Week 4: Termination

Local Consistency revisited

$x = 7$

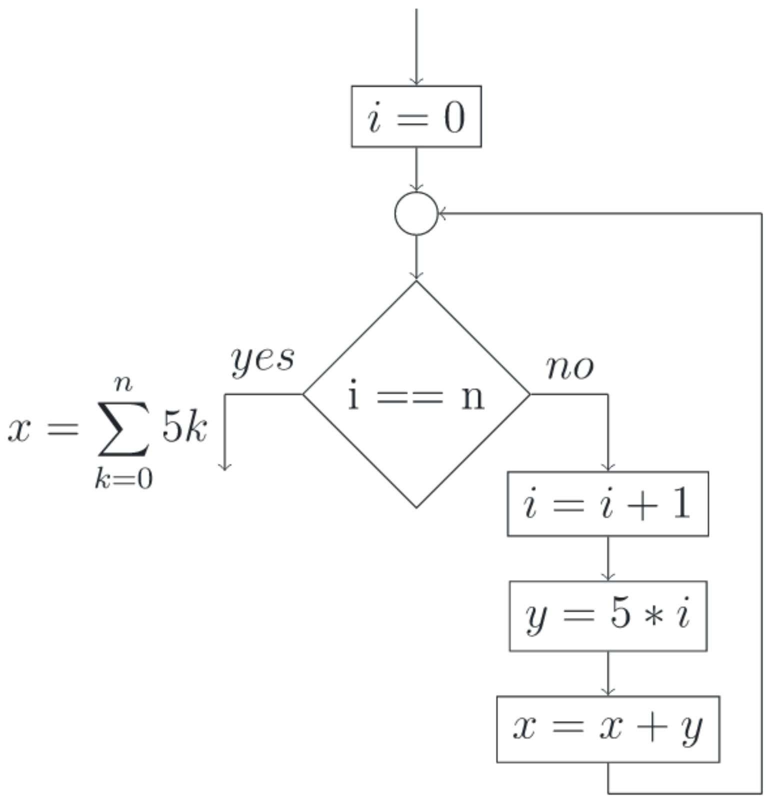
$A \equiv \cancel{x = 5} \land y = 10 \land z = -3$

$$\downarrow$$

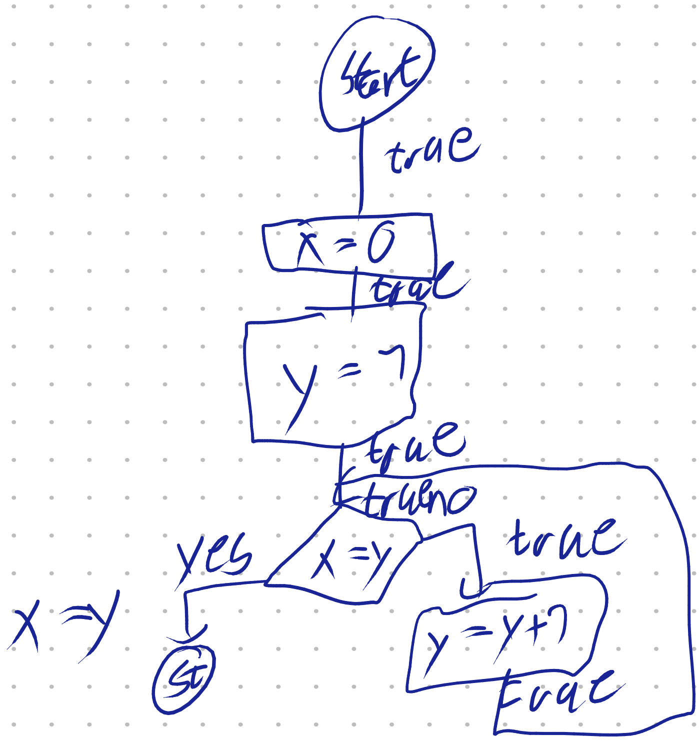
$$\boxed{x = x*x + y}$$

$$\downarrow$$

$B \equiv x > 20 \land \cancel{y \neq 10}$



$i = 0$
i == n
yes
no
$x = \sum_{k=0}^{n} 5k$
$i = i + 1$
$y = 5 * i$
$x = x + y$



Start
true
x = 0
true
y = 7
true
true no
x = y
yes
true
y = y+7
true
x = y

Idea

- Make sure that each loop is executed only finitely often ...
- For each loop, identify an indicator value r, that has two properties

  (1) $r > 0$ whenever the loop is entered;

  (2) r is decreased during every iteration of the loop.

- Transform the program in a way that, alongside ordinary program execution, the indicator value r is computed.
- Verify that properties (1) and (2) hold!

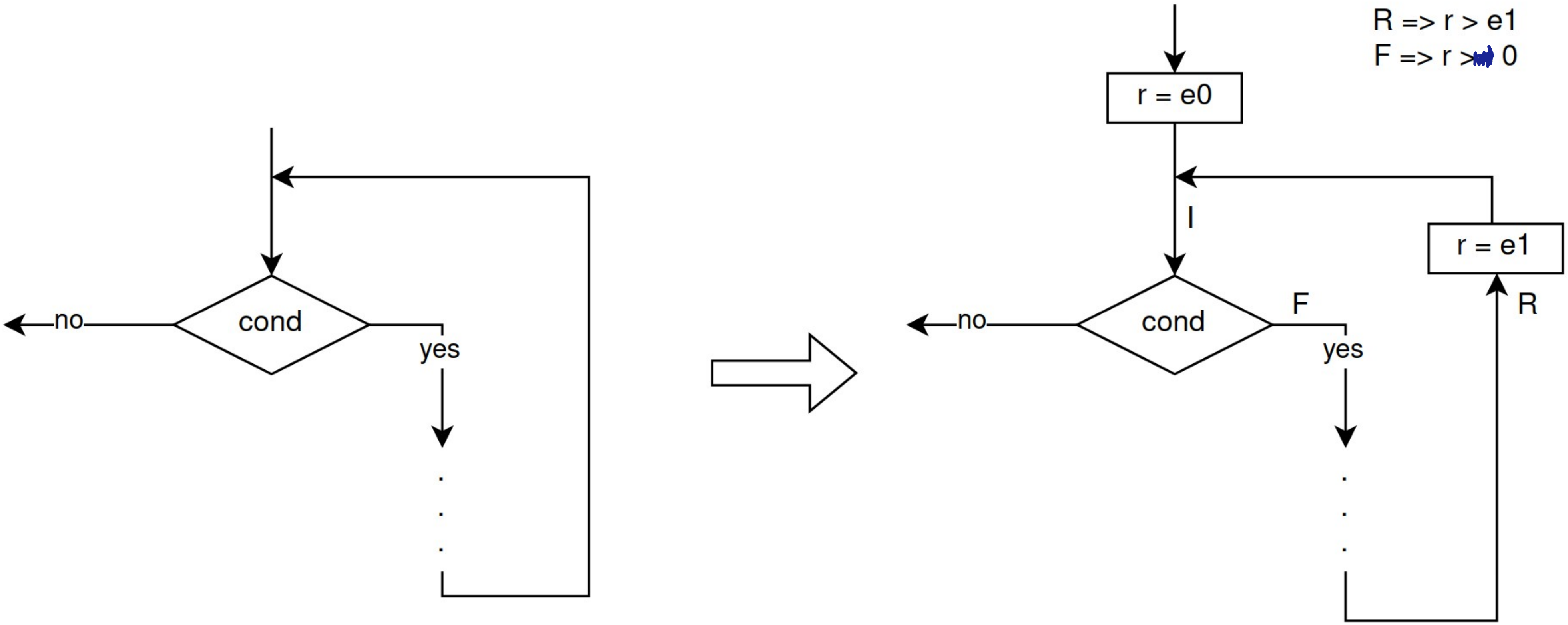
General Method

- For every occurring loop `while (b) s` we introduce a fresh variable `r`.
- Then we transform the loop into:

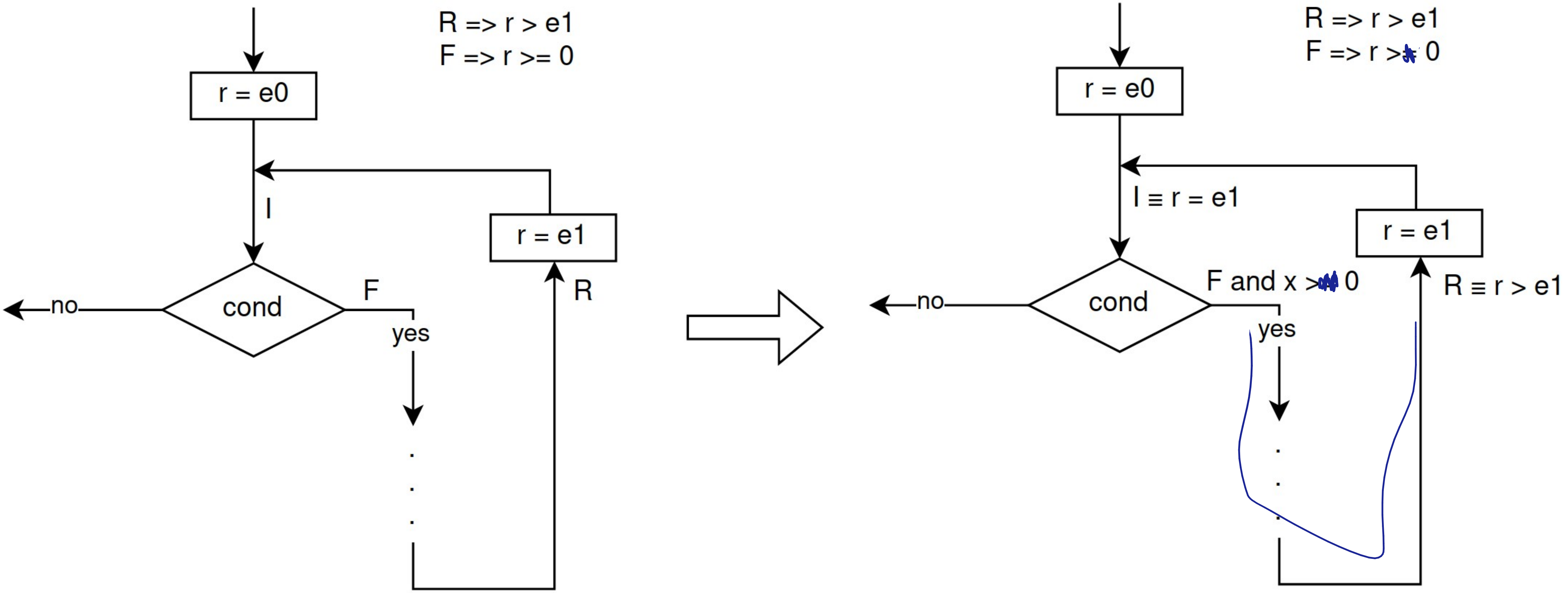
```
r = e0;
while(b) {
        assert(r > 0);
        s;
        assert(r > e1);
        r = e1;
}
```

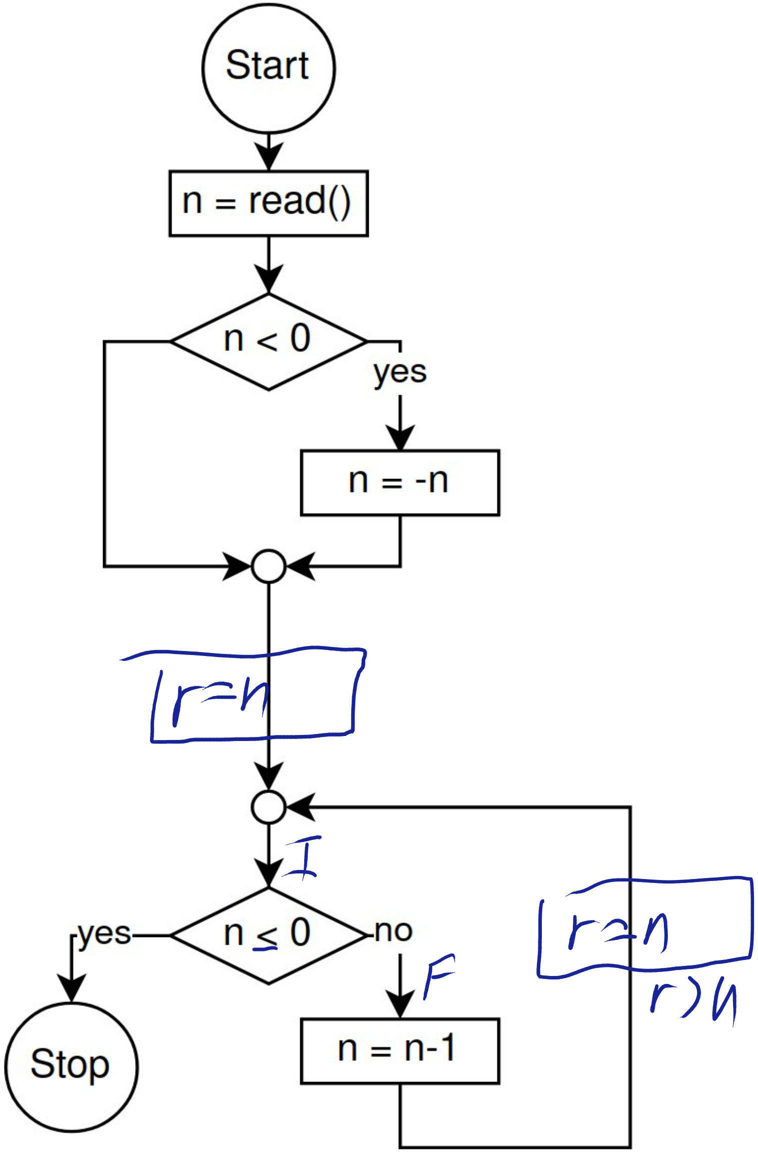
for suitable expressions `e0, e1`.

How to prove termination



How to prove termination





How to prove termination

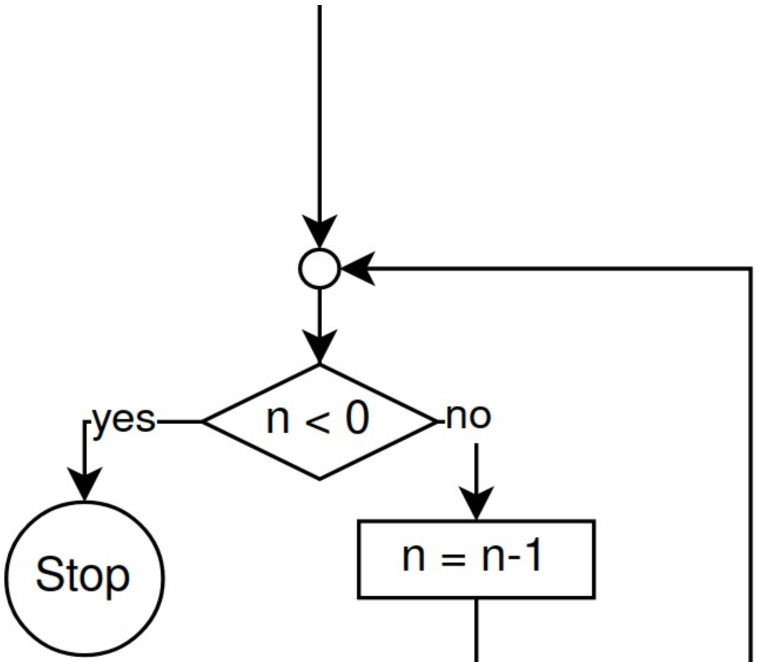
$I \equiv r = n$

$F \equiv r > n-1 \wedge r > 0$

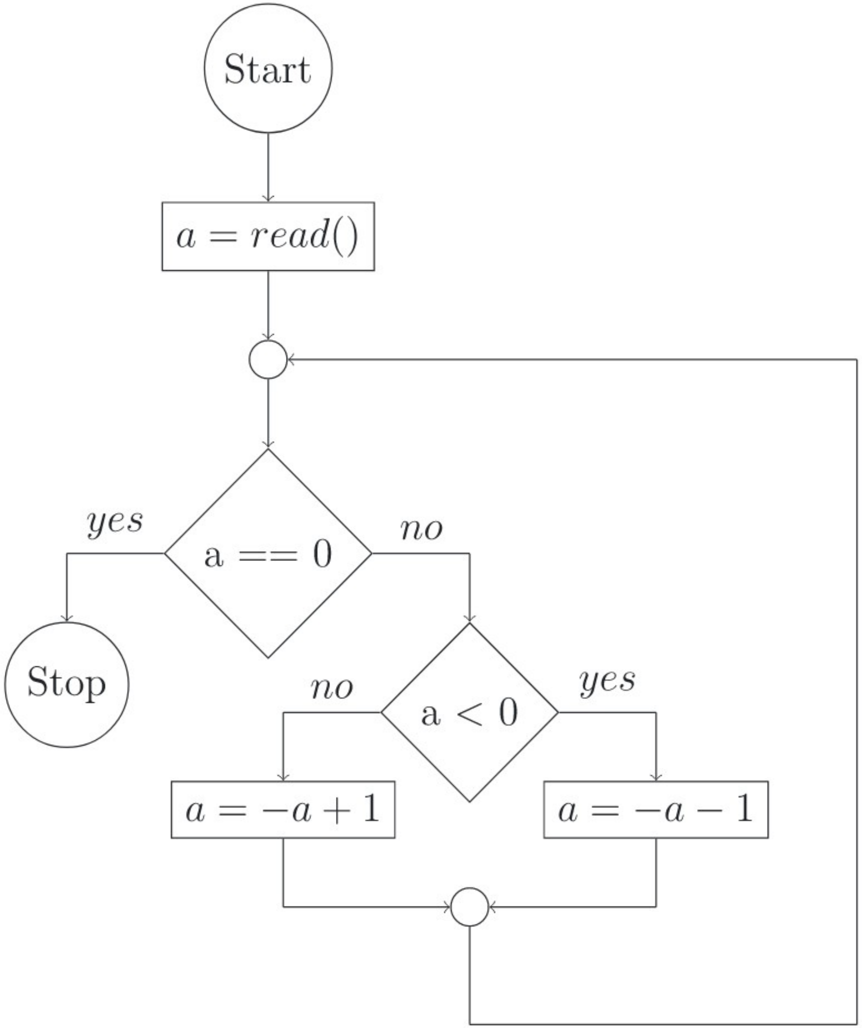
Prüfen auf IC

① $I \wedge n > 0 \Rightarrow F$

How to prove termination

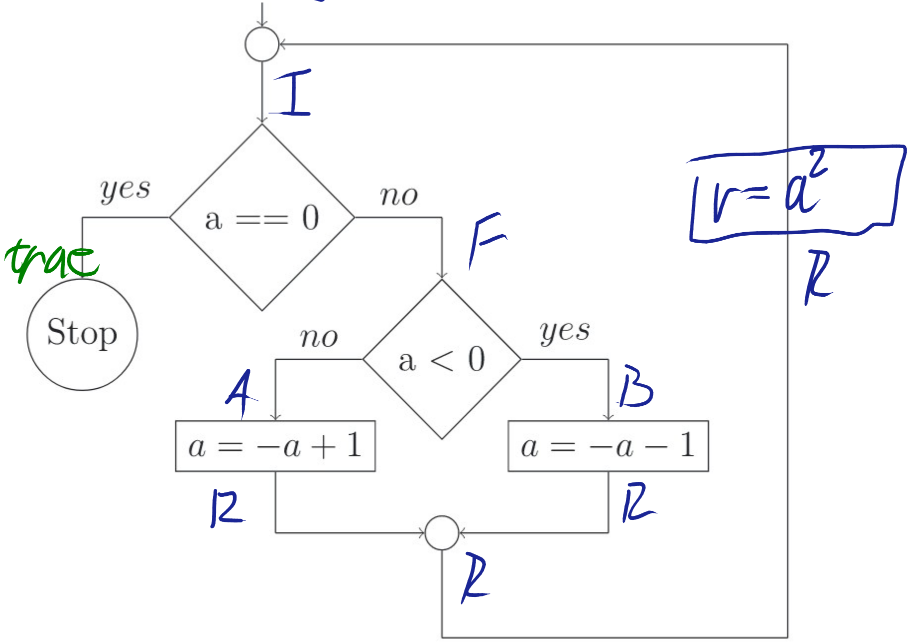


Week 04 Tutorial 03 — A Wavy Approach



Week 04 Tutorial 03 — A Wavy Approach

$\boxed{r=a^2}$



$I \equiv r = a^2$

$R \equiv r > a^2$

$A \equiv r > (1-a)^2 \equiv a^2 - 2a + 1$

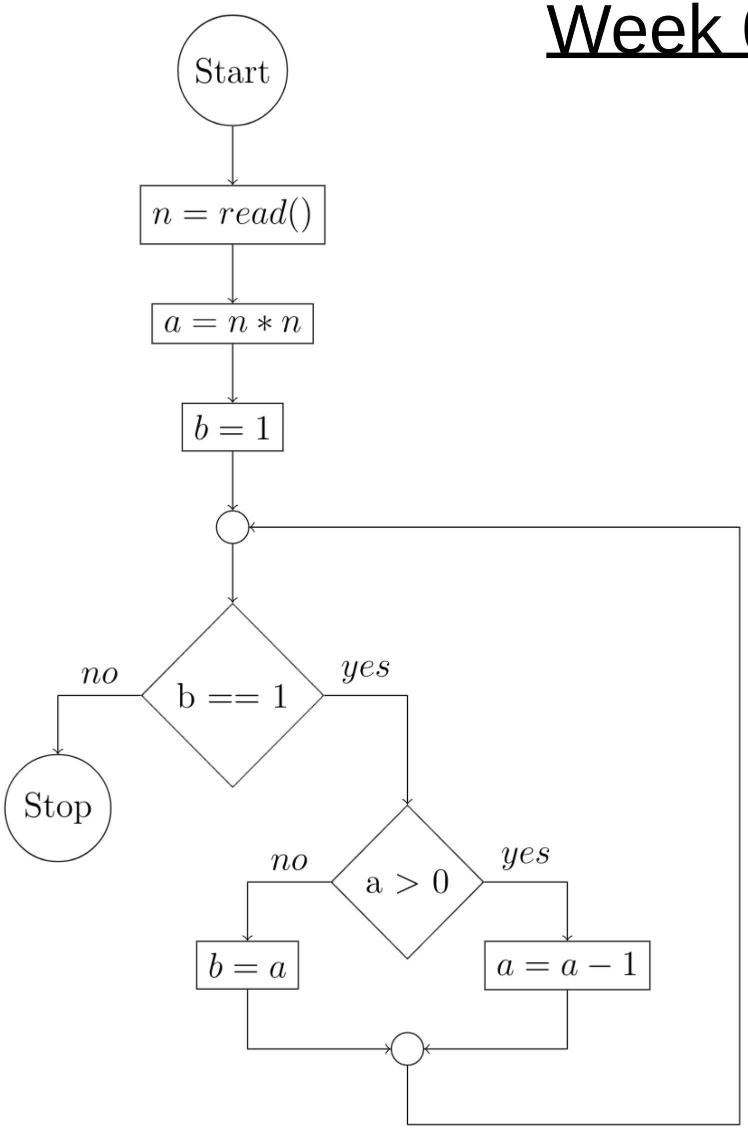
$B \equiv r > (-a-1)^2 \equiv a^2 + 2a + 1$

$F \equiv (a \geq 0 \Rightarrow r > a^2 - 2a + 1)$
$\wedge (a < 0 \Rightarrow r > a^2 + 2a + 1) \wedge r > 0$

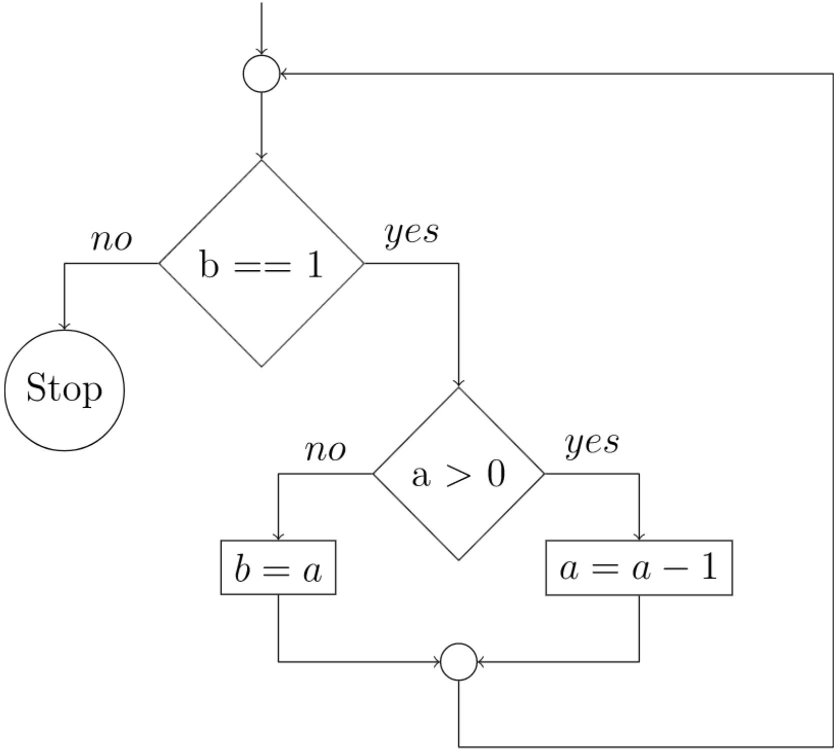
Prüfen auf IC

$I \wedge a \neq 0 \Rightarrow F$ ✓

Week 04 Task 4: Why is this not on Artemis 2.0



Week 04 Task 4: Why is this not on Artemis 2.0



$r = a + b$

$I \equiv r = a + b \wedge a \geq 0$