

### 1. Spectrum

Y-axis=  $E^2 \frac{dI}{dE}$ , means per **area** per **time** per **solid angle** per **energy**.

### 2. Solid Angles

- An “isotropic” distribution

$$\frac{dN}{d\Omega} = \text{constant}$$

- 

$$\frac{dN}{\sin \theta \, d\theta \, d\phi} = 1, \quad [\theta = 0, \pi], \quad \phi = [0, 2\pi]$$

- 

$$\frac{dN}{d\theta} = \sin \theta \int d\phi \frac{dN}{\sin \theta \, d\theta \, d\phi} = 2\pi \sin \theta \quad \text{Not a constant!}$$

- 

$$\frac{dN}{dx} = \frac{dN}{d \cos \theta} = \int d\phi \frac{dN}{\sin \theta \, d\theta \, d\phi} = 2\pi, \quad \cos \theta = [-1, 1]$$

### 3. No directionality

- It is good to express in typical values
- Lamor Radius (Gyro-radius)

$$r_g = \frac{p_{\perp}}{|q|B}$$

- 

$$r_g \simeq 3.3 \, \text{m} \left( \frac{E}{\text{GeV}} \right) \left( \frac{e}{|q|} \right) \left( \frac{1 \, \text{T}}{B} \right)$$

- Size of the Galaxy  $\sim 10 \, \text{kpc}$
- $\text{pc} \simeq 3 \times 10^{16} \, \text{m}$  or about 3 lyr

### 4. CR Composition

- Right plot shows elemental abundance ratio (normalized at Carbon).
- Cosmic ray (CR) composition differs from Solar System abundance.

#### Reasons for the difference:

- Different origins:** CRs come mainly from supernova remnants and stellar winds, not the same material as the Solar System.
- Acceleration bias:** Elements with low first ionization potential (e.g. Mg, Si, Fe) or large charge  $|Z|$  are more efficiently accelerated  $\Rightarrow$  overrepresented in CRs.
- Propagation effect:** CRs interact with interstellar matter and produce secondary nuclei (Li, Be, B), leading to their high abundance.
- Conclusion:** CR composition reflects *acceleration and propagation physics*, not direct stellar nucleosynthesis.

### 5. Cross Section

One particle interaction,

- $\sigma_{AB} n_B L \ll 1$ : very likely to pass through(**optically thin**)
- $\sigma_{AB} n_B L \gg 1$ : very likely to interact(**optically thick**)

Probability:

$$P = 1 - e^{-n\sigma L} \quad (1)$$

where  $\tau = n\sigma L$  is called **Optical Depth**.

Unit: Barn  $1 \text{ barn} = 10^{-28} \text{ m}^2$ .

### 6. Diffusion Model

Diffusion-loss equation,

$$\frac{\partial n}{\partial t} = \nabla \cdot (D \vec{\nabla} n) - \frac{\partial}{\partial E} (n \dot{E}) + Q \quad (2)$$

Diffusion-Convection equation,

$$\frac{\partial}{\partial t} n = \nabla \cdot (D \vec{\nabla} n - \vec{V} n) - \frac{\partial}{\partial E} (n \dot{E}) + Q \quad (3)$$

with momentum loss term  $\dot{p} = -\frac{1}{3}(\nabla \cdot V)p$ .

Rigity:  $R = \frac{p}{q}$ . Motivation: Lamor Radius is proportional to the rigity  $r_g = \frac{v_{\perp}}{q}$ .

Number of particles per phase space:  $f = \frac{dN}{d^3p d^3x}$ .

Differential number density of particles,  $n = \frac{dN}{dp d^3x} = 4\pi p^2 f$ .

From diffusion-loss equations, we can imply

$$D \frac{\partial f}{\partial r} + \frac{V_p \partial f}{3 \partial p} = 0 \Rightarrow df(r, p) = 0 \Rightarrow f(r_1, p_1) = f(r_1, p_1)$$

with definition of flux  $I = vn/(4\pi) = vp^2 f$ , we have relation

$$\frac{I(p)}{vp^2} = \frac{I(p_{ILS})}{v_{LIS} p_{LIS}^2} \quad (4)$$

combining with solar modulation potential  $\phi$ , we have

$$\frac{I(p)}{vp^2} = \frac{I(p + \phi)}{v_{lis}(p + \phi)^2} \quad (5)$$

### 7. CR Secondaries

The full propagation equation,

$$\begin{aligned} \frac{\partial \psi(r, p, t)}{\partial t} = & \underbrace{q(r, p, t)}_{\text{Re-acceleration}} + \underbrace{\nabla \cdot (D_{xx} \nabla \psi - V \psi)}_{\text{Diffusion Convection}} \\ & + \underbrace{\frac{\partial}{\partial p} p^2 D_{pp} \frac{\partial}{\partial p} \frac{1}{p^2} \psi}_{\text{Re-acceleration}} - \underbrace{\frac{\partial}{\partial p} \left[ \dot{p} \psi - \frac{p}{3} (\nabla \cdot V) \psi \right]}_{\text{Continuous Energy Loss}} \\ & - \underbrace{\frac{1}{\tau_f} \psi - \frac{1}{\tau_r} \psi}_{\text{Fragmentation \& Radi. decay Loss}} \end{aligned} \quad (6)$$

For Fragmentation Loss, for the i-th species, it's loss by  $\rightarrow$  j-th species

$$\frac{\partial n_i}{\partial t} = -n_i \left( \frac{\rho}{m} \right)_{ism} \sigma_{i \rightarrow j} v \quad (7)$$

For radioactive decay loss,

$$\frac{\partial n_i}{\partial t} = -n_i \frac{1}{\tau_i}, \tau_i \text{ is the lifetime} \quad (8)$$

For simple case: Leaky box approx, one species dominates the production,  $Q = 0$ . We have relation,

$$\frac{n_i}{T_e} = -\frac{n_i}{T_f} - \frac{n_i}{T_{dec}} + C_i \quad (9)$$

where  $C_i$  is the production of "i" due to other species, then we can get expression of  $n_i$ ,

$$n_i = \frac{C_i}{1/T_e + 1/T_f + 1/T_{dec}} \quad (10)$$

### 8. Collision

We can use Lorentz Invariant  $s$ ,

$$s = (p^\mu + p^\nu)^2 \quad (11)$$

where  $p^\mu = (\frac{E}{c}, p^1, p^2, p^3)$ .

And definition of Differential cross section,

$$\frac{d\sigma_{i \rightarrow j}(T_i, E_j)}{dT_i} \quad (12)$$

Total cross section:

$$\frac{dP}{dt} = n\sigma \quad (13)$$

and differential cross section,

$$\frac{dP}{dt dT} = n \frac{d\sigma}{dT} \quad (14)$$

thus, using differential cross section, the number of  $\bar{p}$  in interaction  $p + p \rightarrow \bar{p} + X$  can be expressed by

$$n_{\bar{p}}(T_{\bar{p}}) = \left( \int_{E_{th}} n_p \frac{d\sigma_{p\bar{p} \rightarrow \bar{p}X}}{dT_{\bar{p}}}(E_p, T_p) dE_p - n_{\bar{p}} \sigma_{\bar{p} \rightarrow X} \right) \frac{X}{m} \quad (15)$$

### 9. Electron-Matter Interaction

A particle interacts with stuff lower energy than itself causes energy loss through following mechanisms. In matter:

- Ionization: Kick off electrons from atoms
- Bremsstrahlung: curved trajectory emits photon

In space:

- Inverse-Compton scattering
- Synchrotron radiation

With electrons/positrons:

- Moller/Bhabba scattering:  $e^- + e^+ + \text{electron scattering}$
- Positron annihilation:  $e^- + e^+ \rightarrow \gamma + \gamma$

### 10. Inverse Compton Scattering

We assume it as the **Thomson**(elastic) scattering, because photon energy in CR frame  $E_{ph} \sim kT \sim 10^{-4} \text{ eV}$ .

In electron rest frame,

$$\begin{pmatrix} E'_{ph} \\ E'_{ph} \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} E_{ph} \\ E_{ph} \end{pmatrix} \quad (16)$$

$$\Rightarrow E'_{ph} = \gamma(1 + \beta)E_{ph} \simeq 2\gamma E_{ph} \quad (17)$$

$$\Rightarrow E_{\gamma} \simeq 2\gamma E'_{\gamma} \simeq 4\gamma^2 E_{ph} \quad (18)$$

Due to the same Lorentz transformation, we know the energy loss of Inverse Compton, is

$$-\frac{dE_e}{dt} = \frac{dE_{\gamma}}{dt} = \frac{dE'_{\gamma}}{dt} \quad (19)$$

Give the expression of power

$$\frac{dE'_{\gamma}}{dt} = \int E'_{\gamma} \frac{c d\sigma}{dE'_{\gamma}} dE'_{\gamma} dn'_{\gamma} \quad (20)$$

working in Thomson limit, the differential cross section is,

$$\frac{d\sigma}{dE'_{\gamma}} = \sigma_t (E'_{ph} - E'_{\gamma}) \Rightarrow \frac{dE'_{\gamma}}{dt'} = c\sigma_t U'_{ph} \quad (21)$$

where  $U'_{ph}$  is the photon energy density(in ERS).

$$\frac{dE}{dt} = \frac{dE'_{\gamma}}{dt} \quad (22)$$

### 11. Positrons

Positrons ( $e^+$ ) are the antimatter counterparts of electrons ( $e^-$ ). They are a component of cosmic rays, but their measured flux is significantly lower than that of protons ( $p$ ) and the combined electron/positron flux ( $e^- + e^+$ ) (Slide 2, 3). The key diagnostic tool for understanding their origin is the **positron fraction**:

$$\text{Positron Fraction} = \frac{\Phi_{e^+}}{\Phi_{e^+} + \Phi_{e^-}}$$

### 12. The Standard Model: Secondary Production

Unlike primary cosmic rays (like protons or electrons) accelerated in sources, positrons are believed to be **secondary particles**. They are produced by the interaction of primary cosmic rays with the interstellar medium (ISM).

#### 12.1 Hadronic Interaction Chain

The production mechanism is a multi-step decay process initiated by high-energy proton-proton collisions (Slide 4, 8, 9):

- Pion Production:** A high-energy proton (from cosmic rays) collides with a proton in the ISM (interstellar gas).

$$p + p \rightarrow \pi^{\pm} + X \quad (\text{where } \pi^0 \text{ also produced})$$

- Pion Decay:** The charged pions ( $\pi^+$  and  $\pi^-$ ) decay into muons ( $\mu^+$  and  $\mu^-$ ).

$$\pi^+ \rightarrow \mu^+ + \nu_{\mu}$$

$$\pi^- \rightarrow \mu^- + \bar{\nu}_{\mu}$$

- Muon Decay:** The muons then decay, producing positrons and electrons.

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_{\mu}$$

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_{\mu}$$

#### 12.2 Predicted Positron Fraction

This secondary production model can be calculated (Slide 10). Because the initial protons have a falling energy spectrum, the resulting secondary positrons also have a falling spectrum. This model predicts that the positron fraction should **decrease** with increasing energy (Slide 11, 12 - black line).

### 13. The Positron Anomaly

The central puzzle in this field is the "positron anomaly," which is a major discrepancy between the theoretical prediction and experimental observation.

#### 13.1 The PAMELA Discovery

**PAMELA** (Payload for Antimatter Matter Exploration and Light-nuclei Astrophysics) experiment published results showing that the positron fraction does not fall with energy. Instead, it **begins to rise** at energies above  $\sim 10 \, \text{GeV}$  (Slide 12). This was a significant anomaly.

#### 13.2 Confirmation by AMS and Fermi

This anomalous rising fraction was not an error. It was subsequently confirmed with higher precision by two other major experiments (Slide 14):

- Fermi-LAT (Large Area Telescope):** Confirmed the rise, even without a magnet, by cleverly using the Earth's magnetic field to separate  $e^+$  and  $e^-$  (Slide 15).
- AMS-02 (Alpha Magnetic Spectrometer):** Provided the most precise measurement to date, confirming the rise up to hundreds of GeV.

### 14. Interpretations and New Sources

The confirmed anomaly means there must be an **additional source** (or sources) of high-energy positrons that the secondary production model does not account for.

- Astrophysical Sources:** The leading candidates are nearby **Pulsar Wind Nebulae (PWNs)** (Slide 17). These are rapidly rotating neutron stars (pulsars) that create a nebula of high-energy electron-positron pairs. These pairs can escape and propagate to Earth, adding to the positron flux.

- Exotic Sources:** The anomaly also generated excitement for potential exotic sources, such as the annihilation or decay of **Dark Matter** particles, which could produce  $e^+e^-$  pairs.

The data from AMS-02 (shown in Slide 17) can be well-fitted by models that include both the standard secondary production (dashed line) and an additional component from pulsars (e.g., Monogem, Vela).



15. Gamma Ray

- **Detection Methods**
  - **Main Principle: Pair Production** ( $\gamma \rightarrow e^+ + e^-$ )
  - **Space Telescopes (e.g., Fermi-LAT)**
    - \* Orbit above atmosphere.
    - \* Use conversion foils to create  $e^+e^-$  pair.
    - \* Tracker reconstructs direction; Calorimeter measures energy.
  - **Ground Telescopes (e.g., LHAASO)**
    - \* Detect "air showers" (particle cascades) in the atmosphere.
    - \* Sample particles that reach the ground.
- **Production Mechanisms**
  - **Leptonic (from Electrons)**
    - \* **Bremsstrahlung:**  $e^-$  "brakes" in a field, emits  $\gamma$ .
    - \* **Inverse Compton:**  $e^-$  "boosts" a low-energy photon to a  $\gamma$ .
  - **Hadronic (from Protons): *The "smoking gun"***
    - \* Proton-proton collision creates a **Neutral Pion** ( $\pi^0$ ).
    - \* Pion instantly decays:  $\pi^0 \rightarrow \gamma + \gamma$ .
- **Key Discoveries**
  - **The "Pion Bump":**
    - \* Unique spectral signature of Hadronic (pion) decay.
    - \* **Fermi-LAT** saw this in Supernova Remnants (SNRs).
    - \* **Proof:** SNRs accelerate protons (cosmic rays).
  - **PeVatrons:**
    - \* Discovered by **LHAASO** at PeV (quadrillion eV) energies.
    - \* These are the Galactic "factories" accelerating particles to the highest energies.

16. UHE Cosmic Ray

- **UHECR Basics**
  - **Definition:** Cosmic rays with energy above the "Knee" ( $> 1$  PeV).
  - **Origin: Extragalactic.**
  - **Reason:** The Milky Way's magnetic field is too weak to contain them (based on Larmor Radius calculation).
- **The Four Big Mysteries**
  - **1. Source:** Unknown.
  - **2. Direction:** Mostly isotropic (uniform).
    - \* Pierre Auger Observatory found a "hotspot" (anisotropy), but no clear source.
  - **3. Composition:** Unknown (Protons? Iron?).
    - \* We measure it from air showers, which depends on interaction models.
    - \* **"Muon Puzzle":** Our models (e.g., QGSJET, SIBYLL) consistently predict fewer muons in the air shower than we actually observe. This implies our models or composition assumptions are wrong.
  - **4. Energy Spectrum Features:**
    - \* **"Ankle":** Point where the spectrum flattens, believed to be the transition from Galactic to Extragalactic cosmic rays.
    - \* **GZK Cutoff:** A sharp drop in particles observed above  $\sim 10^{19.5}$  eV.
- **The GZK Limit**
  - **Theory:** (Greisen-Zatsepin-Kuzmin) High-energy protons will interact with the Cosmic Microwave Background (CMB) photons ( $p + \gamma_{CMB}$ ).
  - **Effect:** This interaction causes the proton to lose energy, creating a "horizon." We can only see UHECR sources from relatively nearby ( $\sim 100$  Mpc).
  - **The Catch:** This cutoff energy only works for protons. If UHECRs are heavy nuclei (like Iron), the cutoff energy is different.
- **Multi-Messenger Solution**
  - **Cosmic Rays:** Are charged, so they are bent by magnetic fields and don't point to their source.
  - **Gamma Rays:** Are neutral, but they get absorbed by background light over long distances ( $\gamma + \gamma \rightarrow e^+e^-$ ).
  - **Neutrinos:** Are neutral and barely interact. They are the best tool to point directly back to the UHECR sources.

17. Acceleration

1. Fermi Acceleration (General Idea)

- After each encounter, particle gains energy:

$$E = \beta^k E_0, \quad N(> E) = N_0 P^k$$

- Eliminate  $k$ :

$$N(> E) \propto E^{\frac{\ln P}{\ln \beta}}, \quad \frac{dN}{dE} \propto E^{-1 + \frac{\ln P}{\ln \beta}}$$

- For first-order Fermi (shock acceleration):

$$\frac{dN}{dE} \propto E^{-2}$$

2. Shock Basics

- Upstream:  $(\rho_1, v_1, P_1)$ , Downstream:  $(\rho_2, v_2, P_2)$
- Conservation:

$$\rho_1 v_1 = \rho_2 v_2, \quad \rho_1 v_1^2 = \rho_2 v_2^2 + P_2, \quad \frac{1}{2} \rho_1 v_1^3 = \frac{1}{2} \rho_2 v_2^3 + \frac{3}{2} P_2 v_2$$

- For strong shocks ( $P_1 \simeq 0$ ):

$$\rho_2 = 4\rho_1, \quad v_1 = 4v_2$$

3. Particle Acceleration at Shocks

- Relative velocity between up/down stream:

$$V = \frac{3}{4}U$$

- Energy gain per crossing:

$$\left\langle \frac{\Delta E}{E} \right\rangle = \frac{2}{3}V, \quad \text{Round trip: } \left\langle \frac{\Delta E}{E} \right\rangle = \frac{4}{3}V$$

- Energy gain factor:  $\beta = 1 + \frac{4}{3}V$ , Escape probability:  $P = 1 - U$
- For small  $U$ :

$$\frac{\ln P}{\ln \beta} \simeq -1 \Rightarrow \frac{dN}{dE} \propto E^{-2}$$

4. Observed Cosmic Ray Spectrum

- Intrinsic (source) index:  $\gamma \approx 2.0 - 2.2$
- After propagation losses:  $\gamma_{\text{obs}} \approx 2.7 - 3.3$

5. Maximum Energy (Hillas Criterion)

- From Faraday's law:  $\nabla \times \vec{\mathcal{E}} = -\partial_t \vec{B}$
- Dimensional estimate:

$$\mathcal{E} \sim BU, \quad E_{\text{max}} = ZeBUL$$

- Example: young SNR  $B \sim 1 \mu\text{G}$ ,  $U \sim 10^4 \text{ km/s}$ ,  $L \sim 1 \text{ pc} \Rightarrow E_{\text{max}} \sim 10^{16} \text{ eV}$
- Hillas plot:  $E_{\text{max}} \approx ZBLU$  distinguishes feasible CR sources.

18. Neutrino

- **History & Discovery**
  - **Proposal (Pauli, 1930):** Solved the "missing energy" in beta decay ( $n \rightarrow p^+ + e^-$ ). The electron's energy was a continuous spectrum, not a fixed value, implying a third, unseen particle (the neutrino) was present.
  - **Discovery (Cowan & Reines, 1956):** Used a nuclear reactor (a powerful  $\bar{\nu}_e$  source) to detect neutrinos via **Inverse Beta Decay** ( $\bar{\nu}_e + p^+ \rightarrow n + e^+$ ).
- **Weak Interaction & Parity Violation**
  - Neutrinos only interact via the weak force.
  - **Wu Experiment (1956):** Observed beta decay from aligned Cobalt-60.
  - **Result:** Electrons were emitted asymmetrically (violating Parity/mirror symmetry).
  - **Conclusion:** The weak force is "left-handed"—it only interacts with **left-handed particles** and **right-handed anti-particles**.
- **The Solar Neutrino Problem**
  - **Experiment (Homestake):** Raymond Davis Jr. used 600 tons of cleaning fluid ( $\text{C}_2\text{Cl}_4$ ) to count solar neutrinos ( $\nu_e + {}^{37}\text{Cl} \rightarrow {}^{37}\text{Ar} + e^-$ ).
  - **Problem:** He only detected 1/3 of the neutrinos predicted by the Standard Solar Model.
- **The Solution: Neutrino Mass**
  - **Neutrino Oscillation:** Discovered by Super-Kamiokande (1998) and confirmed by SNO (2001-02). Neutrinos change "flavor" as they travel (e.g.,  $\nu_e \rightarrow \nu_\mu$ ).

- **Implication:** This oscillation is only possible if neutrinos have **mass**.
- **Significance:** This is physics **Beyond the Standard Model**, which originally assumed neutrinos were massless.

19. GR& Cosmos

• Principles & Observations

- **Cosmological Principle:** Universe is homogeneous and isotropic on large scales.
- **Hubble's Law:** Galaxies are moving away from us.

$$v = H_0 d$$

- **Redshift (z):** Caused by the expansion of space (stretching of light).

$$1 + z = \frac{\lambda_o}{\lambda_e} = \frac{1}{a(t)}$$

where  $a(t)$  is the scale factor (with  $a = 1$  today).

– CMB (Cosmic Microwave Background):

- \* Discovered by Penzias & Wilson (1960).
- \* Perfect blackbody spectrum with  $T = 2.726 \text{ K}$ .
- \* Proves the early universe was hot and dense.

• Friedmann-Lemaître-Robertson-Walker (FLRW) Model

- **Einstein's Equation:** Connects spacetime geometry to energy/matter.

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- **FLRW Metric:** The metric for a homogeneous, isotropic universe.

$$ds^2 = -c^2 dt^2 + a(t)^2 \left[ \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right]$$

(Assuming flat,  $k = 0$ , for this course)

• Cosmic Dynamics (Friedmann Equations)

- **1. Friedmann Eq.:** (Hubble parameter  $H = \dot{a}/a$ )

$$H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2}$$

- **2. Conservation Eq.:** (Fluid equation)

$$\dot{\rho} = -3H(\rho + P)$$

- **3. Acceleration Eq.:**

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$$

• Cosmic Inventory (Components of the Universe)

- **Equation of State:**  $P = w\rho$
- **Density Evolution:**  $\rho \propto a^{-3(1+w)}$ 
  - \* **Matter (Dust):**  $w = 0 \Rightarrow \rho_m \propto a^{-3} \propto (1+z)^3$
  - \* **Radiation:**  $w = 1/3 \Rightarrow \rho_r \propto a^{-4} \propto (1+z)^4$
  - \* **Dark Energy ( $\Lambda$ ):**  $w = -1 \Rightarrow \rho_\Lambda \propto a^0$  (constant)
- **Critical Density:**  $\rho_{cr} = \frac{3H_0^2}{8\pi G}$ .  $\Omega_i = \rho_i / \rho_{cr}$ .
- **Full Friedmann Eq.:**

$$H(z)^2 = H_0^2 \left[ \Omega_m (1+z)^3 + \Omega_r (1+z)^4 + \Omega_k (1+z)^2 + \Omega_\Lambda \right]$$

• Cosmological Probes

- **Standard Candles (Type Ia Supernovae):** Known luminosity ( $L$ ). We measure flux ( $F$ ) to find Luminosity Distance ( $d_L$ ).

$$F = \frac{L}{4\pi d_L^2} \quad \text{where} \quad d_L = (1+z) \int_0^z \frac{dz'}{H(z')}$$

- **Key Discovery (1998):** Supernovae were dimmer (farther) than expected. This implies the expansion is **accelerating**.
- **Deceleration Parameter ( $q_0$ ):** Found to be negative, proving acceleration.

$$q_0 = \frac{1}{2}\Omega_m + \Omega_r - \Omega_\Lambda \approx \frac{\Omega_m}{2} - \Omega_\Lambda < 0$$

- **Hubble Tension:**  $H_0$  measured from the "local" universe (Supernovae) is  $\sim 73$ .  $H_0$  inferred from the "early" universe (CMB) is  $\sim 67$ . This is a major unsolved problem.