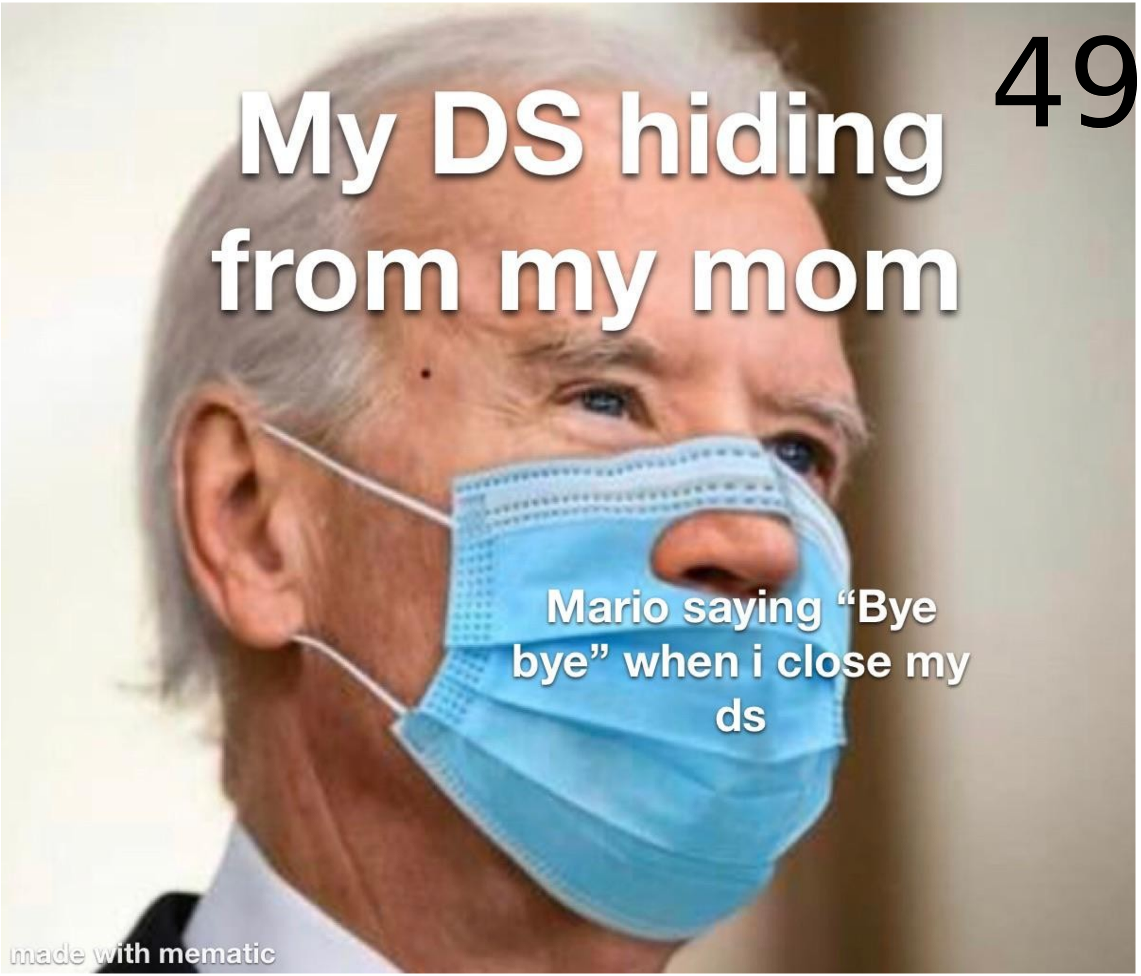
Week 4: Termination



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Quiz

Idea

- Make sure that each loop is executed only finitely often ...
- For each loop, identify an indicator value r, that has two properties

  (1) $r > 0$ whenever the loop is entered;

  (2) r is decreased during every iteration of the loop.

- Transform the program in a way that, alongside ordinary program execution, the indicator value r is computed.
- Verify that properties (1) and (2) hold!

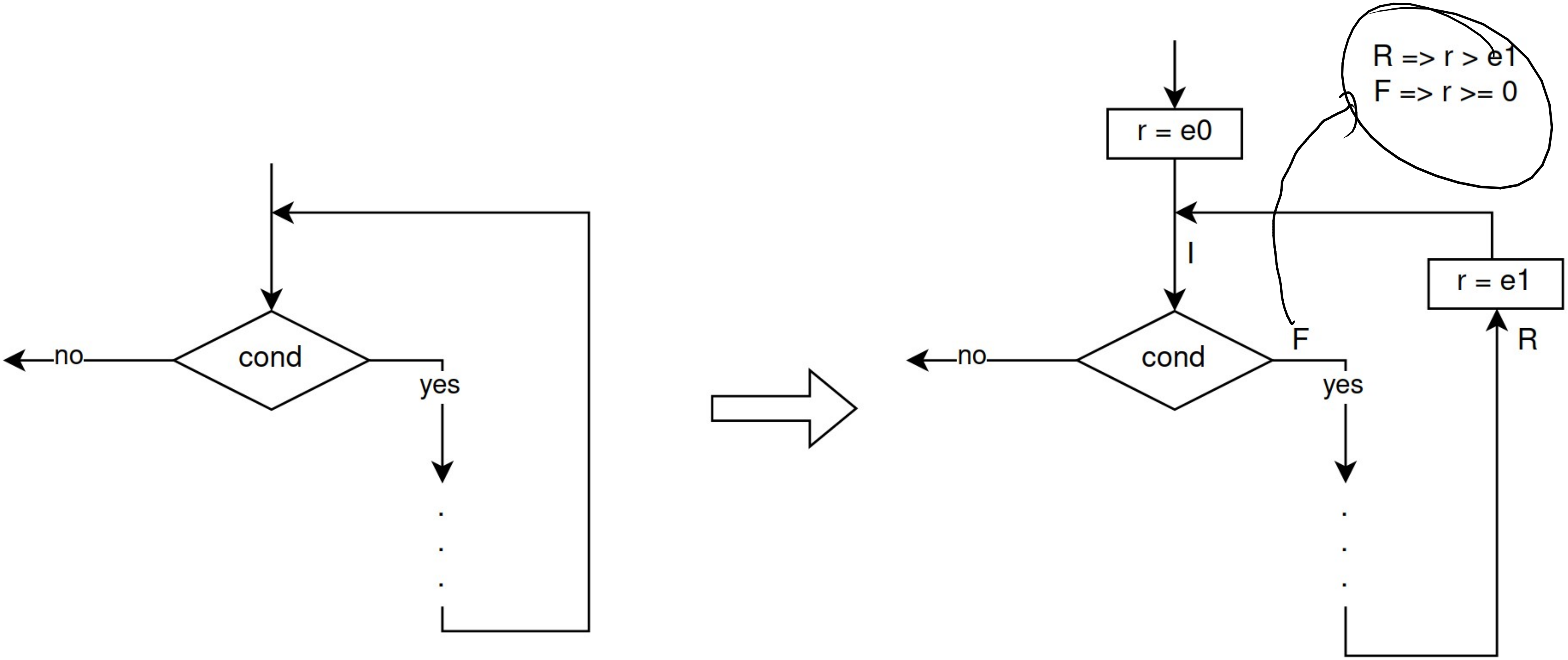
General Method

- For every occurring loop `while (b) s` we introduce a fresh variable `r`.
- Then we transform the loop into:

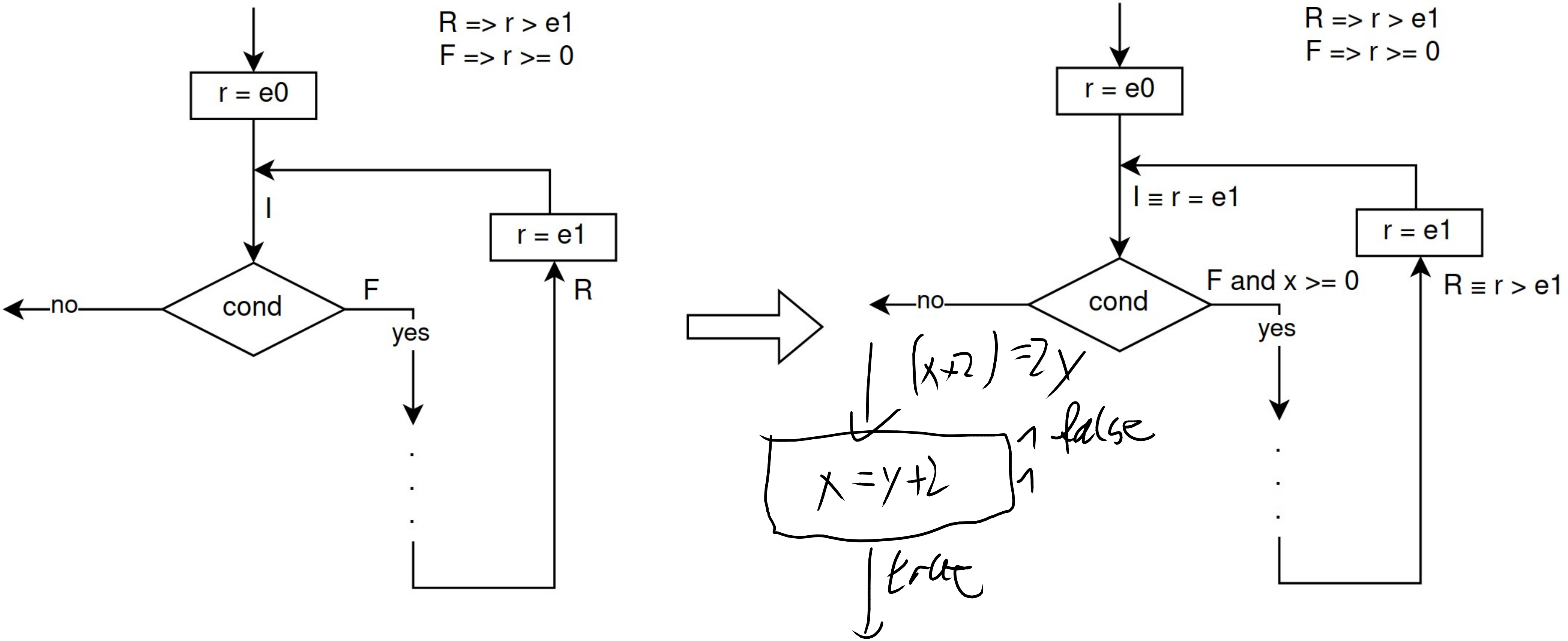
```
r = e0;
while(b) {
        assert(r > 0);
        s;
        assert(r > e1);
        r = e1;
}
```

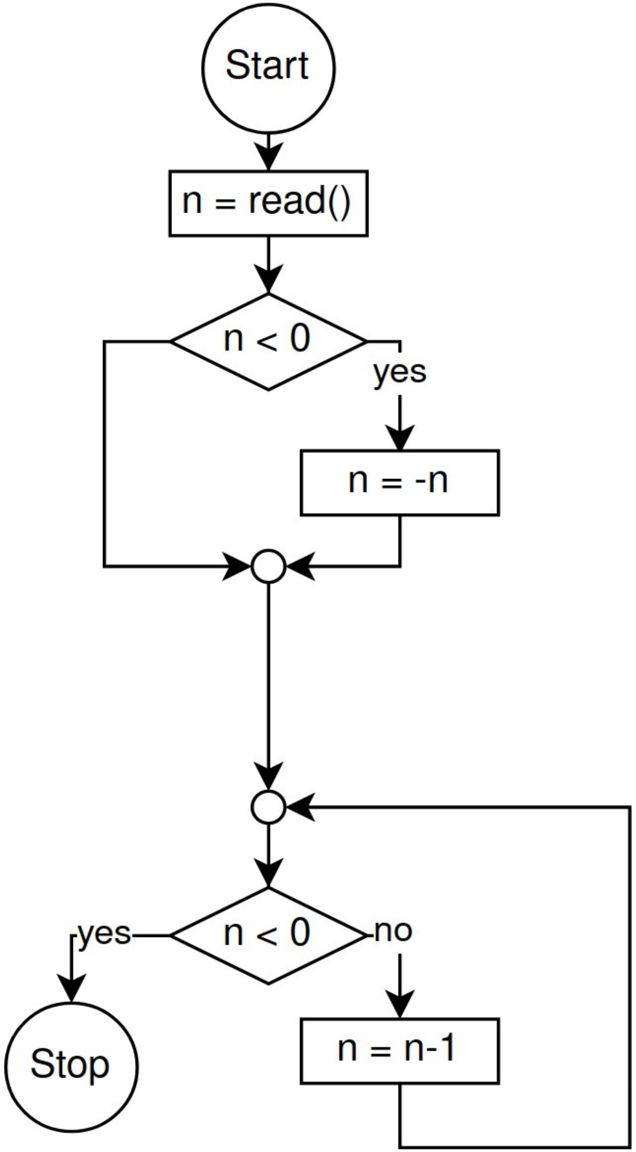
for suitable expressions e0, e1.

How to proof termination



How to proof termination





How to proof termination

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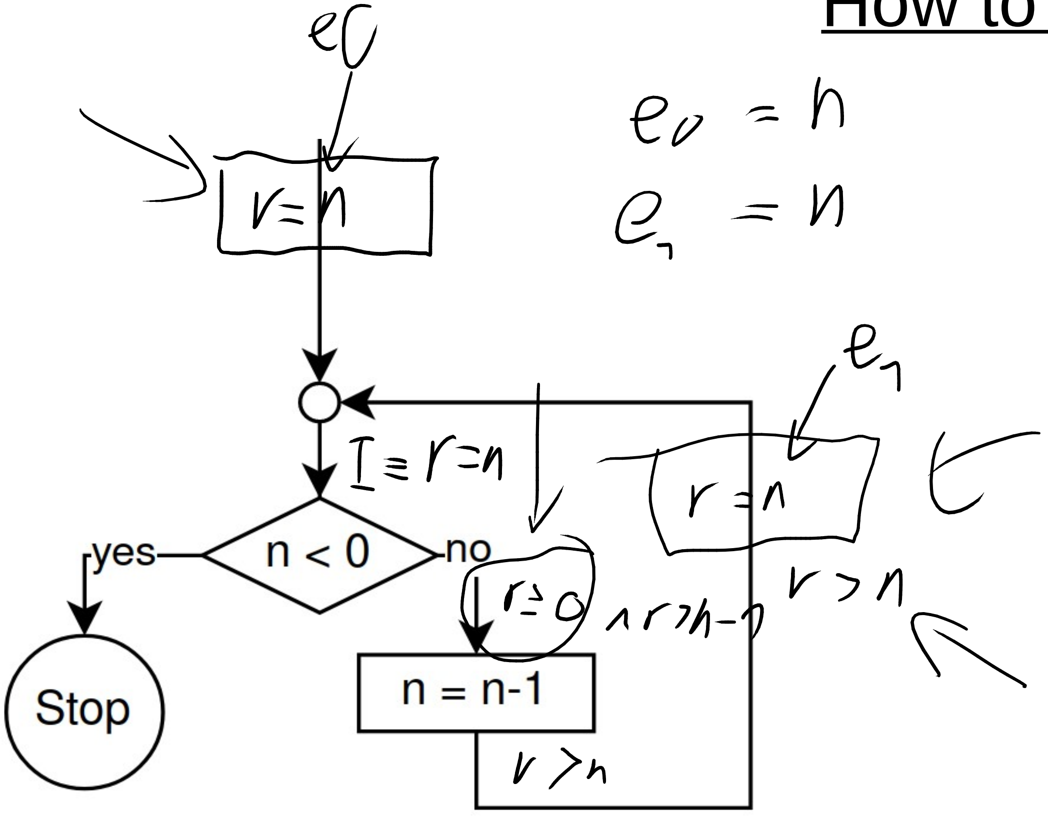
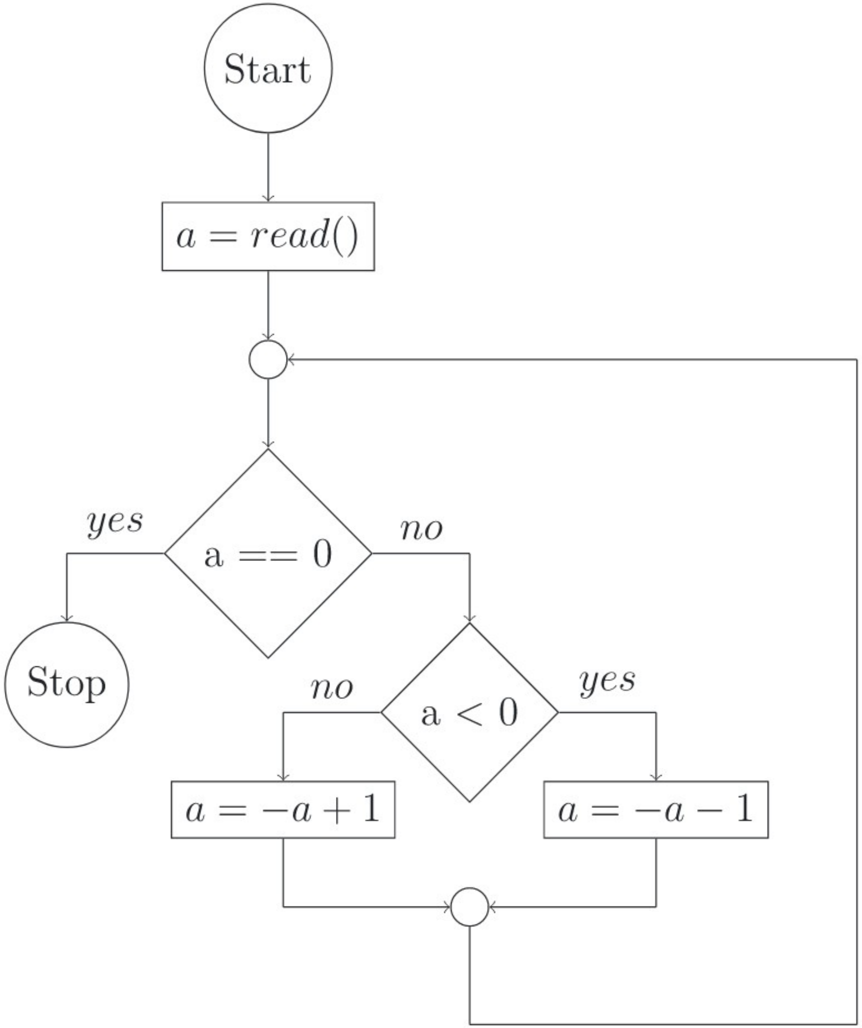
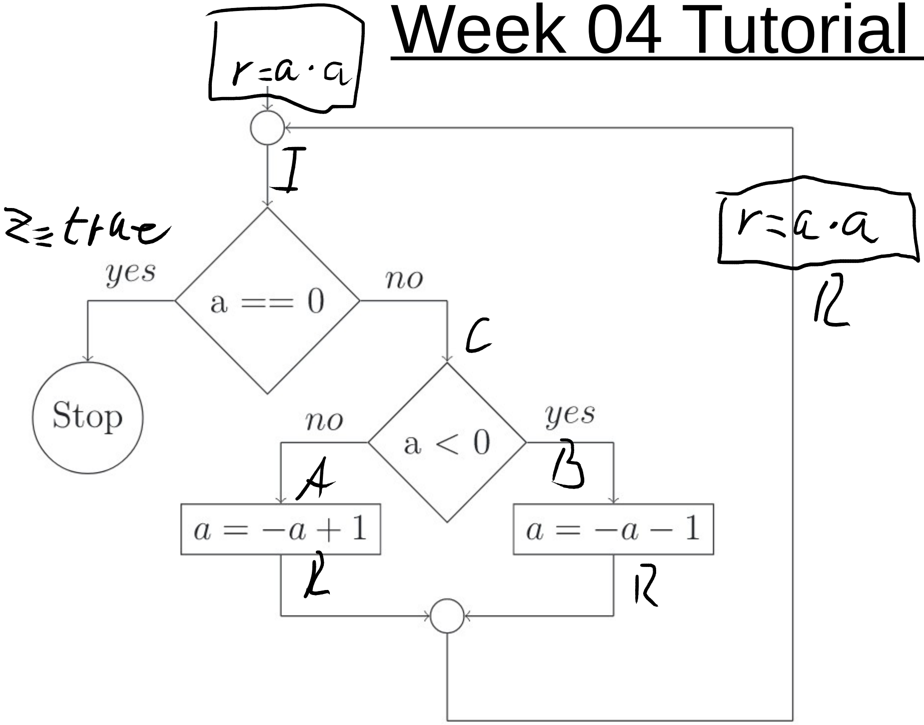


$$\equiv r = n \land n \geq 0 \Rightarrow r \geq 0 \land r > n-1$$

Week 04 Tutorial 03 — A Wavy Approach



Week 04 Tutorial 03 — A Wavy Approach



$I \equiv r = a \cdot a$

$R \equiv r > a \cdot a$

$A \equiv r > (-a+1)^2$

$r > a^2 - 2a + 1$

$B \equiv r > (-a-1)^2$

$r > a^2 + 2a + 1$

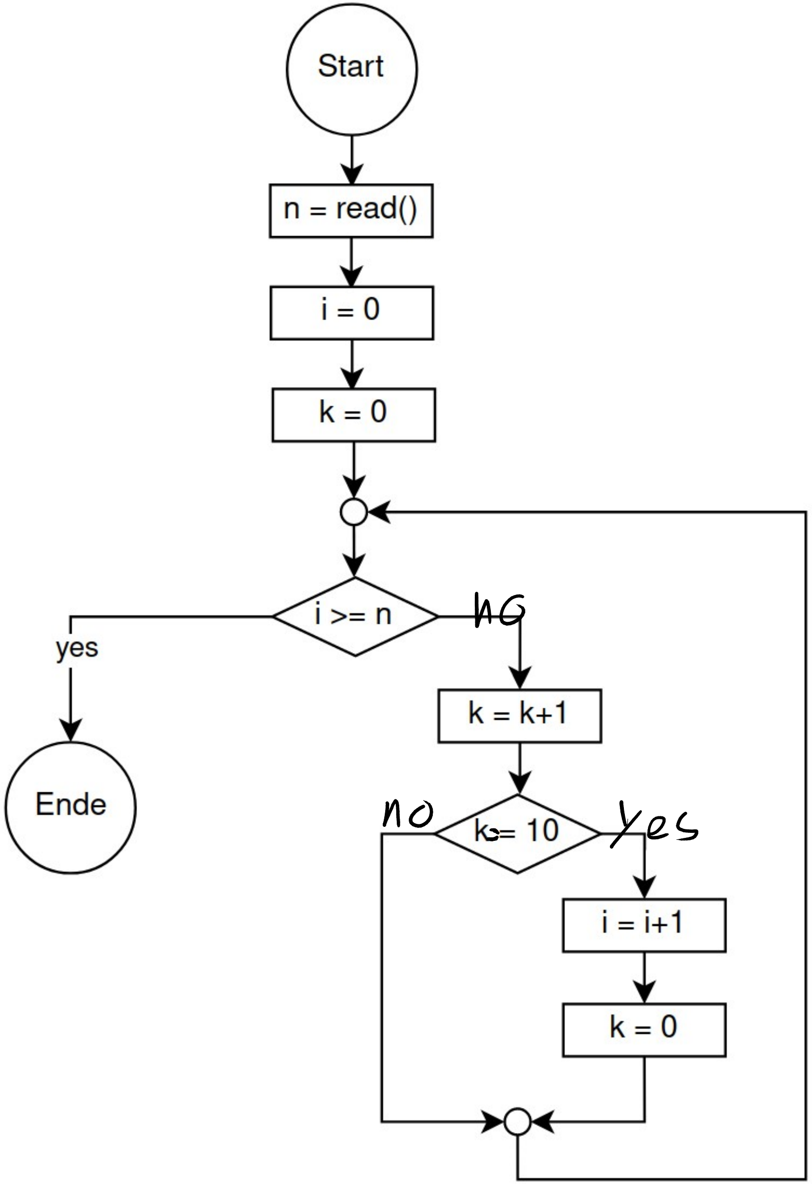
$C \equiv \big((a < 0 \wedge r > a^2 + 2a + 1)$

$\vee (a \geq 0 \wedge r > a^2 - 2a + 1) \big) \wedge r \geq 0$

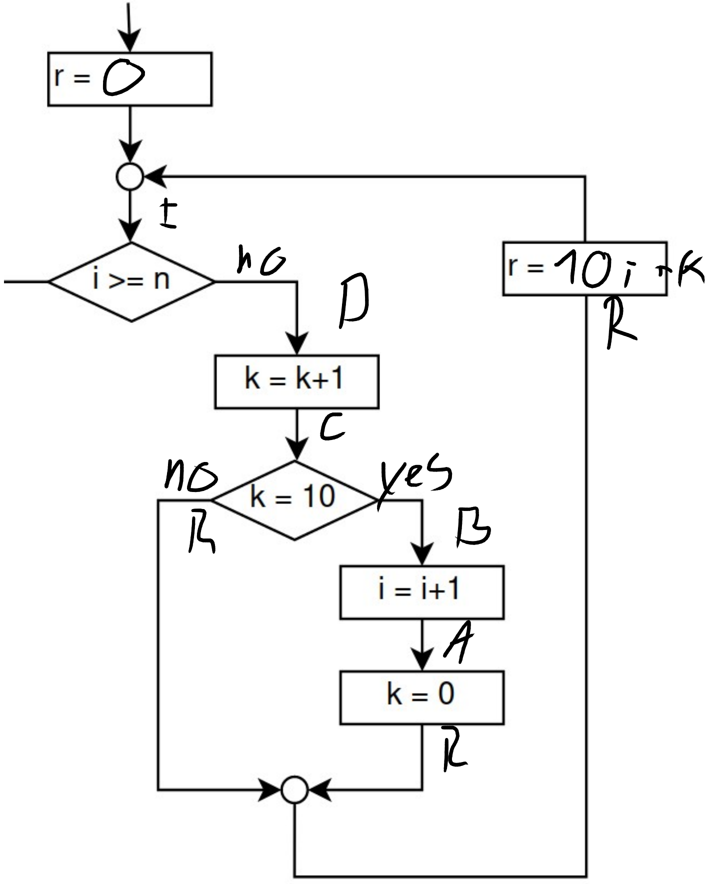
$A' :\equiv WP[\![a = -a+1]\!](R)$

$\equiv$

$r = a \cdot a \wedge a \neq 0 \rightarrow !$



Week 04 Tutorial 04 — Why is this not on Artemis?



$r = 10i + k$
$\wedge\ k \leq 9$
$\wedge\ i < n$

$I \equiv r = 10i + k \quad \wedge\ k \leq 9$

$R \equiv r < 10i + k \quad \wedge\ k \leq 9$

$A \equiv r < 10i$

$B \equiv r < 10i + 10 \quad \Rightarrow B$

$C \equiv (k = 10 \ \wedge\ r < 10i + 10)$
$\vee (\cancel{k \neq 10} \ \wedge\ r < 10i + k \ \wedge\ k \leq 9)$
$\Downarrow$
R

$D \equiv r \leq 10n \ \wedge \Big((k = 9 \ \wedge\ r < 10i + 10)$
$\vee (\cancel{k \neq 9} \ \wedge\ r < 10i + (k+1)) \Big)$
$\wedge\ k \leq 8$

!