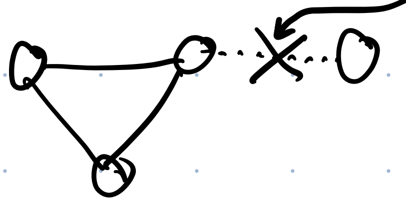
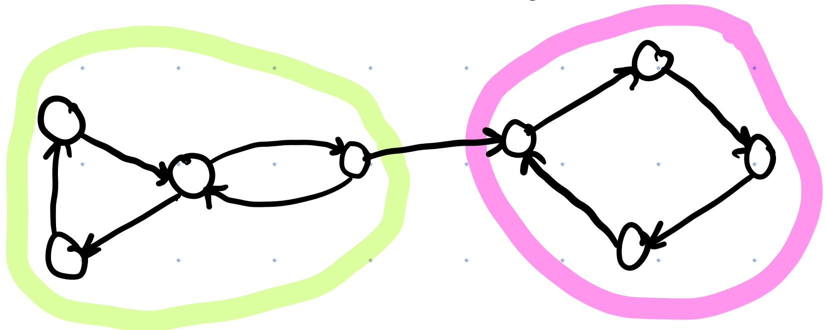
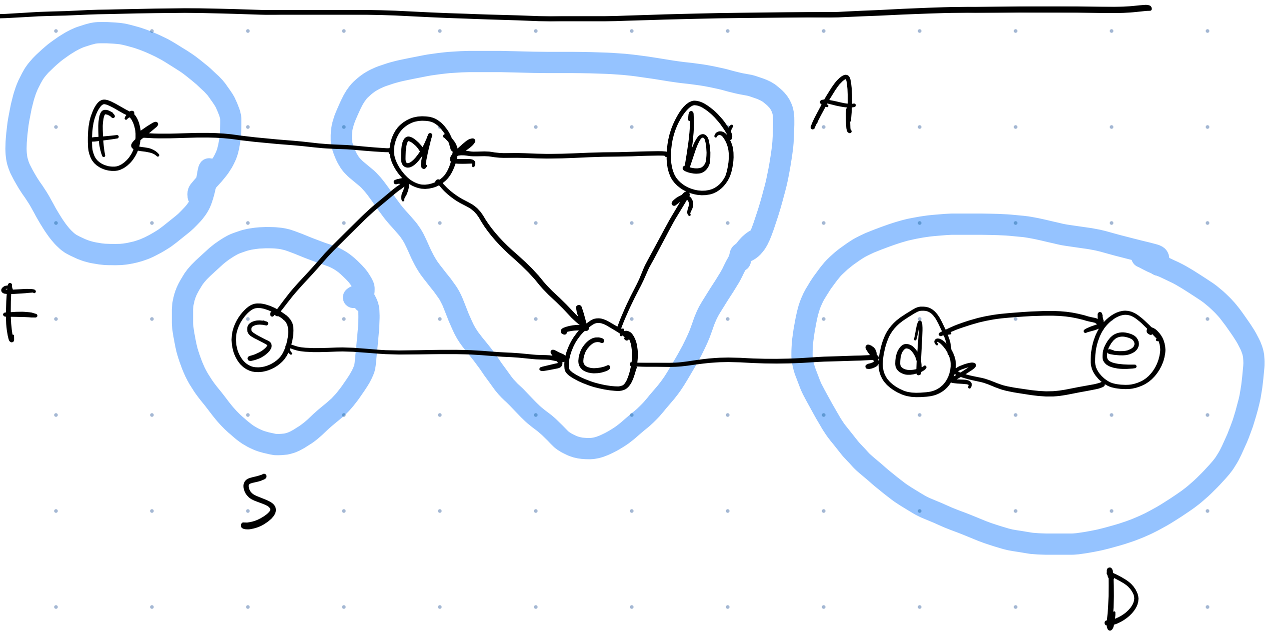
# Компоненты сильной связности
(strongly connected components)

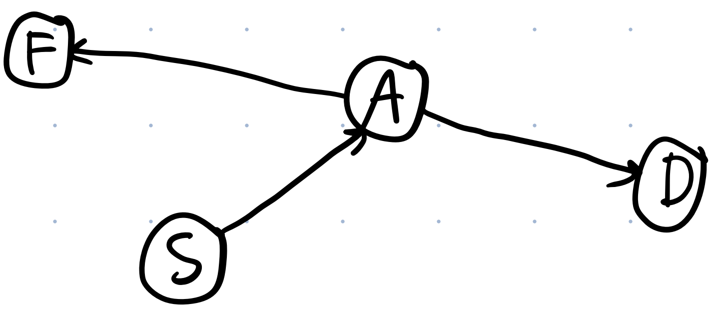
## Связность
(и её отсутствие)



## Сильная связность

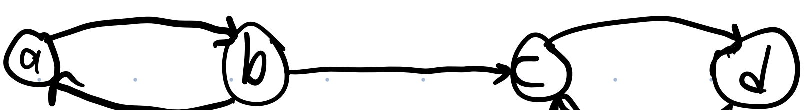





Метаграф (конденсат)

## Алг. поиск КСС



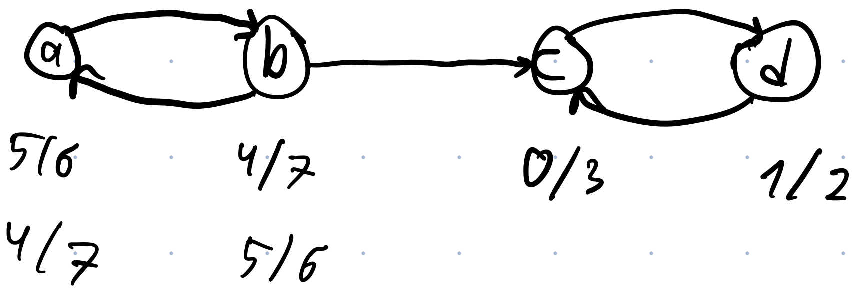

ВРЕМЯ ОТКРЫТИЯ

ВРЕМЯ ЗАКРЫТИЯ

$s_i, f_i$

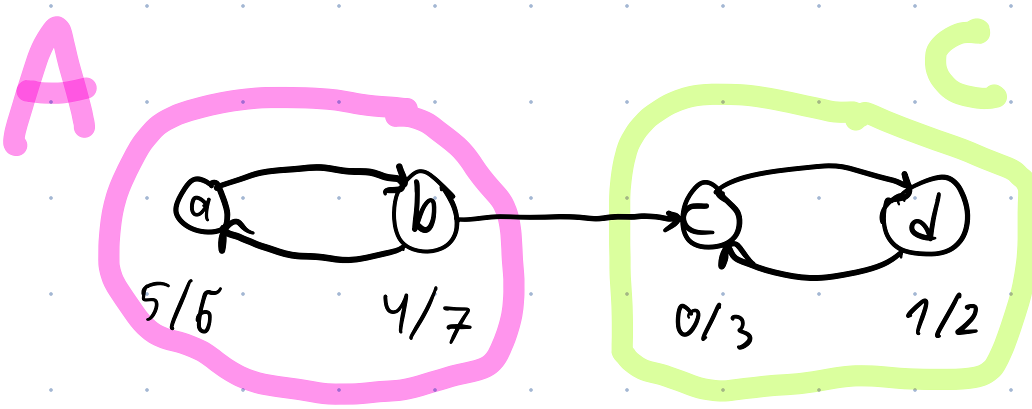
$s_j, f_j$

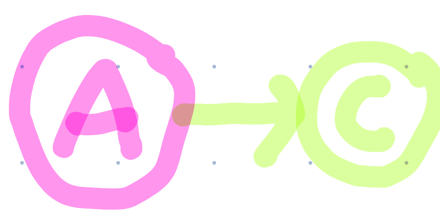
$s_i < s_j < f_j < f_i$

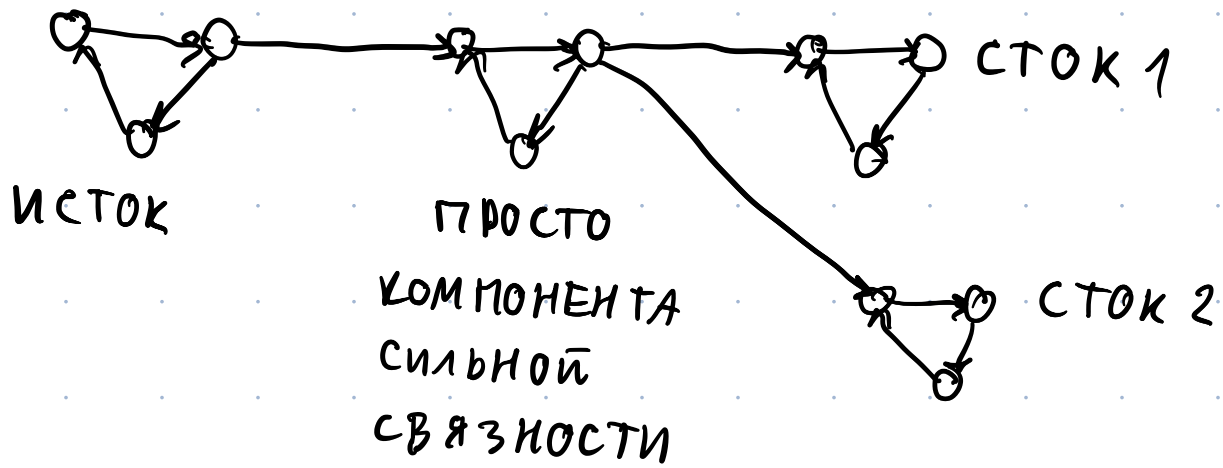





КОНДЕНСАТ:



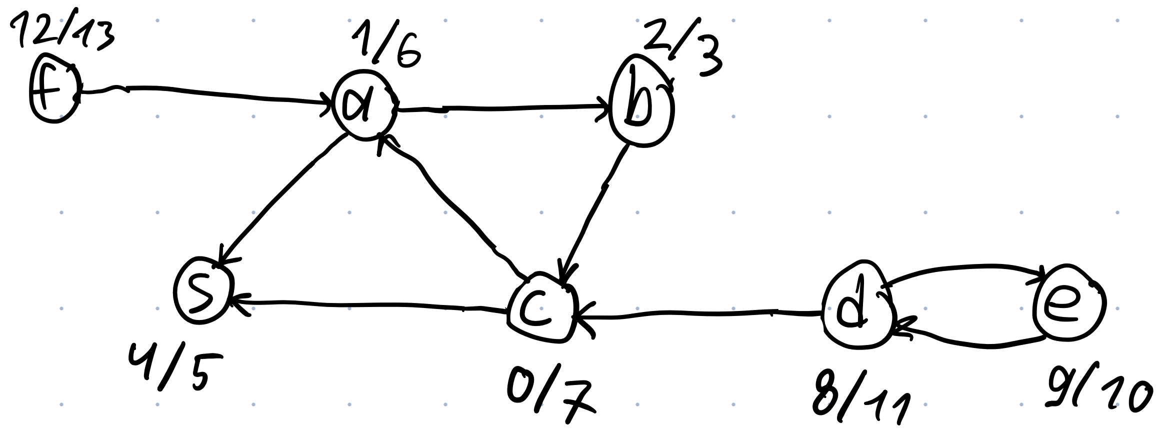



Алг. поиска КСС:

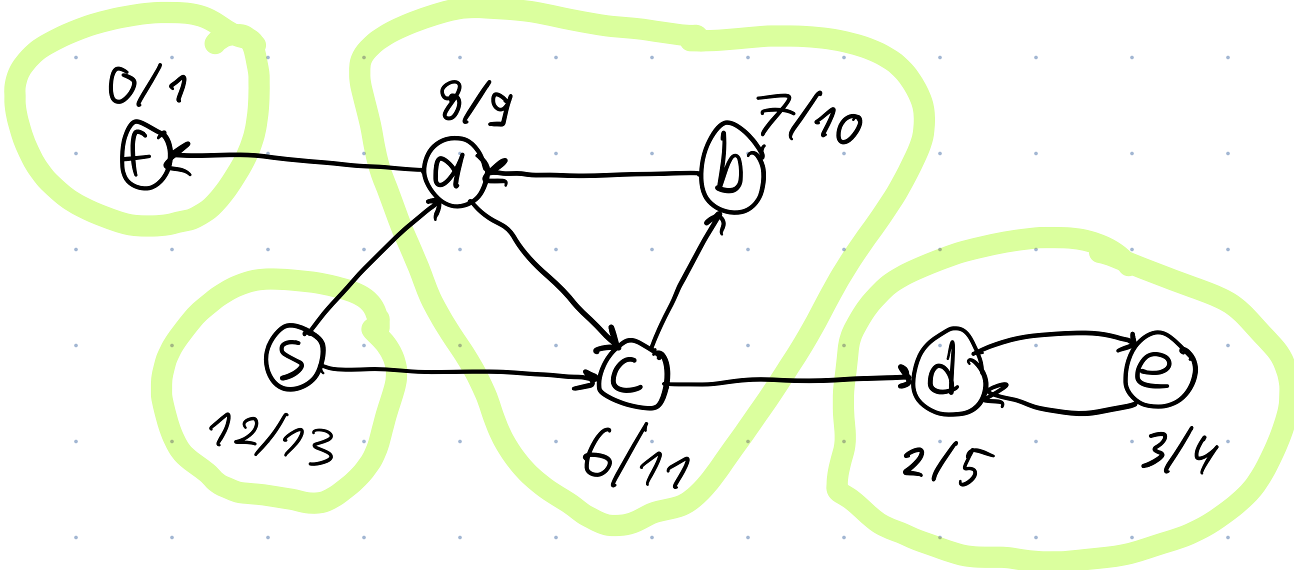
Вход: G

1. $G^T$
2. DFS($G^T$)
3. DFS(G), вершины в порядке убыв. post_time в DFS($G^T$)

1–2)



3)




F: {f}

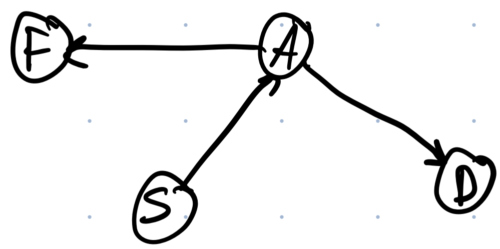
A: {a, b, c}

S: {s}

D: {d, e}

| a | b | c | d | e | f |
|---|---|---|---|---|---|
| A | A | A | D | D | F |

| | A | F | S | D |
|---|---|---|---|---|
| A | 0 | | | |
| F | | 0 | | |
| S | | | 0 | |
| D | | | | 0 |




| | A | D | F | S |
|---|---|---|---|---|
| i → A | 0 | 1 | 1 | −1 |
| D | −1 | 0 | 0 | 0 |
| F | −1 | 0 | 0 | 0 |
| S | 1 | 0 | 0 | 0 |

f ⇝ [ ]

a ⇝ [f, c]

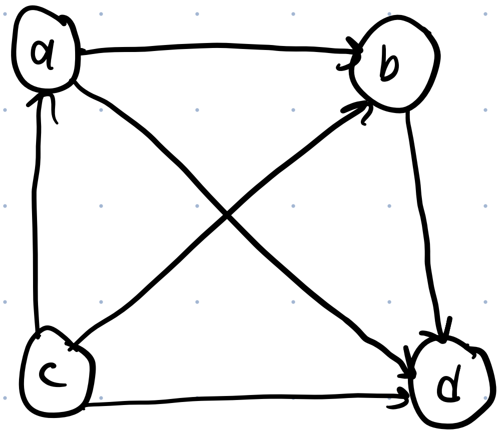
A [
F [
S [
D [

---

ТУРНИР, Т. Е. ПОЛНОСВ. ОРИЕНТ. ГРАФ

( ) на n ВЕРШИНАХ

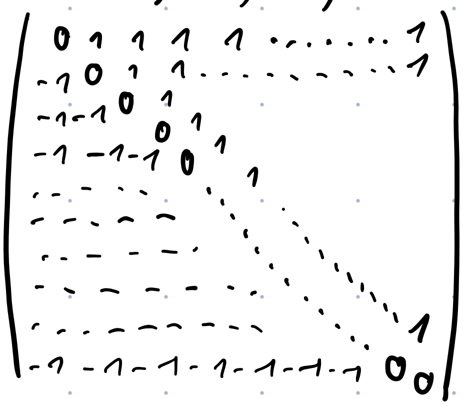
$O(n^2)$

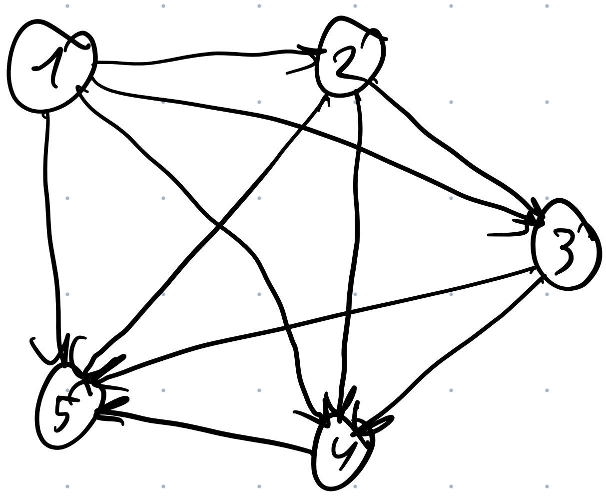


| | a | b | c | d |
|---|---|---|---|---|
| a | 0 | 1 | -1 | 1 |
| b | -1 | 0 | -1 | 1 |
| c | 1 | 1 | 0 | 1 |
| d | -1 | -1 | -1 | 0 |

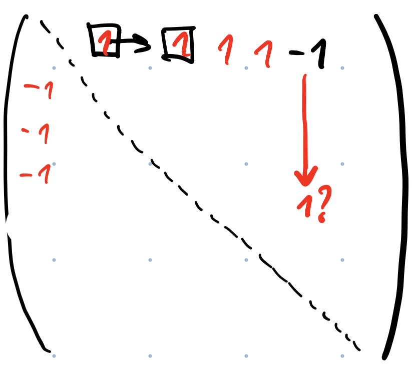
$i \longrightarrow j \qquad A[i,j] = 1$

flags = [ , , , ..., ...]

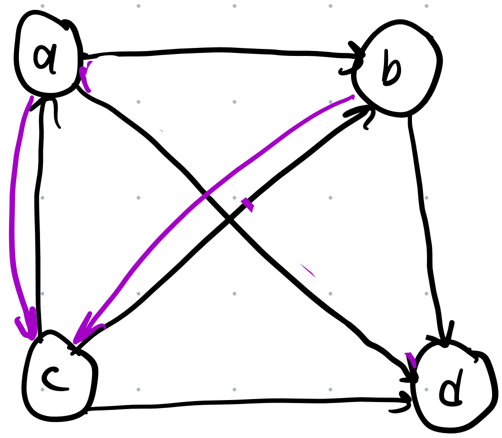




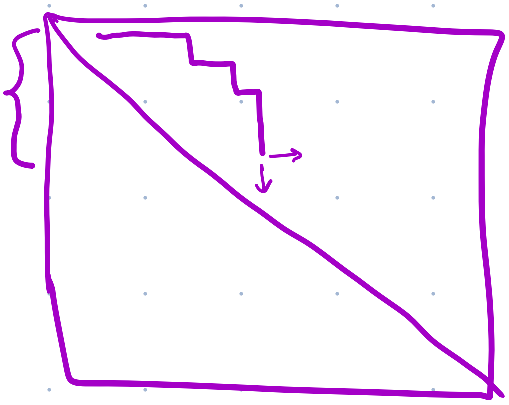
Вход: матр. A

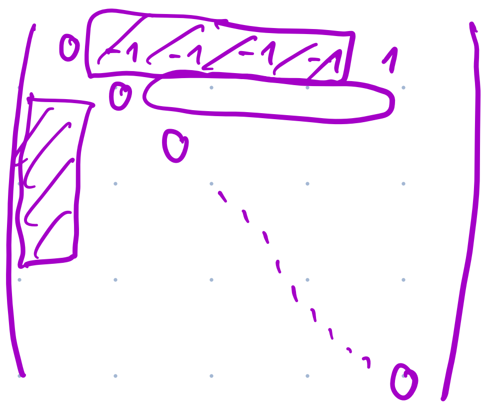


| | a | b | c | d |
|---|---|---|---|---|
| a | 0 | 1 | -1 | 1 |
| b | -1 | 0 | -1 | 1 |
| c | 1 | 1 | 0 | 1 |
| d | -1 | -1 | -1 | 0 |

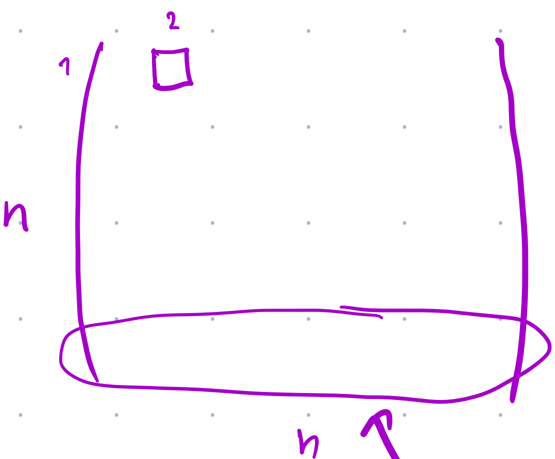



ИЩЕМ ОБЩ. СТОК










1? ⇄ 2?

curr_candidate

O(1)

O(n) – УБР. ВСЕХ КАНД., КР. М.Б 1

O(n) – ДОПРОВЕР. КАНДИД.

O(n) СУММАРН

| | a | b | c | d |
|---|---|---|---|---|
| a | 0 | 1 | -1 | 1 |
| b | -1 | 0 | -1 | 1 |
| c | 1 | 1 | 0 | 1 |
| d | -1 | -1 | -1 | 0 |

I. 1) $\cancel{a} \to b$ $(A[0,1] = 1)$

curr_cand = b

2) curr_cand vs c

$\overset{\shortparallel}{b}$

$\cancel{a} \to b$

curr_cand = b

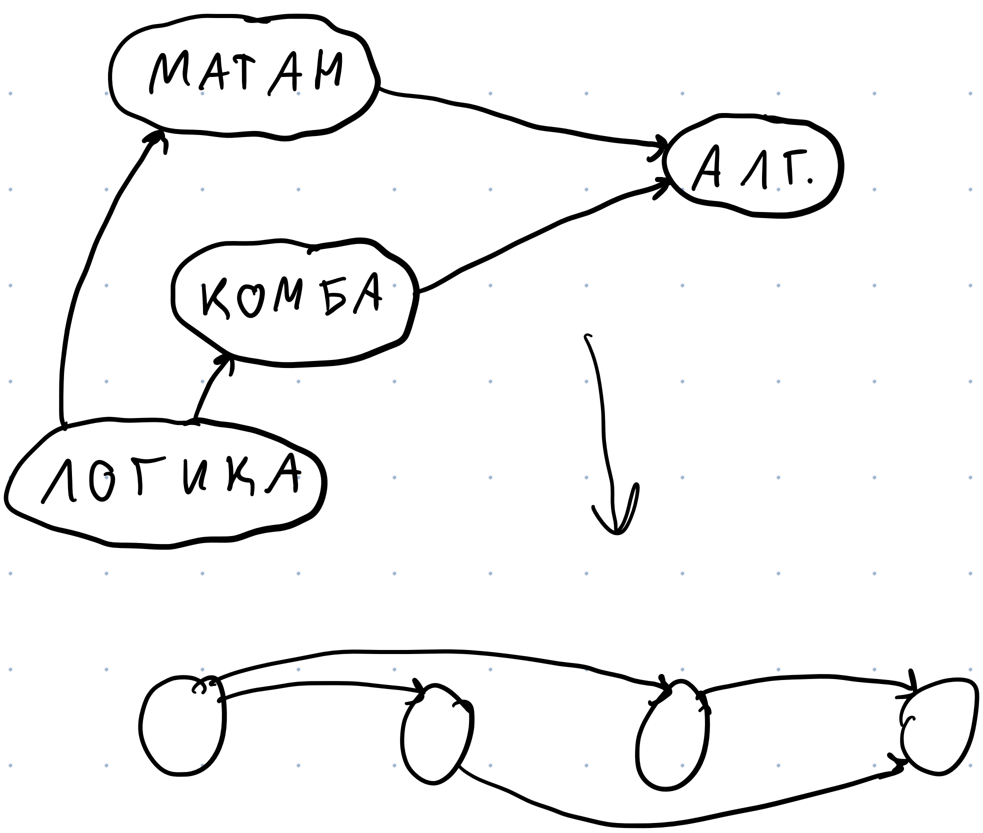
3) b vs d

$\cancel{b} \to d$

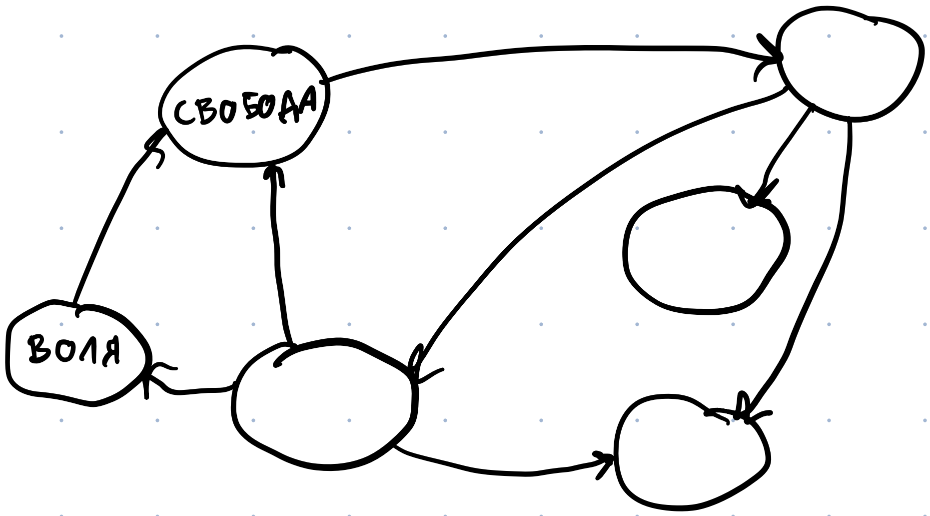
curr_cand = d

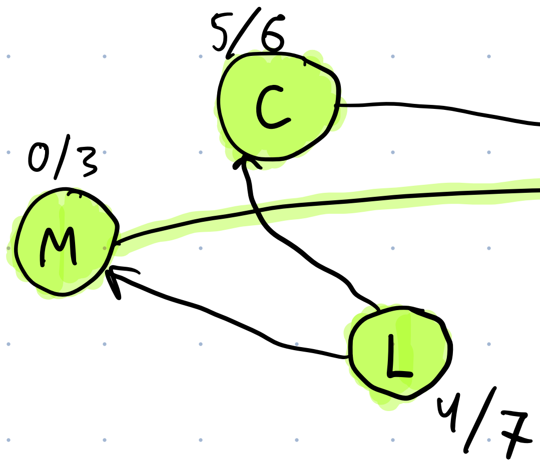
II. ?

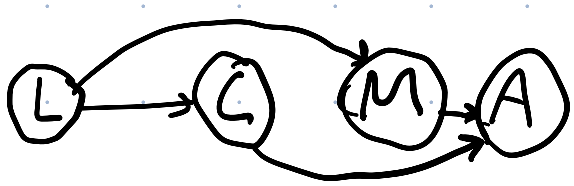
ТОПОЛОГИЧЕСКАЯ СОРТИРОВКА



$$J = \sum_{i=0}^{N} \gamma^{i} c_i \qquad \gamma < 1 \qquad \gamma = 1$$



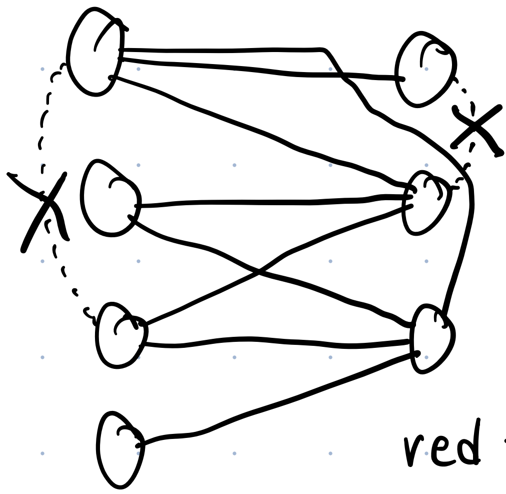

```
top_sort(u)
    u.color = gray
    for all e_uv in E:
        if (v.color == black):
            pass
        if (v.color == gray):
            print("hehe net")
        if (v.color == white):
            top_sort(v)
```

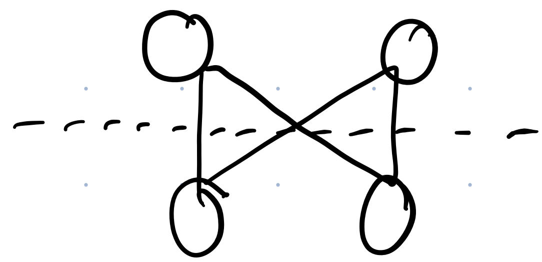
top_sort(v)

u.color = black

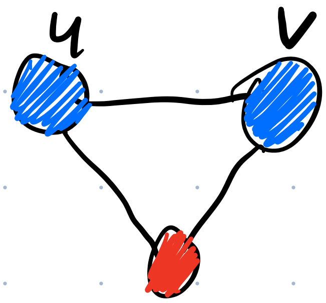
---

## Проверка графа на двудольность



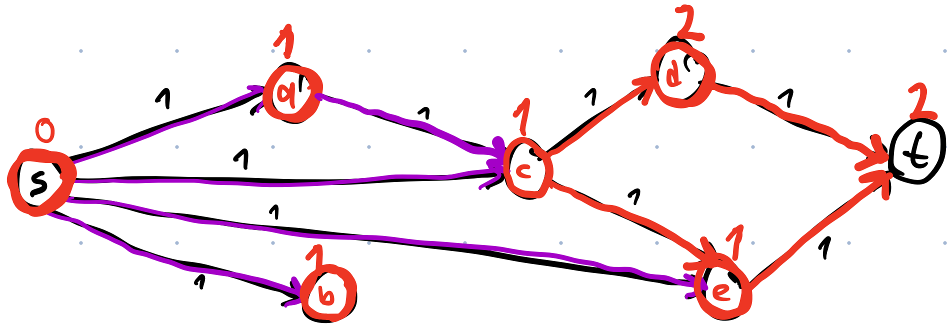


red = not blue




```
bipartite_check(u, color)
    u.color = color
    for e_uv ∈ E:
        if (v.color == color):
            print("not bipartite")
            exit()
        if (v.color == not color):
            pass
        if (v.color == transparent):
            bipartite_check(v, not color)
```

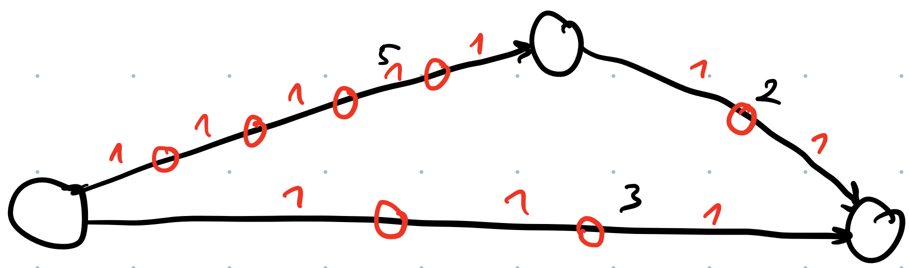
# Кратчайшие пути из s в t




BFS

Сложность: $\Theta(|V|+|E|)$

## А если веса натуральные?



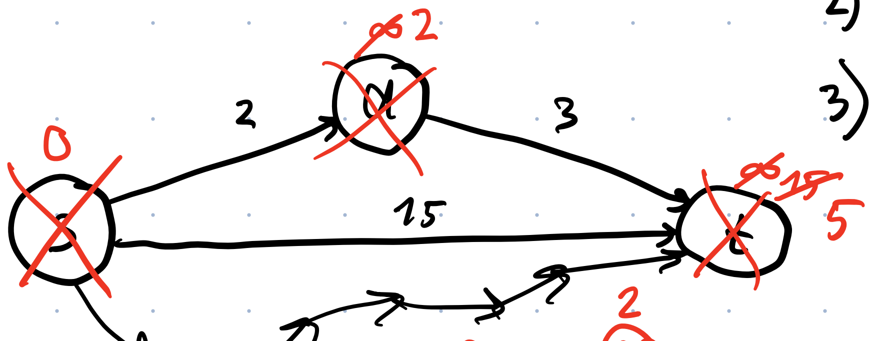
$e_i \le W$

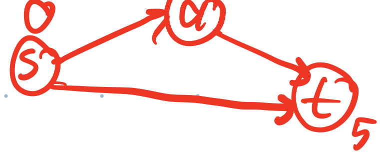
$O(|E|+|V|+|E|W\cdot 2)$

## Алгоритм Дейкстры:

1) $d[\forall v] = +\infty$
2) $d[s] = 0$
3) создаём heap на d

```
for i in range(|V|):
    curr_vertex = heap.extract_min()
```

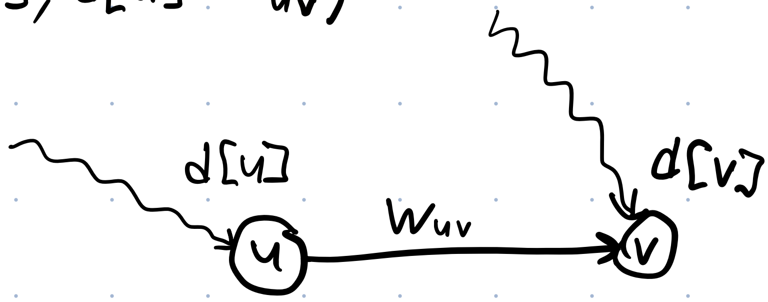





for $e_{curr\_ver,v}$ in E:
    relax($e_{curr\_ver,v}$)

def relax($e_{uv}$)
    // $w_{uv}$, d[u], d[v]

    $d[v] = \min(d[v], d[u] + w_{uv})$




$O((|V| + |E|) \log |V|)$