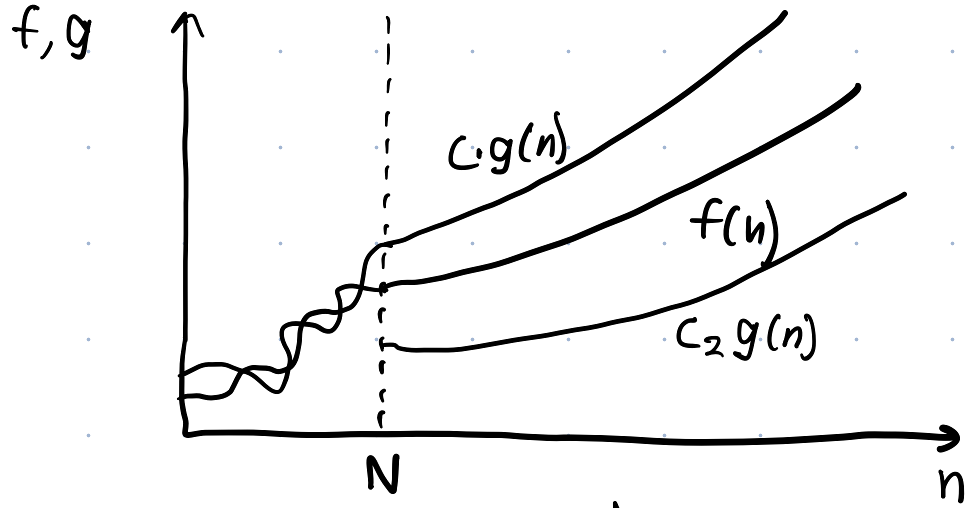
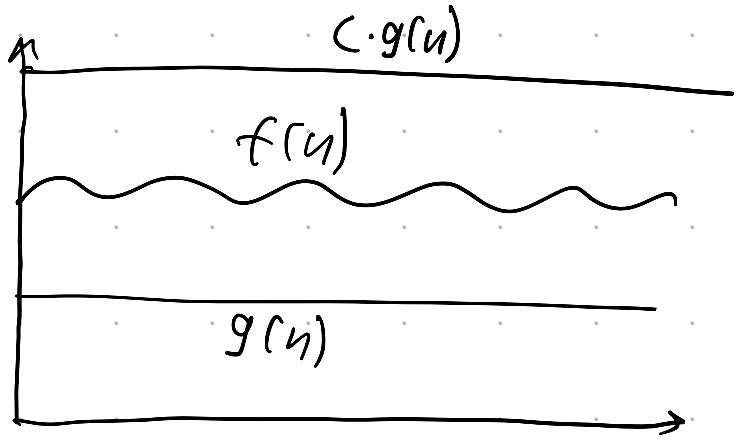
$f : \mathbb{N} \to \mathbb{R}_+$

1) $f(n) = O(g(n))$: "О большое"

$\exists C > 0, \exists N \in \mathbb{N} : \forall n \geq N \hookrightarrow f(n) \leq C g(n)$





2) $f(n) = \Omega(g(n))$:

$\exists C > 0, \exists N \in \mathbb{N} : \forall n > N \hookrightarrow f(n) \geq C \cdot g(n)$

3) $f(n) = \Theta(g(n))$:

$\exists C_1, C_2 > 0, \exists N \in \mathbb{N} : \forall n > N \hookrightarrow C_1 g(n) \leq f(n) \leq C_2 g(n)$

$n \stackrel{?}{=} O(n)$ ✓

$n \stackrel{?}{=} O(n^2)$ ✓

$10n \stackrel{?}{=} O(n)$ ✓

$\varepsilon > 0$

$n^\varepsilon = O(\log n)$

отриц. O:

$\forall C>0 \;\; \forall N \in \mathbb{N} \;\; \exists n \geq N : f(n) > C g(n)$

.... $n^\varepsilon > C \log n$

$\frac{n^\varepsilon}{\log n} > C$

$$\lim_{n\to\infty} \frac{n^\varepsilon}{\log n} = \lim_{n\to\infty} \varepsilon \cdot n^{\varepsilon-1} \cdot \frac{1}{\cancel{\frac{1}{n}}} = \lim_{n\to\infty} \varepsilon \cdot n^\varepsilon = +\infty$$

$f(n)$ – ????

$f(n) = O(n^k)$

$f(n) = \Omega(n^m)$

1) $k, m \in \mathbb{N}$

$f(n) = n^{\frac{3}{2}}$

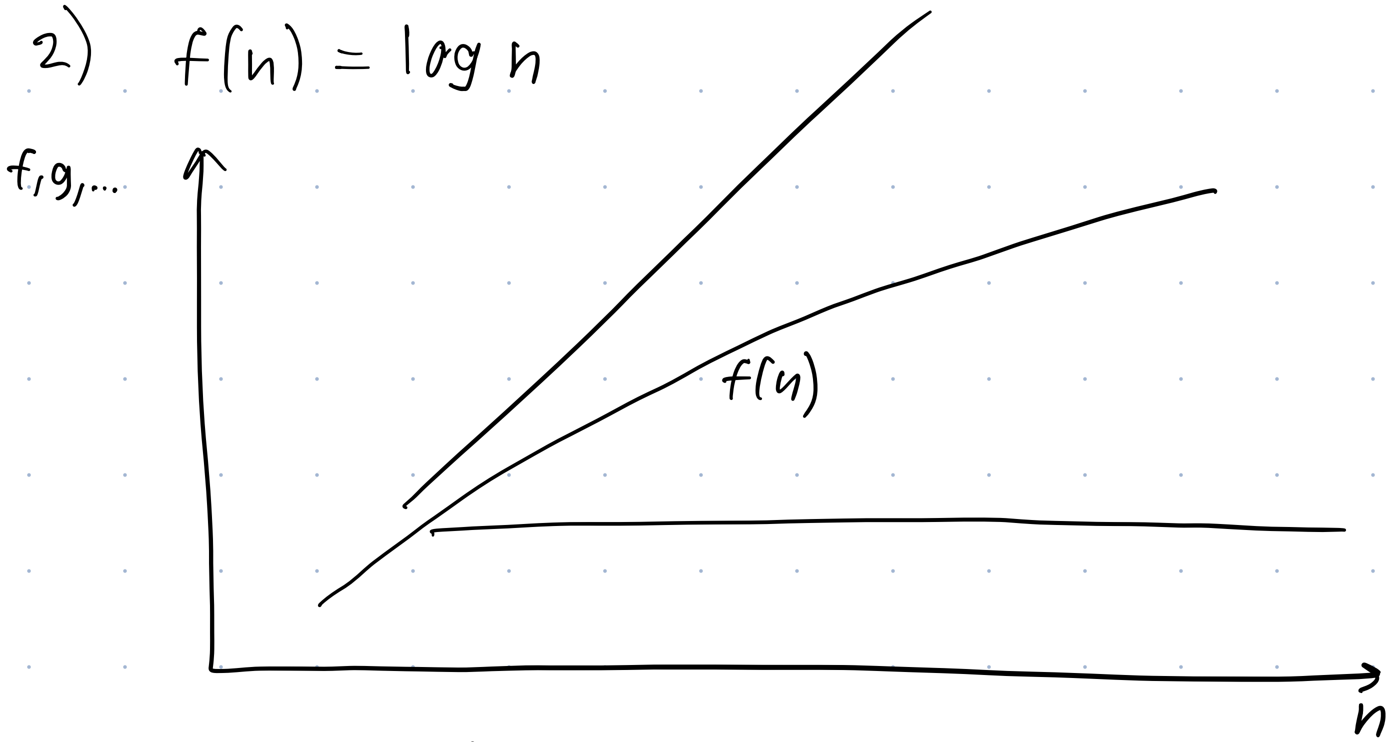
// НАИЛУЧШИЕ ОЦЕНКИ

n^k, $k \in \mathbb{N}$

$g = n^k$ – наил. оц. (для $f(n)$) при $k \in \mathbb{N}$:

$$\begin{cases} f(n) = O(n^k) \\ f(n) \neq O(n^{k-1}) \end{cases}$$

2) $f(n) = \log n$



$f(n) = \Omega(1)$

$f(n) \neq \Omega(n)$ "$\mathbb{N}$"

$f(n) \neq \Omega(n^{\varepsilon})$ $\forall \varepsilon > 0$ "$\mathbb{R}$"

$f(n) \neq O(1)$

$f(n) = O(n)$

$f(n) = O(n^{\varepsilon})$ $\forall \varepsilon > 0$

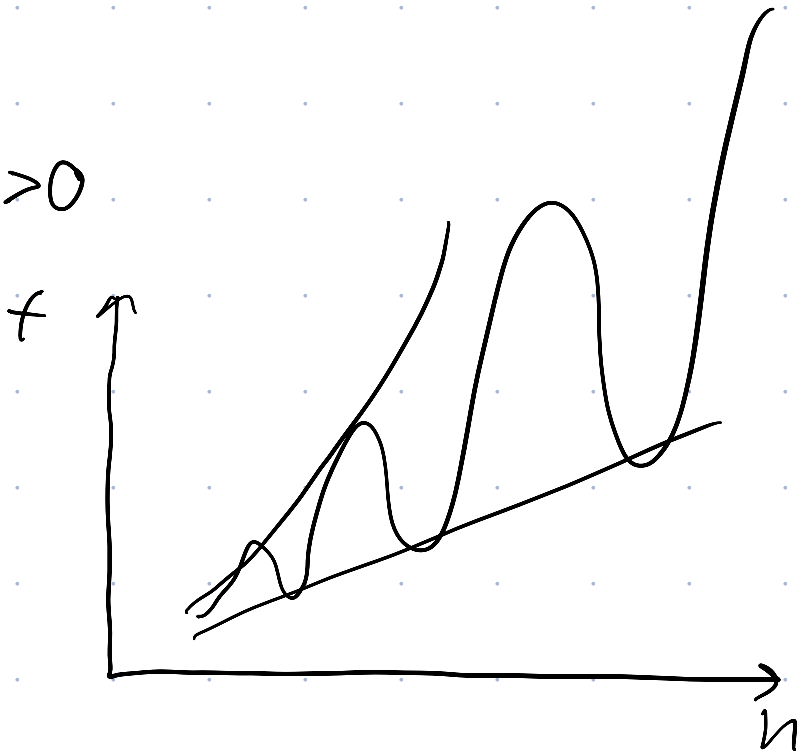
3)

$f(n) = n^{2+\sin n}$

$f(n) = \Omega(n)$

$= O(n^3)$

$|\sin n| \leq 1$



$f(n) = O(g(n))$

$\in$

$$\left\{ n, c\cdot n, \log n, n^{\gamma}, \ldots \right\} \in O(n) \quad \gamma \le 1$$

$$f(n) = \sum_{i=1}^{n} \frac{1}{i(i+1)}$$

ОЦЕНИТЬ $f(n)$

$f(n) = O(g(n))$

$\Omega(h(n))$

$n^r \cdot \log^p n$	2^n

$K \leftarrow$ ВХОД

$\underbrace{\boxed{\;|\;|\;|\;|\;|\;}}_{K \text{ БИТ}}$

$0\,0 \ldots\ldots 0$

$1\,1 \ldots 1$

2^K

$n = \lceil \log K \rceil$

2^{2^n}

$\overset{m}{\longleftrightarrow}$

$\boxed{a_1 \mid \ldots \mid \ldots}$

$000, \ldots, 111$

2^m

$K \qquad \lceil \log_2 K \rceil$

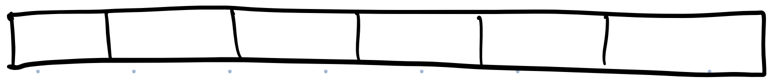
ВХОД: K; ДЛИНА ВХ. $= \lceil \log_2 K \rceil = n$

АЛГ.: $\boxed{\;|\;|\;\ldots\ldots\;|\;}$

$\overset{\longleftrightarrow}{K \text{ БИТ}}$

$O(2^K)$

$\underbrace{00\ldots0, \ldots, 1\ldots1}_{2^K \text{ ШТУК}}$ $\quad O(2^{2^n})$



32 БИТА НА ЧИСЛО

К числа

ДЛИНА ВХ. $= \underbrace{32}K = n$

$$n + \frac{n}{2} + \frac{n}{4} + \dots \le 2n$$

$$f(n) = \sum_{i=1}^{n} \frac{1}{i(i+1)} = 1 - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \frac{1}{3} + \dots + \frac{1}{n} - \frac{1}{n+1} = 1 - \frac{1}{n+1}$$

$$\frac{1}{i(i+1)} = \frac{1}{i} - \frac{1}{i+1}$$

$O(1)$ Θ

$\Omega(1)$

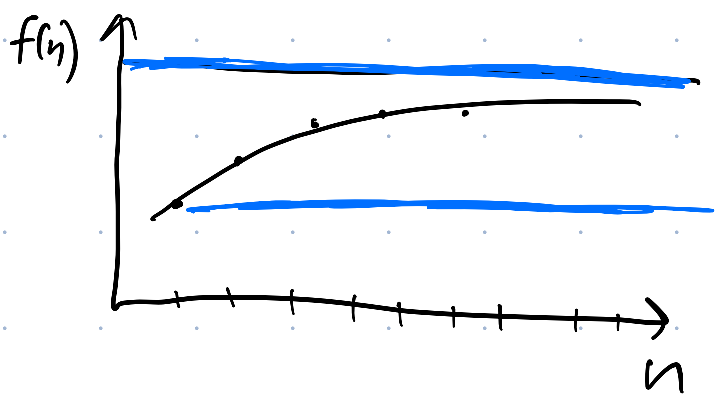
$f(n) \ge \frac{1}{2}$

$g(n) = 1$

$f(n) \ge \frac{1}{2} \cdot 1$

$f(n) = 1 - \frac{1}{n+1} < 1$

$\ge \frac{1}{2}$

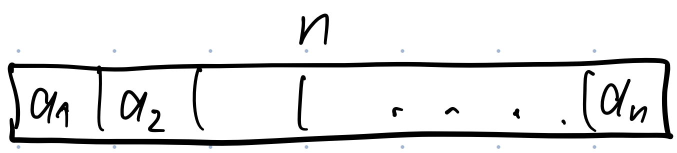


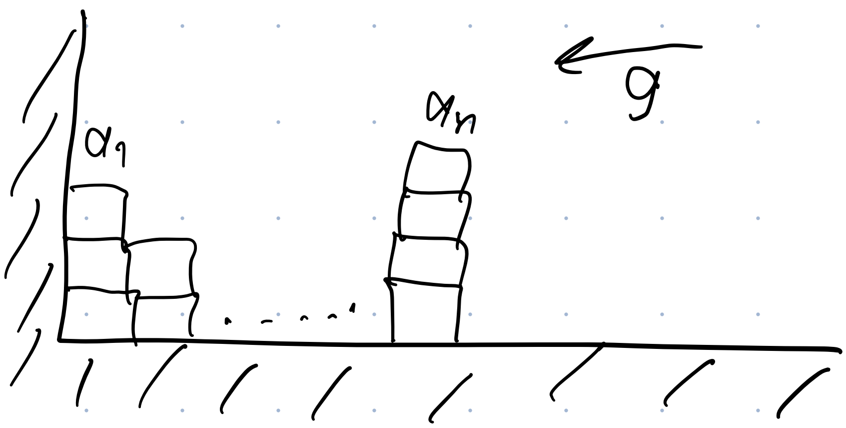
c_1, c_2

$$c_1 \cdot 1 \le f(n) \le c_2 \cdot 1$$

$c_1 = \frac{1}{2}$, $c_2 = 1$

$$f(n) = \sum_{i=1}^{n} \frac{1}{i(i+1)} \leq \sum_{i=1}^{n} \frac{1}{i^2} \leq \sum_{i=1}^{\infty} \frac{1}{i^2} = \frac{\pi^2}{6}$$



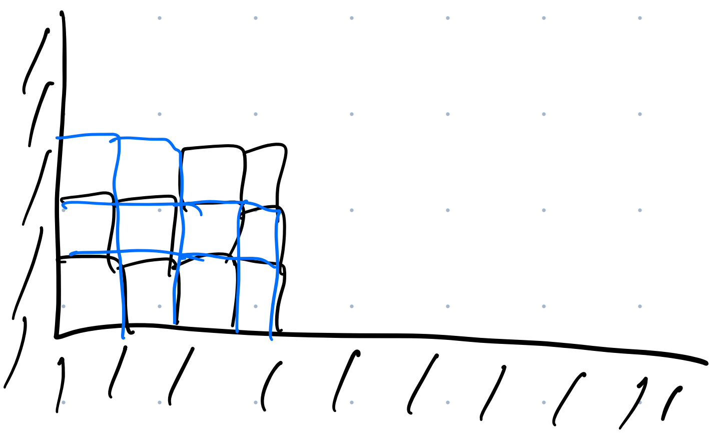


$$s = \frac{g t^2}{2} = 1 \cdot n$$

$$t^2 = \frac{2 \cdot 1}{g} n$$

$$t = C \cdot \sqrt{n}$$

$$t = O(\sqrt{n})$$



0	0	0	0	0	1
1	0	0	0	0	1
1	0	1	1	1	1
1	1	1	1	1	1