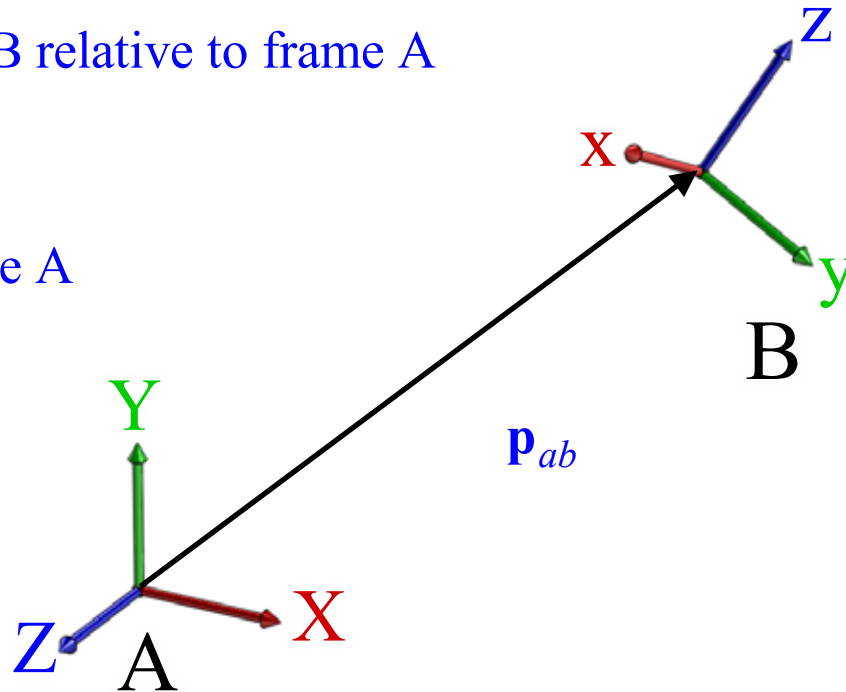# Rigid Body Transformations

Rigid Body Transformations (RBTs) map right-handed, orthonormal frames, to right-handed, orthonormal frames.

In general, a rigid motion consists of a **rotation** and a **translation**.

$\mathbf{p}_{ab} \in \mathbb{R}^3$ is a position vector from the origin of frame A to the origin of frame B

$\mathbf{R}_{ab} \in SO(3)$ is the orientation of frame B relative to frame A

The pair $\left(\mathbf{p}_{ab}, \mathbf{R}_{ab}\right)$ describes the configuration of frame B relative to frame A
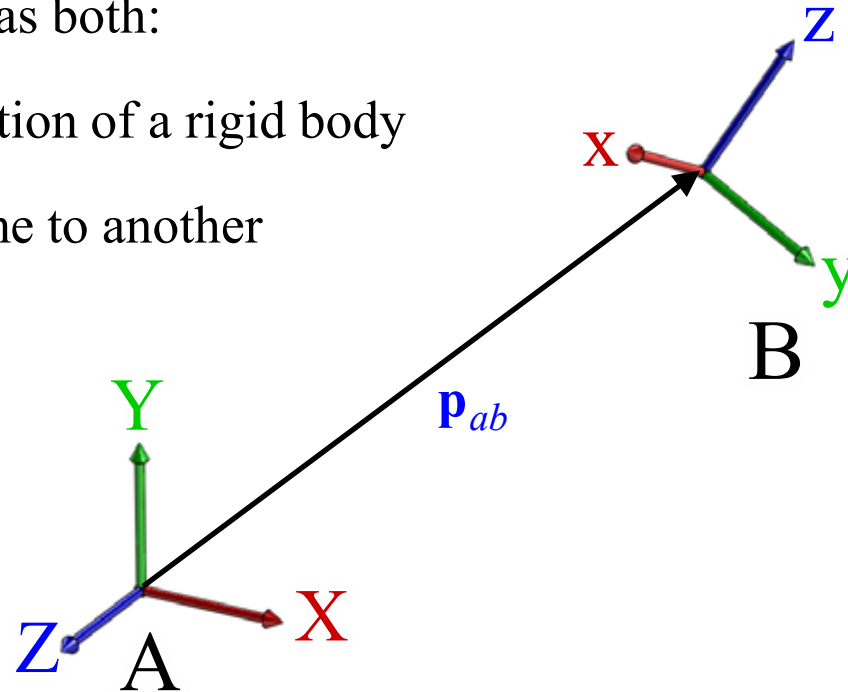
Rigid Body Transformations

The Special Euclidean Group is the product of rotation matrices and translation vectors.

For n = 3:

$$SE(3) = \left\{ (\mathbf{p}, \mathbf{R}) : \mathbf{p} \in \mathbb{R}^3, \mathbf{R} \in SO(3) \right\} = \mathbb{R}^3 \times SO(3)$$

An element $(\mathbf{p}, \mathbf{R})$ in SE(3) serves as both:

1)   Specification of the configuration of a rigid body

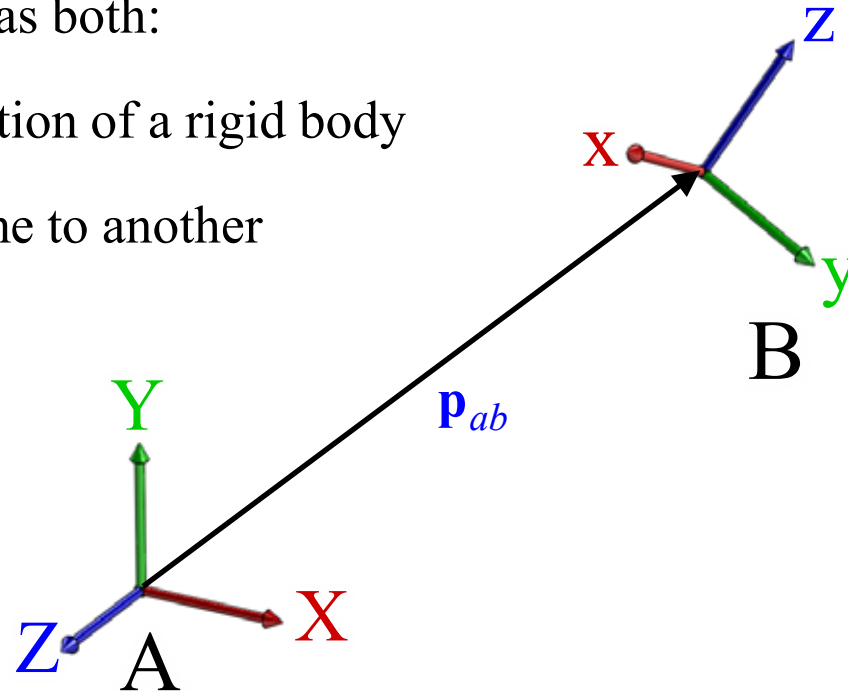2)   Transformation from one frame to another

Rigid Body Transformations

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Rigid Body Transformations

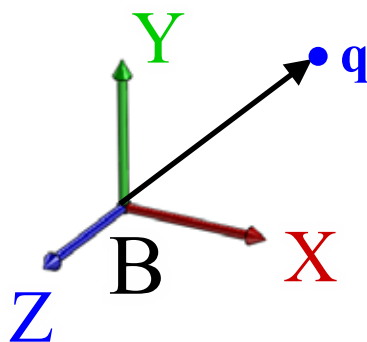To transform points from frame B to frame A, consider the same point define in both frames:

$\mathbf{q}_a \in \mathbb{R}^3$ is the coordinates of $\mathbf{q}$ relative to frame A

$\mathbf{q}_b \in \mathbb{R}^3$ is the coordinates of $\mathbf{q}$ relative to frame B

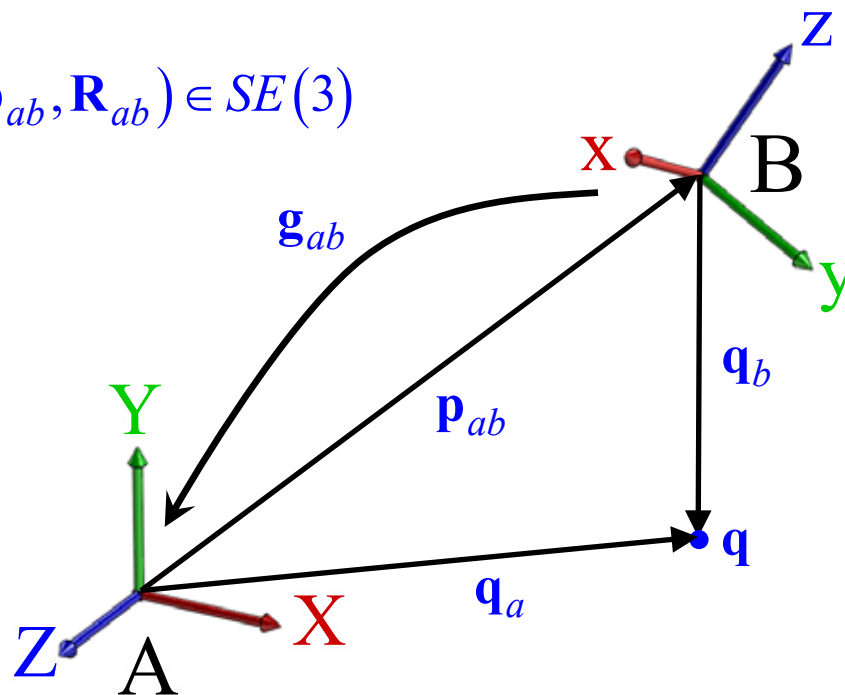Given the point in frame B, we can find the point in frame A using the transformation:

$$\mathbf{q}_a = \mathbf{p}_{ab} + \mathbf{R}_{ab}\mathbf{q}_b$$

We define the configuration of frame B relative to frame A using:

$$\mathbf{g}_{ab} = (\mathbf{p}_{ab}, \mathbf{R}_{ab}) \in SE(3)$$

B frame view

complete system

Rigid Body Transformations

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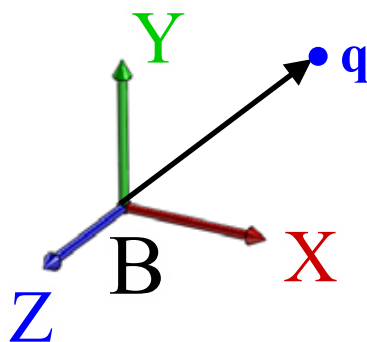
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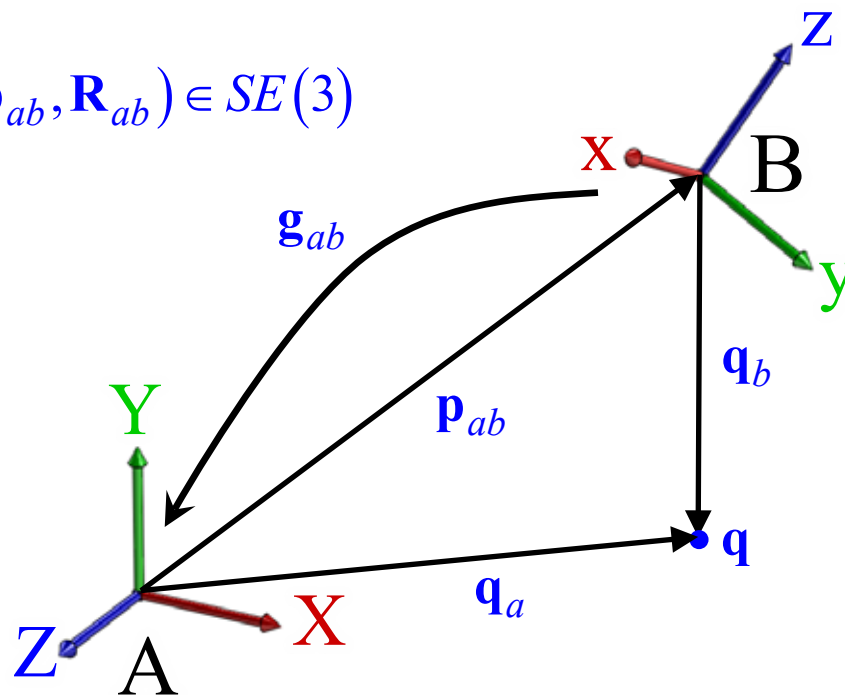Given the point in frame B, we can find the point in frame A using the transformation:

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