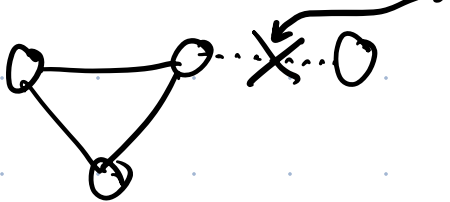


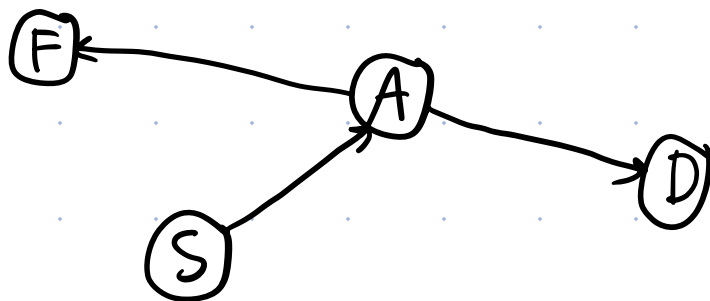
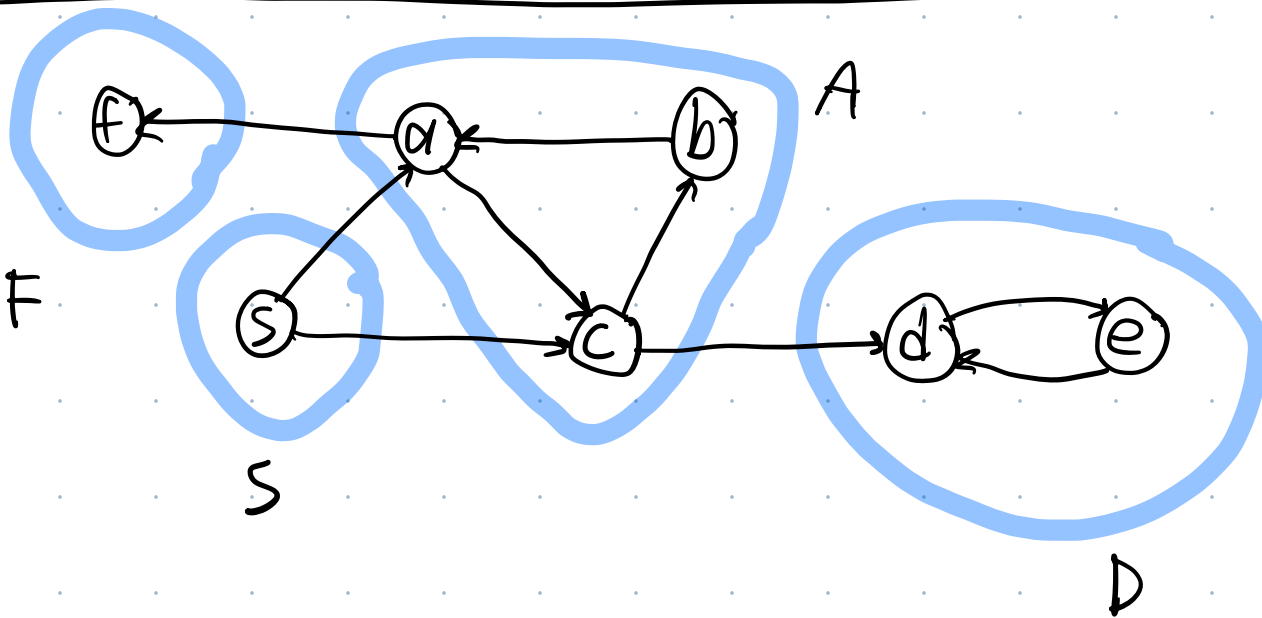
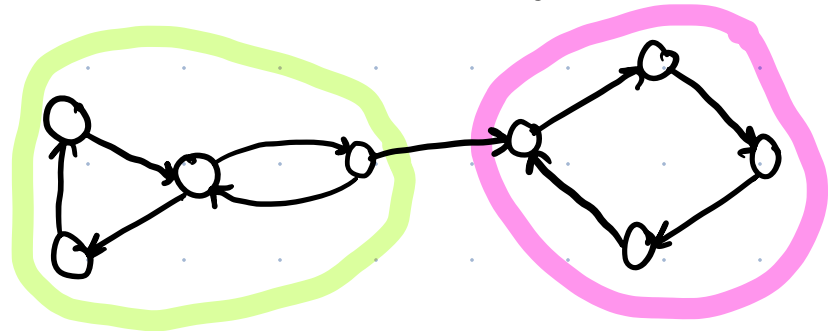
КОМПОНЕНТЫ СИЛЬНОЙ СВЯЗНОСТИ (strongly connected components)

СВЯЗНОСТЬ

(и её отсутствие)



СИЛЬНАЯ СВЯЗНОСТЬ



МЕТАГРАФ
(КОНДЕНСАТ)

АЛГ. ПОИСК КСС





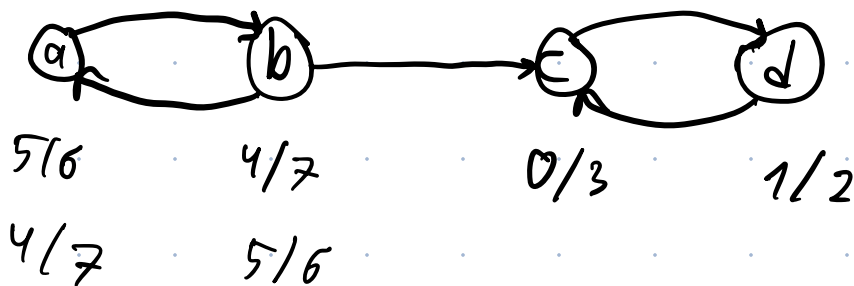
ВРЕМЯ
ОТКРЫТИЯ

ВРЕМЯ
ЗАКРЫТИЯ

s_i, f_i

s_j, f_j

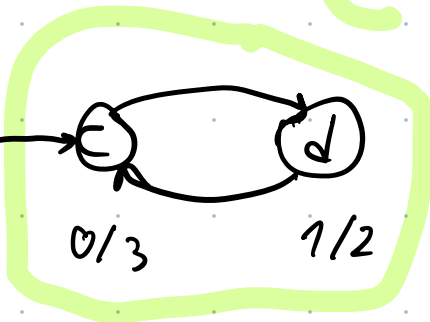
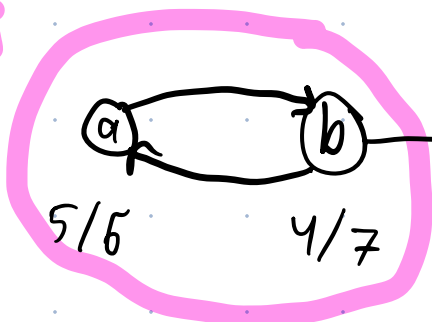
$s_i < s_j < f_j < f_i$

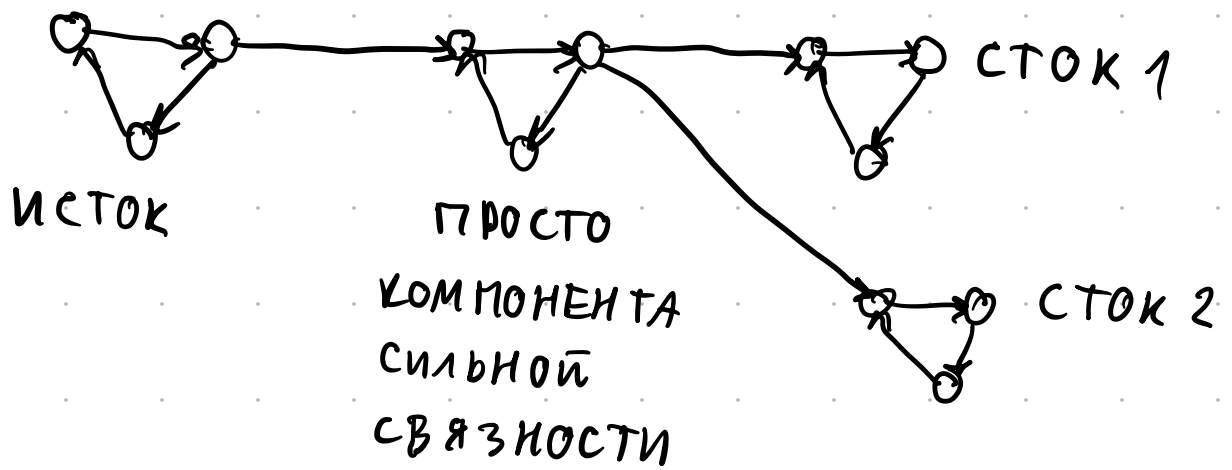


КОНДЕНСАТ:

A

C

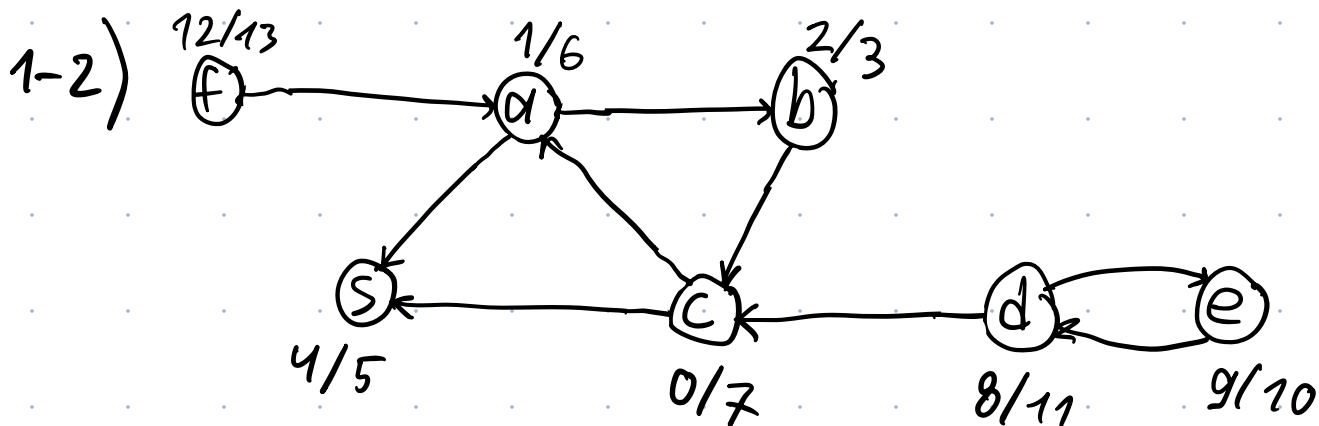




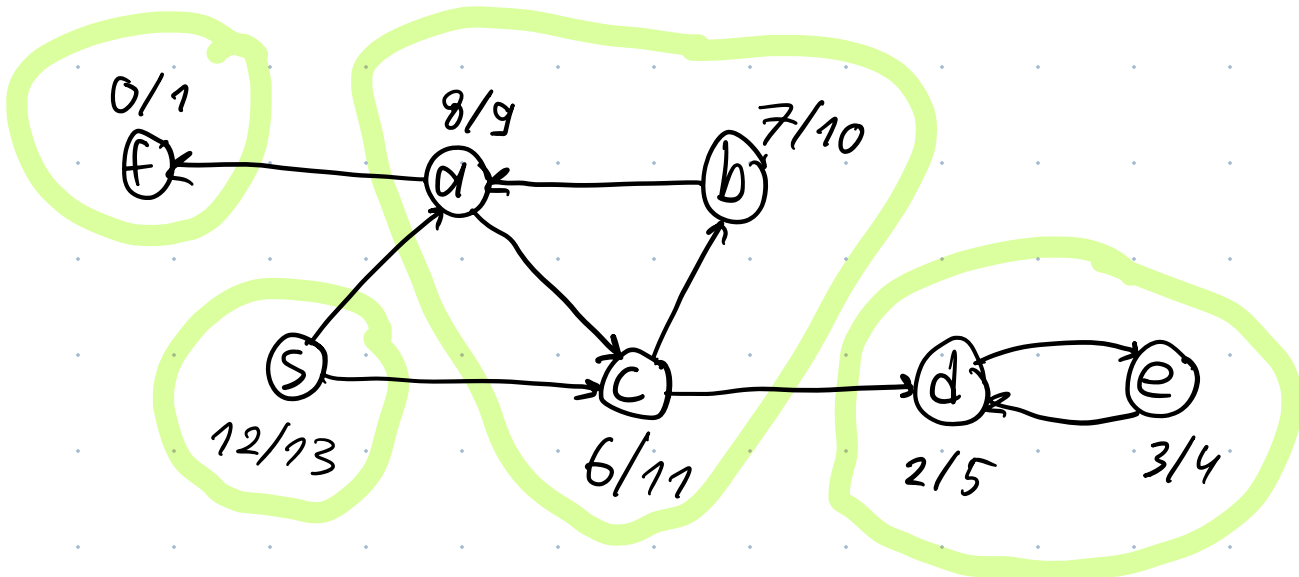
АЛГ. ПОИСКА КСС:

ВХОД: G

1. G^T
2. $\text{DFS}(G^T)$
3. $\text{DFS}(G)$, ВЕРШИНЫ В ПОРЯДКЕ УБЫВ. post_time в $\text{DFS}(G^T)$



3)



$F: \{f\}$

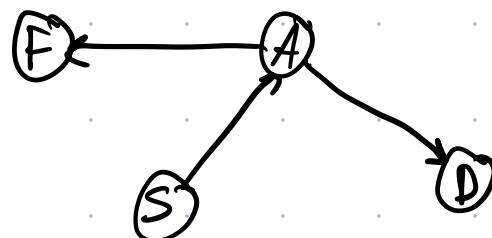
$A: \{a, b, c\}$

$S: \{s\}$

$D: \{d, e\}$

a b c d e f
A A A D D F

	A	F	S	D
A	0			
F		0		
S			0	
D				0



$f \rightsquigarrow []$

$a \rightsquigarrow [f, c]$

A [
 F [
 S [
 D [

i →

	A	D	F	S
A	0	1	1	-1
D	-1	0	0	0
F	-1	0	0	0
S	1	0	0	0

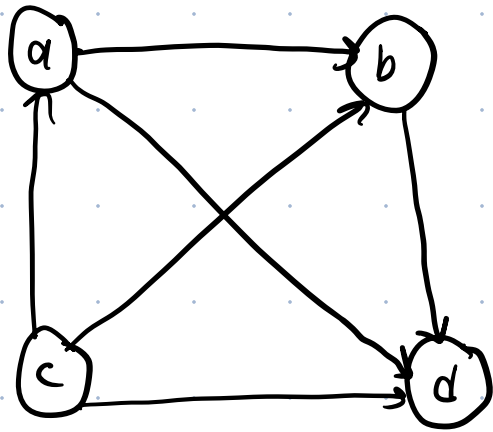
ТУ ДМИ Р, Т. Е. ПОЛНОСВ. ОРИЕНТ. ГРАФ

(

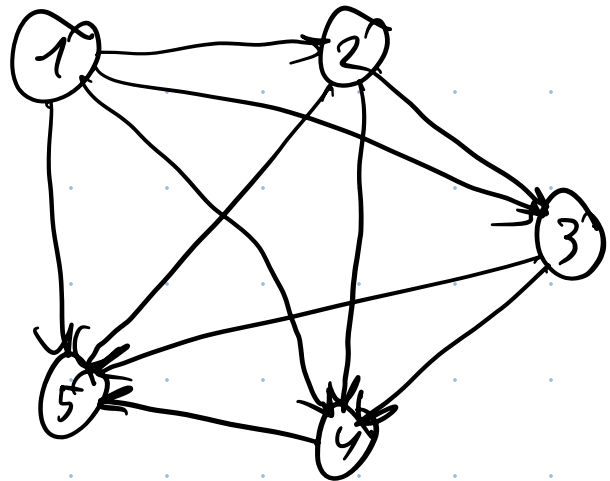
НА n ВЕРШИНАХ

$O(n^2)$

	α	b	c	d
α	0	1	-1	1
b	-1	0	-1	1
c	1	1	0	1
d	-1	-1	-1	0


$$i \rightarrow j \quad A[i, j] = 1$$

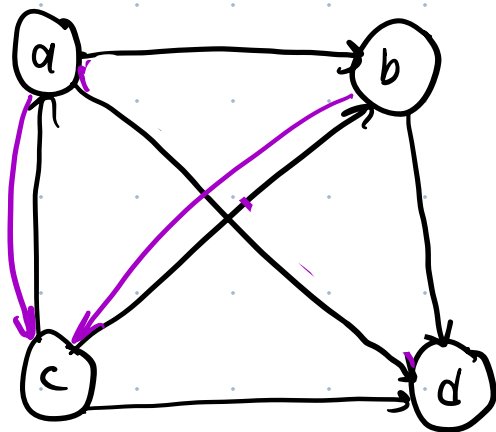
flags = [, , , ..., ...]

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & \dots & \dots & 1 \\ -1 & 0 & 1 & 1 & \dots & \dots & \dots & 1 \\ -1 & -1 & 0 & 1 & \dots & \dots & \dots & \\ -1 & -1 & -1 & 0 & 1 & \dots & \dots & \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & 0 & 0 \end{pmatrix}$$


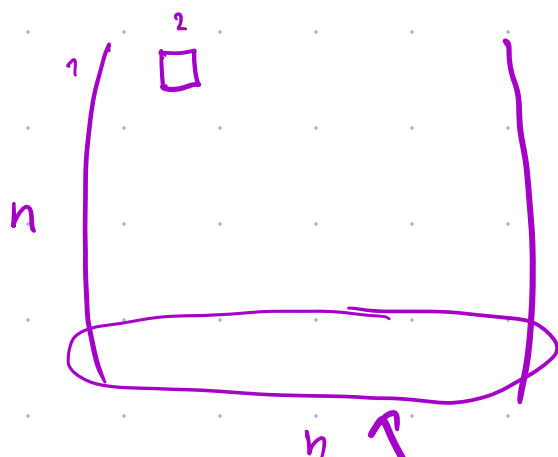
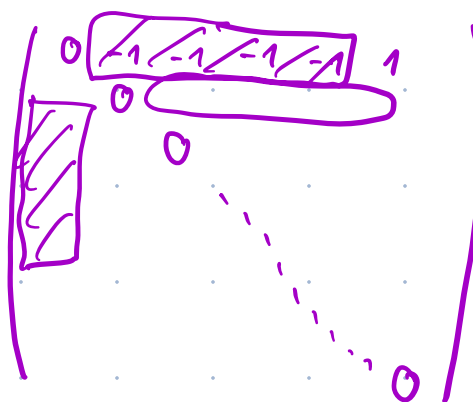
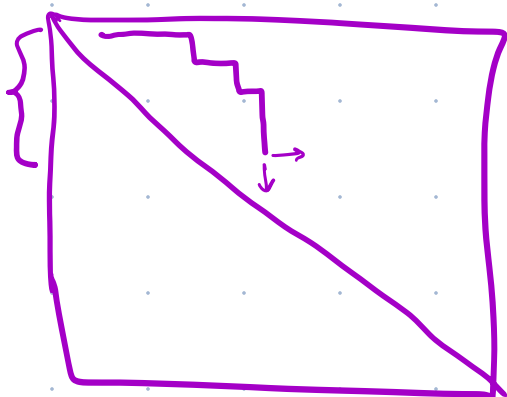
ВХОД: Матр. А

A handwritten diagram of a matrix, likely representing a system of linear equations. The matrix is enclosed in large parentheses. A dashed diagonal line runs from the top-left to the bottom-right. The top-left element is a boxed '2', with an arrow pointing to a boxed '1'. To the right of this, the text '1 1 -1' is written in red. Below the first row, the first column contains three '-1' entries in red. To the right of the matrix, a red arrow points downwards to a '1?' in red.

	a	b	c	d
a	0	1 ₁	-1	1
b	-1	0	-1 ₂	1 ₃
c	1	1	0	1
d	-1	-1	-1	0



ИЩЕМ ОБЩ. СТОК



1? 2?

curr-candidate

$O(1)$

$O(n)$ — УБР. ВСЕХ КАНД., кр.м.б 1

$O(n)$ — ДОПРО ВЕР. КАНДА

$O(n)$ СУММАРН

	a	b	c	d
a	0	1	-1	1
b	-1	0	-1	1
c	1	1	0	1
d	-1	-1	-1	0

I. 1) ~~a~~ \rightarrow b ($A[0,1] = 1$)
curr-cand = b

2) curr-cand vs c
b

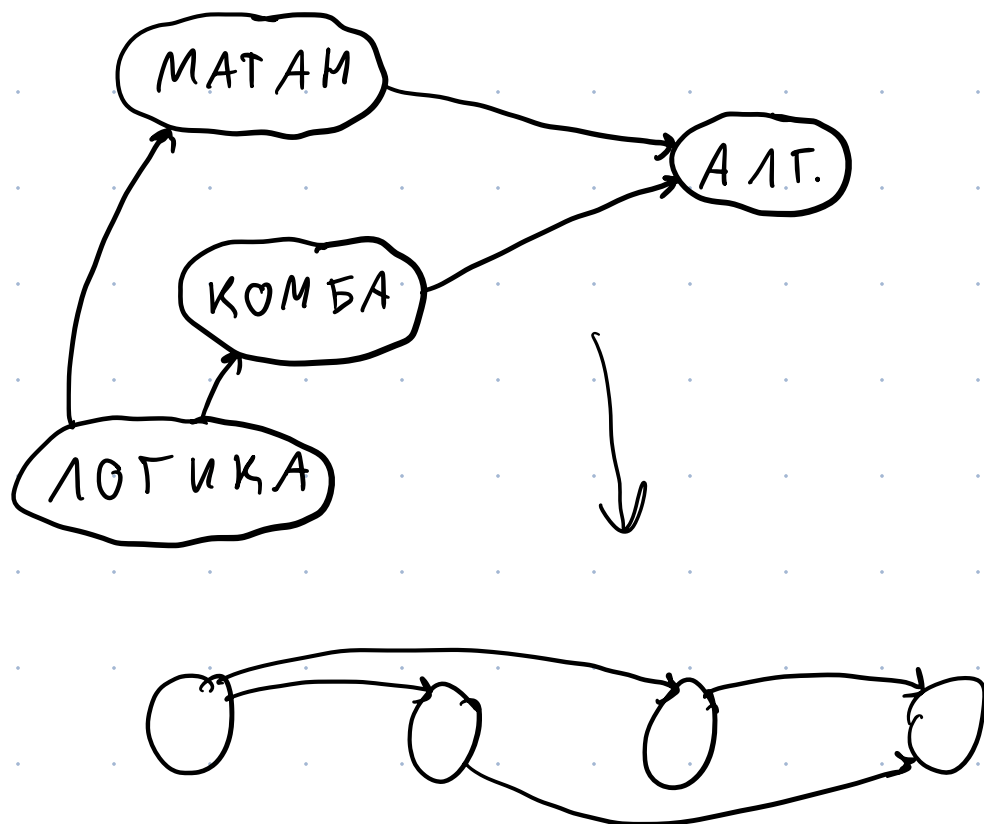
~~a~~ \rightarrow b
curr-cand = b

3) b vs d

~~a~~ \rightarrow d
curr-cand = d

II. ? ✓

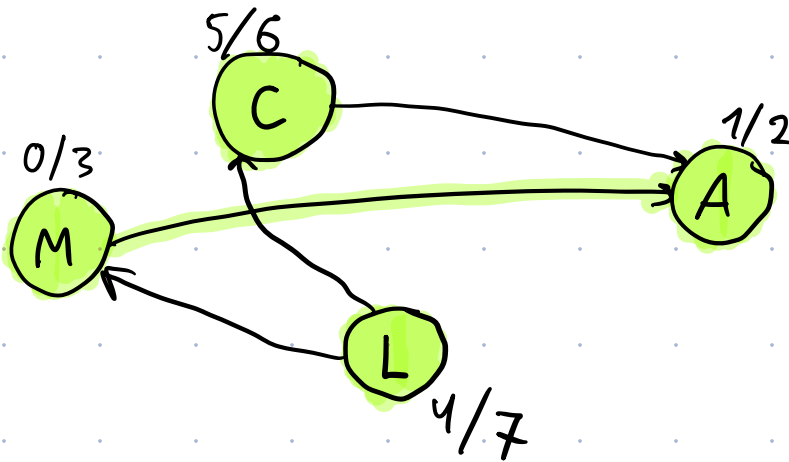
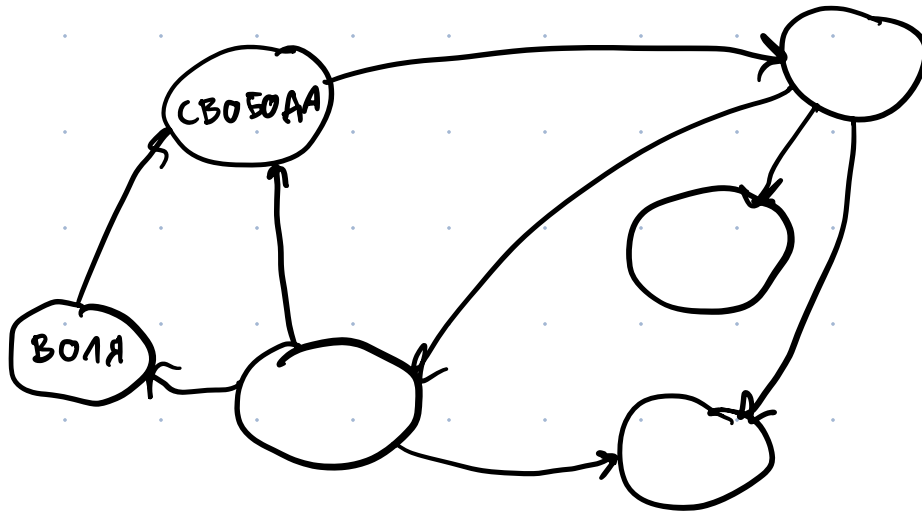
ТОПОЛОГИЧЕСКАЯ СОРТИРОВКА



$$J = \sum_{i=0}^N \gamma^i C_i$$

$$\gamma < 1$$

$$\gamma = 1$$



top-sort(u)

$u.color = gray$

for all e_{uv} in E :

if ($v.color == black$):

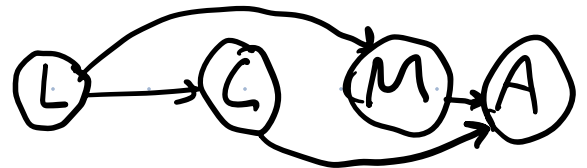
pass

if ($v.color == gray$):

print("hehe net")

if ($v.color == white$):

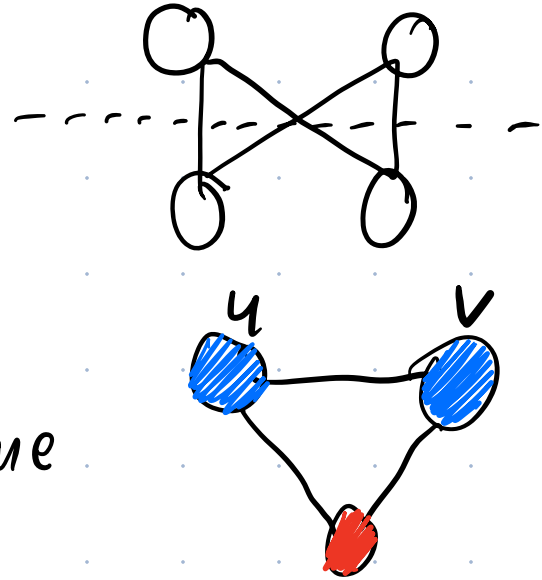
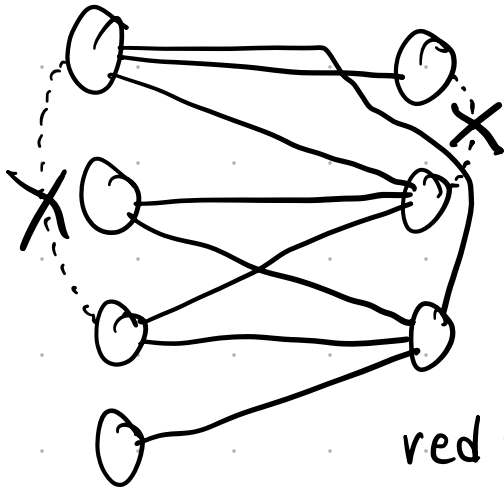
top-sort(v)



top-sort $\neq (v)$

$u.color = black$

ПРОВЕРКА ГРАФА НА ДВУДОЛЬНОСТЬ



bipartite_check($u, color$)

$u.color == color$

for $e_{uv} \in E$:

if ($v.color == color$):

print("not bipartite")

exit()

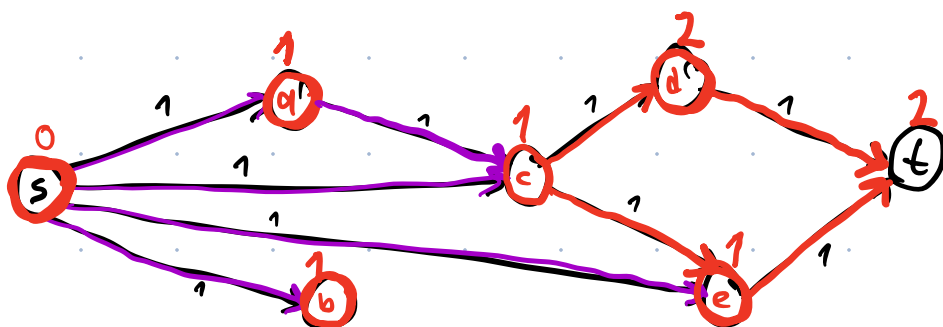
if ($v.color == not\ color$):

pass

if ($v.color == transparent$):

bipartite_check($v, not\ color$)

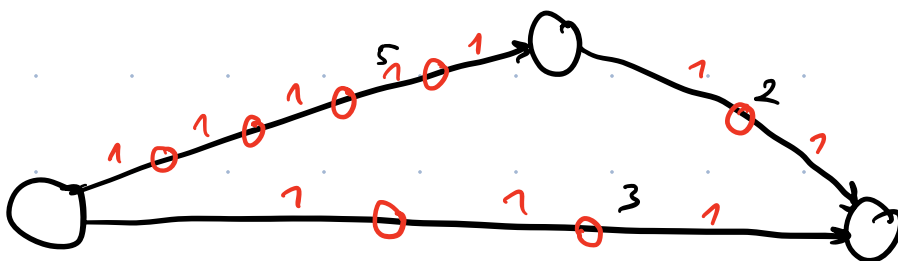
КРАТЧАЙШИЕ ПУТИ ИЗ s В t



BFS

СЛОЖНОСТЬ: $\Theta(|V| + |E|)$

А ЕСЛИ ВЕСА НАТУРАЛЬНЫЕ?



$$e_i \leq W$$

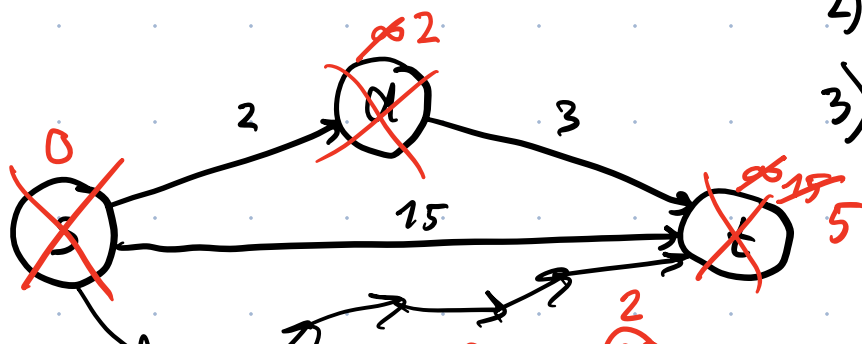
$$O(|E| + |V| + |E|W \cdot 2)$$

АЛГОРИТМ ДЕЙКСТРА :

1) $d[v] = +\infty$

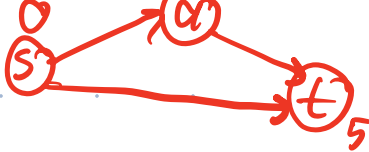
2) $d[s] = 0$

3) ^{СОЗДАЁМ} heap на d



for i in range($|V|$):

curr_vertex =
heap.extract_min()



for $e_{curr_ver, v}$ in E :
 $relax(e_{curr_ver, v})$

def $relax(e_{uv})$

// $w_{uv}, d[u], d[v]$

$d[v] = \min(d[v], d[u] + w_{uv})$



$O((|V| + |E|) \log |V|)$