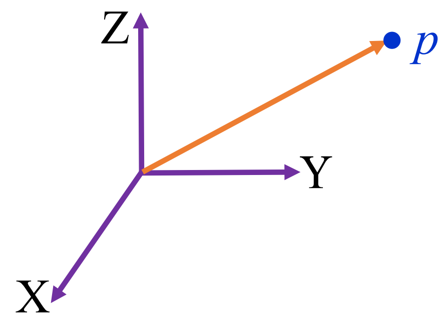
Points and Rigid Bodies



Points: Located w.r.t. orthonormal axis, specified as:

$$\mathbf{p}(t)=\begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} \in \mathbb{R}^3$$

$\in =$ 'is an element of'

$\mathbb{R} =$ 'real numbers'

$\mathbb{R}^3 =$ '3×1 vector of real numbers'

bold = vector or matrix

Rigid Body: Collection of points such that the distance between any two points, **p** and **q**, remains constant, such that:

$$\left\|\mathbf{p}(t)-\mathbf{q}(t)\right\| = \left\|\mathbf{p}(0)-\mathbf{q}(0)\right\| = \text{constant}$$

A rigid body is a subset O of $\mathbb{R}^3$

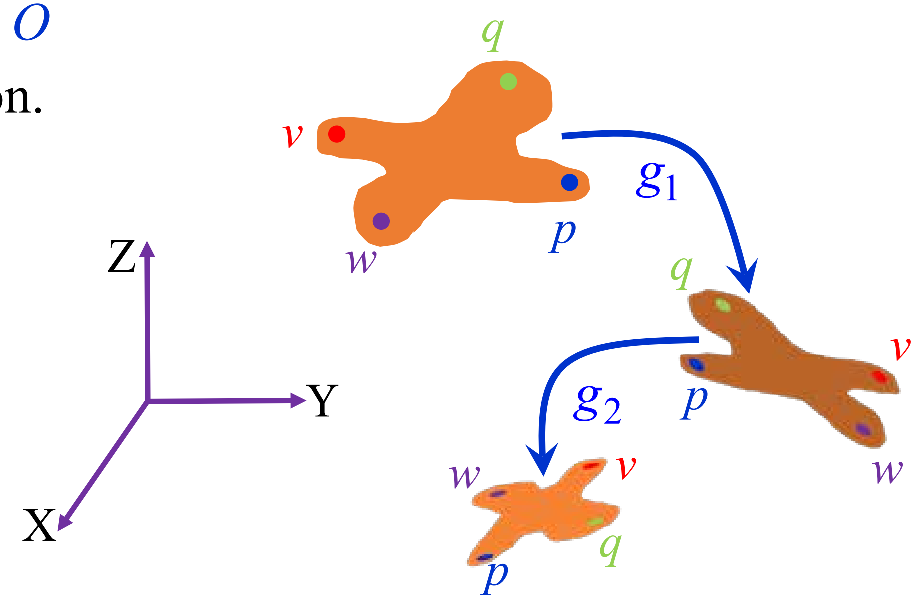
Rigid Motions and Displacements

Rigid Motion: Movement of a body that maintains rigid.
The movement is represented by a continuous family of mappings:

$$g(t): O \to \mathbb{R}^3 \quad \left(\text{the mapping, } g(t) \text{ : maps subset O of } \mathbb{R}^3 \text{ into } \mathbb{R}^3\right)$$

Rigid Displacement: Net movement of body O from one location to another via a rigid motion.

$$g: O \to \mathbb{R}^3 \quad (\text{single net mapping})$$



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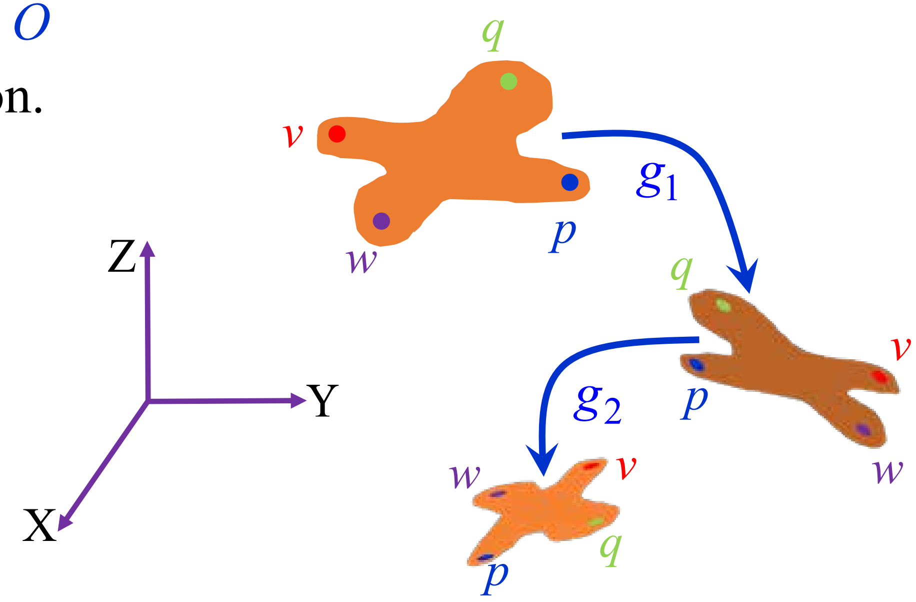


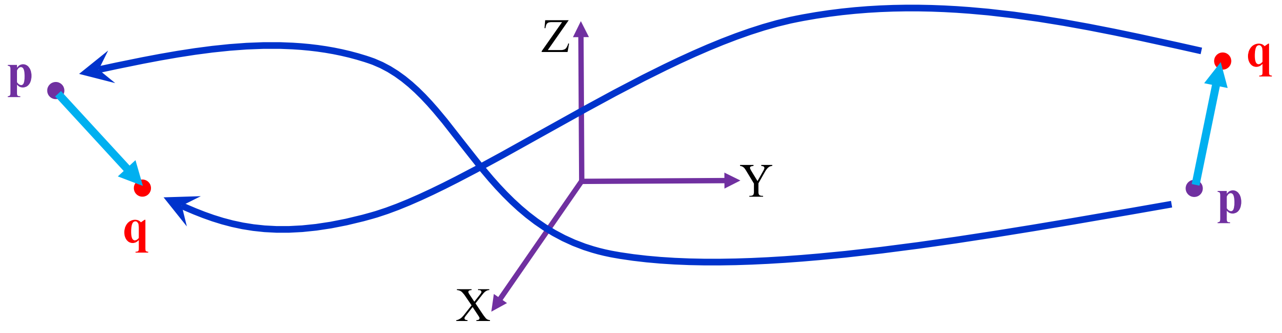
Vector Transformation

Vectors: A vector connecting points $\mathbf{p}$ to $\mathbf{q}$ is defined as the directed line segment from $\mathbf{p}$ to $\mathbf{q}$:

$$\mathbf{v} = \mathbf{q} - \mathbf{p} \quad \text{with} \quad \mathbf{p}, \mathbf{q} \in \mathbb{R}^3$$

A vector has both direction and magnitude and is not attached to the origin. Sometimes called a free vector.

Vector Transform: if $g : O \to \mathbb{R}^3$, and $\mathbf{v} = \mathbf{q} - \mathbf{p}$, then $g_*(\mathbf{v}) = g(\mathbf{q}) - g(\mathbf{p})$



Rigid Body Transformations

Rigid Body Transformations (R.B.T.s)

$g:\mathbb{R}^3 \to \mathbb{R}^3$ is a *Rigid Body Transformation* (*RBT*) If and Only If (IFF):

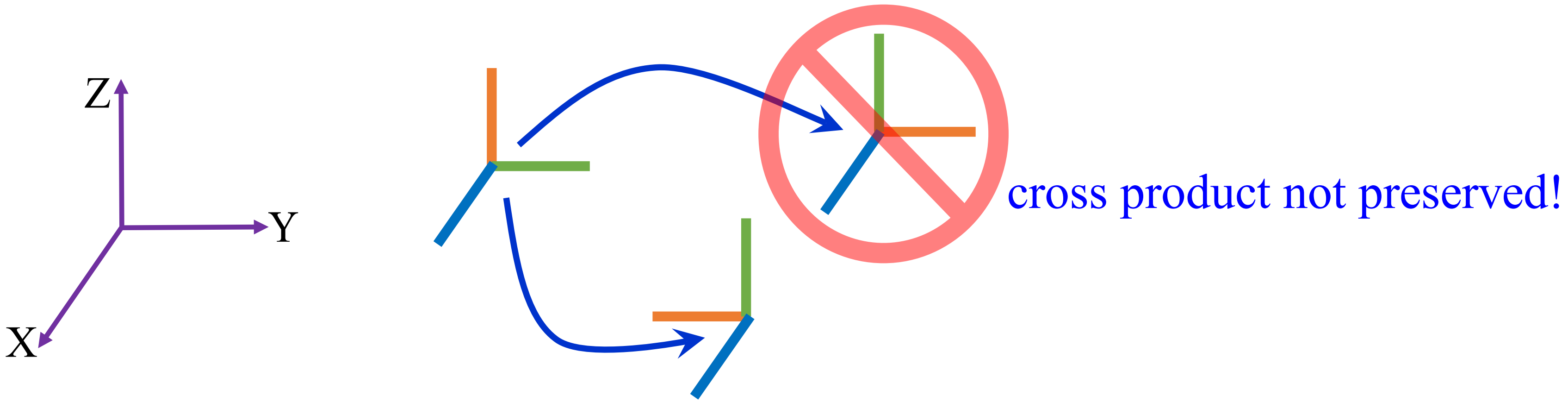
1. Length is preserved. $\|\mathbf{p}(t)-\mathbf{q}(t)\| = \|\mathbf{p}(0)-\mathbf{q}(0)\| =$ for all points $\mathbf{p},\mathbf{q}\in\mathbb{R}^3$

2. Cross product is preserved: $g_*(\mathbf{v}\times\mathbf{w}) = g_*(\mathbf{v})\times g_*(\mathbf{w})$ for all vectors $\mathbf{v},\mathbf{w}\in\mathbb{R}^3$

RBTs transform orthogonal vectors to orthogonal vectors.

RBTs transform orthonormal coordinate frames to orthonormal coordinate frames

This implies that particles in a rigid body can rotate w.r.t. other particles but cannot translate during a rigid motion.



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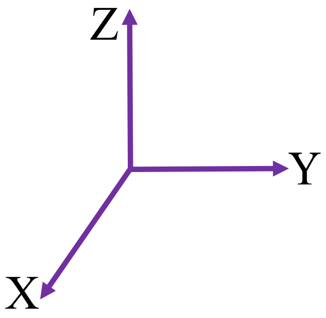
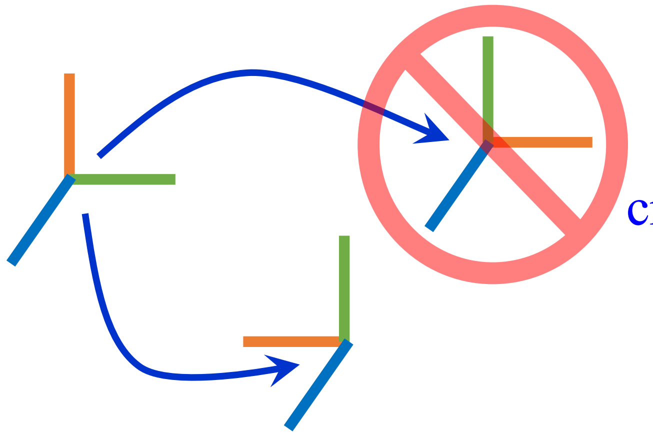


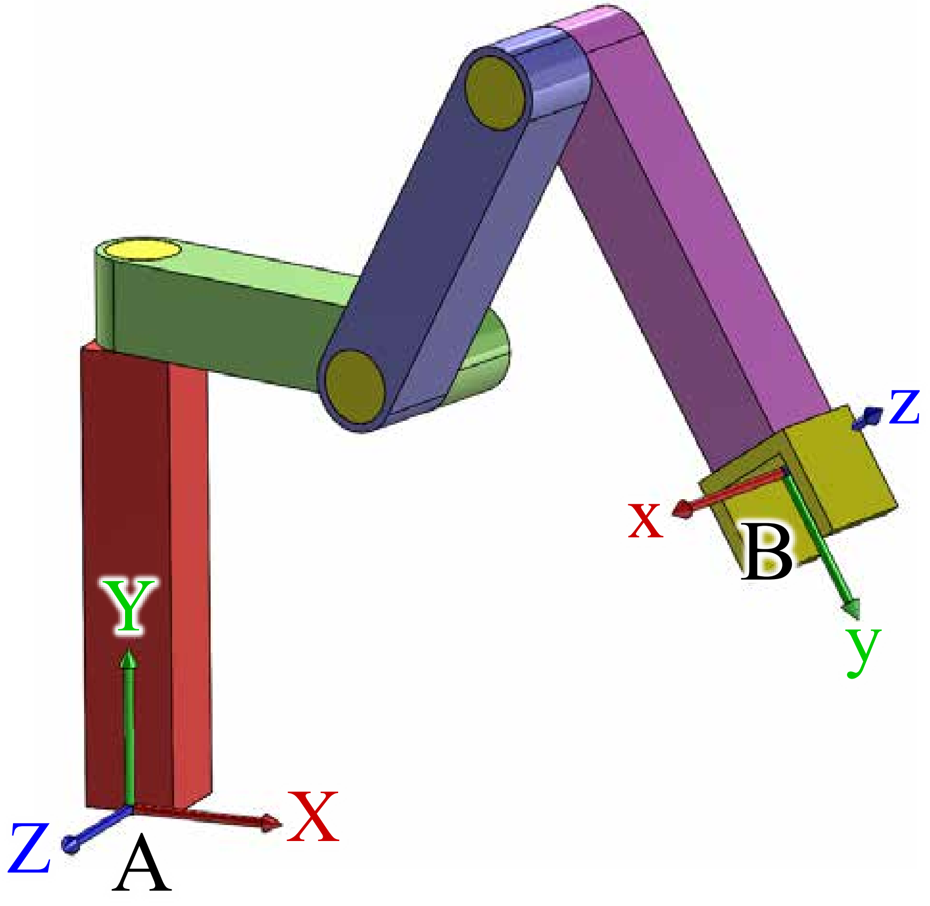


cross product not preserved!

Rigid Body Configuration

Representing a Rigid Body: The **configuration** of a rigid body is represented by attaching a Cartesian coordinate frame to some point of the rigid body. For robots, this is commonly the end-effector.

Body Frame, B
or **Tool Frame, T**
or **Local Frame**
(Letters: **B**, **T**)



Spatial Frame, S
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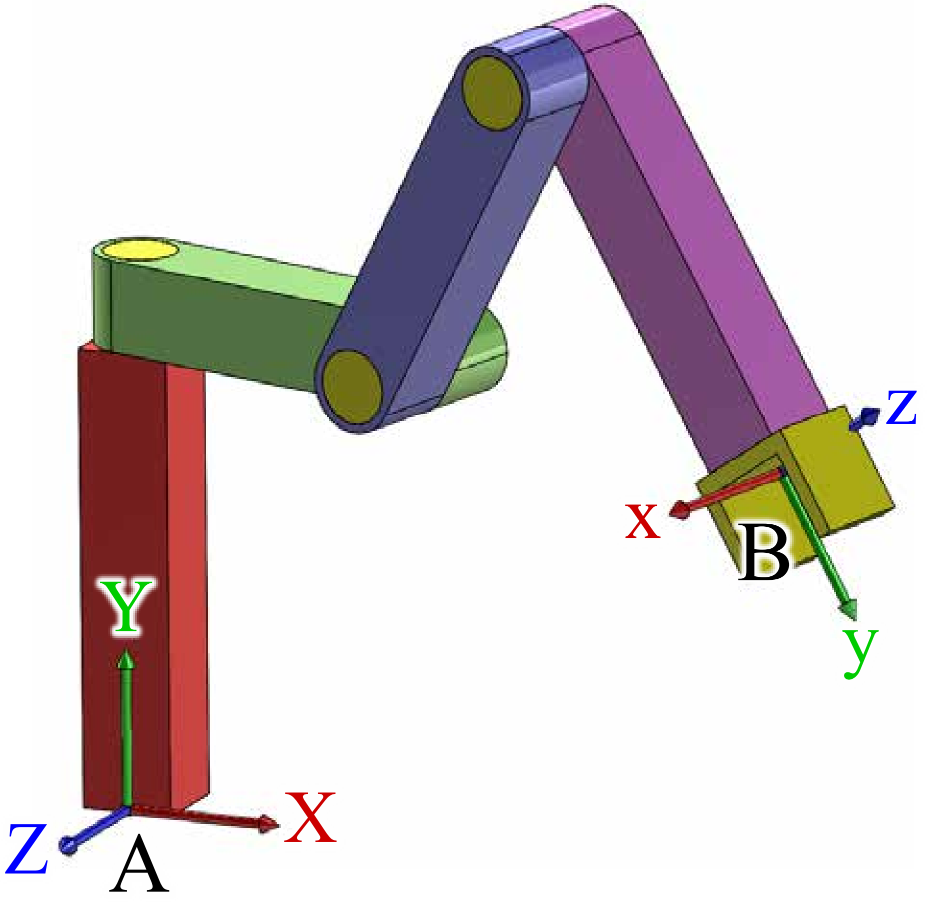

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