

Week 4: Termination



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Quiz

Idea

- Make sure that each loop is executed only finitely often ...
- For each loop, identify an indicator value r , that has two properties
 - (1) $r > 0$ whenever the loop is entered;
 - (2) r is decreased during every iteration of the loop.
- Transform the program in a way that, alongside ordinary program execution, the indicator value r is computed.
- Verify that properties (1) and (2) hold!

General Method

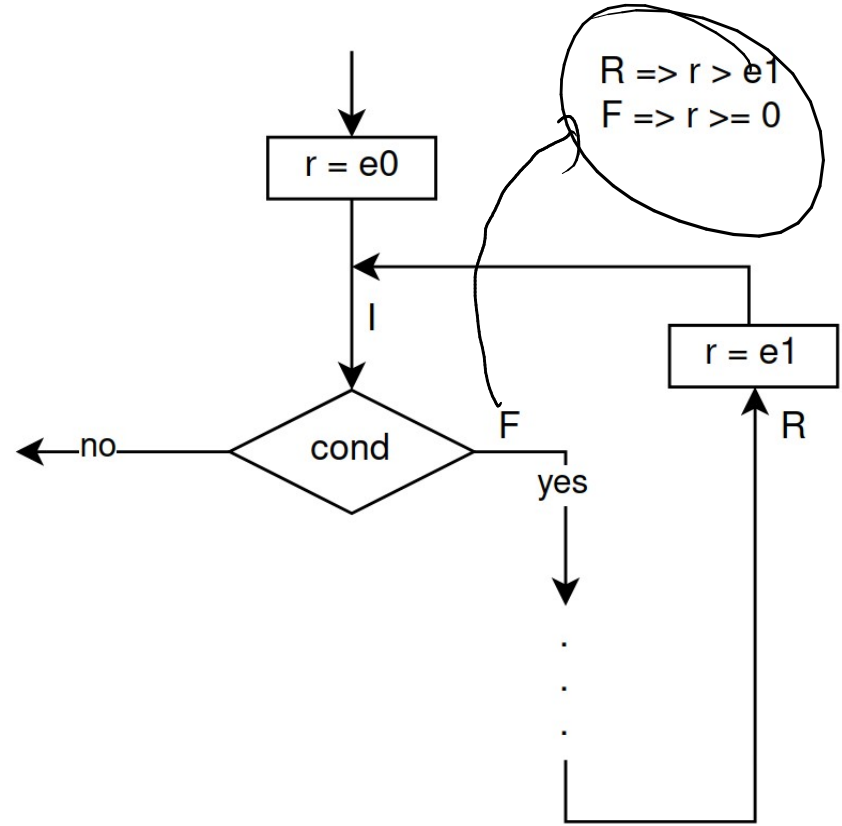
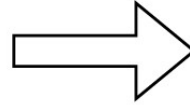
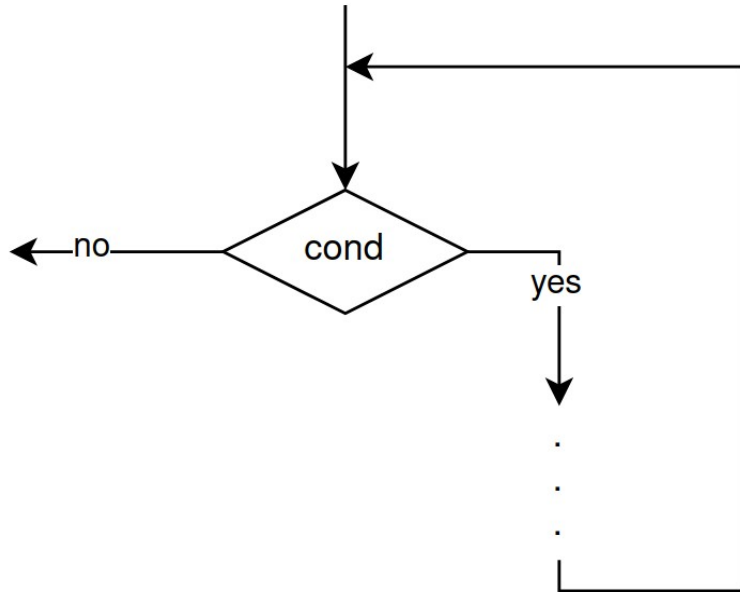
- For every occurring loop `while (b) s` we introduce a fresh variable `r`.
- Then we transform the loop into:

The diagram illustrates the transformation of a `while` loop. It shows a code snippet with line numbers 1 through 7. Hand-drawn annotations include circles around the initialization `r = e0;`, the loop body statements `assert(r > 0);`, `s;`, `assert(r > e1);`, and `r = e1;`, and arrows indicating the flow from the loop condition `while(b)` to the first assertion and from the last statement back to the condition.

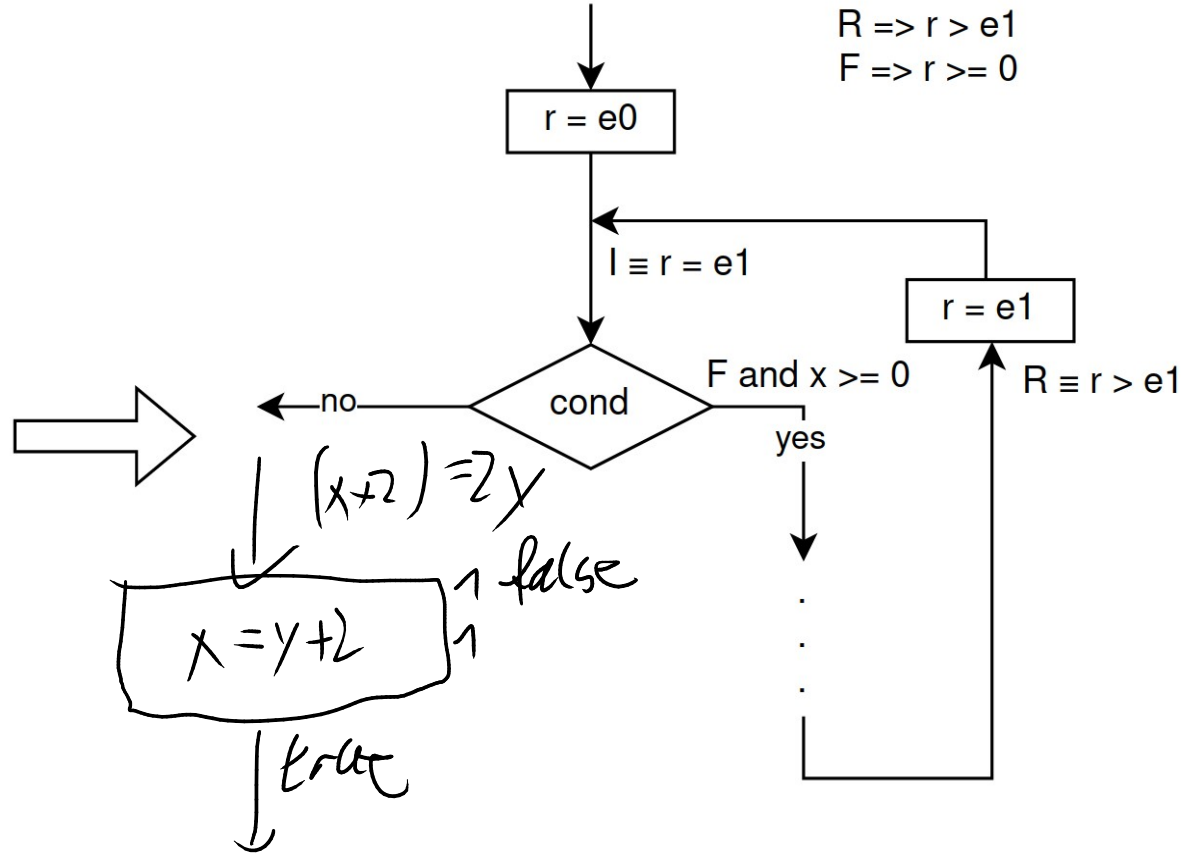
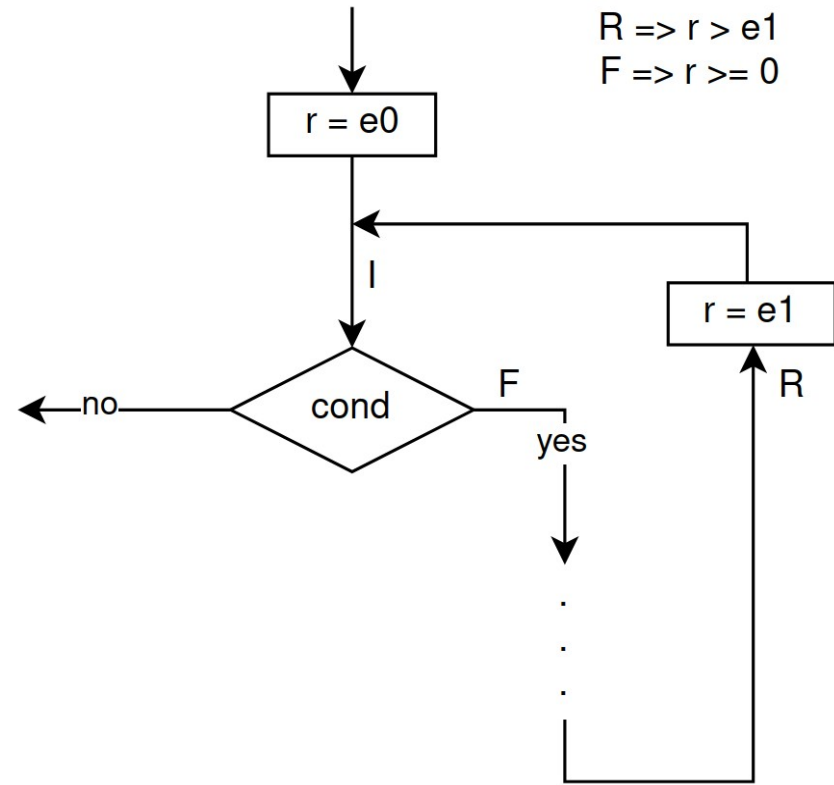
```
1  r = e0;  
2  while(b) {  
3      assert(r > 0);  
4      s;  
5      assert(r > e1);  
6      r = e1;  
7  }
```

for suitable expressions `e0, e1`.

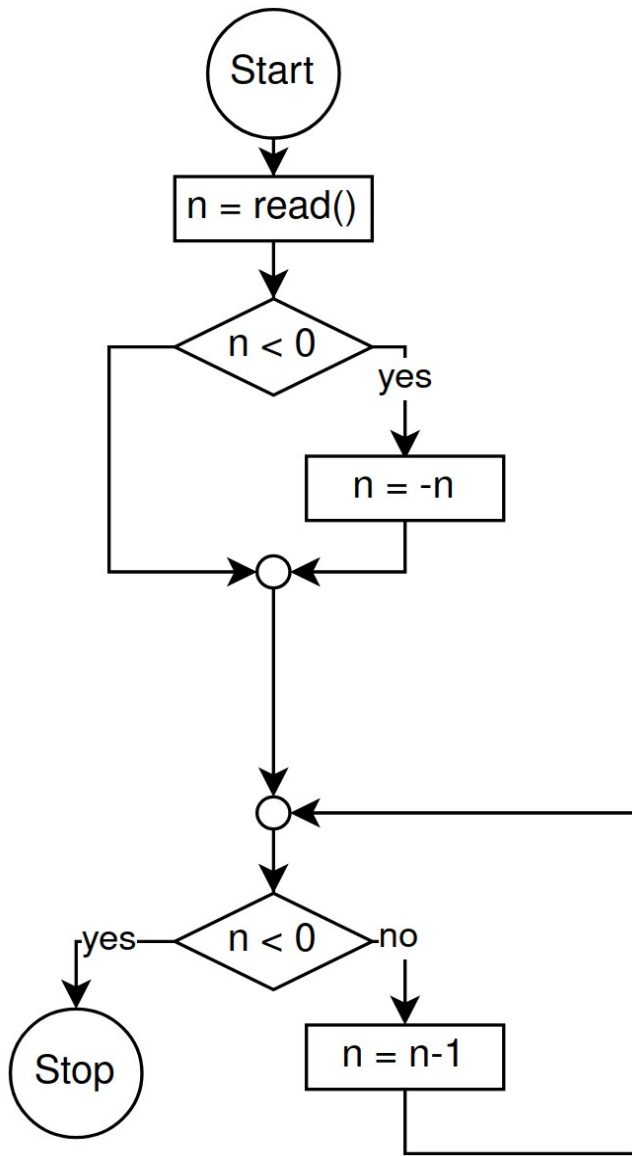
How to proof termination



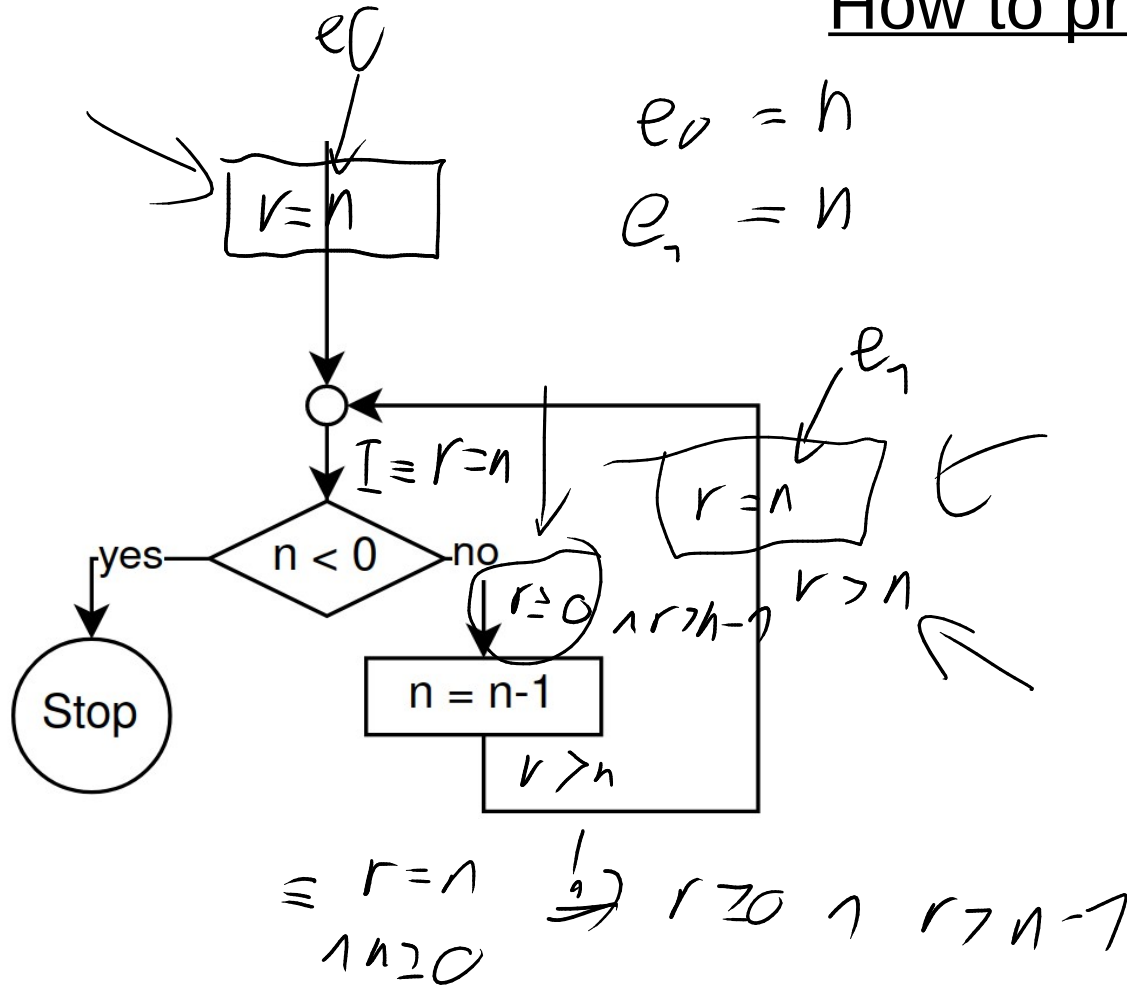
How to proof termination



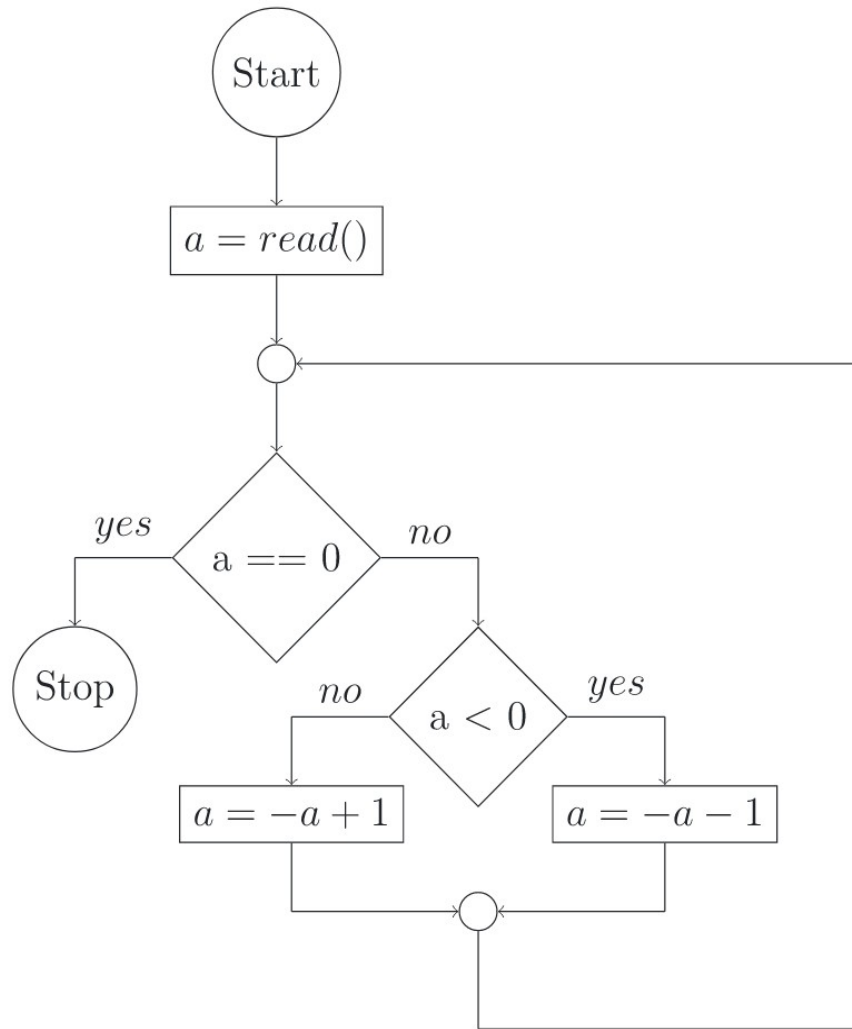
How to proof termination



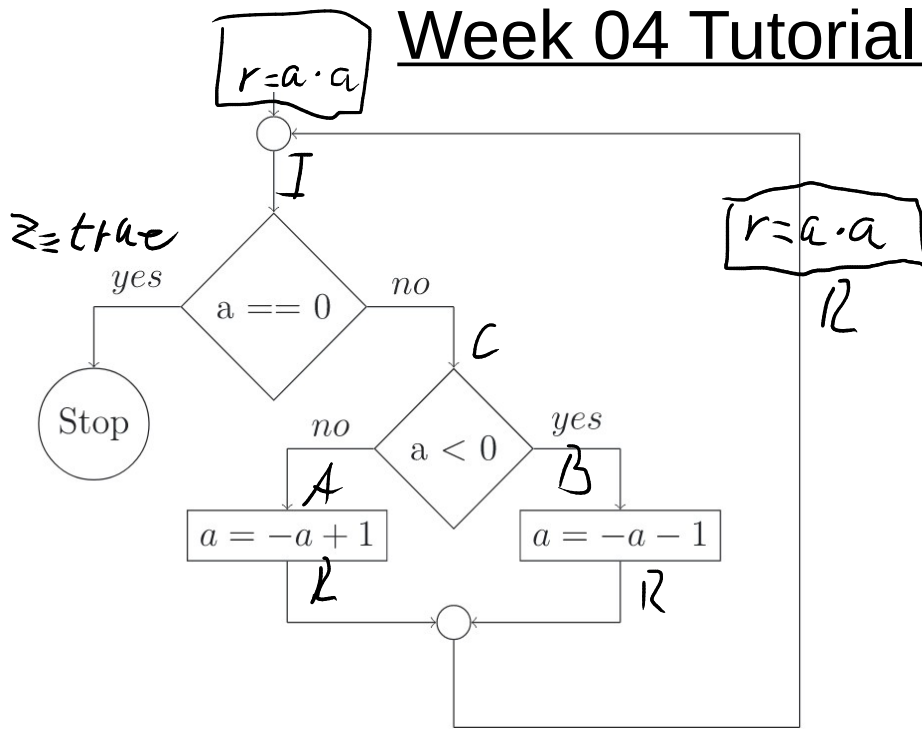
How to proof termination



Week 04 Tutorial 03 — A Wavy Approach



Week 04 Tutorial 03 — A Wavy Approach



$$I \equiv r = a \cdot a$$

$$R \equiv r > a \cdot a$$

$$A \equiv r > (-a+1)^2$$

$$r > a^2 - 2a + 1$$

$$B \equiv r > (-a-1)^2$$

$$r > a^2 + 2a + 1$$

$$C \equiv \left((a < 0 \wedge r > a^2 + 2a + 1) \vee (a \geq 0 \wedge r > a^2 - 2a + 1) \right) \wedge r \geq 0$$

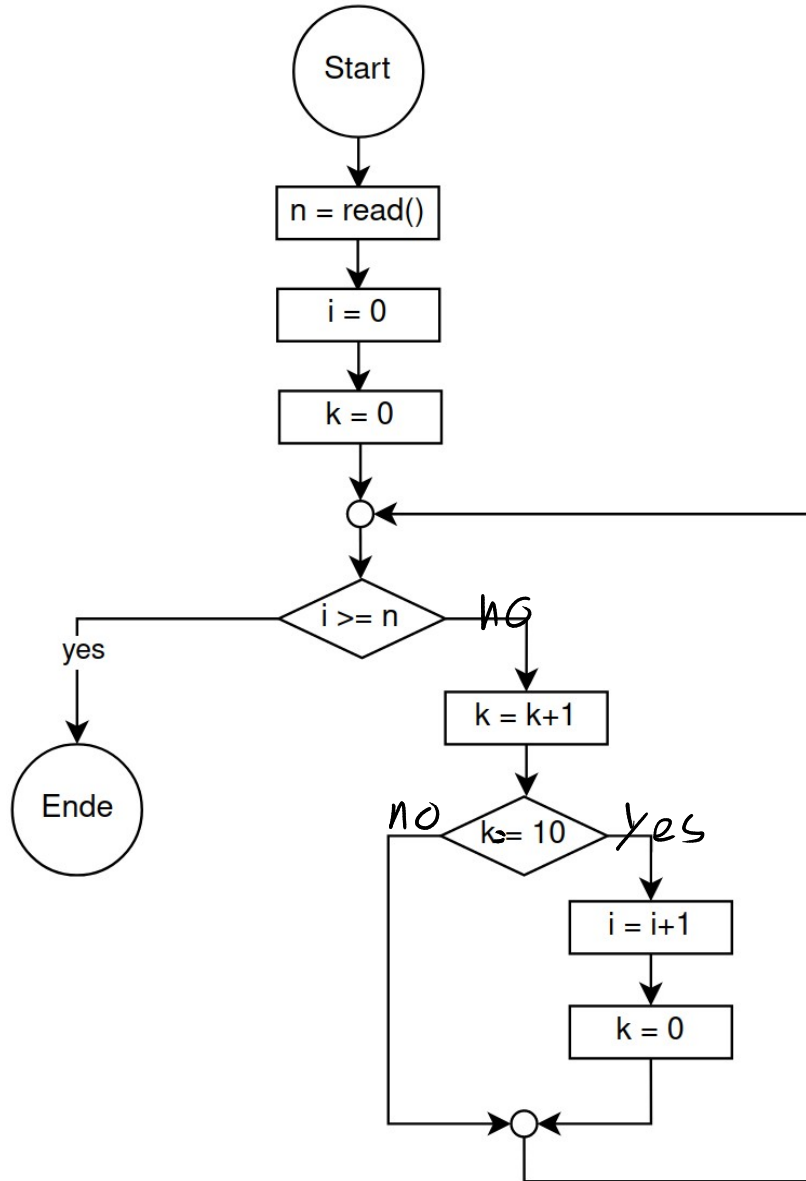
$$r = a \cdot a \wedge a \neq 0$$

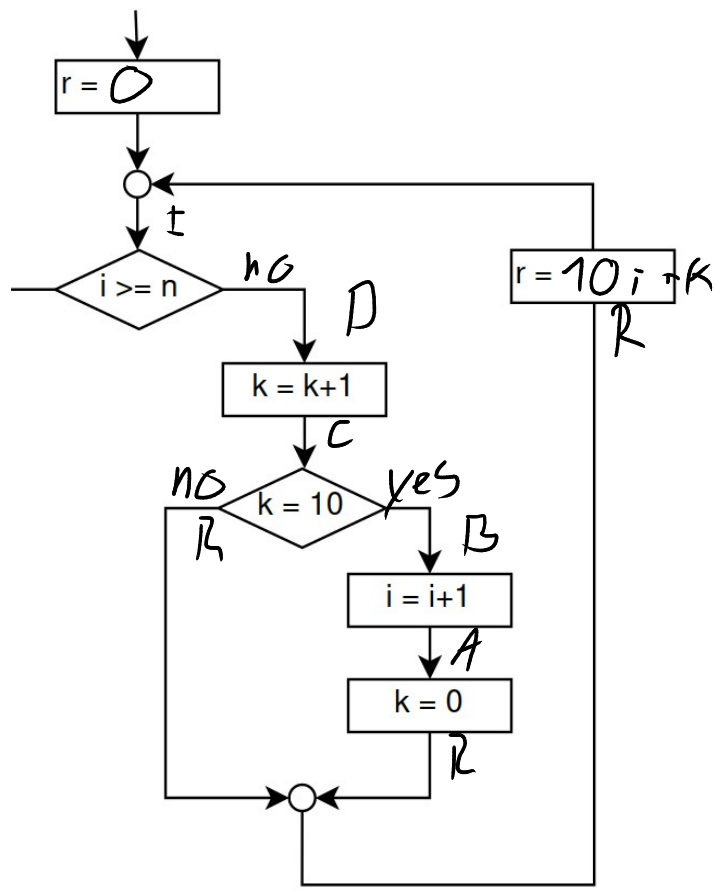
$$A \equiv \text{WP}[a = -a+1](I)$$

$$\equiv$$

Week 04 Tutorial 04

— Why is this not on Artemis?





$$r = 10i + k$$

$$\wedge k \leq 9$$

$$\wedge i < n$$

$$I \equiv r = 10i + k \quad \wedge k \leq 9$$

$$R \equiv r < 10i + k \quad \wedge k \leq 9$$

$$A \equiv v < 10i$$

$$B \equiv r < 10i + 10$$

$$\bar{C} \equiv (k = 10 \wedge r < 10i + 10)$$

$$\vee (k \neq 10 \wedge r < 10i + k \wedge k \leq 9)$$

$$D \equiv r \leq 10n \wedge \left(\begin{array}{l} (k = 9 \wedge r < 10i + 10) \\ \vee (k \neq 9 \wedge r < 10i + k + 1) \end{array} \right)$$

$$\wedge k \leq 8$$

