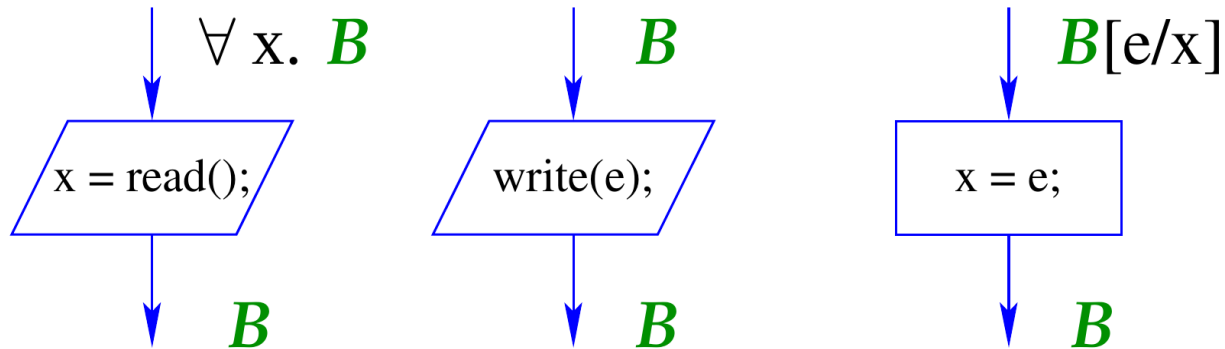


Week 3: Loop Invariants



Week 03 Tutorial 01 — MiniJava 2.0

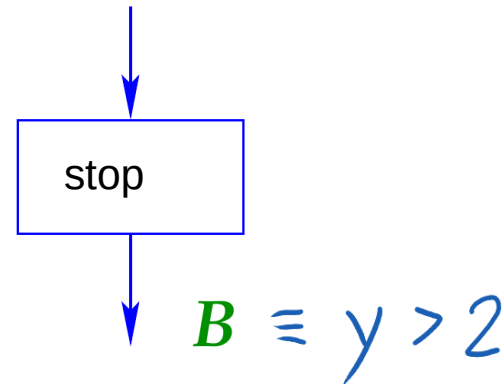
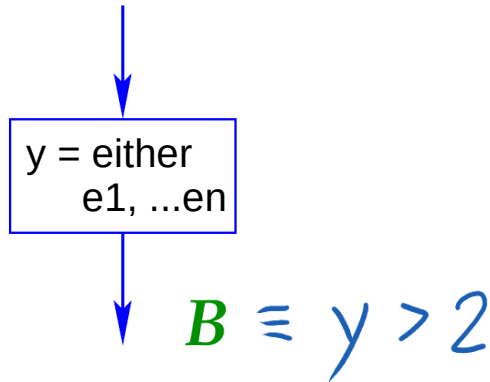
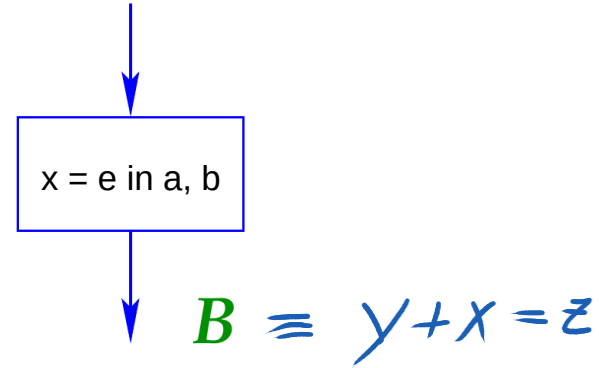
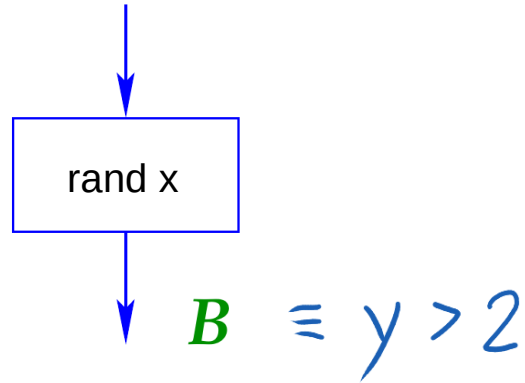


Week 03 Tutorial 01 — MiniJava 2.0

1. **rand** x :
Assigns a random value to variable x ,
2. $x = \text{either } e_0, \dots, e_k$:
Assigns one of the values of the expressions e_0, \dots, e_k to variable x non-deterministically,
3. $x = e \text{ in } a, b$:
Assigns the value **1** to variable x , if the value of expression e is in the range $[a, b]$ and **0** if e is not in the range or the range is empty ($a > b$),
4. **stop**:
Immediately stops the program.

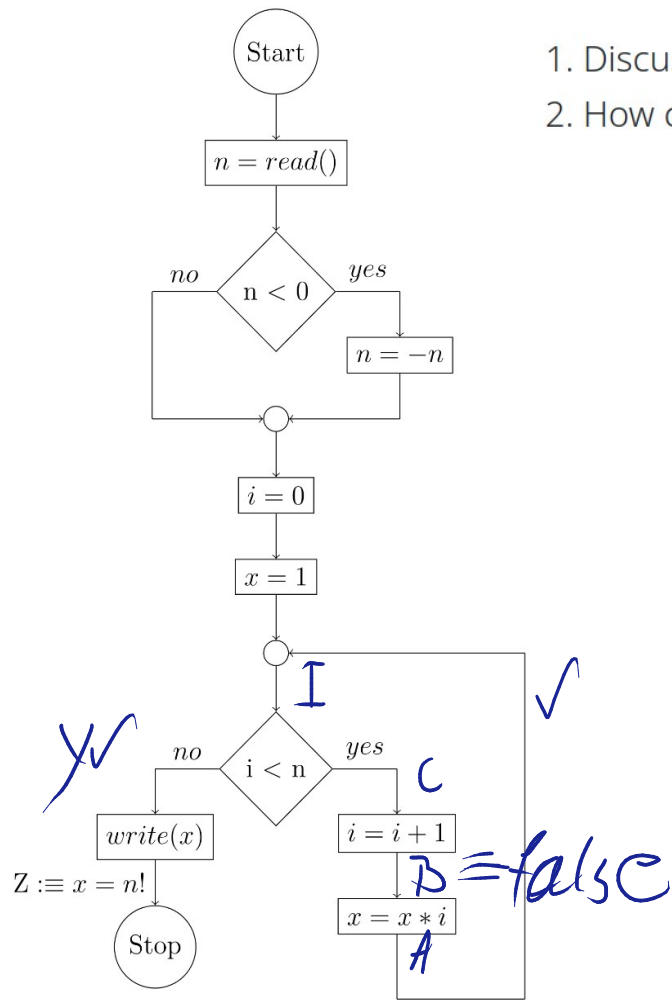
Define the weakest precondition operator $\mathbf{WP}[\![\dots]\!](B)$ for each of these statements.

Week 03 Tutorial 01 — MiniJava 2.0



Week 03 Tutorial 02 — Loop Invariants

1. Discuss the problem that arises when computing weakest preconditions to prove Z .
2. How can you use weakest preconditions to prove Z anyway?

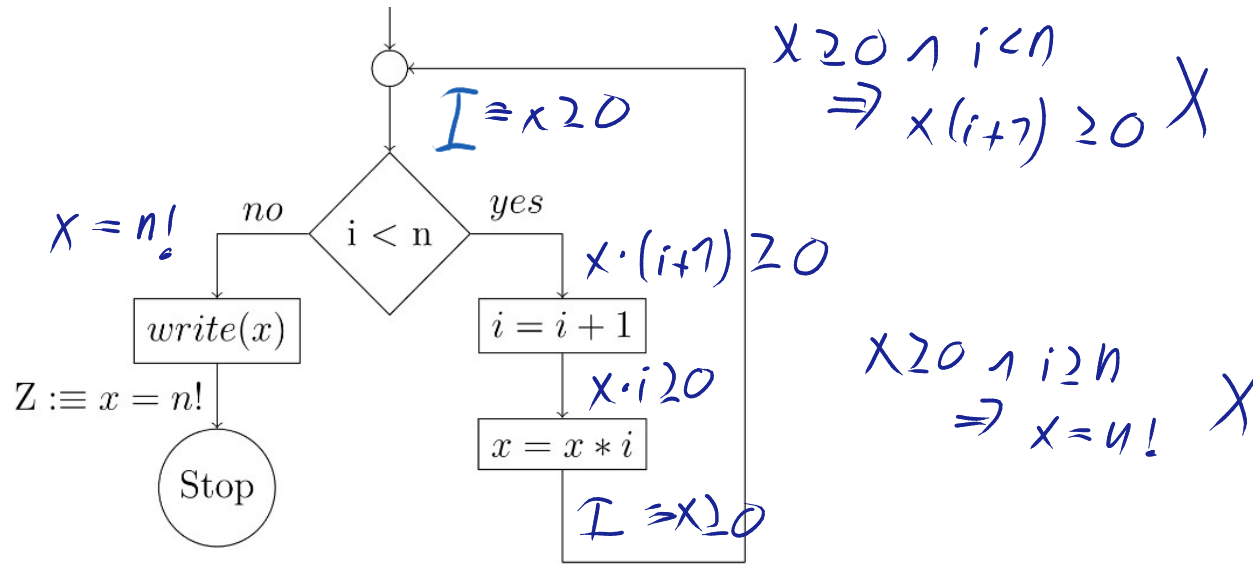


$$I \equiv x \geq 0$$

$$I_2 \equiv i = 0 \wedge x = 1 \wedge n = 0$$

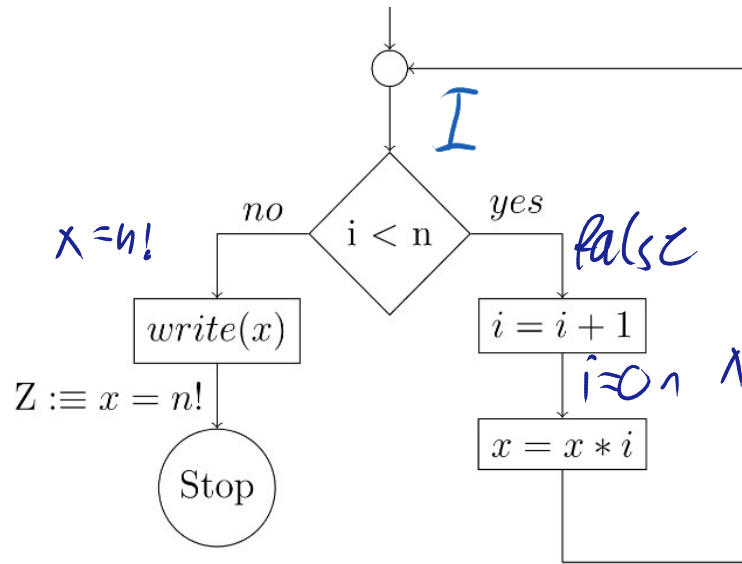
Week 03 Tutorial 02 — Loop Invariants

$$I \equiv x \geq 0$$



Week 03 Tutorial 02 — Loop Invariants

$$I \equiv i=0 \wedge x=1 \wedge n=0$$

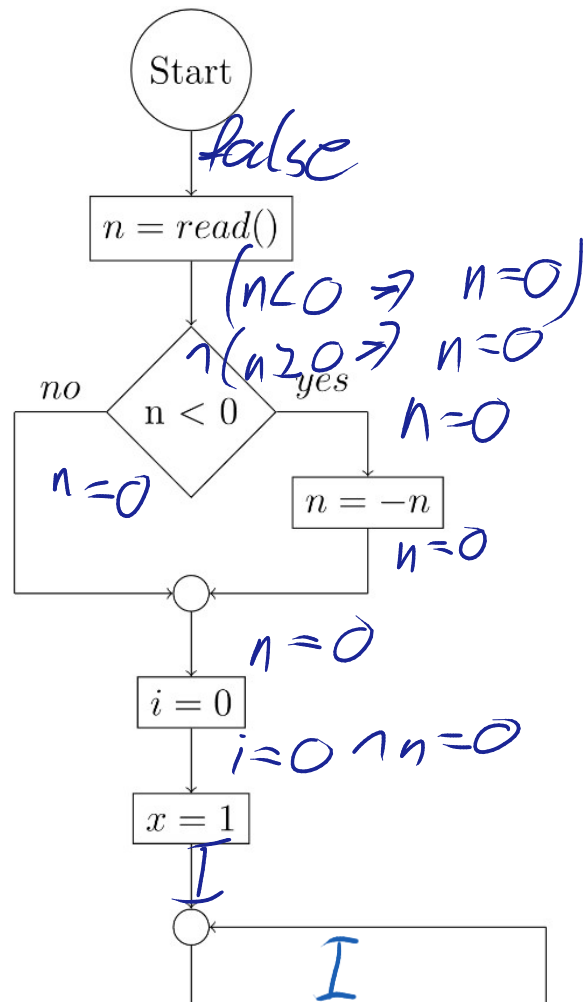


$$I \wedge \underline{i < n} \quad \checkmark$$
$$\Rightarrow \text{false}$$

$$I \wedge \underline{i \geq n} \quad \checkmark$$
$$\Rightarrow x = n!$$

$$i=0 \wedge x=1 \wedge n=0 \equiv \text{false}$$

Week 03 Tutorial 02 — Loop Invariants



$$I \equiv i=0 \wedge x=1 \wedge n=0$$

$$(n < 0 \Rightarrow n=0) = \text{false} \wedge \sim \Rightarrow \text{false}$$

$$\neg(n < 0 \Rightarrow n=0) = \text{false}$$

$$n=0$$

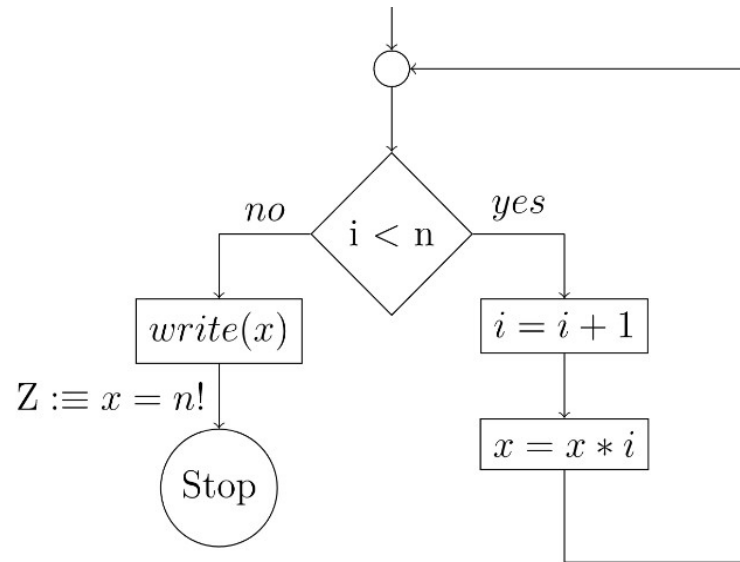
$$i=0 \wedge n=0$$

$$I$$

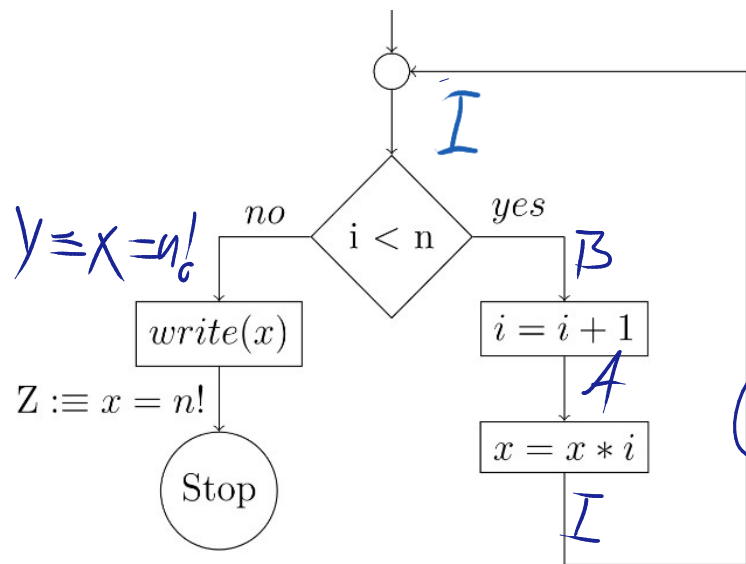
$$I$$

Week 03 Tutorial 02 — Loop Invariants

- a) How has a useful loop invariant be related to Z ?
- b) What happens if the loop invariant is chosen too strong?
- c) What happens if the loop invariant is chosen too weak?



Week 03 Tutorial 02 — Loop Invariants



$$I \equiv x = i! \quad \wedge \quad i \leq n \quad \wedge \quad i \geq 0$$

$$A \equiv x \cdot i = i! \equiv x = (i-1)! \quad \wedge \quad i \leq n \quad \wedge \quad i \geq 0$$

$$B \equiv x = i! \quad \wedge \quad i+1 \leq n \quad \wedge \quad i+1 \geq 0$$

Prüfen auf LC

$$I \wedge i < n \Rightarrow B \quad \checkmark$$

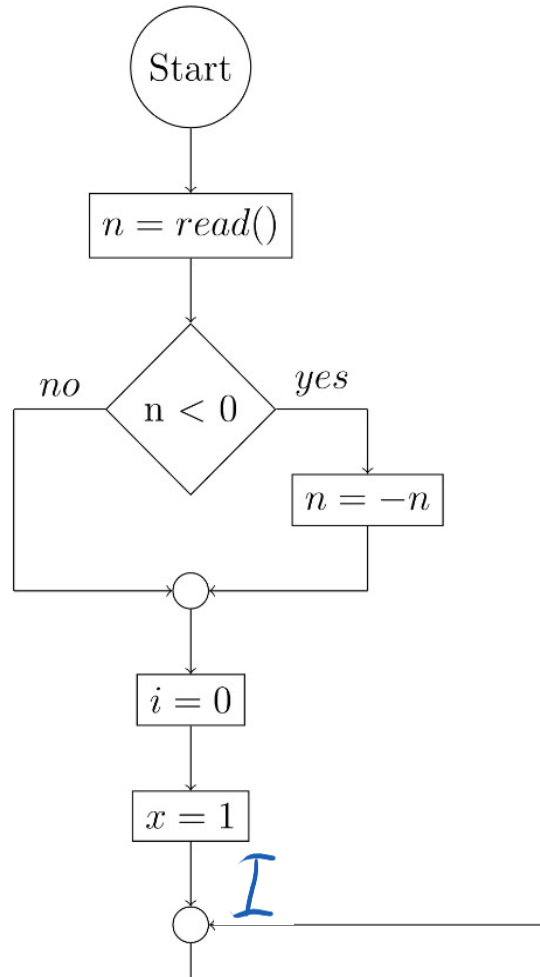
$$I \wedge i \geq n \Rightarrow Y \quad \times$$

Prüfen auf LC

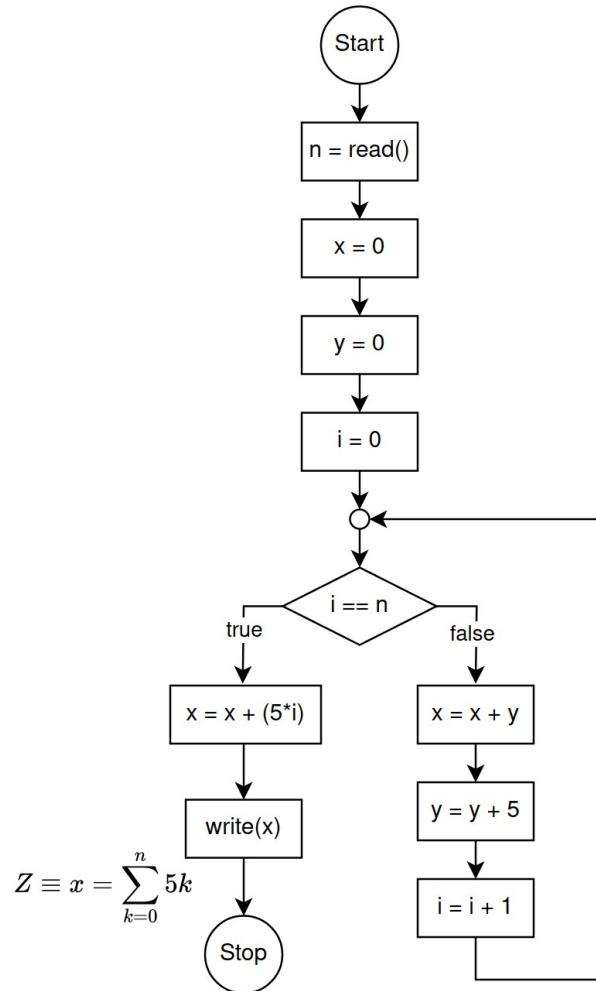
$$(2) \quad I \wedge i < n \Rightarrow B \quad \checkmark$$

$$I \wedge i \geq n \Rightarrow Y \quad \checkmark$$

Week 03 Tutorial 02 — Loop Invariants



Week 03 Task 3: Why is this not on Artemis?



Week 03 Task 3: Why is this not on Artemis?

