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A Novel Pareto Archive Evolution Algorithm with Adaptive Grid Strategy for Multi-objective Optimization Problem

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***Abstract*—Multi-objective evolutionary algorithms usually utilize fixed evolutionary mechanism and the evolutionary operators are static during the process of algorithm evolution. It is easy to cause a simple population structure, unable to exploit the search space fully and trapped in local optimal solution. In this paper, a novel method named Pareto Archive Evolution Strategy (PAES) with adaptive grid strategy (AGS_PAES) which only makes one mutation to create one new solution and use an "archive" which are called Non-Dominated Archive to store the best solution, is introduced. This procedure is completed by a special approach - adaptive grid method, which decides the criterion of the solution to be archived and the place of the grid location the solution would be stored. The Pareto front obtained by the procedure outperforms the classical Multi-objective Genetic Algorithm (MOGA). Simulation results on the standard benchmark problems show that the proposed adaptive scheme has a better convergence and diversity compared with the second generation classical multi-objective evolutionary algorithms.**

Keywords—Multi-objective Problems; Pareto Archived Evolution Strategy; Non-Dominated Archive; adaptive grid strategy; Pareto front

I. Introduction

The multi-objective problems (MOPs), which exists in vast amount of areas, such as industry, manufacturing, transportation, power system, have attracted great concerns from researchers and practitioners [1-3]. MOPs are the important issues for the real application and research areas as more influence factors are in the consideration for the real problems. Variety of objectives and constraints need to be concerned in the modeling and solution process. There were many researchers [4-6] who created various methods, such as combining different single-objective into one objective by weighting method in the early years. Some methods, which are based on the biological evolution, such as the Genetic Algorithm (GA) are introduced to MOPs in recent years. It is difficult to accurately classify the methods of multi-objective evolutionary algorithms (MOEAs), since certain of MOEAs are a combination of multiple mechanisms. MOEAs are roughly divided into three categories as follow.

- *MOEAs based on dominance relation (pareto).* Sanchez-Anguix, Chalumuri, Aydoan and Julian [7] proposed a multi-objective and near Pareto optimal genetic algorithm that aims to provide human decision makers with trade-off opportunities for the allocation of students to supervisors, in which a novel mutation and crossover operators were introduced. The experimental results showed that the components of the genetic algorithm were more apt for the problem than classic components, and that the genetic algorithm was capable of producing allocations that were near Pareto optimal in a reasonable time. Yi, Deb, Dong, Alavi and Wang [8] proposed a novel adaptive mutation operator for the NSGA-III algorithm to solve Big Data optimization problems. Experimental results indicated that NSGA-III with UC and adaptive mutation operator outperformed the other NSGA-III algorithms.
- *MOEAs based on indicators.* An MOEA based on an enhanced inverted generational distance indicator, in which an adaptation method was suggested to adjust a set of reference points based on the indicator contributions of candidate solutions in an external archive, was proposed by Tian, Cheng, Zhang, Cheng and Jin [9] to improve versatility on problems with different shapes of Pareto fronts in most existing MOEAs. Experimental results demonstrated that the proposed algorithm was versatile for solving problems with various types of Pareto fronts, and outperformed several state-of-the-art evolutionary algorithms for multi-objective and many-objective optimization.
- *Decomposition-based MOEAs.* Zhu, Gao, Du, Cheng and Xu [10] proposed a decomposition-based multi-objective optimization approach considering multiple preference points(mprMOEA/D), which was able to find multiple preferred regions in a single run. In the

mprMOEA/D, a subpopulation (SP) was utilized for each reference point to search for the corresponding preferred region and an external population (EP) is maintained to selectively preserve solutions from all the SPs, which was helpful in convergence and diversity preserving. In addition, local crossover coordinate systems, which coincide with the local manifold of the Pareto set, were introduced to obtain robust performance in the mprMOEA/D. Experimental results demonstrated that the effectiveness and efficiency of the mprMOEA/D

As the development of requirement for the application and algorithm performance to the real problems in different domain, the research works are gradually focus on the combination of evolution algorithm with MOPs. Lin, Zhu, Huang, Chen, Ming and Yu [11] proposed a novel hybrid multi-objective immune algorithm with adaptive differential evolution, named ADE-MOIA to solve various kinds of MOPs. A suitable parent selection strategy and a novel adaptive parameter control approach to provide a correct evolutionary direction. A hybrid multi-objective optimization approach named MO-PSO–EDA [12] which combines the particle swarm optimization (PSO) algorithm and the estimation of distribution algorithm (EDA) is developed for solving the reservoir flood control operation problem. In the MO-PSO–EDA, the particle population was divided into several sub-populations and probability model of each sub-population was built. MO-PSO–EDA performs as well as or superior to the other three competitive multi-objective optimization algorithms. Three genetic-based algorithms are proposed for approximating the Pareto-optimal frontier in project scheduling problem where the above three objectives are simultaneously considered [13]. Sarrafha, Rahmati, Niaki and Zaretalab [14] presented a novel algorithm, called multi-objective biogeography based optimization (MOBBO) with tuned parameters to find a near-optimum solution in bi-objective integrated procurement, production, and distribution problem. Liu and Liu [15] used a modified version of the multi-objective ant colony optimization (MOACO) algorithm to solve the unequal area facility layout problem. Ning, Zhang, Liu and Zhang [16] proposed an archive-based multi-objective artificial bee colony optimization algorithm (AMOABC) to solve MOPs. In the AMOABC, an external archive was used to preserve the current obtained non-dominated best solutions, and a novel Pareto local search mechanism was incorporated into the optimization process.

Most of evolutionary algorithms use evolutionary operators to produce new individual under a common principle, such as genetic algorithms, differential evolution algorithms, etc. Some researchers have shown that some operators are more suitable for some types of problems, but not be available in the whole evolution process of an algorithm. For instance, simulated binary crossover (SBX) is widely used in MOEAs, but Deb [17] observed that SBX was unable to deal with problems with variable linkages. Therefore, an efficient evolutionary operator is important for the optimization methods in solving all optimization problems. The invariant evolutionary operators in the whole evolution process of an algorithm may not be the best choice. Since hybrid different search methods [1, 18, 19] fully exploit the search space, it is necessary to employ different evolutionary operators.

Although there are varieties applications in their works, any approaches require the determination of some appropriate parameters for guiding the search in a suitable way. In this paper, a method for solving the Multi-objective Problems which called AGS_PAES is introduced. Multi evolutionary operators which include the simulated binary crossover, polynomial mutation and adaptive grid strategy are employed to enhance the convergence and diversity for the SPEA2. Simulation results on the standard benchmark problems show that the proposed adaptive scheme has a better convergence and diversity compared with SPEA2, NSGA-II and PESA-II.

The reminder of the paper is viewed as follows. In section 2, the key part of the algorithm would be listed, such as the mathematic definition of the MOPs, the Pareto front and non-dominated solution. In section 3, the details of the AGS_PAES algorithm is illustrated, which consists of the procedure carried out and an adaptive grid strategy. Test result is given out in section 4. Finally, conclusions and suggestions for future studies are presented in Section 5.

II. MULTI-OBJECTIVE PROBLEMS(MOPS)

As no single solution can optimize all the objectives at the same time, the solution of a MOP is a set of decision variable vectors rather than a unique solution. Let $\boldsymbol{x}_a, \boldsymbol{x}_b \in \Omega$ be two decision vectors, $\boldsymbol{x}_a$ is said to dominate $\boldsymbol{x}_b$ ($\boldsymbol{x}_a \succ \boldsymbol{x}_b$), if $f_i(\boldsymbol{x}_a) \le f_i(\boldsymbol{x}_b)$ for all $i = 1, 2, \cdots, m$, and $F(\boldsymbol{x}_a) \ne F(\boldsymbol{x}_b)$. A point $\boldsymbol{x}^* \in \Omega$ is called Pareto optimal solution or non-dominated solution if there is no $\boldsymbol{x} \in \Omega$ such that $F(\boldsymbol{x})$ dominates $F(\boldsymbol{x}^*)$. The set of all the Pareto optimal solutions is called the Pareto set, denoted by PS . The set of all the Pareto optimal objective vectors, $PF = \{F(\boldsymbol{x}^*) \mid \boldsymbol{x}^* \in PS\}$, is called the Pareto front. Knowing that it is impossible to find the whole *PS* of continuous MOPs, the purpose is aim at finding a finite set of Pareto optimal vectors which are uniformly scattered along the true *PF* and highly representative of the entire *PF*.

In general, the Multi-objective Problems are illustrated by following format in mathematically.

$$\begin{aligned} &\min/\max\ y = F(x) = (f_1(x), f_2(x), ..., f_n(x)) \\ &sub\ to: g_i(X) \le 0, i = 1,2,...,k_1 \\ &\qquad\quad g_i(X) = 0, i = k_1+1,...,k_2 \\ &where: x = (x_1, x_2, ..., x_j) \in X \\ &\qquad\quad y = (y_1, y_2, ..., y_t) \in Y \end{aligned}$$

In this equation, x is the decision vector, X is the decision space. y is the objective vector, Y is the objective space. The most difficult reason to treat the Multi-objective Problems is that each Objective $f_i(x)$ is related, restrained and even conflicted with each other. In the above of our paper, we mentioned $f_i(x)$, which represents the geographic condition of the new airport or the good distance from the city center or the more-lower spending and so on. And in every MOPs, there are different subjective to confine the result.

III. Pareto Archive Evolution Strategy (PAES) with adaptive grid strategy (AGS_PAES)

A. AGS_PAES

An improved version of SPEA2 is proposed to reasonably set the selection probability of each operator, which is called AGS_PAES. The flow chart of the proposed AGS_PAES algorithm is shown in Figure 1. The operating mechanism of AGS_PAES is as follows.

AGS_PAES algorithm:

Input: Ne: population size
N: archive size
T: maximum number of generations
Output: non-dominated set (NDS)

Step 1: Initialization: generate an initial population P(0) and create an empty archive (external archive) A(0). Set t=0.

Step 2: Fitness assignment: calculate fitness values of individuals in P(t) and A(t).

Step 3: Environment selection: copy all non-dominated individuals in P(t) and A(t) to A(t+1). Update A(t+1) according to the rules of the adaptive grid.

Step 4: Termination: if t>T is satisfied, then stops and outputs NDS. Otherwise, continue.

Step 5: Mating selection: perform binary tournament selection with replacement on A(t+1) in order to fill mating pool. The size of mating pool is Ne.

Step 6: Reproduction: if t=0, randomly select SBX, PM, DE to generate individuals in P(t+1); if t>1, assign minimum selection probability to all the operators, then assign the rest probability according to their contribution to the external archive. Set t=t+1, go to Step 2.

In the whole algorithm, the adaptive grid method plays an important port. This method is introduced for making a decision between the current and mutation definitely. Compared with traditional Multi-objective, the adaptive grid method has two advantages. Firstly, its cost is low. Secondly, it is adaptive that would not require the setting of the niche size parameter crucially. Three evolutionary operators have played different roles in process of SPEA2.

If one of the following conditions is met, the candidate solution will be added to the archive set.

(1) The external archive is empty, the current solution is accepted.

(2) None of the elements contained in the external population dominates the solution wishing to enter, then such a solution is stored in the external archive.

(3) There are solutions in the archive that are dominated by the new candidate solutions, such solutions are removed from the archive. The candidate solution is stored in the external archive.

(4) When the archive is full, it will eliminate the random solution which is in the more crowed grid, divide the grid and add the new solution to the archive.

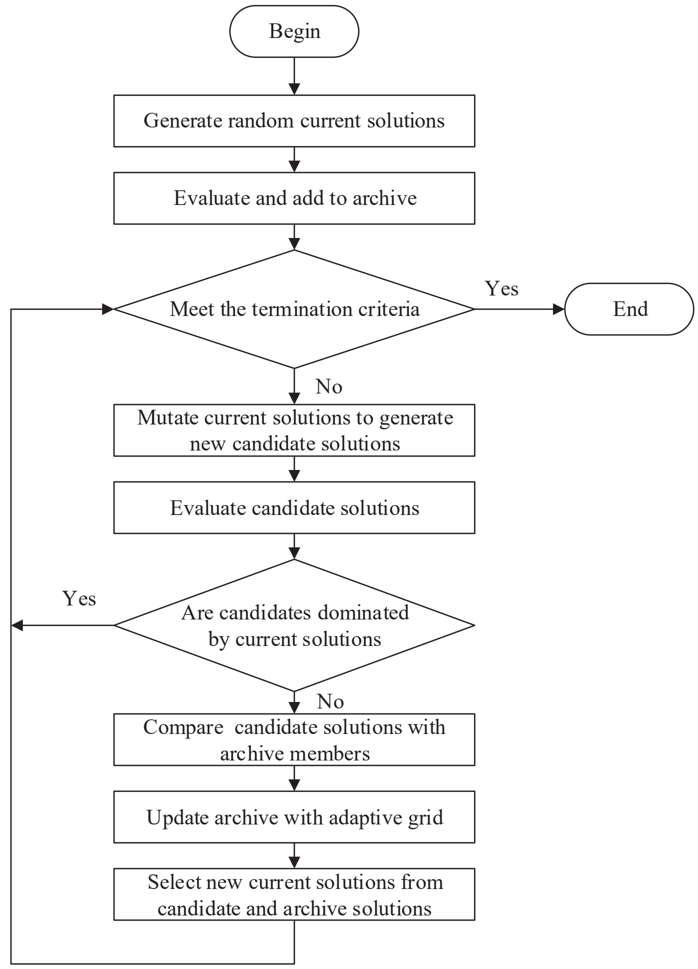

Fig. 1 The flow chart of the AGS_PAES

The adaptive grid is a space formed by hypercubes. The hypercubes have as many components as objective functions. Each hypercube is interpreted as a geographical region that contains an no number of individuals. Take the optimization problem of two objective functions as an example to illustrate the specific implementation process of adaptive grid:

Calculate the boundary of the target space in the current iteration: $(minf_1^k, maxf_1^k \; and \; minf_2^k, maxf_2^k)$, k is the number of iterations.

Calculate the modulus of the grid:

$$\Delta f_1^k = \frac{maxf_1^k - minf_1^k}{M}, \Delta f_2^k = \frac{maxf_2^k - minf_2^k}{M}, R = M \times M$$

Where, R is the number of grids, $M = 32$.

Traverse all the particles in the archive set and determine where they are located in the grid. For particle i, its particle number is:

$$\left[Int\left[\frac{f_1^i - minf_1^k}{\Delta f_1^k}\right] + 1, Int\left[\frac{f_2^i - minf_2^k}{\Delta f_2^k}\right] + 1\right]$$

Calculate the particle density value in the grid, those hypercubes containing more than one particle are assigned a fitness equal to the result of dividing any number x (we used in $x = 10$ our experiments) by the number of particles that they contain.

IV. Simulation Results

A. The basic test case

- *Experimental Setting*

All the algorithms were implemented in MATLAB. AGS_PAES is compared with SPEA2, NSGA-II, PESA-II and MODEA [20]. The simulations were run on a PC with 2.1-GHz CPU and 2-GB RAM. The SBX and PM operators are used in all the algorithms. The tuned parameter values are listed in TABLE I where m is the number of variables. For SPEA2, the population size is 100 and the size of external archive is 100. For NSGA-II, the population size is 100. For PESA-II, the internal population size is 100, the archive size is 100, and the number of hyper-grid cells of each dimension is 32.

TABLE I Parameter settings

Parameters	SPEA2	NSGA-II	PESA-II	AGS_PAES
Cross Probability	0.75	0.75	0.75	0.9
SBX distribution index	20	20	20	20
Mutation Probability	$1/m$	$1/m$	$1/m$	$1/m$
PM distribution index	25	25	25	25

For AGS_PAES, the population size is 100, the size of external archive is 100, the minimum selection probability is 0.05, and CR=1.0 and F=0.5 for DE operation. For ZDT problems, the number of function evaluations is kept at 15000. For DTLZ is kept at 50000. Each algorithm is run 20 times independently for each test instance.

- *Experimental Results*

The mean and standard deviation of the metrics in four algorithms for ZDT problems was showed in TABLE II. In each table cell, the first line was the mean value, the second line was the standard deviation, and the bold was optimal. For GD, AGS_PAES is optimal for all the test instances but ZDT2. For SP, AGS_PAES is optimal for all the test instances but ZDT3. For MS, SPEA2, NSGA-II, MODEA and AGS_PAES are close to one, but AGS_PAES is slightly better than SPEA2, NSGA-II and MODEA. The MS of PESA-II is worst because its boundary solutions are replaced. For IGD and HV, AGS_PAES is optimal for all the test instances. Overall, it is clear from TABLE II that AGS_PAES is better than the others. Because PESA-II just reserves non-dominated solutions in the archive, SPEA2, NSGA-II, MODEA and AGS_PAES reserve dominated solutions when the number of non-dominated solutions is less than the size. So it is also seen from TABLE II that it's useful to reserve dominated solutions for ZDT problems at the early stage.

Statistic values of DTLZ test instances were showed in TABLE III. These four problems have three objectives. For GD, AGS_PAES is optimal for all the test instances but DTLZ1. For SP, AGS_PAES is optimal on DTLZ4. For MS, AGS_PAES is optimal on DTLZ1 and DTLZ3. So AGS_PAES has a better diversity considering SP and MS. For HV, AGS_PAES is the best of all. The solutions obtained by PESA-II are easily gathered together due to that the MS of PESA-II is small. So the IGD of PESA-II is optimal for DTLZ test instances.

TABLE II Mean values and standard deviations of performance indicators on ZDT test instances

Problem	Algorithm	GD	SP	HV	MS	IGD
ZDT1	SPEA2	2.0005e-006	2.0021e-004	3.1321e+000	**8.1326e-002**	1.4325e-003
		4.3495e-006	1.7674e-004	2.7732e-004	**2.3685e-004**	1.2312e-004
	NSGA-II	1.8-932e-005	5.2251e-004	3.2387e+000	8.7821e-002	8.3212e-005
		5.3375e-005	5.3461e-004	5.4561e-004	2.6547e-004	3.0021e-006
	PESA-II	7.1343e-004	5.5674e-002	3.4356e+000	9.23417e-002	5.0456e-005
		6.9216e-005	3.3245e-005	3.1564e-004	1.2111e-004	3.0009e-006
	MODEA	1.0282e-006	3.2012e-004	3.1231e+000	9.3331e-002	5.8790e-005
		3.2312e-007	3.0121e-005	3.2341e-004	3.2109e-004	4.1213e-006
	AGS_PAES	**1.01205e-006**	**1.2646e-004**	**3.1415e+000**	9.3341e-002	**1.2120e-005**
		2.8806e-007	**2.0003e-005**	**8.3498e-006**	1.0017e-005	**1.0224e-006**
ZDT2	SPEA2	**1.8902e-006**	1.9989e-004	3.0212e+000	9.4325e-002	7.0009e-006
		3.1213e-007	1.2131e-004	2.0003e-005	3.2126e-005	3.0111e-007
	NSGA-II	2.1005e-006	5.8221e-004	3.0121e+000	9.3312e-002	4.2342e-005
		7.4300e-007	4.2231e-005	2.2341e-005	1.0121e-004	6.3443e-006
	PESA-II	6.9234e-004	5.0578e-004	3.0981e+000	9.2319e-002	4.2312e-005
		7.3398e-005	3.2231e-005	4.0005e-004	2.2298e-004	8.2145e-006
	MODEA	3.1621e-006	2.0031e-004	3.1121e+000	9.4451e-002	7.2001e-006
		3.2131e-007	1.8798e-004	2.2289e-005	4.2212e-005	3.3321e-007
	AGS_PAES	2.0001e-006	**1.3787e-004**	**3.0021e+000**	**9.3341e-002**	**4.0056e-006**
		2.2289e-007	**1.8848e-005**	**1.1002e-005**	**1.0771e-005**	**2.1209e-007**
ZDT3	SPEA2	3.2312e-005	**2.0321e-004**	**4.2498e+000**	9.3839e-002	3.2871e-005
		2.2219e-006	**1.0019e-004**	**1.2187e-005**	5.1121e-006	1.2387e-005
	NSGA-II	9.1001e-006	7.0214e-004	4.2878e+000	9.1980e-002	3.0123e-004
		4.1387e-007	3.3431e-004	2.2317 e-005	6.1232e-004	5.1231e-005
	PESA-II	6.4559e-004	8.1453e-004	4.2371e+000	9.4079e-002	4.1012e-005
		3.9991e-005	1.0128e-004	2.3406 e-004	9.1421 e-004	5.0012e-006

	MODEA	1.1231e-005	6.2109e-004	4.0238e+000	9.3309e-002	7.0124e-004
		5.2201e-007	3.2981e-004	2.2131 e-005	6.0123e-004	5.3324e-005
	AGS_PAES	**8.0205e-006**	2.3321e-004	4.1089e+000	**9.3365e-002**	**3.0556e-005**
		1.0103e-006	2.4431e-004	1.0002e-005	**6.0143e-005**	**1.1257e-006**
ZDT4	SPEA2	6.0165e-005	2.0205e-004	3.2395e+000	9.0989e-002	4.3006e-005
		2.2221e-006	3.0859e-005	4.8878e-005	1.2341e-005	1.2312e-006
	NSGA-II	4.2213e-005	4.0011e-004	3.2219e+000	9.3459e-002	3.2319e-004
		2.3825e-006	2.1210e-005	4.3459e-005	2.5532e-005	3.1256e-005
	PESA-II	9.0031e-004	8.0511e-004	3.2319e+000	9.5512e-002	3.0001e-004
		5.2219e-005	3.0089e-005	2.0389 e-004	3.2109e-004	8.1767e-005
	MODEA	3.1121e-005	3.1231e-004	3.2239e+000	9.3419e-002	3.3412e-004
		3.2756e-006	3.0032e-005	4.0432e-005	2.0897e-005	3.0001e-005
	AGS_PAES	**1.2210e-005**	**1.3429e-004**	**3.2210e+000**	**9.2377e+000**	**1.2308e-005**
		1.2209e-006	**1.9812e-005**	**1.0019e-005**	**1.2201e-005**	**1.2111e-006**
ZDT6	SPEA2	6.0482e-005	1.3426e-004	2.8545e+000	9.4521e-002	5.2512e-005
		2.2487e-006	2.2109e-005	3.0009e-006	3.3498e-005	1.2109e-006
	NSGA-II	7.2229e-005	2.3319e-004	2.8887e+000	9.3341e-0021	6.1919e-004
		3.4588e-006	2.0312e-005	3.2319 e-006	8.0019 e-005	4.2908e-006
	PESA-II	5.0819e-005	5.1098e-004	2.8561e+000	9.5339e-002	5.0021e-005
		2.0215e-006	2.4509e-005	2.0098e-005	5.0037e-005	2.1488e-006
	MODEA	7.0365e-005	2.2341e-004	2.9981e+000	9.2761e-0021	6.3551e-004
		3.2169e-006	2.2256e-005	3.0005 e-006	8.3209 e-0054	4.1709e-006
	AGS_PAES	**1.8349e-005**	**1.2110e-004**	**2.8421e+000**	**9.4319e-002**	**4.2101e-005**
		1.3347e-006	**1.0338e-005**	**1.0266e-006**	**2.1009e-005**	**1.0055e-006**

TABLE III Mean values and standard deviations of performance indicators on DTLZ test instances

Problem	Algorithm	GD	SP	HV	MS	IGD
DTLZ1	SPEA2	4.1980e-006	2.0098e-002	7.2987e-001	9.3487e-001	7.3341e-004
		6.0321e-007	8.8878e-003	3.4512e-004	4.0016e-004	6.0029e-005
	NSGA-II	5.3309e-006	1.0129e-002	7.2121e-001	9.8246e-001	5.9104e-004
		6.0098e-007	2.2869e-002	9.3371e-004	5.0011e-004	5.2398e-005
	PESA-II	**3.7223e-008**	**1.1619e-002**	8.9008e-002	8.0019e-001	**8.0013e-005**
		5.1410e-009	**2.1465e-002**	8.2319e-004	4.3376e-004	**6.1432e-005**
	MODEA	5.1414e-006	1.3671e-002	7.2989e-001	9.3421e-001	6.2098e-004
		6.0031e-007	2.1176e-002	9.3719e-004	5.3651e-004	5.1219e-005
	AGS_PAES	8.3315e-007	1.1391e-002	**7.3321e-001**	**9.1071e-001**	6.1388e-004
		3.0022e-007	1.3619e-002	**4.1012e-004**	**8.1015e-004**	1.3421e-005
DTLZ2	SPEA2	5.0011e-006	**1.0981e-002**	8.4471e-002	**9.4819e-001**	8.9812e-004
		5.3212e-007	**3.0129e-003**	2.0006e-003	**5.3312e-004**	8.0015e-005
	NSGA-II	4.5419e-005	4.0231e-002	3.3471 e-001	9.4518e-001	1.0129e-003
		3.2312e-006	9.1212e-003	6.0021e-003	3.1318e-004	8.9933e-004
	PESA-II	7.0123e-005	1.4521e-002	2.1219 e-001	6.0123e-001	1.1219e-003
		3.3312e-006	2.1217e-002	5.0918e-003	3.0128e-003	4.0182e-004
	MODEA	4.1091e-005	2.4471e-002	3.2281 e-001	9.4321e-001	1.0001e-003
		3.0371e-006	1.0021e-002	6.0127e-003	3.2277e-004	5.3218e-004
	AGS_PAES	**7.0002e-007**	1.1023e-002	**4.0002e-001**	9.4427e-001	**6.1316e-004**
		6.4532e-007	7.3219e-004	**1.0071e-003**	7.0165e-004	**3.3219e-005**
DTLZ3	SPEA2	2.0087e-005	**1.1287e-002**	8.3998e-002	9.3367e-001	1.0017e-003
		9.7818e-007	**1.0031e-003**	3.0121e-004	9.4418e-005	8.8829e-005
	NSGA-II	9.6678e-006	4.0021e-002	3.3519e-001	9.4519e-001	6.3671e-004
		5.2317e-006	3.0029e-003	5.0126e-003	8.9817e-005	9.9918e-006
	PESA-II	1.4165e-005	2.1791e-002	1.2318 e-001	9.8892e-002	**4.4518e-004**
		2.5618e-006	2.0171e-003	2.0716e-003	3.2012e-002	**2.1319e-005**
	MODEA	1.33318e-005	2.0217e-002	3.5182e-001	9.5419e-001	6.5672e-004
		5.5516e-006	3.1816e-003	5.0018e-003	3.0021e-004	8.2812e-005
	AGS_PAES	**1.1812e-006**	1.2171e-002	**4.0012 e-001**	**9.5487e-001**	6.0121e-004
		3.5619e-007	1.4491e-003	**2.0102e-004**	**9.7761e-005**	5.0071e-004
DTLZ4	SPEA2	3.3345e-005	1.1102e-002	8.6891e-001	**9.4471e-001**	1.2129e-003
		1.0910e-006	2.2171e-003	9.3318e-005	**9.7819e-005**	5.0021e-004
	NSGA-II	5.1218e-005	4.2218e-002	3.2109e-001	9.2987e-001	1.0129e-003
		1.0019e-005	9.8882e-003	9.7829e-005	9.5556e-005	5.1299e-004
	PESA-II	5.2238e-005	4.0021e-002	3.0021e-001	9.4309e-001	**9.3218e-004**
		5.1121e-006	2.2171e-002	2.0003e-003	5.0121e-003	**5.0009e-005**
	MODEA	5.0007e-005	3.0980e-002	3.1218e-001	9.1219e-001	1.0012e-003
		1.0321e-005	1.2121e-002	7.0121e-004	1.2391e-004	5.1515e-004
	AGS_PAES	**1.1210e-005**	**9.6778e-003**	**9.3201e-002**	9.5312e-001	9.8829e-004
		5.3201e-006	**9.0012e-004**	**2.0005e-004**	9.0003e-004	6.4529e-004

V. CONCLUSION AND FUTURE WORK

In this paper, the simple form (1+1)-PAES have been introduced and its time complexity have been shown by counting the times of comparison in the main core of the algorithm-selection and receive/rejection. The key part of the method - adaptive grid method plays a core part in the algorithm, it decides the solution whether to be received or rejected. Furthermore, if the solution is added to the archive, the procedure must judge the grid of solutions in the current archive and decide which solution to be deleted and where the solution to locate. The experimental results demonstrate that the effectiveness and efficiency of the AGS_PAES.

The extension of the simple form $(1+1)-PAES$: $(1+\lambda)-PAES$ (a) and $(\mu+\lambda)-PAES$ (b) are our next step to research. (a) means 1 father solution and λ new off-springs solutions. (b) means μ father solution and λ new off-springs solution. $(1+\lambda)-PAES$ $(\mu+\lambda)-PAES$ will be applied to various fields for the real applications.

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