1. Spectrum

Y-axis= $E^2\frac{dJ}{dE}$, means per **area** per **time** per **solid angle** per **energy**.

2. Solid Angles

- An "isotropic" distribution

$$\frac{dN}{d\Omega} = \text{constant}$$

- $$\frac{dN}{\sin\theta\, d\theta\, d\phi} = 1, \quad [\theta = 0, \pi], \ \phi = [0, 2\pi]$$

- $$\frac{dN}{d\theta} = \sin\theta \int d\phi \frac{dN}{\sin\theta\, d\theta\, d\phi} = 2\pi \sin\theta \quad \text{Not a constant!}$$

- $$\frac{dN}{dx} = \frac{dN}{d\cos\theta} = \int d\phi \frac{dN}{\sin\theta\, d\theta\, d\phi} = 2\pi, \quad \cos\theta = [-1, 1]$$

3. No directionality

- It is good to express in typical values
- Lamor Radius (Gyro-radius)

$$r_g = \frac{p_\perp}{|q|B}$$

- $$r_g \simeq 3.3\,\mathrm{m} \left(\frac{E}{\mathrm{GeV}}\right)\left(\frac{e}{|q|}\right)\left(\frac{1\,\mathrm{T}}{B}\right)$$

- Size of the Galaxy ~ 10 kpc
- pc $\simeq 3 \times 10^{16}\,\mathrm{m}$ or about 3 lyr

4. CR Composition

- Right plot shows elemental abundance ratio (normalized at Carbon).
- Cosmic ray (CR) composition differs from Solar System abundance.

Reasons for the difference:

1. **Different origins:** CRs come mainly from supernova remnants and stellar winds, not the same material as the Solar System.
2. **Acceleration bias:** Elements with low first ionization potential (e.g. Mg, Si, Fe) or large charge $|Z|$ are more efficiently accelerated $\Rightarrow$ overrepresented in CRs.
3. **Propagation effect:** CRs interact with interstellar matter and produce secondary nuclei (Li, Be, B), leading to their high abundance.
4. **Conclusion:** CR composition reflects *acceleration and propagation physics*, not direct stellar nucleosynthesis.

5. Cross Section

One paritcle interaction,

- $\sigma_{AB} n_B L \ll 1$: very likely to pass through(**optically thin**)
- $\sigma_{AB} n_B L \gg 1$: very likely to interact(**optically thick**)

Probability:

$$P = 1 - e^{-n\sigma L} \tag{1}$$

where $\tau = n\sigma L$ is called **Optical Depth**.
Unit: Barn $1barn = 10^{-28} m^2$.

6. Lorentz Factor

(For motion at velocity v along the x-axis)

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \tag{2}$$
$$x' = \gamma(x - vt) \tag{3}$$
$$E' = \gamma(E - vp_x) \tag{4}$$
$$p'_x = \gamma\left(p_x - \frac{vE}{c^2}\right) \tag{5}$$

7. Diffusion Model

Diffusion-loss equation,

$$\frac{\partial n}{\partial t} = \nabla \cdot \left(D\vec{\nabla} n\right) - \frac{\partial}{\partial E}(n\dot{E}) + Q \tag{6}$$

Diffusion-Convection equation,

$$\frac{\partial}{\partial t} n = \nabla \cdot \left(D\vec{\nabla} n - \vec{V} n\right) - \frac{\partial}{\partial E}(n\dot{E}) + Q \tag{7}$$

with momentum loss term $\dot{p} = -\frac{1}{3}(\nabla \cdot V)p$.
Rigity: $R = \frac{p}{q}$. Motivation: Lamor Radius is propotional to the rigity $r_g = \frac{p_\perp}{q}$.
Number of particles per phase space: $f = \frac{\mathrm{d}N}{\mathrm{d}^3p\mathrm{d}^3x}$.

Differential number density of particles, $n = \frac{\mathrm{d}N}{\mathrm{d}p\mathrm{d}^3x} = 4\pi p^2 f$.
From diffusion-loss equations, we can imply

$$D\frac{\partial f}{\partial r} + \frac{V_p}{3}\frac{\partial f}{\partial p} = 0 \implies \mathrm{d}f(r,p) = 0 \implies f(r_1, p_1) = f(r_1, p_1)$$

with definition of flux $I = vn/(4\pi) = vp^2 f$, we have relation

$$\frac{I(p)}{vp^2} = \frac{I(p_{ILS})}{v_{LIS} p_{LIS}^2} \tag{8}$$

combining with solar modulation potential ϕ, we have

$$\frac{I(p)}{vp^2} = \frac{I(p+\phi)}{v_{lis}(p+\phi)^2} \tag{9}$$

8. CR Secondaries

The full propagation euqation,

$$\begin{aligned}\frac{\partial\psi(r,p,t)}{\partial t} &= q(r,p,t) + \overbrace{\nabla \cdot (D_{xx}\nabla\psi - V\psi)}^{\text{Diffusion Convection}} \\ &+ \underbrace{\frac{\partial}{\partial p} p^2 D_{pp} \frac{\partial}{\partial p}\frac{1}{p^2}\psi}_{\text{Re-acceleration}} - \underbrace{\frac{\partial}{\partial p}\left[\dot{p}\psi - \frac{p}{3}(\nabla \cdot V)\psi\right]}_{\text{Continuous Energy Loss}} \\ &- \underbrace{\frac{1}{\tau_f}\psi - \frac{1}{\tau_r}\psi}_{\text{Fragmentation\& Radi. decay Loss}}\end{aligned} \tag{10}$$

For Fragmentation Loss, for the i-th species, it's loss by $\rightarrow$ j-th species

$$\frac{\partial n_i}{\partial t} = -n_i\left(\frac{\rho}{m}\right)_{ism}\sigma_{i\rightarrow j}v \tag{11}$$

For radioactive decay loss,

$$\frac{\partial n_i}{\partial t} = -n_i\frac{1}{\tau_i}, \tau_i \text{ is the lifetime} \tag{12}$$

For simple case: Leaky box approx, one species dominates the production, $Q = 0$. We have relation,

$$\frac{n_i}{T_e} = -\frac{n_i}{T_f} - \frac{n_i}{T_{dec}} + C_i \tag{13}$$

where C_i is the production of "i" due to other species, then we can get expression of n_i,

$$n_i = \frac{C_i}{1/T_e + 1/T_f + 1/T_{dec}} \tag{14}$$

9. Collision

We can use Lorentz Invariant s,

$$s = (p^\mu + p^\nu)^2 \tag{15}$$

where $p^\mu = (\frac{E}{c}, p^1, p^2, p^3)$.
And definition of Differential cross section,

$$\frac{\mathrm{d}\sigma_{i\rightarrow j}}{\mathrm{d}T_i}(T_i, E_j) \tag{16}$$

Total corss section:

$$\frac{\mathrm{d}P}{\mathrm{d}t} = n\sigma \tag{17}$$

and differential cross section,

$$\frac{\mathrm{d}P}{\mathrm{d}t\,\mathrm{d}T} = n\frac{\mathrm{d}\sigma}{\mathrm{d}T} \tag{18}$$

thus, using differential cross section, the number of $\bar{p}$ in interaction $p + p \rightarrow \bar{p} + X$ can be expressed by

$$n_{\bar{p}}(T_{\bar{p}}) = \left(\int_{E_{th}} n_p \frac{\mathrm{d}\sigma_{pp\rightarrow\bar{p}X}}{\mathrm{d}T_{\bar{p}}}(E_p, T_p)\,\mathrm{d}E_p - n_{\bar{p}}\sigma_{\bar{p}\rightarrow X}\right)\frac{X}{m} \tag{19}$$

10. Electron-Matter Interaction

A particle interacts with stuff lower energy than itself causes energy loss through following mechanisms. In matter:

- Ionization: Kick off electrons from atoms
- Bremsstrahlung: curved trajectory emits photon

In space:

- Inverse-Compton scattering
- Synchrotron radiation

With electrons/positrons:

- Moller/Bhabba scattering: $e^- + e^+ +$ electron scattering
- Positron annihilation: $e^- + e^+ \rightarrow \gamma + \gamma$

11. Inverse Compton Scattering

We assume it as the **Thomson**(elastic) scattering, because photon energy in CR frame $E_{ph} \sim kT \sim 10^{-4}eV$.
In electron rest frame,

$$\begin{pmatrix} E'_{ph} \\ E'_{ph} \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix}\begin{pmatrix} E_{ph} \\ E_{ph} \end{pmatrix} \tag{20}$$
$$\implies E'_{ph} = \gamma(1+\beta)E_{ph} \simeq 2\gamma E_{ph} \tag{21}$$
$$\implies E_\gamma \simeq 2\gamma E'_\gamma \simeq 4\gamma^2 E_{ph} \tag{22}$$

Due to the same Lorentz transformation, we know the energy loss of Inverse Compton, is

$$-\frac{\mathrm{d}E_e}{\mathrm{d}t} = \frac{\mathrm{d}E_\gamma}{\mathrm{d}t} = \frac{\mathrm{d}E'_\gamma}{\mathrm{d}t} \tag{23}$$

Give the expression of power

$$\frac{\mathrm{d}E'_\gamma}{\mathrm{d}t} = \int E'_\gamma \frac{c\,\mathrm{d}\sigma}{\mathrm{d}E'_\gamma}\mathrm{d}E'_\gamma\,\mathrm{d}n'_\gamma \tag{24}$$

working in Thomson limit, th edifferential cross section is,

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E'_\gamma} = \sigma_t(E'_{ph} - E'_\gamma) \implies \frac{\mathrm{d}E'_\gamma}{\mathrm{d}t'} = c\sigma_t U'_{ph} \tag{25}$$

where U'_{ph} is the photon energy density(in ERS).

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{\mathrm{d}E'_\gamma}{\mathrm{d}t} \tag{26}$$

12. Positrons

Positrons (e^+) measured flux is significantly lower than that of protons (p) and the combined electron/positron flux ($e^- + e^+$) (Slide 2, 3). The key diagnostic tool for understanding their origin is the **positron fraction**:

$$\text{Positron Fraction} = \frac{\Phi_{e^+}}{\Phi_{e^+} + \Phi_{e^-}}$$

13. The Standard Model: Secondary Production

Positrons are believed to be **secondary particles**. They are produced by the interaction of primary cosmic rays with the interstellar medium (ISM).

13.1 Hadronic Interaction Chain

The production mechanism is a multi-step decay process initiated by high-energy proton-proton collisions (Slide 4, 8, 9):

1. **Pion Production:** A high-energy proton (from cosmic rays) collides with a proton in the ISM (interstellar gas).
$$p + p \rightarrow \pi^\pm + X \quad (\text{where } \pi^0 \text{ also produced})$$
2. **Pion Decay:** The charged pions (π^+ and π^-) decay into muons (μ^+ and μ^-).
$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$
$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$
3. **Muon Decay:** The muons then decay, producing positrons and electrons.
$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$$
$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

13.2 Predicted Positron Fraction

This secondary production model can be calculated. Because the initial protons have a falling energy spectrum, the resulting secondary positrons also have a falling spectrum. This model predicts that the positron fraction should **decrease** with increasing energy.

14. The Positron Anomaly

The central puzzle in this field is the "positron anomaly," which is a major discrepancy between the theoretical prediction and experimental observation.

14.1 The PAMELA Discovery

PAMELA experiment published results showing that the positron fraction does not fall with energy. Instead, it **begins to rise** at energies above ~ 10 GeV. This was a significant anomaly.

14.2 Confirmation by AMS and Fermi

This anomalous rising fraction was not an error. It was subsequently confirmed with higher precision by two other major experiments:

- **Fermi-LAT (Large Area Telescope):** Confirmed the rise, even without a magnet, by cleverly using the Earth's magnetic field to separate e^+ and e^-.
- **AMS-02 (Alpha Magnetic Spectrometer):** Provided the most precise measurement to date, confirming the rise up to hundreds of GeV.

15. Interpretations and New Sources

The confirmed anomaly means there must be an **additional source** (or sources) of high-energy positrons that the secondary production model does not account for.

- **Astrophysical Sources:** The leading candidates are nearby **Pulsar Wind Nebulae (PWNs)** (Slide 17). These are rapidly rotating neutron stars (pulsars) that create a nebula of high-energy electron-positron pairs. These pairs can escape and propagate to Earth, adding to the positron flux.

- **Exotic Sources:** The anomaly also generated excitement for potential exotic sources, such as the annihilation or decay of **Dark Matter** particles, which could produce e^+e^- pairs.

16. Gamma Ray

For detection, at high energy, pair creation dominates the cross section: $\gamma + A \to A + e^+ + e^-$.
For prodection,

- Electron $\to$ matter: Bremsstrahlung gamma rays
- Electron $\to$ radiation: Inverse Compton gamma rays

For IC emission,

$$\frac{\mathrm{d}n}{\mathrm{d}E_\gamma\,\mathrm{d}t} = \int \frac{\mathrm{d}n_e}{\mathrm{d}E_e\,\mathrm{d}\Omega_e}\frac{\mathrm{d}N}{\mathrm{d}E_\gamma\,\mathrm{d}t}\,\mathrm{d}E_e\,\mathrm{d}\Omega_e \tag{27}$$

$$P = E_\gamma \frac{\mathrm{d}N}{\mathrm{d}E_\gamma\,\mathrm{d}t} \tag{28}$$

PP interactions:

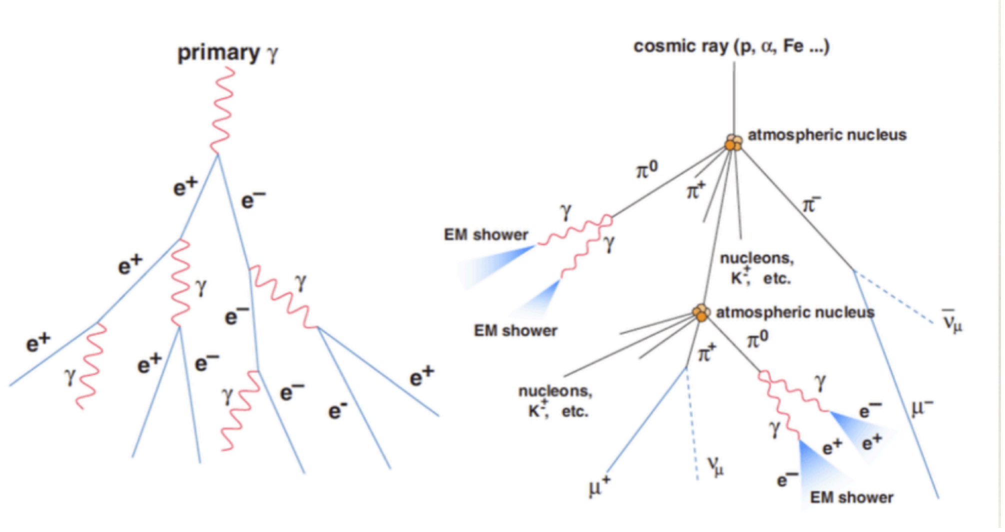

Figure 1: *The lightest hadronic states are pions, so they are the primary products from proton-proton interactions*

16.1 pionic gamma-ray production

$pp \to \pi^0 \to \gamma$ Minimum pion energy to produce a photon with energy E_γ

$$E_{min} = E_\gamma + \frac{m_\pi^2}{4E_\gamma}$$

Gamma-ray emissivity(photons per volume per time per energy):

$$\frac{\mathrm{d}n}{\mathrm{d}E_\gamma} = \iint \frac{\mathrm{d}n_p}{\mathrm{d}E_p}\frac{\mathrm{d}\sigma_{pp\to\pi}}{\mathrm{d}E_\pi} n_{ISM} c \frac{\mathrm{d}N_{\pi\to\gamma}}{\mathrm{d}E_\gamma}\,\mathrm{d}E_\pi\,\mathrm{d}E_p \tag{29}$$

17. UHE Cosmic Ray

- **UHECR Basics**
  - **Definition:** Cosmic rays with energy above the "Knee" (> 1 PeV).
  - **Origin: Extragalactic.**
  - **Reason:** The Milky Way's magnetic field is too weak to contain them (based on Larmor Radius calculation).
- **The Four Big Mysteries**
  - **1. Source:** Unknown.
  - **2. Direction:** Mostly isotropic (uniform).
    - Pierre Auger Observatory found a "hotspot" (anisotropy), but no clear source.
  - **3. Composition:** Unknown (Protons? Iron?).
    - We measure it from air showers, which depends on interaction models.
    - **"Muon Puzzle":** Our models (e.g., QGSJET, SIBYLL) consistently predict fewer muons in the air shower than we actually observe. This implies our models or composition assumptions are wrong.
  - **4. Energy Spectrum Features:**
    - **"Ankle":** Point where the spectrum flattens, believed to be the transition from Galactic to Extragalactic cosmic rays.
    - **GZK Cutoff:** A sharp drop in particles observed above $\sim 10^{19.5}$ eV.
- **The GZK Limit**
  - **Theory:** (Greisen-Zatsepin-Kuzmin) High-energy protons will interact with the Cosmic Microwave Background (CMB) photons ($p + \gamma_{CMB}$).
  - **Effect:** This interaction causes the proton to lose energy, creating a "horizon." We can only see UHECR sources from relatively nearby (~ 100 Mpc).
  - **The Catch:** This cutoff energy only works for protons. If UHECRs are heavy nuclei (like Iron), the cutoff energy is different.
- **Multi-Messenger Solution**
  - **Cosmic Rays:** Are charged, so they are bent by magnetic fields and don't point to their source.
  - **Gamma Rays:** Are neutral, but they get absorbed by background light over long distances ($\gamma + \gamma \to e^+e^-$).
  - **Neutrinos:** Are neutral and barely interact. They are the best tool to point directly back to the UHECR sources.

18. Acceleration

1. Fermi Acceleration (General Idea)

- After each encounter, particle gains energy:

$$E = \beta^k E_0, \quad N(>E) = N_0 P^k$$

- Eliminate k:

$$N(>E) \propto E^{\frac{\ln P}{\ln \beta}}, \quad \frac{dN}{dE} \propto E^{-1+\frac{\ln P}{\ln \beta}}$$

- For first-order Fermi (shock acceleration):

$$\frac{dN}{dE} \propto E^{-2}$$

2. Shock Basics

- Upstream: (ρ_1, v_1, P_1), Downstream: (ρ_2, v_2, P_2)
- Conservation:

$$\rho_1 v_1 = \rho_2 v_2, \quad \rho_1 v_1^2 = \rho_2 v_2^2 + P_2, \quad \tfrac{1}{2}\rho_1 v_1^3 = \tfrac{1}{2}\rho_2 v_2^3 + \tfrac{3}{2}P_2 v_2$$

- For strong shocks ($P_1 \simeq 0$):

$$\rho_2 = 4\rho_1, \quad v_1 = 4v_2$$

3. Particle Acceleration at Shocks

- Relative velocity between up/down stream:

$$V = \frac{3}{4}U$$

- Energy gain per crossing:

$$\left\langle \frac{\Delta E}{E} \right\rangle = \frac{2}{3}V, \quad \text{Round trip: } \left\langle \frac{\Delta E}{E} \right\rangle = \frac{4}{3}V$$

- Energy gain factor: $\beta = 1 + \frac{4}{3}V$, Escape probability: $P = 1 - U$
- For small U:

$$\frac{\ln P}{\ln \beta} \simeq -1 \Rightarrow \frac{dN}{dE} \propto E^{-2}$$

4. Observed Cosmic Ray Spectrum

- Intrinsic (source) index: $\gamma \approx 2.0 - 2.2$
- After propagation losses: $\gamma_{\text{obs}} \approx 2.7 - 3.3$

5. Maximum Energy (Hillas Criterion)

- From Faraday's law: $\nabla \times \vec{\mathcal{E}} = -\partial_t \vec{B}$
- Dimensional estimate:

$$\mathcal{E} \sim BU, \quad E_{\text{max}} = ZeBUL$$

- Example: young SNR $B \sim 1\,\mu$G, $U \sim 10^4$ km/s, $L \sim 1$ pc $\Rightarrow E_{\text{max}} \sim 10^{16}$ eV
- Hillas plot: $E_{\text{max}} \approx ZBLU$ distinguishes feasible CR sources.

19. Neutrino

- **History & Discovery**
  - **Proposal (Pauli, 1930):** Solved the "missing energy" in beta decay ($n \to p^+ + e^-$). The electron's energy was a continuous spectrum, not a fixed value, implying a third, unseen particle (the neutrino) was present.
  - **Discovery (Cowan & Reines, 1956):** Used a nuclear reactor (a powerful $\bar{\nu}_e$ source) to detect neutrinos via **Inverse Beta Decay** ($\bar{\nu}_e + p^+ \to n + e^+$).
- **Weak Interaction & Parity Violation**
  - Neutrinos only interact via the weak force.
  - **Wu Experiment (1956):** Observed beta decay from aligned Cobalt-60.
  - **Result:** Electrons were emitted asymmetrically (violating Parity/mirror symmetry).
  - **Conclusion:** The weak force is "left-handed"—it only interacts with **left-handed particles** and **right-handed anti-particles**.
- **The Solar Neutrino Problem**
  - **Experiment (Homestake):** Raymond Davis Jr. used 600 tons of cleaning fluid (C_2Cl_4) to count solar neutrinos ($\nu_e + {}^{37}Cl \to {}^{37}Ar + e^-$).
  - **Problem:** He only detected 1/3 of the neutrinos predicted by the Standard Solar Model.
- **The Solution: Neutrino Mass**
  - **Neutrino Oscillation:** Discovered by Super-Kamiokande (1998) and confirmed by SNO (2001-02). Neutrinos change "flavor" as they travel (e.g., $\nu_e \to \nu_\mu$).
  - **Implication:** This oscillation is only possible if neutrinos have **mass**.
  - **Significance:** This is physics **Beyond the Standard Model**, which originally assumed neutrinos were massless.

20. GR& Cosmos

- **Principles & Observations**
  - **Cosmological Principle:** Universe is homogeneous and isotropic on large scales.
  - **Hubble's Law:** Galaxies are moving away from us.

$$v = H_0 d$$

  - **Redshift (z):** Caused by the expansion of space (stretching of light).

$$1 + z = \frac{\lambda_o}{\lambda_e} = \frac{1}{a(t)}$$

  where $a(t)$ is the scale factor (with $a = 1$ today).
  - **CMB (Cosmic Microwave Background):**
    - Discovered by Penzias & Wilson (1960).
    - Perfect blackbody spectrum with $T = 2.726$ K.
    - Proves the early universe was hot and dense.
- **Friedmann-Lemaître-Robertson-Walker (FLRW) Model**
  - **Einstein's Equation:** Connects spacetime geometry to energy/matter.

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

  - **FLRW Metric:** The metric for a homogeneous, isotropic universe.

$$ds^2 = -c^2 dt^2 + a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right]$$

  (Assuming flat, $k = 0$, for this course)
- **Cosmic Dynamics (Friedmann Equations)**
  - **1. Friedmann Eq.:** (Hubble parameter $H = \dot{a}/a$)

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

  - **2. Conservation Eq.:** (Fluid equation)

$$\dot{\rho} = -3H(\rho + P)$$

  - **3. Acceleration Eq.:**

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$$

- **Cosmic Inventory (Components of the Universe)**
  - **Equation of State:** $P = w\rho$
  - **Density Evolution:** $\rho \propto a^{-3(1+w)}$
    - **Matter (Dust):** $w = 0 \implies \rho_m \propto a^{-3} \propto (1+z)^3$
    - **Radiation:** $w = 1/3 \implies \rho_r \propto a^{-4} \propto (1+z)^4$
    - **Dark Energy (Λ):** $w = -1 \implies \rho_\Lambda \propto a^0$ (constant)
  - **Critical Density:** $\rho_{cr} = \frac{3H_0^2}{8\pi G}$. $\Omega_i = \rho_i/\rho_{cr}$.
  - **Full Friedmann Eq.:**

$$H(z)^2 = H_0^2 \left[\Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_k(1+z)^2 + \Omega_\Lambda \right]$$

- **Cosmological Probes**
  - **Standard Candles (Type Ia Supernovae):** Known luminosity (L). We measure flux (F) to find Luminosity Distance (d_L).

$$F = \frac{L}{4\pi d_L^2} \quad \text{where} \quad d_L = (1+z)\int_0^z \frac{dz'}{H(z')}$$

  - **Key Discovery (1998):** Supernovae were dimmer (farther) than expected. This implies the expansion is **accelerating**.
  - **Deceleration Parameter (q_0):** Found to be negative, proving acceleration.

$$q_0 = \frac{1}{2}\Omega_m + \Omega_r - \Omega_\Lambda \approx \frac{\Omega_m}{2} - \Omega_\Lambda < 0$$

  - **Hubble Tension:** H_0 measured from the "local" universe (Supernovae) is ~ 73. H_0 inferred from the "early" universe (CMB) is ~ 67. This is a major unsolved problem.