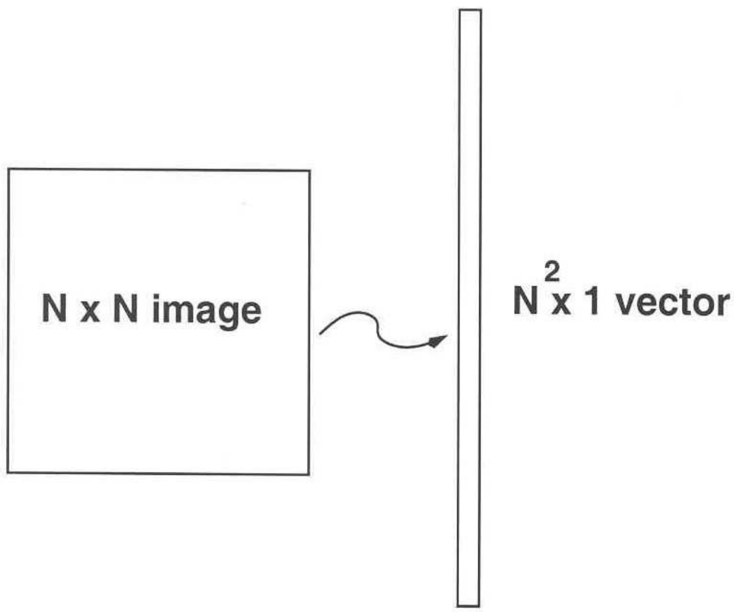
Eigenfaces for Face Detection/Recognition

(M. Turk and A. Pentland, "Eigenfaces for Recognition", *Journal of Cognitive Neuroscience*, vol. 3, no. 1, pp. 71-86, 1991, hard copy)

• Face Recognition

- The simplest approach is to think of it as a template matching problem:



- Problems arise when performing recognition in a high-dimensional space.

- Significant improvements can be achieved by first mapping the data into a *lower-dimensionality* space.

- How to find this lower-dimensional space?

• Main idea behind eigenfaces

- Suppose Γ is an N^2x1 vector, corresponding to an NxN face image I.

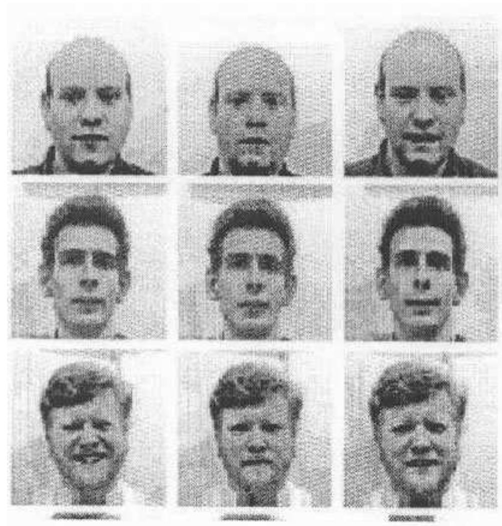
- The idea is to represent Γ (Φ=Γ - mean face) into a low-dimensional space:

$$\hat{\Phi} - mean = w_1 u_1 + w_2 u_2 + \cdots \, w_K u_K \ (K << N^2)$$

Computation of the eigenfaces

Step 1: obtain face images $I_1, I_2, \ldots, I_M$ (training faces)

(**very important:** the face images must be *centered* and of the same *size*)



Step 2: represent every image I_i as a vector Γ_i

Step 3: compute the average face vector Ψ:

$$\Psi = \frac{1}{M} \sum_{i=1}^{M} \Gamma_i$$

Step 4: subtract the mean face:

$$\Phi_i = \Gamma_i - \Psi$$

Step 5: compute the covariance matrix C:

$$C = \frac{1}{M} \sum_{n=1}^{M} \Phi_n \Phi_n^T = AA^T \quad (N^2 \mathrm{x} N^2 \text{ matrix})$$

where $A = [\Phi_1 \; \Phi_2 \cdots \Phi_M] \quad (N^2 \mathrm{x} M \text{ matrix})$

Step 6: compute the eigenvectors u_i of AA^T

The matrix AA^T is very large --> not practical !!

Step 6.1: consider the matrix $A^T A$ (MxM matrix)

Step 6.2: compute the eigenvectors v_i of $A^T A$

$$A^T A v_i = \mu_i v_i$$

What is the relationship between us_i and v_i?

$A^T A v_i = \mu_i v_i$ => $AA^T A v_i = \mu_i A v_i$ =>

$CAv_i = \mu_i A v_i$ or $Cu_i = \mu_i u_i$ where $u_i = Av_i$

Thus, AA^T and $A^T A$ have the same eigenvalues and their eigenvectors are related as follows: $u_i = Av_i$!!

Note 1: AA^T can have up to N^2 eigenvalues and eigenvectors.

Note 2: $A^T A$ can have up to M eigenvalues and eigenvectors.

Note 3: The M eigenvalues of $A^T A$ (along with their corresponding eigenvectors) correspond to the M *largest* eigenvalues of AA^T (along with their corresponding eigenvectors).

Step 6.3: compute the M best eigenvectors of AA^T: $u_i = Av_i$

(**important:** normalize u_i such that $\|u_i\| = 1$)

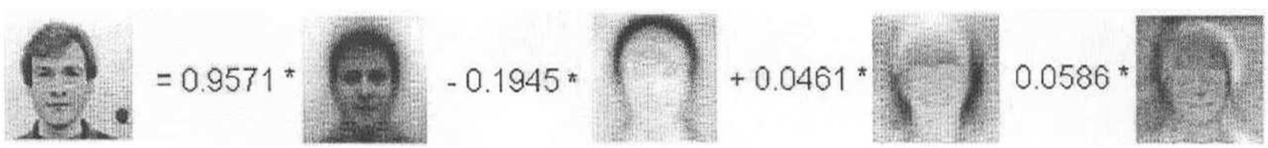
Step 7: keep only K eigenvectors (corresponding to the K largest eigenvalues)

Representing faces onto this basis

- Each face (minus the mean) Φ_i in the training set can be represented as a linear combination of the best K eigenvectors:

$$\hat{\Phi}_i - mean = \sum_{j=1}^{K} w_j u_j, \quad (w_j = u_j^T \Phi_i)$$

(we call the u_j's *eigenfaces*)



- Each normalized training face Φ_i is represented in this basis by a vector:

$$\Omega_i = \begin{bmatrix} w_1^i \\ w_2^i \\ \cdots \\ w_K^i \end{bmatrix}, \quad i = 1, 2, \ldots, M$$

Face Recognition Using Eigenfaces

- Given an unknown face image Γ (centered and of the same size like the training faces) follow these steps:

<u>Step 1:</u> normalize Γ: $\Phi = \Gamma - \Psi$

<u>Step 2:</u> project on the eigenspace

$$\hat{\Phi} = \sum_{i=1}^{K} w_i u_i \quad (w_i = u_i^T \Phi)$$

<u>Step 3:</u> represent Φ as: $\Omega = \begin{bmatrix} w_1 \\ w_2 \\ \cdots \\ w_K \end{bmatrix}$

<u>Step 4:</u> find $e_r = \min_l \|\Omega - \Omega^l\|$

<u>Step 5:</u> if $e_r < T_r$, then Γ is recognized as face l from the training set.

- The distance e_r is called <u>distance within the face space (difs)</u>

<u>Comment:</u> we can use the common Euclidean distance to compute e_r, however, it has been reported that the *Mahalanobis distance* performs better:

$$\|\Omega - \Omega^k\| = \sum_{i=1}^{K} \frac{1}{\lambda_i} (w_i - w_i^k)^2$$

(variations along all axes are treated as equally significant)

Face Detection Using Eigenfaces

- Given an unknown image Γ

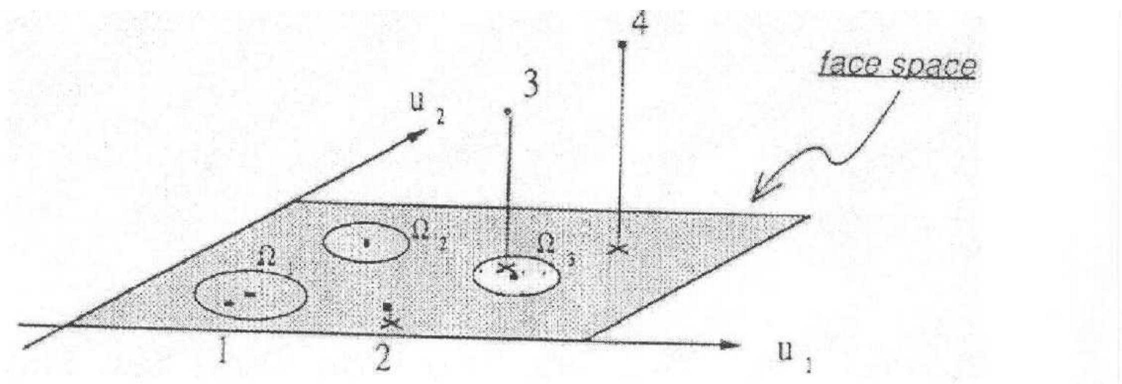
  Step 1: compute $\Phi = \Gamma - \Psi$

  Step 2: compute $\hat{\Phi} = \sum_{i=1}^{K} w_i u_i \quad (w_i = u_i^T \Phi)$

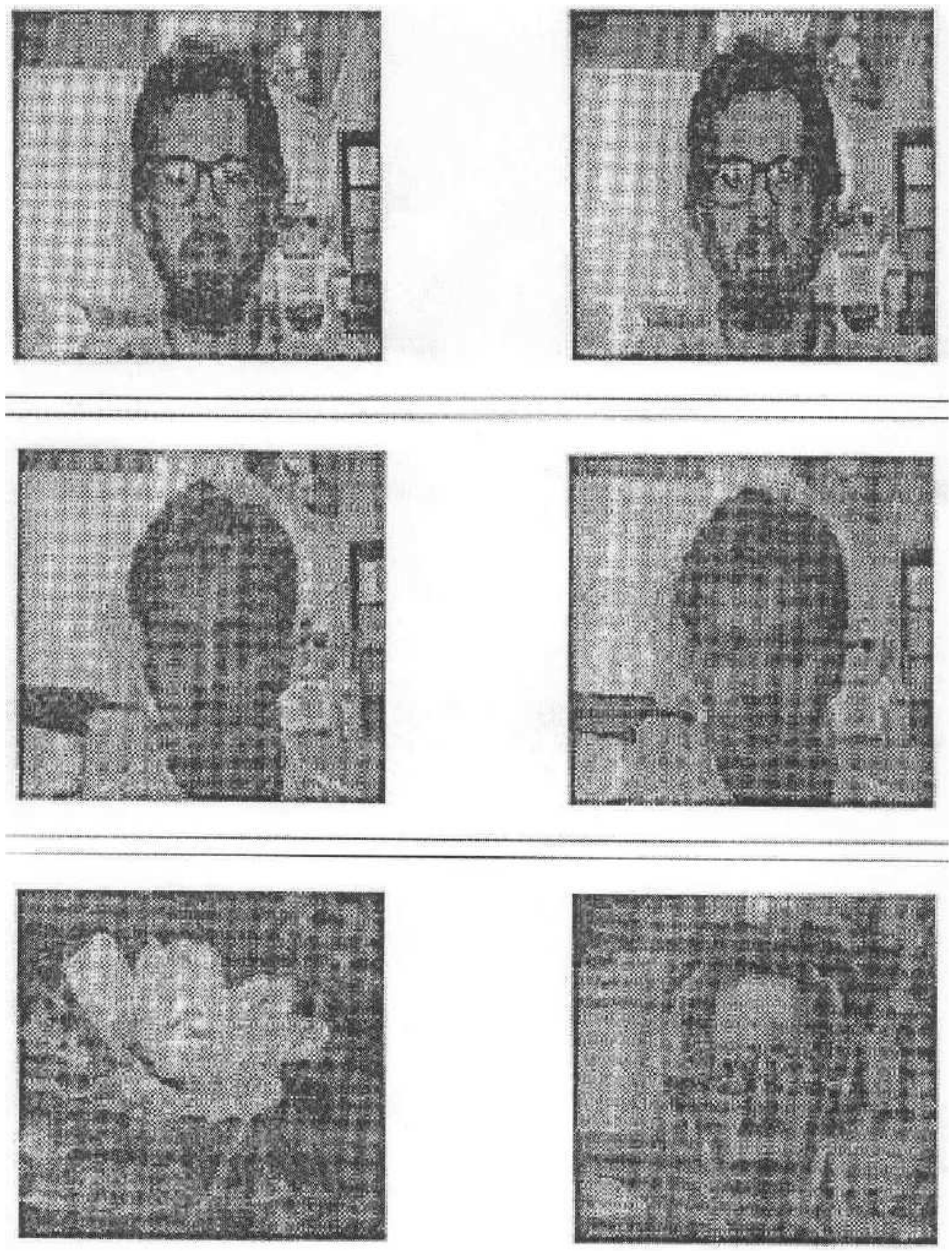
  Step 3: compute $e_d = \|\Phi - \hat{\Phi}\|$

  Step 4: if $e_d < T_d$, then Γ is a face.

- The distance e_d is called distance from face space (dffs)



- Reconstruction of faces and non-faces

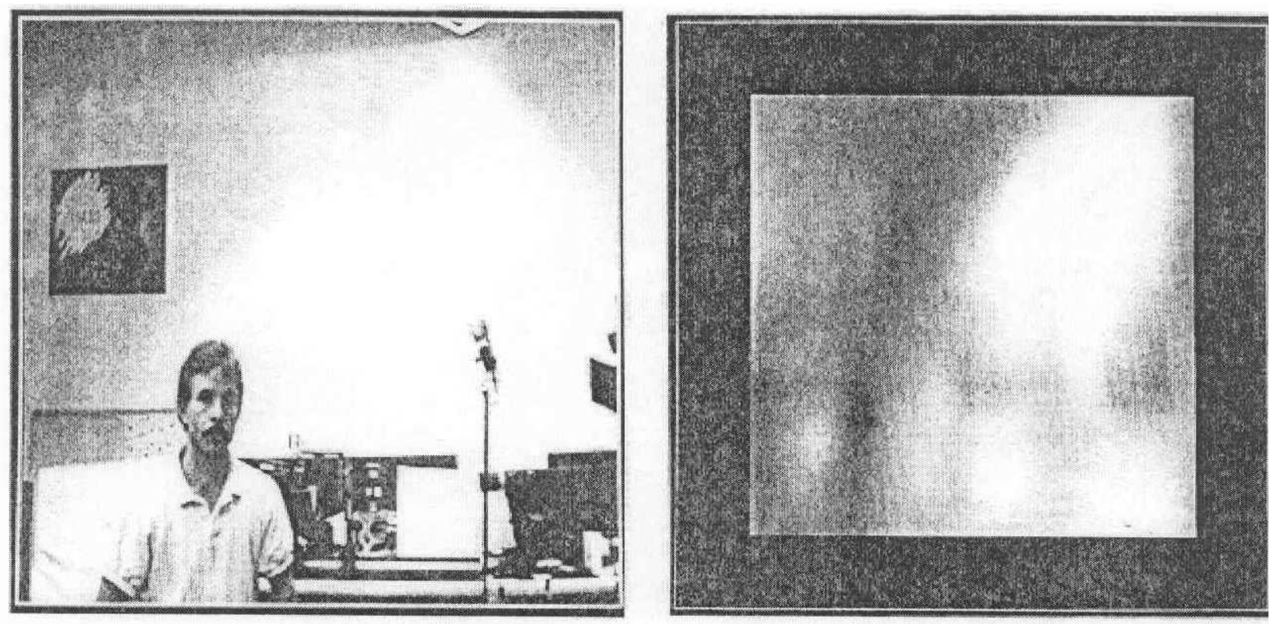


• Time requirements

- About 400 msec (Lisp, Sun4, 128x128 images)

• Applications

- Face detection, tracking, and recognition



• Problems

- Background (deemphasize the outside of the face, e.g., by multiplying the input image by a 2D Gaussian window centered on the face)

- Lighting conditions (performance degrades with light changes)

- Scale (performance decreases quickly with changes to the head size)

  * multiscale eigenspaces
  * scale input image to multiple sizes)

- Orientation (perfomance decreases but not as fast as with scale changes)

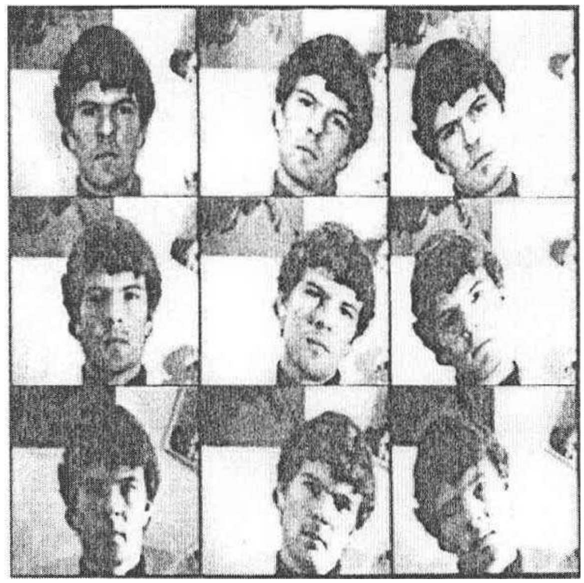
  * plane rotations can be handled
  * out-of-plane rotations more difficult to handle

- **Experiments**

- 16 subjects, 3 orientations, 3 sizes

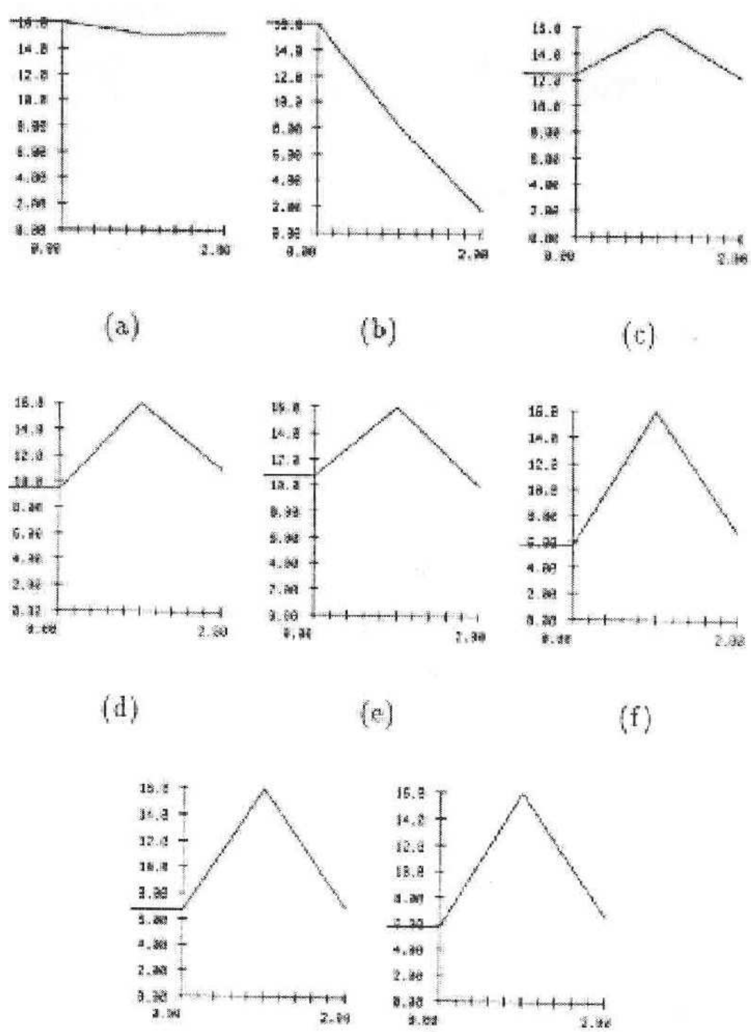
- 3 lighting conditions, 6 resolutions (512x512 ... 16x16)

- Total number of images: 2,592



Experiment 1

* Used various sets of 16 images for training
* One image/person, taken under the same conditions
* Eigenfaces were computed offline (7 eigenfaces were used)
* Classify the rest images as one of the 16 individuals
* No rejections (i.e., no threshold for *difs*)



- Performed a large number of experiments and averaged the results:

96% correct averaged over light variation
85% correct averaged over orientation variation
64% correct averaged over size variation

Experiment 2

- They considered rejections (i.e., by thresholding *difs*)

- There is a tradeoff between correct recognition and rejections.

- Adjusting the threshold to achieve 100% recognition acurracy resulted in:

  * 19% rejections while varying lighting
  * 39% rejections while varying orientation
  * 60% rejections while varying size

Experiment 3

- Reconstruction using partial information