

1. Spectrum

Y-axis= $E^2 \frac{dI}{dE}$, means per **area** per **time** per **solid angle** per **energy**.

2. Solid Angles

- An “isotropic” distribution

$$\frac{dN}{d\Omega} = \text{constant}$$

$$\frac{dN}{\sin \theta \, d\theta \, d\phi} = 1, \quad [\theta = 0, \pi], \, \, \phi = [0, 2\pi]$$

$$\frac{dN}{d\theta} = \sin \theta \int d\phi \frac{dN}{\sin \theta \, d\theta \, d\phi} = 2\pi \sin \theta \quad \textcolor{red}{\text{Not a constant!}}$$

$$\frac{dN}{dx} = \frac{dN}{d \cos \theta} = \int d\phi \frac{dN}{\sin \theta \, d\theta \, d\phi} = 2\pi, \quad \cos \theta = [-1, 1]$$

3. No directionality

- It is good to express in typical values
- Lamor Radius (Gyro-radius)

$$r_g = \frac{p_{\perp}}{|q|B}$$

$$r_g \simeq 3.3 \, \text{m} \left(\frac{E}{\text{GeV}} \right) \left(\frac{e}{|q|} \right) \left(\frac{1 \text{T}}{B} \right)$$

- Size of the Galaxy $\sim 10 \, \text{kpc}$
- $\text{pc} \simeq 3 \times 10^{16} \, \text{m}$ or about 3 lyr

4. CR Composition

- Right plot shows elemental abundance ratio (normalized at Carbon).
- Cosmic ray (CR) composition differs from Solar System abundance.

Reasons for the difference:

- Different origins:** CRs come mainly from supernova remnants and stellar winds, not the same material as the Solar System.
- Acceleration bias:** Elements with low first ionization potential (e.g. Mg, Si, Fe) or large charge $|Z|$ are more efficiently accelerated \Rightarrow overrepresented in CRs.
- Propagation effect:** CRs interact with interstellar matter and produce secondary nuclei (Li, Be, B), leading to their high abundance.
- Conclusion:** CR composition reflects *acceleration and propagation physics*, not direct stellar nucleosynthesis.

5. Cross Section

One paritcle interaction,

- $\sigma_{AB} n_B L \ll 1$: very likely to pass through(**optically thin**)

- $\sigma_{AB} n_B L \gg 1$: very likely to interact(**optically thick**)

Probability:

$$P = 1 - e^{-n\sigma L} \tag{1}$$

where $\tau = n\sigma L$ is called **Optical Depth**.

Unit: Barn $1\text{barn} = 10^{-28} m^2$.

6. Lorentz Factor

(For motion at velocity v along the x-axis)

$$t' = \gamma \left(t - \frac{vx}{c^2} \right) \tag{2}$$

$$x' = \gamma (x - vt) \tag{3}$$

$$E' = \gamma (E - vp_x) \tag{4}$$

$$p'_x = \gamma \left(p_x - \frac{vE}{c^2} \right) \tag{5}$$

7. Diffusion Model

Diffusion-loss equation,

$$\frac{\partial n}{\partial t} = \nabla \cdot \left(D \vec{\nabla} n \right) - \frac{\partial}{\partial E} (n \dot{E}) + Q \tag{6}$$

Diffusion-Convection equation,

$$\frac{\partial}{\partial t} n = \nabla \cdot \left(D \vec{\nabla} n - \vec{V} n \right) - \frac{\partial}{\partial E} (n \dot{E}) + Q \tag{7}$$

with momentum loss term $\dot{p} = -\frac{1}{3}(\nabla \cdot V)p$.

Rigity: $R = \frac{p}{q}$. Motivation: Lamor Radius is propotional to the rigity $r_g = \frac{p_{\perp}}{q}$.

Number of particles per phase space: $f = \frac{dN}{d^3p d^3x}$.

Differential number density of particles, $n = \frac{dN}{dp d^3x} = 4\pi p^2 f$.

From diffusion-loss equations, we can imply

$$D \frac{\partial f}{\partial r} + \frac{V_p}{3} \frac{\partial f}{\partial p} = 0 \implies \text{d}f(r, p) = 0 \implies f(r_1, p_1) = f(r_1, p_1)$$

with definition of flux $I = vn/(4\pi) = vp^2 f$, we have relation

$$\frac{I(p)}{vp^2} = \frac{I(p_{ILS})}{v_{LIS} p_{LIS}^2} \tag{8}$$

combining with solar modulation potential ϕ , we have

$$\frac{I(p)}{vp^2} = \frac{I(p + \phi)}{v_{lis}(p + \phi)^2} \tag{9}$$

8. CR Secondaries

The full propagation euqation,

$$\begin{aligned} \frac{\partial \psi(r, p, t)}{\partial t} = & \overset{\text{Diffusion}}{q(r, p, t)} + \overset{\text{Convection}}{\nabla \cdot (D_{xx} \nabla \psi - V \psi)} \\ & + \overset{\text{Re-acceleration}}{\frac{\partial}{\partial p} p^2 D_{pp} \frac{\partial}{\partial p} \frac{1}{p^2} \psi} - \overset{\text{Continuous Energy Loss}}{\frac{\partial}{\partial p} \left[\dot{p} \psi - \frac{p}{3} (\nabla \cdot V) \psi \right]} \\ & - \overset{\text{Fragmentation\& Radi. decay Loss}}{\frac{1}{\tau_f} \psi - \frac{1}{\tau_r} \psi} \end{aligned} \tag{10}$$

For Fragmentation Loss, for the i-th species, it’s loss by \rightarrow j-th species

$$\frac{\partial n_i}{\partial t} = -n_i \left(\frac{\rho}{m} \right)_{ism} \sigma_{i \rightarrow j} v \tag{11}$$

For radioactive decay loss,

$$\frac{\partial n_i}{\partial t} = -n_i \frac{1}{\tau_i}, \tau_i \text{ is the lifetime} \tag{12}$$

For simple case: Leaky box approx, one species dominates the production, $Q = 0$. We have relation,

$$\frac{n_i}{T_e} = -\frac{n_i}{T_f} - \frac{n_i}{T_{dec}} + C_i \tag{13}$$

where C_i is the production of ”i” due to other species, then we can get expression of n_i ,

$$n_i = \frac{C_i}{1/T_e + 1/T_f + 1/T_{dec}} \tag{14}$$

9. Collision

We can use Lorentz Invariant s ,

$$s = (p^{\mu} + p^{\nu})^2 \tag{15}$$

where $p^{\mu} = (\frac{E}{c}, p^1, p^2, p^3)$.

And definition of Differential cross section,

$$\frac{d\sigma_{i \rightarrow j}}{dT_i}(T_i, E_j) \tag{16}$$

Total corss section:

$$\frac{dP}{dt} = n\sigma \tag{17}$$

and differential cross section,

$$\frac{dP}{dt dT} = n \frac{d\sigma}{dT} \tag{18}$$

thus, using differential cross section, the number of \bar{p} in in-teraction $p + p \rightarrow \bar{p} + X$ can be expressed by

$$n_{\bar{p}}(T_{\bar{p}}) = \left(\int_{E_{th}} n_p \frac{d\sigma_{pp \rightarrow \bar{p}X}}{dT_{\bar{p}}}(E_p, T_p) dE_p - n_{\bar{p}} \sigma_{\bar{p} \rightarrow X} \right) \frac{X}{m} \tag{19}$$

10. Electron-Matter Interaction

A particle interacts with stuff lower energy than itself causes energy loss through following mechanisms. In matter:

- Ionization: Kick off electrons from atoms
- Bremsstrahlung: curved trajectory emits photon

In space:

- Inverse-Compton scattering
- Synchrotron radiation

With electrons/positrons:

- Moller/Bhabba scattering: $e^- + e^+ +$ electron scattering
- Positron annihilation: $e^- + e^+ \rightarrow \gamma + \gamma$

11. Inverse Compton Scattering

We assume it as the **Thomson**(elastic) scattering, because photon energy in CR frame $E_{ph} \sim kT \sim 10^{-4} eV$. In electron rest frame,

$$\begin{pmatrix} E'_{ph} \\ E'_{ph} \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} E_{ph} \\ E_{ph} \end{pmatrix} \tag{20}$$

$$\implies E'_{ph} = \gamma(1 + \beta)E_{ph} \simeq 2\gamma E_{ph} \tag{21}$$

$$\implies E_{\gamma} \simeq 2\gamma E'_{\gamma} \simeq 4\gamma^2 E_{ph} \tag{22}$$

Due to the same Lorentz transformation, we know the en-ergy loss of Inverse Compton, is

$$-\frac{dE_e}{dt} = \frac{dE_{\gamma}}{dt} = \frac{dE'_{\gamma}}{dt} \tag{23}$$

Give the expression of power

$$\frac{dE'_{\gamma}}{dt} = \int E'_{\gamma} \frac{c \, d\sigma}{dE'_{\gamma}} dE'_{\gamma} dn'_{\gamma} \tag{24}$$

working in Thomson limit, th edifferential cross section is,

$$\frac{d\sigma}{dE'_{\gamma}} = \sigma_t (E'_{ph} - E'_{\gamma}) \implies \frac{dE'_{\gamma}}{dt'} = c\sigma_t U'_{ph} \tag{25}$$

where U'_{ph} is the photon energy density(in ERS).

$$\frac{dE}{dt} = \frac{dE'_{\gamma}}{dt} \tag{26}$$

12. The Standard Model: Secondary Production

Positrons are believed to be **secondary particles**. They are produced by the interaction of primary cosmic rays with the interstellar medium (ISM).

12.1 Hadronic Interaction Chain

The production mechanism is a multi-step decay process initiated by high-energy proton-proton collisions (Slide 4, 8, 9):

- Pion Production:** A high-energy proton (from cosmic rays) collides with a proton in the ISM (interstellar gas).

$$p + p \rightarrow \pi^{\pm} + X \quad (\text{where } \pi^0 \text{ also produced})$$

- Pion Decay:** The charged pions (π^+ and π^-) decay into muons (μ^+ and μ^-).

$$\pi^+ \rightarrow \mu^+ + \nu_{\mu}$$

$$\pi^- \rightarrow \mu^- + \bar{\nu}_{\mu}$$

- Muon Decay:** The muons then decay, producing positrons and electrons.

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_{\mu}$$

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_{\mu}$$

12.2 Predicted Positron Fraction

This secondary production model can be calculated. Be-cause the initial protons have a falling energy spectrum, the resulting secondary positrons also have a falling spectrum. This model predicts that the positron fraction should **de-crease** with increasing energy.

13. The Positron Anomaly

The central puzzle in this field is the ”positron anomaly,” which is a major discrepancy between the theoretical pre-diction and experimental observation.

13.1 The PAMELA Discovery

PAMELA experiment published results showing that the positron fraction does not fall with energy. Instead, it **be-gins to rise** at energies above ~ 10 GeV. This was a signif-icant anomaly.

13.2 Confirmation by AMS and Fermi

This anomalous rising fraction was not an error. It was sub-sequently confirmed with higher precision by two other ma-jor experiments:

- Fermi-LAT (Large Area Telescope):** Confirmed the rise, even without a magnet, by cleverly using the Earth’s magnetic field to separate e^+ and e^- .

- AMS-02 (Alpha Magnetic Spectrometer):** Provided the most precise measurement to date, confirming the rise up to hundreds of GeV.

14. Interpretations and New Sources

The confirmed anomaly means there must be an **addi-tional source** (or sources) of high-energy positrons that the secondary production model does not account for.

- Astrophysical Sources:** The leading candidates are nearby **Pulsar Wind Nebulae (PWNs)** (Slide 17). These are rapidly rotating neutron stars (pulsars) that create a nebula of high-energy electron-positron pairs. These pairs can escape and propagate to Earth, adding to the positron flux.

- **Exotic Sources:** The anomaly also generated excitement for potential exotic sources, such as the annihilation or decay of **Dark Matter** particles, which could produce e^+e^- pairs.

15. Gamma Ray

For detection, at high energy, pair creation dominates the cross section: $\gamma + A \rightarrow A + e^+ + e^-$.

For prodection,

- Electron \rightarrow matter: Bremsstrahlung gamma rays
- Electron \rightarrow radiation: Inverse Compton gamma rays

For IC emission,

$$\frac{dn}{dE_\gamma dt} = \int \frac{dn_e}{dE_e d\Omega_e} \frac{dN}{dE_\gamma dt} dE_e d\Omega_e \quad (27)$$

$$P = E_\gamma \frac{dN}{dE_\gamma dt} \quad (28)$$

PP interactions:

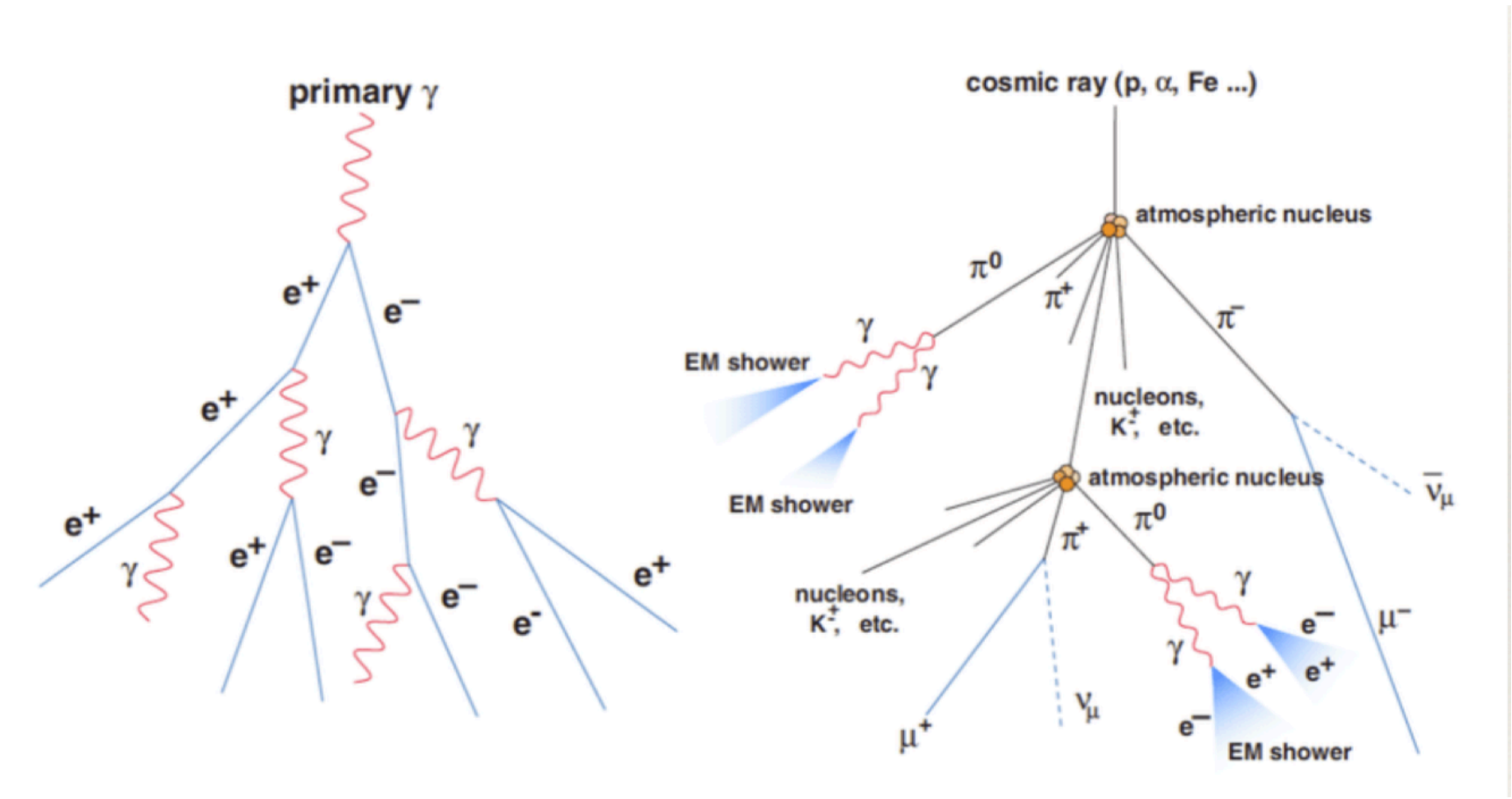


Figure 1: The lightest hadronic states are pions, so they are the primary products from proton-proton interactions

15.1 pionic gamma-ray production

$pp \rightarrow \pi^0 \rightarrow \gamma$ Minimum pion energy to produce a photon with energy E_γ

$$E_{min} = E_\gamma + \frac{m_\pi^2}{4E_\gamma}$$

Gamma-ray emissivity(photons per volume per time per energy):

$$\frac{dn}{dE_\gamma} = \iint \frac{dn_p}{dE_p} \frac{d\sigma_{pp \rightarrow \pi}}{dE_\pi} n_{ISM} c \frac{dN_{\pi \rightarrow \gamma}}{dE_\gamma} dE_\pi dE_p \quad (29)$$

With angular,

$$E_\gamma = \frac{m_\pi}{2} \gamma (1 + \beta \cos \theta')$$

$$\Rightarrow \begin{cases} \frac{dN}{dE_\gamma} = \frac{2}{m_\pi \gamma \beta}, & E_{min}(\beta) < E_\gamma < E_{max}(\beta) \\ \frac{dN}{dE_\gamma} = \frac{2}{m_\pi \gamma \beta} \Theta(E_\gamma - E_{min}) \Theta(E_{max} - E_\gamma) \end{cases} \quad (30)$$

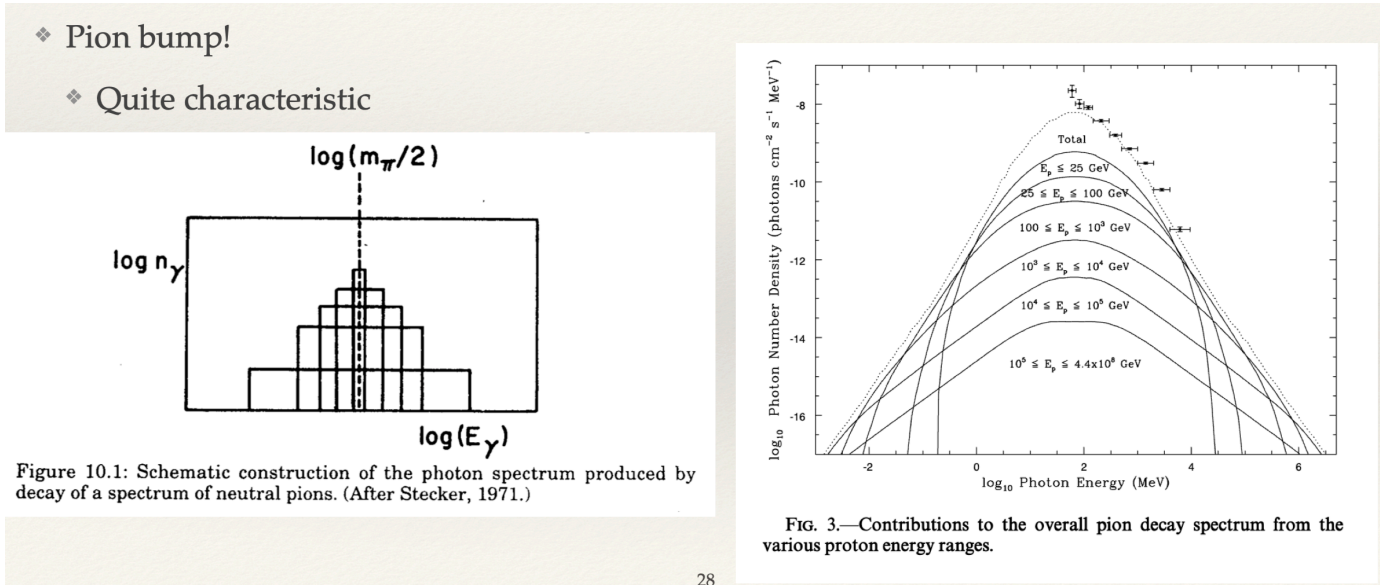


Figure 2: Shape of pion spectrum

16. UHE Cosmic Ray

UHECR Basics

- **Definition:** Cosmic rays with energy above the "Knee" (> 1 PeV).
- **Origin:** Extragalactic.
- **Reason:** The Milky Way's magnetic field is too weak to contain them (based on Larmor Radius calculation).

The Four Big Mysteries

- **1. Source:** Unknown.
- **2. Direction:** Mostly isotropic (uniform).
 - Pierre Auger Observatory found a "hotspot" (anisotropy), but no clear source.
- **3. Composition:** Unknown (Protons? Iron?).
 - We measure it from air showers, which depends on interaction models.
 - **"Muon Puzzle":** Our models (e.g., QGSJET, SIBYLL) consistently predict fewer muons in the air shower than we actually observe. This implies our models or composition assumptions are wrong.
- **4. Energy Spectrum Features:**
 - **"Ankle":** Point where the spectrum flattens, believed to be the transition from Galactic to Extragalactic cosmic rays.
 - **GZK Cutoff:** A sharp drop in particles observed above $\sim 10^{19.5}$ eV.

The GZK Limit

- **Theory:** (Greisen-Zatsepin-Kuzmin) High-energy protons will interact with the Cosmic Microwave Background (CMB) photons ($p + \gamma_{CMB} \rightarrow \Delta^+ \rightarrow p + \pi^0/n + \pi^+$).
- **Effect:** This interaction causes the proton to lose energy, creating a "horizon." We can only see UHECR sources from relatively nearby (~ 100 Mpc).
- **The Catch:** This cutoff energy only works for protons. If UHECRs are heavy nuclei (like Iron), the cutoff energy is different.

Multi-Messenger Solution

- **Cosmic Rays:** Are charged, so they are bent by magnetic fields and don't point to their source.
- **Gamma Rays:** Are neutral, but they get absorbed by background light over long distances ($\gamma + \gamma \rightarrow e^+e^-$).
- **Neutrinos:** Are neutral and barely interact. They are the best tool to point directly back to the UHECR sources.

17. Acceleration

1. Fermi Acceleration (General Idea)

- After each encounter, particle gains energy:

$$E = \beta^k E_0, \quad N(> E) = N_0 P^k$$

- Eliminate k :

$$N(> E) \propto E^{\frac{\ln P}{\ln \beta}}, \quad \frac{dN}{dE} \propto E^{-1 + \frac{\ln P}{\ln \beta}}$$

- For first-order Fermi (shock acceleration):

$$\frac{dN}{dE} \propto E^{-2}$$

2. Shock Basics

- Upstream: (ρ_1, v_1, P_1) , Downstream: (ρ_2, v_2, P_2)
- Conservation:

$$\rho_1 v_1 = \rho_2 v_2, \quad \rho_1 v_1^2 = \rho_2 v_2^2 + P_2, \quad \frac{1}{2} \rho_1 v_1^3 = \frac{1}{2} \rho_2 v_2^3 + \frac{3}{2} P_2 v_2$$

- For strong shocks ($P_1 \simeq 0$):

$$\rho_2 = 4\rho_1, \quad v_1 = 4v_2$$

3. Particle Acceleration at Shocks

- Relative velocity between up/down stream:

$$V = \frac{3}{4}U$$

- Energy gain per crossing:

$$\left\langle \frac{\Delta E}{E} \right\rangle = \frac{2}{3}V, \quad \text{Round trip: } \left\langle \frac{\Delta E}{E} \right\rangle = \frac{4}{3}V$$

- Energy gain factor: $\beta = 1 + \frac{4}{3}V$, Escape probability: $P = 1 - U$
- For small U :

$$\frac{\ln P}{\ln \beta} \simeq -1 \Rightarrow \frac{dN}{dE} \propto E^{-2}$$

4. Observed Cosmic Ray Spectrum

- Intrinsic (source) index: $\gamma \approx 2.0 - 2.2$
- After propagation losses: $\gamma_{\text{obs}} \approx 2.7 - 3.3$

5. Maximum Energy (Hillas Criterion)

- From Faraday's law: $\nabla \times \vec{\mathcal{E}} = -\partial_t \vec{B}$
- Dimensional estimate:

$$\mathcal{E} \sim BU, \quad E_{\text{max}} = ZeBUL$$

- Example: young SNR $B \sim 1 \mu\text{G}$, $U \sim 10^4 \text{ km/s}$, $L \sim 1 \text{ pc} \Rightarrow E_{\text{max}} \sim 10^{16} \text{ eV}$
- Hillas plot: $E_{\text{max}} \approx ZBLU$ distinguishes feasible CR sources.

18. Neutrino

• History & Discovery

- **Proposal (Pauli, 1930):** Solved the "missing energy" in beta decay ($n \rightarrow p^+ + e^-$). The electron's energy was a continuous spectrum, not a fixed value, implying a third, unseen particle (the neutrino) was present.
- **Discovery (Cowan & Reines, 1956):** Used a nuclear reactor (a powerful $\bar{\nu}_e$ source) to detect neutrinos via **Inverse Beta Decay** ($\bar{\nu}_e + p^+ \rightarrow n + e^+$).

• Weak Interaction & Parity Violation

- Neutrinos only interact via the weak force.
- **Wu Experiment (1956):** Observed beta decay from aligned Cobalt-60.
- **Result:** Electrons were emitted asymmetrically (violating Parity/mirror symmetry).
- **Conclusion:** The weak force is "left-handed"—it only interacts with **left-handed particles** and **right-handed anti-particles**.

• The Solar Neutrino Problem

- **Experiment (Homestake):** Raymond Davis Jr. used 600 tons of cleaning fluid (C_2Cl_4) to count solar neutrinos ($\nu_e + {}^{37}\text{Cl} \rightarrow {}^{37}\text{Ar} + e^-$).
- **Problem:** He only detected 1/3 of the neutrinos predicted by the Standard Solar Model.

• The Solution: Neutrino Mass

- **Neutrino Oscillation:** Discovered by Super-Kamiokande (1998) and confirmed by SNO (2001-02). Neutrinos change "flavor" as they travel (e.g., $\nu_e \rightarrow \nu_\mu$).
- **Implication:** This oscillation is only possible if neutrinos have **mass**.
- **Significance:** This is physics **Beyond the Standard Model**, which originally assumed neutrinos were massless.

19. GR& Cosmos

• Principles & Observations

- **Cosmological Principle:** Universe is homogeneous and isotropic on large scales.
- **Hubble's Law:** Galaxies are moving away from us.

$$v = H_0 d$$

- **Redshift (z):** Caused by the expansion of space (stretching of light).

$$1 + z = \frac{\lambda_o}{\lambda_e} = \frac{1}{a(t)}$$

where $a(t)$ is the scale factor (with $a = 1$ today).

- **CMB (Cosmic Microwave Background):**
 - * Discovered by Penzias & Wilson (1960).
 - * Perfect blackbody spectrum with $T = 2.726 \text{ K}$.
 - * Proves the early universe was hot and dense.

• Friedmann-Lemaître-Robertson-Walker (FLRW) Model

- **Einstein's Equation:** Connects spacetime geometry to energy/matter.

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- **FLRW Metric:** The metric for a homogeneous, isotropic universe.

$$ds^2 = -c^2 dt^2 + a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right]$$

(Assuming flat, $k = 0$, for this course)

• Cosmic Dynamics (Friedmann Equations)

- **1. Friedmann Eq.:** (Hubble parameter $H = \dot{a}/a$)

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2}$$

- **2. Conservation Eq.:** (Fluid equation)

$$\dot{\rho} = -3H(\rho + P)$$

- **3. Acceleration Eq.:**

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$$

• Cosmic Inventory (Components of the Universe)

- **Equation of State:** $P = w\rho$
- **Density Evolution:** $\rho \propto a^{-3(1+w)}$
 - * **Matter (Dust):** $w = 0 \Rightarrow \rho_m \propto a^{-3} \propto (1+z)^3$
 - * **Radiation:** $w = 1/3 \Rightarrow \rho_r \propto a^{-4} \propto (1+z)^4$
 - * **Dark Energy (Λ):** $w = -1 \Rightarrow \rho_\Lambda \propto a^0$ (constant)
- **Critical Density:** $\rho_{cr} = \frac{3H_0^2}{8\pi G}$. $\Omega_i = \rho_i / \rho_{cr}$.
- **Full Friedmann Eq.:**

$$H(z)^2 = H_0^2 \left[\Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_k(1+z)^2 + \Omega_\Lambda \right]$$

• Cosmological Probes

- **Standard Candles (Type Ia Supernovae):** Known luminosity (L). We measure flux (F) to find Luminosity Distance (d_L).

$$F = \frac{L}{4\pi d_L^2} \quad \text{where} \quad d_L = (1+z) \int_0^z \frac{dz'}{H(z')}$$

- **Key Discovery (1998):** Supernovae were dimmer (farther) than expected. This implies the expansion is **accelerating**.
- **Deceleration Parameter (q_0):** Found to be negative, proving acceleration.

$$q_0 = \frac{1}{2}\Omega_m + \Omega_r - \Omega_\Lambda \approx \frac{\Omega_m}{2} - \Omega_\Lambda < 0$$

- **Hubble Tension:** H_0 measured from the "local" universe (Supernovae) is ~ 73 . H_0 inferred from the "early" universe (CMB) is ~ 67 . This is a major unsolved problem.