

An Algorithm Based on Monarch Butterfly Optimization with Learning Mechanism and Topological Structure

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Abstract—In the past decades, various attention has been paid to the global optimization problems. The Monarch Butterfly Optimization (MBO) algorithm is an effective meta-heuristic algorithm for the global optimization problems. However, in the MBO, the diversity of the population is lost in the late iteration. The MBO is easy to trap into the local optima. In this study, an algorithm based on MBO with learning mechanism and topological structure, named LTMBO, is proposed to enhance the ability of exploration and exploitation on the global optimization problems. The learning mechanism is present for the migration operator to increase the speed of the iteration. The topological structure is proposed for the butterfly adjusting operator to improve the diversity of the population. The experimental results demonstrated that the efficiency and significance of the proposed LTMBO algorithm.

Keywords—Monarch butterfly optimization; Learning mechanism; Topological structure

I. INTRODUCTION

In the past few years, the global optimization problems have become increasingly complex, which attracted the attention of various researchers. For solving the global optimization problems, researchers had proposed different algorithms. Without loss of generality, a global optimization is defined as $\min f(x)$, $x = [x_1, x_2, \dots, x_d]$, where the objective is to find x in problem domain d , which maximizes/minimizes $f(x)$ [1].

For the traditional optimization methods, it is difficult to solve the optimization problems when the complexity of the optimization problems is expended. Motivated by the reason, the nature-inspired algorithms which are known as evolutionary algorithms (EAs) were applied to solve the complex optimization problems. EAs are inspired by biological systems or physical processes including water wave optimization (WWO) [2], competitive and cooperative particle swarm optimization with information sharing mechanism (CCPSO) [3] and other hybrid EAs [4]. EAs play a key role for solving the optimization.

The monarch butterfly optimization (MBO) [5] is a new population-based meta-heuristic algorithm, which is inspired by the monarch butterfly species [6]. MBO has various advantages, such as the simple structure of the algorithm and

maintains a special evolutionary mechanism of balancing the exploration and exploitation of the algorithm. The MBO has been applied to solving different optimization problems due to the effective performance of itself. Kim and Chae [7] proposed MBO for solving the facility layout problem based on a single loop material handling path. The simulate results on the 11 instances indicated that the proposed approach generates solutions within a reasonable amount of time. In Aravindan and Seshasayanan [8], the authors proposed a method for image denoising based on MBO. The result exhibited that MBO technique is better than the existing and other traditional methods. In addition, MBO is applied to other domains [9].

However, the drawbacks of the MBO are various, such as easily falls into local optima and the diversity of the population is lost in late iteration. In the past decade years, in order to overcome the drawbacks, the researchers have made various improvement. In Cui, Chen and Yin [10], the authors combined the DE and MBO for balancing the exploration and exploitation. In order to enhance the search ability of MBO, Ghetas, Yong, Sumari and Ieee [11] proposed a new algorithm based on MBO and the harmony search, named MBHS. In Wang, Zhao, Deb and Ieee [12], a greedy strategy is incorporated into the migration and a self-adaptive crossover operator is introduced into the butterfly adjusting operator for enhancing the diversity of the population. In this paper, an algorithm based on the learning mechanism and topological structure is proposed, named LTMBO, to improve the diversity of the population and against the algorithm falling into the local optima. In the process of the evolution, a learning mechanism is introduced in the migration operator of the algorithm to increase the speed of the iteration. The topological structure is proposed in the butterfly adjusting operator of the algorithm for maintaining the diversity of the population. In this paper, the primary contributions are introduced as follows.

- A learning mechanism is introduced in the migration operator of the algorithm to increase the speed of the iteration.
- The topological structure is proposed in the butterfly adjusting operator of the algorithm for maintaining the diversity of the population.

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The structure of this paper is organized as follows. The basic MBO is provided in Section 2. The introduction of the proposed LTMBO is given in Section 3. The experiment and performance analysis are introduced in Section 4. The conclusion and future research are given in Section 5.

II. MONARCH BUTTERFLY OPTIMIZATION

A. Migration behavior

MBO is an effective meta-heuristic algorithm based on the migration behavior of monarch butterflies which lived in southern Canada, the northern USA and Mexico. In the MBO, the population of monarch butterfly is divided into two parts. The number of monarch butterflies in *Land 1* is $\text{ceil}(p * NP)(NP1, \text{Subpopulation } 1)$, and the number of the others in *Land2* is $(NP - NP1)(NP2, \text{Subpopulation } 2)$. NP is the population size. p is the ratio of monarch butterflies in *Land1*. t is the current iteration of the population.

B. Migration operator

In *Land1*, the candidate solutions are generated by the migration operator. The migration operator is defined as follows.

$$\begin{cases} x_{i,k}^{t+1} = x_{r1,k}^t & (r \leq p) \\ x_{i,k}^{t+1} = x_{r2,k}^t & (r > p) \end{cases} \quad (1)$$

$$r = \text{rand} * \text{peri} \quad (2)$$

where, $x_{i,k}^{t+1}$ is the k th element of the i th solution at iteration $t + 1$, $x_{r1,k}^t$ is the k th element of the $r1$ th solution at iteration t , $x_{r2,k}^t$ is the k th element of the $r2$ th solution at iteration t . r is a random number in $[0,1]$ based on Eq. (2), and peri is migration rate which is set to 1.2 in the MBO. $r1$ is a random number from *Subpopulation 1*, and $r2$ is a random number from *Subpopulation 2*.

The process of the migration operator is shown as follows.

Algorithm 1: The migration operator	
Begin	
1	for $i=1$ to $NP1$ do
2	for $k=1$ to D do
3	$r = \text{rand} * \text{peri}$
4	if $r \leq p$ then
5	$x_{i,k}^{t+1} = x_{r1,k}^t$
6	else
7	$x_{i,k}^{t+1} = x_{r2,k}^t$
8	end if
9	end for k
10	end for i
End	

C. Butterfly adjusting operator

In *Land2*, the candidate solutions are generated by butterfly the adjusting operator. The migration process is shown as follows.

$$\begin{cases} x_{j,k}^{t+1} = x_{best,k}^t & (\text{rand} \leq p) \\ x_{j,k}^{t+1} = x_{r3,k}^t & (\text{rand} > p) \end{cases} \quad (3)$$

where, $x_{j,k}^{t+1}$ is the k th element of the j th solution at iteration $t + 1$, $x_{best,k}^t$ is the k th element of the *best* solution at iteration t in *Land1* and *Land2*. $x_{r3,k}^t$ is the k th element of the $r3$ th solution at iteration t . rand is a random number in $[0, 1]$. $r3$ is a random number from *Subpopulation 2*.

$$x_{j,k}^{t+1} = x_{best,k}^t + \alpha \times (d_{x_k} - 0.5) \quad (4)$$

$$\alpha = S_{max} / t^2 \quad (5)$$

$$dx = \text{Levy}(x_j^t) \quad (6)$$

where, if $\text{rand} > \text{BAR}$, the k th element of the j th solution at iteration $t + 1$ is updated based on Eq. (4). BAR is the butterfly adjustment rate. In the Eq. (4), α is the weighting factor based on Eq. (5). α is a key parameter for balancing exploration

and exploitation. In Eq. (5), S_{max} is the maximum step size. In Eq. (6), dx is the step size from Le'vy flight.

The butterfly adjusting operator is shown as follows.

Algorithm 2: The butterfly adjusting operator	
Begin	
1	for $i=1$ to $NP2$ do
2	calculate the α and dx
3	for $k=1$ to D do
4	randomly calculate the rand
5	if $\text{rand} \leq p$ then
6	$x_{j,k}^{t+1} = x_{best,k}^t$
7	else
8	$x_{j,k}^{t+1} = x_{r3,k}^t$
9	if $\text{rand} > \text{BAR}$ then
10	$x_{j,k}^{t+1} = x_{best,k}^t + \alpha \times (d_{x_k} - 0.5)$
11	end if
12	end if
13	end for k
14	end for
End	

D. MBO

The process of MBO are shown in algorithm 3.

Algorithm 3: MBO	
Begin	
1	Initialize the value of t , P , NP , $NP1$, $NP2$, MaxGen , BAR , peri , p , S_{max}
2	calculate the fitness of population.
3	$t=0$
4	while ($t < \text{MaxGen}$)
5	Sort the population
6	Divide the population into <i>Subpopulation 1</i> and <i>Subpopulation 2</i>
7	for $i=1$ to $NP1$ do
8	Generate x by Algorithm 1
9	end for i
10	for $j=1$ to $NP2$ do
11	Generate x by Algorithm 2
12	end for j
13	Evaluate the new population
14	$t+1$
15	end while
16	Output the best solution
End	

III. LTMBO

A. Learning mechanism

In this paper, a learning mechanism is proposed for the migration operator. The purpose of the learning mechanism is that increase the speed of the algorithm during the iteration. The learning mechanism is defined as follows.

$$\begin{cases} x_{i,k}^{t+1} = x_{r1,k}^t + \text{rand} * (l - x_{r1,k}^t) & (r \leq p) \\ x_{i,k}^{t+1} = x_{r2,k}^t + \text{rand} * (l - x_{r2,k}^t) & (r > p) \end{cases} \quad (1)$$

$$r = \text{rand} * \text{peri} \quad (2)$$

$$m = \left\lfloor \frac{\text{num_fit} * NP}{\text{Max_fit}} \right\rfloor \quad (3)$$

where, $x_{i,k}^{t+1}$ is the k th element of the i th solution at iteration $t + 1$, $x_{r1,k}^t$ is the k th element of the $r1$ th solution at iteration t , $x_{r2,k}^t$ is the k th element of the $r2$ th solution at iteration t . r is a random number in $[0,1]$ based on Eq. (2), and peri is migration rate which is set to 1.2 in the MBO. $r1$ is a random number from *Subpopulation 1*, and $r2$ is a random number from *Subpopulation 2*. l is the learning operator. m is the number of candidate operators based on Eq. (3), num_fit is the number of the current fitness calculations, Max_fit is the max number of the fitness calculations, the solution of l has the best fitness in m operators. The candidate operators are proposed by the

historically optimal population. The process of the learning mechanism is defined as Figure 1 and algorithm 4.

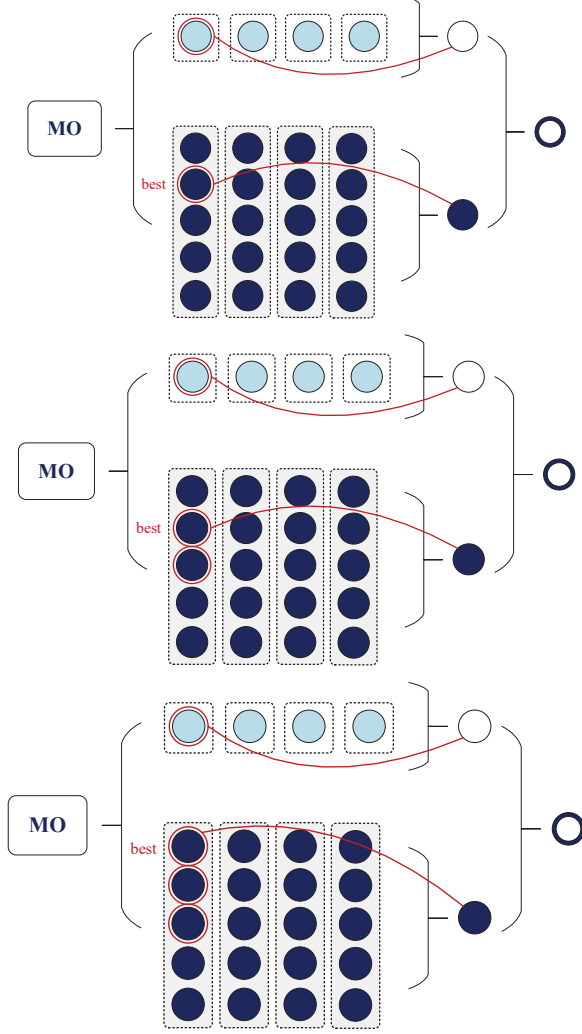


Fig.1. The process of the learning mechanism

Algorithm 4: The learning MO

```

Begin
1  for i=1 to NP1 do
2    for k=1 to D do
3       $r = rand * peri$ 
4      if  $r \leq p$  then
5         $x_{i,k}^{t+1} = x_{r1,k}^t + rand * (l - x_{r1,k}^t)$ 
6      else
7         $x_{i,k}^{t+1} = x_{r2,k}^t + rand * (l - x_{r2,k}^t)$ 
8      end if
9    end for k
10  end for i
End

```

B. Topological structure

In this paper, a topological structure is proposed for the butterfly adjusting operator. The purpose of the topological structure is that improve the diversity of the population during the iteration. The topological structure is defined as follows.

$$\begin{cases} x_{j,k}^{t+1} = x_{best,k}^t & (rand \leq p) \\ x_{j,k}^{t+1} = x_{j,k}^t + normrnd * (x_{j-1,k}^t + x_{j+1,k}^t) & (rand > p) \end{cases} \quad (3)$$

where, $x_{j,k}^{t+1}$ is the k th element of the j th solution at iteration $t + 1$, $x_{best,k}^t$ is the k th element of the *best* solution at iteration t in *Land1* and *Land2*. *rand* is a random number in $[0, 1]$. *normrnd* is a normal distribution random.

The topological structure is defined as Figure 2 and algorithm 5.

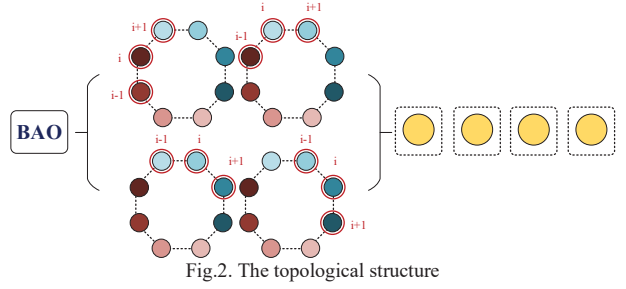


Fig.2. The topological structure

Algorithm 5: The topological BAO

```

Begin
1  for i=1 to NP2 do
2    calculate the  $\alpha$  and  $dx$ 
3    for k=1 to D do
4      randomly calculate the  $rand$ 
5      if  $rand \leq p$  then
6         $x_{j,k}^{t+1} = x_{best,k}^t$ 
7      else
8         $x_{j,k}^{t+1} = x_{j,k}^t + normrnd * (x_{j-1,k}^t + x_{j+1,k}^t)$ 
9      if  $rand > BAR$  then
10        $x_{j,k}^{t+1} = x_{best,k}^{t+1} + \alpha * (d_{x_k} - 0.5)$ 
11     end if
12   end if
13 end for k
14 end for
End

```

C. The process of LTMBO

The details of MBO are outlined in algorithm 6.

Algorithm 6: LTMBO

```

Begin
  Initialize the value of t, P, NP, NP1, NP2, MaxGen,
1   $BAR, peri, p, S_{max}$ 
2  calculate the fitness of population.
3  t=0
4  while (t < MaxGen)
5    Sort the population
6    Divide the population into Subpopulation 1 and
7    Subpopulation 2
8    for i=1 to NP1 do
9      Generate x by Algorithm 4
10   end for i
11   for j=1 to NP2 do
12     Generate x by Algorithm 5
13   end for j
14   Evaluate the new population
15   t+1
16 end while
Output the best solution
End.

```

IV. EXPERIMENT AND PERFORMANCE ANALYSIS

The simulation results in the CEC 2017 benchmark of LTMBO and MBO were listed in Table 1. During the experiment, if the experimental result is less than 10E-8, the error value is directly set to 0. The boldface results in the table represent the best solution among the two algorithms.

TABLE I. THE RESULTS OF THE COMPARED ALGORITHM FOR D=10

Function	Criterion	LTMBO	MBO
1	Mean	1.48E+03	6.40E+05
	Std. Dev.	1.33E+03	4.48E+06
2	Mean	1.06E-04	7.47E+06
	Std. Dev.	1.02E-04	3.11E+07
3	Mean	0.00E+00	4.84E+03
	Std. Dev.	2.24E-07	1.30E+04
4	Mean	4.27E+00	2.79E+01
	Std. Dev.	9.36E+00	5.08E+01
5	Mean	1.26E+01	2.75E+01
	Std. Dev.	4.92E+00	1.03E+01
6	Mean	8.84E-06	4.03E+00
	Std. Dev.	4.32E-06	6.20E+00
7	Mean	2.39E+01	4.00E+01
	Std. Dev.	6.15E+00	1.26E+01
8	Mean	1.34E+01	3.34E+01
	Std. Dev.	5.76E+00	1.66E+01
9	Mean	0.00E+00	2.94E+02
	Std. Dev.	0.00E+00	3.30E+02
10	Mean	4.45E+02	8.90E+02
	Std. Dev.	1.71E+02	3.17E+02
11	Mean	9.74E+00	4.54E+02
	Std. Dev.	4.86E+00	1.45E+03
12	Mean	9.59E+03	1.08E+07
	Std. Dev.	8.78E+03	3.84E+07
13	Mean	9.37E+03	2.57E+05
	Std. Dev.	8.78E+03	1.20E+06
14	Mean	2.79E+03	8.86E+03
	Std. Dev.	4.15E+03	9.27E+03
15	Mean	2.75E+03	9.87E+03
	Std. Dev.	4.93E+03	2.21E+04
16	Mean	1.76E+02	2.39E+02
	Std. Dev.	1.11E+02	1.77E+02
17	Mean	2.03E+01	1.12E+02
	Std. Dev.	1.88E+01	7.44E+01
18	Mean	6.80E+03	4.35E+05
	Std. Dev.	7.90E+03	2.76E+06
19	Mean	4.86E+03	2.78E+04
	Std. Dev.	5.98E+03	9.57E+04
20	Mean	9.04E+00	7.26E+01
	Std. Dev.	1.78E+01	6.05E+01
21	Mean	1.85E+02	2.19E+02
	Std. Dev.	5.47E+01	5.24E+01
22	Mean	1.33E+02	1.71E+02
	Std. Dev.	1.47E+02	3.02E+02
23	Mean	3.27E+02	3.38E+02
	Std. Dev.	1.23E+01	2.02E+01
24	Mean	3.37E+02	3.69E+02
	Std. Dev.	7.97E+01	7.15E+01
25	Mean	4.22E+02	4.44E+02
	Std. Dev.	2.37E+01	5.94E+01
26	Mean	3.45E+02	6.48E+02
	Std. Dev.	1.74E+02	3.07E+02
27	Mean	4.08E+02	5.00E+02
	Std. Dev.	1.93E+01	1.50E-04
28	Mean	4.10E+02	4.99E+02
	Std. Dev.	1.43E+02	3.80E+00
29	Mean	3.03E+02	3.91E+02
	Std. Dev.	3.91E+01	8.26E+01
30	Mean	2.45E+05	1.12E+04
	Std. Dev.	4.62E+05	2.09E+04

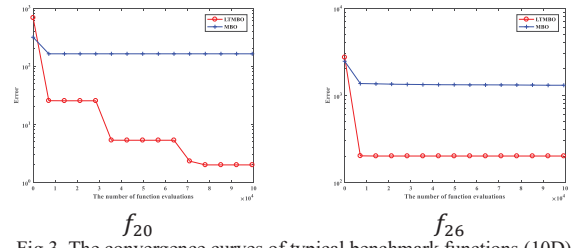
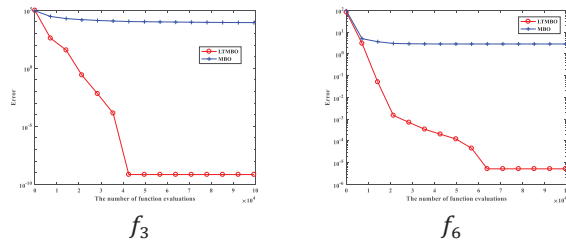


Fig.3. The convergence curves of typical benchmark functions (10D)

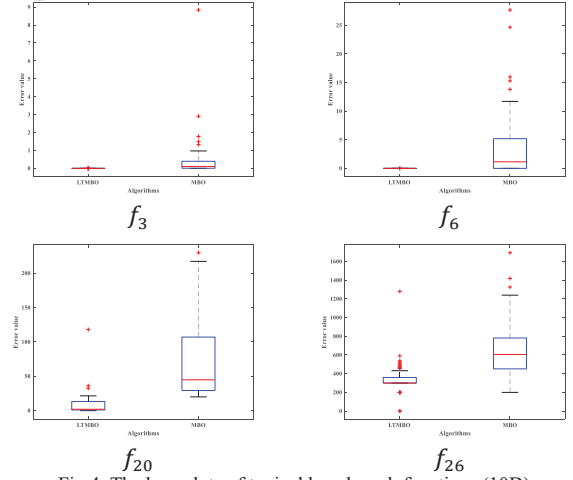


Fig.4. The box-plots of typical benchmark functions (10D)

TABLE II. RANKINGS OBTAINED THROUGH WILCOXON'S TEST

LTMBO VS	R+	R-	<i>p</i> -value	$\alpha = 0.05$	$\alpha = 0.01$
MBO	439	26	2.163E-05	Yes	Yes

To further demonstrate the performance of LTMBO and MBO, the convergence curves of f_3 , f_6 , f_{20} and f_{26} on 10D were showed in Figure 3. As shown in the Figure 3, LTMBO accomplished the fastest speed of the convergence and the best accuracy of the convergence on f_3 , f_6 , f_{20} and f_{26} . The box plots of f_3 , f_6 , f_{20} and f_{26} on 10D were showed in Figure 4. From the box plots, the performance of the HMBO is excellent. The Wilcoxon's test, which compares the algorithms in pairs, is carried out to detect the significant difference between LTMBO and MBO. The statistical analysis results are listed in Table 2. From Table 2, the proposed LTMBO significantly outperforms the MBO with $\alpha = 0.05$ and $\alpha = 0.01$ when D=10 on solving the CEC-2017 benchmark functions. In general, the performance of the LTMBO outperforms the MBO.

V. CONCLUSION AND FUTURE RESEARCH

In this paper, an algorithm based on monarch butterfly optimization with learning mechanism and topological structure is proposed, named LTMBO. In the LTMBO, from the experimental results, the speed of the algorithm during the iteration is effectively increased by the learning mechanism. The diversity of the population is effectually improved by the topological structure. In general, the learning mechanism and topological structure are beneficial for the algorithm to generate the satisfactory solutions in limited time. From the simulated results on CEC-2017 benchmark functions and the Wilcoxon's sign rank test, the performance of LTMBO is better than MBO. The LTMBO is effective and robust.

The future research is conducted in the following directions. First, the expected time analysis is important. Second, the LTMBO is to apply in the scheduling problems. Third, the HBBO-CMA will also be embedded in the machine learning and other research fields.

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