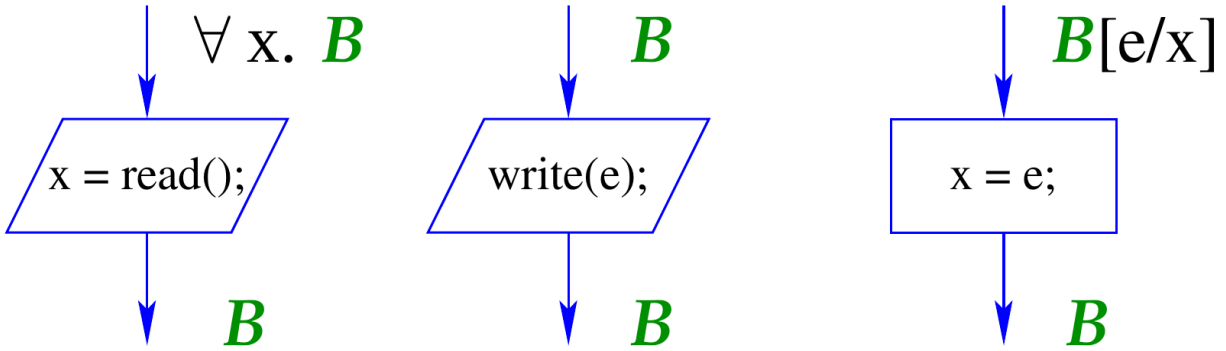
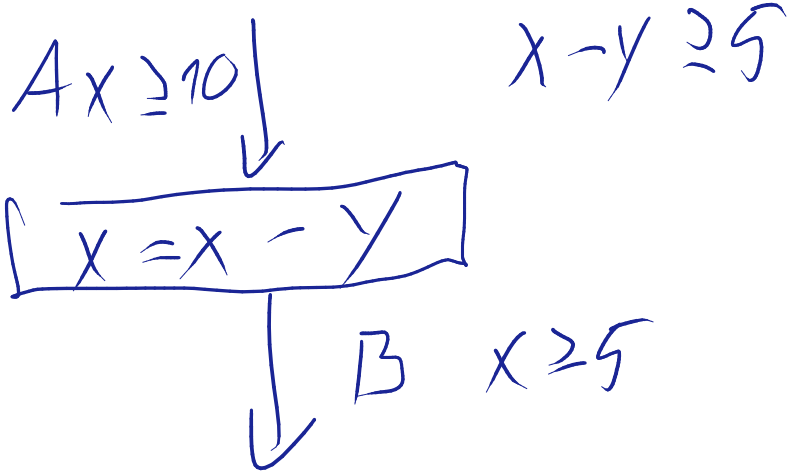
Week 3: Loop Invariants

Week 03 Tutorial 01 — MiniJava 2.0



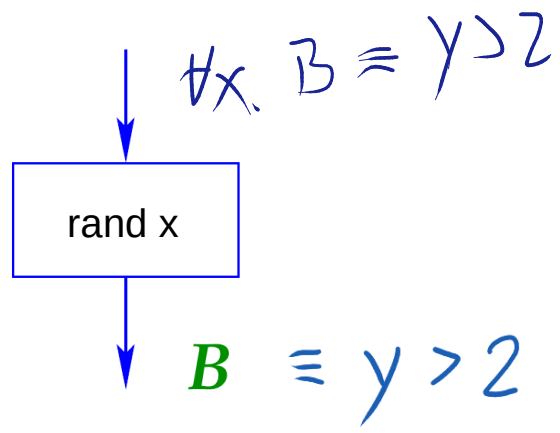


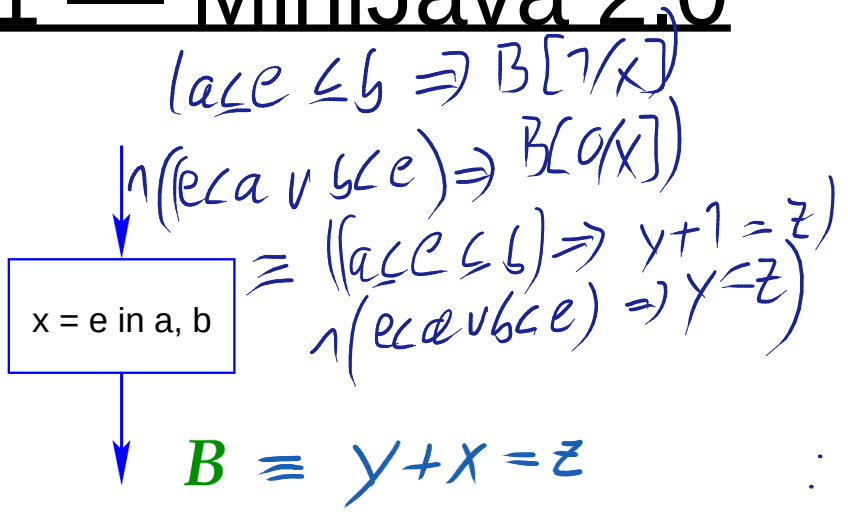
Week 03 Tutorial 01 — MiniJava 2.0

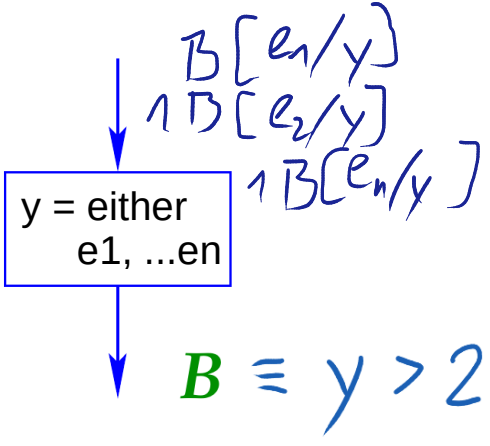
1. `rand x`:
   Assigns a random value to variable x,
2. `x = either` $e_0, \ldots, e_k$:
   Assigns one of the values of the expressions $e_0, \ldots, e_k$ to variable `x` non-deterministically,
3. `x = e in a, b`:
   Assigns the value 1 to variable `x`, if the value of expression `e` is in the range $[a, b]$ and 0 if `e` is not in the range or the range is empty $(a > b)$,
4. `stop`:
   Immediately stops the program.

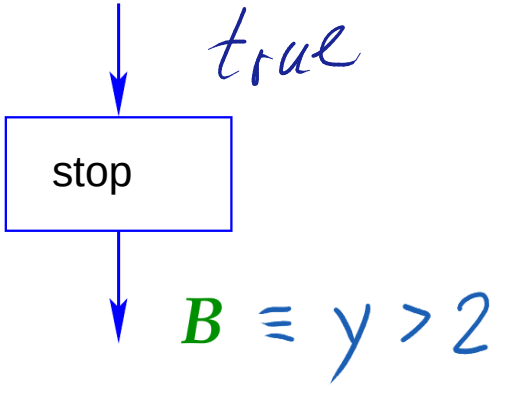
Define the weakest precondition operator $\mathbf{WP}[\![\ldots]\!](B)$ for each of these statements.

Week 03 Tutorial 01 — MiniJava 2.0

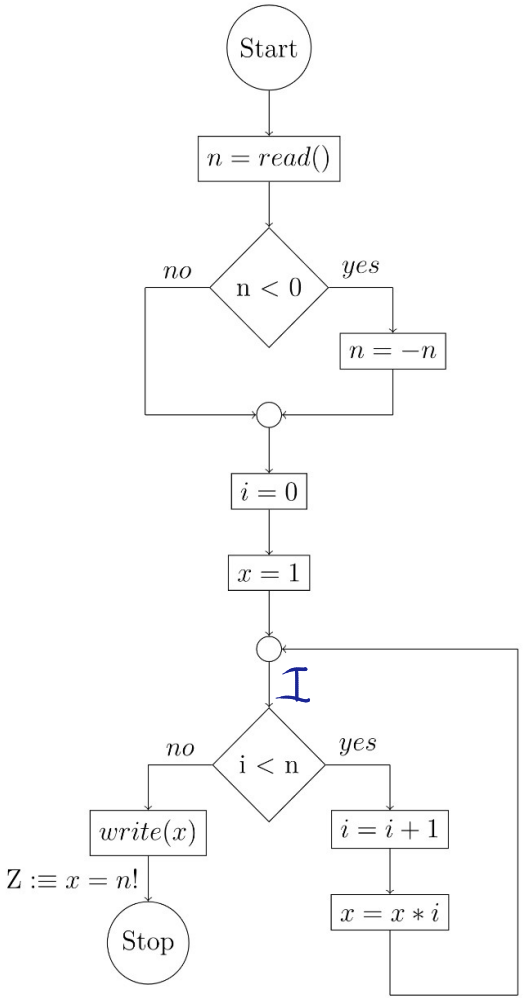






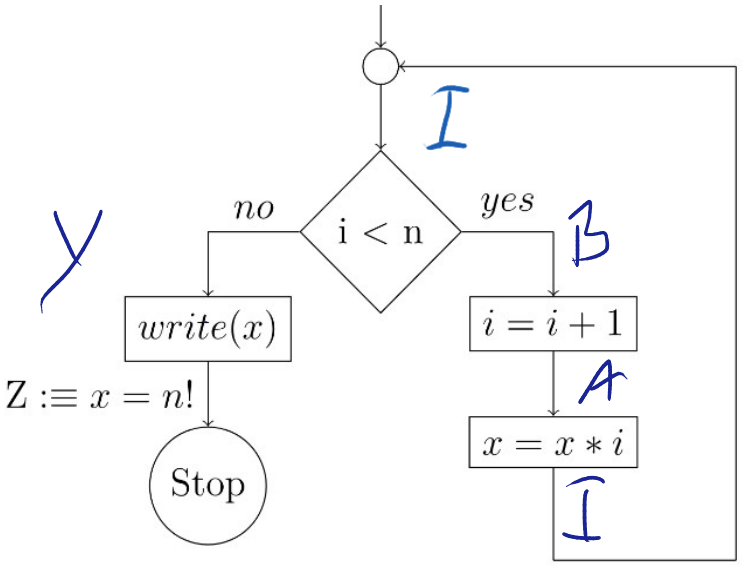


Week 03 Tutorial 02 — Loop Invariants



1. Discuss the problem that arises when computing weakest preconditions to prove Z.
2. How can you use weakest preconditions to prove Z anyway?

Week 03 Tutorial 02 — Loop Invariants



Y

$I :\equiv x \geq 0$

$A :\equiv i \cdot x \geq 0$

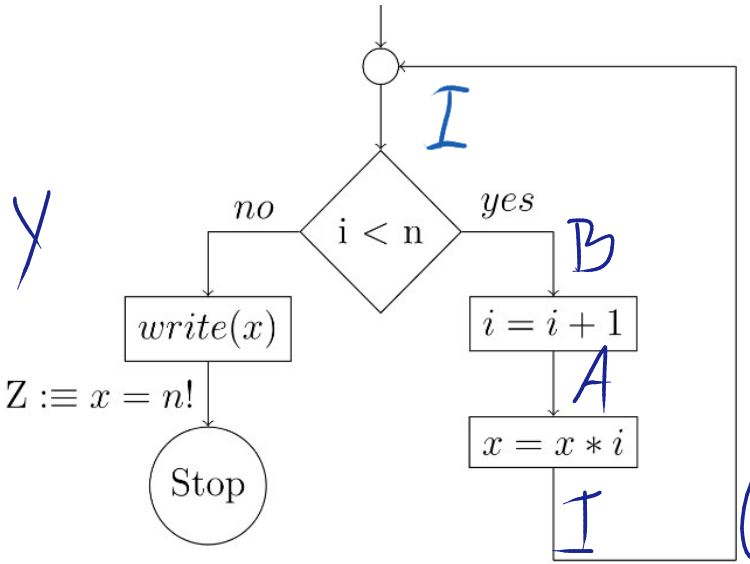
$B :\equiv x \cdot (i+1) \geq 0$

Preferred LC:

$I \wedge i \leq n \stackrel{?}{\Rightarrow} B$ X

$Y :\equiv x = n!$

Week 03 Tutorial 02 — Loop Invariants



$I :\equiv i=0 \wedge x=1 \wedge n=0$

$A :\equiv i=0 \wedge x \cdot i=1 \wedge n=0$

$B :\equiv i+1=0 \wedge x(i+1)=1 \wedge n=0$

Y

A	B	A⇒B
0	0	1
0	1	1
1	0	0
1	1	1

Prüfen auf LC

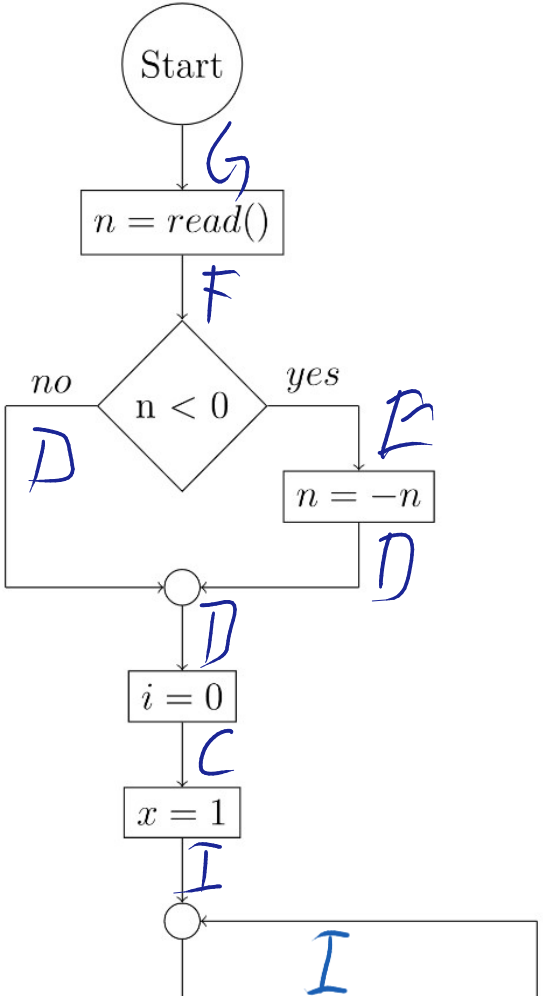
① $I \wedge i<n \Rightarrow B$ ✓

false $\Rightarrow B$

② $I \wedge i \geq n \Rightarrow y$ ✓

$y :\equiv x=n!$

Week 03 Tutorial 02 — Loop Invariants



$I :\equiv i = 0 \wedge x = 1 \wedge n = 0$

$C :\equiv i = 0 \wedge n = 0$

$D :\equiv n = 0$

$E :\equiv n = 0$

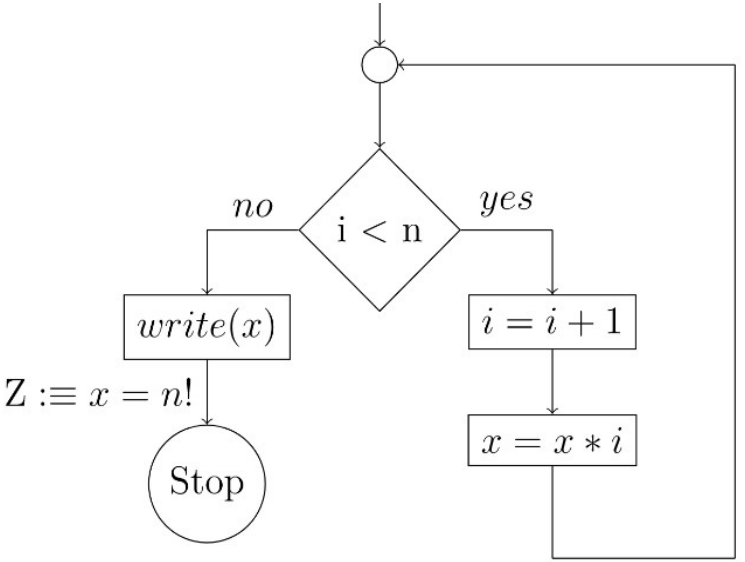
$F :\equiv (n < 0 \Rightarrow n = 0) \wedge \neg(n \geq 0 \Rightarrow n = 0) \equiv \text{false}$

$\equiv \text{false}$

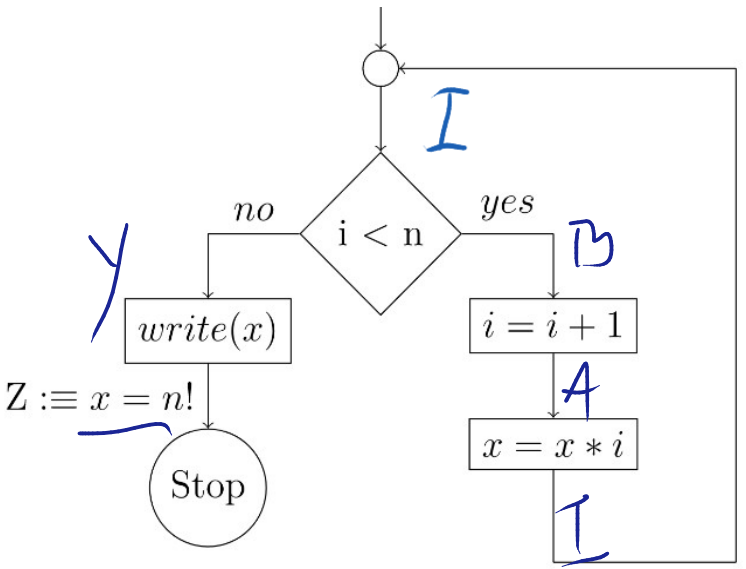
$G :\equiv \neg \text{false}$

Week 03 Tutorial 02 — Loop Invariants

- a) How has a useful loop invariant be related to Z?
- b) What happens if the loop invariant is chosen too strong?
- c) What happens if the loop invariant is chosen too weak?



Week 03 Tutorial 02 — Loop Invariants



$I :\equiv x = i!$ $\wedge\ i \leq n$

$A :\equiv x \cdot i = i!$ $\wedge\ i \leq n$

$B :\equiv x \cdot (i+1) = (i+1)!$ $\wedge\ i < n$

$\equiv x = i!$ $\wedge\ i < n$

(1) Prüfen auf LC

$I \wedge i < n \Rightarrow B$ ✓

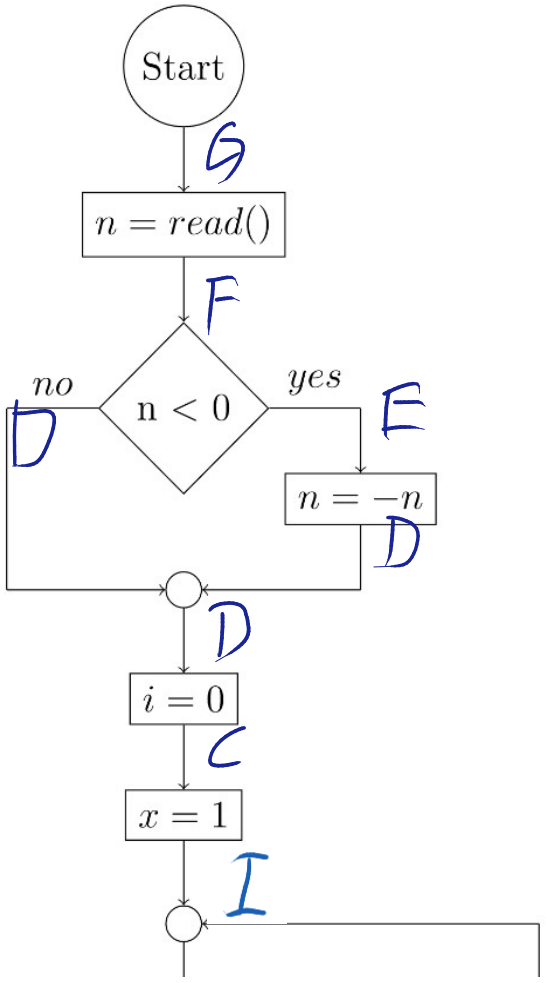
$I \wedge i \geq n \Rightarrow y$ ✗

(2) Prüfen auf LC

$I \wedge i < n \Rightarrow B$ ✓

$I \wedge i \geq n \Rightarrow y$

Week 03 Tutorial 02 — Loop Invariants



$I :\equiv x = i! \wedge i \leq n$

$C :\equiv i! = 1 \wedge i \leq n$

$D :\equiv 0! = 1 \wedge 0 \leq n$

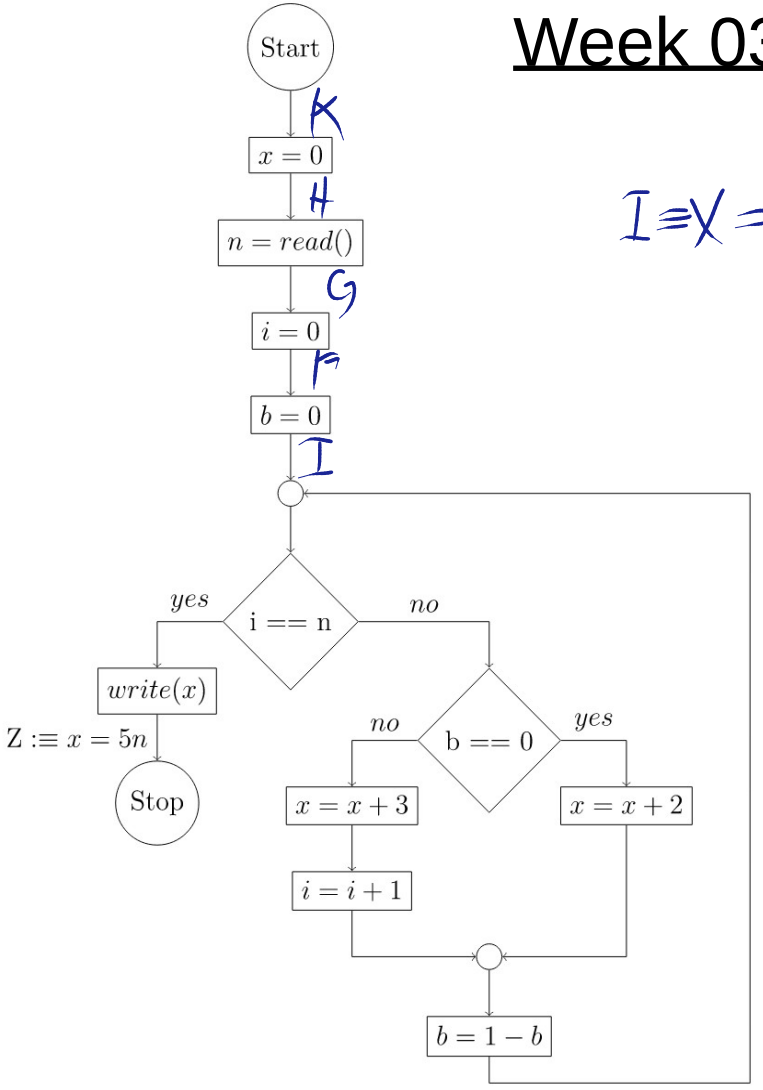
$\equiv 0 \leq n$

$E :\equiv 0 \geq n$

$F :\equiv (n < 0 \Rightarrow 0 \geq n) \wedge (n \geq 0 \Rightarrow n \geq 0) \equiv \text{true}$

$G :\equiv \text{true}$

Week 03 Tutorial 03 — Two b, or Not Two b



$I \equiv X = 5i + 2b \wedge (b = 1 \vee b = 0) \wedge (i = n \Rightarrow b = 0)$

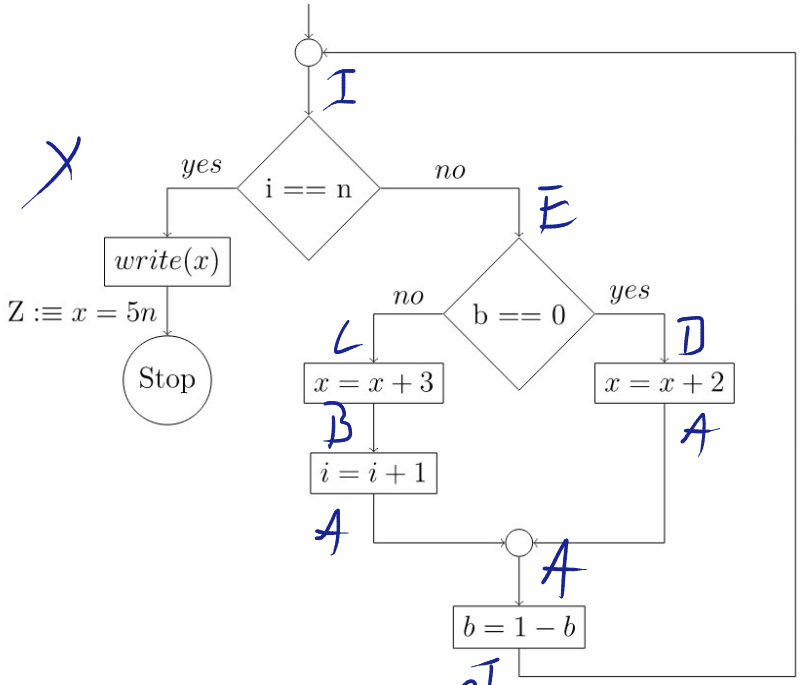
$F \equiv X = 5i$

$G \equiv X = 0$

$H \equiv \forall n. X = 0 \equiv X = 0$

$K \equiv true$

Week 03 Tutorial 03 — Two b, or Not Two b



$I \equiv X = 5i + 26$ $\wedge (b = 1 \vee b = 0) \wedge (i = n \Rightarrow b = 0)$

$A \equiv X = 5i + 2 - 26$ $\wedge (b = 1 \vee b = 0) \wedge (i = n \Rightarrow b = 1)$

$B \equiv X = 5i + 7 - 26$ $\wedge (b = 1 \vee b = 0) \wedge (i + 1 = n \Rightarrow b = 0)$

$C \equiv X = 5i + 4 - 26$ $\wedge (b = 1 \vee b = 0) \wedge (i + 1 = n \Rightarrow b = 1)$

$D \equiv X = 5i - 26$ $\wedge (b = 1 \vee b = 0) \wedge (i = n \Rightarrow b = 1)$

$E \equiv (b = 0 \Rightarrow X = 5i)$

$\wedge (b \neq 0 \Rightarrow X = 5i + 4 - 26)$

$\equiv (b = 0 \Rightarrow X = 5i)$

$\wedge (b = 1 \Rightarrow X = 5i + 2) \wedge (b = 1 \vee b = 0)$

$\equiv (b = 0 \Rightarrow X = 5i \wedge i \neq n) \wedge (b = 1 \vee b = 0)$

$\wedge (b = 1 \Rightarrow X = 5i + 2)$

(1) Prüfen auf LC

$I \wedge i \neq n \Rightarrow E$

$I \wedge i = n \Rightarrow Y$

(2) Prüfen auf LC

$I \wedge i \neq n \Rightarrow E$ ✓

$I \wedge i = n \Rightarrow Y$ ✗

(3) Prüfen auf LC

$I \wedge i \neq n \Rightarrow E$ ✓

$I \wedge i = n \Rightarrow Y$ ✓