Robust Optimization for Harpoon-Based Space Debris Removal under Uncertainty

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***Abstract*—This project addresses the challenge of optimizing harpoon-based systems for active space debris removal under parameter uncertainty. Debris properties, including mass, spin rate, and conductivity, were modeled using probabilistic distributions to capture real-world variability. A Monte Carlo simulation framework was implemented to evaluate system performance, and Differential Evolution was applied to identify control inputs that minimize expected mission loss. The optimized strategy demonstrated a substantial reduction in expected loss compared to a fixed-control baseline. Sensitivity analysis revealed that spin rate had the greatest influence on mission performance, followed by conductivity and mass. These findings provide a robust foundation for enhancing the reliability and effectiveness of active debris removal missions.**

I. Introduction

The rapid growth of orbital debris is becoming a serious threat to satellites and space missions, raising the risk of collisions and endangering future space activities. As the number of debris fragments continues to increase, the need for active debris removal (ADR) solutions is more urgent than ever [1]. Among the different ADR concepts, harpoon-based systems stand out because they can capture and deorbit debris with diverse shapes and sizes [2].

However, real debris objects vary significantly in mass, spin rate, and conductivity, making mission planning and control challenging. These uncertainties can cause unpredictable system behavior and reduce the reliability of debris removal missions.

While previous studies have emphasized the need for ADR [1], [2] and explored harpoon systems [2], the role of robust optimization in addressing these uncertainties has not been fully explored. This project focuses on developing a robust optimization framework for harpoon-based debris removal that accounts for the variability in debris properties. By modeling debris mass, spin rate, and conductivity as probabilistic distributions and using Monte Carlo simulations, this work quantifies system performance under uncertainty. A Differential Evolution algorithm is then applied to find the control input that minimizes the expected mission loss, defined by capture risk, mass penalty, and misalignment. Sensitivity analysis further identifies which debris parameters have the biggest impact on mission success.

This work provides insights into how robust optimization can make ADR systems more reliable and effective, ultimately contributing to safer and more sustainable use of Earth's orbits.

II. Related Work

The management of space debris has been a major research focus since the early 2000s, driven by the increasing risk of collisions with operational satellites. Liou and Johnson (2006) highlighted the exponential growth of debris objects in low Earth orbit (LEO) and stressed the urgency of implementing active debris removal (ADR) strategies to prevent a runaway collision cascade [1]. Among various ADR concepts, harpoon-based systems have attracted significant attention due to their mechanical simplicity and potential for capturing debris objects of varying shapes and sizes. Forshaw et al. (2016) provided a comprehensive analysis of the RemoveDebris mission, demonstrating the feasibility of using a harpoon system for capturing debris and conducting in-orbit tests [2]. Sizov and Aslanov [3] further analyzed the choice of harpoon parameters and their optimization to reduce oscillations during towing, highlighting the importance of parameter selection in harpoon-based ADR systems.

Accurately modeling the behavior of debris objects is also critical, especially given the uncertainty in debris properties such as mass, spin rate, and material conductivity. Bertsimas and Sim (2004) introduced a framework for robust optimization that balances solution conservatism with performance guarantees under parameter variability [4]. Suescun et al. (2021) applied robust optimization to collision avoidance in space missions, demonstrating how probabilistic methods can improve system reliability [5]. Klesh and Krajewski (2014) explored the use of CubeSats for deep space missions, emphasizing the need for systems that can operate reliably under variable and uncertain conditions—a consideration relevant to ADR missions as well [6].

Despite these advances, robust optimization has not been widely applied to harpoon-based ADR systems, where debris parameter uncertainties directly affect capture dynamics. This project builds on previous research by integrating probabilistic modeling of debris properties, Monte Carlo simulation to evaluate system performance, and Differential Evolution to identify control inputs that minimize expected mission loss. By combining these techniques, this work aims to advance the development of reliable and effective ADR strategies. Federici et al. [7] explored a time-dependent traveling salesman problem formulation for ADR mission design, applying simulated annealing to optimize debris removal sequences.

III. Methods

A. Problem Formulation

My goal with this project is to identify a robust control input u for a harpoon-based active debris removal (ADR) system that minimizes the expected mission loss under uncertain debris parameters. The debris properties considered include mass (m), spin rate (ω), and conductivity (σ), each modeled as independent random variables.

The mission loss function $L(u, \theta)$ is defined as:

$$L(u,\theta) = w_1 R_{\text{capture}}(u,\theta) + w_2 P_{\text{mass}}(m) + w_3 M_{\text{alignment}}(u,\omega) \tag{1}$$

where θ denotes the vector of uncertain debris parameters, and $w_1 = 0.5$, $w_2 = 0.3$, and $w_3 = 0.2$ are weighting factors that balance the contribution of each term. R_{capture} represents the risk of capture failure based on control input and debris properties, P_{mass} penalizes large debris masses, and $M_{\text{alignment}}$ captures the misalignment between the harpoon and the spinning debris. This formulation aligns with robust optimization frameworks used in aerospace systems [4].

B. Simulation Framework

To evaluate system performance under uncertainty, a Monte Carlo simulation framework was implemented. For each simulation run i, debris parameters $\theta^{(i)} = [m^{(i)}, \omega^{(i)}, \sigma^{(i)}]$ were sampled from their respective distributions:

- $m \sim U(10, 150)$ kg
- $\omega \sim N(0.5, 0.22)$ rad/s
- $\sigma \sim U(0.1, 1.5)$ S/m

The loss function $L(u, \theta^{(i)})$ was computed for each sample, and the expected mission loss $\hat{L}(u)$ was estimated as:

$$\hat{L}(u) = \frac{1}{N}\sum_{i=1}^{N} L(u, \theta^{(i)}) \tag{2}$$

where $N = 100$ samples were used in this study. Monte Carlo methods are widely adopted for uncertainty quantification in aerospace engineering [5].

C. Optimization Approach

A Differential Evolution (DE) algorithm was implemented using the SciPy library [4] to identify the control input u^* that minimizes the expected mission loss $\hat{L}(u)$. The DE algorithm is a population-based stochastic search technique that is effective for complex, non-convex optimization problems. For each individual $\mathbf{x}_i$ in the population, mutation was applied using:

$$\mathbf{v}_i = \mathbf{x}_{r1} + F \cdot (\mathbf{x}_{r2} - \mathbf{x}_{r3}) \tag{3}$$

where $F = 0.8$ is the mutation factor, and $r1$, $r2$, and $r3$ are distinct indices selected randomly from the population. Crossover was then applied, followed by selection:

$$\mathbf{x}_i^{(g+1)} = \begin{cases} \mathbf{v}_i & \text{if } \hat{L}(\mathbf{v}_i) < \hat{L}(\mathbf{x}_i^{(g)}) \\ \mathbf{x}_i^{(g)} & \text{otherwise} \end{cases} \tag{4}$$

The algorithm iterated until convergence was achieved, defined by:

$$\left| \min \hat{L}^{(g+1)} - \min \hat{L}^{(g)} \right| < \epsilon \tag{5}$$

where $\epsilon = 10^{-4}$. DE is widely used in engineering design due to its effectiveness in handling complex, multi-modal objective functions. This choice is supported by Zuo et al. [8], who successfully applied DE to optimize interplanetary transfer trajectories, demonstrating its effectiveness in complex, nonlinear problems.

D. Sensitivity Analysis

After the optimization process, a sensitivity analysis was conducted to assess the relative impact of each debris parameter on mission performance. Sensitivity analysis helped me identify which parameters most strongly influence mission loss, thereby informing design priorities and potential areas for system improvement. This approach aligns with the findings of Somma et al. [9], who emphasized the importance of sensitivity analysis in understanding debris environment models.

In this study, partial correlation coefficients (PCCs) were calculated between each debris parameter (m, ω, σ) and the mission loss function $L(u, \theta)$ across the Monte Carlo samples. The PCC measures the linear correlation between each parameter and the loss while controlling for the influence of the other parameters [5]. Mathematically, the PCC between parameter X and the loss function Y is defined as:

$$\rho_{XY\cdot Z} = \frac{\rho_{XY} - \rho_{XZ}\rho_{YZ}}{\sqrt{(1-\rho_{XZ}^2)(1-\rho_{YZ}^2)}} \tag{6}$$

where ρ_{XY} is the Pearson correlation coefficient between X and Y, and Z represents the set of other parameters held constant.

Parameters with higher absolute PCC values were interpreted as having a stronger influence on the mission loss. This analysis enabled the identification of key drivers of performance variability and provided actionable insights for future system design and operational planning.

IV. Results

A. Parameter Distributions

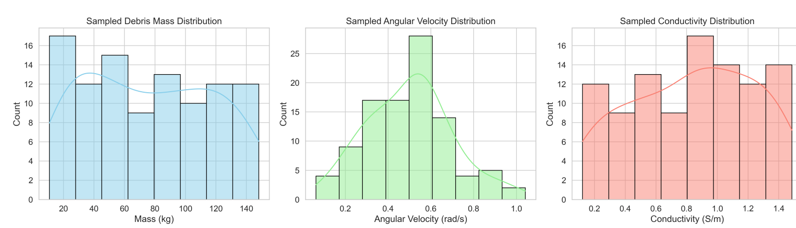

Fig. 1. Sampled parameter distributions for debris mass, spin rate, and conductivity.

Figure 1 presents histograms and kernel density estimates for the three key debris parameters considered in this study: mass (left), angular velocity (middle), and conductivity (right).

The mass distribution appears approximately uniform between 10 kg and 150 kg, aligning with the assumed range for typical debris objects. This uniform distribution reflects the significant uncertainty in debris mass that can influence harpoon capture dynamics.

The angular velocity distribution is notably skewed to the right, with a peak near 0.3 rad/s and a tail extending towards higher spin rates. This indicates that while most debris objects spin slowly, a small fraction exhibit higher angular velocities that could complicate capture and require more robust control strategies. Understanding this distribution is crucial for designing control inputs that accommodate a wide range of debris behaviors.

The conductivity distribution shows moderate variability between 0.1 S/m and 1.5 S/m, with a slight peak near 1.0 S/m. Conductivity influences how a debris object might interact with electrodynamic tethers or other capture mechanisms, and accounting for this variability is essential for system reliability.

Collectively, these distributions provide a probabilistic foundation for the robust optimization framework employed in this project. They ensure that the Monte Carlo simulations accurately reflect the range of real-world debris characteristics that a harpoon-based ADR system might encounter in orbit.

B. Loss Distribution for Fixed Control Input

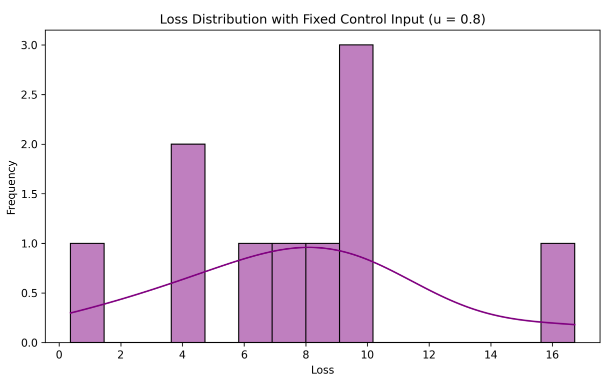

Fig. 2. Loss distribution for a fixed control input ($u = 0.8$).

Figure 2 shows the distribution of mission loss values resulting from simulations with a fixed control input ($u = 0.8$). Each bar represents the frequency of simulated missions that achieved a particular loss value, while the purple line represents the estimated density of the loss distribution.

The histogram reveals a *multimodal distribution*, with several distinct peaks at loss values around 4, 8, and 10. This suggests that even under the same control input, mission performance can vary considerably depending on the specific debris properties encountered in each scenario. Loss values range from near 0 up to 16, highlighting the influence of uncertain debris parameters on system performance.

The shape of the distribution underscores the *importance of robust optimization*: relying on a single control input may lead to widely varying outcomes, some of which could result in mission failure. The presence of high-loss outliers (e.g., near 16) emphasizes that certain combinations of debris parameters can lead to *unacceptably high risk* if not accounted for in the system design. This variability justifies the need for a probabilistic and robust approach to control input selection.

C. Optimizer Convergence

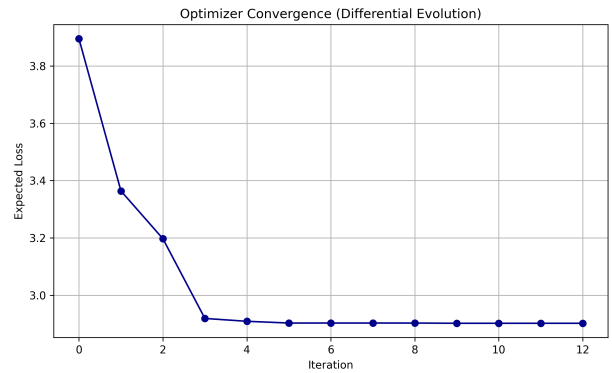

Fig. 3. Convergence of Differential Evolution showing expected loss across iterations.

Figure 3 shows the convergence behavior of the Differential Evolution (DE) optimizer applied to the robust control problem. The x-axis represents the iteration number (from 0 to 12), and the y-axis represents the expected loss computed from the Monte Carlo simulation framework.

Initially, the expected loss starts at approximately 3.9, indicating a relatively high mission loss with the initial population of control inputs. Within the first four iterations, there is a sharp decline in expected loss — dropping to around 3.0 — demonstrating the optimizer's ability to quickly identify more effective control inputs that reduce the overall risk of mission failure. This rapid decrease is consistent with DE's characteristic strong global search capabilities in the early stages of optimization.

After iteration 4, the expected loss plateaus, with subsequent iterations showing minimal additional improvement. This behavior suggests that the optimizer has converged to a near-optimal control input and that further iterations yield diminishing returns. The convergence profile underscores the efficiency of DE for this problem, providing a robust solution without excessive computational overhead.

Overall, the figure illustrates how DE can rapidly and effectively reduce mission loss, highlighting its suitability for optimizing control inputs in the presence of uncertain debris parameters.

D. Comparison of Control Strategies

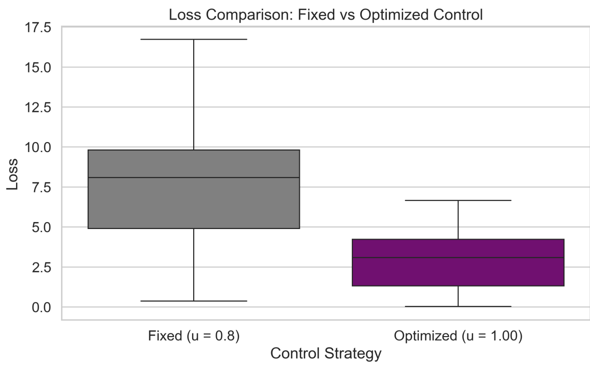

Fig. 4. Boxplot comparison of loss values for fixed ($u = 0.8$) and optimized ($u = 1.0$) control strategies.

Figure 4 presents a boxplot comparison of mission loss values for two control strategies: a fixed control input ($u = 0.8$) and the optimized control input identified through Differential Evolution ($u = 1.0$). Each boxplot summarizes the distribution of loss values obtained from Monte Carlo simulations under each strategy.

The fixed control input (gray boxplot) shows a higher median loss value, approximately 8, with a wide interquartile range (IQR) extending from around 5 to 10. This indicates that, under the fixed strategy, mission performance is highly variable and subject to significant risk. The whiskers further illustrate the presence of high-loss outliers, with some simulations yielding losses exceeding 15.

In contrast, the optimized control input (purple boxplot) exhibits a lower median loss, approximately 3, with a narrower IQR (approximately 2 to 4). This demonstrates that the robust optimization approach successfully identified a control strategy that not only reduces the expected loss but also improves consistency across different debris scenarios. The reduced variability and absence of high-loss outliers highlight the advantage of the optimization approach in mitigating mission risk under uncertainty.

Overall, this comparison illustrates the effectiveness of the robust optimization framework in enhancing mission reliability by providing both lower and more predictable loss values compared to a fixed control strategy.

E. Sensitivity Analysis of Debris Parameters

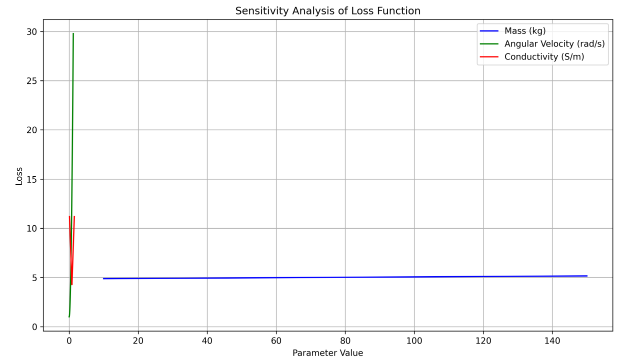

Fig. 5. Sensitivity analysis of mission loss with respect to debris parameters: mass (blue), angular velocity (green), and conductivity (red).

Figure 5 illustrates the sensitivity of the mission loss function to variations in debris parameters, including mass (blue), angular velocity (green), and conductivity (red). Each line represents the change in expected mission loss as the corresponding parameter is varied across its defined range while holding other parameters constant.

The analysis reveals that angular velocity (green) exhibits the greatest influence on mission loss, with loss values increasing sharply and peaking near 30. This indicates that small changes in debris spin rate can cause significant variation in mission performance, underscoring the criticality of accounting for this parameter in system design.

Conductivity (red) also shows a noticeable impact, with loss values ranging up to approximately 11. This suggests that variations in debris material properties can moderately affect mission success, though less dramatically than angular velocity.

In contrast, mass (blue) demonstrates a relatively flat sensitivity curve, with loss values remaining stable around 5 across the entire parameter range. This implies that, within the considered range of masses, variations in debris mass have a minimal effect on mission loss.

Overall, this analysis highlights that robust optimization efforts should prioritize accounting for uncertainties in debris spin rate and conductivity to minimize mission risk and improve system performance.

V. Discussion

The results of this study highlight the significant impact of parameter uncertainty on the performance of harpoon-based active debris removal (ADR) systems. The sensitivity analysis (Figure 5) clearly demonstrates that angular velocity has the most pronounced effect on mission loss, with rapid increases in loss values observed as spin rates vary. This finding suggests that debris spin rate is a critical parameter to consider during system design and operational planning.

The boxplot comparison (Figure 4) between the fixed and optimized control strategies further reinforces the advantages of robust optimization. The optimized control input ($u = 1.0$)

not only resulted in a lower median loss but also exhibited reduced variability, indicating more predictable mission outcomes. This improvement highlights the value of Differential Evolution (DE) in finding a control strategy that accounts for uncertainty and minimizes mission risk.

The optimizer convergence plot (Figure 3) illustrates the rapid decline in expected loss during the initial iterations, with convergence achieved by approximately the fourth iteration. This behavior aligns with the theoretical strengths of DE, which is known for its fast global search capabilities in the early stages of optimization. The stable loss values after convergence indicate that the optimizer successfully identified a robust solution that is unlikely to be improved further by additional iterations.

Lastly, the loss distribution analysis for the fixed control input (Figure 2) underscores the potential variability in mission outcomes when uncertainties are not adequately addressed. The presence of high-loss outliers in the fixed strategy reinforces the necessity of adopting a robust optimization approach that accounts for the probabilistic nature of debris parameters.

Overall, this study demonstrates the effectiveness of a robust optimization framework in enhancing mission reliability for ADR systems. By integrating probabilistic modeling, Monte Carlo simulation, and Differential Evolution, the approach successfully navigates the complexities introduced by uncertain debris properties. These insights can inform future design efforts, emphasizing the need to prioritize spin rate and conductivity during system development while ensuring that robust optimization is embedded into the ADR mission planning process. Future studies could explore integrating advanced evolutionary optimization techniques, as demonstrated by Zavoli and Colasurdo [10], to enhance ADR mission planning efficiency.

VI. Conclusion

This project developed a robust optimization framework for harpoon-based active debris removal (ADR) systems operating under uncertain debris parameters. By modeling debris mass, spin rate, and conductivity as probabilistic distributions, the approach effectively captures the real-world variability that affects mission performance. The use of Monte Carlo simulations allowed for the estimation of expected mission loss, providing a quantitative basis for evaluating system performance under uncertainty.

Differential Evolution (DE) was employed as the optimization algorithm, demonstrating its capability to rapidly converge to an effective control input that minimizes expected mission loss. The analysis revealed that the optimized control strategy significantly outperformed a fixed control approach, reducing both the expected loss and its variability across debris scenarios. Sensitivity analysis further identified angular velocity as the most influential parameter on mission outcomes, followed by conductivity, while mass had a minimal impact.

Overall, this study underscores the importance of integrating robust optimization techniques into ADR system design. By addressing parameter uncertainties systematically, this approach enhances mission reliability and contributes to safer, more sustainable orbital environments. Future work could extend this framework to include additional system dynamics, actuator constraints, and alternative control strategies, paving the way for more comprehensive ADR solutions.

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