2021 IEEE 24th International Conference on Computer Supported Cooperative Work in Design (CSCWD) | 978-1-7281-6597-4/21/$31.00 ©2021 IEEE | DOI: 10.1109/CSCWD49262.2021.9437621

An Orthogonal Learning Design Whale Optimization Algorithm with Clustering Mechanism

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***Abstract*—In this paper, an orthogonal learning (OL) design whale optimization algorithm (WOA) with clustering mechanism, named OLWOA, is proposed to solve the complex continuous problems. In the proposed algorithm, the OL, as an effective strategy to utilize prior search information (experience), is utilized to overcome the disadvantages of the basic WOA, which converges slowly and falls into local optimum easily. The clustering-based mechanism guides the humpback whales to search toward an interesting area by propagating the information of good solutions from one cluster to another cluster. The experimental results reveal the effectiveness and significance of the OL and the clustering-based learning mechanism in the proposed algorithm.**

Keywords—Whale optimization algorithm; Orthogonal learning design; Clustering mechanism; swarm intelligence

I. INTRODUCTION

In recent years, a lot of nature-inspired algorithms have been researched and applied to miscellaneous enormous optimization problems [1], for instance, scheduling [2, 3], industrial manufacturing [4] and aviation [5]. The problems in practical engineering are complex, including non-quadratic, non-convex, non-linear and ruggedness. The original mathematical methods, such as Newton-Raphson method [6], Gradient descent [7] and Simplex algorithm [8] are prone to the problem of combinatorial explosion because of the large solution space to be searched.

Swarm intelligence (SI) is an important branch of the above algorithms. It has been proposed to directly handle the significant and complex optimization algorithms, because it benefits from random property and do not depend on gradient information. Among the series of nature-inspired algorithms, swarm-based mechanism has been widely studied in recent years, in which the critical inspiration is a gather of constantly changing, decentralized, self-organizing agents. These agents interact with their neighbors and surroundings for hunting, foraging, searching, or direction finding [9]. The swarms are usually made up of animals, such as gray wolves, ants, birds and whales [10]. Classical, generic and problem-independent optimization algorithms with competitive and near-optimal results compared to traditional mathematical methods include genetic algorithm (GA) [11], differential evolution (DE) [12], artificial bee colony (ABC) [13], particle swarm optimization (PSO) [10],whale optimization algorithm (WOA) [14] and the other typical hybrid evolution computation algorithm.

WOA is a swarm-based algorithm, which simulates the foraging behavior of humpback whales in oceans [15]. The performance of this method precedes the GA and PSO algorithms for numerical problems [14]. Moreover, in recent years the WOA with several distinguished properties has played a significant role in real-world areas including medical field [16], the synthesis of biodiesel [17], recommendation systems [18], and content-based image retrieval [19]. However, for multimodal functions and hybrid functions, two characteristics widespread problems of swarm intelligence optimizers still exist in the WOA: immature convergence and stagnation in local solutions [20]. Both of these fallings are so persistent they create a negative influence of the final solutions. The taxing problems usually occur when the optimizer not find an appropriate balance between exploratory (global search) and exploitative (local search) inclinations. To mitigate the weaknesses in convergence rate and stagnation, researchers have designed multifarious variations of WOA, since Zhou and Ling proposed a variant of WOA by implementing the Lévy − flight strategy (LWOA) to improve the ability to jump out of local optimal and enhance population diversity in evolutionary process to avoid premature convergence [21]. In [22], a new variant of WOA that combines learning mechanism and variant of hill climbing local search called BMWOA was proposed to improve the exploitation capability. Due to create a proper balance among the exploration and exploitation, the stochastic strategy called β-operator and the neighborhood navigation called N-operator are applied to global search. Chen and Yang proposed a reinforced variant called RDWOA to mitigate the two characteristics widespread problems of swarm intelligence optimizers [15]. There are two strategies which integrate random spare and double adaptive weight in the proposed variant.

The orthogonal design (OD) has been proved to be a commendatory experimental design method, which provided the optimal composition levels for different factors with a reasonable number of experiments. Therefore, the OD has been successfully incorporated in the SI algorithms with multiple parameters and high dimension, including PSO, GA, DE and other algorithms [23]. In the local search phase, the WOA utilizes two mathematical

This work was financially supported by the National Key Research and Development Plan under grant number 2020YFB1713600 and the National Natural Science Foundation of China under grant numbers 62063021. It was also supported by the Lanzhou Science Bureau project (2018-rc-98), Public Welfare Project of Zhejiang Natural Science Foundation (LGJ19E050001), and Project of Zhejiang Natural Science Foundation (LQ20F020011), respectively.

models, encircling prey and spiral bubble-net feeding maneuver, to update the positions of humpback whales towards the best search agent. These two mechanisms are chosen by a probability of 50% for each way. Aiming at mitigating the blindness of random selection, this article proposes an orthogonal learning (OL) for these mechanisms.

The clustering-based learning mechanism has been integrated with SI effectively. The significance of clustering is treated each cluster as a local optimal region in the swarm-based mechanisms. Meanwhile, the information of classy solutions is adequately communicated among different clusters to guide the individuals to carry out development and exploration effectively [24]. In the WOA, each center of cluster is regarded as a leading whale, individual whales located in different clusters search for the leading whale, and then share the classy information with other clusters.

The main modifications are summarized for the basic WOA.

- An OL with two exploitative mechanisms (local search) is proposed to mitigate the blindness of searching for original WOA. The information on search direction of outstanding solution is retained to guide the next search.
- A clustering-based learning strategy is introduced to intensify the cooperation ability of whales from more diverse individuals, and promote collaborative learning.

The remainder of this work is organized as follows. In Section II, the description of traditional WOA and OD are presented. OLWOA is presented in Section III. The experimental study and discussion are presented in Section IV. The conclusion and future research are presented in Section V.

II. Whale Optimization Algorithm and Orthogonal Design

A. *Whale Optimization Algorithm*

The basic operations of the WOA include three phases, searching for prey, encircling prey, and bubble-net attacking method [14]. The first process, searching for prey, is the exploration of WOA. The individual whales follow a random leader to widely find the prey over a whole area. The last two stages are exploitative process, the whale swam toward the global optimum by identifying the position of leading whale. In each loop, the best solution is regarded as the leader. When the population of WOA is initialized, three basic operations are repeated until certain termination criterion (e.g., exhaustion of maximum functional evaluations) is satisfied. In the WOA, the population is a set of real value vectors $x_i = (x_1, \dots, x_D)$, $i = 1, \dots, NP$, where D is the dimension of the objective function, and NP is the size of population.

In the first phase, the position vector of whales is updated with a random leader as:

$$x(t+1) = x_{rand}(t) - A \cdot D \tag{1}$$

$$D = |C \cdot x_{rand}(t) - x(t)| \tag{2}$$

where D denotes the distance among the individuals ($x(t)$) and the random leader ($x_{rand}(t)$), t denotes the current iteration. The A and C denote the coefficient vector, which are calculated as:

$$A = 2ar - a \tag{3}$$

$$C = 2r \tag{4}$$

where r is a random value in [0,1] and the a is linearly dropped from 2 to 0 over the course of iterations.

In the exploitative process, the search agents update their locations by the best leaders. On the one hand, the concise module updates the position of population, and its structure is designed in according to the formula directly as:

$$x(t+1) = x^*(t) - A \cdot D \tag{5}$$

$$D = |C \cdot x^*(t) - x(t)| \tag{6}$$

where D denotes the distance among the individuals ($x(t)$) and the best leader ($x^*(t)$). On the other hand, the author utilizes a spiral to incarnate the peculiar behavior of humpback whales (bubble-net attacking mechanism). The spiral motion is updated according to Eq. (7).

$$x(t+1) = D' \cdot e^{bl} \cdot \cos(2\pi l) + x^*(t) \tag{7}$$

$$D' = |x^*(t) - x(t)| \tag{8}$$

where l is a random number inside [-1,1], b shows a constant. These two mechanisms are performed based on:

$$x(t+1) = \begin{cases} x^*(t) - A \cdot D, & p < 0.5 \\ D' \cdot e^{bl} \cdot \cos(2\pi l) + x^*(t), & p \geq 0.5 \end{cases} \tag{9}$$

B. *Orthogonal Design*

The principal goal of OD is to find the optimal combination of different parameters and levels, and to use less test times [23]. For an experiment with N factors and Q levels for each factor, there are Q^N experimental combinations, the number of which grows exponentially. Therefore, OD is significant for optimization problems. There are two central definitions for OD, orthogonal array (OA) and factor analysis (FA). The OA is denoted as $L_M(Q^N)$, where L is the OA and M is the number of experimental combinations. A concise example of $L_9(3^4)$ is shown as [23],

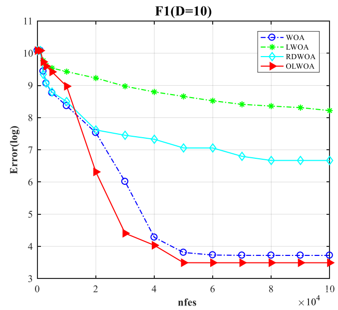
$$L_9(3^4) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 3 & 3 & 3 \\ 2 & 1 & 2 & 3 \\ 2 & 2 & 3 & 1 \\ 2 & 3 & 1 & 2 \\ 3 & 1 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 3 & 3 & 2 & 1 \end{bmatrix}$$

where 1, 2, and 3 denote three levels, and each row represents one test.

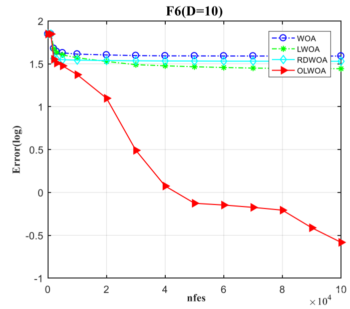
The FA utilizes the combinations of OA to estimate the main effects rapidly according to following formula:

$$S_{ij} = \frac{\sum_{m=1}^{M} f_m \times \emptyset_{mij}}{\sum_{m=1}^{M} \emptyset_{mij}} \quad (10)$$

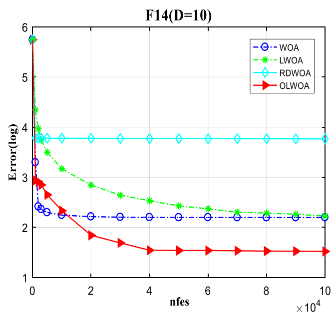
where f_m denote the experimental result of the mth combination, and S_{ij} is the main effect of factor i (i=1,…, N) with level j (j=1,…,Q). The $\emptyset_{mij} = 1$ if the level of the ith factor of the mth combination is j; otherwise, $\emptyset_{mij} = 0$.



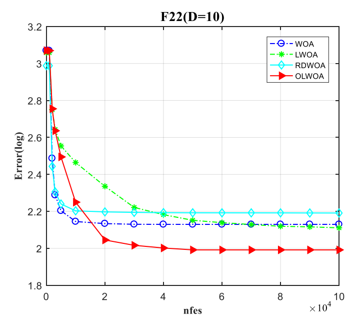
(a) f_1



(b) f_6

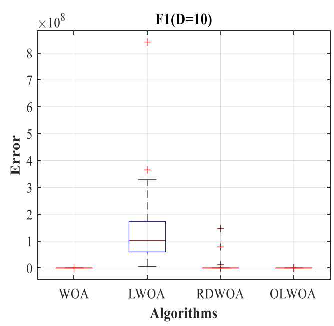


(c) f_{14}

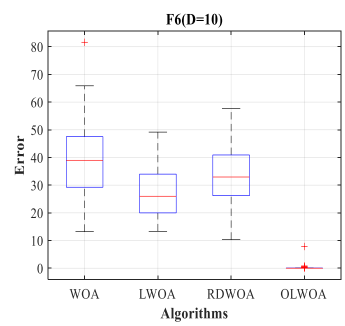


(d) f_{22}

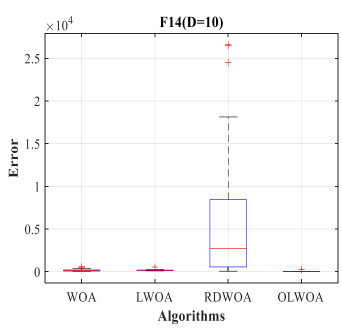
Figure.1 Convergence curves of WOA, LWOA, RDWOA and OLWOA for different CEC 2017 benchmark functions (10D).



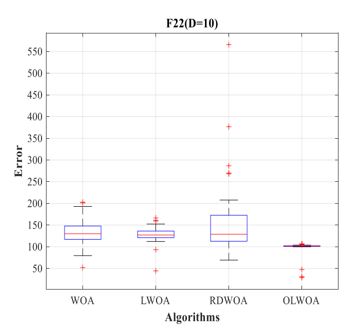
(a) f_1



(b) f_6



(c) f_{14}



(d) f_{22}

Figure.2 Boxplots of WOA, LWOA, RDWOA and OLWOA for different CEC 2017 benchmark functions (10D).

III. Orthogonal Learning Design WOA

A. *Orthogonal Learning*

The most important purpose of OL is to find the most potential search operator on each dimension, so as to exploit deeply. Specifically, the OL mechanism is activated to move the position of whales purposefully when the $stagnated_i$ (the number of stagnations about the ith individual, $i = 1,\dots,N$) exceeds G (maximum number of stagnating generations). One of the motivations is to guide individuals to remedy the defective search

TABLE I. Optimizing Results Tested by Basic WOA, WOA Variants and OLWOA.

Fun	WOA		LWOA		RDWOA		OLWOA	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std
F1	5.25E+03	6.68E+03	1.33E+08	1.84E+08	4.66E+06	2.31E+07	**3.10E+03**	1.43E+04
F3	1.09E+01	2.36E+01	5.20E+02	4.35E+02	3.94E+03	5.81E+03	**8.14E-01**	5.81E+00
F4	2.17E+01	3.17E+01	5.35E+01	5.73E+01	3.86E+01	5.42E+01	**7.97E+00**	1.53E+01
F5	5.56E+01	2.06E+01	4.47E+01	1.50E+01	4.82E+01	2.00E+01	**2.02E+01**	7.20E+00
F6	3.89E+01	1.27E+01	2.76E+01	1.06E+01	3.37E+01	1.13E+01	**2.21E-01**	1.10E+00
F7	7.92E+01	2.26E+01	7.05E+01	1.75E+01	9.93E+01	4.06E+01	**3.76E+01**	9.95E+00
F8	4.36E+01	1.68E+01	2.83E+01	6.82E+00	3.76E+01	1.42E+01	**1.65E+01**	6.78E+00
F9	5.78E+02	4.37E+02	3.01E+02	1.36E+02	8.95E+02	6.69E+02	**5.43E+01**	6.15E+01
F10	1.17E+03	2.71E+02	1.03E+03	2.94E+02	1.08E+03	3.49E+02	**7.14E+02**	2.66E+02
F11	8.98E+01	4.75E+01	1.29E+02	9.64E+01	1.78E+02	7.17E+02	**2.81E+01**	2.08E+01
F12	**1.66E+05**	2.17E+04	3.89E+06	3.88E+06	1.78E+06	2.46E+06	**1.66E+05**	2.55E+05
F13	1.22E+04	1.04E+04	1.63E+04	1.57E+04	1.05E+04	9.60E+03	**4.31E+03**	4.73E+03
F14	1.56E+02	1.10E+02	1.67E+02	6.74E+01	5.81E+03	7.15E+03	**3.30E+01**	3.15E+01
F15	**1.22E+02**	9.43E+01	1.49E+03	1.36E+03	8.01E+03	9.55E+03	1.93E+02	4.86E+02
F16	2.95E+02	1.65E+02	2.63E+02	1.21E+02	3.24E+02	1.87E+02	**2.01E+02**	9.39E+01
F17	1.33E+02	7.76E+01	7.52E+01	2.94E+01	1.24E+02	6.65E+01	**2.87E+01**	2.53E+01
F18	7.80E+03	1.01E+04	2.09E+04	1.52E+04	1.45E+04	1.08E+04	**4.89E+03**	5.33E+03
F19	1.00E+04	1.03E+04	9.90E+03	3.66E+04	7.53E+03	8.66E+03	**2.13E+03**	2.85E+03
F20	1.58E+02	8.05E+01	1.56E+02	6.08E+01	1.88E+02	9.14E+01	**1.84E+01**	1.89E+01
F21	2.15E+02	5.27E+01	1.80E+02	5.95E+01	2.27E+02	5.68E+01	**1.33E+02**	4.66E+01
F22	1.34E+02	2.74E+01	1.28E+02	1.95E+01	1.55E+02	8.16E+01	**9.81E+01**	1.58E+01
F23	3.64E+02	2.54E+01	**3.38E+02**	1.68E+01	3.67E+02	2.70E+01	3.39E+02	2.90E+01
F24	3.85E+02	8.99E+01	3.49E+02	6.18E+01	3.80E+02	8.28E+01	**2.13E+02**	1.25E+02
F25	4.36E+02	5.17E+01	4.68E+02	2.60E+01	4.45E+02	5.68E+01	**4.30E+02**	2.53E+01
F26	8.90E+02	4.48E+02	6.27E+02	3.45E+02	7.59E+02	4.25E+02	**4.95E+02**	2.11E+02
F27	4.76E+02	3.60E+01	4.30E+02	3.76E+01	4.45E+02	3.03E+01	**4.12E+02**	9.83E+00
F28	4.81E+02	1.28E+01	5.93E+02	1.12E+02	5.77E+02	1.82E+02	**4.38E+02**	9.87E+01
F29	4.85E+02	1.20E+02	4.13E+02	7.96E+01	4.79E+02	1.16E+02	**3.26E+02**	5.70E+01
F30	1.01E+04	2.48E+04	9.22E+05	1.15E+06	2.82E+06	4.77E+06	**2.72E+05**	5.46E+05

TABLE II. p-Value of Wilcoxon's Rank-sum Test for D = 10.

OLWOA *vs*	R+	R-	+	≈	−	Z	p-value	α=0.05	α=0.1
WOA	365.00	41.00	26	1	2	-3.689	2.25E-04	Yes	Yes
LWOA	434.00	1.00	28	0	1	-4.681	3.00E-06	Yes	Yes
RDWOA	435.00	0.00	29	0	0	-4.703	3.00E-06	Yes	Yes

in the search space. Eq. (9) is a random strategy, only producing a random parameter p before each update of individual position, thereby bringing about poor exploitation.

In the orthogonal learning stage, each dimension of the solution is regarded as a factor in the OA, so there are obviously D factors. Meanwhile, each factor has two levels in the OA, 0 and 1. Therefore, an OA ($L_M(2^D)$) with two levels is structured by the method mentioned in [23]. If the value of each level equals 0, the mechanism of encircling prey is activated, and the value "1" implies that the mechanism of bubble-net attacking is activated. To illustrate, the following search equation is given:

$$x(t+1) = \begin{cases} x^*(t) - A \cdot D, & L_M(2^D)_{ij} = 0 \\ D' \cdot e^{bl} \cdot \cos(2\pi l) + x^*(t), & L_M(2^D)_{ij} = 1 \end{cases} \quad (11)$$

where $L_M(2^D)_{ij}$ is the level value of combination j with factor i in the $L_M(2^D)$.

B. *Main Framework*

The main framework of OLWOA is shown as follows.

OLWOA main algorithm:	
Input:	N: population size
	T: maximum number of generations
	D: dimension
	n_c: number of clusters
	A and C: coefficient vectors
	G: maximum number of stagnating generations
Step 1:	Initialization: generate an initial population P^0.
Step 2:	Fitness assignment: calculate fitness values of individuals in P^0.
Step 3:	Clustering: cluster N individuals into n_c clusters by *k-means.*
Step 4:	Update location of individuals by basic WOA operations. If the fitness of the updated individuals in different cluster precedes previous value, $stagnated_i = 0$ else $stagnated_i = stagnated_i + 1$.
Step 5:	OL mechanism: When $stagnated_i > G$, structure an OA ($L_M(Q^N)$) here $Q = 2,\ N = D$. If element of the OA is 1, update location of individuals by Eq. 5 and Eq. 6 elseif update location of individuals by Eq. 7 and Eq. 8. Then calculate S_{ij} by Eq. 10, and find its optimum association. Update location of individuals by the optimum association.
Step 6:	Reach the maximum number of iterations, output the optimal solution. Else, the algorithm jumps to step 2.

C. *Computational Complexity*

As shown in main algorithm of OLWOA, a population of size N is clustered into n_c clusters by *k-means*. The computation complexity of this task is O(N*D* n_c), where D is the number of variables. In OL, the construction of an OA needs *O(D)*. Next, the evaluation of the effect of each level on each factor holds *O(M*D)* where M is the number of combinations in OA. The replacement of other individuals in each cluster by the Eq. (1) needs a computation complexity *O(N*D)*, therefore, the computation complexity of the first stage OL is *O(N*D)*. To sum up, the computation complexity of MCCWOA is *O(N*D* n_c)*.

IV. Experimental Study and Discussion

To verify the performance of OLWOA, the simulation experiments on unimodal and multimodal CEC 2017 benchmark problems are adopted. All algorithms are set at same numerical parameters: population size (N) 100 and maximum number of evaluations (FEs) 10×10^4. Meanwhile, each algorithm is tested 51 times independently to obtain fair competition. Table I reports the convergence curves of the mean values about WOA and its variants, Boxplots shows that OLWOA is stable. In conclusion, OLWOA is superior to other comparison algorithms in global performance.

The Wilcoxon-test is utilized to verify the performance difference bet e en OLWOA and other algorithms. Expressly, three pair is e comparisons bet e en OLWOA and others are designed to better visualize the significance of the proposed orthogonal learning strategy and clustering-based learning mechanism. In principle, Wilcoxon-test utilizes the sign test of paired observation data to deduce the probability h en the difference appears. Furthermore, h ether the difference bet ee n the t o means is significant is also determined by the method. From Table II, the R+ represented that the results are outstanding compared ith the other algorithms. R- is the opposite. Particularly, the proposed OLWOA outperforms other comparison algorithms significantly h en solving the test instances, obtaining the better results in terms of mean on 26 out of 29 instances compared ith WOA. The performance of this result for LWOA and RDWOA are 28/29 and 29/29. In Wilcoxon-test, if the p-value is less than the α, that meant there are significant differences in the pair is e algorithms. All the p-values are less than the α on D=10. Therefore, the orthogonal learning strategy and the clustering-based learning mechanism are regarded as an effective direction for improving the performance of the basic WOA.

V. Conclusion and Future Research

According to the experimental results, the ne WOA variant, OLWOA, obtains significant advancement over WOA and its variants. Orthogonal learning mechanism ith predicative ability, instruct WOA to choose the most potential operator h en developing. Mean h ile, the clustering-based learning mechanism utilize different levels of individual information from multiple populations to explore. The proposed OLWOA provides a ne idea for the collaborative learning of t o different but efficient learning mechanisms.

Despite the excellent performance, the follo in g issues need to be solved in future o rk: 1) OL strategy can flexibly s it ch to make the OL strategies adapt to different search stages 2) the clustering-based learning mechanism are clustered in a more reasonable ay (e.g., dynamic adjustment according to the evolutionary state of the population) 3) OLWOA should be applied to solve certain complex scheduling problems (e.g., the flo shop scheduling problems, the job shop scheduling problems, the lot-streaming flo shop scheduling problems and the multi-objective scheduling problems).

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