 \quad (5)$$

We convert the problem into a minimization problem and multiplying (1) by -1, adding slack to (2), multiplying (3) by -1 adding surplus to (4), and turning our general constraints into equalities in 5, We will be using the notation in the book. Our linear program in standard form ends up being:

$$\text{minimize } \hat{z} = -3x_1 - 5x_2 + 4x_3 \quad (1)$$

$$\text{subject to } 7x_1 - 2x_2 - 3x_3 - e_4 = 4 \quad (2)$$

$$2x_1 - 4x_2 - 8x_3 = 3 \quad (3)$$

$$5x_1 - 3x_2 - 2x_3 - s_5 = 9 \quad (4)$$

$$x_1 - e_6 = 1 \quad (5)$$

$$x_2 + s_7 = 7 \quad (6)$$

$$x_1, x_2, x_3, e_4, s_5, e_6, s_7 \geq 0 \quad (7)$$

Or

$$\text{minimize } z = c^T x$$

$$\text{subject to } Ax = b$$

$$x \geq 0$$

Where $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^n$, $x, e, s \in \mathbb{R}^n$ such that:

$$c = [-3 \quad -5 \quad 4 \quad 0 \quad 0 \quad 0 \quad 0]^T \quad A = \begin{bmatrix} 7 & -2 & -3 & -1 & 0 & 0 & 0 \\ 2 & -4 & -8 & 0 & 0 & 0 & 0 \\ 5 & -3 & -2 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$x = [x_1 \quad x_2 \quad x_3 \quad e_4 \quad s_5 \quad e_6 \quad s_7]^T \quad b = [4 \quad 3 \quad 9 \quad 1 \quad 7]^T$$

Problem 2.2 We convert the flowing linear program to standard form: