Formulas for Matrix Exponentials

Simpler, and computationally efficient, formulas are known for matrix exponentials needed for HGTs:

$$\text{Rodrigues' Formula}: \; e^{\hat{\boldsymbol{\omega}}\theta} = \mathbf{I} + \hat{\boldsymbol{\omega}}\sin(\theta) + \hat{\boldsymbol{\omega}}^2\left(1-\cos(\theta)\right)$$

$$\text{For Pure Rotation}: \; e^{\hat{\xi}\theta} = \begin{bmatrix} e^{\hat{\boldsymbol{\omega}}\theta} & \left(\mathbf{I}-e^{\hat{\boldsymbol{\omega}}\theta}\right)\hat{\boldsymbol{\omega}}\mathbf{v} \\ 0\;0\;0 & 1 \end{bmatrix} \quad \boldsymbol{\xi} = \begin{bmatrix} \mathbf{v} \\ \boldsymbol{\omega} \end{bmatrix} \quad \mathbf{v} = -\hat{\boldsymbol{\omega}}\mathbf{q}$$

$$\text{For Pure Translation}: \; e^{\hat{\xi}\theta} = \begin{bmatrix} \mathbf{I} & \theta\mathbf{v} \\ 0\;0\;0 & 1 \end{bmatrix} \quad \boldsymbol{\xi} = \begin{bmatrix} \mathbf{v} \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{v} = \text{Translation Axis}$$

$$\text{HGT Inverse}: \; \overline{\mathbf{g}}^{-1} = \begin{bmatrix} \mathbf{R}^T & -\mathbf{R}^T\mathbf{p} \\ 0\;0\;0 & 1 \end{bmatrix}$$

$\boxed{\mathbf{p}_a(t) = \mathbf{g}_{ab}(\theta)\mathbf{p}_b}$ where $\boxed{\mathbf{g}_{ab}(\theta) = e^{\hat{\xi}\theta}\mathbf{g}_{ab}(0)}$

$\boldsymbol{\omega} \in \mathbb{R}^3 =$ axis of rotation

$\mathbf{q} \in \mathbb{R}^3 =$ any point on $\boldsymbol{\omega}$ defined relative to frame A

$\mathbf{p} \in \mathbb{R}^3 =$ a point on the end of the rotating body