HW+1

Wednesday, January 14, 2026 8:53 PM

ME464+H
W+1+28R...

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Student Name

HW #1: WRITTEN

Box answers and show all work in the space provided. Submit at the start of class on the scheduled due date.

1) Given the rotation matrix $\mathbf{R}^{-1}$ below, find $\mathbf{R}^T$.

$$\mathbf{R}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & -\sqrt{3}/2 \\ 0 & \sqrt{3}/2 & 1/2 \end{bmatrix}$$

$$R^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & -\sqrt{3}/2 \\ 0 & \sqrt{3}/2 & 1/2 \end{bmatrix} \qquad R^{-1} = R^T$$

2) Given the rotation matrix $\mathbf{R}$ below, find $\mathbf{R}^{-1}$ (hint: you don't need to invert the matrix).

$$\mathbf{R} = \begin{bmatrix} 3/4 & -\sqrt{6}/4 & 1/4 \\ \sqrt{6}/4 & 1/2 & -\sqrt{6}/4 \\ 1/4 & \sqrt{6}/4 & 3/4 \end{bmatrix}$$

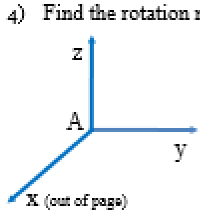
$$R^{-1} = \begin{bmatrix} 3/4 & \sqrt{6}/4 & 1/4 \\ -\sqrt{6}/4 & 1/2 & \sqrt{6}/4 \\ 1/4 & -\sqrt{6}/4 & 3/4 \end{bmatrix} = R^T$$

3) Given the rotation matrix $\mathbf{R}$ below, find $\det(\mathbf{R})$ (hint: you don't need to calculate the determinate).

$$\mathbf{R} = \begin{bmatrix} 3/4 & -\sqrt{6}/4 & 1/4 \\ \sqrt{6}/4 & 1/2 & -\sqrt{6}/4 \\ 1/4 & \sqrt{6}/4 & 3/4 \end{bmatrix}$$

$$\det(R) = +1$$

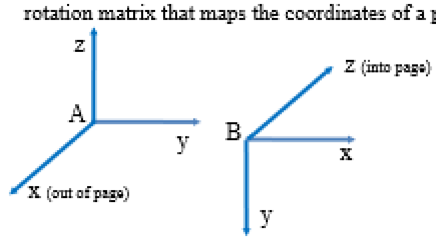
4) Find the rotation matrix that rotates a point 45° about the y-axis.



$$R_y(\theta) = \begin{bmatrix} c(\theta) & 0 & s(\theta) \\ 0 & 1 & 0 \\ -s(\theta) & 0 & c(\theta) \end{bmatrix}$$

$$R_y(45) = \begin{bmatrix} c(45) & 0 & s(45) \\ 0 & 1 & 0 \\ -s(45) & 0 & c(45) \end{bmatrix} = \begin{bmatrix} \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & 1 & 0 \\ -\sqrt{2}/2 & 0 & \sqrt{2}/2 \end{bmatrix}$$

5) Consider the two frames shown below. They are shown apart for clarity, but assume they have the same origin. Find the rotation matrix that maps the coordinates of a point specified in frame B to frame A.



$$R_{ab} = \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

$$x_{AB} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad y_{AB} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}, \quad z_{AB} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

6) Find the point $\mathbf{q}'$ after rotating the point $\mathbf{q} = [3\ \ 1\ \ 2]^T$ using the rotation matrix $\mathbf{R}$ below.

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & -\sqrt{3}/2 \\ 0 & \sqrt{3}/2 & 1/2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \Rightarrow q' = \begin{bmatrix} 3 \\ \frac{1}{2} - \sqrt{3} \\ \frac{\sqrt{3}}{2} + 1 \end{bmatrix}$$

7) Find the matrix $\hat{\omega}$ from applying the hat-operator to the rotation axis $\omega = \left[0\ \ 1/\sqrt{2}\ \ 1/\sqrt{2}\right]^T$.

$$\omega = \begin{bmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} \qquad \hat{\omega} = \begin{bmatrix} 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 0 \\ -1/\sqrt{2} & 0 & 0 \end{bmatrix}$$

8) Find the rotation matrix for rotation about the axis $\omega = \left[0\ \ 1/\sqrt{2}\ \ 1/\sqrt{2}\right]^T$ for a rotation angle $\theta = \pi/3$ using the exponential coordinates for rotation and Rodrigues' formula.

$$\omega = \begin{bmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} \qquad \theta = \pi/3 \qquad \vec{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$e^{\hat{\omega}\theta} = \vec{I} + \hat{\omega}\sin(\theta) + \hat{\omega}^2(1-\cos(\theta))$$

$$e^{\hat{\omega}\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 0 \\ -1/\sqrt{2} & 0 & 0 \end{bmatrix} \underbrace{\sin(\pi/3)}_{\sqrt{3}/2} + \begin{bmatrix} 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 0 \\ -1/\sqrt{2} & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 0 \\ -1/\sqrt{2} & 0 & 0 \end{bmatrix} \underbrace{(1-\cos(\pi/3))}_{1/2}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\sqrt{3}/2\sqrt{2} & \sqrt{3}/2\sqrt{2} \\ \sqrt{3}/2\sqrt{2} & 0 & 0 \\ -\sqrt{3}/2\sqrt{2} & 0 & 0 \end{bmatrix} + \underbrace{\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1/2 & 1/2 \\ 0 & 1/2 & -1/2 \end{bmatrix} (1/2)}_{\begin{bmatrix} -1/2 & 0 & 0 \\ 0 & -1/4 & 1/4 \\ 0 & 1/4 & -1/4 \end{bmatrix}}$$

$$-\sqrt{3}/2\sqrt{2} = -\frac{\sqrt{6}}{4}$$

$$\boxed{e^{\hat{\omega}\theta} = \begin{bmatrix} 1/2 & -\sqrt{6}/4 & \sqrt{6}/4 \\ \sqrt{6}/4 & 3/4 & 1/4 \\ -\sqrt{6}/4 & 1/4 & 3/4 \end{bmatrix}}$$