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A hierarchical guidance strategy assisted fruit fly optimization algorithm with cooperative learning mechanism

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ABSTRACT

The fruit fly optimization algorithm (FOA) has drawn enormous attention from researchers and practitioners in the computation intelligence domain for the benefits of simple implementation mechanism and few parameters tuning requirement of FOA. However, FOA is hard to adapt directly to address complex continuous problems. A hierarchical guidance strategy assisted fruit fly optimization algorithm with cooperative learning mechanism (HGCLFOA) is proposed in this study. The population is divided into elitist and inferior subpopulations with the fitness of objective function. The population center is re-designed as an elitist subpopulation to maintain the diversity of the population. In the olfaction search stage, the hierarchical guidance strategy is introduced for local search according to the difference of solution qualities to assign inferior individuals to elitist individuals on different levels. Meanwhile, the inferior information is applied by the inferior solutions repairing strategy to deflect the prediction of the elitist subpopulation for preventing HGCLFOA from falling into the local optimum. In the vision search stage, a hybrid Gaussian distribution estimation strategy is adopted to extract the elitist information of previous generations to predict the distribution of potential elitist individuals in the next generation. The exploration and exploitation of the HGCLFOA are balanced by the cooperation between elitist subpopulation and inferior subpopulation. A random walk strategy is activated to assist the elitist solutions to jump out the local optimal. The parameters of the HGCLFOA are calibrated by DOE and ANOVA methods. The experimental results demonstrated that the HGCLFOA outperformed the classical FOA and state-of-arts variants of FOA.

1. Introduction

The optimization algorithms, which are based on mathematics and operation science, have attracted huge attention from researchers and practitioners in various domains. In recent years, optimization algorithms have been rapidly promoted and applied in various engineering fields, for instance, aviation manufacturing, artificial intelligence, and production schedule. The real engineering problems have the common features of high complexity, multiple constraints, non-linearity, multi-extreme values, difficulties in modeling. The solution space of the application problems is larger than that of the certain problem solved by the mathematics method. The dimensional disaster problems are accompanied by the increase of the problem scales. Therefore, the black-box optimization problem, which is the blinded operation process, is generated. Under the condition of limited resources, the maximum benefit or the minimum investment is the goal of the optimization algorithms (Zhao, He, & Wang, 2020). The optimization algorithm is a prominent way to obtain the optimum solution.

The fruit fly algorithm (FOA) (Pan, 2012) was proposed by Pan in 2012. Compared with other evolutionary algorithms, FOA is prone to understand and simply achieved by programming. Therefore, it is widely used in various engineering problems and also attracted much attention from researchers in recent years. A series of variants are proposed and applied in neural networks and many other engineering applications. The FOA has shown good performance in solving some simple or specific problems. However, for solving complex problems, such as benchmark CEC 2017, which accompany with the non-separable and rotationally variation properties, the performance of FOA needs to be improved furtherly.

The original FOA has the superior ability in exploitation, while the ability in exploration is weak. Comparing with other swarm intelligence algorithms, the most distinctive feature of the FOA is that all the

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individuals in the population learn information about the center of the population. However, learning from population centers too blindly will reduce population diversity. Therefore, an improved GEDA strategy is considered in the vision search to control the search tendency through the feedback of previous experience. In the progress of the original search, each element of n dimensions individual is mapping from two-dimensional coordinate space to one-dimensional coordinate by using square root and reciprocal calculations. These operations cause the search space to shrink sharply and to gather near the origin coordinates. This is the reason that FOA has extraordinary performance when the solution to the corresponding problem is located in the origin. Though the FOA is a swarm intelligence algorithm, the mechanism of FOA is a single-point search. Especially in the problem model with an intricate fitness landscape, the algorithm is hard to find a promising neighborhood. In the proposed HGCLFOA, the hierarchical guidance strategy is employed in the olfactory. The swarm center is extended to a part of elite individuals, and the fruit fly individuals in the swarm are guided purposely through hierarchical knowledge. This strategy provides different search directions and improves the diversity of the population effectively. The random walk strategy is applied in some elite individuals to jump out the local optima. Integrating the above strategies, the HGCLFOA is proposed in this study, the main contributions are summarized as follows.

In the olfactory search stage, the hierarchical guidance strategy is used to guide the local search around different search directions to enhancing the diversity of the population. The inferior solution repairing strategy (ISR) is introduced to disturb the population and to prevent the population from dropping into the local optimal. Through a feedback machine, the strategies of inferior individuals are adjusted timely for adapting the different search processes.

In the vision search stage, the hybrid GEDA is introduced to guide the evolution of superior solutions in fruit fly swarm. Constructing the probability model to guide the evolution of population efficiency. The random walk strategy is utilized to assist the population to jump out of local optima.

The remainder of this paper is organized as follows. Section 2 is the related Works of this study are given below. The review of the original FOA is described in Section 3 simply. The proposed algorithm HGCLFOA is introduced in detail in Section 4. The results and discussion of experiments are presented in Section 5. Finally, the conclusions and future works are summed up in Section 6.

2. Related works

In the past decades, with the development of optimization algorithms, evolutionary algorithms(EAs) (Kennedy, 2001), which are inspired by biology and phenomena of nature, have been concerned by scholars and engineering practitioners, for instance, particle swarm optimization algorithm(PSO) (Kennedy & Eberhart, 1995; Mirghasemi, Andreae, & Zhang, 2019), genetic algorithm(GA) (Goldberg, 1989; Kim, 2018), differential evolution algorithm (DE) (Storn & Price, 1997; Zhao, Zhao, Wang, & Song, 2020), simulated annealing algorithm (SA) (Johnson, Aragon, McGeoch, & Schevon, 1989; Turanoğlu & Akkaya, 2018), etc. The various evolutionary algorithms are widely applied in vast domains because of the simplicity, flexibility, and efficiency. PSO is inspired by bird swarm in which the individuals are impacted by the combined effect of these factors including inertia, society, and neighbors in the process of seeking the global best solution iteratively. Similarly, the GA simulates the evolutionary process of biology in natural selection. The chromosomes of the GA are recombined by selection, crossover, mutation to achieve the continuous optimization of chromosomes until the number of iterations is satisfied or the fitness value is stabilized. DE algorithms developed from the GA and have the characteristics of simple operation, strong stability and convergence. SA algorithm imitates the solid cooling process, in which the inferior solutions are accepted probability for obtaining the global optimal randomly. In addition to these classic EAs, other evolutionary algorithms are also booming gradually. Ant colony optimization (ACO) (Ruiz & Stützle, 2007) algorithm, through pheromone placement by ants and the volatilization of pheromone, the optimal path is obtained. Biogeography-based optimization (BBO) (Dan, 2009; Zhao et al., 2019) is inspired by the migration and mutation in biology, and the quality of inferior solutions is improved continuously. Water wave optimization (WWO) (Zhao, Zhang, Cao, & Tang, 2020; Zheng & Yu-Jun, 2015) is based on the shallow water wave model, and by controlling the movement of waves, the global optimal is approaching gradually. In artificial bee colony optimization (ABC) (Karaboga & Basturk, 2007), the bee foraging behavior in the swarm is imitated, individuals simulate the behavior of the three types of bee activities, and finally, the best food source is found through the coordination between the populations. The grey wolf algorithm (GWO) (Mirjalili, Mirjalili, & Lewis, 2014) simulates the hierarchical predator–prey mechanism in wolves and obtains the optimal solution by using the cooperation among wolves. Based on the relationship between tree and seed, the tree-seed algorithm(TSA) (Kiran, 2015) is proposed, in which the exploration and exploitation are balanced by a search tendency parameter. As stochastic optimization algorithms, the key to obtaining outstanding performance is to balance the exploration and the exploitation of algorithms combining with different strategies and mechanisms.

The FOA is first proposed for forecasting the financial tendency in (Pan, 2012). Since then, because of the simple principle and good performance, the FOA has been employed widely in many engineering problems (Darvish & Ebrahimzadeh, 2018). For instance, the FOA combined with geometric reasoning approach to reducing production cycle of casting, in the research (Wang et al., 2018) and the performance of the result is valid. Gao and Liu proposed a LSSVM (Gao & Liu, 2018) which is used to classify cancer diagnosis in which the parameters are optimized by a combination of FOA and PSO. Kanarachos (Kanarachos, Griffin, & Fitzpatrick, 2017) proposed a contrast-based fruit fly optimization algorithm, and the algorithm is utilized for structure optimization of truss design. For the identification of dynamic protein complexes, FOA is combined with gene expression profiles, and the results of the experiment demonstrate that FOCA (Lei, Ding, Fujita, & Zhang, 2016) is very effective.

Because of the simple theory, the flexible framework, and ease to nested with various problems, the theory and improvement of FOA have also been closely concentered in recent years. A modifying FOA (MFOA) is proposed by Pan (Pan & Wen-Tsao, 2013), in which the decision variable is determined by the fruit fly individuals in three-dimension space. And an escaping coefficient is introduced to expand the search space in the negative space. In the LGMS_FOA (Shan, Cao, & Dong, 2013), the fruit fly individuals in search space are directly represented by the distance in one dimension space rather than determined by the distance in two-dimension space. And in the search process, the individuals are generated through the nonlinear generation strategy. Compare with the linear generation of candidates in original FOA, the ability to balance the exploration and exploitation are enhanced. Among the algorithms of IFFO (Pan, Sang, Duan, & Gao, 2014), a dynamic parameter is introduced to change the search radius to adapt the iterations. The individuals in the swarm are selectively updated according to the uniform distribution to intensive the local search. The performance of IFFO is superior especially in high dimensions. The trend search strategy is used in CEFOA (Han, Liu, Wang, & Wang, 2018). Through the feedback of food source quality, to guide the search direction to the potential area self-adaptively. At the same time, the co-evolution mechanism is embedded, which assists the swarm escape from local optima. The exploration ability is obviously improved compared with the original FOA. The CEFOA is verified in the clustering parameter problem, and the result shows the algorithm is efficient. In the MCFOA (Zhang, Xu, Yu, Heidari, & Li, 2019), the gaussian mutation is introduced to the original FOA to replace the random generation strategy. At the same time, a chaotic local search is applied for local search and

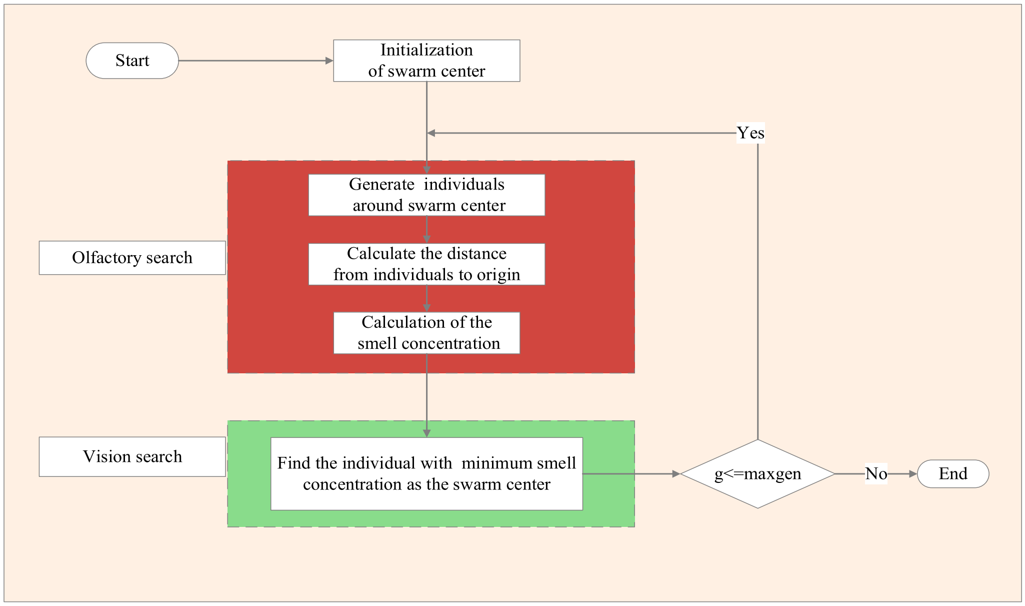


Fig. 1. The flowchart of the original FOA.

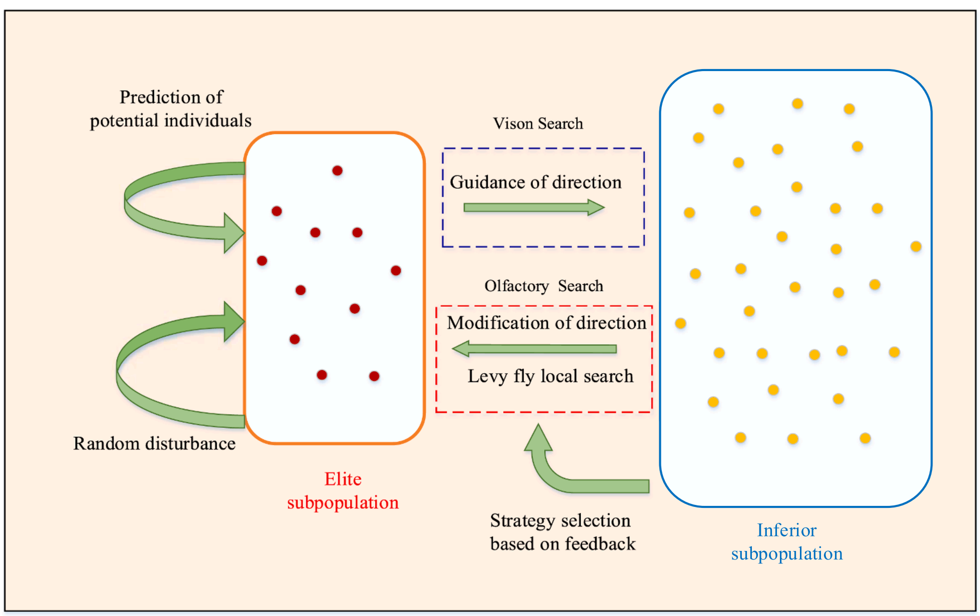
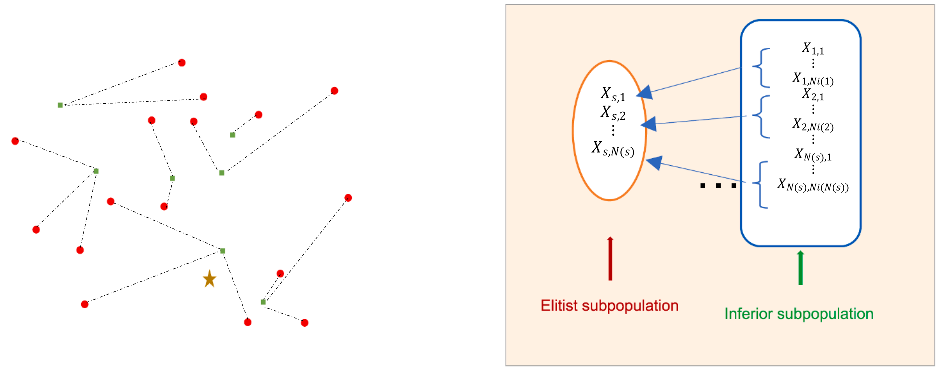


Fig. 2. The total ideas of the proposed HGCLFOA.

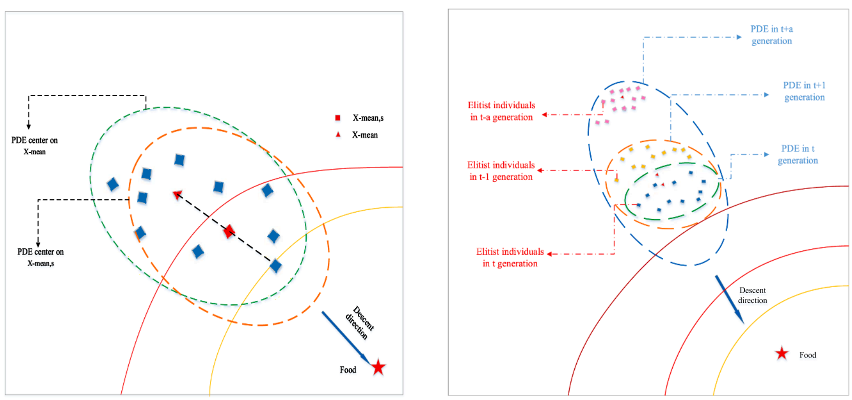
balance the exploration and exploitation. The MCFOA was used in feature selection problems, and the results is verified the accuracy of the CEFOA algorithm. The initialization based on the opposition learning strategy is employed in the SFFO (Sang, Pan, & Duan, 2019) to increase the diversity of the population. The feedback strategy of successful probability is utilized in the olfactory search stage to lead the swarm search to the potential area. A new inertia weight function is utilized to adjust the search domain dynamically to increase the efficiency of local search in the efficient algorithm IFOA (Tian & Li, 2019). And the crossover operation is utilized to increase the diversity of the swarm. The cluster strategy based on cooperative learning is high performance in the high dimensional problems. The different search radius in the two subpopulations is used in the algorithm HACLFOA (Ding, Dong, & Zou, 2019). The different search range is decided by the information in previous generation. The new generation of candidates are utilized the information of the population sufficiently. The cooperative learning strategy in HACLFOA improves the ability to balance the global and local search. And the high performance of HACLFOA is shown in the instance of optimization of multilevel image thresholding. The two versions of variants pFOA_v1 and pFOA_v2 (Iscan, Kiran, & Gunduz, 2019)are proposed, which are combined with Jaya algorithm (Venkata Rao, 2016) in vision search, by using the idea of moving towards the best individuals and far away from the worst individuals. The introduction of information on inferior solutions improves the diversity of the population effectively and improves the ability to escape from the local optimum. The Orthogonal tables are introduced in the GOLFOA (Yang, Chen, Li, Heidari, & Wang, 2020) to guarantee the swarm search in the potential area. And the mutation is applied in the swarm center to perturbation the swarm. The two strategies cooperate with each to adapt to different search stage. In the IAFOA (Wu, Liu, Tian, Zhang, & Xiao,



(a) hierarchical guidance in landscape

(b)the rule of hierarchical

Fig. 3. The hierarchical guidance strategy.



(a) mean shift

(b) covariance adjusting

Fig. 4. The mean shift and the covariance based on different archive.

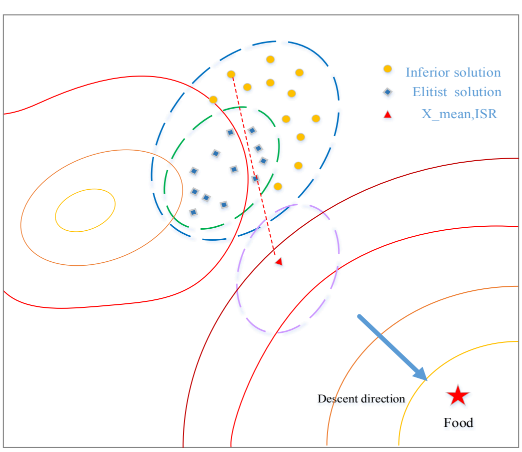


Fig. 5. The simulation of ISR in landscape.

2018), the previous information of superior individuals is used to guide the search, and the adaptive step range is used to satisfied the different requirement of different search stage. And the adaptive mutation and crossover strategy is used to prevent the fruit fly individuals to concentrate in the swarm center excessively.

Although the variants mentioned above have high performance in some specific problems, the clustering of the population center still affects the performance of the fruit fly algorithm on complex problems. These existing variants of FOA still are difficult to solve complex problems adaptively, especially for complex engineering problems with the characteristic non-separable and rotationally variation. With the development of artificial intelligence, experience-based learning strategies are also introduced to the evolutionary algorithm. Instead of adjusting the parameters for a specific problem, the learning strategy can be adapted to more different problem models for gathering and summarizing the experience of previous generations and guiding the search. As far as the author knows, combining with learning mechanism is a research focus recently. Because this mechanism addresses various problems adeptly and efficiently. For the existing research on the combination of FOA and learning mechanism, it is generally based on the simple feedback of changes in fitness values to adjust parameters in every generation. Such learning strategies are not able to adjust the search direction of the population to potential area essentially. The typical knowledge guidance algorithm distribution estimation algorithm (EDA) (Larranga & Lozano, 2001) is one of the most concerned learning algorithms in past decades. Different from traditional EAs which guide the individuals evolving one by one in the population, the EDA predicts the evolutionary trend of the whole population through establishing a mathematical model. The Gaussian distribution estimation algorithm (GEDA) (Shahraki & Tutunchy, 2013) is one of the most popular EDA algorithms in resolving continuous problems. Univariate GEDA, bivariate GEDA, and multivariate GEDA are three different molds of GEDA. About the three types, multivariate GEDA describes the relationship between variables through a probability model, which is most applicable for solving nonlinear and variable non-separable optimization

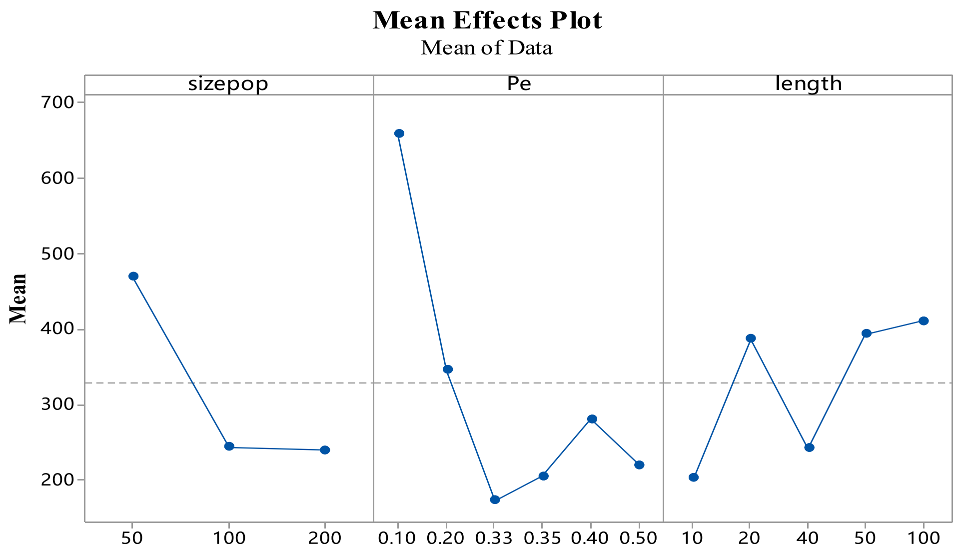


Fig. 6. The mean effects plot.

Table 1
ANOVA results for parameter settings of HGCLFOA.

Source	Sum of squares	Degrees of freedom	Mean Square	F-ratio	P-value
Sizepop	4.23E + 05	2	2.42E + 05	3.58	**0.0372**
Pe	1.22E + 06	5	2.43E + 05	4.11	**0.042**
Length	4.03E + 05	4	1.01E + 05	1.71	0.1679
Sizepop*Ne	1.25E + 06	10	1.25E + 05	2.11	**0.0462**
Sizepop*length	5.30E + 05	8	6.62E + 04	1.12	0.3707
Ne*length	1.32E + 06	20	6.60E + 04	1.12	0.3713
Error	2.37E + 06	40	5.91E + 04		
Total	7.51E + 06	89			

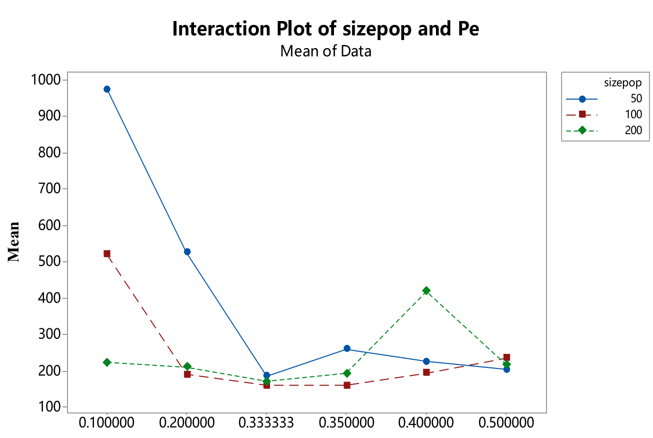


Fig. 7. The interaction plot.

problems. The traditional GEDA is powerful in searching for potential solutions. Although the traditional GEDA is powerful in search of potential solutions, on the one hand, with the increase of iteration numbers, the probability density ellipsoid is perpendicular to the direction of fitness decline; on the other hand, numerous samples are needed for building the accurate probability function model. Many improvements are proposed to overcome the shortcomings mentioned above. The mean shift is used to skip the local optima and improve the quality of solutions (Bosman, Grahl, & Thierens, 2008). The inferior solutions are used to adjust the direction and scale of covariance (Liang, Ren, Lin, Bei, & Hossain, 2017). An external archive is introduced to provide enough sample points and find the global search direction (Liang et al., 2020).

3. The original fruit fly optimization algorithm

The fruit fly is a common insect in daily life, due to the super olfaction and vision for foraging behavior, the food source is quickly found. In the fruit fly population, the individual judges the food source by olfactory sensation, and compares the smell concentration with other partners. Then they approach the individual who is nearest to the food by the visual sense. This process is circulated until they found the food.

Initialization of FOA is generated an initial swarm center, the center is determined by two-dimension coordinates X_S, Y_s. In the olfaction search stage, the individuals generated around the center, the distance of individuals to the origin is$D_i = \sqrt{X_i^2 + Y_i^2}$, the smell is defined as$S_i = 1/D_i$, the fitness of *ith* individuals is calculated byS_i. The best fitness individual is the center of the next generation. The flowchart of FOA is detailed shown in Fig. 1.

4. The proposed HGCLFOA

The original FOA although is easy to understand, the exploration and exploitation of FOA both are limited in solving complex functions problems. In the proposed HGCLFOA, the local search is improved by a hierarchical guidance strategy in the olfactory search stage. In vision search, the global search is guided under the information of previous to accelerate the convergence speed and maintain the diversity. The exploration and exploitation in HGCLFOA are balanced by the cooperation and feedback in the population. In this section, the hierarchical guidance strategy-assisted cooperative learning fruit fly optimization algorithm (HGCLFOA) is introduced in detail in Fig. 2.

4.1. *Hierarchical guidance strategy in olfactory search*

In the olfactory search stage of the original fruit fly optimization algorithm, all individuals learn from the current global optimum, resulting in a reduction of diversity. Thus, a hierarchical guidance

Table 2
Rankings obtained through Wilcoxon test.

D	Algorithms	vs.	+	−	=	R+	R−	Z	P-value	HB	0.05	0.1
10	HGCLFOA	FOA	30	0	0	4.65E + 02	0.00E + 00	−4.78E + 00	**1.73E−06**	**1.557E−05**	**yes**	**yes**
		LGMS_FOA	30	2	0	4.65E + 02	1.40E + 01	−4.78E + 00	**1.73E−06**	**1.557E−05**	**yes**	**yes**
		JS_FOA	30	0	0	4.65E + 02	0.00E + 00	−4.78E + 00	**1.73E−06**	**1.557E−05**	**yes**	**yes**
		HACLFOA	30	0	0	4.65E + 02	0.00E + 00	−4.78E + 00	**1.73E−06**	**1.557E−05**	**yes**	**yes**
		IFFO	30	0	0	4.65E + 02	1.04E + 02	−4.78E + 00	**1.73E−06**	**1.557E−05**	**yes**	**yes**
		SFFO	30	0	0	4.65E + 02	7.15E + 01	−4.78E + 00	**1.73E−06**	**1.557E−05**	**yes**	**yes**
		IFOA	28	2	0	4.61E + 02	0.00E + 00	−4.70E + 00	**2.60E−06**	**1.557E−05**	**yes**	**yes**
		TSA	30	0	0	4.65E + 02	0.00E + 00	−4.78E + 00	**2.00E−06**	**1.557E−05**	**yes**	**yes**
		GWO	29	1	0	4.50E + 02	1.50E + 01	−4.47E + 00	**1.73E−06**	**1.557E−05**	**yes**	**yes**
30	HGCLFOA	FOA	30	0	0	4.65E + 02	0.00E + 00	−4.78E + 00	**1.73E−06**	**1.557E−05**	**yes**	**yes**
		LGMS_FOA	30	0	0	4.65E + 02	0.00E + 00	−4.78E + 00	**1.73E−06**	**1.557E−05**	**yes**	**yes**
		JS_FOA	29	1	0	4.64E + 02	1.00E + 00	−4.76E + 00	**1.73E−06**	**1.557E−05**	**yes**	**yes**
		HACLFOA	30	0	0	4.65E + 02	0.00E + 00	−4.78E + 00	**1.73E−06**	**1.557E−05**	**yes**	**yes**
		IFFO	30	0	0	4.65E + 02	0.00E + 00	−4.78E + 00	**1.73E−06**	**1.557E−05**	**yes**	**yes**
		SFFO	30	0	0	4.65E + 02	0.00E + 00	−4.78E + 00	**1.73E−06**	**1.557E−05**	**yes**	**yes**
		IFOA	24	6	0	4.03E + 02	6.20E + 01	−3.51E + 00	**4.53E−04**	**9.060E−04**	**yes**	**yes**
		TSA	30	0	0	4.65E + 02	0.00E + 00	−4.78E + 00	**2.00E−06**	**1.557E−05**	**yes**	**yes**
		GWO	19	11	0	3.61E + 02	1.04E + 02	−2.64E + 00	**8.22E−03**	**8.220E−03**	**yes**	**yes**
50	HGCLFOA	FOA	30	0	0	4.65E + 02	0.00E + 00	−4.78E + 00	**1.73E−06**	**1.557E−05**	**yes**	**yes**
		LGMS_FOA	30	0	0	4.65E + 02	0.00E + 00	−4.78E + 00	**1.73E−06**	**1.557E−05**	**yes**	**yes**
		JS_FOA	28	2	0	4.62E + 02	3.00E + 00	−4.72E + 00	**2.35E−06**	**1.557E−05**	**yes**	**yes**
		HACLFOA	28	2	0	4.38E + 02	2.70E + 01	−4.23E + 00	**2.37E−05**	**7.110E−05**	**yes**	**yes**
		IFFO	30	0	0	4.65E + 02	0.00E + 00	−4.78E + 00	**1.73E−06**	**1.557E−05**	**yes**	**yes**
		SFFO	30	0	0	4.65E + 02	0.00E + 00	−4.78E + 00	**1.73E−06**	**1.557E−05**	**yes**	**yes**
		IFOA	24	6	0	3.34E + 02	1.31E + 02	−2.09E + 00	**3.68E−02**	**7.360E−02**	**yes**	**yes**
		TSA	30	0	0	4.65E + 02	0.00E + 00	−4.78E + 00	**2.00E−06**	**1.557E−05**	**yes**	**yes**
		GWO	17	13	0	3.31E + 02	1.34E + 02	−2.03E + 00	**4.28E−02**	**7.360E−02**	**yes**	**yes**
100	HGCLFOA	FOA	30	0	0	4.65E + 02	0.00E + 00	−4.78E + 00	**1.73E−06**	**1.557E−05**	**yes**	**yes**
		LGMS_FOA	30	0	0	4.65E + 02	0.00E + 00	−4.78E + 00	**1.73E−06**	**1.557E−05**	**yes**	**yes**
		JS_FOA	28	2	0	4.62E + 02	0.00E + 00	−4.72E + 00	**2.35E−06**	**1.557E−05**	**yes**	**yes**
		HACLFOA	28	2	0	4.25E + 02	0.00E + 00	−3.96E + 00	**7.51E−05**	**2.253E−04**	**yes**	**yes**
		IFFO	29	1	0	4.64E + 02	6.30E + 01	−4.76E + 00	**1.92E−06**	**1.557E−05**	**yes**	**yes**
		SFFO	29	1	0	4.64E + 02	3.20E + 01	−4.76E + 00	**1.92E−06**	**1.557E−05**	**yes**	**yes**
		IFOA	15	15	0	2.29E + 02	2.36E + 02	−7.20E−02	9.43E−01	9.430E−01	no	no
		TSA	30	0	0	4.65E + 02	0.00E + 00	−4.78E + 00	**2.00E−06**	**1.557E−05**	**yes**	**yes**
		GWO	15	15	0	3.33E + 02	1.32E + 02	−2.07E + 00	**3.87E−02**	**7.740E−02**	**yes**	**yes**

strategy is proposed. The population is divided into an elitist population and an inferior population. In the olfactory search phase, the strategy assigns different objects from the elitist population to provide search directions for inferior solutions. The top 33% of the solutions (this part is verified and modified in the experiments) in the population is chosen as the elitist, and the rest part of inferior solutions. A schematic view of the solutions in the hierarchical learning strategy is shown in Fig. 3(b), and (a) is the simulation of the search process of fruit flies with different learning objectives in the landscape.

The allocation rules are closely related to the objective space. Depending on the different initial function values between inferior solutions and superior solutions, a different number of inferior solutions is assigned to each superior solution. Different learning objectives are assigned to different suboptimal solutions. According to different levels by sorting, the high level the elite individual is, the more inferior solutions are assigned to learn from it. In each generation, the inferior solutions are reordered and learning goals are assigned according to the current population status. The number of assigned inferior solutions is calculated as follows.

$$D_n = f(n) - f\left(x_{i,1}\right) \tag{1}$$

$$Ni(n) = round\left\{\left|D_n \Big/ \sum_{i=1}^{N(s)} D_i\right| * (SumNi)\right\} \tag{2}$$

where n is different elitist individual and $n = X_{s,1}, X_{s,2}, \cdots X_{s,N(s)}$, $N(s)$ is the total number of elitist individuals. The $f(n)$ is the fitness value of individual n. $Ni(n)$ is the number of inferior solutions that are guided by the elitist individual n. The $SumNi$ is the summary number of inferior individuals. In every generation, the individuals in the population are restarted sorting.

a) The local search around different elitist individuals

For increasing the efficiency of local search, the levy flight searching strategy (Gandomi, Yang, & Alavi, 2013) is incorporated to simulate the foraging behavior of fruit fly. This disturbed strategy based on Levi's flight, which is a random walk with a short-steps local search and occasional long-steps walks, is applied for a local search around the assigned learning objectives effectively; the inferior individuals generate new locations according to the following formula.

$$X_{i,j}(t+1) = X_{s,n}(t) + \left(levy(\lambda) * \frac{sizepop}{UB - LB}\right)\left(X_{s,n}(t) - X_{i,j}(t)\right) \tag{3}$$

Where $X_{i,j}$ is the jth individuals in the inferior population i. $X_{s,n}$ is the nth individual in the superior populations. $X_{s,n}$ is assigned as the role model of the individual $X_{i,j}$ as shown in Eqs. (1) and (2). $Levy(\lambda)$ is the step size of the random walk based on the Levy flight. $\frac{sizepop}{UB-LB}$ is the shrinkage scale. $X_{s,n}(t) - X_{i,j}(t)$ is a perturbed component and is used to control the search direction.

The foraging trajectories of many animals in nature follow the Levy distribution. In the Levy distribution, the individual always searches with short-distance walking and occasionally searches with long-distance walking. In this way, the fruit fly individual is assisted to execute local search and jump out from local optima. In this paper, the step of Levy is determined by the following.

$$step = \frac{\mu}{|\nu|^{\frac{1}{\beta}}}, \mu\ N\left(0, \sigma_\mu^2\right), \nu\ N\left(0, \sigma_\nu^2\right) \tag{4}$$

Table 3
the result of D = 10.

Function	FOA		LGMSFOA		JSFOA		HACLFOA		IFFO		SFFO		IFOA		TSA		GWO		HGCLFOA	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
1	2.63E+10	3.08E+09	2.94E+10	6.47E+07	1.49E+10	5.00E+09	8.61E+09	2.50E+07	8.04E+09	4.97E+09	8.19E+06	5.44E+09	2.39E+03	2.81E+03	9.38E+09	5.60E+09	7.93E+06	4.66E+07	**6.71E+02**	**1.72E+03**
2	2.90E+17	2.49E+17	7.74E+17	1.45E+16	2.69E+12	3.16E+12	4.00E+10	3.88E+10	5.81E+10	1.38E+11	5.43E+09	2.27E+11	4.88E+02	1.59E+03	6.65E+13	4.39E+14	4.78E+06	1.28E+07	**1.31E−05**	**1.53E−05**
3	3.08E+04	5.37E+04	3.13E+04	2.95E+04	6.29E+04	1.51E+04	4.22E+04	6.04E+03	4.10E+04	2.17E+04	1.53E+03	2.69E+04	1.08E+03	1.38E+03	4.09E+04	2.18E+04	2.92E+02	6.34E+02	**2.50E−05**	**2.42E−05**
4	4.54E+03	4.89E+02	5.32E+03	3.72E+01	1.77E+03	6.81E+02	5.85E+02	3.91E+02	4.34E+02	5.41E+02	4.32E+02	3.92E+02	1.43E+01	2.14E+01	9.67E+02	9.32E+02	1.02E+01	1.27E+01	**1.26E−01**	**1.30E−01**
5	2.04E+02	1.30E+01	2.12E+02	**1.92E+00**	1.35E+02	1.72E+01	4.63E+01	2.44E+01	5.01E+01	1.92E+01	8.58E+01	2.34E+01	1.58E+01	6.04E+00	8.93E+01	2.90E+01	1.17E+01	5.35E+00	**9.58E+00**	4.48E+00
6	1.16E+02	6.19E+00	1.03E+02	4.89E+00	9.67E+01	1.44E+01	2.62E+01	1.93E+01	3.18E+01	1.33E+01	6.17E+01	1.54E+01	**0.00E+00**	0.00E+00	4.26E+01	1.49E+01	3.85E−01	5.06E−01	2.65E−01	5.24E−01
7	2.01E+02	1.46E+01	1.76E+02	4.99E+00	6.83E+02	1.11E+02	2.14E+02	7.67E+01	1.89E+02	8.32E+01	2.74E+02	8.80E+01	2.11E+01	5.50E+00	2.47E+02	9.08E+01	2.64E+01	9.61E+00	**2.01E+01**	**4.27E+00**
8	1.20E+02	1.17E+01	1.34E+02	**1.94E+00**	1.57E+02	1.65E+01	5.86E+01	2.35E+01	5.67E+01	1.80E+01	8.04E+01	2.10E+01	1.45E+01	5.60E+00	7.42E+01	2.42E+01	1.23E+01	5.90E+00	**8.47E+00**	3.69E+00
9	2.38E+03	4.95E+02	2.50E+03	2.45E+02	5.41E+03	1.29E+03	8.57E+02	5.94E+02	9.95E+02	4.92E+02	1.22E+03	6.65E+02	1.66E+01	5.25E+01	1.06E+03	6.34E+02	5.18E+00	1.38E+01	**2.30E−05**	**2.80E−05**
10	4.42E+03	2.53E+02	4.92E+03	**2.57E+01**	2.16E+03	1.84E+02	1.11E+03	4.84E+02	1.19E+03	2.98E+02	1.41E+03	3.03E+02	6.96E+02	2.62E+02	1.43E+03	3.01E+02	**5.76E+02**	2.70E+02	6.23E+02	1.88E+02
11	2.74E+06	3.70E+06	5.59E+07	3.82E+06	1.05E+04	7.26E+03	4.24E+03	3.40E+02	4.47E+03	5.87E+03	3.00E+02	4.99E+03	2.25E+01	4.57E+01	7.60E+03	1.33E+04	2.35E+01	1.98E+01	**7.09E+00**	**4.80E+00**
12	4.13E+09	9.12E+08	3.46E+09	2.91E+07	6.25E+08	3.98E+08	5.27E+08	1.35E+06	5.95E+08	6.44E+08	7.48E+05	6.27E+08	7.96E+05	8.01E+05	3.93E+08	6.00E+08	5.08E+05	6.64E+05	**3.77E+02**	**1.55E+02**
13	1.09E+09	6.98E+08	2.51E+09	2.21E+08	5.29E+07	5.72E+07	3.69E+07	1.38E+06	5.62E+07	4.50E+07	2.02E+05	7.03E+07	9.47E+03	9.73E+03	7.56E+06	3.41E+07	8.08E+03	5.14E+03	**7.70E+01**	**5.77E+01**
14	3.98E+08	3.97E+08	1.99E+09	1.37E+08	2.70E+04	3.41E+04	7.23E+03	7.50E+03	7.54E+03	7.75E+03	7.39E+03	8.21E+03	4.90E+03	6.51E+03	7.53E+03	8.91E+03	9.46E+02	1.52E+03	**2.37E+01**	**6.70E+00**
15	9.91E+06	1.58E+07	5.73E+08	1.23E+08	3.09E+05	5.32E+05	9.95E+03	2.55E+04	1.20E+04	2.17E+04	1.29E+04	3.86E+04	6.94E+03	6.99E+03	4.20E+04	8.98E+04	1.41E+03	1.56E+03	**2.75E+01**	**2.16E+01**
16	1.28E+03	8.97E+01	1.79E+03	**8.63E+00**	7.56E+02	1.76E+02	5.26E+02	2.22E+02	5.21E+02	1.93E+02	5.58E+02	2.14E+02	2.60E+02	1.42E+02	5.05E+02	2.03E+02	9.04E+01	7.44E+01	**1.16E+01**	2.45E+01
17	1.13E+03	4.37E+02	1.49E+03	1.49E+01	5.16E+02	1.38E+02	2.69E+02	1.50E+02	2.84E+02	1.61E+02	2.88E+02	1.26E+02	4.03E+01	4.78E+01	2.35E+02	1.53E+02	4.37E+01	2.76E+01	**3.48E+01**	**7.50E+00**
18	8.78E+09	1.26E+09	1.42E+10	5.22E+07	2.60E+07	3.43E+07	1.35E+08	3.13E+07	1.44E+08	2.34E+08	4.74E+06	2.59E+08	1.42E+04	1.21E+04	6.60E+07	2.27E+08	2.63E+04	1.38E+04	**3.56E+01**	**1.06E+01**
19	5.60E+09	2.45E+09	4.91E+09	9.07E+07	1.58E+06	3.10E+06	9.68E+06	1.31E+05	5.73E+06	1.50E+07	4.49E+04	9.81E+06	1.10E+04	9.26E+03	1.68E+06	1.12E+07	3.71E+03	5.33E+03	**5.74E+00**	**1.61E+00**
20	1.11E+03	2.25E+01	1.12E+03	1.24E+01	3.48E+02	6.12E+01	1.76E+02	1.13E+02	1.84E+02	8.67E+01	2.49E+02	1.05E+02	**2.23E+01**	2.75E+01	2.66E+02	1.20E+02	5.69E+01	4.08E+01	2.63E+01	**1.03E+01**
21	6.22E+02	7.90E+01	7.13E+02	**2.57E+00**	3.17E+02	4.30E+01	2.41E+02	4.82E+01	2.55E+02	4.40E+01	2.68E+02	3.43E+01	2.06E+02	4.46E+01	2.77E+02	4.06E+01	1.94E+02	4.28E+01	**1.28E+02**	4.80E+01
22	2.66E+03	2.91E+02	2.97E+03	1.55E+01	1.85E+03	4.22E+02	7.60E+02	5.16E+02	7.30E+02	4.38E+02	6.91E+02	4.02E+02	1.05E+02	**1.18E+01**	9.76E+02	5.29E+02	1.06E+02	4.09E+00	**9.24E+01**	2.18E+01
23	1.85E+03	2.12E+02	1.98E+03	1.81E+01	4.16E+02	2.35E+01	4.29E+02	4.79E+01	4.27E+02	4.40E+01	4.46E+02	5.18E+01	3.25E+02	1.05E+01	4.45E+02	7.01E+01	3.16E+02	8.28E+00	**3.14E+02**	**6.25E+00**
24	9.41E+02	3.18E+01	9.83E+02	**1.58E+00**	4.62E+02	2.23E+01	4.22E+02	7.85E+01	4.25E+02	5.26E+01	4.76E+02	3.95E+01	3.50E+02	7.54E+01	4.67E+02	7.05E+01	3.42E+02	1.03E+01	**2.38E+02**	1.22E+02
25	2.10E+03	2.20E+02	2.29E+03	**6.23E+00**	1.72E+03	5.11E+02	9.41E+02	3.28E+02	1.02E+03	4.33E+02	7.74E+02	3.82E+02	4.27E+02	5.24E+01	1.11E+03	5.10E+02	4.35E+02	1.63E+01	**4.11E+02**	2.04E+01
26	3.00E+03	6.16E+01	3.10E+03	**8.06E+00**	2.08E+03	3.45E+02	1.12E+03	4.98E+02	1.04E+03	5.12E+02	1.32E+03	3.73E+02	4.75E+02	2.10E+02	1.65E+03	5.66E+02	3.57E+02	1.88E+02	**3.01E+02**	4.82E+01
27	1.95E+03	3.02E+02	2.15E+03	3.27E+01	4.68E+02	2.48E+01	4.93E+02	6.37E+01	4.84E+02	6.09E+01	5.52E+02	5.33E+01	4.27E+02	3.37E+01	5.02E+02	5.15E+01	3.94E+02	2.52E+00	**3.87E+02**	**5.20E+00**
28	1.52E+03	2.11E+02	1.30E+03	9.12E+01	4.98E+02	**2.67E+00**	7.83E+02	2.32E+02	7.99E+02	1.38E+02	8.99E+02	1.79E+02	5.14E+02	1.23E+02	9.55E+02	2.28E+02	5.50E+02	9.76E+01	**3.97E+02**	8.39E+01
29	1.14E+04	7.19E+03	1.97E+03	4.59E+03	7.00E+02	1.14E+02	4.57E+02	1.46E+02	4.77E+02	1.29E+02	5.97E+02	1.45E+02	3.46E+02	6.77E+01	5.62E+02	1.52E+02	2.83E+02	4.76E+01	**2.50E+02**	**1.39E+01**
30	3.76E+08	5.46E+07	4.70E+08	1.68E+07	6.26E+06	6.44E+06	6.84E+06	1.38E+07	3.94E+06	1.19E+07	4.35E+06	7.49E+06	7.14E+05	7.56E+05	2.40E+07	2.90E+07	4.56E+05	6.20E+05	**1.18E+03**	**8.47E+02**

Table 4
the result of D = 30.

Function	FOA		LGMSFOA		JSFOA		HACLFOA		IFFO		SFFO		IFOA		TSA		GWO		HGCLFOA	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
1	8.06E+10	3.96E+09	8.44E+10	1.34E+08	1.75E+11	5.66E+10	9.47E+06	3.72E+07	5.89E+10	1.65E+10	5.30E+10	1.60E+10	5.27E+03	6.34E+03	8.57E+10	2.30E+10	9.25E+08	7.70E+08	**3.31E+03**	**3.89E+03**
2	6.49E+59	1.37E+60	1.83E+61	8.59E+59	3.86E+52	2.67E+53	9.51E+46	6.80E+47	8.76E+45	6.26E+46	1.63E+47	9.79E+47	4.22E+11	7.84E+11	6.07E+52	4.23E+53	1.06E+32	7.55E+32	**5.48E+06**	**1.22E+07**
3	3.15E+07	9.69E+07	1.29E+09	3.26E+08	7.40E+08	4.61E+09	1.52E+05	1.19E+05	1.82E+05	5.03E+04	1.81E+05	6.38E+04	8.44E+04	3.80E+04	2.40E+05	6.25E+04	2.88E+04	1.02E+04	**2.24E+00**	**1.68E+00**
4	2.89E+04	3.37E+03	2.41E+04	1.59E+03	8.34E+04	8.04E+04	1.07E+03	2.76E+03	1.16E+04	5.96E+03	1.15E+04	7.23E+03	9.77E+01	**2.28E+01**	2.62E+04	1.03E+04	1.50E+02	4.17E+01	**6.74E+01**	3.51E+01
5	5.82E+02	2.53E+01	6.17E+02	**2.31E+00**	8.43E+02	2.44E+02	4.41E+02	1.08E+02	3.17E+02	6.36E+01	2.93E+02	7.49E+01	9.89E+01	2.22E+01	4.85E+02	7.66E+01	**8.59E+01**	2.67E+01	1.13E+02	2.43E+01
6	1.39E+02	1.99E+00	1.37E+02	1.52E+00	1.60E+02	3.39E+01	1.01E+02	1.43E+01	3.92E+01	9.93E+00	4.11E+01	1.01E+01	**0.00E+00**	**0.00E+00**	8.82E+01	1.39E+01	4.01E+00	2.53E+00	2.55E+01	7.91E+00
7	9.06E+02	2.80E+01	8.80E+02	9.51E+00	5.06E+03	1.47E+03	1.95E+03	3.17E+02	1.26E+03	3.33E+02	1.28E+03	3.40E+02	**1.05E+02**	2.24E+01	1.89E+03	3.38E+02	1.35E+02	3.20E+01	2.32E+02	5.10E+01
8	4.79E+02	1.95E+01	5.08E+02	**2.90E+00**	7.65E+02	1.72E+02	3.88E+02	9.86E+01	2.91E+02	5.02E+01	2.74E+02	5.56E+01	1.06E+02	2.96E+01	4.20E+02	5.89E+01	**7.37E+01**	2.18E+01	9.26E+01	1.57E+01
9	2.51E+04	2.92E+03	1.72E+04	1.43E+03	5.69E+04	2.96E+04	8.50E+03	7.79E+03	7.16E+03	2.05E+03	7.07E+03	2.51E+03	1.04E+03	7.50E+02	1.29E+04	3.44E+03	**4.68E+02**	3.63E+02	1.53E+03	**7.33E+02**
10	9.72E+03	3.45E+02	9.68E+03	**6.67E+01**	8.73E+03	5.00E+02	4.63E+03	1.61E+03	4.95E+03	5.12E+02	4.99E+03	5.94E+02	**2.99E+03**	4.71E+02	6.51E+03	6.43E+02	3.00E+03	7.88E+02	3.85E+03	5.43E+02
11	1.54E+08	1.58E+08	6.01E+08	1.04E+07	7.77E+04	9.81E+04	7.02E+02	1.82E+03	2.04E+04	1.06E+04	2.17E+04	1.34E+04	1.98E+02	2.81E+02	2.57E+04	1.17E+04	2.89E+02	1.48E+02	**1.37E+02**	**4.43E+01**
12	2.73E+10	1.02E+09	2.71E+10	7.89E+08	3.43E+10	2.54E+10	7.01E+06	1.03E+07	7.52E+09	4.25E+09	8.76E+09	3.70E+09	1.76E+06	1.25E+06	1.36E+10	7.12E+09	4.17E+07	6.46E+07	**1.95E+05**	**1.92E+05**
13	3.91E+10	1.60E+09	4.19E+10	1.44E+09	2.63E+10	2.24E+10	5.94E+05	1.79E+06	5.48E+09	5.33E+09	5.05E+09	4.37E+09	2.28E+04	2.14E+04	1.19E+10	8.65E+09	6.35E+06	2.41E+07	**4.82E+03**	**1.98E+03**
14	5.50E+08	2.03E+08	1.22E+09	1.09E+07	2.69E+07	4.46E+07	4.41E+04	6.80E+04	6.83E+06	7.26E+06	9.08E+06	1.31E+07	1.53E+06	1.53E+06	1.63E+07	2.25E+07	1.54E+05	2.97E+05	**1.48E+02**	**3.31E+01**
15	2.70E+09	1.89E+09	5.41E+09	5.06E+08	6.45E+09	5.78E+09	2.93E+07	2.09E+08	3.42E+09	2.18E+09	2.84E+09	2.07E+09	4.78E+03	5.80E+03	1.26E+09	1.46E+09	2.70E+05	6.88E+05	**6.25E+02**	**2.24E+02**
16	2.31E+04	1.05E+03	2.58E+04	**7.56E+01**	6.41E+03	4.60E+03	2.27E+03	1.04E+03	2.75E+03	1.23E+03	2.79E+03	8.17E+02	1.32E+03	3.14E+02	3.82E+03	1.31E+03	7.27E+02	2.41E+02	**6.22E+02**	2.63E+02
17	1.61E+05	5.27E+04	2.47E+05	1.23E+04	9.73E+04	2.87E+05	1.27E+03	4.87E+02	4.91E+03	8.99E+03	4.46E+03	1.08E+04	7.11E+02	2.61E+02	3.70E+03	5.78E+03	2.59E+02	1.45E+02	**2.23E+02**	**9.56E+01**
18	1.92E+09	1.43E+09	4.59E+09	2.73E+07	1.84E+08	1.16E+08	9.24E+06	3.41E+07	2.74E+07	2.82E+07	3.31E+07	4.76E+07	2.78E+06	3.17E+06	7.77E+07	1.09E+08	6.39E+05	8.94E+05	**1.46E+03**	**5.17E+03**
19	4.40E+09	1.40E+09	5.07E+09	8.96E+08	1.39E+10	1.51E+10	1.92E+04	3.15E+04	3.09E+09	2.31E+09	3.34E+09	2.25E+09	8.90E+03	1.24E+04	2.50E+09	2.51E+09	1.02E+06	2.25E+06	**1.78E+02**	**6.37E+01**
20	2.28E+03	4.26E+02	3.27E+03	3.07E+02	1.50E+03	2.63E+02	1.11E+03	3.40E+02	1.08E+03	2.38E+02	1.18E+03	2.55E+02	5.78E+02	2.49E+02	1.16E+03	2.50E+02	3.22E+02	1.24E+02	**2.72E+02**	1.04E+02
21	1.05E+03	5.47E+01	1.11E+03	**1.61E+01**	9.37E+02	2.31E+02	6.21E+02	9.35E+01	4.55E+02	5.19E+01	4.68E+02	5.04E+01	3.10E+02	2.79E+01	6.54E+02	9.34E+01	**2.80E+02**	2.16E+01	3.00E+02	1.85E+01
22	1.03E+04	3.70E+02	1.09E+04	3.93E+01	8.73E+03	5.59E+02	5.02E+03	1.76E+03	5.12E+03	9.48E+02	5.25E+03	6.99E+02	2.54E+03	1.69E+03	6.78E+03	7.33E+02	2.08E+03	1.59E+03	**6.56E+02**	1.42E+03
23	4.99E+03	6.75E+02	5.16E+03	4.43E+01	1.27E+03	2.75E+02	1.25E+03	2.13E+02	8.89E+02	1.06E+02	8.76E+02	9.88E+01	**4.83E+02**	**3.62E+01**	1.16E+03	2.07E+02	**4.32E+02**	2.83E+01	4.95E+02	4.45E+01
24	2.70E+03	5.63E+01	2.79E+03	**2.05E+00**	1.24E+03	1.62E+02	1.47E+03	2.56E+02	1.03E+03	1.30E+02	1.04E+03	1.13E+02	7.08E+02	7.82E+01	1.30E+03	2.06E+02	**5.21E+02**	5.74E+01	5.42E+02	3.62E+01
25	5.86E+03	6.55E+02	5.44E+03	6.88E+02	4.70E+04	4.25E+04	1.41E+03	1.28E+03	3.13E+03	1.91E+03	2.82E+03	1.33E+03	3.98E+02	1.91E+01	8.87E+03	3.82E+03	4.55E+02	2.30E+01	**3.95E+02**	**1.64E+01**
26	1.29E+04	5.41E+02	1.34E+04	**3.32E+01**	1.13E+04	2.12E+03	5.68E+03	3.05E+03	7.46E+03	1.60E+03	7.61E+03	1.48E+03	2.40E+03	8.54E+02	9.36E+03	1.92E+03	**1.86E+03**	2.43E+02	2.00E+03	1.17E+03
27	6.85E+03	1.00E+03	7.68E+03	3.20E+01	**5.00E+02**	**8.06E−05**	1.76E+03	4.45E+02	8.39E+02	1.37E+02	8.21E+02	1.48E+02	5.54E+02	2.58E+01	1.47E+03	4.14E+02	5.35E+02	1.49E+01	5.01E+02	6.42E+00
28	6.85E+03	4.45E+02	7.19E+03	1.22E+02	5.00E+02	**8.45E−05**	1.36E+03	1.79E+03	3.59E+03	1.12E+03	3.70E+03	1.07E+03	4.27E+02	3.51E+01	7.01E+03	2.12E+03	5.37E+02	6.18E+01	**4.00E+02**	5.15E+01
29	1.84E+05	4.13E+04	2.21E+05	2.57E+03	9.27E+04	3.72E+05	2.46E+03	7.55E+02	1.95E+03	6.75E+02	1.94E+03	8.48E+02	1.02E+03	2.51E+02	1.41E+04	5.36E+04	**7.90E+02**	1.22E+02	**8.97E+02**	**2.50E+02**
30	8.92E+09	9.72E+08	1.01E+10	2.35E+07	2.76E+09	4.01E+09	1.10E+08	5.75E+08	1.42E+09	1.11E+09	1.36E+09	9.27E+08	2.10E+04	4.52E+04	1.49E+09	1.21E+09	4.87E+06	3.78E+06	**1.49E+04**	**7.03E+03**

Table 5
the result of D = 50.

Function	FOA		LGMSFOA		JSFOA		HACLFOA		IFFO		SFFO		IFOA		TSA		GWO		HGCLFOA	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
1	1.32E+11	2.91E+09	1.35E+11	1.72E+08	4.47E+11	1.55E+11	5.81E+07	1.81E+08	1.18E+11	2.40E+10	1.24E+11	2.46E+10	**4.30E+03**	**4.90E+03**	1.92E+11	2.93E+10	4.41E+09	2.27E+09	5.06E+03	7.09E+03
2	2.53E+87	3.27E+87	2.78E+87	6.81E+86	2.87E+104	2.04E+105	8.67E+80	6.15E+81	2.09E+86	1.36E+87	3.67E+87	2.62E+88	1.49E+31	1.07E+32	1.65E+90	1.14E+91	6.84E+50	4.69E+51	**6.91E+13**	**1.94E+14**
3	2.21E+13	2.86E+13	1.17E+14	4.01E+13	1.29E+07	5.25E+07	3.23E+05	2.09E+05	3.34E+05	8.14E+04	3.41E+05	7.11E+04	2.18E+05	6.49E+04	4.36E+05	8.84E+04	7.10E+04	1.69E+04	**1.43E+02**	**6.96E+01**
4	5.26E+04	2.85E+03	5.59E+04	1.19E+02	3.00E+05	2.30E+05	8.13E+02	3.72E+02	3.11E+04	1.32E+04	2.84E+04	1.27E+04	**1.12E+02**	4.95E+01	6.32E+04	1.72E+04	4.75E+02	3.24E+02	1.44E+02	**4.78E+01**
5	8.36E+02	1.39E+01	8.48E+02	**2.53E+00**	1.79E+03	5.12E+02	7.48E+02	1.51E+02	5.84E+02	1.02E+02	5.76E+02	8.23E+01	1.82E+02	3.80E+01	9.07E+02	1.03E+02	**1.78E+02**	3.18E+01	2.36E+02	3.62E+01
6	1.42E+02	1.48E+00	1.42E+02	1.04E+00	2.03E+02	4.84E+01	1.20E+02	1.42E+01	4.65E+01	8.40E+00	4.69E+01	7.45E+00	**0.00E+00**	**0.00E+00**	1.06E+02	1.28E+01	1.18E+01	3.33E+00	4.18E+01	7.69E+00
7	1.48E+03	2.91E+01	1.48E+03	**1.27E+01**	1.01E+04	2.55E+03	4.06E+03	4.78E+02	2.65E+03	5.36E+02	2.53E+03	5.47E+02	**2.28E+02**	4.34E+01	4.23E+03	4.67E+02	3.05E+02	6.07E+01	4.79E+02	9.98E+01
8	8.70E+02	1.58E+01	8.93E+02	**2.77E+00**	1.69E+03	4.23E+02	8.27E+02	1.75E+02	6.57E+02	8.19E+01	6.43E+02	9.76E+01	1.92E+02	3.00E+01	9.17E+02	1.07E+02	**1.86E+02**	3.24E+01	2.50E+02	4.03E+01
9	6.40E+04	5.47E+03	5.27E+04	2.29E+03	1.66E+05	7.08E+04	2.13E+04	2.01E+04	1.80E+04	5.45E+03	1.76E+04	4.42E+03	**4.16E+03**	**2.16E+03**	4.34E+04	9.53E+03	**3.55E+03**	1.99E+03	1.06E+04	2.79E+03
10	1.96E+04	**3.55E+02**	1.64E+04	8.81E+02	1.57E+04	7.24E+02	7.16E+03	2.54E+03	8.01E+03	6.43E+02	8.15E+03	6.97E+02	**4.96E+03**	8.29E+02	1.22E+04	9.68E+02	5.45E+03	1.23E+03	7.29E+03	1.17E+03
11	3.02E+04	5.45E+03	6.38E+05	1.01E+05	1.59E+05	1.12E+05	1.32E+03	3.23E+03	3.36E+04	1.55E+04	3.61E+04	1.40E+04	2.36E+03	3.30E+03	6.62E+04	2.49E+04	1.74E+03	1.23E+03	**2.51E+02**	**7.10E+01**
12	1.33E+11	6.95E+09	1.42E+11	1.92E+08	1.92E+11	1.18E+11	2.67E+07	3.48E+07	5.16E+10	2.31E+10	5.64E+10	2.02E+10	4.81E+06	**2.97E+06**	8.54E+10	2.51E+10	3.23E+08	5.21E+08	**4.78E+06**	3.30E+06
13	1.01E+11	6.87E+09	1.13E+11	2.03E+08	1.08E+11	7.11E+10	1.76E+05	5.41E+05	2.83E+10	1.53E+10	2.82E+10	1.60E+10	**1.02E+04**	1.16E+04	4.36E+10	2.22E+10	7.49E+07	1.01E+08	1.97E+04	**1.06E+04**
14	1.08E+09	1.49E+08	1.44E+09	6.72E+06	1.28E+08	1.42E+08	5.89E+06	2.60E+07	2.95E+07	5.26E+07	3.16E+07	4.60E+07	1.65E+06	1.46E+06	8.83E+07	9.32E+07	5.34E+05	6.31E+05	**3.83E+02**	**1.01E+02**
15	2.16E+10	1.17E+09	2.36E+10	6.03E+07	5.23E+10	3.91E+10	1.18E+04	1.41E+04	1.48E+10	8.21E+09	1.43E+10	7.86E+09	1.11E+04	7.63E+03	1.24E+10	7.87E+09	6.51E+06	1.24E+07	**2.41E+03**	**1.23E+03**
16	1.98E+04	1.47E+03	2.22E+04	8.12E+02	1.75E+04	1.52E+04	3.50E+03	2.20E+03	5.72E+03	1.94E+03	5.65E+03	1.63E+03	2.20E+03	4.98E+02	8.09E+03	2.45E+03	**1.19E+03**	2.98E+02	1.39E+03	**3.43E+02**
17	8.39E+04	5.11E+04	1.59E+05	3.60E+03	6.39E+07	1.41E+08	2.25E+03	5.27E+02	1.11E+05	1.70E+05	1.09E+05	1.44E+05	1.50E+03	2.97E+02	1.86E+05	2.37E+05	1.05E+03	2.74E+02	**1.03E+03**	**2.15E+02**
18	1.32E+09	3.67E+08	1.73E+09	2.26E+08	9.92E+08	1.14E+09	4.80E+07	1.58E+08	2.24E+08	1.49E+08	2.79E+08	2.01E+08	2.75E+06	2.21E+06	3.00E+08	2.55E+08	2.99E+06	3.94E+06	**1.99E+04**	**1.38E+04**
19	1.19E+10	9.02E+08	1.19E+10	6.67E+08	1.75E+10	1.65E+10	1.86E+04	2.53E+04	4.14E+09	3.01E+09	4.08E+09	2.51E+09	1.58E+04	1.16E+04	5.60E+09	4.01E+09	1.40E+06	4.22E+06	**5.88E+02**	**8.62E+02**
20	3.20E+03	1.35E+02	3.29E+03	**2.54E+01**	3.44E+03	5.65E+02	2.01E+03	5.38E+02	2.29E+03	4.44E+02	2.32E+03	3.75E+02	1.36E+03	3.64E+02	2.21E+03	3.87E+02	7.59E+02	2.98E+02	**6.94E+02**	2.02E+02
21	1.81E+03	2.77E+02	1.99E+03	1.19E+02	1.90E+03	4.51E+02	1.05E+03	1.86E+02	7.31E+02	8.10E+01	7.47E+02	9.82E+01	4.56E+02	4.74E+01	1.17E+03	1.13E+02	**3.89E+02**	5.07E+01	4.49E+02	**4.51E+01**
22	1.83E+04	3.13E+02	1.87E+04	**6.48E+01**	1.59E+04	7.59E+02	7.75E+03	2.46E+03	9.11E+03	8.15E+02	8.78E+03	7.58E+02	5.99E+03	7.44E+02	1.26E+04	1.00E+03	**5.86E+03**	9.02E+02	7.50E+03	1.46E+03
23	6.74E+03	5.77E+02	7.32E+03	**2.92E+01**	2.38E+03	4.02E+02	2.12E+03	2.99E+02	1.55E+03	1.90E+02	1.48E+03	1.86E+02	9.05E+02	5.37E+01	2.13E+03	2.94E+02	**5.99E+02**	3.74E+01	8.32E+02	8.08E+01
24	4.38E+03	1.02E+02	4.44E+03	**3.15E+00**	2.19E+03	3.08E+02	2.59E+03	4.44E+02	1.71E+03	1.77E+02	1.70E+03	1.85E+02	1.19E+03	1.56E+02	2.31E+03	3.37E+02	**7.06E+02**	9.47E+01	8.67E+02	6.88E+01
25	1.61E+04	8.21E+02	1.72E+04	8.63E+01	2.06E+05	1.41E+05	2.25E+03	6.71E+03	9.76E+03	4.03E+03	1.08E+04	4.35E+03	5.61E+02	**2.92E+01**	3.57E+04	9.11E+03	8.34E+02	1.39E+02	**5.31E+02**	2.96E+01
26	1.72E+04	3.78E+02	1.77E+04	**3.75E+01**	2.56E+04	6.29E+03	8.82E+03	5.25E+03	1.47E+04	2.09E+03	1.46E+04	2.55E+03	3.95E+03	1.38E+03	2.01E+04	3.40E+03	**3.17E+03**	3.69E+02	3.70E+03	3.13E+03
27	1.54E+04	9.73E+02	1.59E+04	2.10E+02	**5.00E+02**	**6.97E−05**	3.36E+03	1.38E+03	1.69E+03	3.73E+02	1.63E+03	3.01E+02	8.51E+02	1.02E+02	3.56E+03	8.16E+02	7.82E+02	7.79E+01	5.09E+02	6.22E+01
28	1.55E+04	1.63E+03	1.73E+04	3.79E+01	**5.00E+02**	**7.33E−05**	2.46E+03	3.93E+03	8.95E+03	1.82E+03	9.01E+03	1.92E+03	5.13E+02	3.27E+01	1.51E+04	3.11E+03	1.08E+03	3.04E+02	5.04E+02	2.27E+01
29	4.40E+06	2.07E+06	6.35E+06	6.31E+04	4.29E+07	2.19E+08	2.75E+04	1.73E+05	9.96E+04	2.63E+05	8.19E+04	1.79E+05	1.85E+03	**3.16E+02**	2.07E+05	4.50E+05	**1.24E+03**	2.17E+02	1.77E+03	3.73E+02
30	2.11E+10	3.29E+09	2.43E+10	2.43E+08	3.24E+10	2.56E+10	**2.32E+06**	2.90E+06	4.18E+09	3.27E+09	4.87E+09	3.14E+09	5.56E+06	3.12E+07	9.24E+09	4.97E+09	6.88E+07	2.51E+07	3.10E+06	**1.33E+06**

Table 6
the result of D = 100D.

Function	FOA Mean	FOA Std	LGMSFOA Mean	LGMSFOA Std	JSFOA Mean	JSFOA Std	HACLFOA Mean	HACLFOA Std	IFFO Mean	IFFO Std	SFFO Mean	SFFO Std	IFOA Mean	IFOA Std	TSA Mean	TSA Std	GWO Mean	GWO Std	HGCLFOA Mean	HGCLFOA Std
1	2.92E+11	3.69E+09	2.96E+11	2.16E+08	1.07E+12	3.15E+11	1.59E+08	5.34E+08	2.96E+11	4.45E+10	2.97E+11	4.19E+10	**5.90E+03**	8.72E+03	5.15E+11	5.06E+10	2.98E+10	6.79E+09	8.78E+03	**5.06E+03**
2	1.79E+187	**6.55E+04**	9.94E+190	**6.55E+04**	4.72E+222	**6.55E+04**	5.01E+181	**6.55E+04**	9.70E+185	**6.55E+04**	4.10E+185	**6.55E+04**	4.90E+76	3.11E+77	2.85E+193	6.55E+04	1.63E+129	1.17E+130	**3.17E+40**	2.13E+41
3	6.75E+12	1.58E+13	1.51E+14	1.15E+13	8.73E+09	3.13E+10	3.02E+06	9.24E+06	6.96E+05	1.02E+05	7.01E+05	1.06E+05	5.19E+05	1.43E+05	9.26E+05	1.39E+05	2.01E+05	2.26E+04	**1.12E+04**	**2.54E+03**
4	1.43E+05	1.29E+04	1.58E+05	3.33E+02	9.93E+05	7.44E+05	1.26E+03	3.48E+02	7.29E+04	2.63E+04	7.40E+04	2.23E+04	**2.64E+02**	**4.39E+01**	2.08E+05	4.62E+04	2.40E+03	7.59E+02	2.77E+02	4.40E+01
5	1.82E+03	2.20E+01	1.85E+03	**3.77E+00**	4.00E+03	9.65E+02	1.85E+03	3.88E+02	1.31E+03	1.38E+02	1.34E+03	1.48E+02	**5.15E+02**	7.22E+01	2.23E+03	1.63E+02	**5.65E+02**	9.16E+01	6.61E+02	6.34E+01
6	1.37E+02	8.80E−01	1.36E+02	9.40E−01	2.25E+02	4.14E+01	1.40E+02	1.23E+01	5.01E+01	5.91E+00	4.98E+01	6.28E+00	**0.00E+00**	**0.00E+00**	1.27E+02	1.04E+01	**2.76E+01**	3.93E+00	5.39E+01	4.20E+00
7	3.58E+03	3.49E+01	3.57E+03	**2.61E+01**	2.52E+04	6.28E+03	1.01E+04	6.54E+02	6.46E+03	7.18E+02	6.02E+03	8.01E+02	**6.43E+02**	9.39E+01	1.11E+04	6.89E+02	**1.00E+03**	1.08E+02	1.41E+03	2.59E+02
8	1.99E+03	1.93E+01	2.01E+03	**4.90E+00**	3.94E+03	9.08E+02	1.97E+03	3.72E+02	1.50E+03	1.68E+02	1.48E+03	1.40E+02	**5.42E+02**	7.07E+01	2.30E+03	1.83E+02	**5.58E+02**	6.09E+01	7.39E+02	1.03E+02
9	1.03E+05	4.59E+03	1.05E+05	**3.25E+03**	4.23E+05	1.66E+05	4.98E+04	2.67E+04	4.15E+04	6.80E+03	4.22E+04	9.18E+03	**1.45E+04**	3.61E+03	1.12E+05	1.67E+04	**2.31E+04**	1.06E+04	3.76E+04	5.66E+03
10	3.46E+04	4.47E+02	3.52E+04	**1.35E+02**	3.41E+04	1.33E+03	1.55E+04	4.43E+03	1.85E+04	1.36E+03	1.82E+04	1.35E+03	**1.28E+04**	1.18E+03	2.71E+04	1.34E+03	**1.40E+04**	2.57E+03	1.43E+04	1.59E+03
11	3.37E+12	4.99E+12	9.00E+12	5.33E+12	3.64E+06	4.99E+06	1.25E+05	2.64E+05	1.88E+05	6.58E+04	2.14E+05	7.09E+04	5.08E+04	2.54E+04	4.90E+05	1.29E+05	3.62E+04	1.06E+04	**1.42E+03**	**2.42E+02**
12	2.51E+11	5.30E+09	2.59E+11	2.64E+08	6.23E+11	2.42E+11	1.24E+08	1.15E+08	1.32E+11	3.42E+10	1.41E+11	3.94E+10	**1.02E+07**	**5.19E+06**	2.59E+11	4.65E+10	4.96E+09	2.49E+09	2.64E+07	1.14E+07
13	6.20E+10	2.02E+09	6.51E+10	1.09E+08	1.74E+11	7.85E+10	3.06E+05	9.63E+05	4.08E+10	1.55E+10	3.96E+10	1.28E+10	**8.58E+03**	**8.89E+03**	6.64E+10	1.58E+10	3.58E+08	4.19E+08	4.14E+04	1.64E+04
14	1.15E+09	2.60E+08	1.46E+09	8.63E+06	5.19E+08	2.80E+08	1.05E+07	4.88E+07	2.48E+07	1.91E+07	2.59E+07	2.13E+07	1.64E+06	1.11E+06	2.41E+08	1.63E+08	3.18E+06	2.20E+06	**6.88E+03**	**9.83E+03**
15	3.88E+10	1.27E+09	4.12E+10	6.74E+07	7.57E+10	4.07E+10	1.40E+05	9.03E+05	2.68E+10	1.06E+10	2.46E+10	7.39E+09	**4.14E+03**	5.18E+03	3.05E+10	9.77E+09	8.32E+07	1.46E+08	1.20E+04	**5.10E+03**
16	3.50E+04	1.43E+03	3.75E+04	**6.65E+01**	8.01E+04	6.26E+04	5.90E+03	1.89E+03	1.33E+04	3.74E+03	1.24E+04	2.95E+03	4.52E+03	6.67E+02	2.55E+04	6.48E+03	**3.64E+03**	6.49E+02	3.90E+03	5.37E+02
17	1.05E+08	3.26E+07	1.60E+08	7.55E+06	8.89E+08	2.64E+09	1.11E+06	7.87E+06	9.59E+06	9.45E+06	1.22E+07	1.59E+07	3.62E+03	6.49E+02	2.20E+07	3.90E+07	**2.70E+03**	5.48E+02	3.22E+03	**5.86E+02**
18	1.12E+09	1.37E+08	1.44E+09	9.81E+06	1.20E+09	9.03E+08	4.36E+07	1.72E+08	6.70E+07	5.90E+07	9.59E+07	8.66E+07	1.69E+06	9.38E+05	4.26E+08	2.57E+08	3.71E+06	2.18E+06	**1.09E+05**	**4.35E+04**
19	3.78E+10	2.29E+09	4.16E+10	6.47E+07	8.39E+10	4.65E+10	1.16E+05	5.33E+05	2.41E+10	8.79E+09	2.57E+10	9.69E+09	**4.67E+03**	**6.38E+03**	3.20E+10	1.02E+10	8.24E+07	1.58E+08	2.42E+04	1.94E+04
20	7.86E+03	6.20E+02	8.84E+03	**5.05E+01**	8.02E+03	1.11E+03	4.91E+03	1.42E+03	5.05E+03	5.35E+02	5.06E+03	5.00E+02	3.28E+03	5.78E+02	5.32E+03	5.84E+02	**2.42E+03**	7.03E+02	**2.63E+03**	4.09E+02
21	7.27E+03	1.21E+03	8.74E+03	1.61E+02	4.11E+03	9.55E+02	2.39E+03	3.58E+02	1.48E+03	1.34E+02	1.48E+03	1.60E+02	**8.78E+02**	**9.30E+01**	2.84E+03	2.35E+02	7.50E+02	5.97E+01	1.14E+03	1.06E+02
22	3.74E+04	4.04E+02	3.79E+04	**8.48E+01**	3.42E+04	9.99E+02	1.76E+04	5.45E+03	1.96E+04	1.42E+03	1.93E+04	1.30E+03	2.37E+04	1.19E+03	2.89E+04	1.38E+03	**1.56E+04**	3.26E+03	**1.72E+04**	1.88E+03
23	1.28E+04	1.42E+03	1.35E+04	2.17E+02	4.10E+03	6.52E+02	3.75E+03	1.19E+03	2.09E+03	1.53E+02	2.12E+03	1.64E+02	**9.16E+02**	**6.07E+01**	4.14E+03	5.44E+02	**1.10E+03**	7.30E+01	1.60E+03	1.57E+02
24	1.37E+04	1.04E+03	1.43E+04	**1.02E+01**	6.88E+03	9.69E+02	6.46E+03	3.80E+03	3.82E+03	3.40E+02	3.83E+03	3.65E+02	2.53E+03	8.84E+01	8.22E+03	1.30E+03	**1.51E+03**	9.27E+01	**2.51E+03**	3.09E+02
25	3.17E+04	1.26E+03	3.32E+04	5.65E+01	5.12E+05	3.19E+05	2.02E+03	4.39E+02	3.38E+04	9.48E+03	3.20E+04	1.00E+04	8.18E+02	**5.05E+01**	1.03E+05	1.62E+04	2.72E+03	5.26E+02	**8.03E+02**	8.42E+01
26	6.17E+04	1.26E+03	6.38E+04	**8.35E+01**	6.72E+04	8.77E+03	1.44E+04	1.80E+03	3.84E+04	5.49E+03	3.60E+04	4.61E+03	2.09E+04	2.05E+03	6.48E+04	1.04E+04	**1.01E+04**	1.55E+03	1.93E+04	6.23E+03
27	2.09E+04	1.80E+03	2.28E+04	3.64E+01	**5.00E+02**	**7.36E−05**	4.47E+03	4.34E+03	3.72E+03	9.09E+02	3.45E+03	9.04E+02	9.16E+02	8.85E+01	9.45E+03	2.11E+03	1.11E+03	1.06E+02	5.99E+02	2.57E+02
28	3.99E+04	6.69E+02	4.11E+04	5.74E+01	**5.00E+02**	**5.32E−05**	2.86E+03	7.78E+03	3.00E+04	3.01E+03	3.09E+04	3.48E+03	6.25E+02	3.78E+01	5.33E+04	6.71E+03	3.71E+03	9.78E+02	6.19E+02	5.39E+01
29	6.32E+06	1.50E+06	8.31E+06	8.56E+04	3.89E+08	9.40E+08	5.25E+03	**6.10E+02**	5.00E+05	6.32E+05	5.14E+05	7.96E+05	5.23E+03	8.59E+02	3.56E+06	3.97E+06	**4.40E+03**	4.76E+02	**5.07E+03**	6.39E+02
30	5.82E+10	1.89E+09	6.08E+10	8.57E+07	1.41E+11	9.09E+10	4.52E+05	7.09E+05	2.83E+10	1.13E+10	2.53E+10	1.09E+10	**1.69E+04**	**1.17E+04**	4.66E+10	1.19E+10	3.29E+08	2.37E+08	2.24E+06	1.02E+06

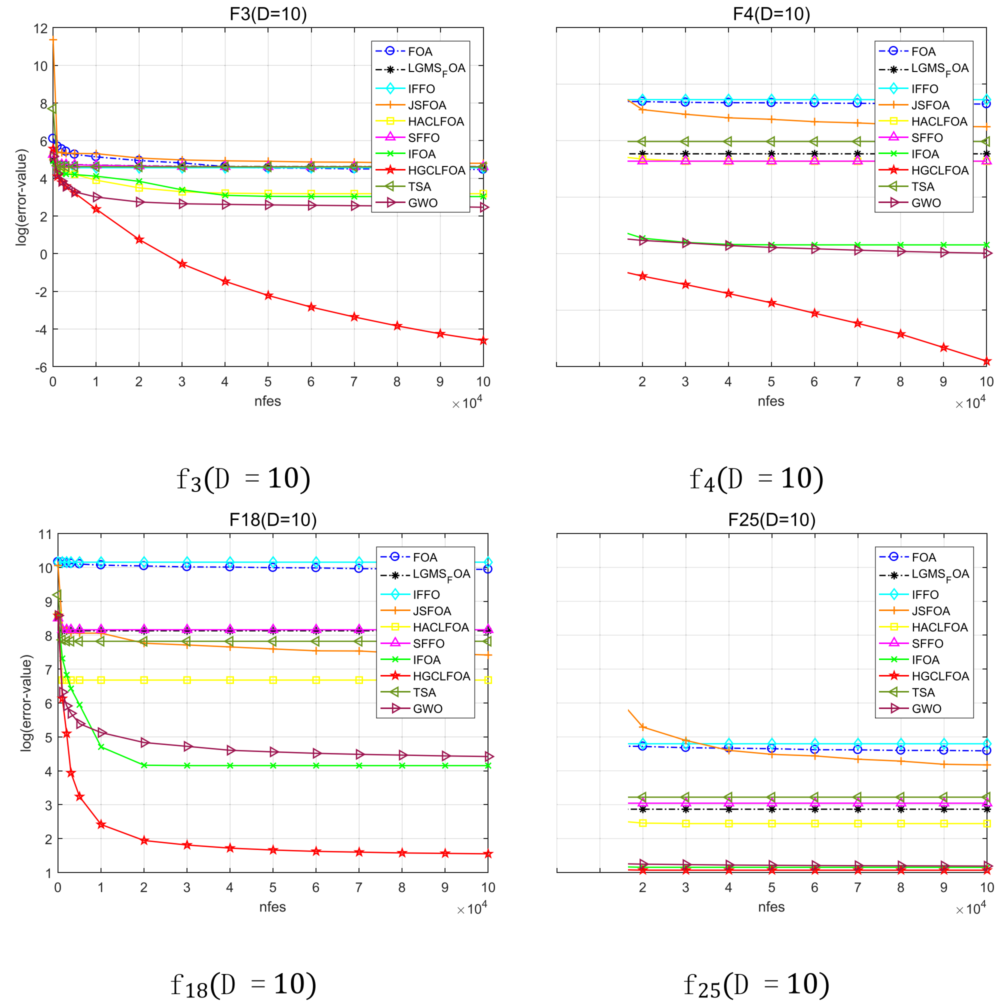


Fig. 8. Convergence curves of eight algorithms (10D).

$$\sigma_\mu = \left[\frac{\Gamma(1+\beta)\sin\left(\frac{\pi\beta}{2}\right)}{\Gamma\left(\frac{1+\beta}{2}\right)\beta 2^{\frac{\beta-1}{2}}}\right]^{\frac{1}{\beta}}, \sigma_\nu = 1 \tag{5}$$

where the parameter β affects the distribution, the smaller the β is, the longer the levy step is executed, a larger β is accompanied by a shorter levy search step (Ingle and Jatoth, 2020).

To further improve the efficiency of search, the β is set as a small value in initialization, and jumps in a big step, to explore a big search region. With the development of iteration, the β is increased; the jump step of HGCLFOA is decreased gradually. The inferior fruit fly individuals exploit and converge to the assigned learning objects. Therefore, the self-adaptive β is implemented in the search process and is defined as follows.

$$\beta(t) = \left[(\beta_{final} - \beta_{initial})^* \frac{g^* sizepop}{\max\_nfes}\right] + \beta_{initial} \tag{6}$$

where *max_nfes* and *g* denotes the maximum number of evaluations and the current iteration number respectively. β_{final} and $\beta_{initial}$ represent the initial and final values of the parameter β The range of βself adaptively changes in $[0.5, 1]$.

b) Inferior Solution Repairing Cooperative strategy

To further improve the efficiency of the search, the ISR strategy is introduced. The ISR cooperative strategy also is a search mechanism based on Gaussian distribution. However, the search ranges and direction are not relying on the covariance matrix. In the ISR cooperative strategy, the location relationship between the superior and the inferior fruit fly individuals is utilized to control the search. The reason for applying this strategy is that the probability model of GEDA is built based on all current elitist fruit fly individuals. All the individuals are guided by the elitist subpopulation. This may lead the population to evolve towards a local optimal direction. The probability density ellipse (PDE)is perpendicular to the direction of the decrease of the fitness value so that the probabilistic model predicts the feasible region inaccurately. In this way, the inferior solution repairing strategy is applied. The search range is extended by utilizing the information of inferior fruit fly in the ISR strategy which increases the diversity of the population and guarantees convergence at the same time. Moreover, the differential vector rather than the covariance to control the range of search in the ISR strategy. The detail of the ISR strategy is detailed depicted in Eqs. (7)–(10).

$$X_{i,j}(t+1) = X_{mr} + Gaussian0, \vec{\delta} \tag{7}$$

$$X_{mr} = X_{s,n} + r^*\left(X_{s,n} - X_{i,j}(t)\right) \tag{8}$$

$$\alpha = 1 - \frac{g^* sizepop}{max\_nfes} \tag{9}$$

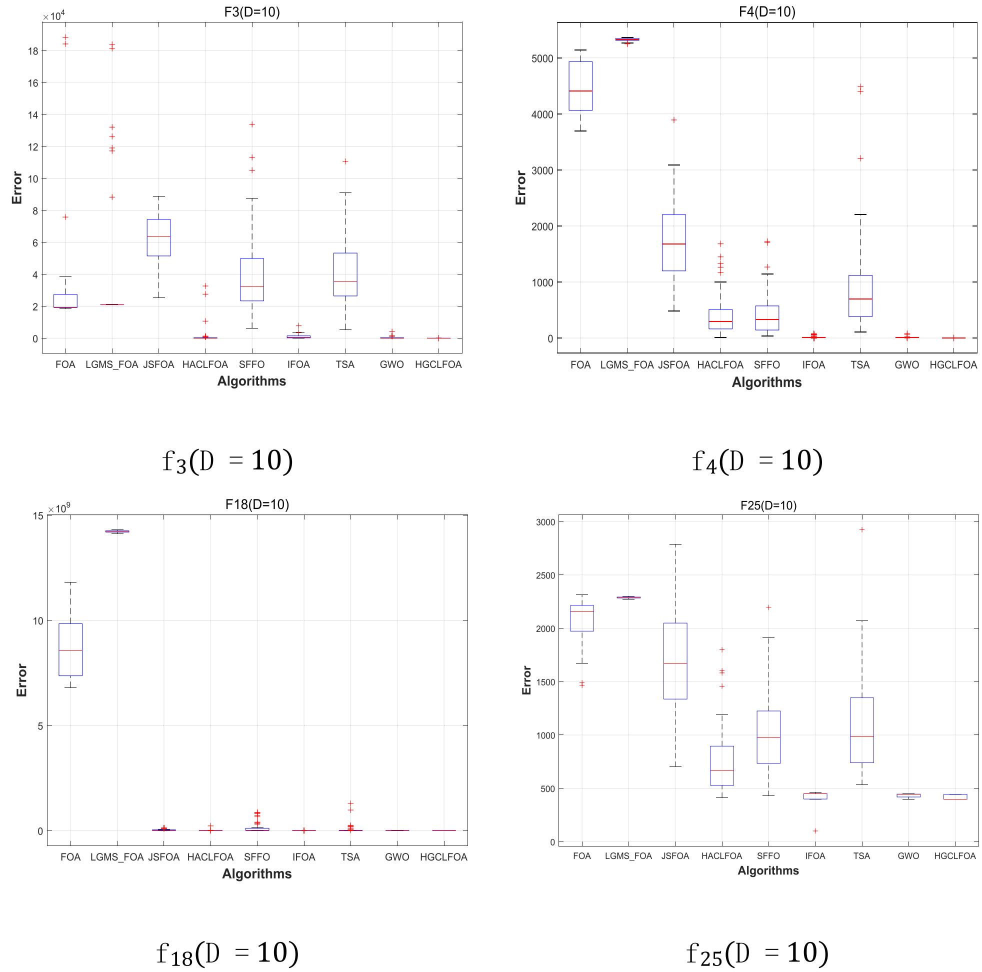


Fig. 9. Box plots compare with eight algorithms (10D).

$$\vec{\delta} = \alpha^* \left| X_{s,i}(t) - X_{s,m}(t) \right| \tag{10}$$

where the $X_{s,i}(t)$ is *ith* individual and $m \neq n \neq i$, the differential vector $X_{s,n} - X_{i,j}(t)$ is used to guarantee the search direction towards gradient descent. The superior candidates are found in the iterative procedure. The distribution range is adjusted by the scale shrinkage factorα.In the process of search, the distribution is assigned from dispersion to aggregation gradually for the balance of exploration and exploitation in HGCLFOA. The search center of the population is adjusted in Eq. (8), which uses inferior solutions to prevent the population moves towards the local optimal. The diagrammatic sketch is shown in Fig. 5.

c) Probability adjustment of selection strategy based on the feedback of object function

The inferior subpopulation is used to enhance the accuracy of the proposed algorithm. Thus, proper allocations of two strategies for inferior fruit fly individuals are also an important issue. Plethoric use of the strategy ISR leads the search of the population to deviate from potential areas. And unsuitable use of local search impacts the accuracy of the HGCALFOA and wastes the number of iterations. Thus, the selection of the strategy for inferior individuals is difficult rather important determination. Thus, a probability selection strategy based on feedback is introduced by utilizing the difference in the objective space.

$$D_i = \sqrt{\left(x_{s,i}^2 - X_{s,1}^2\right)} \tag{11}$$

$$qD_i = \frac{D_i}{\max(D)} \tag{12}$$

$$dfit_i = abs\left(f\left(x_{s,i}\right) - f\left(x_{s,0}\right)\right) \tag{13}$$

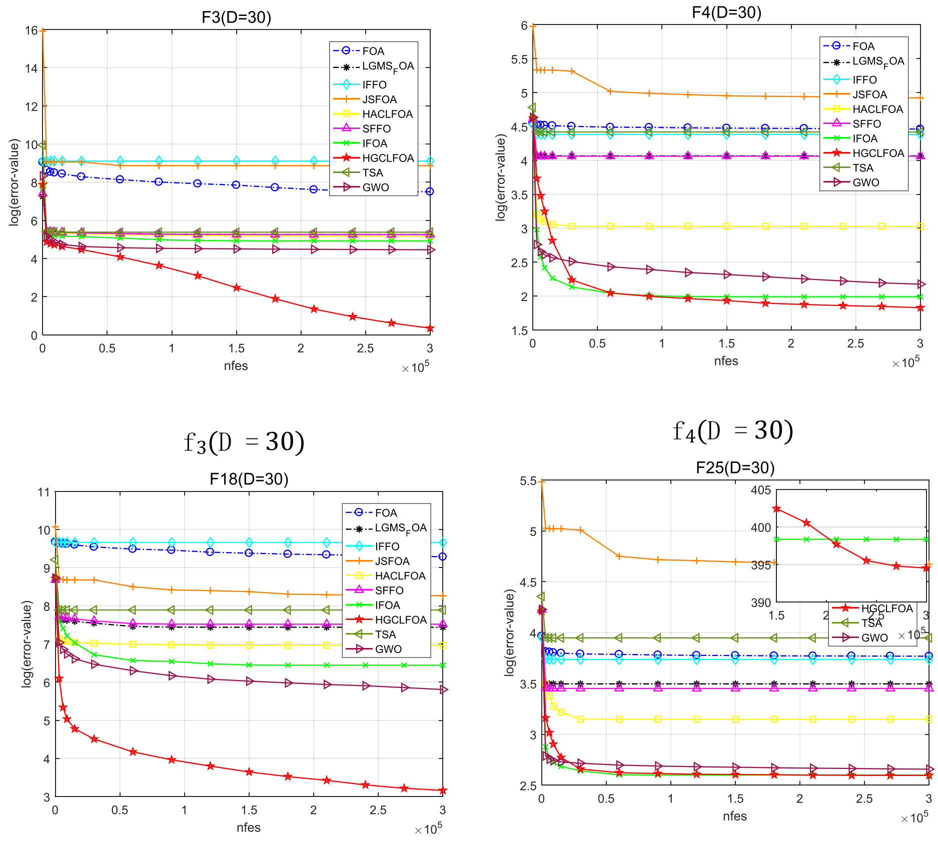
$$qd_i = \frac{dfit_i}{\max(dfit)} \tag{14}$$

$$P_i = \frac{qD_i + e^{-qd_i}}{2} \tag{15}$$

where$D_i$is the Euclidean distance between the *ith*individuals and the current best individual, $dfit_i$ is the absolute value for the difference of *ith* fitness to the fitness of the current best individual$x_{s,0}$. The probability of selection in Eq. (15) is increasing with the increase of relative distance and declines with the increase of relative fitness. Therefore, with the increase of distance and decrement of fitness, the probability of selection is increased which means that the population finds a potential area, tending to local search. In contrast, with the decrement of distance and increase of fitness, the probability of selection is decreased, which means the poor performance of the search direction, tending to ISR strategy.

4.2. Hybrid GEDA guides the elite population in vision search

In the proposed HGCLFOA, the population center is extended to an elitist population in vision search. The elitist population is a crucial role in the evolution which symbolizes the most potential search range for the entire threshold so far and guides the evolution of the population. Hence the hybrid GEDA is applied. This is a learning strategy, which



f3(D = 30)

f4(D = 30)

f18(D = 30)

f25(D = 30)

Fig. 10. Convergence curves of eight algorithms (30D).

analyses and summaries the distribution regulation of the current elite solutions. Then the distribution of potential solutions in the next generation is predicted by the regulation. The traditional GEDA is described following.

$$p(x|\theta) = \frac{1}{2\pi^{\frac{D}{2}}|\sum|^{\frac{1}{2}}}\exp\left(\frac{(x-\mu)^T\sum^{-1}(x-\mu)}{2}\right) \tag{16}$$

$$\mu = \sum_{i=1}^{N(s)} \omega_i X_{s,i} \tag{17}$$

$$\sum = \frac{1}{N(s)}\sum_{i=1}^{N(s)} (X_{s,i}-\mu)(X_{s,i}-\mu)^T \tag{18}$$

where the $X_{s,i}$is theith superior individual in the elitist population, ω_i is the weight coefficient to emphasize the proportion of the better solution.

The traditional GEDA makes a statistical analysis of the distribution regulation by the mean, covariance of the elite individuals of the previous generation. A probability density function is constructed to predict the tendency of distribution in the next generation. The new candidate for the ithsuperior fruit fly individual in the elitist population is updated in each generation by Eq. (19).

$$X_i = \mu + N\left(0, \sum\right) \tag{19}$$

where the mean μ and the covariance are the key roles of GEDA. The mean μ is the center of probability density ellipsoid which describes the search process in search space. The covariance decides the search direction and search range of the probability density ellipsoid.

The crucial problem in the traditional GEDA is that the main search direction of the probability PDE is perpendicular to the gradient descent direction with evolution, and the search range continues to shrink and gather in the center of the population gradually. The search efficiency of GEDA is affected seriously. Thus, the hybrid GEDA is implemented.

To overcome the problem mentioned in GEDA and the premature convergence problem of the original FOA, the mean shift is applied. The original mean information is averaged with the other one individual in the elitist subpopulation to obtain a new center that is slightly offset from the center of the elitist subpopulation. The fine-tuning of the mean based on information integration enhanced the exploration performance and increased the diversity of search space.

$$\mu_s = \frac{\mu + X_{s,j}}{2} \tag{20}$$

where $X_{s,j}$is thejth superior individual chosen from the elitist fruit fly subpopulation.

In addition, a large number of samples are required to structure a probability density function in traditional GEDA, especially for high-dimensional problems. However, too many sample points lead to increasing the number of evaluations and wasting computing resources. In FOA, the greedy strategy is used to update the population which wastes much potential information, fruit fly individuals are covered whether or not the better locations are found. For the above reasons, an external archive is implemented to store the elitist individuals in past $a$generation.

$$Archive^t = X_s^{t-1} \cup X_s^{t-2} \cup \cdots \cup X_s^{t-a} \tag{21}$$

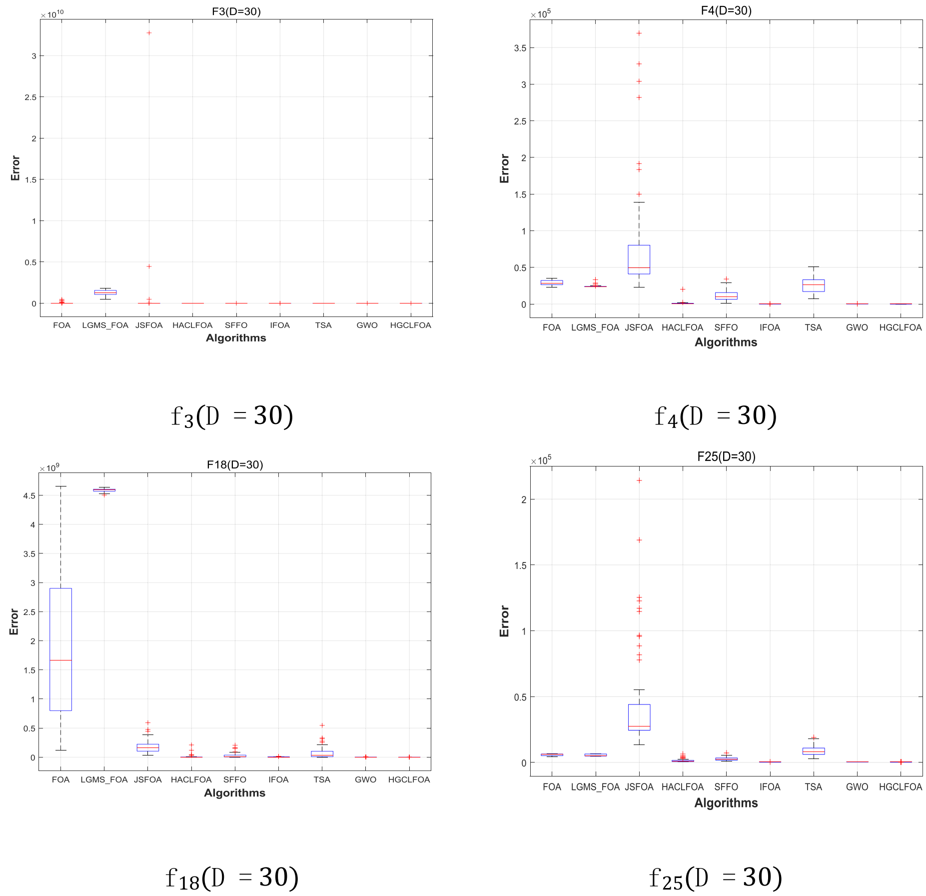


Fig. 11. Box plots compare with eight algorithms (30D).

where the $Archive^t$ istheexternalarchiveinthgeneration, $X_s^{t-a}$is all the superior solutions in $(t-a)th$ generation.

The information in the archive is integrated and analyzed to update the covariance as follows.

$$CA^t = Archive^t \cup X_s^t \tag{22}$$

$$\sum = \frac{1}{|CA^t|}\sum_{i=1}^{|CA^t|}\left(CA_i^t-\mu\right)\left(CA_i^t-\mu\right)^T \tag{23}$$

The probability density ellipsoid extends along the direction of gradient descent with the increase of length of the external archive. This method of covariance adjustment expanded the search range of GEDA effectively, and adjusted the shape of the PDE in the potential direction timely. The Fig. 4 shows the process of means shifts and adjusting the shape of PDE in different archive length respectively. The new promising candidates are denoted by Eq. (24).

$$X_i = \mu_s + N\left(0, \sum\right) \tag{24}$$

4.3. Gaussian random walk strategy

The elite population is a role model for the entire population, leading the evolution of the population. Therefore, when the population is stagnation, it is reactivated by disturbing the elite population.

When the following conditions in Eq. (25) are satisfied, the population is regarded as trapping to local optima. A Gaussian random walk is used to disturb the population and increase the ability of escaping the local optima.

$$\frac{1}{|S|}\sum_{i=1}^{|S|} fX_s(t+1) = \frac{1}{|S|}\sum_{i=1}^{|S|} fX_s(t) \tag{25}$$

$$X_{s,i}(t+1) = GaussianX_{s,j}(t), \vec{\delta} + r^*\left(X_{s,j}(t) - X_{s,i}(t)\right) \tag{26}$$

$$\vec{\delta} = \alpha^*\left|X_{s,i}(t) - X_{s,j}(t)\right| \tag{27}$$

In Eq. (27), the scaling parameter α is reducing gradually, which means the shrink of search scope. r is the random values in [0, 1]. $X_{s,j}(t) - X_i(t)$ is a perturbation term.$X_{s,j}(t)$ is rand selected from superiors individuals. This selection is guaranteed the convergence of the population. The α is same defined in section 4.1. (b).

4.4. The framework of HGCLFOA

The complete procedure of the HGCLFOA is outlined in pseudo-code as follows.

The pseudo-code of HGCLFOA

Input*sizepop, Ne,a,max_nfes, UB, LB,Archive*
sizepopis number of population, Ne is number of elitist number, a is the length of external archive, max_nfes is the number of maximum number of evaluations, UB, LBbound of threshold, n is the length of archived so far.
Output *Smellbest,X_axis*
Smellbestis the global best fitness, X_axis is the correspondingcoordinate for Global Optimal

For $i = 1, 2\cdots, sizepop$
  $X_i = LB + (UB - LB)*rand()$;
End for
  $X_s \leftarrow arg(topNeofsortf(X_i)), X_i \leftarrow arg(restofsortf(X_i))$;
  $g = 0$;
While$nfes <= max\_nfes$

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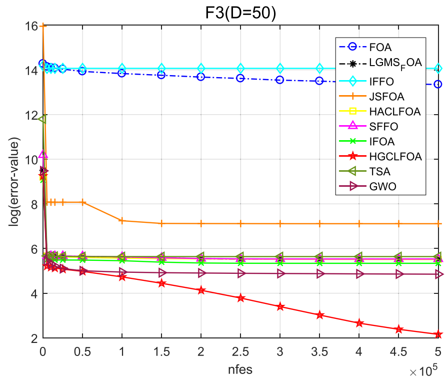
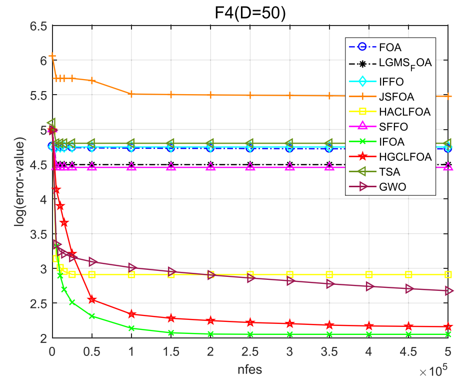
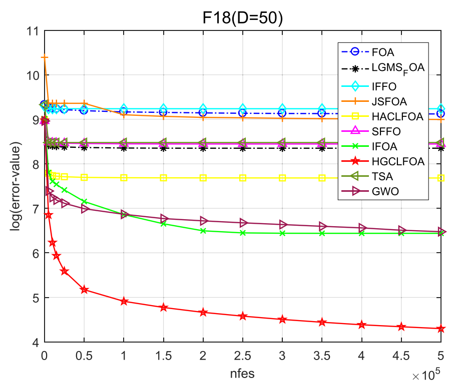


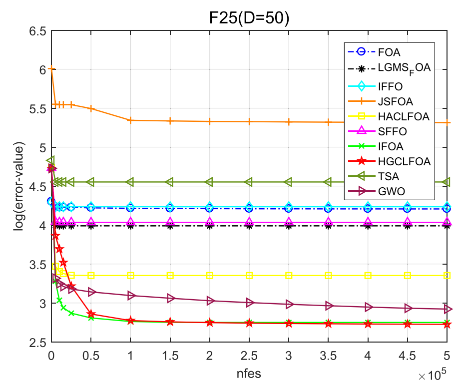
$f_3(D = 50)$



$f_4(D = 50)$



$f_{18}(D = 50)$



$f_{25}(D = 50)$

Fig. 12. Convergence curves of eight algorithms (50D).

(*continued*)

The pseudo-code of HGCLFOA

$g = g + 1$;
Assign elite learning objects to inferior solutions according Eqs. (1) and (2)
According the Eqs. (11)–(15) to decide the probability $P(i)$ of selection.
For $i = 1, 2, \cdots, sizepop - Ne$
If $rand < P(i)$
Updating X_i based on Eqs. (3)–(6);
Else
Updating X_i based on Eq. (7)–(10);
End if
Calculate the fitness
End for
For $i = 1, 2, \cdots Ne$
If Eq. (25) is satisfied
Execute disturbance according Eqs. (26) and (27);
Else
Updating X_s according Eqs. (21)–(24);
Calculate the fitness;
If n < a
$Archive(g+1) = Archive(g) \cup X_s$;
n = n + 1;
Else
$Archive(g+1) = Archive(g) \cup X_s \backslash X_s(g-a)$;
End If
End if
End For
Sort fitness and corresponding coordinates;
Adjusting elite and suboptimal populations;
End while
Return $Smellbest = min(f(X_i) \cup f(X_s))$;
$X\_asxis = arg(min(f(X_i) \cup f(X_s)));)$

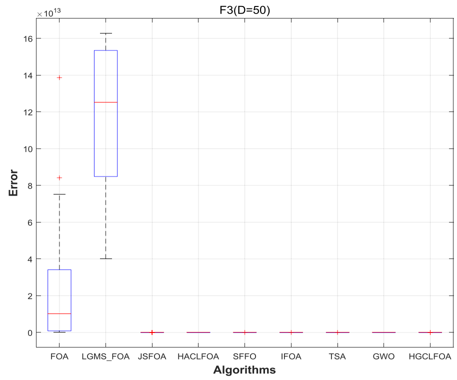
4.5. Time complexity analysis of HGCLFOA

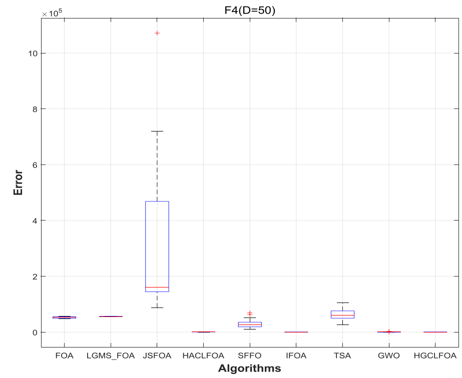
All the algorithms are running in the CEC 2017 benchmark, which stipulated the maximum evolution as $D \times 10^4$. Thus, the time complexity of fitness comparison in the proposed HGCLFOA and variant are the same. In the analysis of computational complexity, the time complexity of HGCLFOA is analyzed by splitting each operation. In the improved olfactory search stage, the positions of individuals are still updated, just learning from different elitist individuals, the time complexity is O (sizepop-Ne). In the vision search of elitist individuals are updating according to historical knowledge, and the time complexity is O(Ne). In the proposed HGCLFOA, the search process is guided by the feedback of historical knowledge rather than complex computation. So, the time complexity is O(sizepop*gen). The gen is the generation that is decided by the maximum evolution.

4.6. Convergence analysis of HGCLFOA

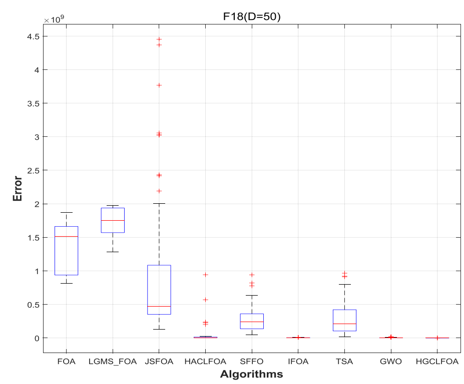
The convergence of original FOA is tried to proof in Zhang, Cui, Wu, Pan, and He (2016), the result shows that it is difficult to convergence for FOA. The main reason is because the coordinates in the search process is almost randomly, the individual fall into local optima and hard to escape. Thus, the global optima did not find. For checking the convergence of the proposed HGCLFOA, the Markov model is used to analyses the performance.

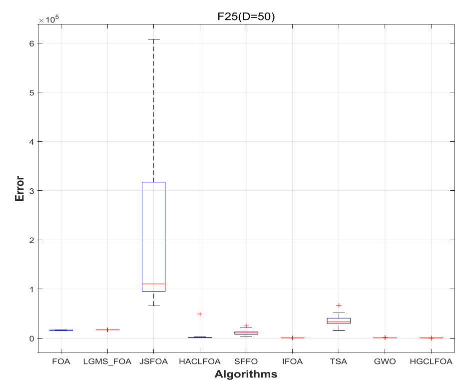
The optimization problem is defined as (x, f). The feasible solution spaces is x, which represents the search range of individuals. The f is the function of fitness. The $\{x(t)\}_{t \to 0}^{t \to \infty}$ is the state sequence of a fruit fly individual, t is the number of generations. The global optimum set was





$f_3(D = 50)$ $f_4(D = 50)$





$f_{18}(D = 50)$ $f_{25}(D = 50)$

.

Fig. 13. Boxplots compare with eight algorithms (50D).

defined as follows.

$$B^* \cong \{x^* : \neg\exists x \neq x^*, f(x) > f(x^*)\}$$

$$\eta^N = \underbrace{\eta \times \eta \times \cdots \times \eta}_{N}$$

Definition 1. *The* $\{x(t)\}_{t\to0}^{t\to\infty}$ *is the random process of HGCLFOA, and the* $\{x(t)\}_{t\to0}^{t\to\infty}$ *has the Markov quality.* $\{x(t)\}$ *is satisfied and only satisfied the following condition, the sequence weakly converges to the global optimum in probability*.

$$\lim_{t\to\infty} P\{x(t)\cap B^* \neq \varnothing\} = 1$$

Definition2. $\{x(t)\}_{t\to0}^{t\to\infty}$ *is the stochastic process from one state transformation to another in discrete time. The progress is memoryless, and the next state* x_{n+1} *only depend on the current state* x_n.

$$P(x_{n+1} = j|x_n = i, x_{n-1} = i_{n-1}, \dots x_0 = i_0) = P(x_{n+1} = j|x_n = i)$$

Thus, $\{x(t)\}_{t\to0}^{t\to\infty}$ is the sequence has the Markov quality. Assume x is a discrete state space, then

$\{x(t)\}_{t}^{t\to\infty}$ is a Markov chain.

Definition 3. *The continuous search space* ξ *is mapped to a finite discrete set* η *due to the limitations of the numerical calculation accuracy in computers. If the population scale is N, the state space is*

Theorem1. *Suppose that* $\{x(t), t = 0, 1, 2\cdots\}$ *is a population sequence generated by HGCLFOA.* $\{x(t), t = 0, 1, 2\cdots\}$ *is a finite homogeneous Markov chain in the state space* η^N.

Proof. *The* $\{x(t), t = 0, 1, 2\cdots\}$ *is finite according to the Definition 3. The population generated by HGCLFOA is independent on the past state and depends on the current state. As Definition 2, the* $\{x(t), t = 0, 1, 2\cdots\}$ *is a finite homogeneous Markov chain in the state space* η^N.

According to the above conditions, assume this condition following:

$$P(t) = P\{x(t)\cap B^* \neq \varnothing\} = P\{x(t) \in B^*\}$$

Thus, the following conditions are valid.

$$P^{'}(t) = P\{x(t)\cap B^* = \varnothing\} = 1 - P\{x(t) \in B^*\}$$

According to the Bayesian conditional probability formula,

$$P^{'}(t+1) = P^{'}(t) = P\{x(t)\cap B^* = \varnothing|x(t)\cap B^* \neq \varnothing\}^* P\{x(t)\cap B^* \neq \varnothing\} + P\{x(t)\cap B^* = \varnothing|x(t)\cap B^* = \varnothing\}^* P\{x(t)\cap B^* = \varnothing\}$$

Due to the greedy selection strategy of the elitist individuals, the best individuals are obtained constantly.

$$P(t+1) = P\{x(t)\cap B^* = \varnothing|x(t)\cap B^* = \varnothing\} = 0$$

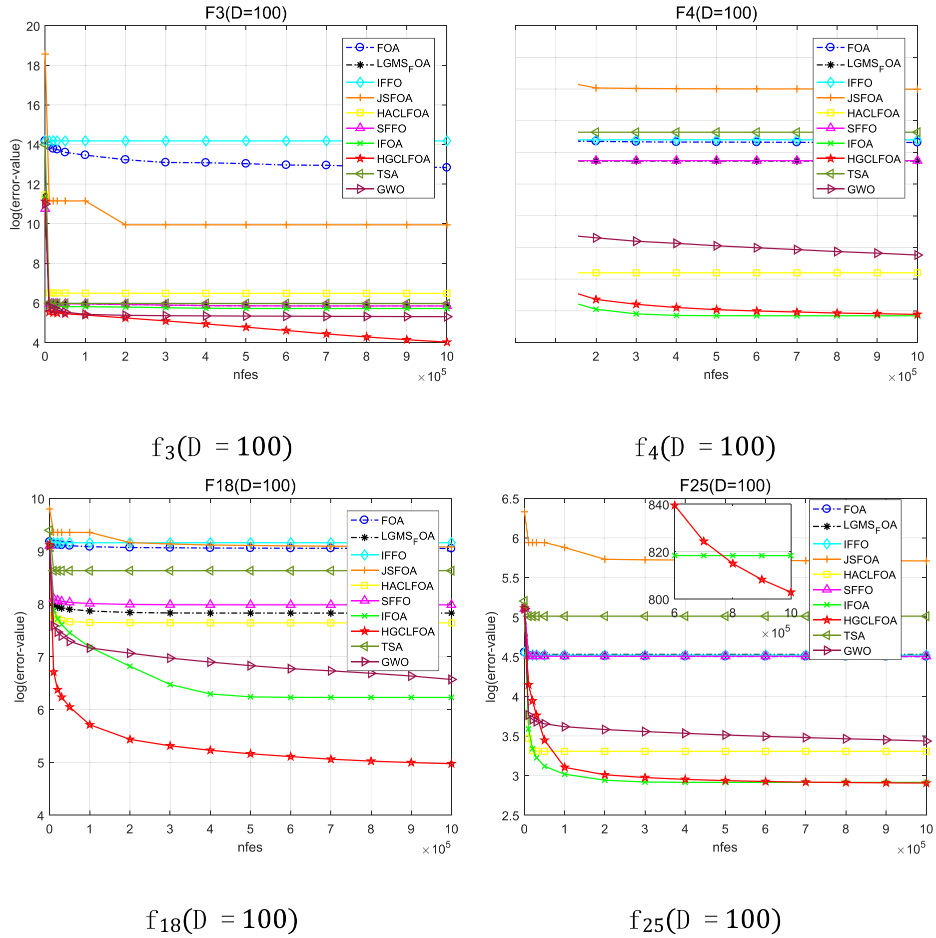


Fig. 14. Convergence curves of eight algorithms (100D).

So that

$$P^{'}(t+1) = P\{x(t)\cap B^{*} = \varnothing | x(t)\cap B^{*} = \varnothing\} * P\{x(t)\cap B^{*} = \varnothing\}$$

Assume ξ is the minimum of $P\{x(t)\cap B^{*} = \varnothing | x(t)\cap B^{*} = \varnothing\}$, so that:

$$0 \leq \xi \leq P\{x(t)\cap B^{*} = \varnothing | x(t)\cap B^{*} = \varnothing\} \leq 1$$

$$0 \leq 1 - P\{x(t)\cap B^{*} \neq \varnothing | x(t)\cap B^{*} = \varnothing\} \leq 1-\xi$$

$$1 - P\{x(t)\cap B^{*} \neq \varnothing | x(t)\cap B^{*} = \varnothing\} = P\{x(t)\cap B^{*} = \varnothing | x(t)\cap B^{*} = \varnothing\}$$

$$0 \leq P^{'}(t+1) \leq (1-\xi)*P^{'}(t) \leq \cdots\cdots \leq (1-\xi)^{t+1}P^{'}(0)$$

When the following condition is satisfied,

$$lim_{t\to\infty}(1-\xi)^{t+1} = 0$$

$$0 \leq P^{'}(0) \leq 1$$

So that,

$$0 \leq lim_{t\to\infty}P^{'}(t) \leq lim_{t\to\infty}(1-\xi)^{t+1}P^{'}(0) = 0$$

Finally, the result is obtained.

$$1 - \lim_{t\to\infty} P(t)^{'} = 1$$

$$\lim_{t\to\infty} P\{x(t)\cap B^{*} \neq \varnothing\} = 1$$

Through the analysis above, the convergence of HGCLFOA is proved.

5. Experimental result analysis and discussion

In this study, the CEC2017 (Wu, Mallipeddi, & Suganthan, 2016) benchmark is applied to verify the performance of the proposed HGCLFOA, which is the widely accepted evaluation criteria for continuous problems. In the test set, four different categories of functions are divided as follows.$f1-f3$ are unimodal functions, $f4-f10$ are simple multimodal functions, $f11-f20$ are hybrid functions, and $f21-f30$ are composition functions. These functions are tested in 10, 30, 50,100D (dimensions) for 51 times respectively. The maximum evolution number is stipulated as $D\times 10^4$.The proposed HGCLFOA is programmed with MATLAB, and runs on a server with a 2.60 GHz Intel(R) Xeon(R) E5-2650CPU, 32 GB of RAM, and 64-bit OS. For communicating with other interested researchers, the code of the HGCLFOA is published on GitHub, the link is https://github.com/AmeliaDing123/Research.

5.1. Parameters analysis

In this study, several vital parameters have a great effect on the performance of the experiment. They are*sizepop*(the sum of individuals in the population),*Pe*(the proportion of elite individuals in the total population),*length*(The length of the external archive). According to experience, their value ranges are respectively set as follows.$sizepop \in \{50,100,200\}$, $Pe \in \{10\%,20\%,33\%,35\%,40\%,50\%\}$, $length \in \{10,20,40,50,100\}$ In order to find the optimal arrangement and combination, DOE experimental method is executed. There are $3\times 6\times 5 = 90$different cases through the parameters combination. In order to avoid the accidental result of the experiment, every combination runs

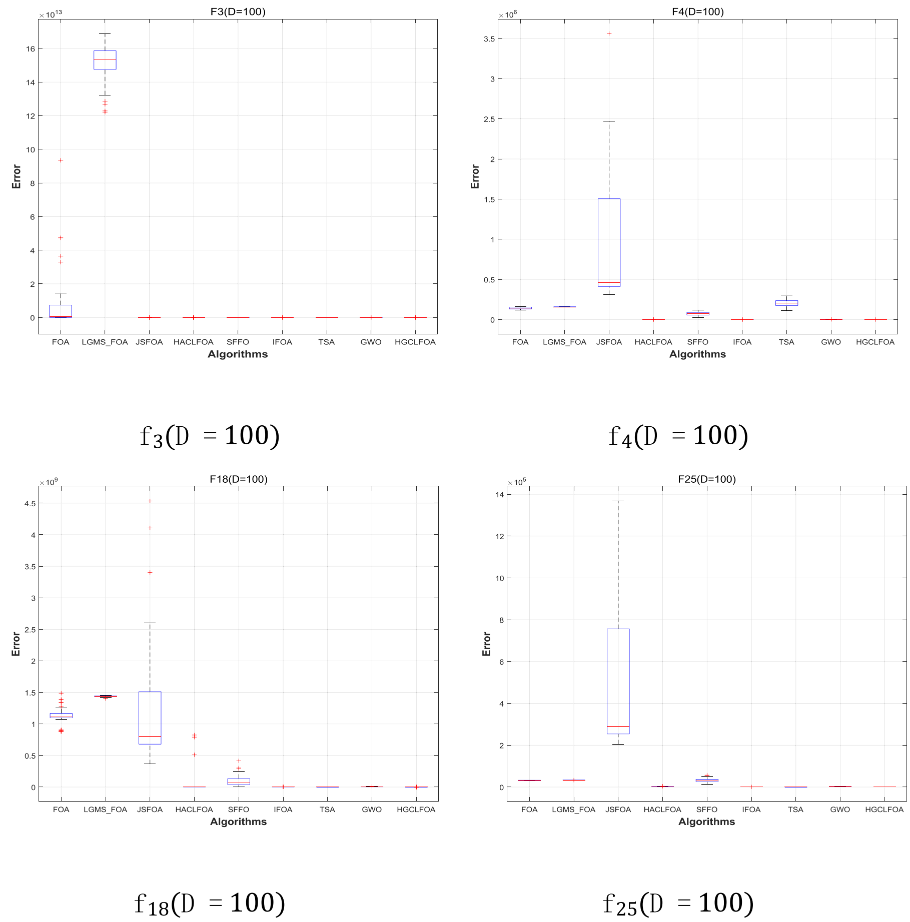


Fig. 15. Boxplots compare with eight algorithms (100D).

Table 7
Friedman test (10D).

Algorithm	Mean rank
FOA	6.93
LGMS_FOA	7.48
JSFOA	5.69
HACLFOA	4.10
IFFO	4.31
SFFO	4.41
IFOA	2.00
TSA	6.57
GWO	2.37
HGCLFOA	1.07
Crit. Diff.$\alpha = 0.05$	2.773
Crit. Diff.$\alpha = 0.10$	2.540

Table 8
Friedman test (30D).

Algorithm	Mean rank
FOA	8.52
LGMS_FOA	9.14
JSFOA	8.31
HACLFOA	4.86
IFFO	5.38
SFFO	5.48
IFOA	2.41
TSA	6.97
GWO	2.31
HGCLFOA	1.62
Crit. Diff.$\alpha = 0.05$	2.773
Crit. Diff.$\alpha = 0.10$	2.540

10 times respectively. As showing in Fig. 6 that these parameters are set as$sizepop = 200$, $Pe = 0.33$, $length = 10$ is the most suitable combination.

The ANOVA (the multivariate analysis of variance) (Shao, Pi, Shao, & Yuan, 2019) experiment is used to analyze the result of DOE. According to the experimental results, the interaction between parameters and the impact of different parameters on the results was analyzed. The detailed results are shown in the Table 1 below.

In Table 1, the $P-value$ of the parameters *sizepop* and *Pe* is less than $\alpha = 0.05$. The α is the confidence level, which means the parameters *sizepop* and *Pe* are more sensitive than other parameters in the HGCLFOA. At the same time, the value of $F-ratio$is indicated that the parameter *Pe* has the greatest impact on the average performance of the proposed HGCLFOA, compares with other factors. Thus, the experiments of the interaction plot need to be employed furthermore to verify the setting of parameters.

The result of the interaction plot is shown in Fig. 7, which expresses that the $sizepop = 100$ and $Pe = 0.33$ are obtained better performance than the other combinations in the main effects plot. Finally, these vital parameters are set as following.$sizepop = 100$, $Pe = 0.33$,$length = 10$.

5.2. Result and discussion

The original FOA, the classical variant LGMS_FOA, other five state-of-arts variants, and two *meta*-heuristic algorithms GWO, TSA are selected to compare with the proposed HGCLFOA. The results express

Table 9
Friedman test (50D).

Algorithm	Mean rank
FOA	7.83
LGMS_FOA	8.87
JSFOA	8.67
HACLFOA	4.67
IFFO	5.47
SFFO	5.50
IFOA	2.37
TSA	7.43
GWO	2.30
HGCLFOA	1.90
Crit. Diff.α = 0.05	2.773
Crit. Diff.α = 0.10	2.540

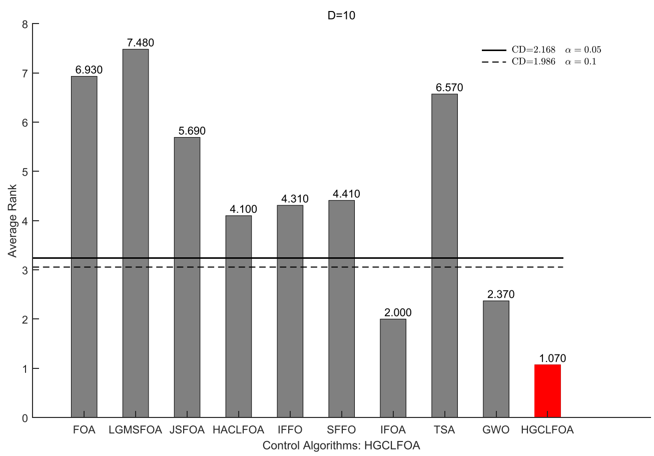


Fig. 16. Ranking for Friedman's test (D = 10).

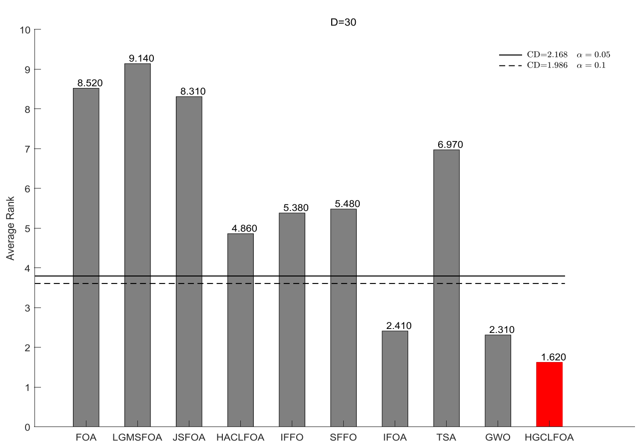


Fig. 17. Ranking for Friedman's test (D = 30).

that the HGCLFOA is valid in the CEC2017benchmark.The detailed experimental data of the mean error and standard deviation in 10, 30, 50,100D is concluded and shown in the Table 3–6, respectively.

The detailed information in these tables shows that the proposed HGCLFOA is superior to the original fruit fly algorithm and other comparing algorithms significantly in the low dimension. To be more specific, the HGCLFOA is only worse than the IFOA in $f6,f20$in the 10 D. In the 30 D, HGCLFOA performs well besides the $f5,f6,f7,f9,f10$ and $f23$. In the 50 D, the HGCLFOA still has superior performance than compared variants on $f2-f3,f11-f12,f14-f21,f23-f26,f29$ functions. In 100 D, the results of the proposed algorithm are remarkable, compared with others except for the IFOA. Although the HGCLFOA does not have too many advantages to comparing with the variant IFOA in 100 D, the HGCLFOA has better performance for the hybrid functions and composition functions than the IFOA.

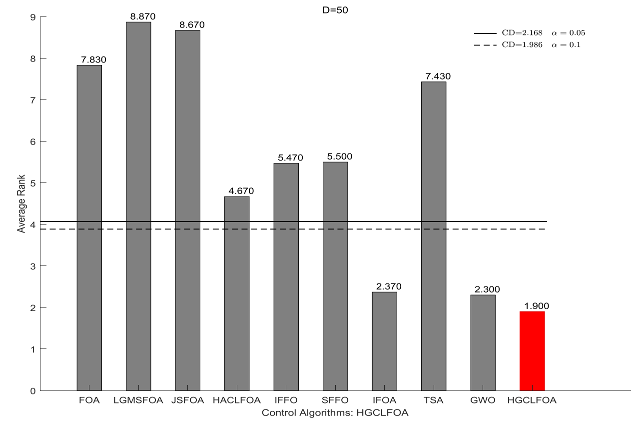


Fig. 18. Ranking for Friedman's test (D = 50).

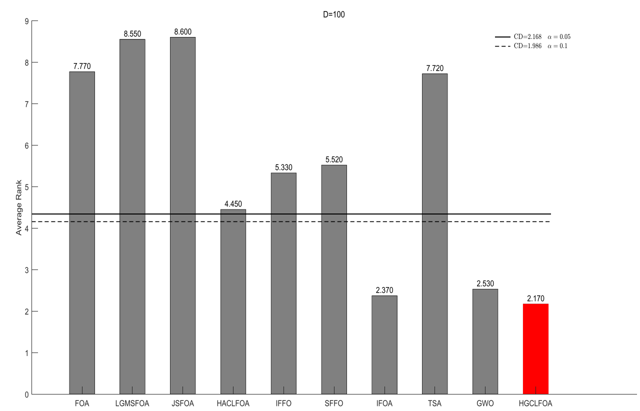


Fig. 19. Ranking for Friedman's test (D = 100).

Table 10
Friedman test (100D).

Algorithm	Mean rank
FOA	7.77
LGMS_FOA	8.55
JSFOA	8.60
HACLFOA	4.45
IFFO	5.33
SFFO	5.52
IFOA	2.37
TSA	7.72
GWO	2.53
HGCLFOA	2.17
Crit. Diff.α = 0.05	2.773
Crit. Diff.α = 0.10	2.540

In order to express the performance of HGCLFOA furthermore, the convergence curves and box plots are drawn to depict the convergence speed and stability. The $f3,f4,f18$, and $f25$ are selected to represent the four different types of functions mentioned above. The Figs. 8 and 10 show that the HGCLFOA converges to the more precise solutions quickly than other variants in the 10 and 30 dimensions. In high dimensions, the proposed algorithm has higher convergence speed than competitors except on the unimodal function. By observing the convergence curves in Figs. 12 and 14, though the convergence speed of HGCLFOA is not fast enough, HGCLFOA always maintains the convergence trend. In practical applications, when the constraint of the number of evaluations is relaxed, HGCLFOA is believed to obtain superior solution. The box plots in Figs. 9, 11, 13, 15 are shown evidence that the HGCLFOA has the best robustness comparing with these completive algorithms. The $P-value$ of the HGCLFOA is far less than 0.05, which means the proposed HGCLFOA is definitely superior to other algorithms in 10, 30, and 50 dimensions. Although the performance of HGCLFOA is slightly inferior in 100 D compared with IFOA, the algorithm is still advantageous for solving certain hybrid functions and composition functions.

a) Comparison using the Wilcoxon Sign Rank Test and Holm-Bonferroni correction

Through further research, the Wilcoxon's sign rank test (Zhao et al., 2018) is used to make statistical analysis in pairwise comparison of experimental results.R^+ is the sum of the rank that FOA is superior to the current comprised algorithm.R^- is the sum of the rank that the current comparison algorithm is superior to FOA. The null hypothesis is supposed that the two algorithms have the same performance. The $P-value$ is the distinctiveness between the two compared algorithms. When the $P-value$is bigger than 0.1that means the null hypothesis is standing and in the confidence interval 99%the two algorithms don't have dramatic differences. Similarly, in the confidence interval 99.5%, when the $P-value$ is bigger than 0.05, the HGCLFOA doesn't have outstanding performance (Shao, Pi, & Shao, 2019). As shown in the Table 2, the $P-value$ of the HGCLFOA is far less than 0.05, which means the proposed HGCLFOA is definitely superior to other algorithms in 10, 30 and 50 dimensions. Although the performance of HGCLFOA is slightly inferior in 100 D compared with IFOA, the algorithm is still advantageous for solving certain hybrid functions and composition functions. The Holm-Bonferroni correction is utilized for increasing the accuracy of the experiment in multiple comparison. In the Table 2, HB is the adjusted $p-value$after the Holm-Bonferroni correction.

b) Comparison using the Friedman Test

The Friedman's test for multi-correlation sample is implemented to check out the eight different variant FOA and the two *meta*-heuristic algorithms GWO, TSA together. In all dimensions, the mean rank of HACLFOA is the best one in all the comparison algorithm. As showing in the Tables 7–9, the performance of HGCLFOA is prominent, and significantly different from most compared algorithms. As presenting in Figs. 16–19, despite the difference between the HGCLFOA and IFOA, HGCLFOA and GWO are not notable, the HGCLFOA is still slightly better than the IFOA and GWO Table 10.

To sum up, the HGCLFOA is feasible and efficient for solving complex function optimization problems, especially for the hybrid functions and the composition functions. The main reason is that elitist individuals improve the efficiency for predicting the potential solutions. This method is a powerful tool for exploration. Thus, the HGCLFOA has good performances in hybrid and composition functions. At the same time, the diversity of FOA is increased through a hierarchical guidance strategy. The accuracy of solutions is improved by the local search obviously, particularly in low dimension. The exploration and exploitation between the elitist population and inferior population are balanced by the collaborative strategy effectively. Although in the high dimension, the performance of the HGCLFOA is deteriorating as the increases of dimension, it still is outstanding in the comparison algorithms. Through analysis of the experimental data and convergence curves, the two reasons are responsible for the unsatisfactory performance in high-dimensional problems. On the one hand, the problem of dimension disaster is recognized as hard to solve in the optimization problem. With the increasing of dimension, the solution space of functions is tremendous and the landscape of function is difficult to explore, especially for complex engineering problems. In the complex benchmark problems CEC 2017, with the non-separable and rotationally variation features, the phenomenon is also common. On the other hand, the proposed algorithm HFCLFOA relies on the experience accumulation of previous generations. With the dimension increasing, the landscape gets more complex. So, more experiences are needed to estimate the landscape, which causes the slowdown of the convergence rate. Due to the limited evaluation times for the CEC benchmark, the advantages of the proposed algorithm are not obvious. From the convergence curve $f12$, $f13$ below that, although the performance of the comparison algorithm IFOA is slightly better than the proposed HGCLFOA in some functions, it does not converge to the optimal solution. The convergence curves show that the proposed algorithm is under a promising search tendency. For some difficult practical problems, in reality, the HGCLFOA has the potential to obtain better solutions by relaxing the constraint of the number of evaluations.

6. Conclusions and future work

In this research, a novel FOA based on cooperative learning is presented to address complex continuous optimization problems. In the olfaction search process, the accuracy of solutions is improved clearly which is attributed to the choice based on feedback. In the vision search stage, the diversity of the population is maintained by the hybrid GEDA. Thus, the HGCLFOA performs well in the exploration stage, especially in certain hybrid functions and composition functions. The HGCLFOA is robust because the evolution of the algorithm is guided by information in previous generations rather than stochastic search. The convergence speed is accelerated by the cooperative learning mechanism in elitist subpopulation and inferior subpopulation. Besides, the individuals escape from local optima availably under the guidance of Gaussian random walk. The results of experiments which are verified in CEC 2017 benchmark, demonstrate that the HGCLFOA is available to solve complex continuous optimization problems and has superior performance compared with the series variants.

In future research, the search accuracy of the HGCLFOA needs to be further improved by adjusting the local search. The convergence speed in high dimensions also needs to be enhanced furthermore. The data driving mechanism is a potential research trend in recent years. Combing with the surrogate models, the potential optimal solutions are predicted effectively. The surrogate models also expedite the convergence speed effectively and save numbers of the evaluation. In applications, the improved FOA can be combined with the support vector machine to effectively optimize various parameters. Meanwhile, the improved FOA is a great potential for addressing scheduling problems.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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