1. Spectrum

Y-axis$= E^2\frac{\mathrm{d}J}{\mathrm{d}E}$, means per **area** per **time** per **solid angle** per **energy**.

2. Cross Section

One paritcle interaction,

- $\sigma_{AB}n_BL \ll 1$: very likely to pass through(**optically thin**)
- $\sigma_{AB}n_BL \gg 1$: very likely to interact(**optically thick**)

Probability:

$$P = 1 - e^{-n\sigma L} \tag{1}$$

where $\tau = n\sigma L$ is called **Optical Depth**.
Unit: Barn $1barn = 10^{-28}m^2$.

3. Diffusion Model

Diffusion-loss equation,

$$\frac{\partial n}{\partial t} = \nabla \cdot \left(D\vec{\nabla}n\right) - \frac{\partial}{\partial E}(n\dot{E}) + Q \tag{2}$$

Diffusion-Convection equation,

$$\frac{\partial}{\partial t}n = \nabla \cdot \left(D\vec{\nabla}n - \vec{V}n\right) - \frac{\partial}{\partial E}(n\dot{E}) + Q \tag{3}$$

with momentum loss term $\dot{p} = -\frac{1}{3}(\nabla \cdot V)p$.
Rigity: $R = \frac{p}{q}$. Motivation: Lamor Radius is propotional to the rigity $r_g = \frac{p_\perp}{q}$.
Number of particles per phase space: $f = \frac{\mathrm{d}N}{\mathrm{d}^3p\mathrm{d}^3x}$.
Differential number density of particles, $n = \frac{\mathrm{d}N}{\mathrm{d}p\mathrm{d}^3x} = 4\pi p^2 f$.
From diffusion-loss equations, we can imply

$$D\frac{\partial f}{\partial r} + \frac{V_p}{3}\frac{\partial f}{\partial p} = 0 \implies \mathrm{d}f\,(r,p) = 0 \implies f(r_1,p_1) = f(r_1,p_1)$$

with definition of flux $I = vn/(4\pi) = vp^2f$, we have relation

$$\frac{I(p)}{vp^2} = \frac{I(p_{ILS})}{v_{LIS}p_{LIS}^2} \tag{4}$$

combining with solar modulation potential ϕ, we have

$$\frac{I(p)}{vp^2} = \frac{I(p+\phi)}{v_{lis}(p+\phi)^2} \tag{5}$$

4. CR Secondaries

The full propagation euqation,

$$\begin{aligned}\frac{\partial \psi(r,p,t)}{\partial t} &= q(r,p,t) + \overset{\text{Diffusion Convection}}{\nabla \cdot (D_{xx}\nabla\psi - V\psi)} \\ &+ \underset{\text{Re-acceleration}}{\frac{\partial}{\partial p}p^2 D_{pp}\frac{\partial}{\partial p}\frac{1}{p^2}\psi} - \underset{\text{Continuous Energy Loss}}{\frac{\partial}{\partial p}\left[\dot{p}\psi - \frac{p}{3}(\nabla \cdot V)\psi\right]} \\ &- \underset{\text{Fragmentation\& Radi. decay Loss}}{\frac{1}{\tau_f}\psi - \frac{1}{\tau_r}\psi}\end{aligned} \tag{6}$$

For Fragmentation Loss, for the i-th species, it's loss by $\rightarrow$ j-th species

$$\frac{\partial n_i}{\partial t} = -n_i(\frac{\rho}{m})_{ism}\sigma_{i\rightarrow j}v \tag{7}$$

For radioactive decay loss,

$$\frac{\partial n_i}{\partial t} = -n_i\frac{1}{\tau_i}, \tau_i \text{ is the lifetime} \tag{8}$$

For simple case: Leaky box approx, one species dominates the production, $Q = 0$. We have relation,

$$\frac{n_i}{T_e} = -\frac{n_i}{T_f} - \frac{n_i}{T_{dec}} + C_i \tag{9}$$

where C_i is the production of "i" due to other species, then we can get expression of n_i,

$$n_i = \frac{C_i}{1/T_e + 1/T_f + 1/T_{dec}} \tag{10}$$