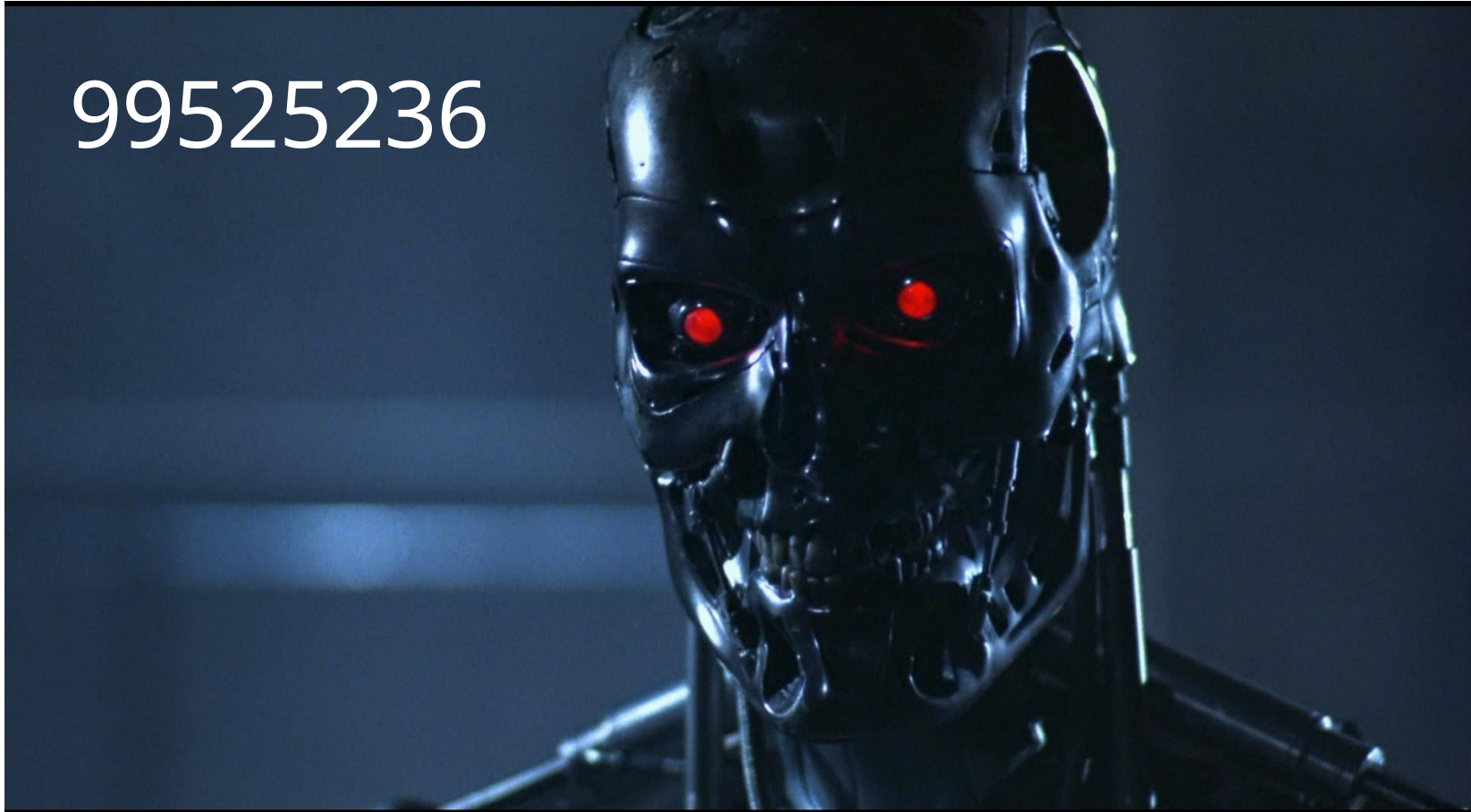


# Week 4: Termination

99525236



# Local Consistency revisited

$$x = 7$$

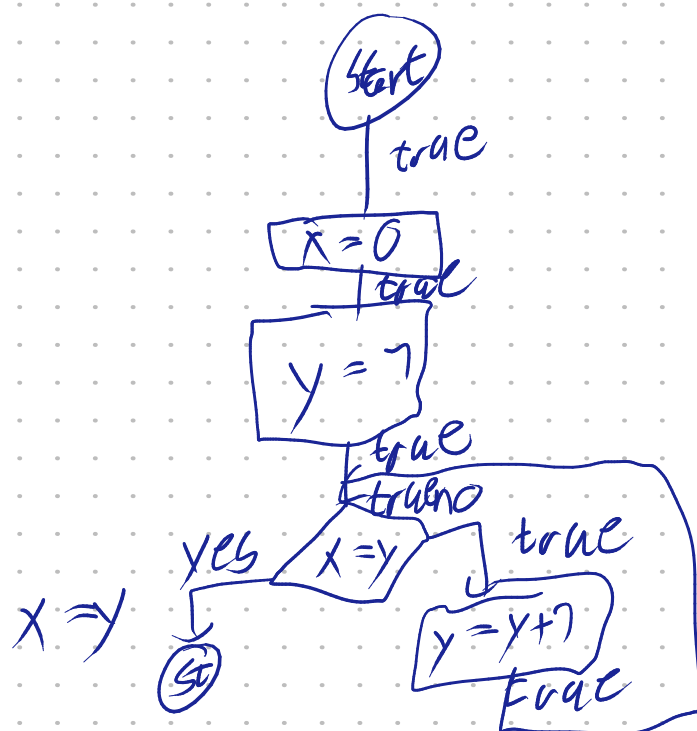
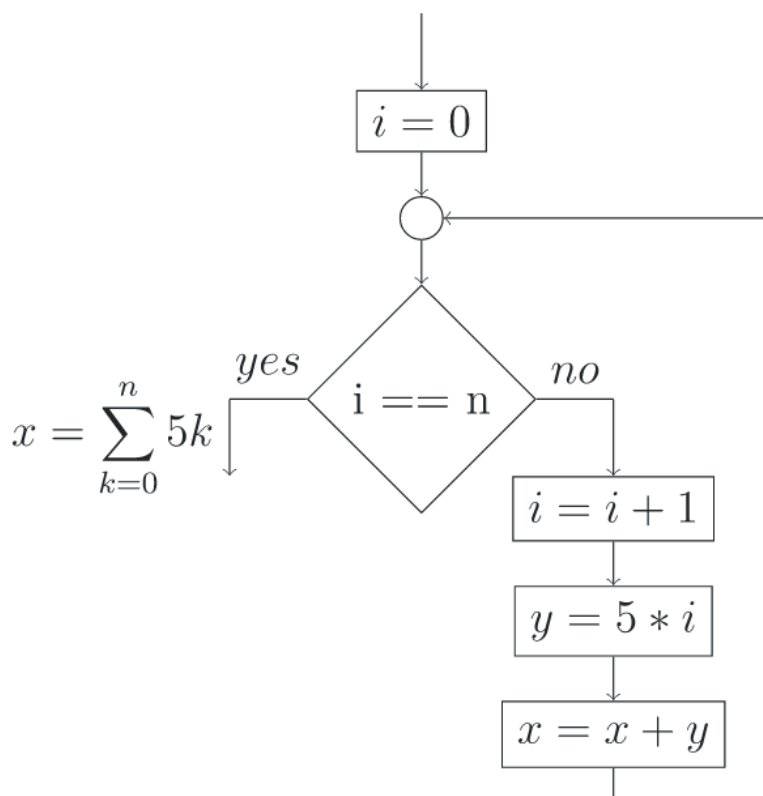


$$A \equiv \cancel{x=5} \wedge y=10 \wedge z=-3$$

$$x = x * x + y$$



$$B \equiv x > 20 \wedge \cancel{y=10} \wedge \cancel{z=-3}$$

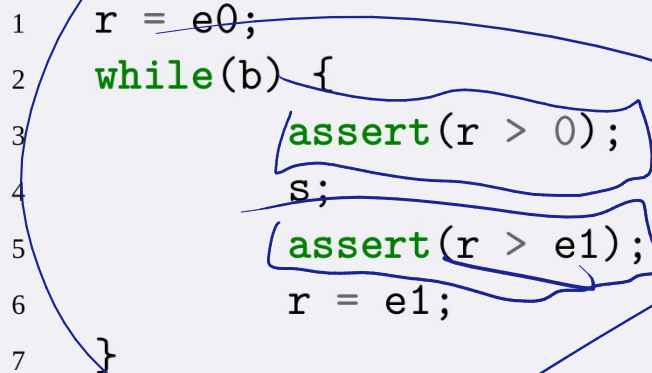


# Idea

- Make sure that each loop is executed only finitely often ...
- For each loop, identify an indicator value  $r$ , that has two properties
  - (1)  $r > 0$  whenever the loop is entered;
  - (2)  $r$  is decreased during every iteration of the loop.
- Transform the program in a way that, alongside ordinary program execution, the indicator value  $r$  is computed.
- Verify that properties (1) and (2) hold!

# General Method

- For every occurring loop `while (b) s` we introduce a fresh variable `r`.
- Then we transform the loop into:

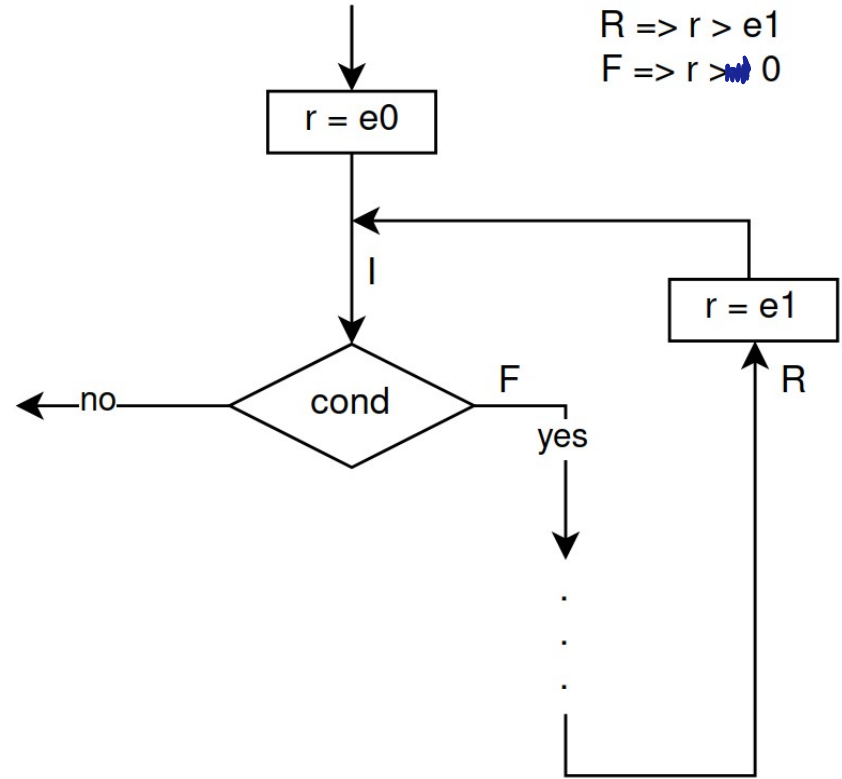
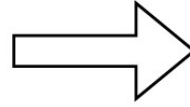
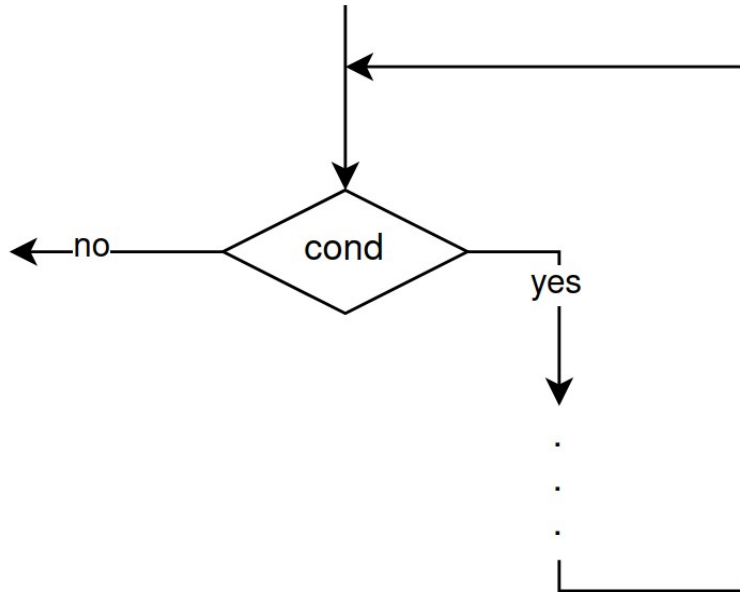


```
1  r = e0;  
2  while(b) {  
3      assert(r > 0);  
4      s;  
5      assert(r > e1);  
6      r = e1;  
7  }
```

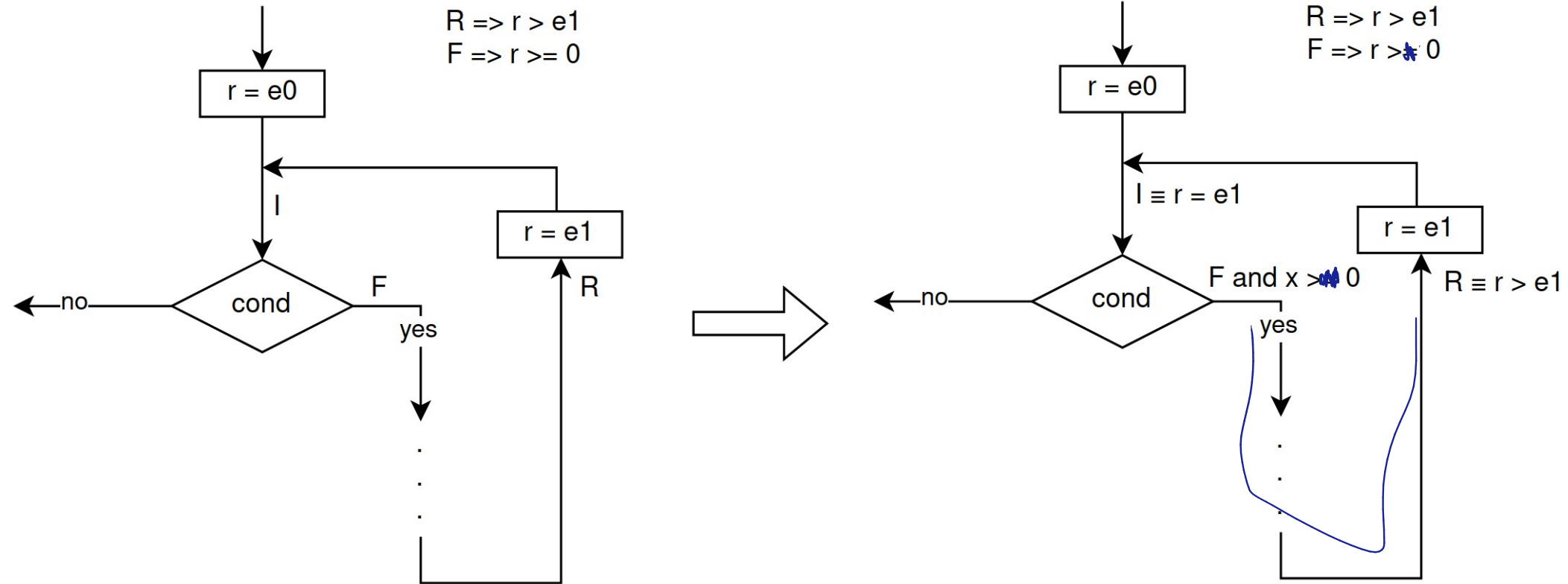
The diagram illustrates the transformation of a `while` loop. A blue oval encloses the entire transformed loop structure from line 1 to line 7. Inside the loop body (lines 3-6), three components are highlighted with blue rounded rectangles: the assertion `assert(r > 0);` on line 3, the statement `s;` on line 4, and the assertion `assert(r > e1);` on line 5. The variable `r` is introduced on line 1 and updated on line 6.

for suitable expressions `e0, e1`.

# How to prove termination



# How to prove termination

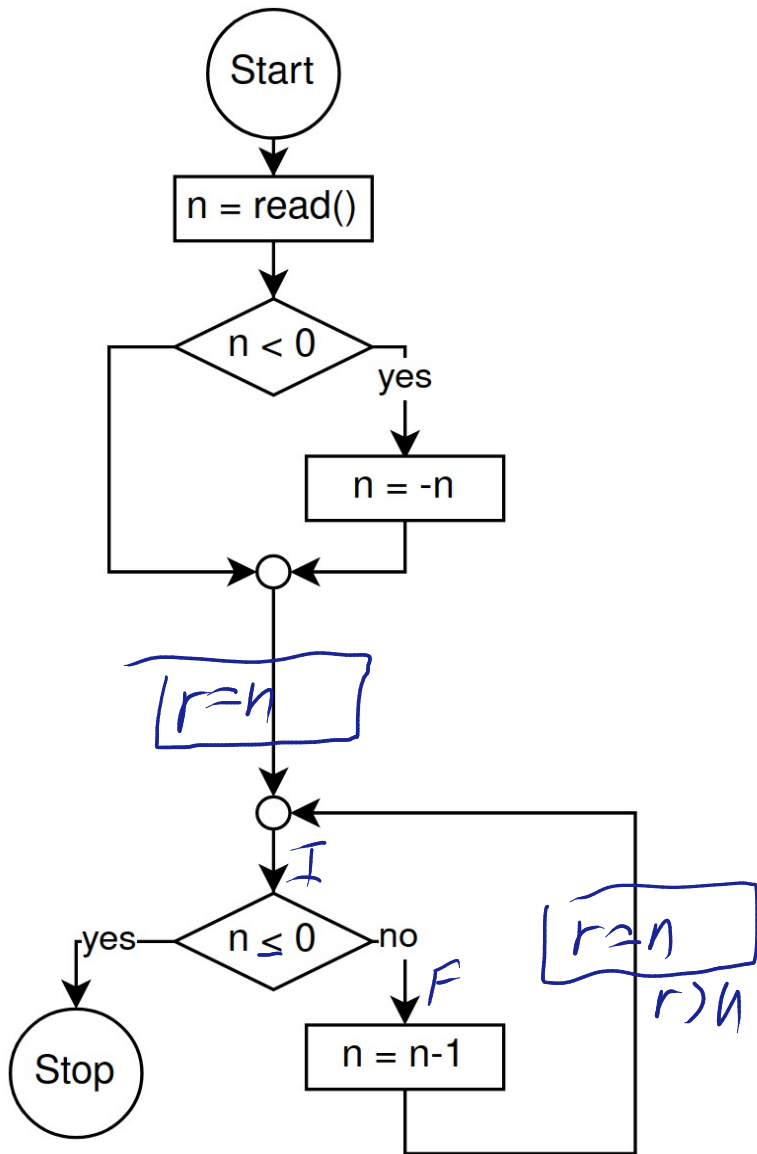


## How to prove termination

$$I \equiv r = n$$
$$F \equiv r > n - 1 \quad \wedge \quad r > 0$$

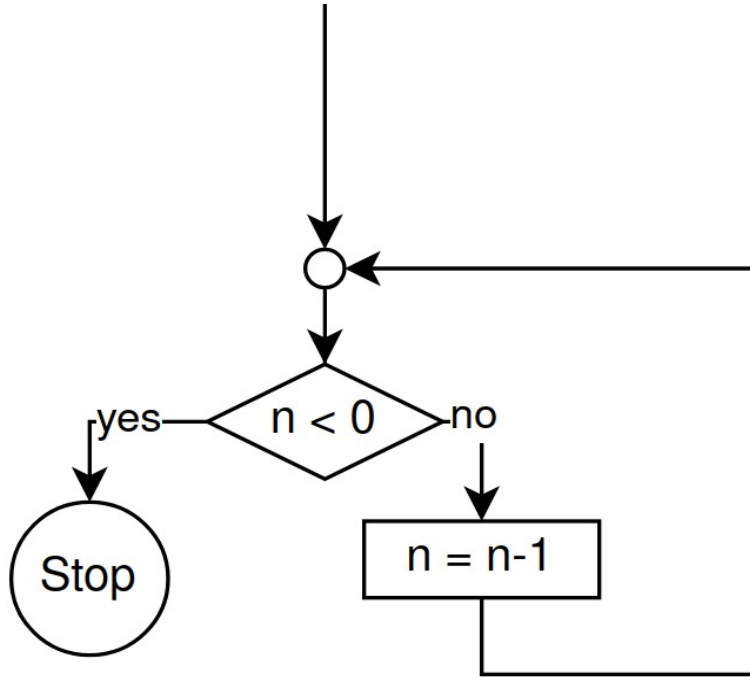
Prüfen auf  $I$

$$\textcircled{1} \quad I \wedge n > 0 \Rightarrow F$$

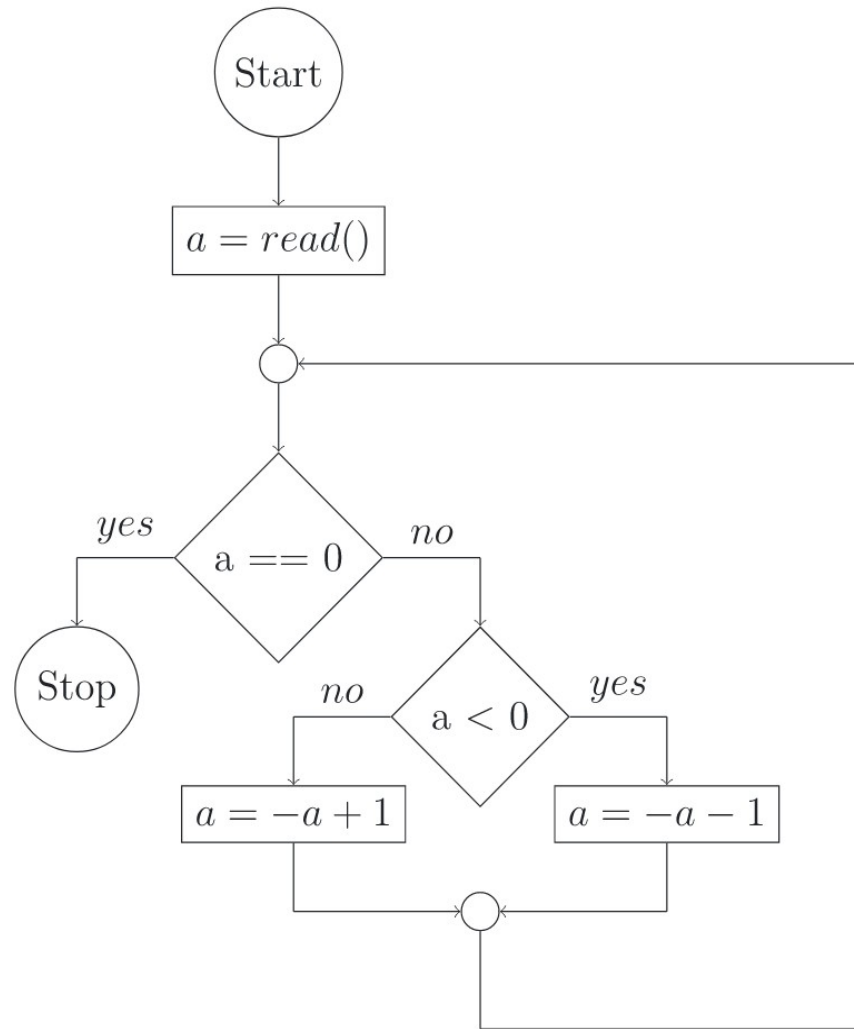




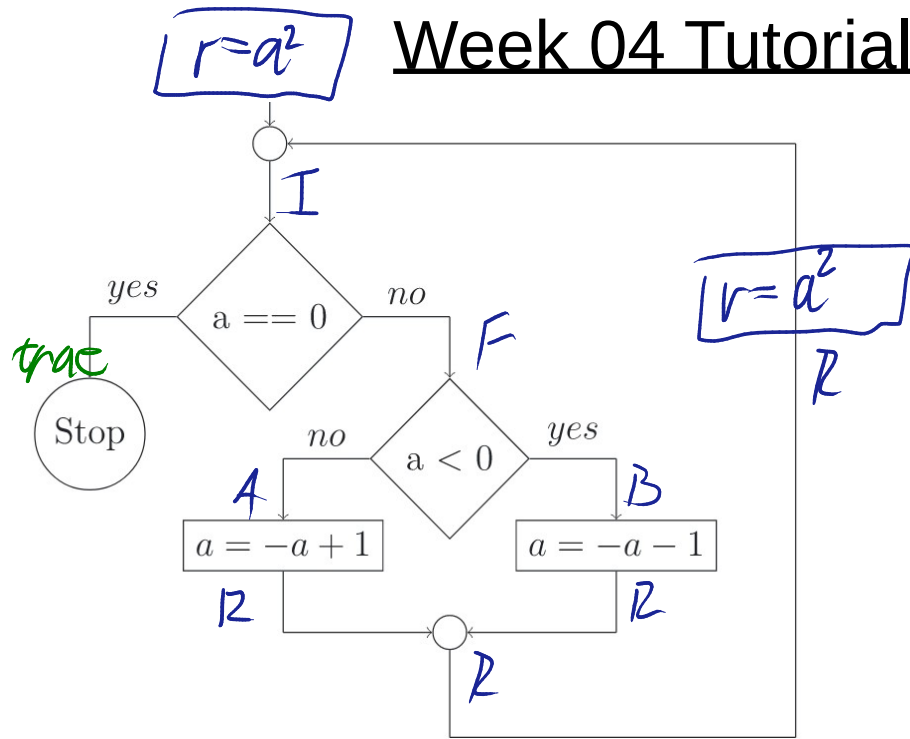
# How to prove termination



# Week 04 Tutorial 03 — A Wavy Approach



# Week 04 Tutorial 03 — A Wavy Approach



$$I \equiv r = a^2$$

$$R \equiv r > a^2$$

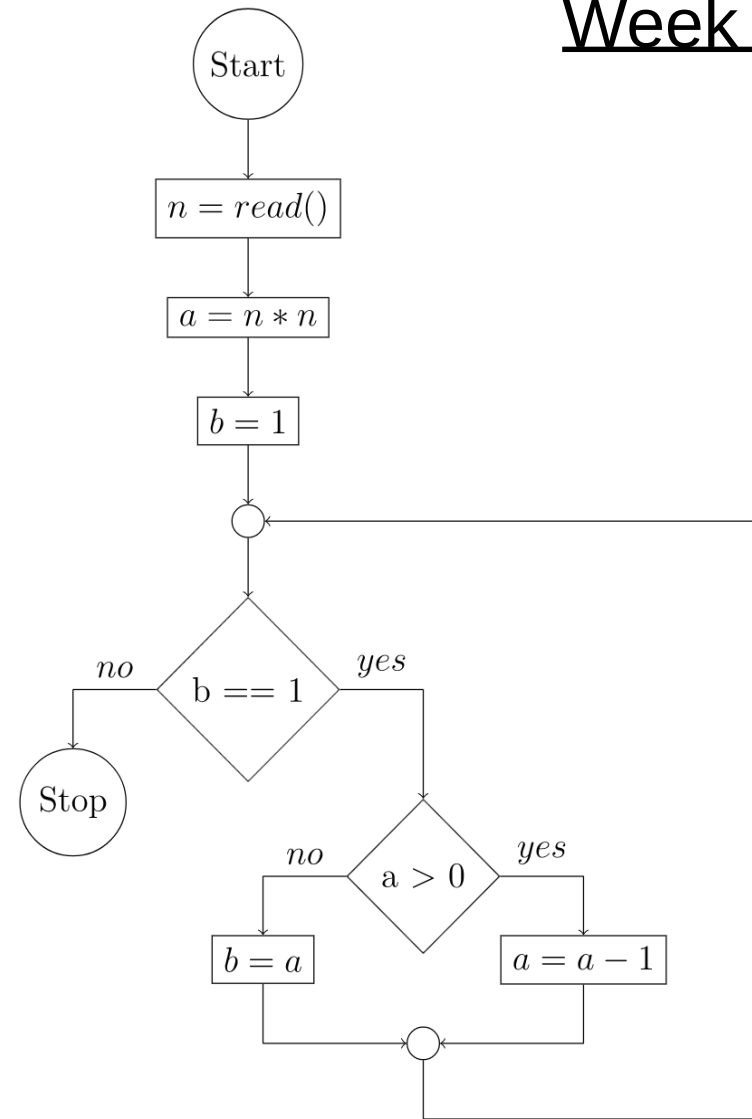
$$A \equiv r > (1-a)^2 \equiv a^2 - 2a + 1$$

$$B \equiv r > (-a-1)^2 \equiv a^2 + 2a + 1$$

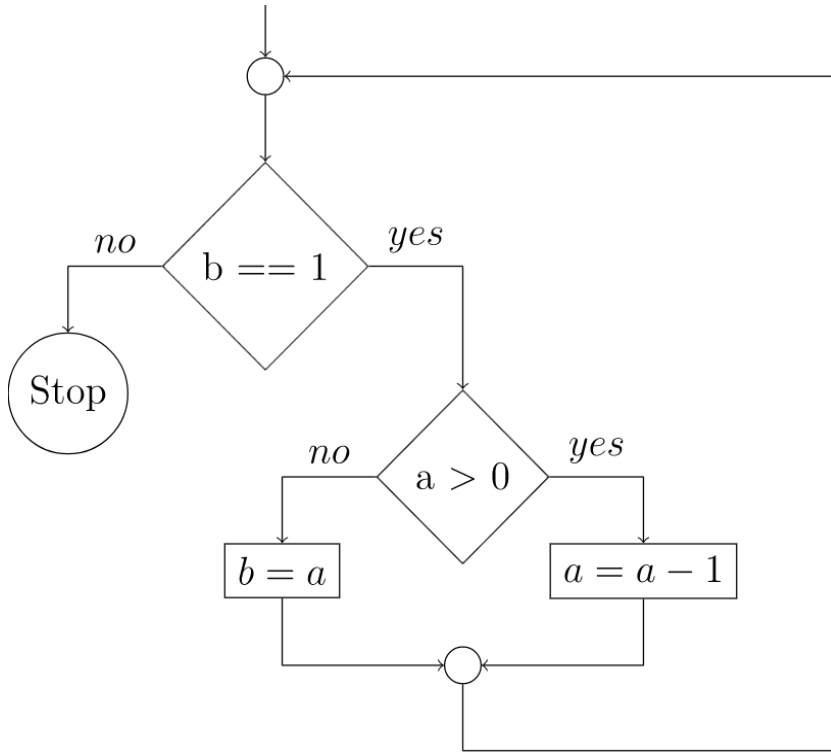
$$F \equiv \left( a \geq 0 \Rightarrow r > a^2 - 2a + 1 \right) \wedge \left( a < 0 \Rightarrow r > a^2 + 2a + 1 \right)$$

Prüfen auf LC  
 $I \wedge a \neq 0 \Rightarrow F$  ✓

## Week 04 Task 4: Why is this not on Artemis 2.0



## Week 04 Task 4: Why is this not on Artemis 2.0



$$r = a + b$$

$$I \equiv r = a + b \quad \wedge \quad a \geq 0$$