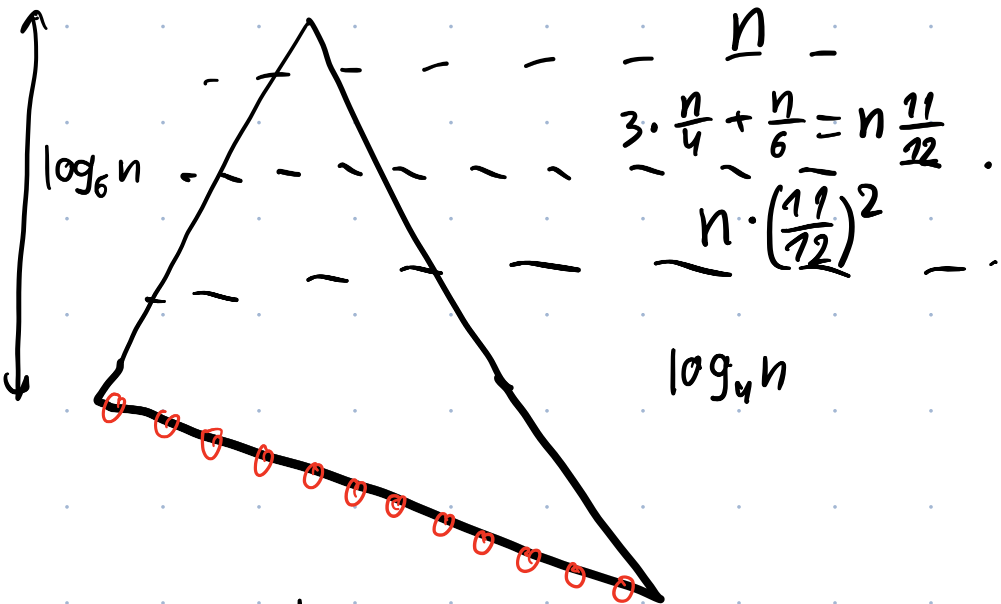
ДЗ 6, задача 4.

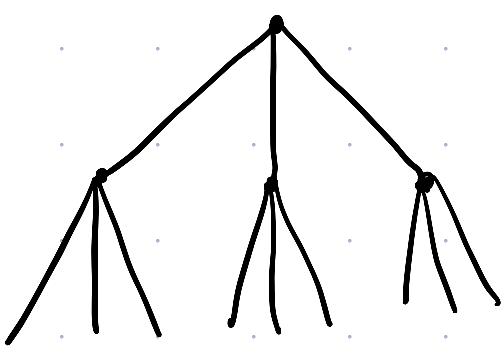
$$T(n) = 3T\left(\frac{n}{4}\right) + T\left(\frac{n}{6}\right) + n \qquad \boxed{T(5) = 2}$$



$$\sum_{k=0}^{n} C_n^k = 2^n$$

$$T(n) \le \sum_{i=0}^{\log_4 n} n\left(\frac{11}{12}\right)^i + 2^{\log_4 n} \cdot T(5) = O(n)$$

$$T(n) = 3T\left(\frac{n}{4}\right) + n \qquad T(n) = T\left(\frac{3n}{4}\right) + n$$



$$T(n) = 3T\left(\frac{n}{4}\right) + n^{\frac{1}{2}} \qquad T(n) = T\left(\frac{3n}{4}\right) + n^{\frac{1}{2}}$$

$c = \log_4 3 \qquad c = \log_{\frac{4}{3}} 1 = 0$

$$f(n) = n^{\frac{1}{2}} = O(n^{\log_4 3 - \varepsilon})$$

$$T(n) = \Theta(\sqrt{n})$$

$$T(n) = \Theta(n^{\log_4 3})$$

$$T(n) \leq 5T(n-1) \leq 5^2 T(n-2) \leq \dots \leq 5^n T(1)$$

$$\geq 5T(n-3)$$

```
def divide(x,y)
    if (x == 0):
        return (0,0)
    q,r = divide(⌊x/2⌋, y)
    q *= 2
    r *= 2
    if (x %2 == 1):
        r += 1
    if (r ≥ y):
        r -= y
        q += 1
    return (q,r)
```

div(11010, 101):

$q,r = \text{div}(1101, 101) = 10, 11$

$q,r = \text{div}(110, 101) = 1, 1$

$q,r = \text{div}(11, 101) = 0, 11$

$q,r = \text{div}(1, 101) = 0, 1$

$q,r = \text{div}(0\ 101) = 0, 0$

return (0,0)

q,r = 0,0

r = 1

return(0,1)

q,r = 0, 10

r = 11

return(0,11)

q,r = 0,110

// r ≥ y (110 ≥ 101)

r = r-y = 110-101 = 1

q += 1 = 1

return (1,1)

q,r = 10, 10

r = 11

return(10,11)

q,r = 100, 110

// r ≥ y

r = 110 - 101 = 1

q = 100 + 1 = 101

return(101,1)

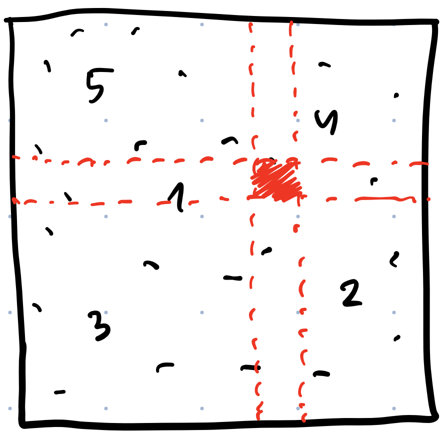
x / y

$\left\lfloor \frac{x}{y} \right\rfloor = q$

$x - y\left\lfloor \frac{x}{y} \right\rfloor = r$

$x = y \cdot q + r$

ЗАДАЧА

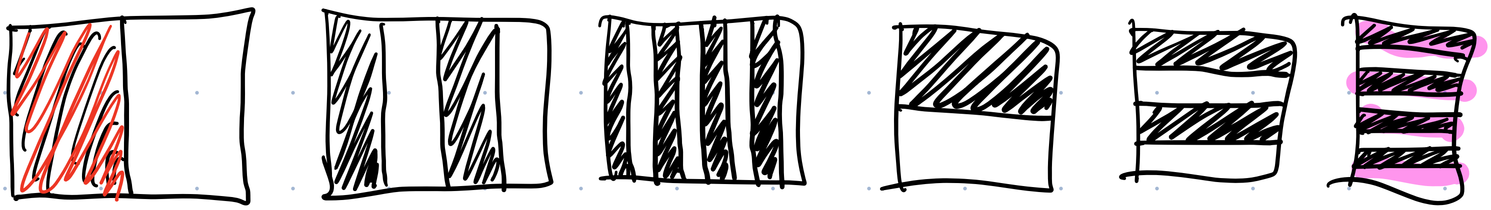


1) 63

2) 8+8 = 16

КОДИРОВКА

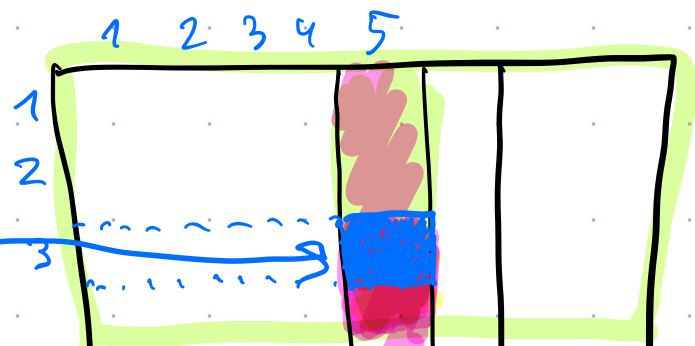
b_1	b_2	...	b_6



$\boxed{1\ *\ *\ *\ *\ *}$ $*1****$ $**1***$

{...} {...} {...} {...} {...} {...}

$\boxed{0\ 1\ 1}\ 1\ 0\ 1$



ДИОФАНТОВЫ УРАВНЕНИЯ

$ax+by=c \qquad a,b,c,x,y \in \mathbb{Z}$

$d=\gcd(a,b)$

$a=k_a d$

$b=k_b d$

$ax+by=(k_a x+k_b y)d=c$

$c \not\vdots\, d \Rightarrow$ РЕШЕНИЙ НЕТ

$c \,\vdots\, d$:

$EE \rightsquigarrow (x_0, y_0): \quad ax_0+by_0=d$

$x_0, y_0 \quad \Big| \cdot \frac{c}{d}$

$x_0' = x_0 \cdot \frac{c}{d}$

$y_0' = y_0 \cdot \frac{c}{d}$

$a \cdot x_0' + b \cdot y_0' = c$) ЧАСТНОЕ РЕШЕНИЕ

$a \cdot \frac{\cancel{c}}{\cancel{d}} \cdot x_0 + b \cdot y_0 \cdot \frac{\cancel{c}}{\cancel{d}} = d \cdot \frac{\cancel{c}}{\cancel{d}}$

ПОЛУЧИМ ОБЩЕЕ РЕШЕНИЕ:

$\begin{cases} x_{общ} = x_ч + t\frac{b}{\gcd(a,b)} \end{cases} \qquad t \in \mathbb{Z}$

$$\begin{cases} y_{общ} = y_ч - t\dfrac{a}{\gcd(a,b)} \end{cases} \qquad t \in \mathbb{Z}$$

ОБЩЕЕ РЕШЕНИЕ

$$ax + by = c$$

$$ax_ч + \cancel{t\frac{b}{\gcd(a,b)}}a + y_ч b - \cancel{t\frac{a}{\gcd(a,b)}}b = c$$

$$ax_ч + by_ч = c$$

$$a\Delta x + b\Delta y = 0$$

$$K_a \underset{\gcd(a,b)}{d}\, \Delta x + K_b d\, \Delta y = 0$$

$$K_a \Delta x = -K_b \Delta y$$

$$\gcd(K_a, K_b) = 1$$

$$\begin{cases} \Delta x = K_b \\ \Delta y = -K_a \end{cases}$$

$$\Delta x = 2K_b$$
$$\Delta y = -2K_a$$

$$\Delta x = sK_b$$
$$\Delta y = -sK_b$$

РЕШИМ КОНКРЕТНОЕ ДИОФ. УР.

$$4x + 12y = 20$$

$$x = y'$$
$$y = x' - \left\lfloor \frac{a}{b} \right\rfloor y'$$

a	b	d	$\lfloor \frac{a}{b} \rfloor$	x	y
4	12	4	0	1	$0 - 0 \cdot 1 = 0$
12	4	4	3	0	$1 - 3 \cdot 0 = 1$

4 0 4 − 1 0

$$\begin{cases} x = 1 \\ y = 0 \end{cases} \quad \Big| \cdot \frac{20}{gcd} = 5$$

$$1 \cdot 4 + 0 \cdot 12 = gcd(4, 12) = 4$$

$$\begin{cases} x_1 = 5 \\ y_1 = 0 \end{cases}$$

$$\begin{cases} x_{общ} = 5 + t\frac{12}{4} = 5 + 3t \\ y_{общ} = 0 - t\frac{4}{4} = -t \end{cases} \quad t \in \mathbb{Z}$$

$$4x_{общ} + 12y_{общ} = 4 \cdot 5 + \cancel{4 \cdot 3 \cdot t} - \cancel{12t} = 20 = 20$$

---

mod 7

0, 1, 2, 3, 4, 5, 6

$5^{-1} \equiv ? \mod 7$

$5 \cdot 5^{-1} \equiv 1 \mod 7$

$5^{-1} \equiv 3 \mod 7$

$5 \cdot 3 \equiv 1 \mod 7$