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A two-stage differential biogeography-based optimization algorithm and its performance analysis

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ABSTRACT

Biogeography-based optimization (BBO) has drawn a lot of attention as its outstanding performance. However, same with certain typical swarm optimization algorithm, BBO severely suffers from premature convergence problem and the rotational variance of migration operator. In this paper, a two-stage differential biogeography-based optimization (TDBBO) is proposed to address the premature convergence problem and alleviate the rotational variance. In the migration operator, the emigration model is selected according to the two-stage mechanism. The constant emigration model is employed to maintain the diversity of population in the early evolutionary process. The sinusoidal emigration model is selected to accelerate the convergence speed in the late evolutionary process. Meanwhile, the BBO/current-to-select/1, which is a rotationally invariant arithmetic crossover operator, is designed to alleviate the rotational variance. The standard mutation operator is replaced by the Gaussian mutation operator to jump out the local optimum effectively. The greedy selection strategy is introduced to accelerate the convergence speed after the migration and the mutation operators. Besides, the convergence performance of TDBBO is analyzed with the Markov model. Compared with the standard BBO and other outstanding BBO variants on CEC 2017 benchmarks, the TDBBO is superior to the state-of-art BBO variants in terms of solution quality, convergence speed and stability. The TDBBO lays a solid foundation for solving optimization problems of expert and intelligent systems.

1. Introduction

Single objective optimization problems, which are also known as bound-constrained problems, can be transferred to complex optimization problems. Thus, single-objective real parameter optimization plays a vital role in optimization problems. The problem is described as follows. For the cost function $f(x)$ and the search space Ω which is constrained by the upper and lower bounds, the global optimum point x^* satisfies the condition that $(x^*) = \min f(x)$, $x = [x_1, x_2, \ldots, x_D]$. It is viewed as a black-box problem which is aimed to find the global optimum point without explicit knowledge of the structure of objective functions.

Research in the expert system (ES) is one of the most prosperous areas within the AI field. In recent years, various applications of ES have been published covering a wide range of functional area and problem domains. ES provides powerful and flexible means for solving the complex problems which are hard to be addressed with traditional methods. In this paper, an expert system for solving the single-objective optimization problems is conducted as the single-objective optimization problems are complex optimization problems.

Heuristic algorithms have received widespread attention as their simplicity, flexibility and gradient-free mechanism. Various meta-heuristic algorithms, such as genetic algorithm (Goldberg & Holland, 1988), particle swarm optimization (Kennedy & Eberhart, 2002), artificial bee colony algorithm (Karaboga & Basturk, 2007), harmony search algorithm (Zong, Kim, & Loganathan, 2001), biogeography-based optimization (Simon, 2008), have been proposed. However, classical meta-heuristics also have certain drawbacks such as premature convergence and insufficient exploration ability. Various enhanced meta-heuristics were designed (Brest, Maučec, & Bošković, 2017; Nabil, 2016; Zhao, Liu, Zhang, & Wang, 2015) to address the premature convergence problem and improve the exploration ability of classical meta-heuristics. In recent years, the meta-heuristic algorithms were suc-

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cessfully applied to practical problems, such as permutation flow shop scheduling problem (Zhao et al., 2016; Zhao, Liu, Zhang, Ma, & Zhang, 2017; Zhao, Zhang, Wang, & Zhang, 2015), job shop scheduling problem (Zhao, Chen, Wang, & Zhang, 2017; Zhao, Shao, Wang, & Zhang, 2016; Zhao, Zhang, Zhang, & Wang, 2015), and no-wait flow shop scheduling problem (Zhao, Liu, Zhang, Ma, & Zhang, 2018).

Biogeography-based Optimization (BBO), which is inspired by the equilibrium theory of biogeography, is a new population-based optimization algorithm proposed by Simon (2008). Biogeography theory is the science for the migration of species between different habitats, as well as the extinction of species. For an optimization problem and some candidate solutions, each habitat represents a candidate solution in multidimensional solution space. As the Biogeography theory said, the superior solutions tend to share their information with the inferior solutions, and the inferior solutions receive the knowledge of the superior solutions. Although the exploitation level of BBO is excellent, the standard BBO has certain drawbacks such as insufficient exploration ability (Guo, Chen, Wang, Mao, & Wu, 2017), rotationally variance (Chen, Tianfield, Du, & Liu, 2016; Simon, Omran, & Clerc, 2014). Consequently, various developments have been proposed.

The development of BBO is roughly divided into three categories: (1) Theoretical research of the BBO algorithm; (2) Enhance the performance of BBO algorithm by modifying the primary operator; (3) Hybrid BBO with other algorithms (including meta-heuristic algorithms and traditional mathematical methods) to combine the advantages of different algorithms. The literature review is as follows.

The linear migration model was introduced by Simon (2008) in the standard BBO. The influence of migration model on optimization was analyzed by Ma (2010). Five migration models, which are inspired by the mathematical model of biogeography, were listed in this work. The results confirm that the nonlinear migration models are generally superior to linear migration models. Compared with ACO, DE, ES, GA, and PSO, BBO exhibits excellent performance of optimization. However, the BBO still requires additional theoretical and empirical investigations. The Markov model of BBO, which gives the theoretical probability of the occurrence of each possible population as the generation count goes to infinity, is derived by Simon, Ergezer, Du, and Rarick (2011). The Markov model is restricted to binary problems in which each solution feature is a bit, but for the issues in which the solution features are integers, the proposed Markov model needs to be extended. The impact of migration rates on BBO was investigated by Guo, Wang, and Wu (2014) with mathematical way. However, the mutation operator is also a significant component of BBO. The mutation rate deserved further theoretical research. The expected hitting time of BBO with different migration models were investigated by Guo, Wang, Ge, Ren, and Mao (2015) with drift analysis. The results indicate that the performance of BBO is enhanced by the migration model which generates a high mutation rate. A mathematical description of the dynamics of BBO was proposed by Ma, Simon, and Fei (2016) from the perspective of statistical mechanics. The theoretical results for BBO were also compared with a simple GA. The above literatures play an essential role in the theoretical development of the BBO algorithm. However, little literature was executed to research the variants of BBO theoretically.

As the crucial operators of BBO, the migration operator and mutation operator were modified by quite a few researchers to improve the performance of BBO. The Gaussian mutation, Cauchy mutation and levy mutation were introduced by Gong, Cai, Ling, and Li (2010) to replace the regular mutation operator. Simulation results indicate that the modified mutation operator significantly enhances the exploration ability of BBO. A blended migration, which is inspired by the crossover in GAs, was designed by Ma and Simon (2011). The experimental results confirm that the blended migration operator further improves the performance of BBO. Inspired by the Laplacian crossover of real-coded genetic algorithms, a Laplacian BBO was proposed by Garg and Deep (2016) and compared to blended BBO. The results show that Laplacian BBO is an efficient algorithm for solving continuous functions and real-world problems. In the above approaches, the design of the migration operator only involves another one solution, which means the current solution learns from one solution in each migration.

A polyphyletic migration operator was designed by Xiong, Li, Chen, Shi, and Duan (2014) to make the candidate learn from more than one solution in each migration of PBBO. An orthogonal learning strategy is employed to search the potential solution around current solutions after the migration operator. Similarly, the orthogonal learning strategy is also introduced into the OXBBO which was proposed by Feng, Liu, Tang, and Yong (2013). In OXBBO, a large dimension is divided into several factors. The orthogonal learning strategy is employed to produce the new candidate. Experimental results reveal that the PBBO and OXBBO maintain the diversity of the population and accelerate the convergence speed effectively. The strategy that the current solution learns from more than one solution improves the exploration ability of standard BBO. The orthogonal learning strategy is also an effective mechanism to enhance the exploitation ability. However, the limitation of the PBBO and OXBBO is that the time complexity needs to be reduced.

The oppositional-based biogeography-based optimization, which calls the opposition-based learning with a probability after migration and mutation operator, was proposed by Ergezer, Dan, and Du (2009). The simulation results demonstrate that the OBBO is valid for solving the high-dimensional problems. An improved Biogeography-based Optimization (IBBO), which combines an enhanced migration operator and a self-adaptive clear duplicate operator, was proposed by Feng, Liu, Zhang, Yang, and Yong (2014) to enhance the diversity of population and accelerate the convergence speed in the later stage of the evolutionary process. Ecogeography-based optimization (EBO) was proposed by Zheng, Ling, and Xue (2014). The EBO employs a local topology to distinguish neighbors and non-neighbors of each island and defines two new migration operators to enrich information sharing between the islands. Besides, a hybrid migration operator with random ring topology, a modified mutation operator, and a self-adaptive Powell's method are rationally integrated into the PRBBO which is proposed by Feng, Liu, Zhang, Yang, and Yong (2016). Various mechanisms were introduced into standard BBO to balance the exploration ability and exploitation ability in the above methods. The simulation results also demonstrate the OBBO, IBBO, EBO, and PRBBO are competitive methods for global optimization. However, the rotational variance leaves the poor performance of BBO for non-separable problems.

The LBBO, which combines periodic re-initialization and local search operators, was proposed by Simon et al. (2014). Compared with other state-of-art algorithms, LBBO provides competitive performance and performs well for the multimodal problem and high-dimensional problems. The covariance matrix based migration was designed by Chen et al. (2016) to alleviate the rotational variance of BBO. Numerical simulation shows that the new approach effectively reduces BBO's dependence on the coordinate system and enhances the rotational invariance.

The performance of BBO has been significantly enhanced by modifying the primary operators. However, not all the optimization problems are solved by using only one algorithm as the no-free-launch theorem (Wolpert & Macready, 1997) assumed. Hybrid algorithms gained wide popularity as they combine advantages of

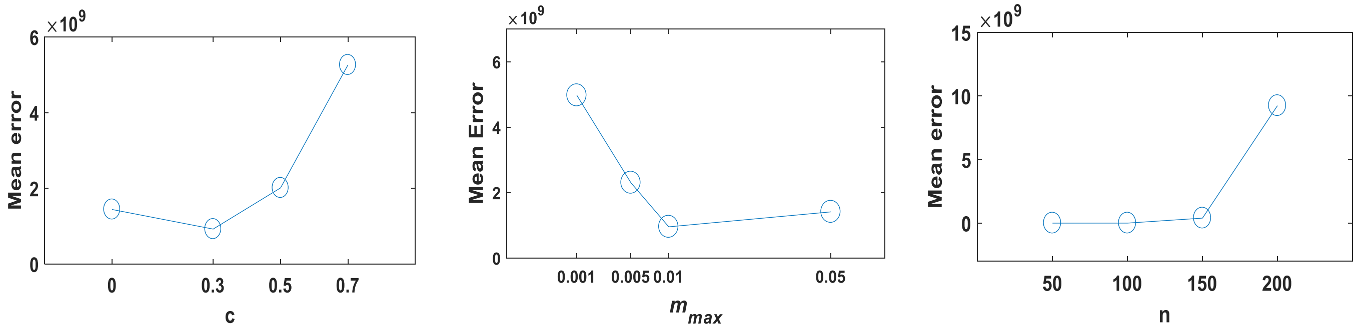


Fig. 1. The trend of parameters.

different algorithms. In the research of BBO, the hybrid approaches are also popular.

DE/BBO was proposed by Gong, Cai, and Ling (2011) to combine the exploration of DE with the exploitation of BBO. In DE/BBO, the crossover operator and mutation operator of DE are embedded into the migration operator of BBO. The biogeography-based particle swarm optimization (BPSO) was proposed by Guo, Li et al. (2014) as the excellent exploration ability of PSO. In BPSO, The entire population is split into several groups. BBO is utilized to search within each subgroup, and PSO is employed for global search. The IWO/BBO was proposed by Khademi, Mohammadi, and Simon (2017) to incorporate feature-sharing among individuals in IWO. The migration operator of BBO was introduced into IWO. The gradient descent strategy was also employed to improve the exploitation ability of IWO. Compared with seven other state-of-art optimization algorithms, IWO/BBO has competitive performance. The MpBBO was proposed by Al-Roomi and El-Hawary (2016) to combine the exploitation ability of BBO and the exploration ability of simulated annealing (SA). In the MpBBO, the inferior candidate will not be selected unless they pass the Metropolis criterion of SA. Therefore, the MpBBO has more immunity against trapping into the local optimums. Several hybrid algorithms which combine other EAs with BBO were designed by Ma, Simon, Fei, Shu, and Chen (2014) to investigate the performance of iteration-level hybrid approach and the algorithm-level hybrid approach. Results indicate that the algorithm-level approach is superior to iteration-level hybrid approach and the best algorithm is competitive with the algorithms from the CEC 2013 competition.

In spite of the fact that the BBO algorithm was extensively studied in recent years, little literature focused on balancing the exploration ability and exploitation ability, as well as enhancing the rotational invariant. This paper presents a TDBBO to address the premature convergence problem and alleviate the rotational variance. In TDBBO, the two-stage mechanism is designed to maintain the diversity of population at the early evolutionary process and accelerate the convergence speed at the late evolutionary process. The BBO/current-to-select/1 is employed to enhance the rotational invariance and learn from more than one solution to maintain the diversity of the population. The Gaussian mutation operator is introduced to make the population jump out of the local optimal effectively. The greedy selection strategy is also employed to accelerate the convergence speed after the migration and mutation operator. As above, TDBBO takes all the factors which affected the performance of the BBO in its consideration. The TDBBO has remarkable advantages in optimization accuracy, convergence speed, and stability. From the perspective of the application, the TDBBO has better performance than other state-of-art algorithms for solving various optimization problems as the most of optimization problems are non-separable problems. The expert system which is based on the TDBBO is also an effective expert system for solving single-objective optimization problems. The contributions of this paper are described as follows.

- In related work, the migration models are employed either linear or nonlinear. In this paper, a two-stage mechanism, which provides constant and sinusoid migration models in the different stage of evolutionary process respectively, is designed to balance the exploration ability and exploitation ability in this paper.
- The BBO/current-to-select/1, which is a rotationally invariant arithmetic crossover operator, is designed to enhance the rotational invariance and make the solution learn from more than one solution in each migration.
- The Markov model of TDBBO is employed to analyze the convergence performance of the TDBBO mathematically. The evolutionary process is mapped to the state transition process in the Markov model, and the convergence performance is also confirmed.
- The proposed TDBBO is tested on CEC 2017 benchmarks and compared with six variants of BBO. The results indicate that the TDBBO performs more effectively and more accuracy than other algorithms. Besides, the optimal combination of parameters is also analyzed by implementing the Taguchi method (Montgomery, 1976)

The remainder of the paper is organized as follows. In Section 2, a brief description of BBO is given. Section 3 describes the TDBBO algorithm. Section 4 analyzes the convergence performance of the TDBBO algorithm. The experimental results, as well as related analysis and discussions, are presented in Section 5. Finally, Section 6 concludes the paper and some future work are suggested.

The notation in this paper is listed as follows:

λ_i	the immigration rate of *i*th habitats
μ_i	the emigration rate of *i*th habitats
I	the maximum immigration rate
E	the maximum emigration rate
n	the maximum count of species
m_{max}	the maximum mutation probability
m_i	the mutation rate of *i*th habitat
P_i	the probability that single habitat just contains *i* species.
F_i	the scaling factor of *i*th habitats
$lamdaScale_i$	the immigration rate of *i*th habitats in TDBBO
Max_NFFEs	the maximum number of fitness evaluations
nfes	the current number of fitness evaluations
$H_i(j)$	the *j*th SIV of the ith habitat
$randn(\mu, \sigma^2)$	the Gaussian distributed random number with mean μ and variance σ^2
$rndreal(0, 1)$	a uniform random number on the interval (0,1)
U_i	the *i*th habitat of the trial habitats
NP	the size of population

Table 1
Parameter setting of competitive algorithm.

Algorithm	Parameter Setting
BBO	$n=50$, $I=1$, $E=1$, $m_{max}=0.005$, keep = 2
B-BBO	$n=50$, $I=1$, $E=1$, $m_{max}=0.005$, $F=0.5$, keep = 2
GBBO	$n=50$, $I=1$, $E=1$, $m_{max}=0.005$, alpha = 0.9
IBBO	$n=50$, $I=1$, $E=1$, $m_{max}=0.005$, $F=0.5$, keep = 2
LXBBO	$n=50$, $I=1$, $E=1$, $m_{max}=0.005$, $a=0$, $b=0.5$, $k=0.95$, keep = 2, $\gamma_{min}=0.1$, $\gamma_{max}=1$
DE/BBO	$n=50$, $I=1$, $E=1$, $F=rndreal(0.1, 1)$, CR = 0.9

Table 2
The parameters for different levels.

Parameters	Levels 1	2	3	4
c	0	0.3	0.5	0.7
m_{max}	0.001	0.005	0.01	0.05
n	50	100	150	200

Table 3
Parameter combinations and Mean error.

No.	Parameter combinations c	m_{max}	n	Mean Error
1	1	1	1	3.62E+02
2	1	2	2	1.27E+05
3	1	3	3	1.40E+08
4	1	4	4	5.62E+09
5	2	1	2	4.90E+06
6	2	2	1	3.65E+02
7	2	3	4	3.67E+09
8	2	4	3	1.34E+07
9	3	1	3	1.37E+08
10	3	2	4	7.92E+09
11	3	3	1	3.62E+02
12	3	4	2	2.19E+03
13	4	1	4	1.97E+10
14	4	2	3	1.28E+09
15	4	3	2	1.63E+06
16	4	4	1	4.69E+02

Table 4
Parameter rank and response values.

Levels	c	m_{max}	n
1	1.44E+09	4.97E+09	3.89E+02
2	9.22E+08	2.30E+09	1.67E+06
3	2.01E+09	9.53E+08	3.94E+08
4	5.26E+09	1.41E+09	9.24E+09
Response value	4.34E+09	4.02E+09	9.24E+09
Rank	2	3	1

2. Biogeography-based optimization

In Biogeography-based Optimization (BBO) (Simon, 2008), suppose that there are an optimization problem and some candidate solutions. Each solution is considered as a habitat with a habitat suitability index (*HSI*). *HSI* is similar to the fitness in other evolutionary algorithms. The features of candidate solutions are called suitability index variables (*SIVs*). New candidates are generated by using migration and mutation. The primary operators of BBO are briefly described as follows.

In migration operator, features are mixed among habitats based on immigration rate λ and emigration rate μ. Each solution has its immigration rate λ_i and emigration rate μ_i. According to linear migration model, λ_i and μ_i are calculated as follows (Simon, 2008).

$$\lambda_i = I \cdot \left(1 - \frac{i}{n}\right) \tag{1}$$

$$\mu_i = E \cdot \left(\frac{i}{n}\right) \tag{2}$$

where I and E are the maximum immigration rate and the emigration rate respectively. i is the species count of the ith habitat. n is the maximum count of species. For convenience, the maximum species count is equal to population size.

P_i is the probability that the single habitat just contains i species, P_i changes from time t to time $(t+\Delta t)$ is given as (Simon, 2008):

$$P_i(t+\Delta t) = P_i(t)(1-\lambda_i\Delta t - \mu_i\Delta t) + P_{i-1}\lambda_{i-1}\Delta t + P_{i+1}\mu_{i+1}\Delta t \tag{3}$$

The above equation holds as one of the following conditions must be met to obtain the i species at the time:

There were i species at time t, and no immigration or emigration occurred between t and $(t + \Delta t)$.

There were $(i-1)$ species at time t, and one species immigrated from other habitat.

There were $(i+1)$ species at time t, and one species emigrated from other habitat.

Assume that Δt is small enough that the probability of more than one immigration or emigration is ignored. Taking the limit of Eq. (3) as $\Delta t \to 0$, gives

$$P_i = \begin{cases} -\lambda_0 P_0 + \mu_1 P_1 & i = 0 \\ -(\lambda_i + \mu_i)P_i + \lambda_{i-1}P_{i-1} + \mu_{i+1}P_{i+1} & 1 \le i \le n-1 \\ -\mu_n P_n + \lambda_{n-1}P_{n-1} & i = n \end{cases} \tag{4}$$

It is noted that Eq. (4) is valid for $i=0, .., n$ when $\mu_0=0$ and $\lambda_n=0$.

Define $P = [P_0 \cdots P_n]$. We can obtain

$$\dot{P} = AP \tag{5}$$

where the matrix A is given as

$$A = \begin{bmatrix} -\lambda_0 & \mu_1 & 0 & \cdots & 0 \\ \lambda_0 & -(\lambda_1+\mu_1) & \mu_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \lambda_{n-2} & -(\lambda_{n-1}+\mu_{n-1}) & \mu_n \\ 0 & \cdots & 0 & \lambda_{n-1} & -\mu_n \end{bmatrix} \tag{6}$$

The steady state value for the probability of the number of each species was given by (Ma, Ni, & Sun, 2009).

$$P_i = \begin{cases} P_0 = \dfrac{1}{1+\sum_{i=1}^{n}\dfrac{\lambda_0\lambda_1\cdots\lambda_{i-1}}{\mu_1\mu_2\cdots\mu_i}} & i = 0 \\ P_i = \dfrac{\lambda_0\lambda_1\cdots\lambda_{i-1}}{\mu_1\mu_2\cdots\mu_i\left(1+\sum_{i=1}^{n}\dfrac{\lambda_0\lambda_1\cdots\lambda_{i-1}}{\mu_1\mu_2\cdots\mu_i}\right)} & 1 \le i \le n \end{cases} \tag{7}$$

From the preceding equations, the condition which is necessary for the limiting probabilities to exist also be shown. Namely, it is essential that $\mu_i \neq 0$ for all i habitat.

As cataclysmic events drastically change the *HSI* of habitat, a mutation operator is employed to modify the features of solutions in BBO. Different from other EAs, the mutation rate m_i of each solution dynamically calculated by migration model in BBO.The mutation rate m_i is expressed as

$$m_i = m_{\max} \cdot \left(\frac{1 - P_i}{P_{max}} \right) \tag{8}$$

where m_{max} is the maximum probability of mutation, which is an user-defined parameter, P_i is the probability that a single habitat just contains i species. $P_{\max} = \max\{P_i, i = 1, 2, \ldots n\}$.The original BBO algorithm is simply described in Algorithm 1.

3. The proposed two-stage differential biogeography-based optimization (TDBBO)

Although the previous variants of BBO are well designed, several problems exist in BBO, such as rotational variance, weak diversity of the population. In this paper, TDBBO is proposed to alleviate the rotational variance and maintain the diversity of the population. In TDBBO, the two-stage mechanism, improved migration operator, Gaussian operator and greedy select strategy are employed to balance the exploration ability and exploitation ability, as well as enhancing the rotational invariant. The primary procedure of TDBBO is explained as follows.

3.1. Improved migration operator

One limitation of BBO is the weak diversity of the population. In standard migration operator, the features of inferior solutions are replaced by the features of good solutions directly. The standard migration operator improves the quality of the inferior solutions and keeps excellent exploitation ability. However, identical features exist in several candidate solutions lead to insufficient diversity of population. Another major drawback of BBO is that it treats each feature independently, That is to say, it is not rotationally invariant. It means that BBO performs poorly on non-separable functions.

It is reasonable to introduce the mutation strategy of DE into BBO because of the superior exploration ability of DE. There are several mutation strategies in DE, and each strategy has its advantages and disadvantages. Among them, the $DE/current-to-rand/1$ frequently appears in literature.

$$DE/current{-}to{-}rand/1: \; U_i = X_i + F \cdot (X_{r1} - X_i) + F \cdot (X_{r2} - X_{r3}) \tag{9}$$

where $i \in \{1, 2, \ldots, n\}$, F is a scaling factor, U_i is the ith trial vector, X_i is the ith candidate solution,X_{r1}, X_{r2} and X_{r3} are three mutually different random solutions selected from the population.

As Das, Abraham, Chakraborty, and Konar (2009) pointed out, the $DE/current-to-rand/1$ is a rotational invariant arithmetic crossover operator. The $BBO/current-to-select/1$, which is designed to keep the exploitation ability of BBO and enhance the rotational invariant, is given as follow.

$$\begin{aligned} BBO/current-to-select/1: \; U_i &= H_i + F_i \cdot (H_k - H_i) \\ &+ F_i \cdot (H_{r1} - H_{r2}) \end{aligned} \tag{10}$$

where H_i is the immigrate habitat, H_k is the emigrate habitat, H_{r1} and H_{r2} are two random selected habitats. The parameter F_i plays an important role in the balance of exploration ability and exploitation ability. All individuals in the population are ordered in an ascending way with respect to the cost fitness value. As Tang, Dong, and Liu (2015) pointed out, compared with the superior individuals, the inferior individuals may be closer to the global minimum. Besides, the small standard deviation leads to the control parameters near their mean values to maintain search efficiency. In this paper, the scaling factor F_i and immigration rate $lamdaScale_i$ are independently generated Gaussian distributed random number in each generation. The variance is set to 0.1 so that F_i and $lamdaScale_i$ near their mean value λ_i to search. F_i and $lamdaScale_i$ are calculated by using Eqs. (11) and (12) until they are in interval (0, 1).

$$F_i = randn(\lambda_i, 0.1) \tag{11}$$

$$lamdaScale_i = randn(\lambda_i, 0.1) \tag{12}$$

where λ_i is the immigration rate of the ith individual,

The pseudo-code for the improved migration operator is given in Algorithm 2.

3.2. Two-stage mechanism

The sinusoidal migration model significantly outperforms the other models regarding the solution accuracy for most functions and displays faster converge speed than others as Ma (2010) declared. However, the single sinusoidal migration model leads to the diversity of population decreased rapidly. In this paper, a two-stage migration model is proposed. Suppose that the individuals spread in the whole search space at the early evolutionary process; Then some potentially promising areas could be exploited in more detail. As the Eq. (14), the emigration rate is a constant and all habitats have the same emigration rate in the first stage of the evolutionary process. In the second stage of the evolutionary process, the emigration rate is calculated by the sinusoid migration model. The better habitat has higher emigration rate than the worse habitat. Therefore, the diversity of the population is high in the early stage, and it gradually decreases with the evolutionary process. The values of λ_i and μ_i are computed using Eqs. (13) and (14).

$$\lambda_i = \frac{1}{2}\left(\cos\left(\frac{i \cdot \pi}{n} \right) + 1 \right) \tag{13}$$

$$\mu_i = \begin{cases} \frac{E}{2} & nfes < T \\ E \cdot \frac{1}{2}\left(-\cos\left(\frac{i \cdot \pi}{n} \right) + 1 \right) & nfes \geq T \end{cases} \tag{14}$$

where $T = c \cdot \text{Max\_NFFEs}$, Max_NFFE is the maximum number of fitness evaluations and the value of c is determined in the parameter calibration experiment. The value of $nfes$ is the current number of fitness evaluations. E is the maximum emigration rate.

3.3. Gaussian mutation operation

The Gaussian, Cauchy, and levy mutation operator were introduced into BBO by Gong et al. (2010) for real space. An improved Gaussian mutation operator is integrated into TDBBO to enhance the exploration ability of TDBBO. The probability density function of the Gaussian distribution is given as follow.

$$f_{\mu,\sigma^2}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\dfrac{(x-\mu)^2}{2\sigma^2}} \tag{15}$$

where μ is the mean and σ^2 is the variance. The Gaussian mutation with $\mu = 0$ and $\sigma = 1$ is described as:

$$H_i(j) = H_i(j) + randn(0, 1) \tag{16}$$

where $H_i(j)$ is the *jth* SIV of the ith habitat and the $randn(0, 1)$ indicates that the Gaussian random number with $\mu = 0$ and $\sigma = 1$.

The rough pseudo-code for the Gaussian mutation is shown in Algorithm 3.

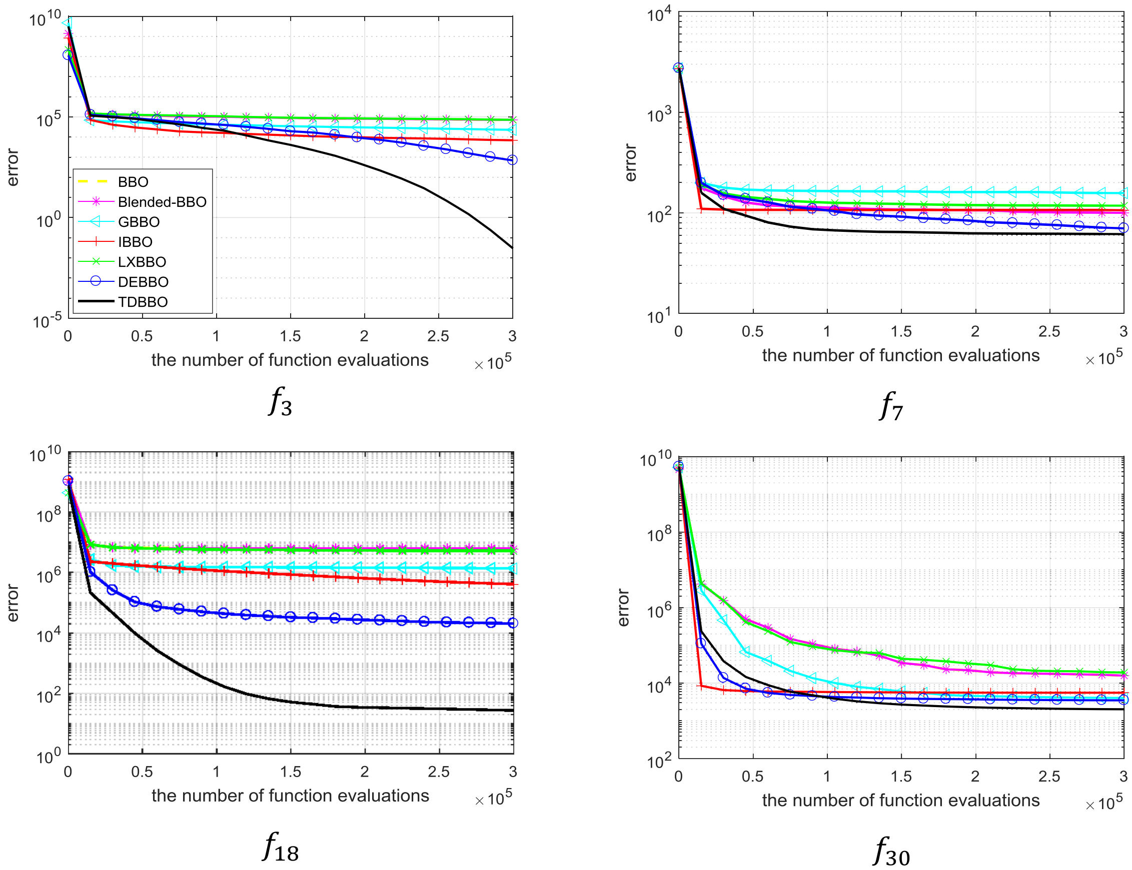


Fig. 2. Convergence curves of some typical benchmark functions (30D).

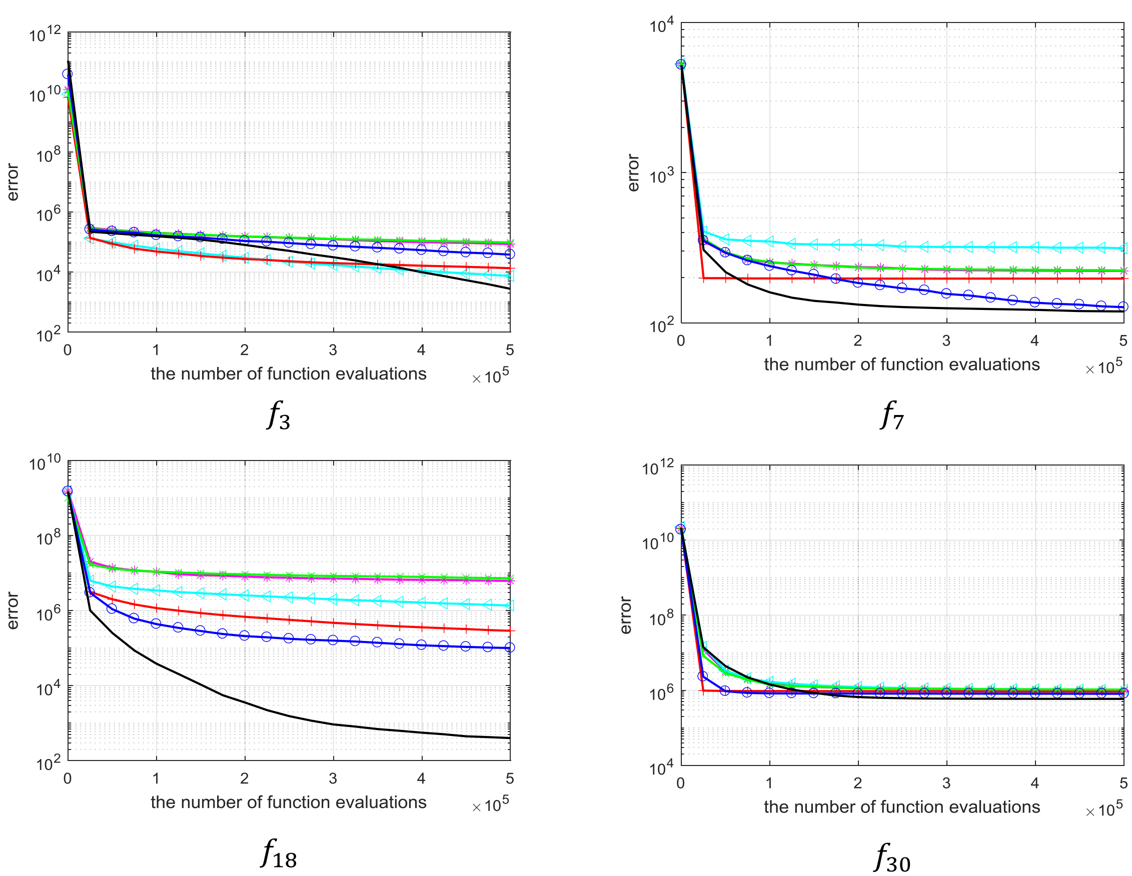


Fig. 3. Convergence curves of some typical benchmark functions (50D).

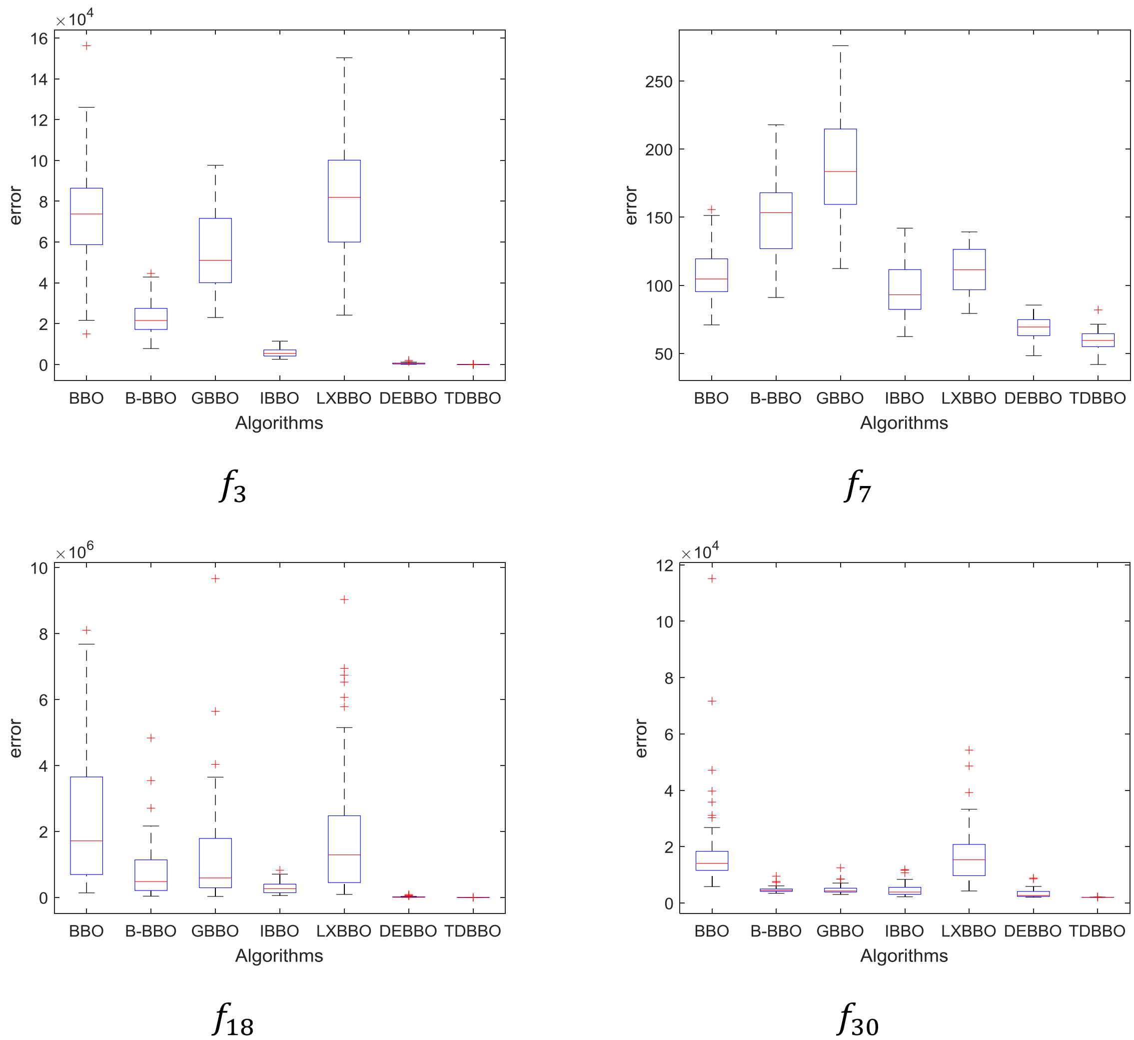


Fig. 4. Boxplots of some typical benchmark functions (30D).

3.4. Proposed TDBBO algorithm

The main steps of TDBBO are summarized as follows. After the initialization of population, the improved migration which contains the two-stage scheme and *BBO/current-to-select/1*, the Gaussian mutation, and greedy selection operator are employed to improve the quality of solutions. The detail description of TDBBO is shown in Algorithm 4. Among the Algorithm 4, H is the population, U is the trial population, NP is the size of the population, F_i is the scaling factor of the ith habitat.

3.5. Comparative study on TDBBO and other variants of BBO

Various modifications of BBO were introduced after Simon (2008) proposed the standard BBO. Blended-BBO and DE/BBO are the most common variants of BBO. In this subsection, the TDBBO is compared with Blended-BBO and DE/BBO to investigate the differences between TDBBO and other variants of BBO from the perspective of design and mechanism.

Migration operator is the most crucial component in BBO. In Blended-BBO, a new feature of the solution is comprised of a feature from another solution and a feature from the current solution. The migration operator of Blended-BBO only involves another one solution, which means that it is easy to fall into local optimum and the rotational variance limits the performance of Blended-BBO. However, the migration operator of TDBBO learns from more than one solution to keep the diversity of population. Besides, the $\mathrm{BBO/current-to-select/1}$ in the migration operator of TDBBO is also a rotationally invariant arithmetic crossover operator which improve the performance of TDBBO on non-separable problems effectively.

DE/BBO is a hybrid algorithm which combines the exploration ability of DE and the exploitation ability of BBO. In DE/BBO, the mutation operator of DE which is based on DE/rand/1 is embedded in the migration operator of BBO. Although TDBBO and DE/BBO are similar in the view of the mechanism, there are some differences between them. Firstly, as the two-stage mechanism, the $\mathrm{BBO/current-to-selected/1}$ which is employed by TDBBO is different in the two stages. In the early evolutionary process, H_k is a random selected habitat. Therefore, both DE/BBO and TDBBO have strength exploration ability. However, the convergence speed of TDBBO is faster than the convergence speed of DE/BBO as H_k is selected according to the emigration rate in the late evolutionary process. Secondly, because the mutation operator is removed from DE/BBO, the probability that the population jump out the local optimum is zero when the population premature convergence.

In summary, the TDBBO is different from other variants of BBO. In Section 5, the CEC 2017 benchmark will be employed to compare the performance of TDBBO and other modifications.

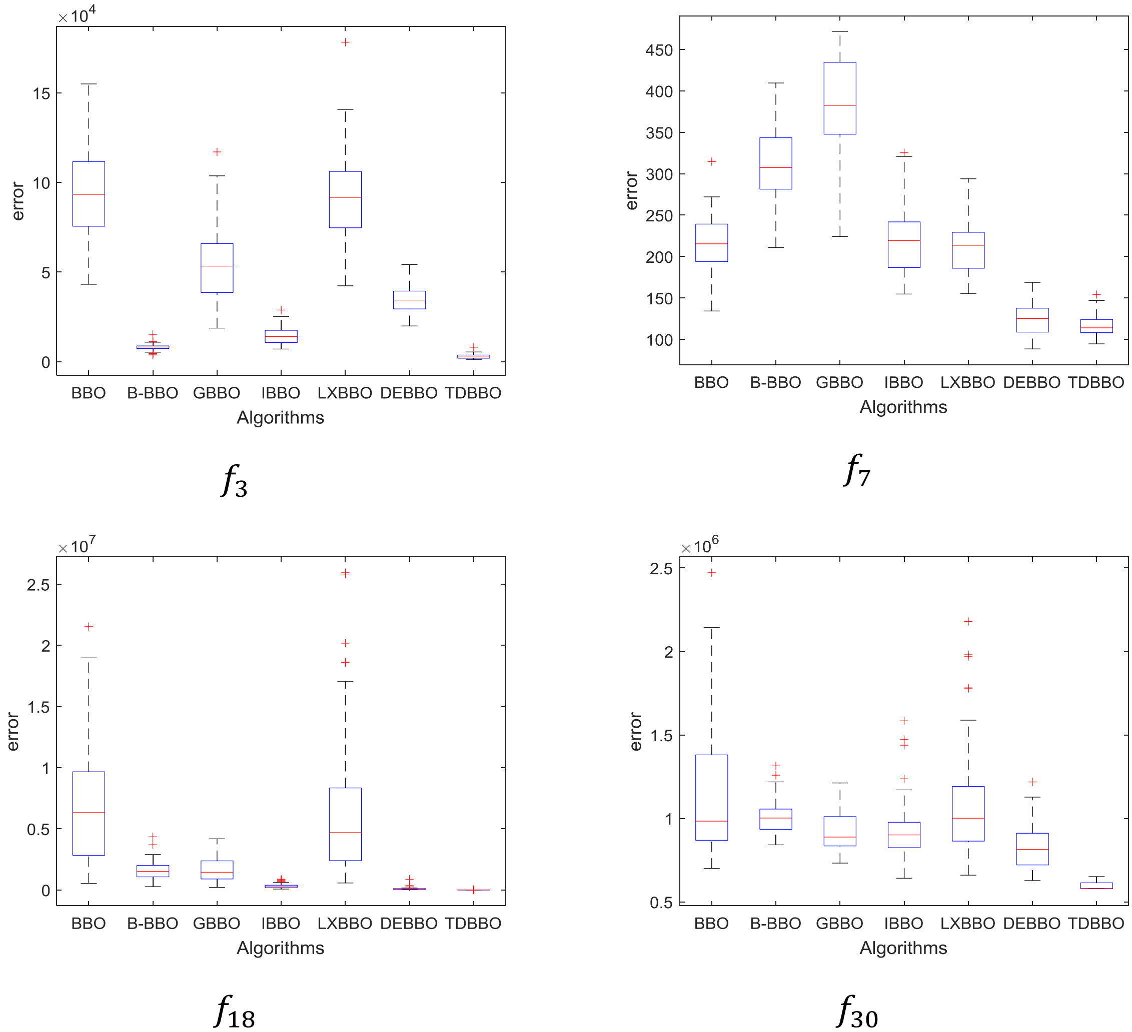


Fig. 5. Boxplots of some typical benchmark functions (50D).

4. Convergence analysis of TDBBO

A Markov model for the standard BBO was presented by Simon et al. (2011) to analyze the probability p_{ij} that the state s_i transfer to s_j. Besides, the Markov chain analysis of differential evolution is also given by Hu, Xiong, Su, and Fang (2014) to investigate the performance of DE. In this paper, the convergence performance of TDBBO is analyzedwith Markov model under the assumptions that all the new solutions are produced before old solutions are replaced, and the solutions are allowed to migrate to themselves.

Definition 1. (Convergence in Probability) (Burton, 1985) Let$\{x(t),\ t=0,\ 1,\ 2\ldots\}$be a population sequence generated by a population-based stochastic algorithm, the stochastic sequence $\{x(t)\}$ weakly converges in Probability to the global optimum. If and only if:

$$\lim_{t\to\infty} P\left\{x(t)\cap^{B^*}\neq\emptyset\right\}=1 \tag{17}$$

B^* is a set of the global optimum of an optimization problem.

Definition 2. For random sequence$\{\boldsymbol{X_n},\boldsymbol{X_n}\in\boldsymbol{S},\ \boldsymbol{n}\geq 0\}$, $\boldsymbol{S}$ is the state space. For$\forall\boldsymbol{n}\geq 0$and any state$\boldsymbol{i},\ \boldsymbol{j},\ \boldsymbol{i}_0,\ldots\ \boldsymbol{i}_{\boldsymbol{n}-1}$, if

$$P(X_n{+}1=j\ |X_n{=}i,X_{n-1}{=}i_{n-1},\ldots X_0=i_0)=P(X_n{+}1{=}j\ |\ X_n=i) \tag{18}$$

$\{X_n,X_n\in S,\ n\geq 0\}$ is a Markov chain. The Eq. (18) may be interpreted as stating that for a Markov chain, the conditional distribution of any future state X_{n+1} is independent of the past states and depends only on the present state X_n.

Definition 3. The continuous search space Ψ is mapped to a finite discrete setΦ, if the population scale is m, the state space is

$$\Phi^m=\underbrace{\Phi\times\Phi\times\ \cdots\ \times\Phi}_{m} \tag{19}$$

Definition 4. M^0,U^0,S^0denote the migration operator, mutation operator and selection operator of the TDBBO respectively. X, Y, Z denote the states of Φ^m. Then, $P\{M^0(X)=Y\}$is the transition probability from X to $Y$by using migration operator.

Definition 5. $f(X^*)$denotes the minimum function value of a population$X$.

Theorem 1. Suppose that $\{\boldsymbol{x}(\boldsymbol{t}),\ \boldsymbol{t}=0,\ 1,\ 2,\ \ldots\}$ is a population sequence generated by TDBBO. $\{\boldsymbol{x}(\boldsymbol{t}),\ \boldsymbol{t}=0,\ 1,\ 2,\ \ldots.\}$ is a finite homogeneous Markov chain on the state space$\boldsymbol{\Phi^m}$.

Proof. According to Definition 3, the state space of $\{x(t),\ t=0,\ 1,\ 2,\ \ldots.\}$ is finite. The next state of TDBBO is independent of iteration times t and depends only on the present state. As Definition 2, the state sequence$\{x(t),\ t=0,\ 1,\ 2,\ \ldots.\}$ is a Markov chain. Then, the $\{x(t),\ t=0,\ 1,\ 2,\ \ldots.\}$ is a finite homogeneous Markov chain on the state space Φ^m.

Property 1. For the TDBBO without selection operator, all states of its population space Φ^mcommunicate. And the one-step transition probabilities from X to $Y$are greater than 0 for all states.

$$P\left\{U^0 \cdot M^0(X) = Y\right\} > 0 \tag{20}$$

Proof. Suppose Φ^m denote the population space of the TDBBO without the selection operator, there are n possible solutions which are represented by y_i, where $i = 1, 2, \ldots, n$, For TDBBO, the population contains N candidate solutions which are represented by x_j. $i = 1, 2, \ldots, N$, we use the notation $\mathfrak{J}_i(s)$ to denote the set of the population indices j as:

$$\begin{aligned}\mathfrak{J}_i(s) &= \left\{j : RA\left(x_i(s) + F \cdot \left(x_j(s) - x_i(s)\right) + F \cdot (x_{r1}(s) - x_{r2}(s))\right)\right.\\ &\left.= x_i(s)\right\}\end{aligned} \tag{21}$$

F is a Gaussian random value, $r1$, $r2$ are random values, and the "RA" rounds the absolute value of X to the nearest integer.

By defining the $y_k(s)$ as the sth feature of y_k and $x_k(s)$ as the sth feature ofx_k, if the migration does not occur. The probability of $y_k(s) = x_k(s)$ is described as:

$$P(y_k(s) = x_k(s)|no\ immigration) = 1 \tag{22}$$

If the $x_k(s)$ is selected for immigration, then the probability that $y_k(s)$ is equal to $x_i(s)$ can be written as:

$$P(y_k(s) = x_i(s)|immigration) = \frac{\sum_{j\in\mathfrak{J}_i(s)} \frac{v_j\mu_j}{N(N-1)}}{\sum_{j=1}^{n} v_j\mu_j} \tag{23}$$

The total probability of $y_k(s) = x_i(s)$ is shown in Eq. (24).

$$\begin{aligned}P(y_k(s) = x_i(s)) &= P(noimmigration) \ \times P(y_k(s)\\ &= x_i(s)|noimmigration)\\ &+ P(immigration) \times P(y_k(s) = x_i(s)|immigration)\\ &= (1-\lambda_k)1_0(y_k(s) - x_i(s)) + \lambda_k \frac{\sum_{j\in\mathfrak{J}_i(s)} \frac{v_j\mu_j}{N(N-1)}}{\sum_{j=1}^{n} v_j\mu_j}\end{aligned} \tag{24}$$

Use $P_{ki}(X)$denotes the probability that immigration result in $y_k = x_i$

$$P_{ki}(X) = \prod_{s=1}^{m}\left[(1-\lambda_k)1_0(y_k(s) - x_i(s)) + \lambda_k \frac{\sum_{j\in\mathfrak{J}_i(s)} \frac{v_j\mu_j}{N(N-1)}}{\sum_{j=1}^{n} v_j\mu_j}\right] \tag{25}$$

Use$U$ to denote the $n \times n$ mutation matrix, where $U_{ij}$is the probability that x_j mutates to x_i, The probability that the kth immigration trial followed by mutation results in $x_i$is denoted as $P_{ki}^{(2)}(X)$,it is written as:

$$P_{ki}^{(2)}(X) = \sum_{j=1}^{n} U_{ij}P_{kj}(X) \tag{26}$$

Then,

$$P_{ki}^{(2)}(Y|X) = \sum_{J\in T}\prod_{k=1}^{N}\prod_{i=1}^{n}\left[P_{ki}^{(2)}(X)\right]^{J_{ki}} > 0 \tag{27}$$

where,

$$T = \left\{J \in R : J_{ki} \in \{0, 1\}, \sum_{i=1}^{n} J_{ki} = 1 forallk, \sum_{k=1}^{N} J_{ki} = Y_i foralli\right\} \tag{28}$$

Hence,

$$P\left\{M^0 \cdot H^0(X) = Y\right\} > 0 \tag{29}$$

That is to say, $\forall X,\ Y \in \Phi^m$, the one-step transition probability from X to $Y$is greater than 0 by using the improved migration operator and mutation operator. So, for the TDBBO without selection operator, all states communicate.

Property 2. Given states$X,\ Y,\ Z \in \Phi^m$,$Z \subset X \cup^Y$, the selection operator belongs to one of the following classes:if $f(Z^*) \neq min(f(X^*), f(Y^*))$,$X$and Ycannot generate Z by select operator, that is,

$$P\left\{S^0(X, Y) = Z\right\} = 0 \tag{30}$$

if$f(Z^*) = min(f(X^*), f(Y^*))$, $X$and Ycan generate Z by select operator, that is,

$$P\left\{S^0(X, Y) = Z\right\} > 0 \tag{31}$$

Theorem 2. Suppose that $\{x(t),\ t = 0,\ 1,\ 2,\ \ldots\}$ is the population sequence generated by TDBBO. Then,$\{x(t),\ t = 0,\ 1,\ 2,\ \ldots\}$converges to global optimum in probability.

Proof. $\forall X,\ Y,\ Z \in \Phi^m$, then the transition probability

$$\begin{aligned}P\{x(t+1) = Z|x(t) = X\} &= P\left\{S^0 \cdot U^0 \cdot M^0(X) = Z\right\}\\ &= \sum_{Y\in\Phi^m} P\left\{U^0 \cdot M^0(X) = Y\right\} \cdot P\left\{S^0(X, Y) = Z\right\}\}\end{aligned} \tag{32}$$

Define B_0 be a population set consist of some populations in which at least one individual is optimum. $B_0 \subset \Phi^m$

$$B_0 = \{X = (x_1, x_2, \ldots, x_m,) \in \Phi^m | x_1 \in B^*, \exists i \in \{1, 2, ..., m\}\} \tag{33}$$

Then, we discuss the transition probability with the following two classes.

Suppose $X \in B_0$,$Z \notin B_0$,$f(Z^*) > min(f(X^*), f(Y^*))$.
According to Property 2, $P\{S^0(X,Y) = Z\} = 0$.
Then, $P\{x(t+1) = Z|x(t) = X\} = 0$.
Suppose $X \in B_0$,$Z \in B_0$,$f(Z^*) = \min(f(X^*), f(Y^*))$.
According to Property 2, $P\{S^0(X,Y) = Z\} > 0$.
From Property 1, $P\{M^0 \bullet H^0(X) = Y\} > 0$.
So, $P\{x(t+1) = Z|x(t) = X\} > 0$.
That is, all states of B_0 communicate.

B_0 is a positive recurrent, irreducible, aperiodic and closed set. According to the properties of the aperiodic, homogeneous Markov chain, the sequence $\{x(t),\ t = 1,\ 2,\ \ldots,\}$ exists a limiting distribution $\pi(Y)$,

$$\lim_{t\to\infty} P\{x(t) = Y\} = \begin{cases}\pi(Y) & Y \in B_0\\ 0 & otherwise\end{cases}$$

$$\text{Then, } \lim_{t\to\infty} P\{x(t) = B_0\} = 1.$$

$$So, \lim_{t\to\infty} P\left\{x(t) \cap^{B^*} \neq \emptyset\right\} = 1 \tag{34}$$

According to Definition 1, TDBBO converges to global optimum in probability.

5. The experiments and comparisons

In this paper, the special session on single objective bound constrained real-parameter optimization of CEC2017 (Liang, Qu, & Suganthan, 2013) are employed to test the performance of TDBBO. The functions are divided into four categories: $f_1 - f_3$ are unimodal functions, $f_4 - f_{10}$ are simple multimodal functions, $f_{11} - f_{20}$ are hybrid functions, and $f_{21} - f_{30}$ are composition functions. The algorithms were executed 51 times on each of the benchmark functions for D=10, 30, 50, 100. Error values and standard deviations smaller than 1e-8 are taken as zero. The maximum fitness of function evaluations is set to D × 10, 000 based on the guidelines provided in the special session of CEC 2017 (Awad, Ali, Liang, Qu, & Suganthan, 2016) to make a fair comparison. The contrast algorithms were carefully re-implemented in C++ programming language and run on a PC with a 3.4 GHZ Intel(R) Core(TM) i7-6700 CPU, 8GB of RAM and 64-bit OS.

Table 5
The results of seven algorithms (10-dimensional benchmark functions).

Function	Criterion	BBO	B-BBO	GBBO	IBBO	LXBBO	DEBBO	TDBBO
1	Mean	3.61E+04	6.33E+02	5.94E+02	1.30E+03	4.42E+04	**0.00E+00**	**0.00E+00**
	Std. Dev.	2.40E+04	8.49E+02	8.02E+02	1.90E+03	3.91E+04	**0.00E+00**	**0.00E+00**
2	Mean	1.15E+02	2.31E+04	8.96E+05	**0.00E+00**	3.36E+02	**0.00E+00**	**0.00E+00**
	Std. Dev.	4.86E+02	9.24E+04	2.76E+06	**0.00E+00**	9.47E+02	**0.00E+00**	**0.00E+00**
3	Mean	4.50E+03	2.11E+03	4.24E+03	**0.00E+00**	4.25E+03	**0.00E+00**	**0.00E+00**
	Std. Dev.	3.59E+03	1.21E+03	2.76E+03	2.82E−14	3.18E+03	**0.00E+00**	**0.00E+00**
4	Mean	8.94E+00	5.58E+00	5.81E+00	1.83E+00	7.87E+00	2.87E+00	**0.00E+00**
	Std. Dev.	1.48E+01	8.86E+00	7.35E+00	3.95E−01	1.37E+01	7.06E−01	**0.00E+00**
5	Mean	8.08E+00	1.58E+01	1.63E+01	8.84E+00	8.51E+00	**2.69E+00**	3.58E+00
	Std. Dev.	3.33E+00	5.04E+00	6.10E+00	3.83E+00	3.75E+00	1.23E+00	**1.06E+00**
6	Mean	1.25E−01	9.41E−04	1.15E−02	3.65E−08	1.27E−01	**0.00E+00**	**0.00E+00**
	Std. Dev.	6.31E−02	2.62E−03	3.04E−02	1.10E−08	6.75E−02	**0.00E+00**	**0.00E+00**
7	Mean	2.26E+01	2.63E+01	3.13E+01	1.74E+01	2.24E+01	1.56E+01	**1.45E+01**
	Std. Dev.	5.36E+00	5.57E+00	5.62E+00	3.23E+00	5.53E+00	1.48E+00	**1.29E+00**
8	Mean	1.04E+01	1.19E+01	1.32E+01	1.11E+01	1.10E+01	**3.15E+00**	3.89E+00
	Std. Dev.	3.77E+00	4.04E+00	6.01E+00	4.85E+00	4.12E+00	1.44E+00	**1.27E+00**
9	Mean	1.27E+00	1.06E+01	1.88E+01	**0.00E+00**	1.98E+00	**0.00E+00**	**0.00E+00**
	Std. Dev.	2.97E+00	1.29E+01	2.09E+01	**0.00E+00**	5.90E+00	**0.00E+00**	**0.00E+00**
10	Mean	3.20E+02	5.92E+02	6.58E+02	4.18E+02	4.01E+02	**9.97E+01**	1.67E+02
	Std. Dev.	1.97E+02	1.69E+02	2.22E+02	2.22E+02	1.93E+02	**8.49E+01**	9.00E+01
11	Mean	9.60E+00	1.40E+02	4.62E+02	3.25E+00	1.16E+01	1.16E+00	**0.00E+00**
	Std. Dev.	6.30E+00	2.41E+02	7.00E+02	2.02E+00	2.60E+01	8.65E−01	**0.00E+00**
12	Mean	1.43E+06	1.10E+06	1.21E+06	3.38E+04	1.17E+06	**1.35E+01**	1.69E+01
	Std. Dev.	1.92E+06	9.79E+05	1.09E+06	6.68E+04	1.52E+06	2.88E+01	**1.55E+01**
13	Mean	1.02E+04	9.96E+03	9.24E+03	7.25E+03	1.36E+04	4.35E+00	**1.00E+00**
	Std. Dev.	8.83E+03	4.11E+03	6.15E+03	6.59E+03	1.06E+04	2.26E+00	**1.21E+00**
14	Mean	7.26E+03	4.96E+03	6.77E+03	4.65E+03	1.06E+04	6.05E−01	**0.00E+00**
	Std. Dev.	7.41E+03	3.00E+03	5.49E+03	5.35E+03	8.60E+03	6.85E−01	**0.00E+00**
15	Mean	6.97E+03	2.33E+03	3.52E+03	5.02E+03	7.11E+03	2.63E−01	**2.01E−02**
	Std. Dev.	7.70E+03	2.37E+03	4.81E+03	6.12E+03	8.01E+03	4.15E−01	**7.13E−02**
16	Mean	1.80E+02	2.38E+02	2.29E+02	9.06E+01	1.78E+02	**5.12E−01**	5.85E−01
	Std. Dev.	1.33E+02	1.25E+02	1.43E+02	1.08E+02	1.34E+02	2.92E−01	**1.80E−01**
17	Mean	4.49E+01	2.18E+01	4.67E+01	1.86E+01	3.07E+01	**3.32E−01**	1.56E+00
	Std. Dev.	5.38E+01	2.16E+01	4.11E+01	2.52E+01	3.40E+01	**3.78E−01**	7.27E−01
18	Mean	9.15E+03	6.32E+03	6.42E+03	1.09E+04	1.43E+04	2.07E−01	**8.34E−02**
	Std. Dev.	7.69E+03	5.08E+03	6.99E+03	9.71E+03	1.03E+04	2.61E−01	**1.37E−01**
19	Mean	8.85E+03	4.61E+03	3.94E+03	6.61E+03	8.21E+03	**1.01E−02**	2.17E−02
	Std. Dev.	8.18E+03	3.64E+03	3.59E+03	6.60E+03	9.04E+03	**9.62E−03**	1.48E−02
20	Mean	6.36E+00	1.10E+01	1.65E+01	9.22E+00	6.13E+00	8.68E−02	**3.67E−02**
	Std. Dev.	4.86E+00	9.07E+00	1.19E+01	1.14E+01	4.23E+00	1.92E−01	**1.01E−01**
21	Mean	2.02E+02	1.79E+02	2.00E+02	1.92E+02	2.00E+02	1.81E+02	**1.58E+02**
	Std. Dev.	**3.61E+01**	5.51E+01	4.71E+01	4.73E+01	3.91E+01	4.47E+01	5.22E+01
22	Mean	1.03E+02	1.06E+02	1.08E+02	9.95E+01	1.01E+02	9.82E+01	**9.61E+01**
	Std. Dev.	**1.07E+00**	1.44E+01	1.49E+01	8.58E+00	1.35E+01	1.39E+01	1.94E+01
23	Mean	3.16E+02	3.38E+02	3.38E+02	3.11E+02	3.15E+02	3.05E+02	**3.04E+02**
	Std. Dev.	5.43E+00	1.41E+01	1.30E+01	4.77E+00	5.01E+00	**1.65E+00**	2.00E+00
24	Mean	3.36E+02	3.12E+02	3.58E+02	3.26E+02	3.35E+02	3.02E+02	**2.56E+02**
	Std. Dev.	4.80E+01	1.25E+02	7.24E+01	5.67E+01	**4.79E+01**	8.06E+01	1.07E+02
25	Mean	4.31E+02	4.37E+02	4.41E+02	4.33E+02	4.28E+02	4.25E+02	**3.99E+02**
	Std. Dev.	2.33E+01	1.99E+01	1.76E+01	2.13E+01	2.32E+01	2.27E+01	**6.31E+00**
26	Mean	4.24E+02	4.73E+02	5.79E+02	3.18E+02	3.57E+02	3.00E+02	**2.94E+02**
	Std. Dev.	2.68E+02	2.42E+02	3.31E+02	1.28E+02	1.57E+02	**0.00E+00**	4.16E+01
27	Mean	3.99E+02	4.32E+02	4.21E+02	3.95E+02	3.99E+02	3.91E+02	**3.90E+02**
	Std. Dev.	6.21E+00	3.00E+01	1.64E+01	2.25E+00	5.84E+00	2.46E+00	**1.63E+00**
28	Mean	5.30E+02	4.58E+02	4.73E+02	4.87E+02	5.23E+02	4.16E+02	**3.11E+02**
	Std. Dev.	1.25E+02	1.60E+02	1.45E+02	1.47E+02	1.23E+02	1.45E+02	**5.51E+01**
29	Mean	2.80E+02	3.22E+02	3.23E+02	2.69E+02	2.78E+02	2.44E+02	**2.40E+02**
	Std. Dev.	3.35E+01	3.87E+01	3.77E+01	2.36E+01	3.85E+01	6.22E+00	**3.78E+00**
30	Mean	5.81E+05	4.85E+05	5.33E+05	1.69E+05	5.74E+05	9.45E+02	**3.98E+02**
	Std. Dev.	6.80E+05	6.19E+05	6.24E+05	3.88E+05	6.39E+05	6.70E+02	**2.02E+00**

5.1. Parameter setting

5.1.1. Parameter setting of competitive algorithm

In this paper, The TDBBO is compared with six other algorithms to evaluate the performance of TDBBO. The first algorithm is the standard BBO. Other compared algorithms are state-of-the-art variants of BBO. The competitive algorithms are as follows: BBO (Simon, 2008), Blended-BBO (Ma & Simon, 2011), GBBO (Sharma, Sharma, & Sharma, 2017), IBBO (Feng et al., 2014), LXBBO (Garg & Deep, 2016), DE/BBO (Gong et al., 2011). The parameters for the compared algorithms are shown in Table 1.

5.1.2. Parameter analysis of TDBBO

The parameter setting plays an important role in the performance of algorithms. Namely, it is crucial to conduct parameter calibration experiment. There are seven parameters in TDBBO: F (scale factor) is an adaptive parameter; n (population size); I (maximum immigration rate); E (maximum emigration rate); maximum species count; $m_{\max}$ (maximum mutation rate); c (control factor). Generally, I and E are set to1, the maximum species count is equal to n. Therefore, three parameters (n, $m_{\max}$, c) need to be analyzed by implementing the Taguchi method (Montgomery, 1976). In the experiment, the CEC 2017 benchmark functions with D = 30 are

Table 6
The results of seven algorithms (30-dimensional benchmark functions).

Function	Criterion	BBO	B-BBO	GBBO	IBBO	LXBBO	DEBBO	TDBBO
1	Mean	1.37E+05	2.63E+03	3.80E+03	5.11E+03	1.36E+05	2.72E+03	**0.00E+00**
	Std. Dev.	5.37E+04	1.92E+03	3.70E+03	5.41E+03	4.12E+04	3.24E+03	**0.00E+00**
2	Mean	2.10E+14	9.14E+07	2.47E+11	**0.00E+00**	5.72E+13	1.01E+01	**0.00E+00**
	Std. Dev.	1.46E+15	2.80E+08	7.18E+11	**0.00E+00**	3.39E+14	1.80E+01	**0.00E+00**
3	Mean	7.43E+04	2.30E+04	5.53E+04	5.87E+03	8.28E+04	5.14E+02	**5.23E−02**
	Std. Dev.	2.63E+04	7.97E+03	1.97E+04	2.17E+03	3.11E+04	3.66E+02	**8.65E−02**
4	Mean	9.85E+01	9.85E+01	7.28E+01	4.72E+01	1.01E+02	7.55E+01	**2.30E+01**
	Std. Dev.	3.01E+01	1.90E+01	3.70E+01	2.95E+01	2.69E+01	**1.52E+01**	2.89E+01
5	Mean	5.71E+01	1.05E+02	1.15E+02	6.05E+01	6.06E+01	**2.63E+01**	3.12E+01
	Std. Dev.	1.36E+01	1.93E+01	2.01E+01	1.61E+01	1.42E+01	6.89E+00	**6.38E+00**
6	Mean	1.37E−01	4.14E−02	1.80E−05	4.82E−07	1.29E−01	2.41E−08	**1.14E−08**
	Std. Dev.	3.43E−02	4.53E−02	1.25E−04	2.94E−07	3.16E−02	1.52E−07	**3.69E−08**
7	Mean	1.09E+02	1.52E+02	1.87E+02	9.64E+01	1.11E+02	6.84E+01	**5.98E+01**
	Std. Dev.	1.98E+01	3.13E+01	3.93E+01	1.94E+01	1.70E+01	8.40E+00	**6.63E+00**
8	Mean	6.05E+01	8.15E+01	1.00E+02	6.17E+01	6.03E+01	**2.79E+01**	3.23E+01
	Std. Dev.	1.18E+01	1.45E+01	1.84E+01	1.56E+01	1.31E+01	8.02E+00	**6.35E+00**
9	Mean	3.36E+02	9.35E+02	1.28E+03	4.19E+00	3.06E+02	**0.00E+00**	1.76E−03
	Std. Dev.	2.87E+02	5.56E+02	5.08E+02	6.96E+00	2.70E+02	**0.00E+00**	1.24E−02
10	Mean	**2.22E+03**	2.71E+03	3.13E+03	2.76E+03	2.39E+03	2.73E+03	2.32E+03
	Std. Dev.	5.19E+02	4.97E+02	5.29E+02	4.73E+02	6.23E+02	3.76E+02	**2.93E+02**
11	Mean	2.16E+03	8.51E+02	1.23E+03	3.37E+01	1.79E+03	1.44E+01	**1.14E+01**
	Std. Dev.	2.48E+03	8.00E+02	1.11E+03	2.80E+01	1.68E+03	1.38E+01	**1.17E+01**
12	Mean	3.09E+06	7.30E+05	7.69E+05	6.92E+04	2.73E+06	1.19E+04	**1.45E+03**
	Std. Dev.	2.13E+06	4.57E+05	4.14E+05	5.64E+04	2.18E+06	5.93E+03	**7.69E+02**
13	Mean	5.14E+04	1.48E+04	1.24E+04	8.88E+03	8.96E+04	7.38E+03	**3.48E+01**
	Std. Dev.	3.52E+04	9.25E+03	8.64E+03	1.02E+04	2.64E+05	7.19E+03	**1.01E+01**
14	Mean	1.84E+06	1.05E+06	1.44E+06	6.89E+04	1.72E+06	**1.55E+01**	2.06E+01
	Std. Dev.	1.92E+06	7.79E+05	1.21E+06	1.31E+05	2.07E+06	**9.31E+00**	1.01E+01
15	Mean	2.82E+04	2.66E+03	2.51E+03	5.16E+03	2.46E+04	4.64E+01	**9.91E+00**
	Std. Dev.	2.10E+04	2.53E+03	2.94E+03	6.94E+03	2.04E+04	2.00E+02	**2.36E+00**
16	Mean	1.16E+03	1.25E+03	1.22E+03	8.46E+02	1.07E+03	**1.71E+02**	2.83E+02
	Std. Dev.	3.24E+02	3.19E+02	2.97E+02	3.61E+02	3.23E+02	1.58E+02	**1.58E+02**
17	Mean	4.85E+02	5.34E+02	5.94E+02	3.47E+02	5.47E+02	**3.70E+01**	4.08E+01
	Std. Dev.	2.35E+02	2.16E+02	2.25E+02	1.71E+02	2.17E+02	2.75E+01	**8.49E+00**
18	Mean	2.44E+06	8.53E+05	1.26E+06	2.96E+05	2.00E+06	1.31E+04	**3.00E+01**
	Std. Dev.	2.18E+06	9.37E+05	1.71E+06	1.78E+05	2.16E+06	1.63E+04	**3.65E+00**
19	Mean	2.16E+04	3.62E+03	4.30E+03	1.19E+04	2.49E+04	6.18E+02	**1.04E+01**
	Std. Dev.	1.46E+04	2.53E+03	5.14E+03	1.29E+04	1.69E+04	3.16E+03	**1.95E+00**
20	Mean	4.99E+02	5.04E+02	5.56E+02	4.54E+02	4.56E+02	**2.67E+01**	4.24E+01
	Std. Dev.	2.30E+02	1.88E+02	2.43E+02	1.96E+02	1.84E+02	4.60E+01	**3.48E+01**
21	Mean	2.69E+02	3.03E+02	3.10E+02	2.61E+02	2.65E+02	**2.31E+02**	2.32E+02
	Std. Dev.	1.72E+01	1.86E+01	2.24E+01	1.43E+01	1.41E+01	**6.98E+00**	7.12E+00
22	Mean	1.98E+03	5.26E+02	1.61E+03	1.31E+03	1.70E+03	**1.00E+02**	**1.00E+02**
	Std. Dev.	1.46E+03	1.17E+03	1.76E+03	1.59E+03	1.42E+03	**0.00E+00**	**0.00E+00**
23	Mean	4.22E+02	5.19E+02	5.08E+02	4.09E+02	4.28E+02	3.78E+02	**3.76E+02**
	Std. Dev.	1.76E+01	4.73E+01	4.55E+01	1.65E+01	1.59E+01	**7.90E+00**	9.24E+00
24	Mean	4.96E+02	7.57E+02	7.90E+02	4.77E+02	4.96E+02	4.54E+02	**4.49E+02**
	Std. Dev.	2.48E+01	7.94E+01	1.10E+02	1.94E+01	2.00E+01	7.97E+00	**6.80E+00**
25	Mean	3.93E+02	4.03E+02	4.10E+02	3.89E+02	3.91E+02	3.87E+02	**3.87E+02**
	Std. Dev.	1.25E+01	1.83E+01	2.18E+01	7.30E+00	9.62E+00	3.39E−01	**9.33E−02**
26	Mean	1.93E+03	2.31E+03	2.56E+03	1.69E+03	1.99E+03	**1.23E+03**	1.29E+03
	Std. Dev.	1.87E+02	1.74E+03	1.73E+03	3.02E+02	2.53E+02	**8.69E+01**	1.66E+02
27	Mean	5.35E+02	5.93E+02	5.98E+02	5.17E+02	5.33E+02	5.06E+02	**4.95E+02**
	Std. Dev.	1.34E+01	3.02E+01	3.44E+01	9.21E+00	1.34E+01	**6.26E+00**	7.90E+00
28	Mean	4.23E+02	3.91E+02	3.72E+02	3.53E+02	4.26E+02	3.49E+02	**3.13E+02**
	Std. Dev.	**2.06E+01**	2.81E+01	4.29E+01	6.09E+01	2.49E+01	6.08E+01	3.45E+01
29	Mean	8.37E+02	9.00E+02	9.72E+02	6.73E+02	9.17E+02	**4.13E+02**	5.01E+02
	Std. Dev.	2.20E+02	2.15E+02	2.00E+02	1.69E+02	2.31E+02	**2.53E+01**	3.38E+01
30	Mean	1.92E+04	4.70E+03	4.87E+03	4.68E+03	1.75E+04	3.38E+03	**2.01E+03**
	Std. Dev.	1.77E+04	1.04E+03	1.60E+03	2.29E+03	1.06E+04	1.51E+03	**2.84E+01**

adopted to test the combinations of parameters. The results of the experiment are given in Table 3. The various values of c, $m_{\max}$, n are shown in Table 2. The combinations of parameters and corresponding average errors obtained by TDBBO are listed in Table 3. The rank of each parameter is described in Table 4. Meanwhile, the trends of each parameter for different levels are shown in Fig. 1.

From Table 4, n is the most significant parameter among all the parameters, that is, n plays an important role on TDBBO. A small n improve the solution accuracy obtained by TDBBO. The c is ranked second, c is also an important factor in TDBBO. c=0 means that the two-stage mechanism is not employed by TDBBO. A large c would diversify the population, and a small c will accelerate the evolutionary speed. The $m_{\max}$ is ranked third. A large $m_{\max}$ increases the diversity of the population. However, the large $m_{\max}$ slows down the convergence speed of TDBBO. Fig. 1 shows the trend of parameters and all the parameters which have large effect on the performance of TDBBO. As the results of parameter calibration experiment and above analysis, the parameters in TDBBO are suggested as follows: n=50, c=0.3, $m_{\max}$=0.01.

Table 7
The results of seven algorithms (50-dimensional benchmark functions).

Function	Criterion	BBO	B-BBO	GBBO	IBBO	LXBBO	DEBBO	TDBBO
1	Mean	3.24E+05	**4.96E+02**	1.26E+03	5.97E+03	3.12E+05	3.15E+03	1.26E+03
	Std. Dev.	1.03E+05	**5.25E+02**	1.60E+03	6.20E+03	1.24E+05	3.56E+03	2.55E+03
2	Mean	1.68E+24	1.14E+14	8.02E+16	9.56E+01	7.64E+28	4.11E+09	**0.00E+00**
	Std. Dev.	8.45E+24	7.11E+14	4.41E+17	6.42E+01	5.40E+29	2.34E+10	**0.00E+00**
3	Mean	9.50E+04	8.04E+03	5.51E+04	1.43E+04	9.19E+04	3.44E+04	**2.83E+03**
	Std. Dev.	2.53E+04	1.95E+03	2.04E+04	4.62E+03	2.70E+04	7.29E+03	**1.30E+03**
4	Mean	1.52E+02	1.20E+02	1.13E+02	6.78E+01	1.44E+02	7.37E+01	**6.75E+01**
	Std. Dev.	4.58E+01	**3.52E+01**	4.98E+01	4.69E+01	5.53E+01	5.18E+01	4.21E+01
5	Mean	1.19E+02	2.07E+02	2.09E+02	1.43E+02	1.27E+02	**6.39E+01**	6.96E+01
	Std. Dev.	2.57E+01	2.51E+01	3.41E+01	3.25E+01	2.41E+01	**1.24E+01**	1.31E+01
6	Mean	1.43E−01	9.74E−02	4.24E−03	3.59E−06	1.45E−01	**1.26E−07**	1.65E−06
	Std. Dev.	2.53E−02	6.01E−02	1.61E−02	5.81E−06	2.78E−02	**7.02E−07**	2.35E−06
7	Mean	2.15E+02	3.13E+02	3.86E+02	2.20E+02	2.13E+02	1.24E+02	**1.17E+02**
	Std. Dev.	3.47E+01	4.31E+01	5.08E+01	3.99E+01	3.10E+01	1.94E+01	**1.28E+01**
8	Mean	1.21E+02	2.13E+02	2.23E+02	1.42E+02	1.23E+02	**5.97E+01**	7.09E+01
	Std. Dev.	2.36E+01	2.28E+01	3.23E+01	3.75E+01	2.18E+01	1.32E+01	**1.15E+01**
9	Mean	1.15E+03	6.00E+03	4.97E+03	5.94E+02	8.68E+02	4.48E−01	**1.21E−01**
	Std. Dev.	6.74E+02	1.20E+03	1.46E+03	7.40E+02	5.53E+02	8.53E−01	**2.85E−01**
10	Mean	4.22E+03	4.71E+03	5.03E+03	5.23E+03	**4.04E+03**	5.26E+03	4.79E+03
	Std. Dev.	5.07E+02	7.52E+02	6.79E+02	8.07E+02	6.77E+02	4.84E+02	**4.75E+02**
11	Mean	6.03E+03	1.84E+03	2.62E+03	7.73E+01	5.05E+03	4.71E+01	**4.60E+01**
	Std. Dev.	5.72E+03	1.21E+03	2.45E+03	3.05E+01	4.93E+03	**9.16E+00**	1.04E+01
12	Mean	8.26E+06	1.37E+06	1.31E+06	8.19E+05	7.63E+06	2.23E+05	**1.93E+04**
	Std. Dev.	4.26E+06	4.41E+05	4.82E+05	4.29E+05	4.12E+06	1.42E+05	**1.52E+04**
13	Mean	8.64E+04	1.56E+03	2.06E+03	3.27E+03	8.99E+04	2.26E+03	**1.71E+02**
	Std. Dev.	4.01E+04	2.38E+03	2.58E+03	4.45E+03	4.80E+04	3.17E+03	**4.72E+01**
14	Mean	4.80E+06	1.92E+06	1.63E+06	6.72E+04	3.94E+06	2.79E+03	**5.00E+01**
	Std. Dev.	3.54E+06	1.13E+06	1.05E+06	4.70E+04	2.96E+06	4.89E+03	**1.05E+01**
15	Mean	4.33E+04	8.62E+03	7.90E+03	2.96E+03	5.60E+04	2.67E+03	**4.24E+01**
	Std. Dev.	3.03E+04	4.85E+03	5.33E+03	3.93E+03	4.25E+04	3.04E+03	**6.94E+00**
16	Mean	1.80E+03	1.69E+03	1.84E+03	1.55E+03	1.81E+03	**7.29E+02**	7.55E+02
	Std. Dev.	3.95E+02	4.16E+02	4.66E+02	4.13E+02	4.01E+02	2.65E+02	**2.12E+02**
17	Mean	1.40E+03	1.56E+03	1.51E+03	1.01E+03	1.32E+03	**4.13E+02**	5.15E+02
	Std. Dev.	3.34E+02	3.32E+02	3.52E+02	3.53E+02	3.57E+02	1.61E+02	**1.55E+02**
18	Mean	7.17E+06	1.62E+06	1.64E+06	3.13E+05	6.94E+06	9.45E+04	**3.53E+02**
	Std. Dev.	4.99E+06	7.88E+05	1.03E+06	1.96E+05	6.42E+06	1.24E+05	**3.50E+02**
19	Mean	2.90E+04	1.69E+04	1.52E+04	1.54E+04	3.01E+04	1.26E+04	**2.22E+01**
	Std. Dev.	1.36E+04	5.77E+03	7.02E+03	1.07E+04	1.59E+04	7.68E+03	**3.97E+00**
20	Mean	1.09E+03	1.06E+03	1.15E+03	1.04E+03	1.14E+03	**2.52E+02**	3.37E+02
	Std. Dev.	3.61E+02	2.85E+02	3.09E+02	3.43E+02	3.78E+02	1.43E+02	**1.40E+02**
21	Mean	3.25E+02	4.22E+02	4.32E+02	3.41E+02	3.30E+02	**2.69E+02**	2.70E+02
	Std. Dev.	2.27E+01	2.76E+01	4.19E+01	2.57E+01	2.46E+01	1.39E+01	**1.11E+01**
22	Mean	5.19E+03	6.05E+03	6.05E+03	5.96E+03	4.96E+03	5.53E+03	**4.89E+03**
	Std. Dev.	6.38E+02	**5.91E+02**	1.27E+03	8.68E+02	5.97E+02	1.24E+03	1.80E+03
23	Mean	5.89E+02	7.55E+02	7.76E+02	5.65E+02	5.88E+02	4.90E+02	**4.86E+02**
	Std. Dev.	3.21E+01	7.54E+01	6.93E+01	3.74E+01	2.81E+01	1.48E+01	**1.27E+01**
24	Mean	6.58E+02	1.20E+03	1.34E+03	6.23E+02	6.55E+02	5.61E+02	**5.61E+02**
	Std. Dev.	3.16E+01	1.31E+02	1.62E+02	3.11E+01	3.22E+01	1.37E+01	**1.33E+01**
25	Mean	5.53E+02	6.01E+02	5.74E+02	5.19E+02	5.46E+02	**5.11E+02**	5.40E+02
	Std. Dev.	2.48E+01	**1.15E+01**	2.91E+01	3.94E+01	3.05E+01	3.96E+01	3.52E+01
26	Mean	2.84E+03	3.73E+03	5.77E+03	2.60E+03	2.87E+03	**1.78E+03**	1.81E+03
	Std. Dev.	3.25E+02	3.00E+03	1.41E+03	3.42E+02	3.56E+02	1.73E+02	**1.67E+02**
27	Mean	7.18E+02	1.04E+03	1.12E+03	6.36E+02	7.37E+02	5.41E+02	**5.18E+02**
	Std. Dev.	6.97E+01	1.21E+02	1.41E+02	5.17E+01	7.46E+01	1.93E+01	**1.21E+01**
28	Mean	5.10E+02	5.58E+02	5.07E+02	4.89E+02	5.13E+02	**4.89E+02**	4.92E+02
	Std. Dev.	2.88E+01	2.53E+01	**2.03E+01**	2.06E+01	2.09E+01	2.23E+01	2.06E+01
29	Mean	1.19E+03	1.27E+03	1.47E+03	9.91E+02	1.12E+03	**3.80E+02**	4.53E+02
	Std. Dev.	2.79E+02	3.83E+02	3.41E+02	3.02E+02	3.01E+02	**7.33E+01**	1.25E+02
30	Mean	1.13E+06	1.01E+06	9.27E+05	9.36E+05	1.09E+06	8.30E+05	**5.92E+05**
	Std. Dev.	3.73E+05	1.05E+05	1.22E+05	1.86E+05	3.39E+05	1.35E+05	**1.85E+04**

5.2. Experimental result

5.3. Analysis and discussion

In this paper, the standard BBO and five variants of BBO which have achieved excellent performance in some cases are selected to be compared with TDBBO to evaluate the effectiveness of TDBBO. Respectively, Tables 5–8 show the mean error and standard deviation of the seven algorithms with D=10, 30, 50, 100. For D=10, TDBBO is outperformed by BBO, Blended-BBO, GBBO, IBBO, and LXBBO on 0 of 30 functions. In addition, TDBBO is significantly superior to DE/BBO on 23 of 30 functions. It is worth mentioning that TDBBO finds the global optimum on f_1-f_4, f_6, f_9, f_{11}, f_{14} with D=10. For D=30, TDBBO is surpassed by BBO, Blended-BBO, GBBO, IBBO, and LXBBO on 1, 0, 0, 0, 0 functions respectively. By comparing to DE/BBO, the error values of TDBBO are smaller on 20 functions. To detect the stable of TDBBO, all the benchmark functions with $D=50$,100 are also executed. From Tables 7 to 8, the performance of TDBBO is better than other variants of BBO on the most functions. As above, TDBBO significantly outperforms other vari-

Table 8
The results of seven algorithms (100-dimensional benchmark functions).

Function	Criterion	BBO	B-BBO	GBBO	IBBO	LXBBO	DEBBO	TDBBO
1	Mean	1.13E+06	1.03E+05	4.73E+03	1.18E+04	1.06E+06	6.26E+03	**4.56E+03**
	Std. Dev.	5.32E+05	6.56E+04	4.47E+03	1.25E+04	2.59E+05	8.45E+03	**3.73E+03**
2	Mean	2.53E+101	6.43E+45	2.32E+31	3.29E+15	1.55E+110	7.11E+62	**6.95E+01**
	Std. Dev.	1.79E+102	4.41E+46	1.64E+32	2.33E+16	1.10E+111	3.56E+63	**4.97E+01**
3	Mean	1.82E+05	5.52E+04	**4.47E+04**	5.54E+04	1.85E+05	2.11E+05	9.13E+04
	Std. Dev.	4.30E+04	**8.60E+03**	1.47E+04	1.46E+04	4.08E+04	2.69E+04	1.07E+04
4	Mean	2.99E+02	3.04E+02	2.47E+02	1.95E+02	2.83E+02	2.05E+02	**1.77E+02**
	Std. Dev.	5.08E+01	4.87E+01	6.05E+01	4.01E+01	4.94E+01	**2.36E+01**	4.49E+01
5	Mean	3.18E+02	5.73E+02	6.18E+02	3.91E+02	3.23E+02	**1.72E+02**	1.92E+02
	Std. Dev.	4.65E+01	5.59E+01	5.63E+01	6.78E+01	5.30E+01	3.18E+01	**2.75E+01**
6	Mean	1.58E−01	3.24E−01	4.39E−02	2.86E−02	1.56E−01	**4.59E−05**	1.25E−04
	Std. Dev.	2.27E−02	1.66E−01	2.49E−02	3.55E−02	2.30E−02	**1.56E−04**	5.33E−04
7	Mean	6.12E+02	9.59E+02	1.08E+03	6.39E+02	6.05E+02	**3.02E+02**	3.09E+02
	Std. Dev.	6.73E+01	1.27E+02	1.38E+02	8.98E+01	7.00E+01	6.12E+01	**2.09E+01**
8	Mean	3.30E+02	5.93E+02	6.31E+02	3.86E+02	3.24E+02	**1.71E+02**	1.83E+02
	Std. Dev.	4.07E+01	6.10E+01	6.57E+01	6.69E+01	5.26E+01	3.07E+01	**2.64E+01**
9	Mean	5.21E+03	1.71E+04	1.84E+04	6.23E+03	5.05E+03	**1.17E+01**	1.22E+01
	Std. Dev.	1.89E+03	1.73E+03	2.85E+03	2.62E+03	2.03E+03	**1.09E+01**	3.17E+01
10	Mean	1.09E+04	1.20E+04	1.23E+04	1.28E+04	**1.09E+04**	1.47E+04	1.22E+04
	Std. Dev.	1.06E+03	1.33E+03	1.18E+03	1.43E+03	7.91E+02	1.22E+03	**7.11E+02**
11	Mean	5.79E+04	1.22E+04	1.28E+04	2.54E+02	5.57E+04	**1.56E+02**	2.63E+02
	Std. Dev.	2.28E+04	4.27E+03	6.57E+03	6.66E+01	2.12E+04	**3.85E+01**	5.89E+01
12	Mean	2.55E+07	8.28E+06	2.92E+06	9.88E+05	2.60E+07	5.36E+05	**1.88E+05**
	Std. Dev.	1.20E+07	1.22E+06	7.06E+05	3.57E+05	1.21E+07	2.16E+05	**8.09E+04**
13	Mean	3.47E+04	4.08E+03	3.80E+03	3.43E+03	3.54E+04	**2.31E+03**	2.72E+03
	Std. Dev.	1.24E+04	**2.22E+03**	2.60E+03	3.84E+03	1.14E+04	2.46E+03	2.61E+03
14	Mean	6.92E+06	7.17E+05	8.05E+05	1.55E+05	6.13E+06	1.18E+05	**4.46E+02**
	Std. Dev.	2.94E+06	1.75E+05	3.30E+05	5.85E+04	3.07E+06	7.24E+04	**3.96E+02**
15	Mean	1.95E+04	7.58E+02	1.29E+03	2.23E+03	3.72E+04	1.43E+03	**4.72E+02**
	Std. Dev.	9.79E+03	6.46E+02	1.24E+03	2.23E+03	1.21E+05	1.68E+03	**5.47E+02**
16	Mean	4.06E+03	4.39E+03	4.54E+03	3.79E+03	4.06E+03	2.55E+03	**2.34E+03**
	Std. Dev.	6.24E+02	6.97E+02	6.98E+02	7.47E+02	5.04E+02	4.61E+02	**3.44E+02**
17	Mean	3.07E+03	3.18E+03	3.45E+03	2.89E+03	3.09E+03	**1.59E+03**	1.69E+03
	Std. Dev.	4.74E+02	5.26E+02	4.76E+02	5.77E+02	5.05E+02	4.02E+02	**3.71E+02**
18	Mean	5.66E+06	7.38E+05	7.94E+05	3.54E+05	5.49E+06	6.82E+05	**4.66E+04**
	Std. Dev.	2.67E+06	2.78E+05	4.01E+05	1.42E+05	2.62E+06	3.48E+05	**3.29E+04**
19	Mean	2.16E+04	9.62E+02	1.69E+03	2.01E+03	2.02E+04	1.86E+03	**3.37E+02**
	Std. Dev.	1.05E+04	9.36E+02	2.03E+03	2.49E+03	7.76E+03	2.89E+03	**6.93E+02**
20	Mean	3.01E+03	2.90E+03	3.24E+03	2.62E+03	2.93E+03	**1.74E+03**	1.99E+03
	Std. Dev.	5.34E+02	4.88E+02	5.43E+02	5.13E+02	5.49E+02	3.74E+02	**2.81E+02**
21	Mean	5.86E+02	7.65E+02	8.06E+02	6.30E+02	5.87E+02	**4.05E+02**	4.12E+02
	Std. Dev.	5.50E+01	7.12E+01	6.82E+01	7.27E+01	5.06E+01	3.19E+01	**2.37E+01**
22	Mean	**1.19E+04**	1.46E+04	1.41E+04	1.46E+04	1.19E+04	1.54E+04	1.35E+04
	Std. Dev.	1.01E+03	1.35E+03	1.23E+03	1.27E+03	1.06E+03	1.19E+03	**8.57E+02**
23	Mean	7.62E+02	8.87E+02	9.35E+02	8.08E+02	7.56E+02	**6.73E+02**	7.21E+02
	Std. Dev.	3.46E+01	6.04E+01	7.68E+01	4.70E+01	3.94E+01	**2.42E+01**	2.62E+01
24	Mean	1.29E+03	1.54E+03	1.59E+03	1.25E+03	1.30E+03	**1.05E+03**	1.06E+03
	Std. Dev.	5.64E+01	8.15E+01	9.71E+01	6.96E+01	5.37E+01	3.66E+01	**3.14E+01**
25	Mean	8.15E+02	8.12E+02	8.18E+02	**7.61E+02**	8.21E+02	7.72E+02	7.85E+02
	Std. Dev.	6.73E+01	**3.49E+01**	4.07E+01	6.68E+01	6.39E+01	6.38E+01	5.82E+01
26	Mean	7.57E+03	9.90E+03	1.44E+04	7.28E+03	7.67E+03	**4.99E+03**	5.13E+03
	Std. Dev.	5.67E+02	7.79E+03	4.48E+03	6.64E+02	5.80E+02	3.85E+02	**3.67E+02**
27	Mean	8.39E+02	1.16E+03	1.25E+03	7.66E+02	8.31E+02	6.54E+02	**6.32E+02**
	Std. Dev.	5.99E+01	1.14E+02	1.14E+02	4.96E+01	5.53E+01	**2.14E+01**	2.27E+01
28	Mean	6.28E+02	6.72E+02	5.87E+02	5.56E+02	6.30E+02	5.53E+02	**5.46E+02**
	Std. Dev.	4.03E+01	**1.91E+01**	3.41E+01	2.77E+01	4.10E+01	2.20E+01	3.85E+01
29	Mean	3.51E+03	3.84E+03	4.02E+03	3.21E+03	3.15E+03	**1.53E+03**	1.91E+03
	Std. Dev.	4.61E+02	6.01E+02	5.54E+02	5.83E+02	5.15E+02	**3.37E+02**	3.76E+02
30	Mean	6.07E+04	1.58E+04	7.96E+03	5.99E+03	6.77E+04	5.64E+03	**3.44E+03**
	Std. Dev.	2.12E+04	3.24E+03	2.81E+03	3.31E+03	2.83E+04	2.24E+03	**1.57E+03**

ants of BBO with D=10, 30. However, it is clear that performance of TDBBO decreases with the dimension increase. To our surprise, TDBBO obtains the global optimum values on f_2 with D=50. As above analysis, TDBBO outperforms other compared algorithms on the CEC 2017 benchmark especially on low-dimensional functions.

The Friedman's test is carried out to rank the TDBBO and compared algorithms. As Figs. 6–8 show, TDBBO has the best ranking among the algorithms. The Bonferroni–Dunn's test method is introduced to calculate the critical difference for comparing the differences of the algorithms with $\alpha = 0.05$ and 0.1. It is clear that TDBBO significantly outperforms other variants of BBO except for DE/BBO. The Wilcoxon's test, which compares the algorithms in pairs, is introduced to detect the significant difference between TDBBO and other algorithms. Table 9 shows the statistical analysis result of the Wilcoxon's test between TDBBO and other variants of BBO on the functions with different dimensions. In Table 9, R+ is the sum of the rank that TDBBO outperforms than another algorithm in the current row, and R− is the sum of the levels that TDBBO another algorithm in the current row outperforms than TDBBO. A yes means that TDBBO is significantly superior to other algorithms in the current row. Table 9 summarizes the statistical results consider TDBBO as the control Algorithm. From Table 9,

Table 9
Rankings obtained through Wilcoxon's test.

Dimension	TDBBO vs.	R+	R−	p-value	$\alpha = 0.05$	$\alpha = 0.1$
10D	BBO	465	0	0.000	Yes	Yes
	Blended-BBO	465	0	0.000	Yes	Yes
	GBBO	465	0	0.000	Yes	Yes
	IBBO	378	0	0.000	Yes	Yes
	LXBBO	465	0	0.000	Yes	Yes
	DEBBO	253	72	0.015	Yes	Yes
30D	BBO	454	11	0.000	Yes	Yes
	Blended-BBO	465	0	0.000	Yes	Yes
	GBBO	465	0	0.000	Yes	Yes
	IBBO	435	0	0.000	Yes	Yes
	LXBBO	465	0	0.000	Yes	Yes
	DEBBO	299	107	0.029	Yes	Yes
50D	BBO	452	13	0.000	Yes	Yes
	Blended-BBO	446	19	0.000	Yes	Yes
	GBBO	435	0	0.000	Yes	Yes
	IBBO	458	7	0.000	Yes	Yes
	LXBBO	451	14	0.000	Yes	Yes
	DEBBO	322	113	0.024	Yes	Yes

TDBBO provides higher R+ values than R− with D=10, 30, 50. According to the Wilcoxon's test with $\alpha = 0.05$ and $\alpha = 0.1$, the performance of TDBBO is significantly better than other variants of BBO on solving the CEC 2017 benchmark functions.

TDBBO achieves better accuracy, stability than other variants of BBO. However, according to the Friedman's test, both TDBBO and DE/BBO significantly outperform other algorithms with D=10, 30, 50. The reason is that both TDBBO and DE/BBO are hybrid algorithms of DE and BBO. That is to say, all of them combined the exploitation ability of BBO and the exploration ability of DE. However, the performance of TDBBO and DE/BBO are different as the way of the combination is different. As the result of Wilcoxon's test shows, TDBBO performs significantly better than DE/BBO with D=10, 30, 50.

Figs. 2 and 3 show the convergence curves of TDBBO and compared algorithms on f_3, f_7, f_{18}, and f_{30}, with D=30, 50. The four functions represent four types of CEC 2017 benchmark functions respectively. As Fig. 2 shows, although the convergence speed of TDBBO is slower than some other variants of BBO in the early evolutionary stage, the TDBBO obtains the best solution accuracy in all the functions. Especially on f_{18} the result obtained by TDBBO is significantly superior to compared algorithms. The excellent performance of TDBBO is caused by the two-stage mechanism. In the early evolutionary process, the emigration model is a constant model, namely, H_k is a randomly selected habitat. Therefore, the convergence speed of TDBBO is slower than some other compared algorithms. In the later evolutionary process, H_k is selected by the sinusoidal emigration model. The superior habitats share information to the inferior habitats. Therefore, the convergence speed of TDBBO is accelerated. A similar conclusion is also drawn from Fig. 3. For the functions with D=50, TDBBO significantly outperforms than compared algorithms on f_{18}. From Figs. 2 and 3 and Table 5–8, TDBBO has the best performance than other compared algorithms for solving hybrid functions. Correspondingly, the boxplots of f_3, f_7, f_{18}, and f_{30} with D=30, 50 are shown in Figs. 4 and 5. The standard variance of TDBBO is significantly smaller than other algorithms on the functions. As above, the high stability of the TDBBO results from the combination of the two-stage mechanism, BBO/current-to-select/1 migration operator, Gaussian mutation operator, and greedy select strategy.

From the previous experimental series and analysis, TDBBO outperforms the compared algorithms on CEC 2017 benchmarks. The reason is described as follows. Most of the variants of BBO are designed to improve one factor of standard BBO. For example, the IBBO employs the improved migration operator and self-adaptive

Algorithm 1
The primary procedure of standard BBO.

Generate the initial population H.
Evaluate the fitness of individuals in H.
while the halting criterion is not satisfied **do**
  calculate λ_i, μ_i and m_i for each individual H_i.
  for $i = 1$ to NP **do**
    for $j = 1$ to D **do**
      if $rndreal(0, 1) < \lambda_i$ **then** // $rndreal(0, 1)$ is a uniform random number on the interval (0,1).
        select H_k with probability $\propto \mu_k$.
        $H_i(j) = H_k(j)$.
      end if
    end for
  end for
  for $i = 1$ to NP **do**
    for j= 1 to D **do**
      if $rndreal(0, 1) < m_i$ **then**
        replace $H_i(j)$ with a randomly generated SIV.
      end if
    end for
  end for
end while

Algorithm 2
Improved migration operator of TDBBO.

for $i = 1$ to NP **do**
  Select H_k with probability $\propto \mu_k$.
  Select uniform randomly $r_1 \neq r_2 \neq k \neq i$.
  for $j = 1$ to D **do**
    if $rndreal(0, 1) < lamdaScale_i$ **then**
      $U_i(j) = H_i(j) + F_i \bullet (H_k(j) - H_i(j)) + F_i \bullet (H_{r1}(j) - H_{r2}(j))$.
    else
      $U_i(j) = H_i(j)$
    end if
  end for
end for

Algorithm 3
Gaussian mutation operator.

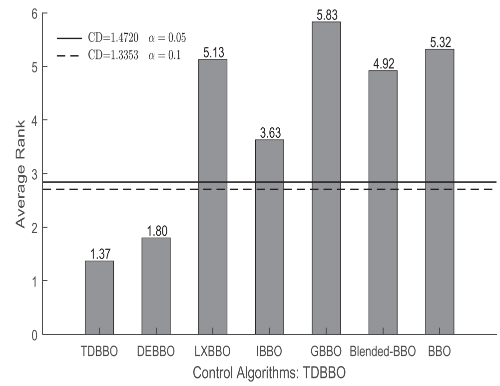
for $i = 1$ to n **do**
  for $j = 1$ to D **do**
    if $rndreal(0, 1) < m_i$ **then**
      $U_i(j) = U_i(j) + randn(0, 1)$.
    end if
  end for
end for

clear duplicate operator to enhance the exploration ability of standard BBO. However, the exploitation ability of IBBO is slightly worse than the exploitation ability of TDBBO. In contrary, the GBBO is prone to stagnation in local optima as the population learns from the global best optimal in migration operator. The Blended-BBO and LXBBO suffer from premature convergence as they are designed to learn from only one another solution. The DE/BBO balances exploration ability and exploitation ability effectively. However, The DE/BBO removes the mutation operator of BBO. Therefore, the probability that the population jumps out the local optimum is zero if the premature convergence is occurred. Besides, the above modifications do not focus on the rotational variance of standard BBO.

Compared with the above variants of BBO, the TDBBO is designed to enhance the performance from different views. The performance of TDBBO comes from the combination between the two-stage mechanism, BBO/current-to-select/1, Gaussian mutation operator and the greedy select strategy. The combination balances the exploration ability and exploitation ability of TDBBO efficiently. Besides, the TDBBO is more suitable for the non-separable problem as BBO/current-to-select/1 alleviate the rotational variance. As Awad et al. (2016) pointed out, the most problems in CEC 2017 benchmark are non-separable problems. Therefore, the con-

Algorithm	Mean Rank
TDBBO	1.37
DEBBO	1.80
LXBBO	5.13
IBBO	3.63
GBBO	5.83
Blended-BBO	4.92
BBO	5.32
Crit. Diff. α=0.05	1.4720
Crit. Diff. α=0.10	1.3353

(a)

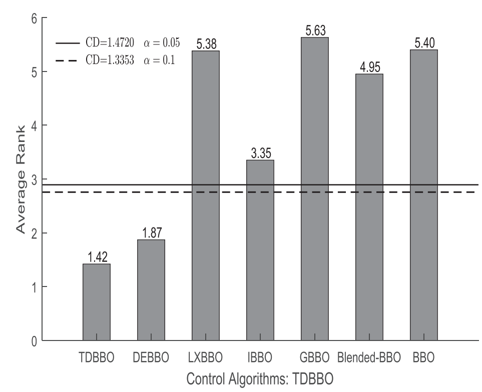


(b)

Fig. 6. Rankings obtained through Friedman's test and graphical representation of Bonferroni–Dunn's procedure (10D).

Algorithm	Mean Rank
TDBBO	1.42
DEBBO	1.87
LXBBO	5.38
IBBO	3.35
GBBO	5.63
Blended-BBO	4.95
BBO	5.40
Crit. Diff. α=0.05	1.4720
Crit. Diff. α=0.10	1.3353

(a)

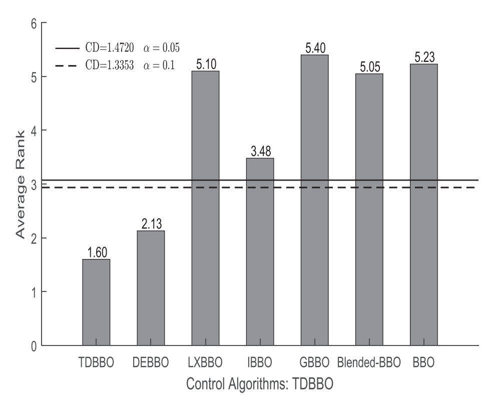


(b)

Fig. 7. Rankings obtained through Friedman's test and graphical representation of Bonferroni–Dunn's procedure (30D).

Algorithm	Mean Rank
TDBBO	1.60
DEBBO	2.13
LXBBO	5.10
IBBO	3.48
GBBO	5.40
Blended-BBO	5.05
BBO	5.23
Crit. Diff. α=0.05	1.4720
Crit. Diff. α=0.10	1.3353

(a)



(b)

Fig. 8. Rankings obtained through Friedman's test and graphical representation of Bonferroni–Dunn's procedure (50D).

clusion that TDBBO outperforms other modifications of BBO on non-separable problems is verified by the experimental results.

The limitation of the proposed TDBBO is parameter tuning. In fact, although the population-based algorithms provide excellent results, they are l very sensitive to algorithm parameters such as the mutation probability and population size. In TDBBO, the optimal combination of parameters in CEC 2017 benchmarks is obtained by Taguchi method. However, the optimal combination of parameters in TDBBO is not suitable for all the problems. The optimal combination of parameters should be reset in other problems.

6. Conclusions and future research

In this paper, a two-stage differential biogeography-based optimization (TDBBO) algorithm is proposed to address the premature convergence problem and alleviate the rotational variance of

Algorithm 4
The main procedure of TDBBO.

generate the initial solutions *H*.
evaluate the fitness of each individual in initial solutions *H*.
while the halting criterion is not satisfied **do**
  for each individual, map the fitness to the number of species.
  calculate the immigration rate λ_i and the emigration rate μ_i with Eq. (13) and (14).
  calculate F_i and$lamdaScale_i$ with Eq. (11) and (12).
  modify the population with the Migration operator shown in Algorithm 2.
  modify the population with the Mutation operator shown in Algorithm 3.
  evaluate the fitness of each individual in *U*.
  for $i = 1$ to *NP* **do**
    evaluate the trial habitats U_i.
    if $U_i$is better than H_i **then**
      $H_i = U_i$
    end if
  end for
end while

standard BBO. In the TDBBO, inspired by DE/current-to-rand/1, a BBO/current-to-select/1 is designed with the aim of improving the exploration ability and enhancing the rotational invariant of BBO. Meanwhile, a two-stage migration model is designed to keep the exploration ability in the early evolutionary process and accelerate the convergence speed in the late evolutionary process. Besides, the Gaussian mutation operator is employed to guide the population jump out the local optimal effectively. The convergence performance is also confirmed theoretically through the Markov model. The experimental results based on CEC 2017 benchmark show that the performance of TDBBO is significantly superior to BBO, LXBBO, GBBO, IBBO on almost all the benchmark functions with all dimensions. In summary, TDBBO is an effective and robust algorithm.

For the future work, several directions are suggested. Firstly, the urban transit network design problem (UTNDP) (Buba & Lee, 2018) is concerned with the development of a set of transit routes and corresponding schedules on an existing road network with known demand points and travel time. It is an NP-hard combinatorial optimization problem. Secondly, Economic Load Dispatch (ELD) (Al-Betar & Awadallah, 2018), which is an application in the power system, is a non-convex and non-linear optimization problem tackled by distributing the needed generations between the generating units properly to minimize the cost of fuel for each unit. In recent years, a wide variety of heuristics and meta-heuristics were designed to find the near-optimal solutions of UTNDP and ELD. Thirdly, the k-NN algorithm is one of the simplest and most effective algorithms in machine learning. The value of parameter k has a significant impact on the performance of classification. Various evolutionary algorithms, such as the standard BBO (Gholamreza Khademi, Mohammadi, Dan, & Hardin, 2015), DE (Bui, Nguyen, Hoang, & Klempe, 2017), PSO (Tharwat, Mahdi, Elhoseny, & Hassanien, 2018), were employed to find optimal parameter k for improved k-NN algorithm. As a modification of standard BBO, the TDBBO is an alternative algorithm for solving the above problems. Finally, the goal of job shop scheduling problem (Zhao et al., 2015), which is one of the most difficult combinatorial optimization problems, is to schedule the jobs on the machines to minimize the completion time needed for processing all jobs. The job shop scheduling problem is currently investigated by the authors of this paper.

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References

Al-Betar, M. A., & Awadallah, M. A. (2018). Island bat algorithm for optimization. *Expert Systems with Applications, 107*, 126–145.
Al-Roomi, A. R., & El-Hawary, M. E. (2016). Metropolis biogeography-based optimization. *Information Sciences, 360*, 73–95.
Awad, N., Ali, M., Liang, J., Qu, B., & Suganthan, P. (2016). Problem definitions and evaluation criteria for the CEC 2017 special session and competition on single objective real-parameter numerical optimization.
Brest, J., Maučec, M. S., & Bošković, B. (2017). Single objective real-parameter optimization: Algorithm jSO. In *Evolutionary computation* (pp. 1311–1318).
Buba, A. T., & Lee, L. S. (2018). A differential evolution for simultaneous transit network design and frequency setting problem. *Expert Systems with Applications, 106*, 277–289.
Bui, D. T., Nguyen, Q. P., Hoang, N. D., & Klempe, H. (2017). A novel fuzzy K-nearest neighbor inference model with differential evolution for spatial prediction of rainfall-induced shallow landslides in a tropical hilly area using GIS. *Landslides, 14*, 1–17.
Burton, R. M. (1985). Pointwise properties of convergence in probability. *Statistics & Probability Letters, 3*, 315–316.
Chen, X., Tianfield, H., Du, W. L., & Liu, G. H. (2016). Biogeography-based optimization with covariance matrix based migration. *Applied Soft Computing, 45*, 71–85.
Das, S., Abraham, A., Chakraborty, U. K., & Konar, A. (2009). Differential evolution using a neighborhood-based mutation operator. *IEEE Transactions on Evolutionary Computation, 13*, 526–553.
Ergezer, M., Dan, S., & Du, D. (2009). Oppositional biogeography-based optimization. In *IEEE international conference on systems, man and cybernetics* (pp. 1009–1014).
Feng, Q., Liu, S., Tang, G., & Yong, L. (2013). Biogeography-based optimization with orthogonal crossover. *Mathematical Problems in Engineering, 2013*(2013-6-4), 211–244.
Feng, Q., Liu, S., Zhang, J., Yang, G., & Yong, L. (2016). Improved biogeography-based optimization with random ring topology and Powell's method. *Applied Mathematical Modelling, 41*, 630–649.
Feng, Q. X., Liu, S. Y., Zhang, J. K., Yang, G. P., & Yong, L. Q. (2014). Biogeography-based optimization with improved migration operator and self-adaptive clear duplicate operator. *Applied Intelligence, 41*, 563–581.
Garg, V., & Deep, K. (2016). Performance of Laplacian Biogeography-Based Optimization Algorithm on CEC 2014 continuous optimization benchmarks and camera calibration problem. *Swarm and Evolutionary Computation, 27*, 132–144.
Goldberg, D. E., & Holland, J. H. (1988). Genetic algorithms and machine learning. *Machine Learning, 3*, 95–99.
Gong, W. Y., Cai, Z. H., & Ling, C. X. (2011). DE/BBO: A hybrid differential evolution with biogeography-based optimization for global numerical optimization. *Soft Computing, 15*, 645–665.
Gong, W. Y., Cai, Z. H., Ling, C. X., & Li, H. (2010). A real-coded biogeography-based optimization with mutation. *Applied Mathematics and Computation, 216*, 2749–2758.
Guo, W., Wang, L., Ge, S. S., Ren, H., & Mao, Y. (2015). Drift analysis of mutation operations for biogeography-based optimization. *Soft Computing, 19*, 1881–1892.
Guo, W., Wang, L., & Wu, Q. (2014a). An analysis of the migration rates for biogeography-based optimization. *Information Sciences, 254*, 111–140.
Guo, W. A., Chen, M., Wang, L., Mao, Y. F., & Wu, Q. D. (2017). A survey of biogeography-based optimization. *Neural Computing & Applications, 28*, 1909–1926.
Guo, W. A., Li, W. Z., Zhang, Q., Wang, L., Wu, Q. D., & Ren, H. L. (2014b). Biogeography-based particle swarm optimization with fuzzy elitism and its applications to constrained engineering problems. *Engineering Optimization, 46*, 1465–1484.
Hu, Z. B., Xiong, S. W., Su, Q. H., & Fang, Z. X. (2014). Finite Markov chain analysis of classical differential evolution algorithm. *Journal of Computational and Applied Mathematics, 268*, 121–134.
Karaboga, D., & Basturk, B. (2007). A powerful and efficient algorithm for numerical function optimization: Artificial bee colony (ABC) algorithm. *Journal of Global Optimization, 39*, 459–471.
Kennedy, J., & Eberhart, R. (2002). Particle swarm optimization. In *IEEE international conference on neural networks, 1995. Proceedings: 1944* (pp. 1942–1948).
Khademi, G., Mohammadi, H., Dan, S., & Hardin, E. C. (2015). Evolutionary optimization of user intent recognition for transfemoral amputees. In *Biomedical circuits and systems conference* (pp. 1–4).
Khademi, G., Mohammadi, H., & Simon, D. (2017). Hybrid invasive weed/biogeography-based optimization. *Engineering Applications of Artificial Intelligence, 64*, 213–231.
Liang, J. J., Qu, B. Y., & Suganthan, P. N. (2013). Problem definitions and evaluation criteria for the CEC 2014 special session and competition on single objective real-parameter numerical optimization.
Ma, H., Ni, S., & Sun, M. (2009). Equilibrium species counts and migration model tradeoffs for biogeography-based optimization. In *Decision and control, 2009 held jointly with the 2009 chinese control conference. Cdc/ccc 2009. Proceedings of the IEEE conference on* (pp. 3306–3310).

Ma, H. P. (2010). An analysis of the equilibrium of migration models for biogeography-based optimization. *Information Sciences, 180*, 3444–3464.

Ma, H. P., & Simon, D. (2011). Blended biogeography-based optimization for constrained optimization. *Engineering Applications of Artificial Intelligence, 24*, 517–525.

Ma, H. P., Simon, D., & Fei, M. R. (2016). Statistical mechanics approximation of biogeography-based optimization. *Evolutionary Computation, 24*, 427–458.

Ma, H. P., Simon, D., Fei, M. R., Shu, X. Z., & Chen, Z. X. (2014). Hybrid biogeography-based evolutionary algorithms. *Engineering Applications of Artificial Intelligence, 30*, 213–224.

Montgomery, D. C. (1976). *Design and analysis of experiments*. New York-London-Sydney: John Wiley & Sons.

Nabil, E. (2016). A modified flower pollination algorithm for global optimization. *Expert Systems with Applications, 57*, 192–203.

Sharma, P. K., Sharma, H., & Sharma, N. (2017). Gbest inspired Biogeography Based Optimization algorithm. In *IEEE international conference on power electronics, intelligent control and energy systems* (pp. 1–6).

Simon, D. (2008). Biogeography-based optimization. *IEEE Transactions on Evolutionary Computation, 12*, 702–713.

Simon, D., Ergezer, M., Du, D. W., & Rarick, R. (2011). Markov models for biogeography-based optimization. *IEEE Transactions on Systems Man and Cybernetics Part B-Cybernetics, 41*, 299–306.

Simon, D., Omran, M. G. H., & Clerc, M. (2014). Linearized biogeography-based optimization with re-initialization and local search. *Information Sciences, 267*, 140–157.

Tang, L. X., Dong, Y., & Liu, J. Y. (2015). Differential evolution with an individual-dependent mechanism. *IEEE Transactions on Evolutionary Computation, 19*, 560–574.

Tharwat, A., Mahdi, H., Elhoseny, M., & Hassanien, A. E. (2018). Recognizing human activity in mobile crowdsensing environment using optimized k-NN algorithm. *Expert Systems with Applications, 107*, 32–44.

Wolpert, D. H., & Macready, W. G. (1997). No free lunch theorems for optimization. *IEEE Trans on Evolutionary Computation, 1*, 67–82.

Xiong, G., Li, Y., Chen, J., Shi, D., & Duan, X. (2014). Polyphyletic migration operator and orthogonal learning aided biogeography-based optimization for dynamic economic dispatch with valve-point effects. *Energy Conversion & Management, 80*, 457–468.

Zhao, F., Chen, Z., Wang, J., & Zhang, C. (2017a). An improved MOEA/D for multi-objective job shop scheduling problem. *International Journal of Computer Integrated Manufacturing, 30*, 616–640.

Zhao, F., Liu, H., Zhang, Y., Ma, W., & Zhang, C. (2018). A discrete Water Wave Optimization algorithm for no-wait flow shop scheduling problem. *Expert Systems with Applications, 91*, 347–363.

Zhao, F., Liu, Y., Shao, Z., Jiang, X., Zhang, C., & Wang, J. (2016a). A chaotic local search based bacterial foraging algorithm and its application to a permutation flow-shop scheduling problem. *International Journal of Computer Integrated Manufacturing, 29*, 962–981.

Zhao, F., Liu, Y., Zhang, C., & Wang, J. (2015a). A self-adaptive harmony PSO search algorithm and its performance analysis. *Expert Systems with Applications, 42*, 7436–7455.

Zhao, F., Liu, Y., Zhang, Y., Ma, W., & Zhang, C. (2017b). A hybrid harmony search algorithm with efficient job sequence scheme and variable neighborhood search for the permutation flow shop scheduling problems. *Engineering Applications of Artificial Intelligence, 65*, 178–199.

Zhao, F., Shao, Z., Wang, J., & Zhang, C. (2016b). A hybrid differential evolution and estimation of distribution algorithm based on neighbourhood search for job shop scheduling problems. *International Journal of Production Research, 54*, 1039–1060.

Zhao, F., Zhang, J., Wang, J., & Zhang, C. (2015b). A shuffled complex evolution algorithm with opposition-based learning for a permutation flow shop scheduling problem. *International Journal of Computer Integrated Manufacturing, 28*, 1220–1235.

Zhao, F., Zhang, J., Zhang, C., & Wang, J. (2015c). An improved shuffled complex evolution algorithm with sequence mapping mechanism for job shop scheduling problems. *Expert Systems with Applications, 42*, 3953–3966.

Zheng, Y. J., Ling, H. F., & Xue, J. Y. (2014). Ecogeography-based optimization: Enhancing biogeography-based optimization with ecogeographic barriers and differentiations. *Computers & Operations Research, 50*, 115–127.

Zong, W. G., Kim, J. H., & Loganathan, G. V. (2001). A New Heuristic Optimization Algorithm: Harmony Search. *Simulation Transactions of the Society for Modeling & Simulation International, 76*, 60–68.