Week 3: Loop Invariants

wp = $x - y \geq 5$

$x \geq 10$

$\boxed{x = x - y}$

$x \geq 5$

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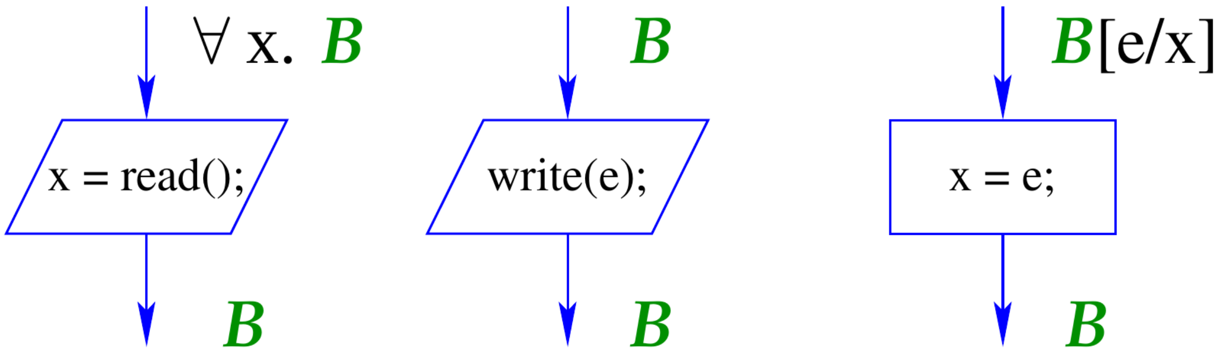
Quiz

Week 03 Tutorial 01 — MiniJava 2.0



Week 03 Tutorial 01 — MiniJava 2.0

1. `rand x`:
   Assigns a random value to variable x,
2. `x = either` $e_0, \dots, e_k$:
   Assigns one of the values of the expressions $e_0, \dots, e_k$ to variable `x` non-deterministically,
3. `x = e in a, b`:
   Assigns the value 1 to variable `x`, if the value of expression `e` is in the range $[a, b]$ and 0 if `e` is not in the range or the range is empty $(a > b)$,
4. `stop`:
   Immediately stops the program.

Define the weakest precondition operator $\mathbf{WP}[\![\ldots]\!](B)$ for each of these statements.

Week 03 Tutorial 01 — MiniJava 2.0

$\forall x.\ B$

rand x

B

$(a \le e \le b \Rightarrow B[1/x])$
$\wedge (a > e \vee b < e) \Rightarrow B[0/x]$

x = e in a, b

B

$\{e_1, e_2, \ldots\}$

$B[e_1/x]$
$\wedge\ B[e_2/x]$
$\wedge\ B[e_3/x]$
$\wedge\ \ldots$

x = either e1, ...en

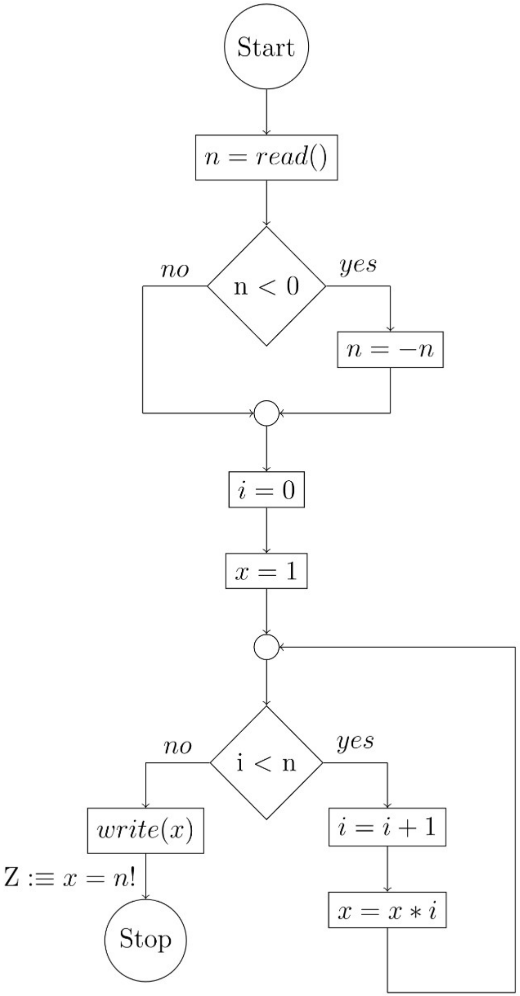
B

true

stop

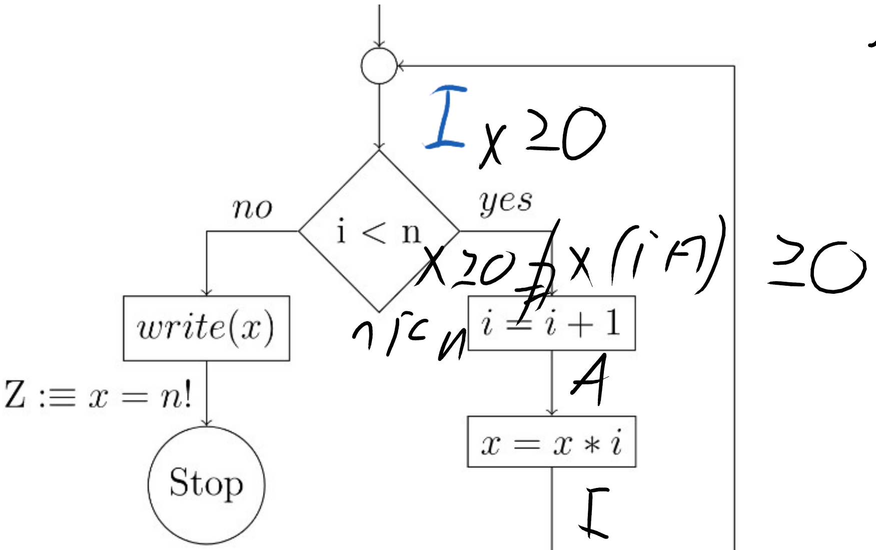
B

Week 03 Tutorial 02 — Loop Invariants



1. Discuss the problem that arises when computing weakest preconditions to prove Z.
2. How can you use weakest preconditions to prove Z anyway?

Week 03 Tutorial 02 — Loop Invariants

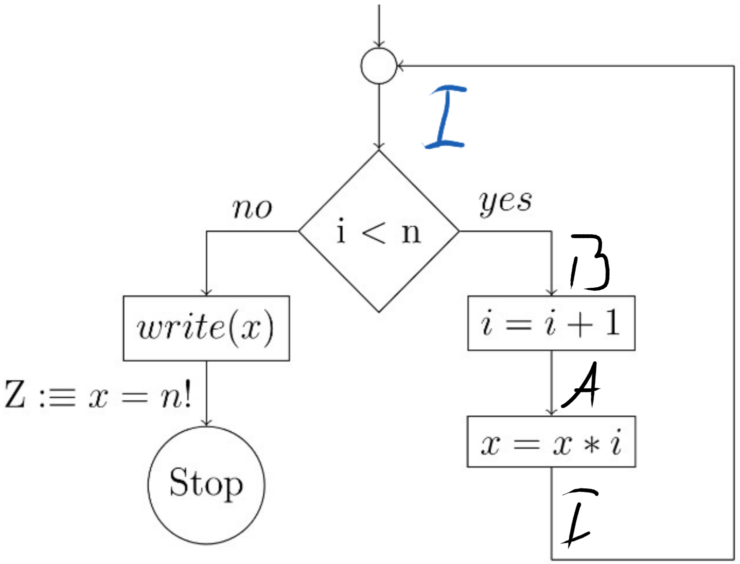


$I :\equiv x \geq 0$

$A :\equiv WP[\![x = x \cdot i]\!](I) \equiv x \cdot i \geq 0$

$B :\equiv x(i+1) \geq 0$

Week 03 Tutorial 02 — Loop Invariants



$I :\equiv i = 0 \land x = 1 \land n = 0$

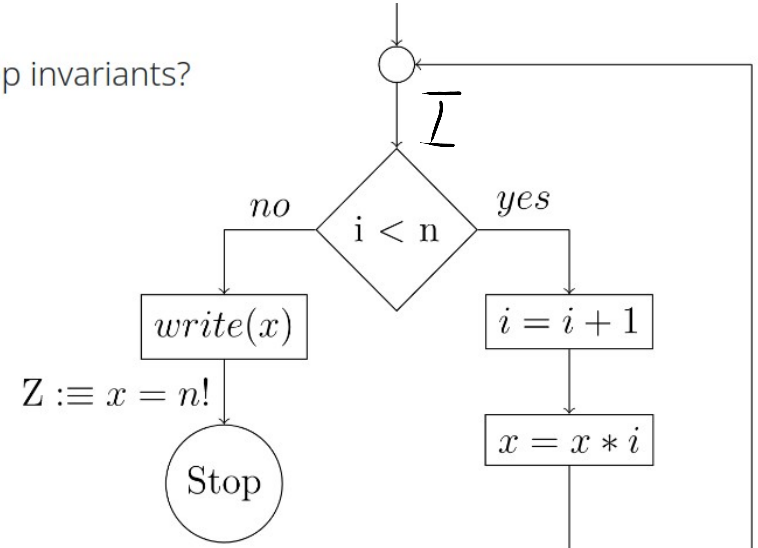
$A :\equiv i = 0 \land x i = 1 \land n = 0$

$B :\equiv i+1 = 0 \land x(i+1) = 1 \land n = 0$

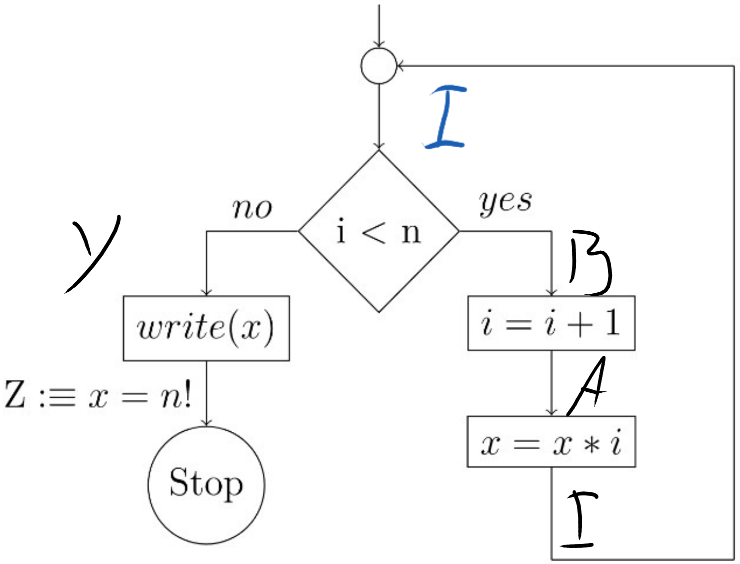
$SP[\![i < n]\!](I) \equiv I \land i < n$

Week 03 Tutorial 02 — Loop Invariants

- a) How has a useful loop invariant be related to Z?
- b) What happens if the loop invariant is chosen too strong?
- c) What happens if the loop invariant is chosen too weak?
- d) Can you give a meaningful lower and upper bound for useful loop invariants?



Week 03 Tutorial 02 — Loop Invariants



$Y \equiv Z$

$I :\equiv x = i! \land i \leq n$

$A :\equiv x \cdot i = i! \land i \leq n$

$B :\equiv x \cdot ((i+1) = (i+1)! \land i+1 \leq n$

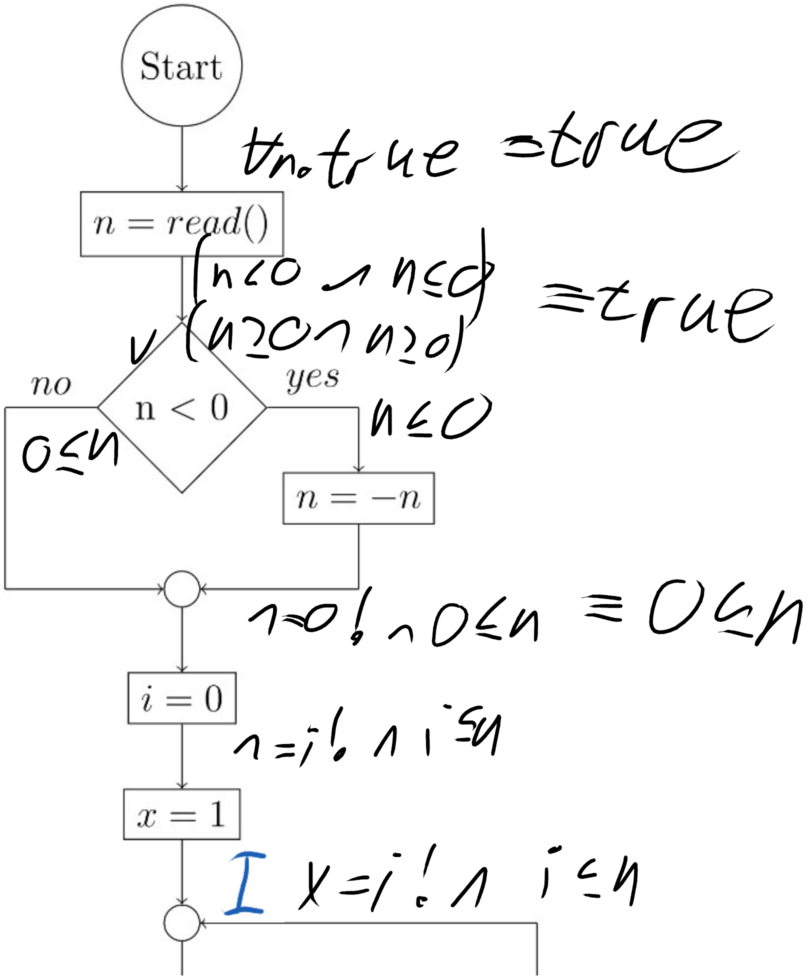
$\equiv x \cdot (i+1) = (i+1) \cdot i!$ ...

$\equiv x = i! \land i+1 \leq n$

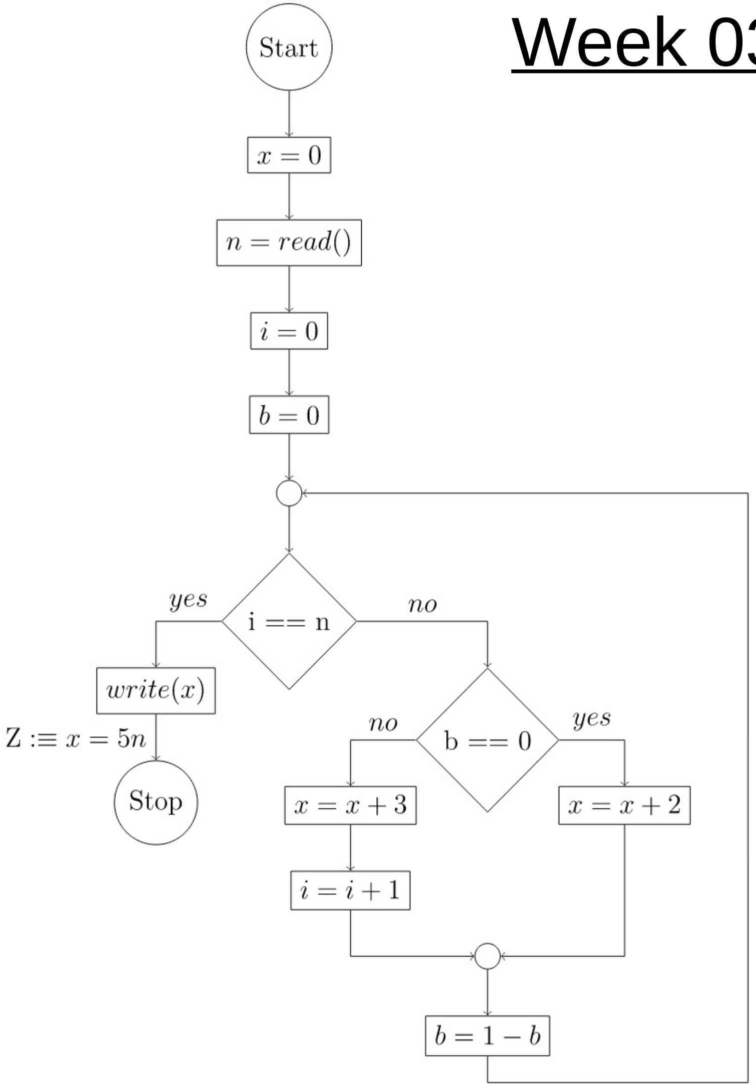
$SP[i<n](I) \overset{?}{\equiv} x = i! \land i<n \overset{!}{\underset{?}{\Rightarrow}} x = i! \land i+1 \leq n$

$SP[i \geq n](I) \overset{?}{\equiv} x = i! \land i \leq n \land i \geq n \equiv x = i! \land i \overset{?}{=} n \Rightarrow x = n!$

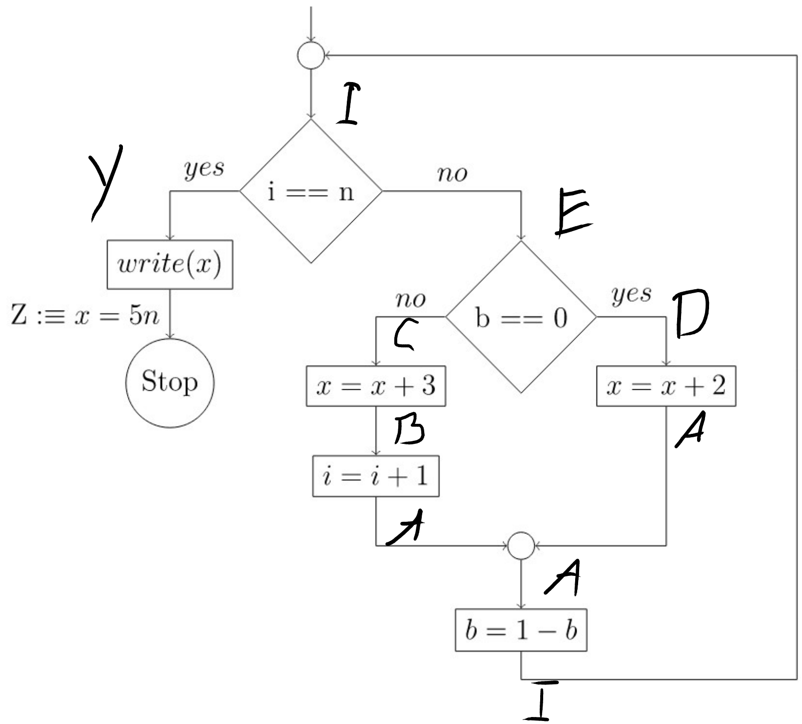
Week 03 Tutorial 02 — Loop Invariants



Week 03 Tutorial 03 — Two b, or Not Two b



Week 03 Tutorial 03 — Two b, or Not Two b



$I \equiv \quad X = 5i + 2b \ \wedge (b=1 \vee b=0) \wedge (i=n \Rightarrow b=0)$

$A :\equiv \quad X = 5i + 2 - 2b \wedge (b=1 \vee b=0) \wedge (i=n \Rightarrow b=1)$

$B :\equiv \quad X = 5i + 7 \quad -2b \wedge (b=1 \vee b=0) \wedge (i+1=n \Rightarrow b=1)$

$C :\equiv \quad X = 5i + 4 \ -2b \wedge (b=1 \vee b=0) \wedge (i+1=n \Rightarrow b=1)$

$D :\equiv \quad X = 5i - 2b \wedge (b=1 \vee b=0) \wedge (i=n \Rightarrow b=1)$

$E :\equiv (b=0 \wedge x = 5i)$
$\vee (b \neq 0 \wedge x = 5i + 4 - 2b)$

$\rightarrow \ \equiv (b=0 \wedge x = 5i)$
$\vee (b=1 \wedge X = 5i+2)$

$\rightarrow \ \equiv (b=0 \wedge X = 5i \wedge i = n \Rightarrow b=1)$
$\vee (b=1 \wedge X = 5i+2 \wedge i+1=n \Rightarrow b=1)$

$Y \equiv Z$

Die beiden hatte ich während dem Tutorium vertauscht

damit $b=0 \wedge a=n \Rightarrow b=1$
gelten kann muss $i \neq n$ gelten

$$\equiv (b=0 \wedge x=5i \wedge i \neq n)$$
$$\vee (b=1 \wedge x=5i+2)$$

Prüfen auf local consistency:

$I \wedge i \neq n \Rightarrow E$ ✓