GRB Note

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CONTENTS

I. POWER DENSITY SPECTRUM

A. Definition

As shown on Wikipedia, for signal $x(t)$, we firstly take Fourier transformation

$$\hat{x}(f) = \int_{-\infty}^{\infty} e^{-i2\pi ft} x(t) dx, \tag{I.1}$$

then we define the **energy spectral density**

$$\bar{S}_{xx} \equiv |\hat{x}(f)|^2. \tag{I.2}$$

Also, for discrete-time signal, we have

$$\bar{S}_{xx}(f) = \lim_{N\to\infty} (\Delta t)^2 \left| \sum_{n=-N}^{N} x_n e^{-2\pi f n \Delta t} \right|^2 \tag{I.3}$$

$P(f_j)$, is then calculated by choosing an appropriate normalization A. For example,

$$P(f_i) = A|\mathrm{DFT}(f_j)|^2 = \frac{2\Delta T}{N} |\mathrm{DFT}(f_j)|^2 \tag{I.4}$$

B. Motivation

From Beloborodov's work[1], we start to use Fourier transformation to research GRB's properties. At first, we need to figure out why should use PDS. Here we list some points to explain why PDS.

- PDS is suitable for transients(**pulse-like** signals) whose energy is concentrated around on time window, which is corresponding to the GRB. That means it's easy to do Fourier transformation on GRB signal.
- Noise discrimination: Background noise tends to be Gaussian and show up as white noise in the PDS, whereas real signals exhibit characteristic "red noise" behavior. The PDS helps distinguish genuine variations from random noise.
- Identify periodic signals: Very low-frequency features in the PDS may reveal periodic or quasiperiodic signals that provide clues about the central engine. These signals would be difficult to detect directly in the light curve.
- The type of signal can show intrinsic distribution, which means $\mathcal{P}(f) \propto f^{-\alpha}$. So we can classify different types of signals.

C. Processing the Signals

We can do the Fourier transformation on the signal by following steps

- From the event list form an evenly sampled time series x_t with a bin time size Δt;
- Break this into M non-overlapping intervals of length N;
- Compute the periodogram of each interval;
- Average the M periodograms.

$$\left(1 - \mathrm{Re}\ \langle\psi_{n\mathbf{k}}|\frac{\partial}{\partial\omega}\Sigma(\omega)\Big|_{E_{n\mathbf{k}}^{QP}}|\psi_{n\mathbf{k}}\rangle\right)^{-1} \tag{I.5}$$

[1] A. M. Beloborodov, B. E. Stern, and R. Svensson, Astrophys. J. Lett. **508**, L25 (1998), arxiv:astro-ph/9807139.