```
def func(n):
1    for i in range(n):
2        for j in range(n):
3            for k in range(j):
                 print("dorova")
4            for l=1; l<n; l*=2:
                 print("dorova")
```

АЛЬТ. ВАР. ЗАПИСИ

$$\sum_{m:\ 1 \leq 2^m \leq n} 1$$

$$f(n) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \left[\sum_{k=0}^{j-1} 1 + \sum_{m=0}^{\lfloor \log_2 n \rfloor} 1 \right] =$$

$$= \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} j + \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (\lfloor \log_2 n \rfloor + 1) =$$

$$= \sum_{i=0}^{n-1} \frac{(n-1)(n-1+1)}{2} + n^2(\lfloor \log_2 n \rfloor + 1) =$$

$$= \frac{n^2(n-1)}{2} + n^2(\lfloor \log_2 n \rfloor + 1) = \Theta(n^3)$$

$$= \frac{n^3}{2} + \ldots \leq n^3 \Rightarrow O(n^3)$$

$$\geq \frac{n^3}{3} \Rightarrow \Omega(n^3)$$

$$f(n) = \log n! = \sum_{i=1}^{n} \log i \le \sum_{i=1}^{n} \log n = n \log n = O(n \log n)$$

$$n! \le n^n$$

$$f(n) = \sum_{i=1}^{n} \log i \ge \sum_{i=\lfloor \frac{n}{2} \rfloor}^{n} \log i \ge \sum_{i=\lfloor \frac{n}{2} \rfloor}^{n} \log \frac{n}{2} = \lceil \frac{n}{2} \rceil \log \frac{n}{2} =$$

$$= \lceil \frac{n}{2} \rceil (\log n - \log 2) = \Omega(n \log n)$$

$$f(n) = \Theta(n \log n)$$

ЧИСЛА ФИБОНАЧЧИ

$\sum_{i_1=1}^{100} \ldots \sum_{i_5=1}^{100} F_{i_1+\ldots i_5}$ НА ДОСУГЕ

$F_0 = 0$

$F_1 = 1$ $\qquad 0, 1, 1, 2, 3, 5, 8, \ldots$

$F_{k+1} = F_k + F_{k-1}$

F_k

1.
```
def Fib(k):
    if (k==0):
        return 0
    if (k==1)
        return 1
    return Fib(k-1)+F(k-2)
```

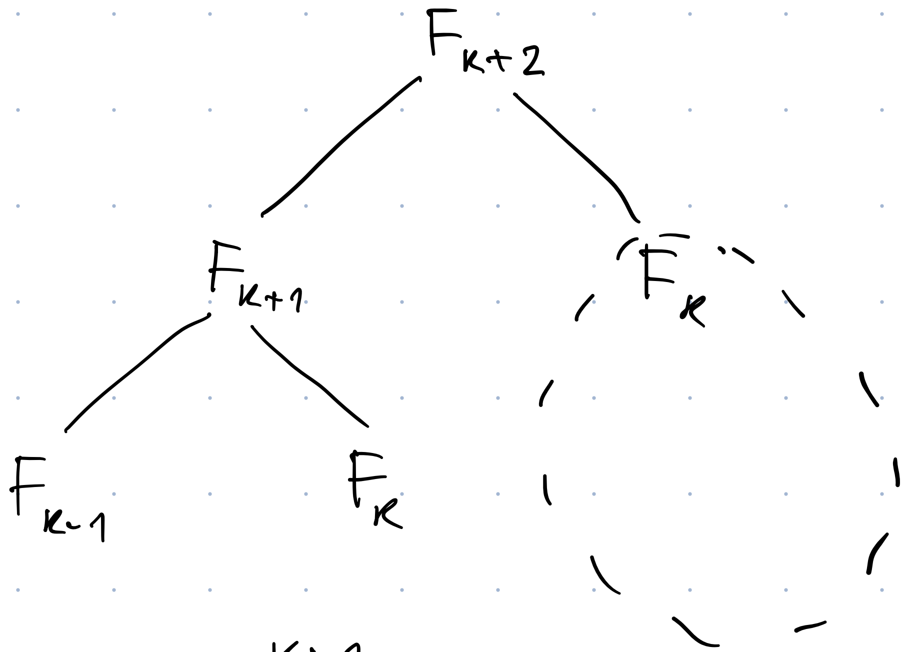
k - вход

$n = \lceil \log_2 k \rceil$

$k = 2^n$

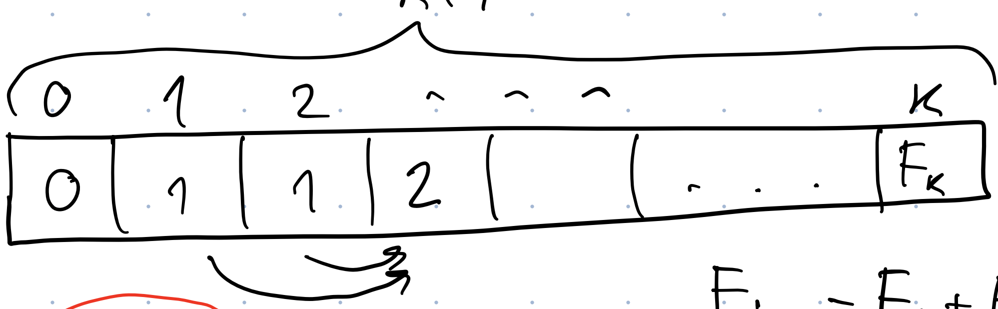
$$f(k) = f(k-1) + f(k-2) + 1 \underset{k>2}{\geq} 2f(k-2) \geq 4f(k-4) \geq$$

$$\geq 2^{\frac{k-3}{2}} f(3) = C \cdot 2^{\frac{k}{2}} = C \cdot 2^{\frac{2^n}{2}} = C \cdot 2^{2^{n-1}}$$



$n = \log_2 k$

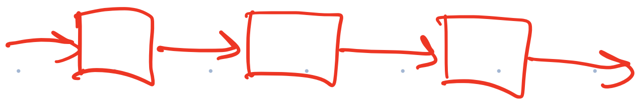
2.



$F_{j+1} = F_j + F_{j-1}$

$\Theta(2^n)$ // ВРЕМЯ И ПАМЯТЬ

$\Theta(k)$



0:	a	∅	∅
1:	b	a	∅
2:	c	b	a
3:	d	c	b
4:	e	d	c

3. $a^b = \underbrace{a \cdot a \cdot \ldots \cdot a}_{b}$ $b-1$ умн.

$a^{11} = \underbrace{a \cdot \ldots \cdot a}_{10 \text{ умн}} =$

1) $a^1 \underset{4}{\cdot} a^2 \underset{5}{\cdot} a^8$

a^2 1
a^4 2
a^8 3

2) $(a^5)^2 \cdot a$

$((a^2)^2 \cdot a)^2 \cdot a$

1 2 3 4 5

$b_2 = \underset{a}{\underset{\rightarrow}{1}}01\ldots010$

"0": $(curr)^2$

"1": $(curr)^2 \cdot a$

a^5

$5 = 101$

$a^1 \to a^2 \to a^4 \to a^5$

$5_2 = \underbrace{1}\ 0\ 1$

curr $= 1$ $\quad 1^2 \cdot a = a$

curr $= a$

curr $=$ curr$^2 \longrightarrow a^2$

curr $=$ curr$^2 \cdot a \longrightarrow (a^2)^2 \cdot a = a^5$

1 1 0 0 1 0 1 0 0

$((((a)^2 \cdot a)^2)^2)^2 \cdot a$

ПРИМЕР

$12 = 1100$

$2^{12} = 4096$

1	1	0	0
1	2	3	4

$c = 1$

1) $c \to c^2 \cdot 2$

$c = 1^2 \cdot 2 = 2$

2) $c \to c^2 \cdot 2$

$c = 2^2 \cdot 2 = 8$

3) $c \to c^2$

$c = 64$

4) $c \to c^2$

$c = 4096$

$$a^{\boxed{b_n \cdots\cdots b_0}} = a^{2^n \cdot b_n + 2^{n-1} \cdot b_{n-1} + \ldots + 2^0 \cdot b_0} =$$

$$b_i = \begin{cases} 0 \\ 1 \end{cases}$$

$$= a^{2^n \cdot b_n} \cdot a^{2^{n-1} b_{n-1}} \cdot \ldots \cdot a^{2^0 \cdot b_0} =$$

$$= a^{2^n b_n}$$

b_n	b_{n-1}	$\ldots$	b_0
1	0	0 ...	0

$$a^{10000}$$

$$((a^1)^2)^2$$

$$F_{k+2} = F_{k+1} + F_k$$

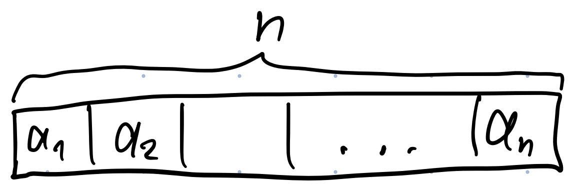
```
import time
print(time.time())
```

$$\begin{pmatrix} F_{k+1} \\ F_k \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}}_{A} \begin{pmatrix} F_k \\ F_{k-1} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^2 \begin{pmatrix} F_{k-1} \\ F_{k-2} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^k \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

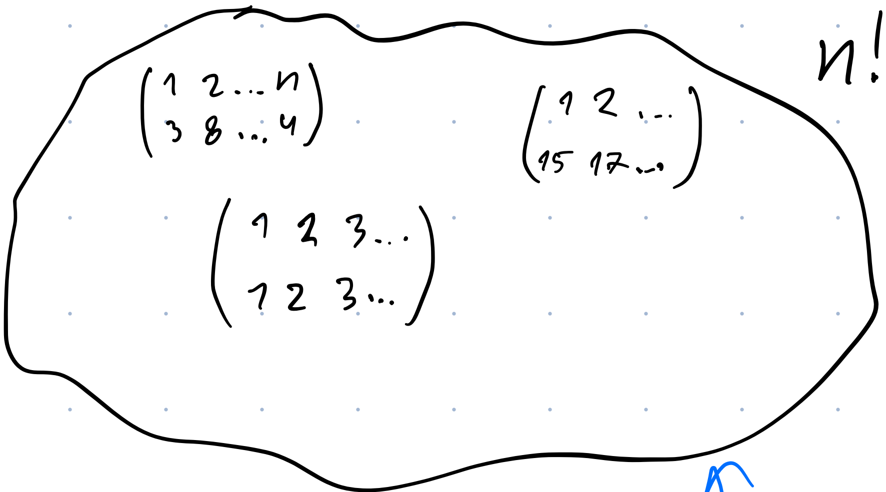
A^k $\quad \Theta(\lceil \log k \rceil)$

$\Theta(n)$ $\quad \Theta(2^n)$ $\quad \Theta(2^{2^n})$

ОЦЕНКА СНИЗУ СОРТИРОВОК СРАВНЕНИЯМИ

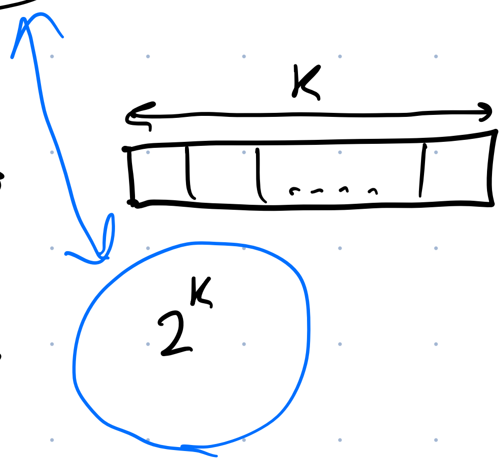


$$a_i \overset{?}{\vee} a_j$$



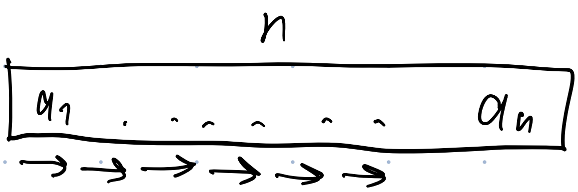
$n!$ элементов

$$k\left\{\begin{array}{l} i_1 \; ? \; j_1 \to 0/1 \\ i_2 \; ? \; j_2 \to 0/1 \\ \dots \\ i_k \; ? \; j_k \to 0/1 \end{array}\right.$$

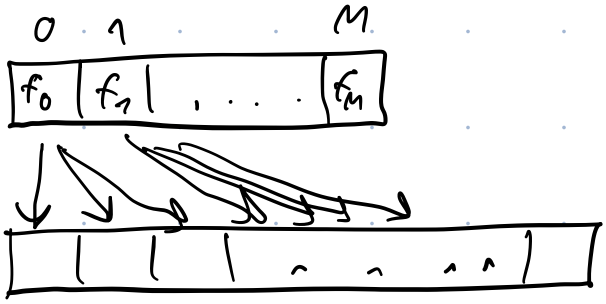


$$2^k \geq n! \quad // \log_2$$

$$k \geq \lceil \log_2 n! \rceil = \Omega(n \log n)$$



$0 \le a_i \le M$

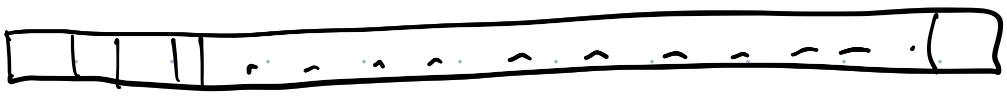


$O(n+M+n+M)$

$O(n+M)$ ВРЕМЯ

$O(M)$ ПАМЯТЬ

$M = 4\,000\,000\,000$



```
def counting_sort(a, M):
    freq = [0|0|....|0]
```

$\underbrace{\qquad\qquad}_{M}$

$O(M)$

```
    for i in range(len(a)):
        freq[a[i]] += 1
```

$O(n)$

```
    curr_el = 0

    for i in range(M):
        for j in range(freq[i]):
            a[curr_el] = i
            curr_el += 1
```

$O(n+M)$