ME464 HW 2

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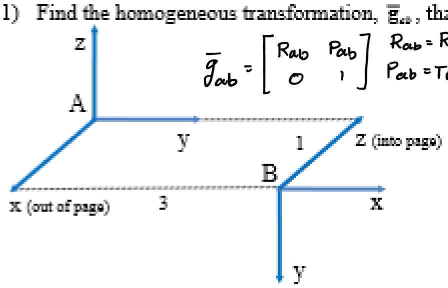
W+2+28H...

Paul Martin
Student Name

HW #2: WRITTEN

Box answers and show all work in the space provided. Submit at the start of class on the scheduled due date.

1) Find the homogeneous transformation, $\overline{g}_{ab}$, that transforms coordinates of a point from frame B to frame A shown.



$\overline{g}_{ab} = \begin{bmatrix} R_{ab} & P_{ab} \\ 0 & 1 \end{bmatrix}$ R_{ab} = Rotation, P_{ab} = Translation

$R_{ab} = \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$ $P_{ab} = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}$

$$\overline{g}_{ab} = \begin{bmatrix} 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2) Assume a rigid body transformation, g_{ab}, is defined by the rotation matrix $\mathbf{R}_{ab}$ and the translation $\mathbf{p}_{ab}$ as defined below. Find the homogeneous representation of g_{ab}.

$$\mathbf{R}_{ab} = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \mathbf{p}_{ab} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

$$\overline{g}_{ab} = \begin{bmatrix} 0 & 0 & -1 & 2 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$R^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$

3) Consider the rigid body transformation given below. Find $\overline{g}_{ab}^{-1}$.

$$\overline{g}_{ab} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\overline{g}_{ab}^{-1} = \begin{bmatrix} R^T & -R^T P \\ 0 & 1 \end{bmatrix}$ $R^T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ $-R^T P = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$

$\overline{g}_{ab} = \begin{bmatrix} [R] & P \\ 0 & 1 \end{bmatrix}$ $\overline{g}_{ab}^{-1} = \begin{bmatrix} 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

4) Consider the rigid body transformation given below. Find $\overline{g}_{ab}^{-1}\overline{g}_{ab}$.

$$\overline{g}_{ab} = \begin{bmatrix} 0 & 0 & -1 & 7 \\ 0 & 1 & 0 & 8 \\ 1 & 0 & 0 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$-R^T P = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} = \begin{bmatrix} -9 \\ -8 \\ 7 \end{bmatrix}$ $\overline{g}_{ab}^{-1} = \begin{bmatrix} 0 & 0 & 1 & -9 \\ 0 & 1 & 0 & -8 \\ -1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$\overline{g}_{ab}^{-1}\overline{g}_{ab} = \begin{bmatrix} 0 & 0 & 1 & -9 \\ 0 & 1 & 0 & -8 \\ -1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & -1 & 7 \\ 0 & 1 & 0 & 8 \\ 1 & 0 & 0 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5) Consider the rigid body transformation $\overline{g}_{ab}$ below that maps points from frame B to frame A. The coordinates of a point $\mathbf{p}$ in the B frame is also given. Find the coordinates of the point in the A frame.

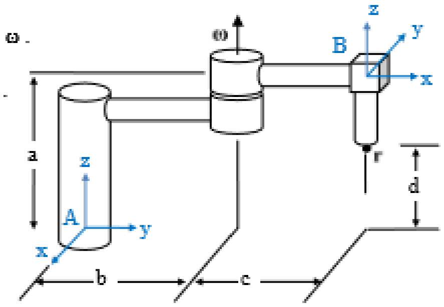
$$\overline{g}_{ab} = \begin{bmatrix} 0 & 1 & 0 & -2 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{p}_b = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

$$\overline{P}_a = \begin{bmatrix} 0 & 1 & 0 & -2 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 2 \\ 1 \end{bmatrix} \leftarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$\overline{P}_a = \begin{bmatrix} -1 \\ -2 \\ 2 \end{bmatrix}$

$\overline{P}_a = \overline{g}_{ab} \cdot P_b$

For the remaining problems, consider the 1 DOF robot arm shown in the home position. Point $\mathbf{r}$ is fixed to tool frame B which rotates about axis $\boldsymbol{\omega}$.



6) Find the rotation axis, $\boldsymbol{\omega} \in \mathbb{R}^3$, and a point on the rotation axis, $\mathbf{q} \in \mathbb{R}^3$.

$$\omega = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad q = \begin{bmatrix} 0 \\ b \\ a \end{bmatrix} = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix}$$

7) Find the twist, ξ, for the rotation about the axis $\boldsymbol{\omega}$.

$$\xi = \begin{bmatrix} \omega \\ v \end{bmatrix} \qquad v = -\omega \times q \qquad -\omega \times q = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix} = \begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix}$$

$$\xi = \begin{bmatrix} 0 \\ 0 \\ 1 \\ b \\ 0 \\ 0 \end{bmatrix}$$

8) Find the wedge ($\hat{\xi}$) version of the twist, ξ, from the previous problem.

$$\hat{\xi} = \begin{bmatrix} \hat{\omega} & v \\ 0 & 0 \end{bmatrix} \qquad \hat{\omega} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \hat{\xi} = \begin{bmatrix} 0 & -1 & 0 & b \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

9) Find the initial position, $\mathbf{r}_a(0)$, of point $\mathbf{r}$ as viewed from the inertial frame A. Also find the position, $\mathbf{r}_b$, of point $\mathbf{r}$ as viewed from body frame B.

$$r_a(0) = [\;] \qquad r_b = [\;]$$

$$r_a(0) = \begin{bmatrix} 0 \\ a+b \\ d \end{bmatrix} \qquad r_b = \begin{bmatrix} 0 \\ 0 \\ -(a-d) \end{bmatrix}$$

10) Find the home configuration, $\bar{g}_{ab}(0)$, of body frame B with respect to inertial frame A.

$$\underset{4\times 4}{\bar{g}_{ab}(0)} = \begin{bmatrix} R_{ab}(0) & P_{ab}(0) \\ 1 & 0 \end{bmatrix}$$

$$R_{ab}(0) = \begin{array}{c} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ x \quad y \quad z \end{array} \qquad P_{ab}(0) = \begin{bmatrix} 0 \\ b+c \\ a \end{bmatrix}$$

$$\bar{g}_{ab}(0) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & b+c \\ 0 & 0 & 1 & a \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

11) Give the formula for the rigid body transformation, $\mathbf{g}_{ab}(\theta)$, that maps points fixed to the tool frame B to the inertial frame A as a function of rotation angle θ about rotation axis $\boldsymbol{\omega}$.

$$g_{ab}(\theta) = e^{\hat{\xi}\theta} = \begin{bmatrix} \underset{3\times 3}{e^{\hat{\omega}\theta}} & \underset{3\times 1}{(I - e^{\hat{\omega}\theta}) \cdot \hat{\omega} v} \\ 0 & 1 \end{bmatrix}$$

$$\hat{\omega} v = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix}$$

$$(I - e^{\hat{\omega}\theta}) \cdot \hat{\omega} v = \left[\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \right] \cdot \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-\cos\theta & \sin\theta & 0 \\ -\sin\theta & 1+\cos\theta & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} b\sin\theta \\ b + b\cos\theta \\ 0 \end{bmatrix}$$

$$e^{\hat{\omega}\theta} = I + \hat{\omega}\sin\theta + \hat{\omega}^2(1-\cos\theta)$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \sin\theta + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} (1-\cos\theta)$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\sin\theta & 0 \\ \sin\theta & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} -1+\cos\theta & 0 & 0 \\ 0 & -1+\cos\theta & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$e^{\hat{\xi}\theta} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & b\sin\theta \\ \sin\theta & \cos\theta & 0 & b + b\cos\theta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$