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A multipopulation cooperative coevolutionary whale optimization algorithm with a two-stage orthogonal learning mechanism

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ABSTRACT

This paper designed a multipopulation cooperative coevolutionary framework with a two-stage orthogonal learning (OL) mechanism for the whale optimization algorithm (MCCWOA) to improve the performance of the whale optimization algorithm (WOA). In the framework, a prediction model of the neighborhood structure is established by discovering the guidance information of the following iteration process in the objective space at the first-stage OL. In the second-stage OL, an auxiliary vector pool with various features in the decision space is introduced to guide the candidates falling in the stagnant status to conduct more valuable exploration. According to the domain knowledge of the candidates, the population is divided into the elite population, the intermediate population, and the inferior population. The information of the subpopulations has interacted with the corresponding historical populations in the evolution processes to enhance the ability of cooperative coevolution among individuals. A standard set of comprehensive benchmark cases and three engineering cases are utilized to verify the advantages of the proposed algorithm. The results of the statistical analysis, diversity analysis, and convergence analysis testified that the MCCWOA outperforms the 15 state-of-the-art algorithms regarding efficiency and significance.

1. Introduction

The optimization problems are an important domain in management science, computer science, and engineering applications with the development of knowledge-based systems since various real problems could be modeled as mathematical expressions [1,2]. With the increase of the problem scales, the challenge of the solution methods is increased accordingly. Traditional mathematical methods, integer programming [3], linear programming [4], and dynamic programming [5] are efficient methods to address small-scale problems. However, the approaches have inferior performance when dealing with large-scale optimization problems arising with the rapid development of manufacturing [6]. In recent years, disparate mechanisms of metaheuristics have been presented and utilized for addressing complex continuous optimization problems. Furthermore, swarm intelligence (SI) has been widely studied and proven to be an effective method to address large-scale problems, including continuous optimization problems and combinatorial optimization problems [7,8]. The critical inspiration of SI is a gathering of constantly changing, decentralized, and self-organizing agents, which interact with neighbors and surroundings by hunting, foraging, searching, or direction-finding [9]. The advantage of SI lies in the cooperative evolution between individuals. Additionally, the unique learning mechanism is the cornerstone of individual coevolution [8,9]. Therefore, it is meaningful to study the learning mechanisms of intelligent optimization algorithms and set up a framework of coevolution.

The whale optimization algorithm (WOA) is a promising swarm-based algorithm, that simulates the foraging behavior of humpback whales in oceans and was originally proposed by Mirjalili [10]. The peculiar hunting is patterned into three stages including searching for prey, encircling prey, and bubble-net attacking method. It is significant to identify a leading whale that allows other individuals to learn from it. In this sense, the WOA is different from other SI algorithms because it has greater potential to benefit from collective intelligence. The WOA performs a heuristic search by iterating around the leaders with characteristic properties. A leader or solution is identified to guide others to swim toward it in the first stage as a crucial diversity mechanism. In the latter two stages, the spiral model and the encircled method are simulated to conduct the exploitation via

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information of the best leader whales. The promising novelty of the WOA lies in its switching mode between exploration and exploitation, which is guided by the learning mechanism of leaders with different characteristics [11]. Each of the leaders is regarded as a potential solution or local-optima region to propagate the information of the leaders in other individuals. In this regard, the WOA is similar to the canonical SI algorithm, partial swarm optimization (PSO) [12]. Compared with the mainstream differential evolution (DE) [13], the novelty of the WOA is that it replaces the crossover and mutation mechanism in the DE with a novel predator–prey model. As with any SI algorithm, a proper balance between exploration and exploitation must be obtained for WOA, PSO, and DE in the search process. However, the critical issue that affects the performance of all SI algorithms has not been effectively solved in the WOA. There are three main reasons for the problem in the WOA.

First, the local search mechanism in the WOA still has some deficiencies to be improved and perfected. The performance of the WOA mainly depends on several exploitation parameters, which are randomly selected according to normal distribution or Gaussian distribution [14]. The parameters are neither self-adaptive nor self-learning. Particularly, the transition of exploration and exploitation is controlled by the parameters A and p (as shown in Eqs. (3) and (11), respectively, in Section 3.1). The parameter control is crucial for the performance of SI algorithms [9]. Although several parameters in the WOA are changed linearly with the number of iterations, it is not enough. In this paper, the parameters controlling local search switching are obtained by a prediction model based on orthogonal learning (OL) to adjust dynamically according to the state and timing of the algorithm.

Second, it is highly controversial to simply construct offspring via the best experience in evolution [15]. The individuals uninterruptedly approach the presumptive global best solution to strengthen the impact of the best leader whale at the later stage of the algorithm, rather than the random leaders [11]. In certain worse cases, it will lead the population to evolve toward the local optimum because the selected leader is not the global optimum but the local optimum. As a result, massive offspring will be crowded around the bad leader. On the other hand, many computational costs are also spent in a region that should not be searched deeply. This is one of the reasons that the kind of SI algorithm easily converges prematurely. The above observations indicate that there is an urgent need for an effective method to adopt the information of leaders with different characteristics to construct new individuals.

Third, it is well known that another reason for premature convergence is the rapid loss of population diversity. Furthermore, promoting diversity has often been seen as an essential step to improve the performance of SI [12,13]. In the original WOA, the method of identifying the random leaders is blind. The leaders are determined simply by a random individual in the current population. Although the diversity of the algorithm is enhanced to a certain extent, this method cannot produce a more promising solution because of its strong randomness. In this paper, the information of solutions at the evolution process is utilized to promote and maintain diversity in the population.

Fortunately, the multipopulation strategy has been proven to promote the diversity of the population in SI algorithms effectively and to alleviate the problem of premature convergence [16]. To date, the multipopulation strategy has been successfully incorporated in various studies, including PSO [17,18], DE [19], and other algorithms [20–22]. The multipopulation strategy plays an active role in improving this type of SI algorithm, such as the WOA [23]. Furthermore, orthogonal design (OD) is a valid experimental design method that provides which provided the optimal composition levels for different factors with reasonable experiments [24]. The OL strategy formed by the orthogonal experimental design (OED) has been proven to exhibit distinctive orthogonal testability and prediction ability [24]. In the WOA, the most potential operators for individuals are predicted via the OL to reduce the randomness in exploitation and strengthen the diversity of exploration.

In this paper, a multipopulation cooperative coevolutionary WOA with a two-stage OL (MCCWOA) is designed to attack the above problems and improve the performance of the WOA. In the first-stage OL, a prediction model of the neighborhood structure is established by discovering the guidance information in the whole performance space to guide the evolutionary direction of the population. In the second-stage OL, an auxiliary vector pool with various features in the decision space is introduced to guide the candidates falling in the stagnant status to conduct more valuable exploration. The contributions of this paper are generalized as follows.

(1) A two-stage OL framework based on the multipopulation of the WOA is proposed to enhance exploration and exploitation in the WOA.
(2) The performance of the MCCWOA is evaluated with 15 other state-of-the-art optimization approaches on the standard test suite and three kinds of engineering optimization problems in the real world. The comparison results demonstrate the effectiveness of the proposed MCCWOA.

The remainder of the paper is structured as follows. The literature closely related is reviewed in Section 2. The proposed MCCWOA is described in Section 3. The experimental results are presented and discussed in Section 4. Three kinds of engineering optimization problems in the real world are addressed via the MCCWOA in Section 5. Finally, the conclusions and outlook for the future study are presented in Section 6.

2. Literature review and motivation

Metaheuristic optimization algorithms can be classified into four main categories: evolutionary algorithms, physic-based algorithms, SI-based algorithms, and human-based algorithms [25,26]. The first category has been inspired by the idea of evolution in nature. These algorithms are based on the theories of Darwin. This theory is an optimization process, aiming at improving an organisms' ability to survive in a dynamic environment. The second category includes physic-based optimization algorithms, in which the motion of particles is inspired by the laws of physics in magnetic fields, gravitational forces among the particles of galaxies, electron charge transfer, chemical reactions, and so on. The third category of meta-heuristic optimizers is swarm-based algorithms (SI). These algorithms often use the swarm of particles, in which a single particle fails to be detected and compared. The particles find the response using group communications. The fourth category is human-based algorithms. These algorithms usually simulate human behavior in life. A large number of researches have addressed optimization algorithms. In this regard, the most important studies and algorithms are briefly listed in Table 1.

2.1. The whale optimization algorithm

Compared with other SI algorithms, the WOA has the advantages of pure principle, fewer adjustment parameters, and active searchability. Hence, the WOA has been widely used in many fields [1]. Harikarthik et al. [101] adopted the WOA to detect software errors. Additionally, the WOA was employed as a trainer to learn the multilayer perceptron neural network to solve the non-linear regression problem [102]. Social learning and wavelet

Table 1
Literature of meta-heuristic optimization algorithms.

	Algorithm full name	Arbitrary name	Author (s)	Year
Evolutionary	Genetic Algorithm [27]	GA	J.H. Holland	1975
	Simulated Annealing [28]	SA	S. Kirkpatrick, C.D. Gelatt, M.P. Vecchi	1983
	Tabu Search [29]	TS	F. Glover	1989
	Genetic programming [30]	GP	J. Koza	1992
	Evolution Strategy [31]	ES	I. Rechenberg	1994
	Memetic Algorithm [32]	MA	N.J. Radcliffe, P.D. Surry	1994
	Cultural Algorithm [33]	CA	R.G. Reynolds	1994
	Differential Evolution [13]	DE	R. Storn, K. Price	1997
	Evolutionary Programming [34]	EP	X. Yao, Y. Liu, G. Lin	1999
	CoEvolutionary Algorithm [35]	CoEa	Y.K. Kim, J.Y. Kim, Y. Kim	2000
	Gradient Evolution Algorithm [36]	GEA	A. Sinha, D. Goldberg	2003
	Imperialistic Competitive Algorithm [37]	ICA	E. Atashpaz-Gargari, C. Lucas	2007
	Biogeography-Based Optimization [38]	BBO	D. Simon	2008
	States of Matter Search [39]	SMS	E. Cuevas, A. Echavarría, M.A. Ramírez	2014
	Sine Cosine Algorithm [40]	SCA	S. Mirjalili	2016
	Multi-level Cross Entropy Optimizer [41]	MCEO	F. MiarNaeimi, G. Azizyan, M. Rashki	2018
Physic-based	Small-World Optimization Algorithm [42]	SWOA	H. Du, X. Wu, J. Zhuang	2006
	Magnetic Optimization Algorithm [43]	MOA	M.H. Tayarani, N.M.R. Akbarzadeh. T.	2008
	Charged System Search [44]	CSS	A. Kaveh, S. Talatahari	2010
	Black Hole [45]	BH	A. Hatamlou	2012
	Ideal Gas Molecular Movement [46]	IGMM	H. Varaee, M.R. Ghasemi	2015
	Water Evaporation Optimization [47]	WEO	A. Kaveh, T. Bakhshpoori	2016
	Multi-Verse Optimizer [48]	MVO	S. Mirjalili, S.M. Mirjalili, A. Hatamlou	2017
	Equilibrium Optimization Algorithm [49]	EOA	A.R. Yıldız, H. Özkaya, M. Yıldız, S. Bureerat, B.S. Yıldız, S.M. Sait	2020
	Chaotic Henry Gas Solubility Optimization [50]	CHGSO	B.S. Yıldz, N. Pholdee, N. Panagant, S. Bureerat, A.R. Yildiz, S.M. Sait	2020
Swarm Intelligence (SI)	Particle Swarm Optimization [12]	PSO	J. Kennedy, R. Eberhart	1995
	Termite Colony Optimization [51]	TCO	M. Roth, S. Wicker	2005
	Ant Colony Optimization [52]	ACO	M. Dorigo, M. Birattari, T. Stutzle	2006
	Shuffled Frog-Leaping [53]	SFL	M. Eusuff, K. Lansey, F. Pasha	2006
	Monkey Search [54]	MS	A. Mucherino, O. Seref	2007
	Dolphin Partner Optimization [55]	DPO	S. Yang, J. Jiang, G. Yan	2009
	Firefly Algorithm [56]	FAA	Yang X-S	2010
	Bird Mating Optimizer [57]	BMO	A. Askarzadeh	2012
	Fruit Fly Optimization [58]	FFO	W.T. Pan	2012
	Lion Pride Optimizer [59]	LPO	B. Wang, X.P. Jin, B. Cheng	2012
	Krill Herd [60]	KH	A.H. Gandomi, A.H. Alavi	2012
	Cuckoo Search [61]	CS	A.H. Gandomi, X.S. Yang, A.H. Alavi	2013
	Grey Wolf Optimizer [62]	GWO	S. Mirjalili, S.M. Mirjalili	2014
	Soccer League Competition Algorithm [63]	SLCA	N. Moosavian, B. Kasaee Roodsari	2014
	Ant Lion Optimizer [64]	ALO	S. Mirjalili	2015
	Dragonfly Algorithm [65]	DA	S. Mirjalili	2015
	Moth–flame Optimization [66]	MFO	S. Mirjalili	2015
	Whale Optimization Algorithm [67]	WOA	S. Mirjalili, A. Lewis	2016
	Salp Swarm Algorithm [68]	SSA	S. Mirjalili, A.H. Gandomi, S.Z. Mirjalili, S. Saremi, H. Faris, S.M. Mirjalili	2017
	Grasshopper Optimization Algorithm [69]	GOA	S. Saremi, S. Mirjalili	2017
	Chaotic Whale Optimization Algorithm [70]	CWOA	D. Oliva, M. Abd El Aziz, A. Ella Hassanien	2017
	Harris Hawks Optimization [71]	HHO	A. Heidari, S. Mirjalili, H. Faris, I. Aljarah, M. Mafarja, H. Chen	2019
	Flying Squirrel Optimizer [72]	FSO	F. Miarnaeimi, G. Azizyan, N. Shabakhty, M. Rashki	2019
	Conscious Neighborhood-based Crow Search Algorithm [73]	CCSA	H. Zamani, M.H. Nadimi-Shahraki, A.H. Gandomi,	2019
	Quantum-based Avian Navigation Optimizer Algorithm [74]	QANA	H. Zamani, M.H. Nadimi-Shahraki, A.H. Gandomi	2021
	Elite Opposition-based Learning Grasshopper Optimization Algorithm [75]	EOBL-GOA	B.S. Yildiz, N. Pholdee, S. Bureerat, A.R. Yildiz, S.M. Sait	2021
	Starling Murmuration Optimizer [76]	SMO	H. Zamani, M.H. Nadimi-Shahraki, A.H. Gandomi	2022
	Gaze Cues Learning-based Grey Wolf Optimizer [77]	GGWO	M.H. Nadimi-Shahraki, S. Taghian, S. Mirjalili, H. Zamani, A. Bahreininejad	2022
Human-based	Harmony Search [78]	HS	Z.W. Geem, J.H. Kim, G. V Loganathan	2001
	Group Search Optimizer [79]	GSO	S. He, Q.H. Wu, J.R. Saunders	2006
	Seeker Optimization Algorithm [80]	SOA	C. Dai, Y. Zhu, W. Chen	2006
	Imperialist Competitive Algorithm [37]	ICA	E. Atashpaz-Gargari, C. Lucas	2007
	League Championship Algorithm [81]	LCA	A.H. Kashan	2009
	Firework Algorithm [82]	FA	Y. Tan, Y. Zhu	2010
	Social Emotional Optimization [83]	SEO	Y. Xu, Z. Cui, J. Zeng	2010
	Brain Storm Optimization [84]	BSO	Y. Shi	2011
	Teaching Learning Based Optimization [85]	TLBO	R. V. Rao, V.J. Savsani, D.P. Vakharia	2011
	Anarchic Society Optimization [86]	ASO	A. Ahmadi-Javid	2011
	Mine Blast Algorithm [87]	MBA	A. Sadollah, A. Bahreininejad, H. Eskandar, M. Hamdi	2013
	Social-based Algorithm [88]	SBA	F. Ramezani, S. Lotfi	2013

(continued on next page)

Table 1 (*continued*).

Cultural Evolution Algorithm [89]	CEA	H.C. Kuo, C.H. Lin	2013
Cohort Intelligence [90]	CI	J. Kulkarni, I.P. Durugkar, M. Kumar	2013
Soccer League Competition Algorithm [91]	SLC	N. Moosavian, B.K. Roodsari	2014
Interior Search Algorithm [92]	ISA	A.H. Gandomi	2014
Group Counseling Optimization Algorithm [93]	GCO	M.A. Eita, M.M. Fahmy	2014
Exchange Market Algorithm [94]	EMA	N. Ghorbani, E. Babaei	2014
Social Group Optimization [95]	SGO	S. Satapathy, A. Naik	2016
Colliding Bodies Optimization [96]	CBO	A. Kaveh	2016
Ideology Algorithm [97]	IA	T.T. Huan, A.J. Kulkarni, J. Kanesan, C.J. Huang, A. Abraham	2016
Social Learning Optimization [98]	SLO	Z.Z. Liu, D.H. Chu, C. Song, X. Xue, B.Y. Lu	2016
Socio Evolution and Learning Optimization Algorithm [99]	SELO	M. Kumar, A.J. Kulkarni, S.C. Satapathy	2018
Volleyball Premier League Algorithm [100]	VPL	R. Moghdani, K. Salimifard	2018

mutation is incorporated into the WOA for forecasting water resources demand by Guo et al. [103]. Li et al. [104] designed an extreme learning machine based on the WOA to evaluate the model of the insulated gate bipolar transistor. In intelligent medical, a hybrid algorithm based on the WOA was proposed to diagnose equipment failure by Hassan et al. [105]. He et al. [106] designed an improved WOA to search the best parameter adaptively in a stochastic resonance system. A hybrid system based on the WOA was proposed for wind speed forecasting by Wang et al. [107]. El-Fergany et al. [108] adopted the WOA to improve the accuracy of the fuel cell model. The WOA was adopted to design a hybrid photovoltaic-biowaste-fuel cell system by Sun et al. [109]. Got et al. [110] developed a hybrid filter-wrapper approach based on the WOA for tackling the feature selection problem.

Firstly, given the shortcomings of parameters in the WOA, many self-tuning strategies about parameters have been proposed to mitigate them. Oliva et al. [70] utilized a chaos-integrated WOA to calculate and automatically adapt internal parameters, which avoided falling into local optimum and speeded up the convergence to a certain extent in the optimization process. The CWOA was also proposed to estimate various parameters of solar cells and photovoltaic modules. Yousri and Allam [14] integrated ten chaos maps to tune the parameters of the original WOA. The researchers tested the influence of the chaos maps for each parameter by adjusting these parameters. The proposed algorithm also optimized the accuracy of the parameter estimation of the permanent magnet synchronous motor. Elhosseini and Haikal [111] proposed a parameter optimization method to balance exploration and exploitation in WOA. The authors introduced the inertia weight strategy to adjust the sensitivity of parameters to the algorithm. Similarly, Chakraborty et al. [112] introduced a selection parameter to balance the global and local search phase.

Second, certain new mutation strategies have been proposed to enhance the search efficiency of WOA. Mafarja and Mirjalili [113] proposed a binary variant of the WOA to search the best feature subsets for classification purposes. The crossover and mutation operators are integrated into the variant to improve both exploration and exploitation. Tournament and roulette wheel selection mechanisms are employed to replace the random operator in the searching process. Chen et al. [114] utilized a double adaptive weight strategy to improve the early exploratory search trend and the later exploitation behavior. Heidari et al. [11] proposed an associative learning mechanism combined with □-hill climbing in the WOA. The authors introduced a mutation operator via the social behavior of whales to enhance the exploitation tendency of the underlying optimizer. Wang and Chen [115] incorporated chaotic and multi-swarm strategies into WOA to perform parameter optimization and feature selection.

Third, some hybridization strategies have been incorporated in the WOA to promote the diversity of the population. Sun and Wang [116] proposed a chaotic method based on the concept of chaos in the WOA. The chaotic characteristic was utilized to improve the diversity of the population and the egocentricity of agent search. Abdel-Basset et al. [117] presented a hybrid WOA integrated with local search strategy, swap mutation, and insert-reversed block operation to address the permutation flow shop scheduling problem. The individuals escaped from the local optima via the swap operation. Furthermore, the diversity of the population was promoted by the insert-reversed block operation. The same authors also proposed an incorporated WOA with Tabu search to handle the quadratic assignment problem [118]. The diversity of initial WOA was improved by defining a set of neighborhood solutions. Zhou et al. [119] presented an improved WOA based on a Lévy flight trajectory (LWOA), which enhanced the capability of jumping out of the local optima. Chen and Li [120] applied chaos mechanism and opposition-based learning method to the based WOA. The opposition-based learning method updated the position of candidate solutions in the second half of the algorithm iteration to improve the diversity of the population. Tu et al. [1] proposed an enhanced WOA, which embedded a communication mechanism and biogeography-based optimization algorithm to promote the global optimal search-ability and the exploitation tendency of the WOA. Zhang and Wen [121] provided an individual-based updating way instead of the dimension-based updating one of WOA to reduce the computational complexity. The authors solved effectively the high-dimensional high-dimensional optimization problems.

However, there are very few research attainments on the complex learning mechanism based on humpback whale social behavior and rich information from multiple swarms in the WOA. This paper focuses on the improvement of the above cooperation.

2.2. Multipopulation mechanism

This paper instructed a multipopulation mechanism in the WOA to enhance the population diversity of the algorithm. Kennedy [122] firstly described the idea of multipopulation and introduced an improved PSO with a neighborhood topology. The experimental results indicated that the large neighborhood was more robust for certain simple problems. On the other hand, a PSO with a small neighborhood structure shows a better performance in tackling complex problems. Shortly afterward Kennedy and Mendes studied the effect of different neighborhood topologies on the performance of PSO [123].

After the idea of multipopulation, many researchers have been proposed improved SI algorithms. Zamani et al. [76] presented an effective starling murmuration optimizer (SMO), which introduced a dynamic multipopulation construction and three new search strategies, separating, diving, and whirling. The same authors developed a quantum-based avian navigation optimizer algorithm (QANA), which was inspired by the extraordinary precision navigation of migratory birds during long-distance aerial paths. In the QANA, the population is distributed by partitioning into multipopulation to explore the search space effectively [74]. Pan et al. [124] proposed an adaptive multi-group salp swarm algorithm (AMSSA), which reconciled three new communication strategies in the process of evolution. Wu et al. [125] proposed a modified FMO (modFMO) algorithm to extend the lifetime of wireless networks effectively. Yildiz et al. [50] developed a Henry gas solubility optimization algorithm (HGSO) to solve real-life engineering optimization problems. Oliva et al. [70] developed a chaotic whale optimization algorithm (CWOA) for the parameters estimation of solar cells. Wang et al. [126] developed an adaptive multipopulation WOA based on the Pareto strategy to tackle the cloud manufacturing service issue. Niu et al. [127] applied a master–slave model, in which a swarm is composed of one master swarm and several slave swarms, to realize the cooperation between populations in PSO. The authors creatively designed two subpopulations with different functions. The slave swarms maintained the diversity of particles by executing a single PSO or its variants independently. The master swarm evolved according to its knowledge and the knowledge of the slave swarms. Wang et al. [128] presented a dynamic multi-swarm PSO that incorporated heterogeneous comprehensive learning with two mutation operators. The subpopulation of this PSO variant was generated by a comprehensive learning strategy with the global best solution. Otherwise, an improved dynamic multi-swarm strategy was especially utilized to construct the exploration subpopulation exemplar. The authors designed a special population renewal model to prevent a loss of diversity. Wang and Chen [115] proposed an improved WOA to perform parameter optimization and feature selection concurrently for the support vector machine. The chaotic and multi-swarm strategies were combined to form the proposed algorithm. Chen et al. [129] presented an improved variant of the basic sine cosine algorithm with a multipopulation mechanism and OL. The multi-swarm scheme with three sub-strategies was performed to improve the exploration capabilities of the algorithm. Sato et al. [130] proposed a multi-swarm differential evolutionary particle swarm optimization to address the problem of energy networks in a smart city. Ang et al. [131] introduced two evolution phases (the current swarm evolution and memory swarm evolution) in PSO to offer multiple operators to improve the robustness of the algorithm. In addition, there are some excellent algorithms based on multipopulation, a conscious neighborhood-based crow search algorithm (CCSA) [73], a hybridizing of WOA and moth–flame optimization algorithm (MFO) [132], a binary moth–flame optimization (B-MFO) [133], a migration-based moth–flame optimization (M-MFO) [134], an improved grey wolf optimizer (I-GWO) [135], a multi-trial vector-based differential evolution algorithm (MTDE) [136], an improved moth–flame optimization (I-MFO) [137], an elite opposition-based learning grasshopper optimization method (EOBL-GOA) [75], and other algorithms [25, 49,138–142]. Whereas, few researchers divided populations in SI algorithms according to different individual information dynamically to enhance the ability to exchange information and improve population diversity.

2.3. Orthogonal design

The principal goal of OD is to find the optimal combination of different parameters and levels to utilize fewer test times [15]. For an experiment with N factors and Q levels for each factor, there are Q^N experimental combinations, evidently the number of which grows exponentially. It is inefficient to find the optimal combination in this way because the number of combinations to be tested is excessively massive when the number of factors and levels is relatively large. Hence, OD is significant for optimization problems with various parameters. OD consists of the following two principal contents.

(1) Orthogonal array (OA): the original researchers constructed a mutually orthogonal Latin square by a graphical method, denoted as $L_M(Q^N)$, where L is the OA and M is the number of experimental combinations. J satisfies in Eq. (1). The method of OA is shown in Algorithm 1.

$$N \leq \frac{Q^J - 1}{Q - 1} \tag{1}$$

The OA with four factors and three levels per factor is shown as follows.

$$L_9(3^4) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 3 & 3 & 3 \\ 2 & 1 & 2 & 3 \\ 2 & 2 & 3 & 1 \\ 2 & 3 & 1 & 2 \\ 3 & 1 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 3 & 3 & 2 & 1 \end{bmatrix}$$

where 1, 2, and 3 denote three levels and each row represents one test and constitutes a total of nine combinations. The meaning of the OA, with the first row as an example, refers to a special experimental combination in which the level of the four factors is 1. The other rows have analogous meanings. There is a typical feature about OA, any sub-columns of an OA are also an OA. Therefore, arbitrary three columns in OA are used to construct a new three-factor OA.

(2) Factor analysis (FA): the FA estimates the effect of each level on each factor by utilizing the combinations of OA to determine the optimal level for each factor. The calculation process is presented as follows.

$$S_{ij} = \frac{\sum_{m=1}^{M} f_m \times \varnothing_{mij}}{\sum_{m=1}^{M} \varnothing_{mij}} \tag{2}$$

where f_m denotes the experimental result of the mth combination, and S_{ij} is the main effect of factor $i(i = 1, \ldots, N)$ with level $j(j = 1, \ldots, Q)$. The $\varnothing_{mij} = 1$ if the level of the ith factor of the mth combination is j; otherwise, $\varnothing_{mij} = 0$.

Algorithm 1 Construction of orthogonal array $L_M(Q^N)$

Input: N(factors), Q(levels for each factor).
Output: L(the orthogonal Array), M(the number of experimental combinations).

For $k = 1{:}J$ **do**
  $j = \frac{Q^{k-1}-1}{Q-1} + 1$.
  For $i = 1{:}Q^J$ **do**
    $p_{i,j} = \left(\frac{i-1}{Q^{j-k}} + 1\right) mod\ Q$.
  End For
End For
For $i = 2{:}J$ **do**
  $j = \frac{Q^{k-1}-1}{Q-1} + 1$.
  For $u = 1{:}j-1$ **do**
    For $v = 1{:}Q-1$ **do**
      $p_{j+(u-1)(Q-1)+n} = \left(p_{u\times v} + p_j\right) \bmod Q$.
    End For
  End For
End For
$p_{i,j} = p_{i,j} + 1, 1 \leq i \leq M$ and $1 \leq j \leq N$.

2.4. Motivation

The three problems mentioned earlier in Sections 1 and 2.1 about the WOA are critical factors that restrict the improvement of the algorithm. First, the defective local search is likely to be caused by Eq. (11) in Section 3.1. The encircling prey and bubble-net attacking mechanism are activated randomly because the parameter p has neither prior knowledge guidance nor a closed-loop learning mechanism. Besides, these two different mutation mechanisms are the alternation of two different neighborhood structures. The random search operator only offers noise of the distance on each dimension of x^{best} as lacking an effective neighborhood structure guiding. This is because the best searching neighborhoods of the dimensions are usually different [15]. Hence, it is difficult for whales to find a better prey location purposefully by relying on inferior exploitation in the local search stage. The best combination neighborhood structures are motivated urgently to guide the local search by observation.

Second, another problem in the local search phase is to only use the best individual information in the current population when constructing the offspring as shown in Eqs. (7) and (9) (Section 3.1). It is important to make better use of the information of partial solutions with inferior performance to weaken premature convergence. The problem is more obvious in multimodal function and hybrid function. Another mentioned problem regarding the construction of offspring is population diversity lost too quickly caused by randomly identified leading whales in Eq. (3) (Section 3.1). According to the original WOA, random leading whales are hardly utilized in the late iteration, because the condition of activating the operation is more and more difficult to be satisfied with the decreasing parameter A. This phenomenon will further reduce population diversity.

From the above observations, the multipopulation strategy is adopted to enhance the information exchange between different individuals and to promote population diversity. Different historical information and the best information are utilized to update each subpopulation. The best search operator in each dimension is predicted to enhance the exploitation ability efficiently by the OD strategy. From this perspective, the first and third disadvantages aforementioned in Section 1 are alleviated by the strategy. Simultaneously, OD is utilized to design the optimal neighborhood structures and to predict more potential search operators in each dimension of the individual. Another purpose of OD is to design a set of construction vectors that make efficient use of different information from multiple populations and obtain more useful information to guide the exploration and exploitation stage efficiently. These construction vectors are not only generated by the best individuals but also include the information of certain inferior individuals and even the information of the worst individuals, which alleviates the second problem mentioned above to a certain extent.

The notion of utilizing the multipopulation and OD strategy in this mechanism is not new to SI research but is not fully explored. Compared with other SI algorithms, WOA has more explicit local search and global search mechanisms. On the one hand, two local search mechanisms with different neighborhood structures are cleverly predicted by the OD strategy to select the optimal search scheme for each dimension of the individuals. Note that, certain research (e.g., artificial bee colony algorithm based on the OL [143], OLPSO [24], OL improved sine and cosine algorithm with multipopulation strategy (OMGSCA) [129]) focus on population updating utilizing the best solution information. Otherwise, the researchers proposed a brain storm optimization based on OL to enhance the learning efficiency and considered the benefits of the whole search space [15]. In this paper, the proposed framework not only uses the information of the whole search experience to guide the individual update but also predicts the neighborhood structure of the individual. On the other hand, in different subpopulations, the information of the leading whale not only spreads rapidly within the subpopulation but also exchanges widely among subpopulations. Finally, a decision mechanism is utilized to make the two mechanisms work cooperatively.

3. Proposed approach

3.1. The basic WOA

The basic operations of the WOA include three phases, searching for prey, encircling prey, and the bubble-net attacking method [10]. The three basic operations are repeated until a certain termination criterion (e.g., exhaustion of maximum functional evaluations) is satisfied, when the population of WOA is initialized. In the WOA, the population is a set of real value vectors $x_i = (x_1, \ldots, x_D)$, $i = 1, \ldots, N$, where D is the dimension of the objective function, and N is the size of the population. After the random initialization of the population, the following strategies are implemented.

(1) Searching for prey (exploration phase): The individual whales follow a random leader to widely find the prey over a whole area. The learning mechanism from random whales enhances the communication between different whales, emphasizes exploration, and allows the WOA to carry out a global search. The position vector of whales is updated with a random leader as follows:

$$x_{i,t+1} = x_t^{rand} - A_{i,t} \times D_{i,t} \tag{3}$$

$$D_{i,t} = \left| C_{i,t} \times x_t^{rand} - x_{i,t} \right| \tag{4}$$

where $x_{i,t}$ and $x_{i,t+1}$ represent i the individual solution at the tth iteration and the $(t+1)$th iteration, respectively, and x_t^{rand} is the random individual solution at the tth iteration. $D_{i,t}$ denotes the distance among the individuals ($x_{i,t}$) and the random leader (x_t^{rand}). $A_{i,t}$ and $C_{i,t}$ denote the coefficient vector at the tth iteration, which are calculated as:

$$A_{i,t} = 2a_t r_{i,t} - a_t \tag{5}$$

$$C_{i,t} = 2r_{i,t} \tag{6}$$

where $r_{i,t}$ is a random value in [0,1], and a_t is linearly dropped from 2 to 0 throughout the iterations.

(2) Encircling prey (exploitation phase): In contrast to the exploration phase, the position of an individual in the exploitation phase is updated to the optimum thus far instead of a randomly chosen search agent. The strategy of learning to the global optimum accelerates the convergence of the population and makes the population evolve to the optimal solution quickly. The best search agent (x_t^{best}) is defined by the current best candidate solution, then the other humpback whales recognize the location of the search agent and encircle it. The concise module updates the position of the population, and its structure is designed according to the formula directly as:

$$x_{i,t+1} = x_t^{best} - A_{i,t} \times D_{i,t} \tag{7}$$

$$D_{i,t} = \left| C_{i,t} \times x_t^{best} - x_{i,t} \right| \tag{8}$$

where x_t^{best} represents the best search agent at the tth iteration, and the $D_{i,t}$ denotes the distance among the individuals ($x_{i,t}$) and the best leader (x_t^{best}) at the tth iteration.

(3) Bubble-net attacking mechanism (exploitation phase): Another interesting point is the characteristic way of attacking prey, a bubble-net attacking mechanism. The positions of other whales are updated according to the helix-shaped movement of the best leader near the target position. The method is mathematically formulated as follows:

$$x_{i,t+1} = D'_{i,t} \times e^{b \times l_{i,t}} \times \cos\left(2\pi l_{i,t}\right) + x_{i,t}^{best} \tag{9}$$

$$D'_{i,t} = \left| x_t^{best} - x_{i,t} \right| \tag{10}$$

where $l_{i,t}$ is a random number inside [−1,1] and b denotes a constant. $D'_{i,t}$ represents the distance between the individuals ($x_{i,t}$) and the best leader (x_t^{best}) at the tth iteration.

Algorithm 2 Main procedures of the MCCWOA

1: Randomly initialize the positions of whales P^0.
2: Get the best whale x^{best}.
3: $iteration = 0$, $Spop_{elite} \leftarrow \emptyset$, $Spop_{middle} \leftarrow \emptyset$, $Spop_{infer} \leftarrow \emptyset$.
4: **While** $iteration < Max\_Iteration$ **do**
5: Calculate the fitness values in $P^{iteration}$.
6: **For** each individual $x_i^{iteration}$ **do**, set $trial_i = 0$ **End For**
7: Group $P^{iteration}$ into M clusters with centers $C = \{C_1, C_2, \ldots, C_M\}$ by k-means.
8: Sort individuals in different clusters in ascending order based on fitness.
9: Set the top fifth of individuals in each cluster as $Hpop_{elite}$.
10: Set the bottom fifth of individuals in each cluster as $Hpop_{infer}$.
11: Set the other three-fifths of individuals in each cluster as $Hpop_{middle}$.
12: **For** each cluster **do**
13: Update each cluster center and inferior cluster center by utilizing **Algorithm 3**.
14: Update other individuals by the Eq. (3).
15: Replace cluster center and inferior cluster center with the best individual and suboptimal individual.
16: **End For**
17: Evaluate the population $P^{iteration}$.
18: **For** each x_i **do**
19: **If** $x_i^{interation}$ is worse than $x_i^{interation-1}$ **then** $trial_i = trial_i + 1$
20: **Else** $trial_i = 0$ **End If**
21: **End For**
22: **If** $trial_i >$ T **then** update everyone x_i by utilizing **Algorithm 4**.
23: **End If**
24: Sort individuals in different clusters in ascending order based on fitness.
25: Set the top fifth of individuals in each cluster as $Spop_{elite}$.
26: Set the bottom fifth of individuals in each cluster as $Spop_{infer}$.
27: Set the other three-fifths of individuals in each cluster as $Spop_{middle}$.
28: Update $Spop_{elite}$, $Spop_{middle}$ and $Spop_{infer}$ by $Hpop_{elite}$, $Hpop_{middle}$ and $Hpop_{infer}$.
29: **End While**

Note that whales hunt in the above ways simultaneously. Hence, the researchers model the simultaneous behavior by the following formula:

$$x_{i,t+1} = \begin{cases} x_t^{best} - A_{i,t} \cdot D_{i,t}, & p < 0.5 \\ D'_{i,t} \cdot e^{b \times l_{i,t}} \cdot \cos\left(2\pi l_{i,t}\right) + x_{i,t}^{best}, & p \geq 0.5 \end{cases} \tag{11}$$

where p is inside (0, 1) and shows the chance of each strategy. Finally, the new individuals are evaluated and compared with the existing best agent (x_t^{best}), and the better one is kept as the new leader.

3.2. Main framework of the MCCWOA

The main framework of the MCCWOA is shown in Algorithm 2, where N individuals are randomly generated in Line 1 as the initial population. The procedures mainly consist of the following parts: (1) original WOA operations, including the prey searching (Line 14); (2) individual clustering (Line 7); (3) the first-stage OL (Line 13); (4) the second-stage OL (Line 14); (5) subpopulation updating (Lines 24–28). In all the above sections, two OL strategies at different stages are key operations to improve algorithm performance. By recording the historical population (Lines 9–11), the dominant individual information is preserved in the evolutionary process and the information exchange between subpopulations is strengthened.

In the first-stage OL, any cluster-center and inferior cluster-center approach the global best leader through two different neighborhood structures. The first-stage OL does not change two mutation operators of the local search but has guidance information when selecting the neighborhood structure to avoid losing the original evolutionary characteristics of the WOA. In the second-stage OL, a construction vector from the auxiliary vector pool is randomly chosen by stagnant individuals for information exchange to jump out of the stagnant state.

3.3. The clustering mechanism

The individuals with high similarity are clustered in the same cluster through the clustering mechanism to considerably enrich the diversity of clusters. In the iterative update process of the population, many costs are spent on updating and evaluating similar individuals, which leads to a slow convergence speed and easily falls into a local optimum. Particularly, this problem is more obvious in the later iterations. Different update strategies for the cluster-centers and other individuals in the clusters are adopted to develop the most promising cluster-centers more deeply. The original WOA searching mechanisms are utilized for non-cluster centers to explore other areas more widely. In more detail, only representative cluster centers and inferior cluster centers are selected for in-depth development since the internal similarity of each cluster is very high after clustering. Other individuals in the cluster are updated by operators with a strong exploration ability. Eq. (3) is an efficient operator for the WOA algorithm to jump out of the local optimum. Other individuals can move closer to it by randomly determining a leading whale. One motivation is to randomly determine the leader whales to update the other individuals with a large number and a high degree of similarity in each cluster and to increase the diversity of the population.

3.4. Auxiliary vectors pool

In the second-stage OL, a new exemplar is constructed by utilizing an auxiliary vector and the individuals whose stagnation times exceed the thresholds. If OL only utilizes a single fixed guidance vector (e.g., x^{best}), the evolution of the individual is likely to deteriorate as the number of iterations continues to increase. The reasons are as follows. On the one hand, the individuals to be updated are the local minimum point which has been stagnated for many generations. If the point wants to be jumped out of the local minimum, it must add a large disturbance instead of simply updating through a single guidance vector, so that the useful information is lost too quickly. On the other hand, a single guidance vector may be biased, and it is easy to lead the population into another local-optimum region. An auxiliary vector pool was constructed to escape the deep stagnation state. The auxiliary vector pool is defined as $x_{exe} = \{x_{exe1}, x_{exe2}, x_{exe3}, x_{exe4}\}$, and a detailed description is as follows.

(1) Convergence vector with a global best individual: $x_{exe1} = x^{best} - A \times D$, where x^{best} is the current best solution in the population, A denotes the coefficient vector by Eq. (5), and D denotes the distance among the stagnant individuals (x_i) and the global best leader (x^{best}) by Eq. (8). The vector utilizes the efficient shrinking mechanism in the local search to retain the information of the best individual.

(2) Convergence vector with cluster centers: $x_{exe2} = D' \times e^{bl} \times \cos(2\pi l) + C_m$, where C_m is the cluster center of the mth ($m = 1, \ldots, M$) cluster [as shown in Algorithm 2 Line 7], and D' denotes the distance among the stagnant individuals (x_i) and a random cluster center (C_m) by Eq. (12). b and l have the same definition as in Eq. (9). The information of the cluster center is utilized to make the stagnant individuals approach the optimal value in the form of the spiral curve.

$$D' = |C_m - x_i| \tag{12}$$

(3) Diversity vector with random individual information: $x_{exe3} = x^{rand} - A \times D$, where x^{rand} is the random individual, A has the same definition as one in x_{exe1}, and D denotes the distance among the stagnant individuals (x_i) and the random leader (x^{rand}) by Eq. (4). The vector assists the stagnant individuals in diverging and jumping out of the local optimum.

(4) Diversity vector with opposition-based learning (OBL): $x_{exe4} = rand \times (U - L) - x_i$, where U and L are the upper and lower bounds for solving the problems. This vector is designed to increase the diversity of the population.

3.5. The first-stage OL

The main procedures of the first-stage OL is shown in Algorithm 3. The goal of the first-stage OL is to select the best search neighborhood structure on each dimension of cluster centers and inferior cluster centers. One motivation is to offset the defective search by adjusting the search neighborhood structure. Eq. (11) is a random operator, that searches a blind neighborhood structure on each dimension of the cluster centers and inferior cluster centers by randomly activating one of the two local search operations. As a result, the exploitation ability is inferior in the WOA.

In Algorithm 3, each dimension of a cluster center or inferior cluster center is regarded as a factor in the OA [as shown in Section 2.3], therefore there are D factors, and each factor has two levels, representing two different search strategies. Then, the OA $L_M\left(2^D\right)$ with D factors and two levels per factor is constructed via Algorithm 1. The value of '1' for each level represents an encircling prey mechanism, and the value of '2' for each level represents a bubble-net attacking mechanism. The following search equation is given to illustrate the first-stage OL:

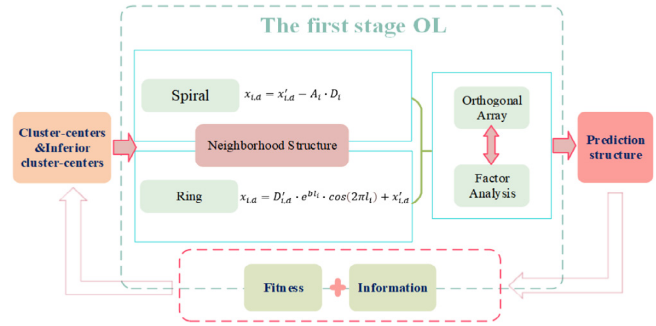


Fig. 1. The abridged general view of the first stage of OL.

$$x_{i,d} = \begin{cases} x'_{i,d} - A_i \times D_i, & L_M\left(2^D\right)_{jm} = 1(\text{spiral}) \\ D'_{i,d} \times e^{bl_i} \times \cos(2\pi l_i) + x'_{i,d}, & L_M\left(2^D\right)_{jm} = 2(\text{ring}) \end{cases} \tag{13}$$

where $L_M(2^D)_{jm}$ is the level value of the mth combination with the jth factor in the $L_M(2^D)$ OA, $x_{i,d}$ is the dth dimension of a cluster center or inferior cluster center x'_i for the mth combination.

Then, the level values corresponding to different factors were selected to make up a number $M = 2^{\lceil \log 2(D+1) \rceil}$ of the experimental combinations by Eq. (13) (Line 2 in Algorithm3). Next, the updated best individuals x_b is recorded and the predictive solution x_p with the best level on each factor is constructed (Lines 3–5). Finally, the new cluster center or inferior cluster center x_i is determined from x_b and x_p (Line 6), the process of the learning mechanism is shown in Fig. 1.

3.6. The second-stage OL

The prime goal is to assist x_i escape the local optimum by finding the optimal combination of x_i and x_{exe} at different levels in each dimension. Each of the D dimensions is regarded as a factor in the OA, therefore there are D factors, and each factor has two levels, i.e., 1 and 2, representing two different selected vectors (x_i and x_{exe}). There are $M = 2^{\lceil \log 2(D+1) \rceil}$ orthogonal combinations with D factors and two levels per factor in the OA (Line 1 in Algorithm 4). A straightforward approach is used to update x_i(Line 2 in Algorithm 4) by Eq. (14). Here, for each factor, if the level value is 1, the corresponding factor selects a stagnant vector (x_i); otherwise, selects auxiliary vector (x_{exe}).

$$x_{i,d} = \begin{cases} x_{i,d}, L_M\left(2^D\right)_{jm} = 1(\text{stagnant vector}) \\ x_{exe,d}, L_M\left(2^D\right)_{jm} = 2(\text{auxiliary vector}) \end{cases} \tag{14}$$

where $L_M(2^D)_{jm}$ has the same definition in the first-stage OL, $x_{i,d}$ is the dth dimension of a stagnation x_i for combination m, and $x_{exe,d}$ is the dth dimension of the selected auxiliary vector x_{exe}.

In detail, a two-level OA $L_M\left(2^D\right)$ with a number $M = 2^{\lceil \log 2(D+1) \rceil}$ of tested solutions is constructed by Algorithm 1 (Line 1). The exemplar from the auxiliary vector pool is utilized as x_{exe} and the current individuals whose stagnation times exceed the threshold are regarded as x_i(Line 2). Then, each candidate solution from OA is evaluated to record the best solution x_b (Line 3). The predictive solution x_p is generated according to the best level on each factor combination (Lines 4 and 5), and then the better solution between x_b and x_p is regarded as new x_i (Line 6). The process of the learning mechanism is shown in Fig. 2.

Algorithm 3 The first-stage OL

Input: C (cluster), x^{best} (current best whales).
Output: Cluster center and inferior cluster center.
1: Construct a $L_M(2^D)$ OA where $M = 2^{\lceil log_2(D+1)\rceil}$, following the procedures as shown in Algorithm 1.
2: Select the corresponding level value according to OA and design M experiment combinations. If the level value in OA is 1, the corresponding factor select $L_{id} =$ spiral; otherwise, select $L_{id} =$ ring.
3: Evaluate all updated individuals $x_j (1 \le j \le M)$ and record the best individuals x_b.
4: Calculate the effect of each level on each factor according to Eq. (2) and find the best level for each factor.
5: Construct the predictive solution x_p based on Step 4 and evaluate x_p.
6: Compare x_b and x_p, and select the better individual as a new cluster center or inferior cluster center.
7: **Return** cluster center and inferior cluster center.

Algorithm 4 The second-stage OL

Input: x_i(the individuals whose stagnation times exceed the threshold), x_{exe}(selected exemplar).
Output: x_i
1: Construct a $L_M(2^D)$ OA where $M = 2^{\lceil log_2(D+1)\rceil}$, following the procedures as shown in Algorithm 1.
2: Select the corresponding level value according to OA and design M experiment combinations. If the level value in the OA is 1, the corresponding factor select x_i; otherwise, select x_{exe}.
3: Evaluate all updated individuals $x_j (1 \le j \le M)$ and record the best individuals x_b.
4: Calculate the effect of each level on each factor according to Eq. (2) and find the best level for each factor.
5: Construct the predictive solution x_p based on Step 4 and evaluate x_p.
6: Compare x_b and x_p, and select the better individual as a new individual x_i.
7: **Return** x_i.

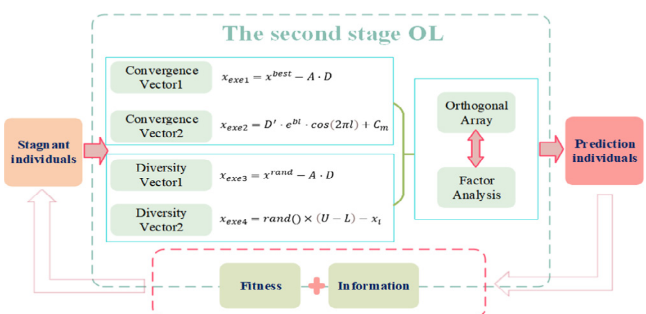


Fig. 2. The abridged general view of the second stage of OL.

3.7. Multipopulation mechanism

The main procedures of the multipopulation mechanism are given in Algorithm 2 (Lines 9–11 and 25–28). First, the population is divided into three subpopulations according to the fitness value after clustering, including the elite population, intermediate population, and inferior population. The elite population contains the top fifth individuals with the best fitness value in each cluster, and the inferior population includes the bottom fifth of the individuals with the worst fitness in each cluster; naturally, the other three-fifths of the individuals are assigned to the intermediate population (Lines 9–11). The divided population is set as the corresponding initial historical populations ($Hpop_{elite}$, $Hpop_{infer}$, and $Hpop_{middle}$) to guide the next generation of individuals to update. When the population is updated for one generation, the individuals in each cluster are sorted, and the population is divided according to similar methods ($Spop_{elite}$, $Spop_{infer}$, and $Spop_{middle}$) in Algorithm 2 (Lines 24–27).

Next, the subpopulations are updated by the corresponding historical populations in Eq. (15) (Line 28 in Algorithm 2),

$$x^*_{i,t+1} = \begin{cases} x^*_{i,t} + w_g \times \left(x^{best}_{i,t} - x^*_{i,t}\right), Q^t_E \le \theta \times Q^0_E \\ x^*_{i,t} + w_c \times \left(x^*_{rand,t} - x^*_{i,t}\right) \\ +w_c \times \left(x^{H*}_{rand1,t} - x^{H*}_{rand2,t}\right), otherwise, \end{cases} \tag{15}$$

where w_g and w_c follow Cauchy distribution and Gaussian distribution respectively. In the exploration and exploitation stage, different mutation operators can be used in global search and local search to improve the performance of the algorithm. Yao et al. [144] pointed out that the Cauchy distribution is more likely to produce a child far from the parent than the Gaussian distribution, which will increase the diversity of the population. In addition, the Cauchy distribution has a longer and flatter tail than the Gaussian distribution, which will increase the search range and perform better in the exploration of the algorithm. Through the above analysis, the Cauchy distribution random number w_c is used in the exploration phase of the MCCWOA to improve the ability of global search, and the Gaussian distribution random number w_g is used in the exploitation phase of the MCCWOA to enhance the ability of the local search. Additionally, in Eq. (15), x^*_{rand} is a random individual from the three subpopulations ($Spop_{elite}$, $Spop_{infer}$, or $Spop_{middle}$), x^{H*}_{rand1} and x^{H*}_{rand2} are different random individuals from the three historical populations ($Hpop_{elite}$, $Hpop_{infer}$, or $Hpop_{middle}$), the $x^{best}_{i,t}$ is the best leader. The Q^t_E is a quasi-entropy which was proposed by Cao et al. [145] to characterize the diversity of the population, and θ is the reduction ratio of Q_E.

The algorithm enters the exploitation stage when the fitness values of a considerable number of individuals in the population are almost the same. According to the concept of pseudo-entropy [145], this paper proposes a similar method to judge whether the algorithm has entered the exploitation stage. The specific approach is as follows. Assuming the total number of individuals in the current population is N, individuals are divided into two categories. The first category satisfies the following relationship.

$$f(x_1) = f(x_2) = f(x_i) = \cdots = f(x_{k-1}) = f(x_k) \tag{16}$$

where $f(x_i)$ is the fitness function value of the ith individual ($i = 1, 2, 3, \ldots, N, 1 \leq k \leq N$). The remaining $(N-k)$ individuals belong to the second category. Therefore, the entropy Q_E of a population consists of two parts, which are denoted as Q_{E1} and Q_{E2}, respectively.

$$Q_E = \underbrace{(-\sum_{i=1}^{k} P_i \cdot \log P_i)}_{Q_{E1}} + \underbrace{(-\sum_{i=k+1}^{N} P_i \cdot \log P_i)}_{Q_{E2}} \tag{17}$$

For individuals in the first category, $P_i = \frac{\sum_{i=1}^{k} x_i}{\sum_{i=1}^{N} x_i}$, the second category is $P_i = \frac{\sum_{i=k+1}^{N} x_i}{\sum_{i=1}^{N} x_i}$. For the initial population, the value of Q_E may only depend on Q_{E2}, because the fitness values corresponding to each individual in the initial population are generally not equal. The value of Q_E is relatively large currently. However, as the number of iterations of the algorithm increases, the Q_E value will gradually decrease because the fitness values of a considerable number of individuals in the population are already completely equal at the later stage of the evolution of the algorithm. Supposing the entropy of the initial population is Q_E^0, and the entropy of the population after t iteration is Q_E^t. The algorithm enters the exploitation phase when the condition $Q_E^t \leq \theta \times Q_E^0$ is satisfied in Eq. (15). The value of θ is set at 0.5 according to the previous experiment.

3.8. Complexity analysis

As shown in Algorithm 2, the population size N is clustered into M clusters (Line 7). The computational complexity of this task is $O(N \times D \times M)$, where D is the number of variables. The sorting population (Line 8) has a computational complexity of $O(N)$. In cluster updating, the construction of an OA needs $O(D)$ to replace each cluster-center and inferior cluster-center (Line 1 in Algorithm 3). Next, the evaluation of the effect of each level on each factor (Line 4 in Algorithm 3) holds $O(D \times M)$, where M is the number of combinations in OA. The replacement of other individuals in each cluster requires a computation complexity $O(N \times D)$; therefore, the computation complexity of the first stage of OL is $O(N \times D)$. The second stage of OL is triggered only when the stagnation times of individuals exceed the threshold (Line 22 in Algorithm 2), and the computational complexity of this task is also $O(N \times D)$. The computational complexity of collaborative updating among the three subpopulations is $O(N \times D)$. In summary, the computational complexity of the MCCWOA is $O(N \times D \times M)$.

4. Experiments and comparisons

In this study, the performance of the proposed algorithm and the comparison algorithms are evaluated on a set of benchmark functions from CEC2017 [146]. These are 29 diverse and difficult minimization benchmarks in total with dimensions $D = 10$, 30, 50, and 100. Particularly, these benchmarks also include a challenging class of functions, the composition functions ($f_{21} - f_{30}$), which exhibit four unique properties: multi-modal, non-separable, asymmetrical, and with different properties around different local optima. To ensure the accuracy and fairness of the experiment, MCCWOA and other algorithms are executed using the same maximum number of function evaluations ($Max\_FEs = D \times 10000$) as the termination conditions. On the one hand, all the algorithms are run 51 times independently under the same experimental environment. The termination conditions are provided in the CEC2017 test suite. The experiments are run in the Windows Server 2016 Datacenter under the hardware environment of Intel (R) Xeon (R) Gold 5115 CPU @ 2.40 GHz processor and 16.0 GB of RAM. The programming language is MATLAB 2016b. On the other hand, the parameter settings of all comparative methods are used as the corresponding suggested values of the original papers.

In Section 4.1, the parameter settings are described. The roles of the two-stage OL and the multipopulation mechanism are verified separately in Section 4.2. In Section 4.3, the proposed MCCWOA is compared with several basic WOA variants and representative algorithms.

4.1. Calibration of the parameters

The design of experiments (DOE) approach is employed to analyze the control parameters on the impact of the proposed MCCWOA in detail. In this study, the parameters of the local search are self-learning through the first stage of OL. In the exploration stage, the parameters are self-adapting at the beginning of the iteration. Furthermore, the performance of the proposed algorithm is affected by three numerical parameters: (1) population size N, (2) stagnation threshold T, (3) the number of clusters M. In the DOE approach, the potential values (levels) of each parameter (factor) are determined according to certain preliminary experiments and the relevant literature [1]. The final levels chosen for the factors are: $N = \{30, 50, 70, 100, 120\}$, $T = \{10, 15, 20, 25, 30\}$, and $m = \{3, 5, 7, 10, 12\}$. There are $5 \times 5 \times 5 = 125$ different parameter combinations in total. A full factorial design is considered to calibrate the parameters and analyze the influence of different parameters on the performance of the MCCWOA. All of the 125 combinations above have the same termination criterion as *Max_FEs*. Each combination is run 5 times independently on 29 10-dimensional and 30-dimensional functions. In summary, there are $125 \times 29 \times 2 \times 5 = 36250$ results in this section. The experimental results are investigated via multifactor analysis of variance (ANOVA), which demonstrates whether interactions between the parameters are significant [147].

Given space limitations, the 36 250 results are not fully listed in this paper. The representative 25-combination test results are shown. The 25 combinations are selected via the Taguchi method of design [148]. The average error values yielded by the MCCWOA are listed in Table 2. The average error values (AVE) and total average error values (TAVE) show the performance of each combination. The results of ANOVA are shown in Table 3. In ANOVA, the F-ratio is an indicator that reflects the importance of each parameter when the p-value is close to zero. A large F-ratio indicates that the analyzed factor has a considerable effect on the response variable.

As observed in Table 3, all of the parameters significantly affect the performance of the proposed algorithm because the three p-values are smaller than 0.05. Among these three parameters, the parameter N yields the greatest F-ratio, which indicates that it has an important effect on the response variable. The main effects plots of all parameters are shown in Fig. 3. As observed in Fig. 3, $N = 100$ obtains good performance, and $N = 30{,}120$ yields the worst results. On the one hand, a large value of N leads to a great similarity between individuals, and the algorithm costs many costs to make the whale population search the same location, which makes the algorithm easily trapped in the local optimum and decreases its exploitation ability. On the other hand, a small value of N makes the clustering mechanism unable to divide the individuals carrying different information. As a result, the multipopulation mechanism does not make a real contribution to the MCCWOA, which reduces the exploration ability of the

Table 2
Parameter combinations and average error values.

No.	Parameter combinations			AVE		TAVE
	N	T	M	$D = 10$	$D = 30$	
1	30	30	3	8.9205E+01	3.7059E+02	2.2990E+02
2	30	25	5	9.2443E+01	4.0474E+02	2.4859E+02
3	30	20	7	9.3096E+01	4.0474E+02	2.4892E+02
4	30	15	10	9.3815E+01	4.0887E+02	2.5134E+02
5	30	10	12	9.3997E+01	4.1006E+02	2.5203E+02
6	50	30	5	8.8131E+01	3.7059E+02	2.2936E+02
7	50	25	7	7.9665E+01	3.2669E+02	2.0318E+02
8	50	20	10	8.2075E+01	3.3759E+02	2.0983E+02
9	50	15	12	8.4479E+01	3.4840E+02	2.1644E+02
10	50	10	3	8.9027E+01	3.6957E+02	2.2930E+02
11	70	30	7	7.8943E+01	3.3759E+02	2.0827E+02
12	70	25	10	7.7952E+01	3.3150E+02	2.0473E+02
13	70	20	12	7.6684E+01	3.3009E+02	2.0339E+02
14	70	15	3	7.7091E+01	3.3324E+02	2.0517E+02
15	70	10	5	7.7947E+01	3.4158E+02	2.0976E+02
16	100	30	10	7.8471E+01	3.2961E+02	2.0404E+02
17	100	20	12	7.8017E+01	3.2014E+02	1.9908E+02
18	100	25	3	7.7012E+01	3.1061E+02	1.9381E+02
19	100	15	5	7.7018E+01	3.1107E+02	1.9404E+02
20	100	10	7	7.8890E+01	3.1993E+02	1.9941E+02
21	120	30	12	7.9056E+01	3.2261E+02	2.0083E+02
22	120	25	3	7.9618E+01	3.3019E+02	2.0490E+02
23	120	20	5	8.2877E+01	3.5107E+02	2.1697E+02
24	120	15	7	8.2029E+01	3.4124E+02	2.1163E+02
25	120	10	10	8.1890E+01	3.2961E+02	2.0575E+02

Table 3
Results of ANOVA for parameters calibration of MCCWOA.

Source	Sum of squares	Degrees of freedom	Mean square	F-ratio	p-value
N	53.122	2	18.562	690.34	**0.0000**
T	2.447	2	0.186	26.72	**0.0000**
M	0.760	4	1.020	46.89	**0.0000**
$N*T$	0.282	5	0.074	0.73	0.5702
$N*M$	0.330	6	0.056	0.60	**0.0004**
$T*M$	0.406	6	0.098	0.74	0.0508
Error	2013.7	36 190	0.095		
Total	2558.1	36 220			

algorithm. As also seen in Table 3, the parameter M ranks second. Meanwhile, it is observed from Fig. 3 that $M = 3$ achieves the best performance, and $M = 5$ yields the worst results. A small value of M reflected the significance of the clustering mechanism dividing the population into more characteristic clusters, which is conducive to the advantage of the first-stage OL. From Table 3, the parameter T is the last significant parameter. As seen in Fig. 3, $T = 25$ achieves the best performance of the proposed algorithm, and $T = 10$ leads to the worst results. The opportunity to switch between exploration and exploitation is determined via the value of T. A small value of T causes the second-stage OL of the MCC-WOA to be activated prematurely. Thereby, the diversity of the population will decrease accordingly in Algorithm 2. Conversely, if the value of T is too large, the second-stage OL is activated too late to restrict the exploitation ability of the algorithm. Although, the calibration experiment looks for the balance between the two to realize the best performance of the MCCWOA.

However, if there are significant interactions between parameters, the main effects plot is not meaningful [147]. Table 3 shows that the $N * M$ interaction is significant since the $p-$ value is less than the $\alpha = 0.05$ level. Hence, the 2-level interaction between the factors is investigated to examine the interaction plot whether it is satisfied with the judgments in the main effects plot. The interaction plot for its interaction is depicted in Fig. 4. From this figure, it can be seen that the results of Fig. 4 are in accordance with the conclusions above. Hence, according to the experiment and analyses above, the suggested parameters are summarized as follows: $N = 100$, $T = 25$, and $M = 3$.

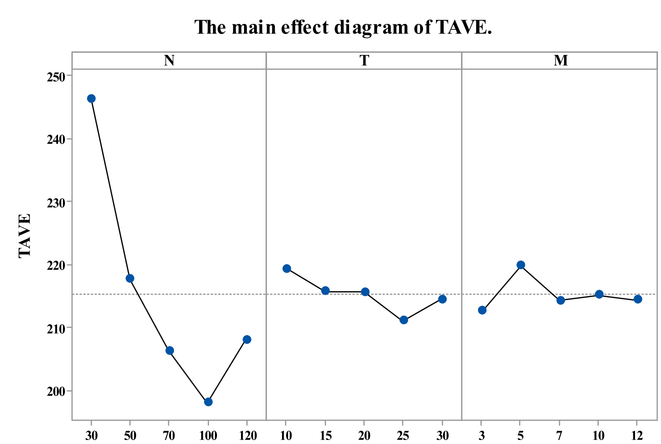


Fig. 3. The tendency of parameters of the MCCWOA.

4.2. Effect of each improvement on MCCWOA

Each main improvement (strategy) of the performance in MC-CWOA is verified in this section. The solutions obtained by MCC-WOAs have contrasted with those without the multipopulation strategy or OL strategies on four kinds of functions (unimodal functions, multimodal functions, hybrid functions, and composition functions) with $D = 10$ as shown in Table 4. The MCCWOAs refer to complete MCCWOA (with two-stage OL strategy and

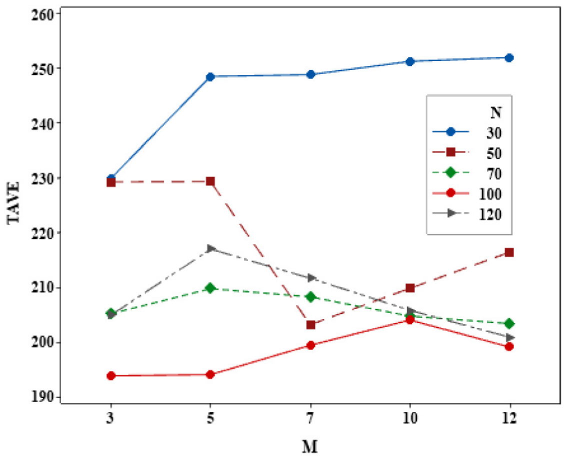


Fig. 4. Interaction plots of N and M.

multipopulation strategy), MCCWOA-M (only with multipopulation strategy), MCCWOA-F (only with the first-stage OL strategy), and MCCWOA-S (only with the second-stage OL strategy). For a clear comparison of the experimental results, the visualization of the experimental results is shown in Fig. 5. The horizontal axis is the number of test functions, and the vertical axis is the normalized value of the mean. If the visualization line was lower than the others, the performance of the algorithm was better than the compared algorithm. The MCCWOA is compared with four algorithms from Table 4 and Fig. 5, and the following observations are obtained.

(1) MCCWOA displays a clear performance advantage over WOA in terms of the mean and standard values on many functions. In particular, the MCCWOA has an order of magnitude advantage over the MCCWOAs on unimodal functions and multimodal functions.

(2) MCCWOA-F performs nearly equivalently to MCCWOA-S. In most instances, these two variants show similar mean and standard values, but they are much better than WOA. MCCWOA-M shows outstanding performance on almost all the test functions. On the other hand, MCCWOA still outperforms these three variants on most of the test instances. These observations show that both the OL and multipopulation strategies engendered a positive impact on the performance improvement of WOA.

The series of experiments are conducted on four kinds of test functions in 30 dimensions to further investigate how the three strategies affect the exploration and exploitation of the WOA. The convergence speed and diversity of the population are contrasted on the MCCWOAs and WOA. The population deviation is denoted as Eq. (18) [149].

$$diversity = \frac{1}{N \times L} \times \sum_{i=1}^{N} \sqrt[2]{\sum_{j=1}^{D} (p_{ij} - \overline{p_j})^2} \tag{18}$$

where N is the size of the population, L is the length of the longest diagonal in the searching space, D is the dimensionality of the problem, p_{ij} is the jth value of the ith particle and $\overline{p_j}$ is the jth value of the average point $\overline{p}$.

The dispersion level of the population is demonstrated via the variations in diversity. A larger diversity indicates that the population has higher diversity, which is beneficial for exploration capability. In contrast, a smaller diversity shows that the population has closely converged to a certain region, which is conducive to exploitation ability. The comparison results of the MCCWOAs and WOA on four kinds of test functions are plotted in Fig. 6 in terms of convergence speed and diversity changing trend.

It is observed that MCCWOAs have more promising performance on maintaining the diversity of the population than WOA on the four kinds of test functions from Fig. 6. Additionally, MCCWOAs demonstrate more favorable performance than the WOA on all functions in terms of solution accuracy. The convergency speed of MCCWOAs is not as good as WOA in the early search stage on f_1, f_6, and f_{16}. However, within the standard cutoff criterion specified by the CEC2017 test function, the accuracy of MCCWOA is significantly better than that of the WOA. This property is attributed to the multipopulation with the two-stage OL. Furthermore, the comparison results among the MCCWOA-M, MCCWOA-F, and MCCWOA-S indicate that the second-stage OL strategy shows promising diversity of the population to improve the exploration of the algorithm. MCCWOA-F and MCCWOA-S converge to a global optimal solution with a higher speed than MCCWOA-M on the four kinds of functions. The phenomena validate the hypothesis that the two-stage OL strategy has a pleasurable performance for improving the convergence speed of the algorithm.

From the comparison, it is observed that the population diversity of MCCWOA-F falls much more rapidly than that of MCCWOA-S and MCCWOA-M on the unimodal function and multimodal function. Furthermore, MCCWOA-F also achieves more accurate solutions than MCCWOA-M on these kinds of functions. Nevertheless, MCCWOA not only has a higher convergence speed but also achieves more accurate solutions than those found with MCCWOA-M, MCCWOA-F, and MCCWOA-S on all kinds of functions. The results verify that the first stage OL operator can indeed help a particle detect a more promising position via the prediction model of the neighborhood structure when the swarm has been trapped in a local optimum. On the other hand, the information-sharing capability of the population is improved by the multipopulation strategy.

The findings of the feasibility analysis of the test functions are shown in Fig. 7. In the figure, row (a) and row (b) demonstrate the three-dimensional location distribution and two-dimensional location distribution of MCCWOA and WOA respectively. The plots of the three-dimensional location distribution are the final test results of each function. The plots of the two-dimensional location distribution correspond to one, two, three, and four quarters of the maximum evaluation times. The green stars are utilized to characterize the position of the optimal solutions. The current solutions are shown by the red dots. It is well known that unimodal functions are suitable to substantiate the exploitation powers of methods because they have only one best solution. Compared with unimodal functions, there are a large number of local optimizations in multimodal cases, and they may increase with increasing dimensionality. Hence, the multimodal functions are utilized to test the explorative ability of an algorithm and the ability to jump out of local optimizations. In Fig. 7(a), it is observed that the individual trajectory fluctuates strongly in the first half of the search due to the stable exploratory behavior of MCCWOA. The trend of the individual trajectory implies that MCCWOA can reach most of the search space. On the three-dimensional location distribution of multimodal functions ($f_4 - f_{10}$), the red dots of MCCWOA cover more search space than that of WOA. Beyond that, the red dots of MCCWOA on f_4 and f_7 are coincident with the green star. Contrary to the MCCWOA, the red dots of WOA on these functions differed significantly from the green star (optimal solution). This shows that MCCWOA does not drop into local optimality and obtains a better solution. In summary, the observed results suggest that the improvement of WOA is very effective. MCCWOA has significantly improved exploration ability and exploitation ability compared with WOA.

Table 4
Result of the Mean and Std. values obtained by WOA, MCCWOAs, and MCCWOA.

Fun	WOA		MCCWOA-M		MCCWOA-F		MCCWOA-S		MCCWOA	
	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
f_1	5.25E+03	6.69E+03	**0.00E+00**	0.00E+00	5.94E−03	4.41E−03	2.15E−02	5.06E−03	**0.00E+00**	0.00E+00
f_3	1.10E+01	2.36E+01	1.03E−05	6.02E−08	1.06E−07	9.18E−08	6.54E−07	3.38E−07	**0.00E+00**	0.00E+00
f_4	2.18E+01	3.18E+01	0.00E+00	0.00E+00	7.15E−01	5.23E−01	5.51E−01	1.10E−01	**0.00E+00**	0.00E+00
f_5	5.56E+01	2.06E+01	3.73E+00	4.48E−01	9.29E+00	4.02E+00	7.98E+00	1.70E+00	**3.30E+00**	1.09E+00
f_6	3.89E+01	1.28E+01	**0.00E+00**	0.00E+00	5.05E−04	2.54E−04	6.02E−04	7.92E−05	**0.00E+00**	0.00E+00
f_7	7.92E+01	2.27E+01	**1.28E+01**	9.86E−01	1.59E+01	9.27E−01	1.55E+01	7.16E−01	1.31E+01	1.24E+00
f_8	4.37E+01	1.69E+01	3.73E+00	4.59E−01	8.29E+00	2.07E+00	5.97E+00	9.95E−01	**3.15E+00**	1.27E+00
f_9	5.78E+02	4.37E+02	**0.00E+00**	0.00E+00	7.14E−10	2.98E−10	9.90E−10	9.60E−11	**0.00E+00**	0.00E+00
f_{10}	1.18E+03	2.72E+02	**3.10E+01**	1.72E+01	4.96E+02	1.34E+02	4.18E+02	2.21E+02	5.74E+01	6.14E+01
f_{11}	8.99E+01	4.75E+01	**5.01E−04**	8.67E−04	2.16E+00	1.06E+00	3.74E+00	6.32E−01	5.85E−02	2.36E−01
f_{12}	2.53E+04	2.17E+04	**2.29E−01**	1.40E−01	2.70E+02	6.83E+01	1.02E+02	5.45E+01	2.44E+00	1.66E+01
f_{13}	1.22E+04	1.04E+04	9.69E−01	4.11E−01	1.38E+01	5.23E+00	1.80E+01	6.39E+00	**5.95E−01**	1.11E+00
f_{14}	1.57E+02	1.10E+02	**0.00E+00**	0.00E+00	1.86E+01	4.64E+00	1.72E+01	5.20E+00	1.59E−07	1.13E−06
f_{15}	1.23E+02	9.44E+01	**9.37E−04**	7.39E−04	2.47E+00	4.09E−01	3.53E+00	1.55E+00	1.02E−03	1.99E−03
f_{16}	2.96E+02	1.66E+02	2.22E−01	5.10E−02	3.09E+00	9.43E−01	4.17E+01	6.75E+01	**9.87E−02**	9.59E−02
f_{17}	1.33E+02	7.74E+01	2.63E−01	1.10E−01	1.36E+01	4.61E+00	1.40E+01	8.11E+00	**2.61E−01**	3.44E−01
f_{18}	7.81E+03	1.02E+04	**7.19E−04**	6.37E−05	7.40E+00	2.91E+00	1.04E+01	8.28E+00	2.54E−03	1.32E−02
f_{19}	1.00E+04	1.03E+04	**5.35E−04**	8.65E−04	2.19E+00	3.62E−01	2.47E+00	1.22E+00	9.78E−04	4.01E−03
f_{20}	1.59E+02	8.06E+01	**0.00E+00**	0.00E+00	1.69E+01	8.42E+00	1.05E+00	9.78E−01	**0.00E+00**	0.00E+00
f_{21}	2.15E+02	5.28E+01	1.00E+02	2.63E−13	1.71E+02	6.13E+01	1.35E+02	6.07E+01	**9.22E+01**	2.72E+01
f_{22}	1.35E+02	2.74E+01	4.01E+01	5.29E+01	1.01E+02	5.63E−01	1.01E+02	5.22E−01	**2.96E+01**	3.37E+01
f_{23}	3.65E+02	2.55E+01	3.07E+02	2.40E+00	3.06E+02	2.15E+00	3.10E+02	1.10E+00	**2.76E+02**	8.97E+01
f_{24}	3.86E+02	8.99E+01	**7.11E+01**	5.00E+01	2.54E+02	1.34E+02	3.35E+02	1.33E+00	9.11E+01	2.78E+01
f_{25}	4.36E+02	5.17E+01	3.13E+02	1.64E+02	3.98E+02	3.22E−01	4.28E+02	2.64E+01	**3.03E+02**	1.29E+02
f_{26}	8.90E+02	4.48E+02	1.67E+02	1.53E+02	3.00E+02	1.75E−06	3.00E+02	1.13E−05	**8.44E+01**	9.67E+01
f_{27}	4.76E+02	3.60E+01	**3.89E+02**	1.68E+00	3.95E+02	2.36E+00	3.95E+02	1.70E+00	**3.89E+02**	1.57E+00
f_{28}	4.82E+02	1.29E+01	3.00E+02	0.00E+00	4.06E+02	1.83E+02	3.39E+02	6.68E+01	**1.77E+02**	1.44E+02
f_{29}	4.84E+02	1.37E+02	2.41E+02	4.40E+00	2.54E+02	5.55E+00	2.66E+02	2.40E+01	**2.35E+02**	9.04E+00
f_{30}	4.45E+03	1.00E+04	4.04E+02	9.23E+00	4.94E+02	5.51E+01	4.98E+02	4.60E+01	4.06E+02	1.31E+01

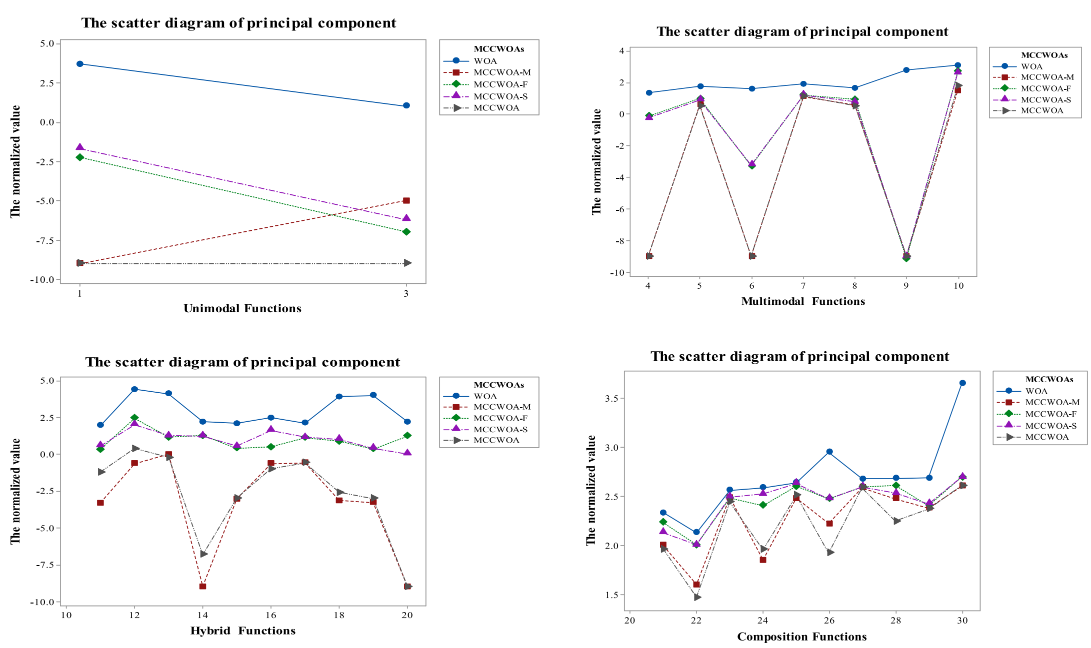
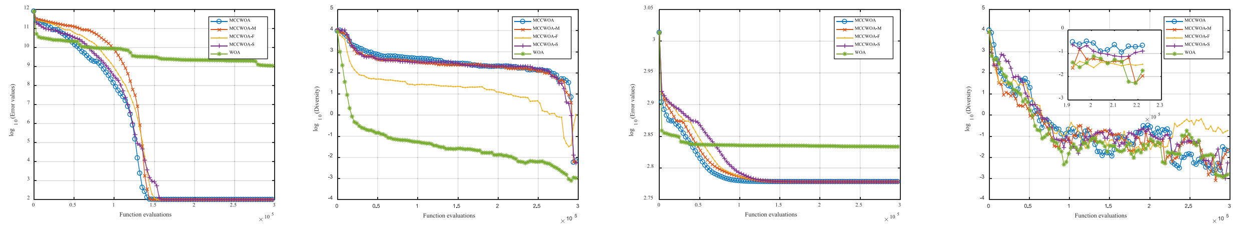


Fig. 5. The visualization of MCCWOA and MCCWOAS results.

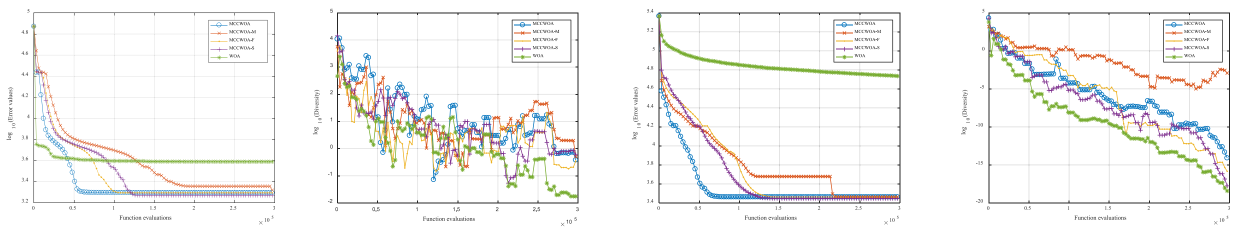
In conclusion, compared with any of the incomplete algorithms of MCCWOA, MCCWOA has better optimization performance. Hence, MCCWOA can effectively maximize the advantages of different strategies to overcome certain problems of WOA and obtain the best optimization performance. This also means that all the main improvements in this paper are effective and indispensable and that MCCWOA can obtain a better balance between exploration and exploitation.

On the one hand, it is expected that using the multipopulation strategy and the second-stage OL strategy in the MCCWOA increases the diversity and opportunity of finding the global optimum position by exploring undiscovered regions. On the other hand, the MCCWOA efficiently exploits around a promising area through the first-stage OL strategy. This experiment analyzes the exploration and exploitation abilities of the MMCCWOA to solve the hybrid functions.

Different from the diversity measurement method in Eq. (18), a dimension-wise diversity measurement approach [73,74,76] was used to analyze the impact of the exploration and exploitation. The whales are scattered and explores the search space when the distance value within the dimension was large. In



(a) f_1-fitness (unimodal function) (b) f_1-diversity (unimodal function) (a) f_6-fitness (multimodal function) (b) f_6-diversity (multimodal function)



(a) f_{16}-fitness (hybrid function) (b) f_{16}-diversity (hybrid function) (a) f_{26}-fitness (composition function) (b) f_{26}-diversity (composition function)

Fig. 6. Comparisons of convergence speed and diversity of the strategies on the four kinds of test functions.

contrast, the whales are placed close to each other for converging to a condensed area when the value is reduced. The dimension-wise diversity of the MCCWOA in the current iteration t (diversity (t)) is calculated by Eq. (19),

$$Div(t) = \frac{1}{D}\sum_{d=1}^{D} Div_d \tag{19}$$

where the Div_d is the diversity of the dth dimension in Eq. (20).

$$Div_d = \frac{1}{N}\sum_{i=1}^{N} median(x_d) - x_{id} \tag{20}$$

where the x_{id} is the value of the dth dimension of whale x_i, and the $median(x_d)$ is the median value of the dth dimension for all N whales. The percent of the exploration and exploitation abilities is calculated by Eqs. (21) and (22),

$$Exploration_t(\%) = \frac{Div(t)}{Div_{max}} \times 100 \tag{21}$$

$$Exploitation_t(\%) = \frac{|Div\ (t) - Div_{max}|}{Div_{max}} \times 100 \tag{22}$$

where the Div_{max} is the maximum diversity found by Eq. (19) in all iterations, $Exploration_t$ and $Exploitation_t$ represent the measurement in the tth iteration. Fig. 8 shows the obtained results of exploration and exploitation abilities of the MCCWOA in each iteration for the hybrid function (f_{17}) on different dimensions. Due to space constraints, all the test results are not displayed. The most representative function hybrid function (f_{17}) is selected to analyze the results. The function (f_{17}) is composed of five basic functions defined in CEC2017 [150], which has multimode properties. The curves plotted in Fig. 8 prove that the MCCWOA has an outstanding exploitation ability to solve f_{17} from hybrid group functions with $D = 10, 30, 50, 100$. In the initial iterations, the MCCWOA mostly explores the search space by the second-stage OL strategy and the multipopulation strategy with the clustering mechanism because the individual whales from different populations can find a promising area by increasing cooperation in information sharing. Then, it gradually exploits some whales located in the promising area. Finally, many whales are highly exploited by the first-stage OL strategy in the final iterations. This behavior proves the exploitation ability of the MCCWOA.

There are many local optima in multi-modal functions (e.g. f_5, f_8, and f_{10} in Fig. 7) which is essential for evaluating the exploration behavior. As observed in Fig. 8, the results on function f_{17} show that the percentage of exploration of the MCCWOA is more than its exploitation in the early iterations. Another explicit phenomenon is that with the increase of the dimensions, the exploration of the MCCWOA gradually increases, especially during the later stage of iteration. That is precisely because the local optima in multi-modal functions exponentially increase when the dimension is increased. Thus, in the later stage of the algorithm iteration, the ability of exploration is improved to find the promising areas. From the perspective of the MCCWOA, this is due to the multipopulation strategy with the clustering mechanism that moves the whales from inferior regions, while the second-stage OL strategy also increases the diversity and corrects the whales trapped in the local optima. This behavior proves that the MCCWOA can keep a balance between exploration and exploitation since the MCCWOA can select the different search strategies consciously by the framework of cooperative coevolutionary.

4.3. Comparison results with other promising algorithms

The 15 state-of-the-art algorithms, including the initial WOA, 10 variants of WOA (LWOA [119], RDWOA [114], BMWOA [11], EWOA [1], ACWOA [111], CCMWOA [151], HWGO [152], WOAmGWO [153], BWOA [154], and HWOAG [121]), two promising OL-based algorithms (OLPSO [24] and OLBSO [15]), and other algorithms (GWO [62] and CMA-ES [155]) are considered for comparison with the proposed method to further confirm the effectiveness of MCCWOA. The parameters of the above algorithms are set according to the corresponding original literature. The simple description and specific parameter settings of the comparison algorithms are presented to in Table 5. As observed in Table 5, the comparison algorithms are representative and comparable since some of the algorithms are variants of the WOA, and others are promising OL algorithms.

The mean value (Mean) and standard deviation (Std) metrics of MCCWOA and the compared algorithms are adopted as the evaluation criteria in Tables 6 to 9. Mean and Std represent the optimization ability and the stability of an algorithm respectively. For the results, the smaller the mean value is, the higher the ranking of the algorithm. If the Mean values of the two algorithms are the same, their Std values are compared. The best entries are in bold. Furthermore, the visualization of the experimental results is shown in Fig. 9. The horizontal axis is the number of test functions and the vertical axis is the normalized values of the Mean. If the

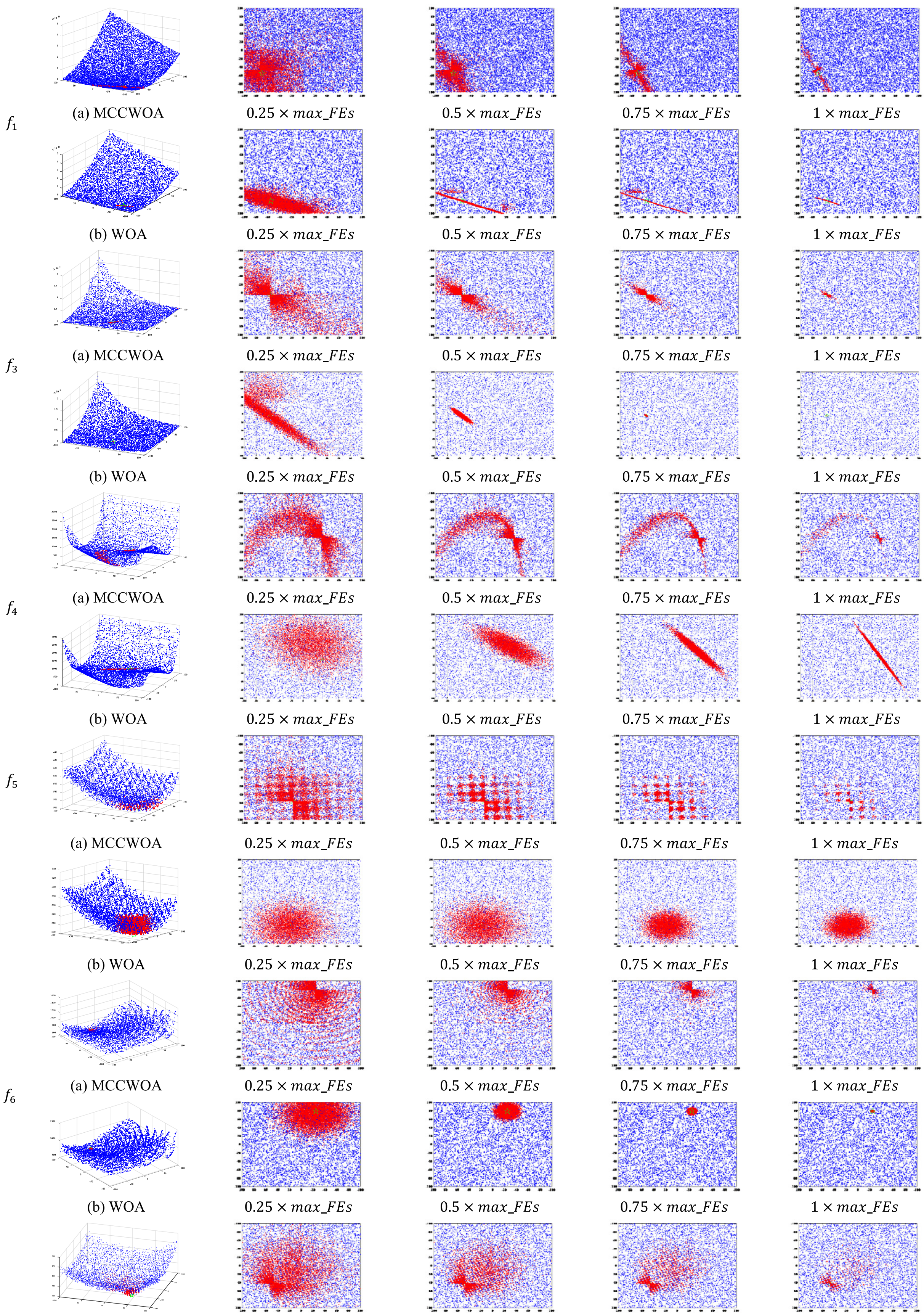


Fig. 7. Three-dimensional location distribution and two-dimensional location distribution of MCCWOA and WOA.

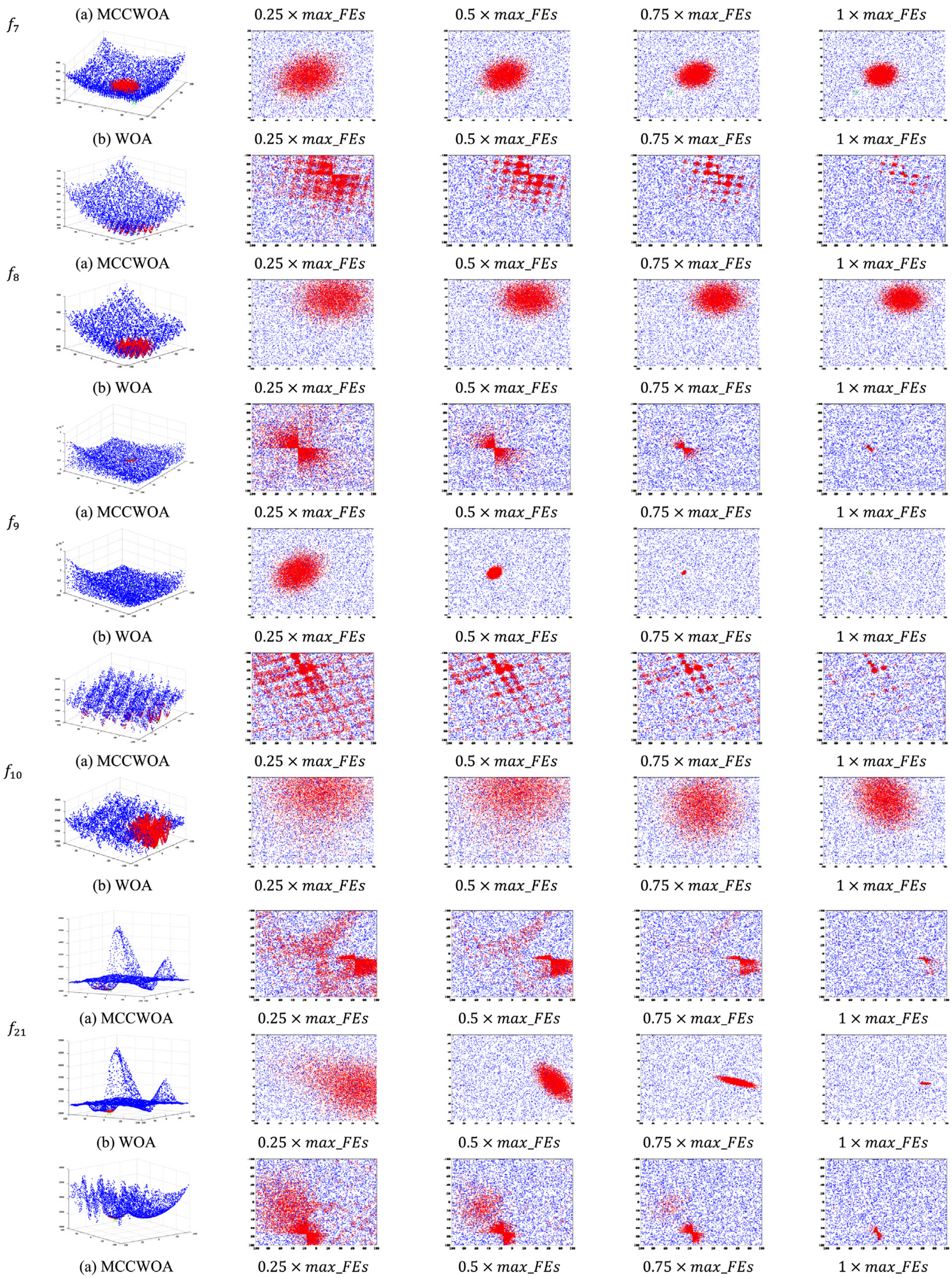


Fig. 7. (*continued*).

visualization line was lower than the others, the performance of the algorithm was better than the compared algorithms. From the optimization results, the MCCWOA wins other compared algorithms intuitively in most functions on different dimensions. Table 6 shows the comparison results of the 16 algorithms on the 10-dimensional functions. On the 27 functions, the MCCWOA has achieved the best performance. As observed in Tables 7 and 8, the optimization results of the MCCWOA for all 2 unimodal functions (f_1 and f_3) and 10 hybrid functions ($f_{11} - f_{20}$) on the 30-dimensional and 50-dimensional functions are always better

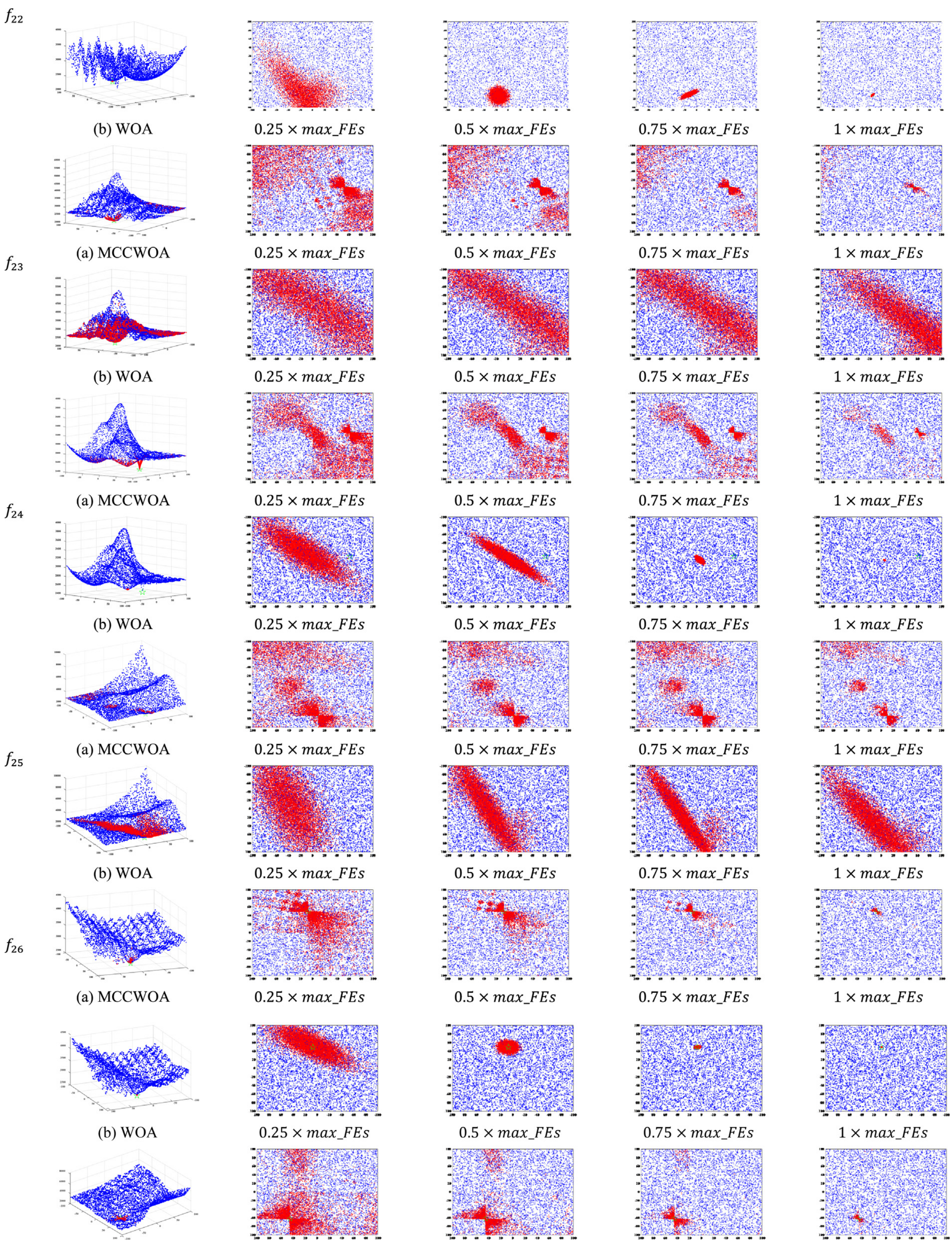


Fig. 7. (*continued*).

than or equal to those of any of the other algorithms. Hence, the MCCWOA has a stronger local search ability because the unimodal functions verify the exploitation power of the algorithm precisely in Section 4.2. On the 100-dimensional functions, the MCCWOA wins 12 functions in other algorithms. On the 7 multimodal functions ($f_4 - f_{10}$), the optimization results of the MCCWOA are always better than those of any of the other algorithms on the 10-dimensional problems. The results of the MCCWOA for multimodal functions on other dimensions are better than those of any of the 13 algorithms except OLPSO and ACWOA. According to Section 4.2, the MCCWOA has a better ability of global search and jump out of local optimizations than other algorithms.

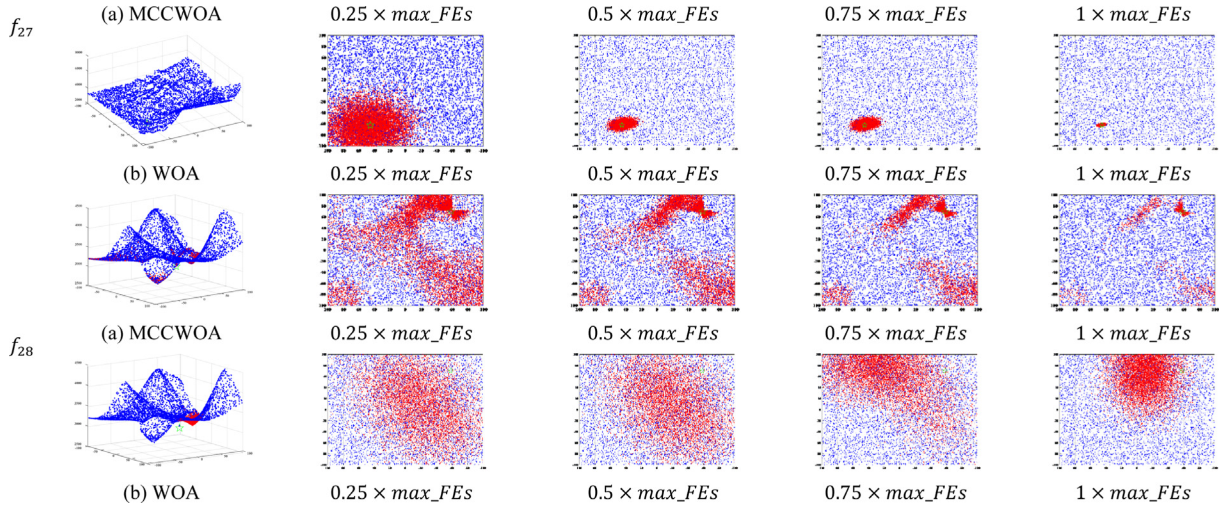


Fig. 7. (*continued*).

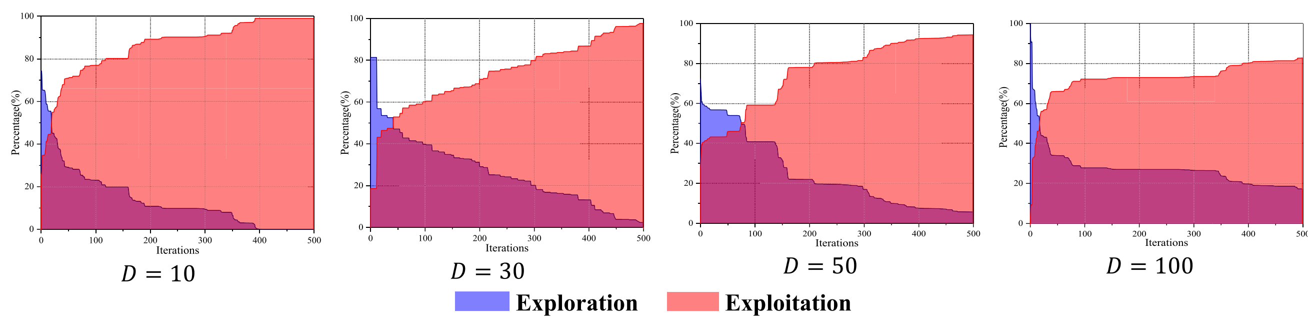


Fig. 8. Exploration and exploitation behavior analysis in f_{17}.

Meanwhile, the convergence of the MCCWOA is carried out on the four kinds of dimension benchmark functions. The results of the 4 representative functions are selected for illustration. These functions are f_3 (unimodal function), f_4 (multimodal function), f_{14} (hybrid function), and f_{22} (composition function). The convergence plots of the functions are shown in Fig. 10. Error-values represent the average difference between the best value obtained by the algorithm and the optimal value of the function after 51 independent runs. As shown in Fig. 10, MCCWOA has an obvious convergence speed compared with other algorithms on f_3, f_4, and f_{14}, which indicates the outstanding advantage of MCCWOA in convergence performance. Although the advantage of MCCWOA is not obvious compared with the BWOA and CCMWOA on f_{22}, MCCWOA maintains a continuous downward trend and obtains a higher precision solution than that of any of the 15 comparison algorithms. Additionally, a distinct separation between MCCWOA and other algorithms on the convergence rate for the late stage of population evolution is obvious on f_3 and f_4, which implicitly indicates that the operating mechanism of the proposed algorithm is effective. In general, the convergence qualities of MCCWOA on the 4 functions are much better than those of any comparison algorithms. The results further verify the effectiveness of the coevolutionary multipopulation mechanism and two-stage orthogonal learning strategy.

The stability of MCCWOA is verified via box plots of the 4 functions on different dimensions. Box plots adopt five statistics: minimum, the first quartile, median, the third quartile, and the maximum value to describe a set of data. The box plots can also roughly see whether the data have symmetry, the distribution of the dispersed degree of information, such as special distributions, can be used for the comparison of several samples. The sides of the two sides of the boxes correspond to the top and bottom quartiles of the data. The black line in the boxes is the position of the median value. An extension line is created between the top quartile and the maximum, which is called a "whicker". Similarly, build an extension line between the bottom quartile and the minimum. The points outside the extension line represent outliers. Therefore, the narrower the box and the fewer outliers mean that the data tend to be stable. The ordinate of the box plots represents the normalized error values. Hence, the closer the box is to the bottom, the better the performance of the algorithm for the considered minimization problems. In Fig. 11, the results of MCCWOA are marked in red rectangles to clearly observe the performance of MCCWOA. As observed in Fig. 11, the boxes of MCCWOA on f_3 are not only the narrowest box but also the closest to the bottom as well as other algorithms on four different dimensions. Therefore, the results of the MCCWOA on f_3 are not only accurate but also stable. Although the narrowest boxes are not shown on some box plots of f_4, f_{14} and f_{22}, the boxes position of the MCCWOA is the closest to the bottom, so the performance of the proposed algorithm is better than other comparison algorithms.

Despite the excellent performance, similar to all swarm intelligence algorithms, the MCCWOA can only search the approximate optimal solution in a limited time, rather than the absolute optimal solution in the real sense. As observed and Table 6, the optimal solutions of f_1, f_3, f_4, f_6, f_9, and f_{20} on 10-dimensional functions are searched via MCCWOA. However, the optimal solutions of the other 23 functions have not been found accurately. Furthermore, only the optimal solutions of f_1 and f_3 are searched on 30-dimensional, 50-dimensional, and 100-dimensional functions. Hence, the proposed MCCWOA recommendation is suitable

Table 5
Specific parameter settings of the algorithms.

Algorithms	Authors	Year	Description	Parameter settings
CMA-ES	Hansen, et al. [155]	2003	Covariance Matrix Adaptation	$N = D, x_{mean} = rand\,(n, 1)\,, \sigma = 0.5,$ $\lambda = 4 + floor\,(3\log(N))\,, \mu = ceil\left(\frac{\lambda - 1}{2}\right),$ $cc = \left(\frac{4}{N+2}\right), cs = \left(\frac{\mu + 2}{\mu + N + 3}\right), eigeneval = 0$
GWO	Mirjalili, et al. [62]	2014	Grey Wolf Optimizer	$a_{max} = 2, a_{min} = 0$
LWOA	Ling, et al. [119]	2018	Levy WOA	$a_{max} = 2, a_{min} = 0, p = rand$
WOA	Mirjalili, et al. [10]	2016	Whale Optimization Algorithm	$b = 1, a_{max} = 2, a_{min} = 0, l \in [-1, 1]\,, p = rand$
RDWOA	Chen, et al. [114]	2019	Double Adaptive Random Spare WOA	$b = 1, a_{max} = 2, a_{min} = 0, l \in [-1, 1]\,, p = rand$
OLPSO	Zhan, et al. [24]	2011	Orthogonal Learning Particle Swarm Optimization	$\omega = 0.9, c = 2.0, G = 5, V_{MAXd} = 0.2 * Range$
OLBSO	Ma, et al. [15]	2020	Orthogonal Learning Brain Storm Optimization	$N = 100, m = 5, P_{replace} = 0.2, P_{one} = 0.8,$ $P_{one-center} = 0.7, P_{two-center} = 0.7, L_{Pg} = 4,$ $L_T = 15$
BMWOA	Heidari, et al. [11]	2020	WOA with Hill Climbing Local Search	$\beta = 0.005, bw = 0.5, b = 1, a_{max} = 2, a_{min} = 0,$ $l \in [-1, 1]\,, p = rand$
EWOA	Tu, et al. [1]	2021	Hybrid WOA and Biogeography-based Optimization	$dt = 1, E = 1, I = 1, Keep = 2,$ $Number\ \ of\ \ habitats = 30$
ACWOA	Elhosseini, et al. [111]	2019	Improving the Parameters A and C in WOA	$b = 1, a_{max} = 2, a_{min} = 0, l \in [-1, 1]\,, p = rand,$ $r = rand$
CCMWOA	Luo, et al. [151]	2019	Improved WOA with chaos-induced initialization, chaotic exploitation, and Gaussian mutation	$a \in [0, 2]\,, m = 1500$
HWGO	Korashy, et al. [152]	2019	Hybrid WOA and GWO	$b = 1, a_{max} = 2, a_{min} = 0, l \in [-1, 1]\,, p = rand$
WOAmGWO	Majeed and Patri [153]	2018	Hybridizing WOA and Modified GWO	$b = 1, a_{max} = 2, a_{min} = 0, l \in [-1, 1]\,, p = rand$
BWOA	Chen, et al. [154]	2019	Balanced WOA	$b = 1, a_1 \in [0, 2]\,, a_2 \in [-2, -1]\,, m = 2500$
HWOAG	Zhang and Wen [121]	2021	Hybrid WOA with Gathering Strategies	$N = 40, Max\_FEs = N \times MaxDT$

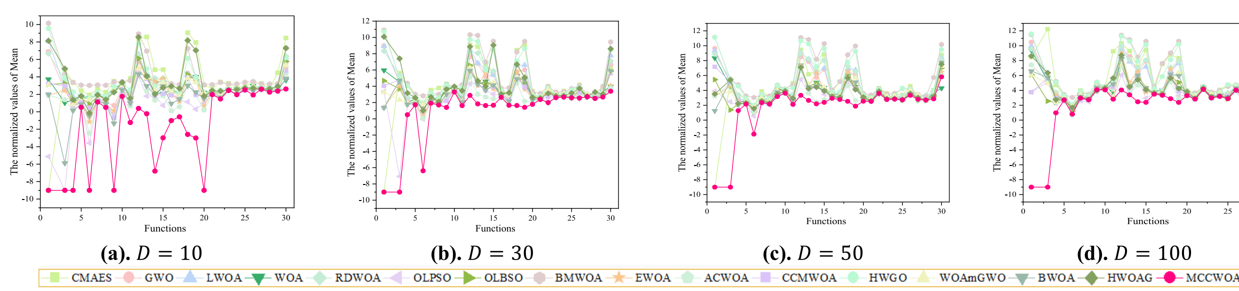


Fig. 9. The visualization of experimental results.

for some practical problems that do not require high accuracy but need to obtain results in a limited time.

The Friedman-test is conducted to make a statistical comparison among the 16 algorithms in Table 10. The Friedman-test is a nonparametric statistical test for determining significant differences in multiple (related) samples [1]. The detailed comparison results of MCCWOA and other algorithms are shown in Table 10. There is no doubt that MCCWOA ranks first on 10-dimensional functions, followed by ACWOA, BWOA, OLPSO, and other algorithms. The top four seed algorithms are still MCCWOA, ACWOA, BWOA, and OLPSO on 30-dimensional and 50-dimensional functions. As observed in Fig. 12, with the increase in dimensions, the advantage of OLPSO gradually gives prominence to ACWOA and BWOA. Despite this, the MCCWOA is still the best algorithm among all approaches, which can prove the superiority of MCCWOA in the evaluation framework. The two reference lines in Fig. 12 represent whether there are significant differences between algorithms under 95% and 90% confidence intervals. The difference in MCCWOA is significant compared with CMA-ES, GWO, LWOA, WOA, RDWOA, OLBSO, BMWOA, EWOA, CCMWOA, HWGO, WOAmGWO, and HWOAG under 95% and 90% confidence intervals for all dimensions. Although there is no significant difference between MCCWOA, OLPSO, and ACWOA, the mean rank of MCCWOA is the smallest in all comparison algorithms. In general, there are significant differences in multiple algorithms, which means that MCCWOA has strong optimization power on complex optimization problems.

The Wilcoxon sign rank test is utilized to further verify the performance difference between MCCWOA and other comparison algorithms. The 15 pairwise comparisons between MCCWOA and others are designed to better visualize the significance of the proposed OL strategy and multipopulation mechanism. In principle, the Wilcoxon sign rank test utilizes the sign test of paired observation data to deduce the probability when the difference appears [121]. Furthermore, whether the difference between the two means is significant is also determined by the method. The significance levels of the test are set to 0.1 and 0.05. The results of the test are shown in Tables 11 to 14. $R+$ is the number of

Table 6
The results of all algorithms on $D = 10$.

Fun	CMA-ES	GWO	LWOA	WOA	RDWOA	OLPSO	OLBSO	BMWOA	ACWOA	EWOA	CCMWOA	HWGO	WOA-mGWO	BWOA	HWOAG	MCCWOA
	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$
f_1	**0.00E+00**$_{0.00E+00}$	7.93E+06$_{4.66E+07}$	1.33E+08$_{1.84E+08}$	5.25E+03$_{6.68E+03}$	4.66E+06$_{2.31E+07}$	7.50E−06$_{3.95E−05}$	1.36E+03$_{1.88E+03}$	1.41E+10$_{4.35E+09}$	7.30E+01$_{1.86E+02}$	1.36E+05$_{1.26E+05}$	1.68E+03$_{2.23E+03}$	3.49E+09$_{9.03E+08}$	1.19E+03$_{1.38E+03}$	1.01E+02$_{1.67E+02}$	1.34E+08$_{1.34E+08}$	**0.00E+00**$_{0.00E+00}$
f_3	5.55E+04$_{1.40E+03}$	2.92E+02$_{6.34E+02}$	5.20E+02$_{4.35E+02}$	1.09E+01$_{2.36E+01}$	3.94E+03$_{5.81E+03}$	**0.00E+00**$_{0.00E+00}$	1.73E+03$_{1.59E+03}$	2.07E+04$_{3.95E+03}$	2.48E+01$_{1.17E+02}$	2.78E+02$_{2.55E+02}$	9.41E+02$_{8.31E+02}$	8.79E+03$_{2.95E+03}$	**0.00E+00**$_{0.00E+00}$	1.31E−06$_{7.80E−07}$	8.54E+04$_{1.02E+05}$	**0.00E+00**$_{1.09E+00}$
f_4	2.26E+03$_{1.85E+03}$	1.02E+01$_{1.27E+01}$	5.35E+01$_{5.73E+01}$	2.17E+01$_{3.17E+01}$	3.86E+01$_{5.42E+01}$	3.87E+00$_{9.57E+00}$	1.25E+01$_{1.79E+01}$	2.30E+03$_{1.49E+03}$	1.54E+00$_{3.96E+01}$	6.02E+00$_{2.28E+00}$	5.05E+00$_{1.20E+00}$	1.94E+02$_{5.88E+01}$	6.87E+00$_{1.59E+01}$	1.44E+00$_{9.61E−01}$	2.40E+01$_{1.75E+01}$	**0.00E+00**$_{0.00E+00}$
f_5	2.62E+02$_{9.07E+00}$	1.17E+01$_{5.35E+00}$	4.47E+01$_{1.50E+01}$	5.56E+01$_{2.06E+01}$	4.82E+01$_{2.00E+01}$	1.15E+01$_{4.75E+00}$	3.60E+01$_{1.45E+01}$	1.12E+03$_{1.02E+03}$	6.90E+00$_{3.92E+01}$	9.44E+00$_{3.15E+00}$	1.46E+01$_{7.19E+00}$	7.17E+01$_{8.71E+00}$	3.61E+01$_{1.14E+01}$	7.36E+00$_{2.21E+00}$	6.26E+01$_{2.42E+01}$	**3.30E+00**$_{1.24E+00}$
f_6	8.34E+01$_{1.47E+01}$	3.85E−01$_{5.06E−01}$	2.76E+01$_{1.06E+01}$	3.89E+01$_{1.27E+01}$	3.37E+01$_{1.13E+01}$	2.65E−04$_{1.21E−03}$	7.57E+00$_{5.33E+00}$	1.06E+03$_{1.01E+03}$	3.47E−03$_{3.52E+00}$	7.26E−02$_{6.42E−02}$	2.98E+00$_{2.72E+00}$	3.88E+01$_{5.58E+00}$	2.57E+01$_{9.09E+00}$	3.49E−01$_{2.82E−01}$	7.28E−01$_{4.11E−01}$	**0.00E+00**$_{1.27E+00}$
f_7	4.10E+02$_{3.51E+00}$	2.64E+01$_{9.61E+00}$	7.05E+01$_{1.75E+01}$	7.92E+01$_{2.26E+01}$	9.93E+01$_{4.06E+01}$	2.16E+01$_{6.26E+00}$	5.71E+01$_{1.96E+01}$	1.19E+03$_{1.00E+03}$	1.76E+01$_{7.59E+00}$	1.72E+01$_{3.24E+00}$	3.10E+01$_{8.00E+00}$	1.67E+02$_{2.28E+01}$	6.30E+01$_{2.46E+01}$	1.69E+01$_{5.26E+00}$	8.63E+01$_{2.29E+01}$	**1.31E+01**$_{0.00E+00}$
f_8	1.69E+02$_{1.21E+01}$	1.23E+01$_{5.90E+00}$	2.83E+01$_{6.82E+00}$	4.36E+01$_{1.68E+01}$	3.76E+01$_{1.42E+01}$	1.09E+01$_{4.54E+00}$	3.14E+01$_{1.21E+01}$	1.09E+03$_{1.00E+03}$	7.59E+00$_{7.58E+00}$	6.65E+00$_{2.92E+00}$	1.80E+01$_{7.44E+00}$	6.52E+01$_{8.72E+00}$	2.08E+01$_{8.82E+00}$	8.22E+00$_{2.34E+00}$	2.49E+01$_{1.52E+01}$	**3.15E+00**$_{6.14E+01}$
f_9	3.17E+03$_{3.27E+02}$	5.18E+00$_{1.38E+01}$	3.01E+02$_{1.36E+02}$	5.78E+02$_{4.37E+02}$	8.95E+02$_{6.69E+02}$	6.77E−01$_{2.20E+00}$	1.99E+02$_{1.55E+02}$	2.62E+03$_{1.20E+03}$	1.97E−01$_{7.52E+00}$	1.33E+00$_{2.17E+00}$	2.02E−01$_{4.63E−01}$	6.98E+02$_{1.65E+02}$	2.44E+02$_{1.25E+02}$	4.63E−02$_{1.42E−01}$	1.71E+02$_{1.07E+02}$	**0.00E+00**$_{2.36E−01}$
f_{10}	1.76E+03$_{2.04E+02}$	5.76E+02$_{2.70E+02}$	1.03E+03$_{2.94E+02}$	1.17E+03$_{2.71E+02}$	1.08E+03$_{3.49E+02}$	3.70E+02$_{1.49E+02}$	9.92E+02$_{3.44E+02}$	2.87E+03$_{1.16E+03}$	2.81E+02$_{1.53E+02}$	3.37E+02$_{1.91E+02}$	7.31E+02$_{2.35E+02}$	1.51E+03$_{1.65E+02}$	1.12E+03$_{2.82E+02}$	3.06E+02$_{1.07E+02}$	2.29E+03$_{3.87E+02}$	**5.74E+01**$_{1.66E+01}$
f_{11}	6.54E+03$_{1.64E+03}$	2.35E+01$_{1.98E+01}$	1.29E+02$_{9.64E+01}$	8.98E+01$_{4.75E+01}$	1.78E+02$_{7.17E+02}$	6.26E+00$_{3.79E+00}$	5.50E+01$_{4.83E+01}$	3.61E+03$_{3.10E+03}$	4.60E+00$_{1.54E+02}$	9.47E+00$_{3.72E+00}$	2.19E+01$_{1.36E+01}$	3.38E+02$_{1.51E+02}$	5.14E+01$_{2.85E+01}$	9.79E+00$_{4.24E+00}$	3.52E+01$_{1.74E+01}$	**5.85E−02**$_{1.11E+00}$
f_{12}	2.36E+08$_{5.07E+08}$	5.08E+05$_{6.64E+05}$	3.89E+06$_{3.88E+06}$	1.66E+05$_{2.17E+04}$	1.78E+06$_{2.46E+06}$	1.23E+04$_{1.18E+04}$	1.30E+06$_{1.19E+06}$	8.30E+08$_{5.14E+08}$	3.23E+04$_{5.32E+04}$	8.53E+04$_{1.21E+05}$	5.27E+04$_{1.73E+05}$	6.18E+07$_{3.09E+07}$	8.44E+04$_{9.28E+04}$	2.08E+04$_{1.63E+04}$	3.65E+08$_{3.22E+08}$	**2.44E+00**$_{1.13E−06}$
f_{13}	3.79E+08$_{1.28E+09}$	8.08E+03$_{5.14E+03}$	1.63E+04$_{1.57E+04}$	1.22E+04$_{1.04E+04}$	1.05E+04$_{9.60E+03}$	6.30E+01$_{6.92E+01}$	9.29E+03$_{7.57E+03}$	9.16E+06$_{1.33E+07}$	2.48E+02$_{5.32E+04}$	8.78E+03$_{6.87E+03}$	5.91E+03$_{5.66E+03}$	8.62E+04$_{6.21E+04}$	7.43E+03$_{5.38E+03}$	8.97E+02$_{5.53E+02}$	1.20E+04$_{8.52E+03}$	**5.95E−01**$_{1.99E−03}$
f_{14}	6.47E+04$_{6.04E+01}$	9.46E+02$_{1.52E+03}$	1.67E+02$_{6.74E+01}$	1.56E+02$_{1.10E+02}$	5.81E+03$_{7.15E+03}$	6.11E+01$_{1.66E+02}$	2.23E+03$_{3.07E+03}$	1.21E+03$_{1.09E+03}$	2.11E+01$_{5.32E+04}$	3.27E+03$_{3.89E+03}$	6.74E+01$_{2.90E+01}$	8.53E+02$_{8.46E+02}$	3.21E+02$_{4.43E+02}$	9.02E+01$_{2.24E+01}$	1.07E+02$_{5.08E+01}$	**1.59E−07**$_{9.59E−02}$
f_{15}	6.87E+04$_{5.65E+00}$	1.41E+03$_{1.56E+03}$	1.49E+03$_{1.36E+03}$	1.22E+02$_{9.43E+01}$	8.01E+03$_{9.55E+03}$	5.35E+00$_{4.92E+00}$	3.34E+03$_{4.44E+03}$	5.01E+03$_{3.77E+03}$	7.22E+01$_{3.14E+02}$	6.72E+03$_{1.03E+04}$	1.18E+02$_{1.24E+02}$	2.55E+03$_{3.07E+03}$	2.14E+03$_{2.02E+03}$	8.39E+02$_{4.76E+02}$	5.71E+02$_{4.58E+02}$	**1.02E−03**$_{3.44E−01}$
f_{16}	5.98E+02$_{3.55E+00}$	9.04E+01$_{7.44E+01}$	2.63E+02$_{1.21E+02}$	2.95E+02$_{1.65E+02}$	3.24E+02$_{1.87E+02}$	1.08E+02$_{1.04E+02}$	2.52E+02$_{1.37E+02}$	1.66E+03$_{1.12E+03}$	1.30E+00$_{6.93E+01}$	1.12E+02$_{1.04E+02}$	7.82E+01$_{7.75E+01}$	2.54E+02$_{9.35E+01}$	2.73E+02$_{1.35E+02}$	8.57E+00$_{5.67E+00}$	8.61E+02$_{1.96E+02}$	**9.87E−02**$_{1.32E−02}$
f_{17}	8.91E+02$_{3.35E+01}$	4.37E+01$_{2.76E+01}$	7.52E+01$_{2.94E+01}$	1.33E+02$_{7.76E+01}$	1.24E+02$_{6.65E+01}$	1.86E+01$_{2.32E+01}$	6.76E+01$_{4.48E+01}$	1.31E+03$_{1.08E+03}$	3.66E+00$_{6.88E+01}$	1.46E+01$_{1.29E+01}$	4.90E+01$_{1.94E+01}$	1.16E+02$_{2.30E+01}$	6.62E+01$_{2.85E+01}$	2.85E+01$_{4.61E+00}$	4.71E+02$_{1.39E+02}$	**2.63E−01**$_{4.01E−03}$
f_{18}	1.16E+09$_{2.21E+09}$	2.63E+04$_{1.38E+04}$	2.09E+04$_{1.52E+04}$	7.80E+03$_{1.01E+04}$	1.45E+04$_{1.08E+04}$	1.36E+01$_{1.42E+01}$	1.84E+04$_{1.73E+04}$	1.76E+07$_{2.50E+07}$	1.22E+03$_{1.16E+03}$	9.53E+03$_{1.01E+04}$	8.29E+03$_{7.46E+03}$	1.33E+06$_{1.51E+06}$	6.63E+03$_{7.44E+03}$	9.60E+02$_{6.90E+02}$	1.51E+08$_{1.38E+08}$	**2.54E−03**$_{0.00E+00}$
f_{19}	8.63E+07$_{4.82E+08}$	3.71E+03$_{5.33E+03}$	9.90E+03$_{3.66E+04}$	1.00E+04$_{1.03E+04}$	7.53E+03$_{8.66E+03}$	1.96E+00$_{1.72E+00}$	4.81E+03$_{6.53E+03}$	7.85E+03$_{8.79E+03}$	7.98E+01$_{1.15E+03}$	4.94E+03$_{5.45E+03}$	9.13E+01$_{2.56E+02}$	7.69E+03$_{9.07E+03}$	2.41E+03$_{2.75E+03}$	1.76E+02$_{1.28E+02}$	1.08E+07$_{2.32E+07}$	**9.78E−04**$_{2.72E+01}$
f_{20}	1.16E+03$_{2.34E−02}$	5.69E+01$_{4.08E+01}$	1.56E+02$_{6.08E+01}$	1.58E+02$_{8.05E+01}$	1.88E+02$_{9.14E+01}$	9.43E+00$_{1.22E+01}$	7.57E+01$_{4.85E+01}$	1.27E+03$_{1.06E+03}$	1.60E+00$_{1.15E+03}$	4.54E+00$_{6.33E+00}$	4.80E+01$_{2.10E+01}$	1.31E+02$_{2.86E+01}$	1.35E+02$_{5.54E+01}$	3.35E+01$_{8.08E+00}$	6.35E+01$_{3.10E+01}$	**0.00E+00**$_{3.37E+01}$
f_{21}	4.78E+02$_{6.55E+00}$	1.94E+02$_{4.28E+01}$	1.80E+02$_{5.95E+01}$	2.15E+02$_{5.27E+01}$	2.27E+02$_{5.68E+01}$	1.44E+02$_{6.05E+01}$	1.50E+02$_{5.90E+01}$	1.28E+03$_{1.04E+03}$	1.01E+02$_{9.66E+01}$	1.89E+02$_{4.31E+01}$	1.09E+02$_{3.69E+01}$	1.54E+02$_{2.53E+01}$	1.85E+02$_{6.08E+01}$	9.80E+01$_{1.40E+01}$	2.61E+02$_{4.52E+01}$	**9.22E+01**$_{8.97E+01}$
f_{22}	2.55E+03$_{2.68E+02}$	1.06E+02$_{4.09E+00}$	1.28E+02$_{1.95E+01}$	1.34E+02$_{2.74E+01}$	1.55E+02$_{8.16E+01}$	9.19E+01$_{2.88E+01}$	1.06E+02$_{4.25E+00}$	2.35E+03$_{1.36E+03}$	5.97E+01$_{4.48E+01}$	1.03E+02$_{2.31E+01}$	9.79E+01$_{2.31E+01}$	2.93E+02$_{1.02E+02}$	1.01E+02$_{1.06E+00}$	**4.48E+01**$_{3.29E+01}$	2.39E+02$_{1.03E+02}$	1.11E+02$_{4{\cdot}01E+00}$
f_{23}	1.81E+03$_{3.64E+01}$	3.16E+02$_{8.28E+00}$	3.38E+02$_{1.68E+01}$	3.64E+02$_{2.54E+01}$	3.67E+02$_{2.70E+01}$	3.14E+02$_{5.98E+00}$	3.48E+02$_{1.82E+01}$	1.43E+03$_{1.02E+03}$	3.10E+02$_{1.12E+02}$	3.14E+02$_{4.99E+00}$	3.16E+02$_{7.28E+00}$	3.88E+02$_{2.78E+01}$	3.92E+02$_{2.52E+01}$	3.05E+02$_{2.24E+01}$	3.21E+02$_{8.70E+00}$	**2.76E+02**$_{1.29E+02}$
f_{24}	6.03E+02$_{2.03E+01}$	3.42E+02$_{1.03E+01}$	3.49E+02$_{6.18E+01}$	3.85E+02$_{8.99E+01}$	3.80E+02$_{8.28E+01}$	2.91E+02$_{1.13E+02}$	3.19E+02$_{1.15E+02}$	1.54E+03$_{1.05E+03}$	1.37E+02$_{1.18E+02}$	3.39E+02$_{3.51E+01}$	3.16E+02$_{7.28E+00}$	3.71E+02$_{5.65E+01}$	3.46E+02$_{1.39E+02}$	1.04E+02$_{5.21E+01}$	3.50E+02$_{5.20E+01}$	**9.11E+01**$_{9.67E+01}$
f_{25}	7.97E+02$_{1.13E+03}$	4.35E+02$_{1.63E+01}$	4.68E+02$_{2.60E+01}$	4.36E+02$_{5.17E+01}$	4.45E+02$_{5.68E+01}$	4.22E+02$_{2.38E+01}$	4.31E+02$_{2.06E+01}$	2.40E+03$_{1.36E+03}$	4.04E+02$_{1.25E+02}$	4.36E+02$_{2.11E+01}$	**2.71E+02**$_{1.12E+02}$	5.77E+02$_{4.42E+01}$	4.27E+02$_{2.21E+01}$	3.98E+02$_{1.17E−01}$	5.26E+02$_{8.72E+01}$	3.23E+02$_{1.57E+00}$
f_{26}	2.40E+03$_{7.16E+01}$	3.57E+02$_{1.88E+02}$	6.27E+02$_{3.45E+02}$	8.90E+02$_{4.48E+02}$	7.59E+02$_{4.25E+02}$	3.12E+02$_{1.62E+02}$	4.89E+02$_{3.07E+02}$	2.91E+03$_{1.37E+03}$	2.78E+02$_{1.31E+02}$	3.60E+02$_{1.00E+02}$	3.10E+02$_{2.81E+01}$	7.42E+02$_{7.72E+01}$	6.96E+02$_{3.21E+02}$	2.82E+02$_{7.13E+01}$	5.28E+02$_{9.91E+01}$	**8.44E+01**$_{1.44E+02}$
f_{27}	1.70E+03$_{6.69E+01}$	3.94E+02$_{2.52E+00}$	4.30E+02$_{3.76E+01}$	4.76E+02$_{3.60E+01}$	4.45E+02$_{3.03E+01}$	3.98E+02$_{4.69E+00}$	4.17E+02$_{6.84E+01}$	1.54E+03$_{1.05E+03}$	3.93E+02$_{7.65E+01}$	4.78E+02$_{4.52E+01}$	3.98E+02$_{3.64E+00}$	4.66E+02$_{2.01E+01}$	4.84E+02$_{3.99E+01}$	3.93E+02$_{1.68E+00}$	4.01E+02$_{4.36E+00}$	**3.89E+02**$_{9.04E+00}$
f_{28}	5.00E+02$_{7.24E−13}$	5.50E+02$_{9.76E+01}$	5.93E+02$_{1.12E+02}$	4.81E+02$_{1.28E+01}$	5.77E+02$_{1.82E+02}$	3.55E+02$_{1.31E+02}$	4.96E+02$_{1.16E+02}$	1.50E+03$_{1.00E+03}$	2.99E+02$_{7.12E+01}$	4.94E+02$_{5.25E+00}$	3.13E+02$_{2.81E+01}$	4.99E+02$_{1.75E+00}$	4.47E+02$_{1.55E+02}$	2.53E+02$_{1.10E+02}$	4.31E+02$_{1.08E+02}$	**1.77E+02**$_{1.31E+01}$
f_{29}	3.02E+04$_{3.60E+04}$	2.83E+02$_{4.76E+01}$	4.13E+02$_{7.96E+01}$	4.85E+02$_{1.20E+02}$	4.79E+02$_{1.16E+02}$	2.65E+02$_{2.20E+01}$	3.55E+02$_{5.67E+01}$	1.67E+03$_{1.11E+03}$	2.53E+02$_{6.46E+01}$	2.85E+02$_{2.75E+01}$	2.85E+02$_{3.15E+01}$	4.19E+02$_{3.90E+01}$	3.55E+02$_{6.94E+01}$	2.52E+02$_{1.03E+01}$	8.15E+02$_{1.55E+02}$	**2.35E+02**$_{0.00E+00}$
f_{30}	2.84E+08$_{1.59E+08}$	4.56E+05$_{6.20E+05}$	9.22E+05$_{1.15E+06}$	1.01E+04$_{2.48E+04}$	2.82E+06$_{4.77E+06}$	1.22E+05$_{3.34E+05}$	6.10E+05$_{4.81E+05}$	2.02E+07$_{1.44E+07}$	2.36E+04$_{5.41E+04}$	1.10E+05$_{1.85E+05}$	5.22E+04$_{2.15E+05}$	1.57E+06$_{1.40E+06}$	2.08E+05$_{5.15E+05}$	6.56E+03$_{6.10E+03}$	1.96E+07$_{2.98E+07}$	**4.06E+02**$_{1.09E+00}$

Table 7
The results of all algorithms on $D = 30$.

Fun	CMA-ES	GWO	LWOA	WOA	RDWOA	OLPSO	OLBSO	BMWOA	ACWOA	EWOA	CCMWOA	HWGO	WOA-mGWO	BWOA	HWOAG	MCCWOA
	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$
f_1	**0.00E+00**$_{0.00E+00}$	9.25E+08$_{7.70E+08}$	8.20E+08$_{1.77E+08}$	9.20E+05$_{5.95E+06}$	2.11E+08$_{7.85E+08}$	2.33E+03$_{3.12E+03}$	4.70E+04$_{3.22E+04}$	8.02E+10$_{3.29E+09}$	2.51E+01$_{6.98E+01}$	5.81E+06$_{2.79E+06}$	1.11E+04$_{1.40E+04}$	5.34E+10$_{6.46E+09}$	1.81E+03$_{1.80E+03}$	2.32E+01$_{4.28E+01}$	1.20E+10$_{3.84E+09}$	**0.00E+00**$_{0.00E+00}$
f_3	4.10E+05$_{2.55E+04}$	2.88E+04$_{1.02E+04}$	4.31E+04$_{7.26E+03}$	3.16E+04$_{1.62E+04}$	2.02E+05$_{6.25E+04}$	8.49E−08$_{1.84E-07}$	4.82E+03$_{8.85E+03}$	9.45E+04$_{1.66E+03}$	4.94E+04$_{4.90E+04}$	1.11E+04$_{3.98E+03}$	6.24E+04$_{1.31E+04}$	1.11E+05$_{1.48E+04}$	2.13E+02$_{3.56E+02}$	5.20E+04$_{6.95E+03}$	2.50E+07$_{6.51E+07}$	**0.00E+00**$_{0.00E+00}$
f_4	1.69E+04$_{4.41E+03}$	1.50E+02$_{4.17E+01}$	4.26E+02$_{1.73E+02}$	1.06E+02$_{2.86E+01}$	4.26E+02$_{4.40E+02}$	6.46E+01$_{1.98E+01}$	7.22E+01$_{2.09E+01}$	2.34E+04$_{2.89E+03}$	8.07E+01$_{2.51E+01}$	1.26E+02$_{1.75E+01}$	9.85E+01$_{2.76E+01}$	1.06E+04$_{1.92E+03}$	8.63E+01$_{2.79E+01}$	5.93E+01$_{2.01E+01}$	1.48E+03$_{5.65E+02}$	**3.18E+00**$_{8.16E+00}$
f_5	5.71E+02$_{1.57E+01}$	8.59E+01$_{2.67E+01}$	2.60E+02$_{3.31E+01}$	2.80E+02$_{5.92E+01}$	3.09E+02$_{7.51E+01}$	6.53E+01$_{2.31E+01}$	1.85E+02$_{4.73E+01}$	1.07E+03$_{1.50E+03}$	**5.17E+01**$_{1.35E+01}$	6.52E+01$_{1.52E+01}$	9.41E+01$_{2.53E+01}$	4.38E+02$_{2.46E+01}$	1.86E+02$_{3.82E+01}$	7.52E+01$_{1.10E+01}$	3.80E+02$_{3.45E+01}$	5.24E+01$_{1.35E+01}$
f_6	9.05E+01$_{4.08E+00}$	4.01E+00$_{2.53E+00}$	6.46E+01$_{4.41E+00}$	6.52E+01$_{1.10E+01}$	6.55E+01$_{1.05E+01}$	9.20E−01$_{4.58E-01}$	1.31E+01$_{6.37E+00}$	6.69E+02$_{1.50E+03}$	1.11E+00$_{3.23E+00}$	1.79E+00$_{7.56E-01}$	2.47E+01$_{8.14E+00}$	8.72E+01$_{6.13E+00}$	5.34E+01$_{6.79E+00}$	1.94E+01$_{3.17E+00}$	1.01E+01$_{4.44E+00}$	**4.12E−07**$_{2.31E-06}$
f_7	3.39E+03$_{2.54E+01}$	1.35E+02$_{3.20E+01}$	4.77E+02$_{6.88E+01}$	5.24E+02$_{7.94E+01}$	5.90E+02$_{1.24E+02}$	8.84E+01$_{1.36E+01}$	2.28E+02$_{3.80E+01}$	1.45E+03$_{1.50E+03}$	8.74E+01$_{1.86E+01}$	1.22E+02$_{2.15E+01}$	1.29E+02$_{3.78E+01}$	1.42E+03$_{1.19E+02}$	4.93E+02$_{7.90E+01}$	1.35E+02$_{1.59E+01}$	4.73E+02$_{5.86E+01}$	**8.29E+01**$_{1.41E+01}$
f_8	6.06E+02$_{1.42E+01}$	7.37E+01$_{2.18E+01}$	1.79E+02$_{2.39E+01}$	2.05E+02$_{5.24E+01}$	2.75E+02$_{6.94E+01}$	5.71E+01$_{1.85E+01}$	1.73E+02$_{4.39E+01}$	9.97E+02$_{1.50E+03}$	**4.84E+01**$_{1.43E+01}$	4.93E+01$_{1.18E+01}$	9.66E+01$_{2.86E+01}$	3.92E+02$_{2.09E+01}$	1.40E+02$_{2.85E+01}$	7.08E+01$_{1.17E+01}$	1.27E+02$_{3.27E+01}$	5.07E+01$_{1.27E+01}$
f_9	1.95E+04$_{1.39E+03}$	4.68E+02$_{3.63E+02}$	5.18E+03$_{8.88E+02}$	5.85E+03$_{2.33E+03}$	9.73E+03$_{3.75E+03}$	1.47E+02$_{9.48E+01}$	5.16E+03$_{2.21E+03}$	1.72E+04$_{1.50E+03}$	4.23E+02$_{4.04E+02}$	4.72E+02$_{2.32E+02}$	1.57E+02$_{5.63E+02}$	1.41E+04$_{1.52E+03}$	3.12E+03$_{7.51E+02}$	1.03E+03$_{2.90E+02}$	4.26E+03$_{1.65E+03}$	**2.50E+01**$_{4.53E+01}$
f_{10}	7.79E+03$_{2.63E+02}$	3.00E+03$_{7.88E+02}$	5.35E+03$_{9.44E+02}$	5.10E+03$_{1.07E+03}$	5.36E+03$_{8.88E+02}$	2.55E+03$_{4.96E+02}$	4.73E+03$_{7.02E+02}$	9.19E+03$_{1.80E+03}$	2.29E+03$_{3.18E+02}$	2.67E+03$_{4.55E+02}$	3.80E+03$_{6.96E+02}$	7.46E+03$_{2.51E+02}$	4.25E+03$_{5.79E+02}$	2.47E+03$_{3.33E+02}$	9.19E+03$_{5.41E+02}$	**1.98E+03**$_{2.98E+02}$
f_{11}	4.05E+03$_{1.31E+03}$	2.89E+02$_{1.48E+02}$	6.50E+02$_{2.77E+02}$	2.31E+02$_{2.80E+02}$	1.42E+03$_{2.39E+03}$	6.77E+01$_{3.23E+01}$	1.77E+02$_{6.21E+01}$	1.50E+04$_{2.80E+03}$	7.58E+01$_{3.24E+01}$	1.92E+02$_{6.85E+01}$	1.43E+02$_{4.51E+01}$	7.18E+03$_{1.42E+03}$	1.37E+02$_{3.88E+01}$	1.04E+02$_{2.58E+01}$	2.90E+03$_{1.42E+03}$	**4.17E+01**$_{1.73E+01}$
f_{12}	1.25E+09$_{2.02E+09}$	4.17E+07$_{6.46E+07}$	2.95E+08$_{2.28E+08}$	2.84E+06$_{2.62E+06}$	1.84E+08$_{2.42E+08}$	2.00E+04$_{2.13E+04}$	3.59E+06$_{2.85E+06}$	2.17E+10$_{4.32E+09}$	1.40E+05$_{1.60E+05}$	2.01E+06$_{1.23E+06}$	1.24E+06$_{8.50E+05}$	5.64E+09$_{1.26E+09}$	1.81E+06$_{9.73E+05}$	8.20E+05$_{5.38E+05}$	7.18E+08$_{4.25E+08}$	**7.53E+02**$_{4.56E+02}$
f_{13}	7.29E+08$_{1.59E+09}$	6.35E+06$_{2.41E+07}$	6.84E+06$_{1.63E+07}$	1.75E+04$_{1.48E+04}$	1.15E+08$_{6.13E+08}$	2.66E+04$_{2.18E+04}$	2.40E+04$_{1.68E+04}$	1.77E+10$_{5.78E+09}$	1.03E+03$_{1.50E+03}$	1.24E+04$_{8.79E+03}$	1.83E+04$_{1.04E+04}$	3.10E+09$_{1.08E+09}$	5.39E+04$_{3.19E+04}$	1.80E+04$_{6.90E+03}$	4.54E+04$_{2.61E+04}$	**6.87E+01**$_{2.61E+01}$
f_{14}	4.93E+06$_{1.81E+06}$	1.54E+05$_{2.97E+05}$	1.27E+06$_{1.11E+06}$	3.85E+04$_{3.07E+04}$	1.66E+06$_{1.86E+06}$	3.31E+03$_{4.23E+03}$	2.51E+04$_{3.67E+04}$	9.09E+06$_{5.89E+06}$	4.74E+02$_{2.50E+03}$	5.37E+05$_{3.85E+05}$	3.19E+03$_{4.60E+03}$	9.57E+05$_{4.44E+05}$	6.70E+03$_{5.73E+03}$	3.44E+03$_{2.19E+03}$	9.89E+03$_{3.74E+04}$	**4.25E+01**$_{1.64E+01}$
f_{15}	5.22E+05$_{2.33E+06}$	2.70E+05$_{6.88E+05}$	2.49E+05$_{4.19E+05}$	1.35E+04$_{1.35E+04}$	7.53E+05$_{4.43E+06}$	1.09E+03$_{9.70E+02}$	8.89E+03$_{7.39E+03}$	3.37E+09$_{1.24E+09}$	2.80E+02$_{1.83E+02}$	4.68E+03$_{5.15E+03}$	8.73E+03$_{7.96E+03}$	3.03E+08$_{1.46E+08}$	2.92E+04$_{1.83E+04}$	1.35E+04$_{5.82E+03}$	1.11E+09$_{9.21E+08}$	**4.60E+01**$_{2.44E+01}$
f_{16}	4.19E+03$_{1.66E+02}$	7.27E+02$_{2.41E+02}$	2.11E+03$_{5.42E+02}$	1.81E+03$_{7.85E+02}$	1.89E+03$_{5.47E+02}$	8.85E+02$_{2.67E+02}$	1.48E+03$_{3.08E+02}$	5.65E+03$_{2.49E+03}$	4.41E+02$_{1.58E+02}$	9.51E+02$_{2.82E+02}$	9.51E+02$_{2.88E+02}$	3.08E+03$_{3.04E+02}$	1.59E+03$_{3.03E+02}$	6.16E+02$_{1.23E+02}$	1.08E+03$_{3.21E+02}$	**4.32E+02**$_{1.19E+02}$
f_{17}	6.51E+02$_{6.38E-08}$	2.59E+02$_{1.45E+02}$	9.62E+02$_{3.37E+02}$	1.14E+03$_{3.28E+02}$	1.13E+03$_{3.36E+02}$	3.03E+02$_{1.62E+02}$	7.29E+02$_{2.51E+02}$	6.48E+03$_{8.34E+03}$	7.43E+01$_{4.38E+01}$	3.44E+02$_{1.74E+02}$	3.35E+02$_{1.66E+02}$	1.36E+03$_{1.68E+02}$	7.33E+02$_{2.81E+02}$	1.42E+02$_{4.08E+01}$	1.62E+03$_{3.60E+02}$	**4.55E+01**$_{1.44E+01}$
f_{18}	2.74E+08$_{6.48E+08}$	6.39E+05$_{8.94E+05}$	3.58E+06$_{5.06E+06}$	1.61E+05$_{1.44E+05}$	4.16E+06$_{5.51E+06}$	8.76E+04$_{9.15E+04}$	2.70E+05$_{2.07E+05}$	1.63E+08$_{1.17E+08}$	2.37E+04$_{7.90E+04}$	6.24E+05$_{8.37E+05}$	1.44E+05$_{1.28E+05}$	1.26E+07$_{5.34E+06}$	1.25E+05$_{9.73E+04}$	7.85E+04$_{3.65E+04}$	4.92E+06$_{6.99E+06}$	**4.32E+01**$_{1.48E+01}$
f_{19}	1.09E+09$_{3.47E+09}$	1.02E+06$_{2.25E+06}$	4.04E+06$_{5.44E+06}$	1.28E+04$_{1.43E+04}$	1.19E+05$_{5.08E+05}$	1.19E+04$_{1.31E+04}$	2.98E+04$_{5.22E+04}$	3.28E+09$_{6.71E+08}$	1.40E+02$_{1.16E+02}$	4.21E+03$_{4.67E+03}$	8.46E+03$_{1.09E+04}$	4.42E+08$_{2.13E+08}$	1.33E+05$_{6.02E+04}$	6.67E+04$_{7.27E+04}$	1.20E+05$_{1.01E+05}$	**2.56E+01**$_{1.06E+01}$
f_{20}	2.58E+03$_{4.62E-06}$	3.22E+02$_{1.24E+02}$	6.75E+02$_{1.63E+02}$	8.97E+02$_{2.32E+02}$	9.27E+02$_{2.12E+02}$	3.90E+02$_{1.51E+02}$	5.73E+02$_{2.21E+02}$	1.92E+03$_{1.64E+03}$	1.22E+02$_{6.10E+01}$	3.47E+02$_{1.57E+02}$	4.60E+02$_{1.50E+02}$	8.11E+02$_{1.06E+02}$	7.05E+02$_{1.82E+02}$	2.15E+02$_{5.94E+01}$	4.53E+02$_{1.65E+02}$	**5.97E+01**$_{5.34E+01}$
f_{21}	1.06E+03$_{1.27E+01}$	2.80E+02$_{2.16E+01}$	4.80E+02$_{5.45E+01}$	4.65E+02$_{5.20E+01}$	4.64E+02$_{6.03E+01}$	2.62E+02$_{1.27E+01}$	4.06E+02$_{6.11E+01}$	1.33E+03$_{1.56E+03}$	2.46E+02$_{1.34E+01}$	2.56E+02$_{1.28E+01}$	2.88E+02$_{2.32E+01}$	5.96E+02$_{2.26E+01}$	3.85E+02$_{3.46E+01}$	2.69E+02$_{1.13E+01}$	3.46E+02$_{4.18E+01}$	**2.42E+02**$_{2.72E+01}$
f_{22}	9.99E+03$_{4.16E+02}$	2.08E+03$_{1.59E+03}$	3.68E+03$_{2.14E+03}$	4.56E+03$_{1.63E+03}$	5.33E+03$_{1.97E+03}$	1.20E+03$_{1.34E+03}$	3.03E+03$_{2.56E+03}$	9.36E+03$_{1.86E+03}$	1.01E+02$_{1.89E+00}$	2.89E+03$_{1.12E+03}$	4.67E+02$_{3.70E+01}$	6.01E+03$_{6.97E+02}$	3.99E+03$_{1.81E+03}$	**1.00E+02**$_{0.00E+00}$	1.33E+03$_{7.14E+02}$	**1.00E+02**$_{0.00E+00}$
f_{23}	1.62E+03$_{1.82E+01}$	4.32E+02$_{2.83E+01}$	8.57E+02$_{1.01E+02}$	8.72E+02$_{1.38E+02}$	8.61E+02$_{1.28E+02}$	4.23E+02$_{1.98E+01}$	6.10E+02$_{6.73E+01}$	1.96E+03$_{1.64E+03}$	4.04E+02$_{1.63E+01}$	4.41E+02$_{1.99E+01}$	4.67E+02$_{3.70E+01}$	9.41E+02$_{4.62E+01}$	1.01E+03$_{1.23E+02}$	4.41E+02$_{1.82E+01}$	6.61E+02$_{8.67E+01}$	**4.00E+02**$_{4.15E+01}$
f_{24}	2.63E+03$_{2.71E+01}$	5.21E+02$_{5.74E+01}$	8.85E+02$_{1.07E+02}$	8.80E+02$_{1.00E+02}$	9.28E+02$_{1.05E+02}$	5.05E+02$_{2.98E+01}$	6.78E+02$_{6.89E+01}$	2.12E+03$_{1.61E+03}$	4.87E+02$_{3.45E+01}$	5.23E+02$_{2.56E+01}$	5.08E+02$_{3.24E+01}$	1.12E+03$_{6.22E+01}$	1.11E+03$_{9.83E+01}$	5.03E+02$_{1.45E+01}$	6.80E+02$_{6.26E+01}$	**4.72E+02**$_{1.40E+01}$
f_{25}	1.87E+03$_{1.63E+03}$	4.55E+02$_{2.30E+01}$	5.27E+02$_{3.21E+01}$	4.16E+02$_{2.86E+01}$	5.52E+02$_{1.37E+02}$	3.90E+02$_{7.23E+00}$	4.07E+02$_{2.26E+01}$	5.27E+03$_{1.50E+03}$	3.88E+02$_{1.62E+00}$	4.29E+02$_{2.45E+01}$	4.12E+02$_{1.98E+01}$	3.89E+03$_{6.70E+02}$	3.89E+02$_{8.01E+00}$	3.88E+02$_{2.46E+00}$	1.45E+03$_{4.64E+02}$	**3.87E+02**$_{5.68E+00}$
f_{26}	1.43E+04$_{1.33E+02}$	1.86E+03$_{2.43E+02}$	5.78E+03$_{1.01E+03}$	4.82E+03$_{1.31E+03}$	5.57E+03$_{1.42E+03}$	1.48E+03$_{6.67E+02}$	3.38E+03$_{1.27E+03}$	1.20E+04$_{2.61E+03}$	9.41E+02$_{5.57E+02}$	2.19E+03$_{2.96E+02}$	1.74E+03$_{7.59E+02}$	7.31E+03$_{5.98E+02}$	5.55E+03$_{1.55E+03}$	**3.43E+02**$_{4.30E+02}$	5.31E+03$_{6.40E+02}$	3.55E+02$_{3.37E+02}$
f_{27}	**5.00E+02**$_{9.23E-10}$	5.35E+02$_{1.49E+01}$	8.05E+02$_{1.70E+02}$	**5.00E+02**$_{8.32E-05}$	9.45E+02$_{1.66E+02}$	5.27E+02$_{1.08E+01}$	4.96E+02$_{7.63E+00}$	1.11E+03$_{1.76E+03}$	5.30E+02$_{1.78E+01}$	**5.00E+02**$_{2.34E-04}$	5.75E+02$_{4.50E+01}$	**5.00E+02**$_{7.07E-05}$	1.22E+03$_{2.16E+02}$	5.50E+02$_{1.19E+01}$	5.92E+02$_{3.33E+01}$	5.11E+02$_{1.24E+01}$
f_{28}	5.00E+02$_{9.33E-08}$	5.37E+02$_{6.18E+01}$	6.71E+02$_{7.72E+01}$	5.00E+02$_{8.17E-05}$	7.64E+02$_{2.67E+02}$	3.44E+02$_{4.20E+01}$	4.33E+02$_{2.44E+01}$	1.17E+03$_{2.30E+03}$	3.94E+02$_{4.33E+01}$	5.00E+02$_{2.70E-04}$	4.56E+02$_{2.22E+01}$	5.00E+02$_{9.81E-05}$	3.94E+02$_{3.43E+01}$	4.07E+02$_{7.81E+00}$	1.94E+03$_{4.46E+02}$	**3.15E+02**$_{3.75E+01}$
f_{29}	4.78E+03$_{7.16E+00}$	7.90E+02$_{1.22E+02}$	2.56E+03$_{7.49E+02}$	2.18E+03$_{6.00E+02}$	2.42E+03$_{4.45E+02}$	7.59E+02$_{1.42E+02}$	1.45E+03$_{2.71E+02}$	1.01E+04$_{8.33E+03}$	5.30E+02$_{6.80E+01}$	7.77E+02$_{1.69E+02}$	8.92E+02$_{1.94E+02}$	2.06E+03$_{3.98E+02}$	1.62E+03$_{3.01E+02}$	7.84E+02$_{8.92E+01}$	1.06E+03$_{2.09E+02}$	**4.70E+02**$_{3.56E+01}$
f_{30}	1.65E+06$_{3.72E+06}$	4.87E+06$_{3.78E+06}$	6.00E+07$_{7.91E+07}$	9.59E+03$_{1.77E+04}$	1.28E+07$_{2.80E+07}$	6.96E+03$_{2.74E+03}$	7.18E+05$_{4.50E+05}$	2.76E+09$_{1.34E+09}$	5.79E+03$_{2.84E+03}$	1.28E+04$_{6.26E+03}$	4.01E+05$_{5.16E+05}$	2.98E+08$_{1.12E+08}$	5.46E+05$_{3.09E+05}$	7.07E+05$_{4.61E+05}$	3.92E+08$_{3.07E+08}$	**2.41E+03**$_{5.36E+02}$

Table 8
The results of all algorithms on $D = 50$.

Fun	CMA-ES	GWO	LWOA	WOA	RDWOA	OLPSO	OLBSO	BMWOA	ACWOA	EWOA	CCMWOA	HWGO	WOA-mGWO	BWOA	HWOAG	MCCWOA
	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$
f_1	**0.00E+00**$_{0.00E+00}$	4.41E+09$_{2.27E+09}$	2.10E+09$_{6.86E+08}$	2.19E+08$_{6.99E+08}$	7.49E+08$_{3.72E+09}$	7.60E+03$_{9.18E+03}$	2.82E+05$_{1.72E+05}$	1.35E+11$_{1.04E+00}$	1.67E+03$_{4.17E+03}$	3.57E+08$_{1.11E+08}$	1.47E+07$_{1.47E+07}$	1.33E+11$_{1.17E+10}$	3.91E+03$_{3.29E+03}$	1.77E+01$_{2.67E+01}$	3.25E+03$_{3.80E+03}$	**0.00E+00**$_{0.00E+00}$
f_3	3.20E+05$_{3.93E+03}$	7.10E+04$_{1.69E+04}$	1.08E+05$_{1.24E+04}$	3.81E+04$_{1.23E+04}$	3.51E+05$_{1.07E+05}$	2.79E+02$_{4.67E+02}$	2.32E+01$_{8.06E+00}$	2.82E+05$_{1.30E-02}$	8.70E+04$_{1.04E+05}$	3.55E+04$_{8.34E+03}$	1.50E+05$_{2.72E+04}$	2.41E+05$_{2.90E+04}$	6.48E+03$_{2.74E+03}$	1.83E+05$_{2.15E+04}$	2.57E+05$_{7.03E+04}$	**0.00E+00**$_{0.00E+00}$
f_4	2.33E+03$_{2.86E+03}$	4.75E+02$_{3.24E+02}$	9.87E+02$_{3.58E+02}$	2.83E+02$_{2.51E+02}$	2.30E+03$_{2.22E+03}$	8.33E+01$_{3.98E+01}$	1.51E+02$_{4.72E+01}$	5.72E+04$_{4.77E-07}$	1.91E+02$_{4.57E+01}$	3.72E+02$_{5.67E+01}$	2.67E+02$_{5.36E+01}$	3.56E+04$_{5.89E+03}$	1.58E+02$_{5.54E+01}$	1.43E+02$_{2.93E+01}$	1.66E+02$_{5.29E+01}$	**1.89E+01**$_{3.29E+01}$
f_5	1.35E+03$_{3.27E+01}$	1.78E+02$_{3.18E+01}$	3.75E+02$_{3.07E+01}$	3.96E+02$_{5.37E+01}$	5.93E+02$_{1.17E+02}$	**1.19E+02**$_{2.21E+01}$	3.95E+02$_{1.03E+02}$	1.82E+03$_{5.07E-08}$	1.63E+02$_{2.08E+01}$	2.07E+02$_{2.49E+01}$	2.21E+02$_{4.43E+01}$	8.32E+02$_{4.12E+01}$	3.25E+02$_{4.45E+01}$	1.78E+02$_{1.99E+01}$	2.00E+02$_{3.96E+01}$	1.43E+02$_{2.48E+01}$
f_6	9.37E+01$_{9.47E+00}$	1.18E+01$_{3.33E+00}$	7.58E+01$_{6.04E+00}$	7.37E+01$_{8.08E+00}$	8.13E+01$_{1.33E+01}$	3.58E+00$_{1.29E+00}$	1.24E+01$_{4.57E+00}$	1.11E+03$_{5.69E-09}$	1.32E+01$_{1.23E+01}$	1.20E+01$_{2.92E+00}$	3.74E+01$_{7.09E+00}$	1.07E+02$_{4.42E+00}$	5.91E+01$_{6.94E+00}$	3.38E+01$_{2.88E+00}$	3.63E+01$_{6.46E+00}$	**1.33E−02**$_{5.87E-02}$
f_7	7.51E+03$_{9.34E+01}$	3.05E+02$_{6.07E+01}$	9.25E+02$_{9.21E+01}$	1.02E+03$_{8.98E+01}$	1.13E+03$_{2.61E+02}$	**1.85E+02**$_{2.41E+01}$	4.36E+02$_{9.00E+01}$	2.50E+03$_{1.33E-07}$	3.06E+02$_{6.04E+01}$	3.92E+02$_{3.86E+01}$	2.35E+02$_{4.09E+01}$	3.13E+03$_{2.36E+02}$	1.00E+03$_{1.63E+02}$	3.66E+02$_{3.96E+01}$	3.71E+02$_{7.34E+01}$	2.24E+02$_{3.68E+01}$
f_8	1.44E+03$_{3.60E+01}$	1.86E+02$_{3.24E+01}$	3.83E+02$_{4.50E+01}$	4.09E+02$_{7.74E+01}$	6.00E+02$_{1.42E+02}$	**1.01E+02**$_{1.62E+01}$	3.36E+02$_{9.26E+01}$	1.81E+03$_{1.31E-06}$	1.64E+02$_{2.77E+01}$	2.08E+02$_{3.26E+01}$	2.02E+02$_{4.55E+01}$	8.29E+02$_{3.25E+01}$	3.39E+02$_{3.56E+01}$	1.80E+02$_{2.01E+01}$	1.91E+02$_{2.98E+01}$	1.42E+02$_{2.40E+01}$
f_9	3.75E+04$_{7.44E+03}$	3.55E+03$_{1.99E+03}$	1.79E+04$_{3.73E+03}$	1.47E+04$_{4.19E+03}$	2.57E+04$_{7.70E+03}$	1.81E+03$_{6.20E+02}$	1.28E+04$_{4.78E+03}$	5.41E+04$_{3.93E-04}$	6.12E+03$_{2.05E+03}$	5.13E+03$_{1.25E+03}$	2.32E+03$_{2.62E+03}$	4.75E+04$_{4.98E+03}$	1.11E+04$_{1.46E+03}$	5.77E+03$_{1.07E+03}$	3.13E+03$_{3.15E+03}$	**1.47E+03**$_{7.97E+02}$
f_{10}	1.43E+04$_{2.57E+01}$	5.45E+03$_{1.23E+03}$	8.97E+03$_{1.72E+03}$	9.75E+03$_{1.90E+03}$	1.02E+04$_{1.56E+03}$	4.42E+03$_{5.39E+02}$	7.76E+03$_{8.50E+02}$	1.64E+04$_{6.29E+02}$	4.50E+03$_{4.70E+02}$	6.65E+03$_{7.27E+02}$	6.60E+03$_{8.39E+02}$	1.37E+04$_{3.49E+02}$	7.34E+03$_{8.12E+02}$	4.68E+03$_{4.70E+02}$	8.52E+03$_{2.79E+03}$	**4.18E+03**$_{4.87E+02}$
f_{11}	9.40E+04$_{5.18E+03}$	1.74E+03$_{1.23E+03}$	9.86E+02$_{2.21E+02}$	3.61E+02$_{1.42E+02}$	3.00E+03$_{3.07E+03}$	1.39E+02$_{4.12E+01}$	3.10E+02$_{8.46E+01}$	5.76E+04$_{5.61E+04}$	3.33E+02$_{1.66E+02}$	7.56E+02$_{2.83E+02}$	3.15E+02$_{2.18E+02}$	2.18E+04$_{3.64E+03}$	2.07E+02$_{5.10E+01}$	5.42E+02$_{1.02E+02}$	8.75E+02$_{5.67E+02}$	**1.30E+02**$_{2.92E+01}$
f_{12}	2.80E+09$_{5.68E+09}$	3.23E+08$_{5.21E+08}$	1.29E+09$_{6.08E+08}$	2.29E+07$_{3.01E+07}$	1.04E+09$_{1.94E+09}$	2.92E+05$_{1.53E+05}$	1.24E+07$_{5.88E+06}$	1.23E+11$_{6.55E-01}$	4.46E+06$_{4.80E+06}$	2.24E+07$_{7.10E+06}$	1.27E+07$_{8.84E+06}$	4.68E+10$_{8.32E+09}$	1.24E+07$_{6.58E+06}$	1.28E+07$_{5.10E+06}$	1.22E+07$_{6.36E+06}$	**2.17E+03**$_{8.77E+02}$
f_{13}	1.45E+06$_{2.96E+06}$	7.49E+07$_{1.01E+08}$	1.40E+08$_{1.16E+08}$	2.64E+04$_{2.66E+04}$	1.75E+08$_{1.20E+09}$	4.15E+03$_{6.54E+03}$	3.99E+04$_{2.49E+04}$	6.94E+10$_{5.54E+08}$	4.31E+03$_{3.41E+03}$	3.76E+04$_{1.53E+04}$	2.00E+04$_{1.05E+04}$	1.74E+10$_{3.38E+09}$	6.54E+04$_{6.77E+04}$	2.85E+04$_{7.10E+03}$	1.74E+04$_{8.39E+03}$	**4.45E+02**$_{1.68E+02}$
f_{14}	3.90E+06$_{3.16E+06}$	5.34E+05$_{6.31E+05}$	2.18E+06$_{2.14E+06}$	1.13E+05$_{1.47E+05}$	1.37E+07$_{2.67E+07}$	6.66E+04$_{8.88E+04}$	8.54E+04$_{7.16E+04}$	1.50E+08$_{4.70E+07}$	2.64E+02$_{7.64E+01}$	1.92E+06$_{1.27E+06}$	3.40E+04$_{2.68E+04}$	1.34E+07$_{5.60E+06}$	3.54E+04$_{2.67E+04}$	3.50E+04$_{2.17E+04}$	2.64E+04$_{2.28E+04}$	**1.54E+02**$_{4.32E+01}$
f_{15}	6.91E+08$_{3.43E+09}$	6.51E+06$_{1.24E+07}$	9.59E+06$_{3.68E+07}$	2.27E+04$_{2.56E+04}$	1.09E+08$_{4.05E+08}$	3.01E+03$_{2.91E+03}$	1.24E+04$_{1.04E+04}$	1.92E+10$_{3.09E+09}$	5.45E+02$_{2.61E+02}$	9.51E+03$_{5.36E+03}$	9.04E+03$_{5.12E+03}$	4.34E+09$_{1.34E+09}$	2.78E+04$_{1.79E+04}$	9.73E+03$_{2.83E+03}$	8.42E+03$_{3.98E+03}$	**2.44E+02**$_{7.56E+01}$
f_{16}	6.68E+03$_{4.50E+03}$	1.19E+03$_{2.98E+02}$	4.05E+03$_{9.45E+02}$	3.39E+03$_{1.36E+03}$	3.79E+03$_{1.16E+03}$	1.57E+03$_{2.47E+02}$	2.42E+03$_{5.35E+02}$	1.17E+04$_{4.42E+03}$	9.26E+02$_{2.03E+02}$	1.52E+03$_{4.22E+02}$	1.51E+03$_{4.32E+02}$	6.06E+03$_{4.43E+02}$	2.21E+03$_{4.11E+02}$	1.20E+03$_{1.86E+02}$	1.61E+03$_{4.49E+02}$	**8.36E+02**$_{2.26E+02}$
f_{17}	1.50E+05$_{8.10E+05}$	1.05E+03$_{2.74E+02}$	2.26E+03$_{5.48E+02}$	2.65E+03$_{6.00E+02}$	3.09E+03$_{7.88E+02}$	9.93E+02$_{2.83E+02}$	1.94E+03$_{3.78E+02}$	1.49E+05$_{2.66E+07}$	6.57E+02$_{1.49E+02}$	1.21E+03$_{3.49E+02}$	1.14E+03$_{2.88E+02}$	6.27E+03$_{1.86E+03}$	1.94E+03$_{4.05E+02}$	9.33E+02$_{1.54E+02}$	1.18E+03$_{2.39E+02}$	**5.96E+02**$_{1.57E+02}$
f_{18}	2.24E+06$_{2.26E+06}$	2.99E+06$_{3.94E+06}$	8.54E+06$_{7.62E+06}$	9.32E+05$_{8.39E+05}$	1.92E+07$_{2.26E+07}$	1.10E+05$_{7.97E+04}$	8.30E+05$_{6.53E+05}$	6.02E+08$_{3.73E+08}$	3.98E+04$_{1.77E+05}$	3.63E+06$_{2.42E+06}$	4.95E+05$_{3.47E+05}$	6.06E+07$_{1.82E+07}$	3.04E+05$_{1.27E+05}$	4.01E+05$_{1.54E+05}$	4.80E+05$_{4.50E+05}$	**3.39E+02**$_{2.59E+02}$
f_{19}	8.76E+05$_{8.85E+05}$	1.40E+06$_{4.22E+06}$	5.56E+06$_{5.86E+06}$	2.97E+04$_{2.13E+04}$	3.41E+06$_{1.32E+07}$	1.55E+04$_{1.11E+04}$	2.09E+04$_{2.14E+04}$	9.20E+09$_{2.01E+09}$	4.45E+02$_{4.41E+02}$	1.75E+04$_{8.73E+03}$	1.54E+04$_{8.84E+03}$	1.74E+09$_{5.24E+08}$	4.87E+05$_{2.35E+05}$	1.24E+05$_{1.12E+05}$	1.29E+04$_{8.44E+03}$	**7.49E+01**$_{2.30E+01}$
f_{20}	2.47E+03$_{3.59E-03}$	7.59E+02$_{2.98E+02}$	1.44E+03$_{2.40E+02}$	1.79E+03$_{4.21E+02}$	1.89E+03$_{4.11E+02}$	1.01E+03$_{2.30E+02}$	1.41E+03$_{3.85E+02}$	3.79E+03$_{2.96E+02}$	4.29E+02$_{1.00E+02}$	7.97E+02$_{2.76E+02}$	1.13E+03$_{2.83E+02}$	2.12E+03$_{1.33E+02}$	1.45E+03$_{2.89E+02}$	6.21E+02$_{1.43E+02}$	1.26E+03$_{4.16E+02}$	**3.49E+02**$_{1.15E+02}$
f_{21}	1.62E+03$_{3.43E+01}$	3.89E+02$_{5.07E+01}$	8.18E+02$_{7.42E+01}$	8.32E+02$_{8.32E+01}$	8.23E+02$_{1.54E+02}$	**3.26E+02**$_{1.73E+01}$	5.35E+02$_{1.03E+02}$	2.34E+03$_{1.50E+02}$	3.63E+02$_{2.61E+01}$	3.93E+02$_{3.04E+01}$	4.10E+02$_{4.35E+01}$	1.04E+03$_{3.69E+01}$	6.46E+02$_{5.95E+01}$	3.76E+02$_{1.98E+01}$	3.88E+02$_{4.34E+01}$	3.28E+02$_{2.09E+01}$
f_{22}	1.96E+04$_{2.31E-01}$	5.86E+03$_{9.02E+02}$	1.10E+04$_{1.77E+03}$	1.01E+04$_{2.05E+03}$	1.12E+04$_{1.44E+03}$	5.12E+03$_{8.04E+02}$	8.43E+03$_{1.47E+03}$	1.67E+04$_{7.42E+02}$	3.91E+03$_{2.29E+03}$	7.58E+03$_{8.34E+02}$	7.11E+03$_{1.64E+03}$	1.39E+04$_{3.86E+02}$	8.00E+03$_{7.41E+02}$	**3.51E+03**$_{2.44E+03}$	7.69E+03$_{2.35E+03}$	3.87E+03$_{2.15E+03}$
f_{23}	4.07E+03$_{5.91E-06}$	5.99E+02$_{3.74E+01}$	1.56E+03$_{1.65E+02}$	1.59E+03$_{1.86E+02}$	1.47E+03$_{1.54E+02}$	**5.73E+02**$_{3.83E+01}$	9.42E+02$_{1.13E+02}$	3.30E+03$_{2.49E+02}$	6.62E+02$_{4.75E+01}$	7.30E+02$_{4.39E+01}$	6.97E+02$_{6.69E+01}$	1.67E+03$_{5.21E+01}$	1.74E+03$_{2.09E+02}$	6.98E+02$_{3.46E+01}$	7.51E+02$_{6.65E+01}$	5.95E+02$_{3.52E+01}$
f_{24}	3.41E+03$_{4.02E+01}$	7.06E+02$_{9.47E+01}$	1.50E+03$_{1.80E+02}$	1.49E+03$_{1.55E+02}$	1.61E+03$_{1.83E+02}$	**6.38E+02**$_{2.57E+01}$	9.77E+02$_{1.06E+02}$	3.90E+03$_{4.15E+02}$	7.81E+02$_{8.20E+01}$	8.25E+02$_{4.04E+01}$	6.97E+02$_{6.69E+01}$	1.97E+03$_{1.08E+02}$	1.71E+03$_{2.10E+02}$	7.63E+02$_{2.79E+01}$	8.03E+02$_{8.54E+01}$	6.56E+02$_{3.85E+01}$
f_{25}	2.45E+03$_{3.04E+03}$	8.34E+02$_{1.39E+02}$	1.06E+03$_{1.42E+02}$	7.11E+02$_{1.98E+02}$	1.78E+03$_{9.78E+02}$	**5.19E+02**$_{3.55E+01}$	4.93E+02$_{4.08E+01}$	1.79E+04$_{1.82E-07}$	5.75E+02$_{4.20E+01}$	7.73E+02$_{4.11E+01}$	6.51E+02$_{2.83E+01}$	2.00E+04$_{2.24E+03}$	5.68E+02$_{2.94E+01}$	5.38E+02$_{2.13E+01}$	5.71E+02$_{3.67E+01}$	5.33E+02$_{4.47E+01}$
f_{26}	3.63E+04$_{7.09E+02}$	3.17E+03$_{3.69E+02}$	9.81E+03$_{1.56E+03}$	9.61E+03$_{2.09E+03}$	1.19E+04$_{1.89E+03}$	2.61E+03$_{4.73E+02}$	5.27E+03$_{2.22E+03}$	1.78E+04$_{1.37E+02}$	3.39E+03$_{9.88E+02}$	4.37E+03$_{5.07E+02}$	3.33E+03$_{7.66E+02}$	1.60E+04$_{1.33E+03}$	1.05E+04$_{8.85E+02}$	3.32E+03$_{1.19E+03}$	3.81E+03$_{6.33E+02}$	**2.15E+03**$_{1.42E+03}$
f_{27}	**5.00E+02**$_{1.19E-09}$	7.82E+02$_{7.79E+01}$	1.72E+03$_{7.13E+02}$	**5.00E+02**$_{1.12E-04}$	2.35E+03$_{4.96E+02}$	7.05E+02$_{6.90E+01}$	5.05E+02$_{2.03E+01}$	2.87E+03$_{4.48E+03}$	9.54E+02$_{1.34E+02}$	**5.00E+02**$_{2.32E-04}$	8.83E+02$_{1.38E+02}$	**5.00E+02**$_{9.11E-05}$	2.76E+03$_{6.31E+02}$	8.74E+02$_{5.94E+01}$	1.06E+03$_{1.77E+02}$	6.23E+02$_{6.28E+01}$
f_{28}	5.00E+02$_{1.77E-08}$	1.08E+03$_{3.04E+02}$	1.49E+03$_{2.86E+02}$	5.00E+02$_{1.04E-04}$	2.13E+03$_{8.95E+02}$	4.99E+02$_{9.87E+01}$	**4.90E+02**$_{1.05E+01}$	1.50E+03$_{1.33E+04}$	5.81E+02$_{7.34E+01}$	5.00E+02$_{2.24E-04}$	6.54E+02$_{6.29E+01}$	5.00E+02$_{7.53E-05}$	5.10E+02$_{1.75E+01}$	5.19E+02$_{1.87E+01}$	5.50E+02$_{3.42E+01}$	**4.90E+02**$_{2.74E+01}$
f_{29}	1.11E+04$_{1.84E+03}$	1.24E+03$_{2.17E+02}$	6.88E+03$_{2.00E+03}$	4.90E+03$_{2.49E+03}$	5.47E+03$_{3.40E+03}$	9.26E+02$_{5.70E+03}$	2.52E+03$_{4.75E+02}$	5.50E+05$_{2.44E+06}$	8.29E+02$_{1.83E+02}$	1.46E+03$_{2.41E+02}$	1.42E+03$_{3.03E+02}$	9.24E+03$_{2.31E+03}$	2.67E+03$_{4.73E+02}$	1.64E+03$_{1.86E+02}$	1.59E+03$_{3.09E+02}$	**7.13E+02**$_{1.19E+02}$
f_{30}	5.81E+08$_{6.95E+08}$	6.88E+07$_{2.51E+07}$	2.52E+08$_{1.03E+08}$	**1.99E+04**$_{5.07E+04}$	1.43E+08$_{1.66E+08}$	8.72E+05$_{5.46E+05}$	1.00E+07$_{2.42E+06}$	1.51E+10$_{4.54E+09}$	1.04E+06$_{1.91E+05}$	2.97E+06$_{6.03E+05}$	3.10E+07$_{8.64E+06}$	3.08E+09$_{7.33E+08}$	1.79E+07$_{1.90E+06}$	5.39E+07$_{9.99E+06}$	3.26E+07$_{7.88E+06}$	6.51E+05$_{1.00E+05}$

Table 9
The results of all algorithms on $D = 100$.

Fun	CMA-ES	GWO	LWOA	WOA	RDWOA
	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$
f_1	$2.91E+08_{7.66E+09}$	$2.98E+10_{6.79E+09}$	$8.83E+09_{1.48E+09}$	$2.57E+09_{3.34E+09}$	$3.68E+09_{1.10E+10}$
f_3	$1.68E+12_{2.69E+13}$	$2.01E+05_{2.26E+04}$	$2.36E+05_{1.43E+04}$	$1.54E+05_{2.34E+04}$	$8.67E+05_{1.43E+05}$
f_4	$5.21E+04_{4.46E+04}$	$2.40E+03_{7.59E+02}$	$2.95E+03_{6.14E+02}$	$1.11E+03_{5.95E+02}$	$5.52E+03_{4.02E+03}$
f_5	$1.62E+03_{1.06E+02}$	$5.65E+02_{9.16E+01}$	$8.93E+02_{5.35E+01}$	$8.78E+02_{9.25E+01}$	$1.33E+03_{3.47E+02}$
f_6	$9.74E+01_{1.53E+01}$	$2.76E+01_{3.93E+00}$	$7.65E+01_{7.10E+00}$	$7.08E+01_{7.32E+00}$	$1.02E+02_{1.54E+01}$
f_7	$9.90E+03_{1.62E+02}$	$1.00E+03_{1.08E+02}$	$2.49E+03_{9.03E+01}$	$2.55E+03_{1.09E+02}$	$2.80E+03_{4.50E+02}$
f_8	$2.56E+03_{2.17E+02}$	$5.58E+02_{6.09E+01}$	$1.08E+03_{5.73E+01}$	$1.04E+03_{1.05E+02}$	$1.41E+03_{3.31E+02}$
f_9	$8.73E+04_{2.08E+04}$	$2.31E+04_{1.06E+04}$	$3.44E+04_{7.43E+03}$	$2.53E+04_{6.17E+03}$	$5.52E+04_{2.37E+04}$
f_{10}	$2.48E+04_{3.01E+03}$	$1.40E+04_{2.57E+03}$	$2.04E+04_{4.07E+03}$	$2.41E+04_{3.64E+03}$	$2.42E+04_{2.51E+03}$
f_{11}	$1.78E+09_{9.25E+10}$	$3.62E+04_{1.06E+04}$	$7.44E+04_{1.42E+04}$	$6.38E+03_{4.10E+03}$	$9.17E+04_{9.77E+04}$
f_{12}	$2.09E+10_{5.75E+10}$	$4.96E+09_{2.49E+09}$	$3.71E+09_{1.21E+09}$	$1.55E+08_{1.70E+08}$	$8.84E+09_{1.03E+10}$
f_{13}	$1.97E+09_{1.25E+10}$	$3.58E+08_{4.19E+08}$	$6.95E+07_{2.35E+07}$	$3.24E+04_{1.76E+04}$	$1.38E+08_{5.99E+08}$
f_{14}	$1.45E+07_{2.50E+08}$	$3.18E+06_{2.20E+06}$	$3.61E+06_{2.14E+06}$	$4.35E+05_{2.19E+05}$	$1.13E+07_{1.25E+07}$
f_{15}	$2.69E+09_{3.06E+10}$	$8.32E+07_{1.46E+08}$	$1.44E+07_{1.13E+07}$	$9.72E+03_{7.88E+03}$	$1.13E+08_{4.73E+08}$
f_{16}	$1.04E+04_{1.31E+04}$	$3.64E+03_{6.49E+02}$	$1.16E+04_{1.99E+03}$	$7.48E+03_{3.15E+03}$	$1.04E+04_{3.02E+03}$
f_{17}	$3.42E+07_{6.67E+08}$	$2.70E+03_{5.48E+02}$	$7.33E+03_{1.72E+03}$	$9.95E+03_{6.63E+03}$	$1.05E+04_{8.04E+03}$
f_{18}	$6.20E+08_{2.85E+09}$	$3.71E+06_{2.18E+06}$	$4.03E+06_{1.93E+06}$	$5.97E+05_{2.81E+05}$	$1.17E+07_{1.25E+07}$
f_{19}	$2.05E+08_{4.61E+09}$	$8.24E+07_{1.58E+08}$	$4.32E+07_{2.55E+07}$	$1.98E+04_{4.79E+04}$	$8.49E+07_{6.06E+08}$
f_{20}	$9.50E+03_{3.74E+02}$	$2.42E+03_{7.03E+02}$	$3.76E+03_{5.56E+02}$	$4.51E+03_{5.67E+02}$	$4.78E+03_{7.76E+02}$
f_{21}	$4.74E+03_{1.79E+02}$	$7.50E+02_{5.97E+01}$	$2.05E+03_{1.69E+02}$	$1.88E+03_{1.72E+02}$	$2.11E+03_{2.83E+02}$
f_{22}	$2.94E+04_{3.81E+03}$	$1.56E+04_{3.26E+03}$	$2.25E+04_{3.48E+03}$	$2.60E+04_{3.66E+03}$	$2.59E+04_{2.93E+03}$
f_{23}	$4.63E+03_{1.58E+02}$	$1.10E+03_{7.30E+01}$	$3.07E+03_{2.54E+02}$	$3.01E+03_{3.29E+02}$	$3.02E+03_{3.36E+02}$
f_{24}	$1.19E+04_{2.60E+02}$	$\mathbf{1.51E+03}_{9.27E+01}$	$4.23E+03_{4.89E+02}$	$4.86E+03_{4.94E+02}$	$5.22E+03_{5.50E+02}$
f_{25}	$5.61E+04_{1.97E+05}$	$2.72E+03_{5.26E+02}$	$2.70E+03_{4.60E+02}$	$1.52E+03_{3.28E+02}$	$4.60E+03_{1.69E+03}$
f_{26}	$3.12E+04_{9.12E+03}$	$1.01E+04_{1.55E+03}$	$3.16E+04_{2.66E+03}$	$3.40E+04_{6.10E+03}$	$3.48E+04_{5.08E+03}$
f_{27}	$5.15E+02_{5.55E+02}$	$1.11E+03_{1.06E+02}$	$3.19E+03_{1.77E+03}$	$5.00E+02_{8.81E-05}$	$4.80E+03_{1.13E+03}$
f_{28}	$5.00E+02_{1.40E-06}$	$3.71E+03_{9.78E+02}$	$2.71E+03_{6.28E+02}$	$5.00E+02_{1.04E-04}$	$7.38E+03_{2.90E+03}$
f_{29}	$1.03E+04_{3.02E+15}$	$4.40E+03_{4.76E+02}$	$1.40E+04_{2.74E+03}$	$1.24E+04_{5.00E+03}$	$1.45E+04_{5.73E+03}$
f_{30}	$4.18E+09_{1.92E+10}$	$3.29E+08_{2.37E+08}$	$6.88E+08_{3.78E+08}$	$1.40E+08_{5.80E+08}$	$1.44E+09_{2.17E+09}$

Fun	OLPSO	OLBSO	BMWOA	ACWOA	EWOA	CCMWOA
	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$
f_1	$5.75E+03_{7.93E+03}$	$1.26E+06_{4.77E+05}$	$2.97E+11_{1.04E+00}$	$2.66E+07_{2.84E+07}$	$1.21E+10_{2.03E+09}$	$5.76E+03_{6.38E+03}$
f_3	$1.36E+05_{2.47E+05}$	$3.60E+02_{6.87E+01}$	$3.67E+05_{1.30E-02}$	$1.48E+05_{2.55E+05}$	$1.63E+05_{1.74E+04}$	$4.55E+05_{4.54E+04}$
f_4	$1.81E+02_{5.15E+01}$	$2.40E+02_{5.16E+01}$	$1.61E+05_{4.77E-07}$	$2.53E+02_{8.73E+01}$	$2.42E+03_{3.71E+02}$	$9.75E+02_{1.64E+02}$
f_5	$\mathbf{2.96E+02}_{3.22E+01}$	$9.57E+02_{2.37E+02}$	$2.81E+03_{5.07E-08}$	$4.48E+02_{6.41E+01}$	$8.70E+02_{5.60E+01}$	$5.55E+02_{8.28E+01}$
f_6	$8.94E+00_{1.99E+00}$	$1.57E+01_{6.02E+00}$	$1.12E+03_{5.69E-09}$	$\mathbf{4.78E+00}_{7.60E+00}$	$3.98E+01_{4.12E+00}$	$4.75E+01_{3.80E+00}$
f_7	$\mathbf{4.01E+02}_{5.01E+01}$	$9.42E+02_{1.90E+02}$	$4.53E+03_{1.33E-07}$	$6.87E+02_{1.03E+02}$	$1.72E+03_{1.74E+02}$	$5.94E+02_{2.05E+02}$
f_8	$\mathbf{2.89E+02}_{6.16E+01}$	$9.20E+02_{2.67E+02}$	$2.94E+03_{1.31E-06}$	$4.42E+02_{6.76E+01}$	$9.21E+02_{5.97E+01}$	$5.80E+02_{9.34E+01}$
f_9	$\mathbf{8.69E+03}_{1.99E+03}$	$2.81E+04_{9.66E+03}$	$1.04E+05_{3.93E-04}$	$1.63E+04_{4.97E+03}$	$2.61E+04_{4.13E+03}$	$1.05E+04_{5.63E+03}$
f_{10}	$\mathbf{1.14E+04}_{1.86E+03}$	$1.61E+04_{1.43E+03}$	$3.43E+04_{6.29E+02}$	$1.06E+04_{6.69E+02}$	$2.23E+04_{1.31E+03}$	$1.48E+04_{1.19E+03}$
f_{11}	$7.81E+02_{1.88E+02}$	$1.45E+03_{1.94E+02}$	$3.98E+05_{5.61E+04}$	$1.56E+04_{2.19E+04}$	$1.07E+04_{2.37E+03}$	$2.77E+04_{1.23E+04}$
f_{12}	$1.16E+06_{4.77E+05}$	$2.70E+07_{8.18E+06}$	$2.61E+11_{6.55E-01}$	$2.06E+07_{2.08E+07}$	$1.15E+09_{2.11E+08}$	$3.10E+08_{1.27E+08}$
f_{13}	$\mathbf{2.38E+03}_{1.37E+03}$	$3.07E+04_{1.07E+04}$	$6.51E+10_{5.54E+08}$	$4.58E+03_{1.03E+04}$	$1.20E+05_{1.00E+05}$	$2.32E+04_{8.15E+03}$
f_{14}	$4.59E+05_{2.47E+05}$	$4.49E+05_{2.18E+05}$	$2.96E+08_{4.70E+07}$	$4.94E+04_{6.73E+04}$	$3.68E+06_{1.42E+06}$	$8.63E+05_{5.51E+05}$
f_{15}	$8.81E+02_{8.90E+02}$	$1.12E+04_{4.27E+03}$	$3.94E+10_{3.09E+09}$	$1.72E+04_{1.32E+04}$	$5.82E+03_{2.47E+03}$	$1.21E+04_{5.40E+03}$
f_{16}	$3.07E+03_{6.35E+02}$	$4.98E+03_{7.67E+02}$	$3.33E+04_{4.42E+03}$	$3.37E+03_{5.28E+02}$	$4.55E+03_{7.79E+02}$	$3.96E+03_{7.57E+02}$
f_{17}	$2.63E+03_{4.66E+02}$	$4.02E+03_{6.34E+02}$	$4.26E+07_{2.66E+07}$	$\mathbf{2.10E+03}_{3.19E+02}$	$3.90E+03_{5.97E+02}$	$2.95E+03_{4.49E+02}$
f_{18}	$8.01E+05_{6.20E+05}$	$7.19E+05_{2.81E+05}$	$1.09E+09_{3.73E+08}$	$1.45E+05_{8.75E+04}$	$6.99E+06_{4.08E+06}$	$1.40E+06_{9.78E+05}$
f_{19}	$1.80E+03_{2.41E+03}$	$1.18E+05_{1.21E+05}$	$4.01E+10_{2.01E+09}$	$6.52E+02_{4.09E+02}$	$8.47E+03_{4.52E+03}$	$7.63E+03_{5.92E+03}$
f_{20}	$2.32E+03_{7.82E+02}$	$4.03E+03_{6.09E+02}$	$7.76E+03_{2.96E+02}$	$\mathbf{1.58E+03}_{1.85E+02}$	$2.82E+03_{4.97E+02}$	$3.49E+03_{5.56E+02}$
f_{21}	$\mathbf{5.79E+02}_{7.08E+01}$	$1.21E+03_{1.78E+02}$	$4.07E+03_{1.50E+02}$	$6.69E+02_{4.78E+01}$	$1.16E+03_{5.93E+01}$	$7.93E+02_{1.40E+02}$
f_{22}	$1.20E+04_{1.37E+03}$	$1.77E+04_{1.34E+03}$	$3.54E+04_{7.42E+02}$	$\mathbf{1.18E+04}_{1.76E+03}$	$2.37E+04_{1.41E+03}$	$1.59E+04_{1.33E+03}$
f_{23}	$8.61E+02_{7.43E+01}$	$1.53E+03_{2.18E+02}$	$5.93E+03_{2.49E+02}$	$\mathbf{7.77E+02}_{2.64E+02}$	$1.46E+03_{7.63E+01}$	$1.25E+03_{1.62E+02}$
f_{24}	$2.89E+03_{1.59E+02}$	$1.98E+03_{2.25E+02}$	$9.67E+03_{4.15E+02}$	$2.83E+03_{2.27E+02}$	$2.43E+03_{1.37E+02}$	$1.62E+03_{1.32E+02}$
f_{25}	$7.61E+02_{1.06E+02}$	$7.60E+02_{5.61E+01}$	$3.36E+04_{1.82E-07}$	$2.03E+03_{3.05E+02}$	$2.69E+03_{2.96E+02}$	$1.75E+03_{2.27E+02}$
f_{26}	$\mathbf{7.66E+03}_{5.12E+02}$	$1.38E+04_{4.25E+03}$	$5.73E+04_{1.37E+02}$	$9.67E+03_{1.85E+03}$	$1.84E+04_{1.42E+03}$	$1.03E+04_{1.28E+03}$
f_{27}	$\mathbf{1.33E+02}_{6.22E+02}$	$5.00E+02_{2.19E-04}$	$3.57E+03_{4.48E+03}$	$1.74E+03_{2.23E+02}$	$5.00E+02_{2.40E-04}$	$1.39E+03_{1.27E+02}$
f_{28}	$5.76E+02_{5.95E+01}$	$5.00E+02_{8.74E+00}$	$9.22E+03_{1.33E+04}$	$2.16E+03_{6.04E+02}$	$5.00E+02_{1.98E-04}$	$2.18E+03_{6.34E+02}$
f_{29}	$\mathbf{3.28E+03}_{2.78E+02}$	$5.45E+03_{6.05E+02}$	$6.95E+06_{2.44E+06}$	$3.66E+03_{4.38E+02}$	$6.01E+03_{6.93E+02}$	$5.53E+03_{7.45E+02}$
f_{30}	$\mathbf{1.66E+05}_{9.13E+03}$	$1.46E+06_{4.94E+05}$	$5.87E+10_{4.54E+09}$	$2.50E+06_{1.45E+06}$	$6.22E+06_{6.44E+06}$	$1.88E+07_{1.73E+07}$

Fun	HWGO	WOA-mGWO	BWOA	HWOAG	MCCWOA
	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$	Mean$_{std.}$
f_1	$3.93E+11_{2.17E+10}$	$1.18E+06_{4.09E+05}$	$3.95E+06_{5.38E+06}$	$4.08E+08_{1.64E+08}$	$\mathbf{0.00E+00}_{0.00E+00}$
f_3	$5.99E+05_{4.57E+04}$	$8.45E+04_{5.56E+01}$	$5.32E+05_{3.38E+04}$	$2.29E+06_{3.74E+06}$	$\mathbf{0.00E+00}_{0.00E+00}$
f_4	$1.23E+05_{1.21E+04}$	$2.78E+02_{5.00E+01}$	$5.55E+02_{5.00E+01}$	$5.57E+02_{6.82E+01}$	$\mathbf{9.62E+00}_{2.58E+01}$
f_5	$1.99E+03_{6.48E+01}$	$8.15E+02_{6.85E+01}$	$5.65E+02_{4.23E+01}$	$5.31E+02_{8.54E+01}$	$5.03E+02_{6.23E+01}$
f_6	$1.26E+02_{3.84E+00}$	$6.40E+01_{4.04E+00}$	$5.13E+01_{2.68E+00}$	$4.82E+01_{4.89E+00}$	$6.35E+00_{3.89E+00}$
f_7	$7.79E+03_{4.57E+02}$	$2.65E+03_{2.72E+02}$	$1.41E+03_{2.34E+02}$	$9.52E+02_{3.49E+02}$	$9.18E+02_{1.60E+02}$
f_8	$2.07E+03_{6.45E+01}$	$9.37E+02_{7.95E+01}$	$7.98E+02_{4.73E+01}$	$5.81E+02_{8.37E+01}$	$5.31E+02_{7.95E+01}$
f_9	$1.34E+05_{7.54E+03}$	$2.72E+04_{2.90E+03}$	$2.40E+04_{1.94E+03}$	$8.80E+03_{6.53E+03}$	$1.22E+04_{2.30E+03}$
f_{10}	$3.06E+04_{5.66E+02}$	$1.57E+04_{1.23E+03}$	$1.36E+04_{5.70E+02}$	$2.08E+04_{6.09E+03}$	$1.19E+04_{7.07E+02}$
f_{11}	$2.37E+05_{2.08E+04}$	$1.31E+03_{1.72E+02}$	$6.81E+04_{1.29E+04}$	$4.42E+05_{1.05E+05}$	$\mathbf{6.51E+02}_{1.13E+02}$
f_{12}	$1.82E+11_{1.90E+10}$	$8.26E+07_{2.03E+07}$	$1.90E+08_{5.57E+07}$	$5.32E+08_{2.41E+08}$	$\mathbf{1.12E+04}_{7.21E+03}$
f_{13}	$3.86E+10_{3.91E+09}$	$3.65E+04_{1.26E+04}$	$2.62E+04_{4.95E+03}$	$3.01E+04_{8.84E+03}$	$2.49E+03_{5.68E+02}$
f_{14}	$7.88E+07_{1.71E+07}$	$3.02E+05_{1.26E+05}$	$1.03E+06_{4.50E+05}$	$2.61E+05_{1.44E+05}$	$\mathbf{2.89E+02}_{6.52E+01}$
f_{15}	$1.44E+10_{2.62E+09}$	$3.23E+04_{1.23E+04}$	$2.13E+04_{4.86E+03}$	$1.21E+04_{4.87E+03}$	$\mathbf{2.42E+02}_{4.71E+01}$
f_{16}	$1.76E+04_{1.27E+03}$	$5.20E+03_{7.24E+02}$	$3.80E+03_{2.96E+02}$	$4.45E+03_{6.64E+02}$	$\mathbf{3.05E+03}_{2.39E+02}$
f_{17}	$7.30E+05_{5.02E+05}$	$4.02E+03_{7.40E+02}$	$2.55E+03_{2.33E+02}$	$3.09E+03_{5.52E+02}$	$2.21E+03_{2.38E+02}$
f_{18}	$1.33E+08_{3.49E+07}$	$4.92E+05_{1.76E+05}$	$1.39E+06_{3.97E+05}$	$4.48E+05_{1.73E+05}$	$\mathbf{7.55E+02}_{7.42E+02}$
f_{19}	$1.57E+10_{3.18E+09}$	$2.54E+06_{1.08E+06}$	$9.01E+05_{4.95E+05}$	$1.76E+04_{1.32E+04}$	$\mathbf{2.41E+02}_{4.26E+01}$
f_{20}	$5.51E+03_{2.50E+02}$	$3.73E+03_{4.69E+02}$	$2.25E+03_{2.54E+02}$	$6.78E+03_{3.10E+02}$	$1.98E+03_{2.94E+02}$
f_{21}	$2.46E+03_{7.59E+01}$	$1.92E+03_{2.00E+02}$	$1.05E+03_{9.06E+01}$	$8.41E+02_{9.79E+01}$	$6.67E+02_{5.14E+01}$
f_{22}	$3.12E+04_{6.19E+02}$	$1.69E+04_{1.27E+03}$	$1.53E+04_{6.09E+02}$	$2.00E+04_{5.45E+03}$	$1.34E+04_{9.29E+02}$
f_{23}	$3.42E+03_{1.18E+02}$	$3.25E+03_{2.96E+02}$	$1.68E+03_{7.53E+01}$	$1.32E+03_{1.46E+02}$	$9.51E+02_{8.84E+01}$
f_{24}	$5.81E+03_{1.51E+02}$	$4.04E+03_{6.40E+02}$	$2.20E+03_{3.06E+02}$	$1.70E+03_{2.18E+02}$	$1.52E+03_{1.21E+02}$
f_{25}	$5.76E+04_{5.18E+03}$	$8.03E+02_{5.92E+01}$	$1.22E+03_{6.84E+01}$	$1.02E+03_{8.41E+01}$	$\mathbf{7.56E+02}_{5.97E+01}$
f_{26}	$5.25E+04_{2.40E+03}$	$2.77E+04_{2.21E+03}$	$1.96E+04_{1.64E+03}$	$1.04E+04_{2.81E+03}$	$1.10E+04_{2.55E+03}$
f_{27}	$5.00E+02_{8.76E-05}$	$5.01E+03_{1.45E+03}$	$1.33E+03_{3.03E+02}$	$1.32E+03_{2.67E+02}$	$9.52E+02_{8.36E+01}$
f_{28}	$5.00E+02_{9.56E-05}$	$6.24E+02_{3.99E+01}$	$1.03E+03_{8.60E+01}$	$7.96E+02_{6.95E+01}$	$\mathbf{4.39E+02}_{1.21E+02}$
f_{29}	$1.44E+05_{6.25E+04}$	$6.01E+03_{6.87E+02}$	$5.44E+03_{4.56E+02}$	$4.75E+03_{5.70E+02}$	$3.32E+03_{4.30E+02}$
f_{30}	$2.51E+10_{3.87E+09}$	$1.15E+07_{2.88E+06}$	$5.60E+07_{1.54E+07}$	$1.09E+07_{4.79E+06}$	$3.45E+05_{1.08E+03}$

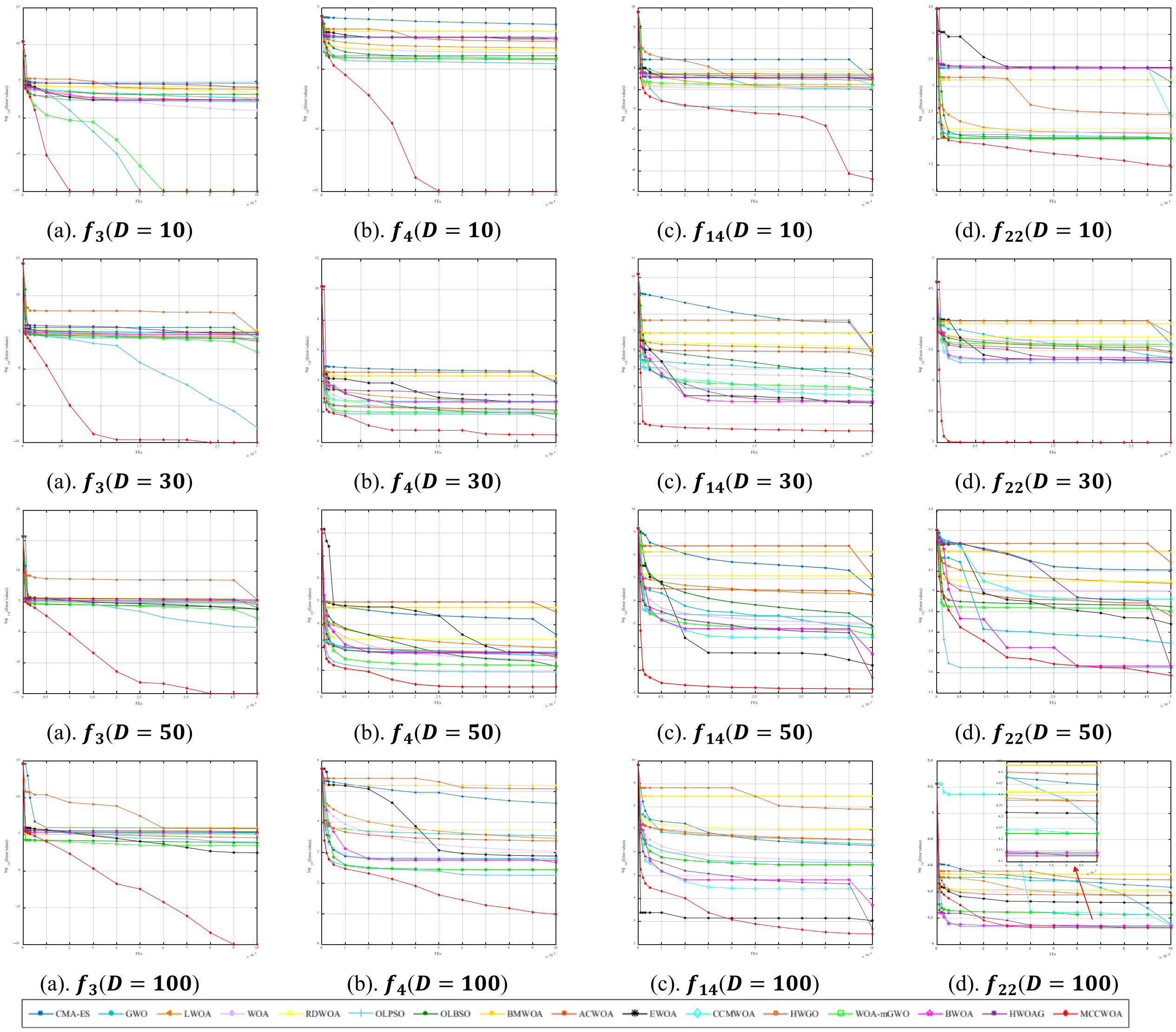


Fig. 10. The convergence curves of all algorithms on $D=$ 10,30,50,100.

Table 10
Friedman-Test results of seven algorithms.

Algorithms	Mean rank			
	$D=10$	$D=30$	$D=50$	$D=100$
CMA-ES	14.74	13.50	13.30	13.64
GWO	7.69	7.81	7.34	7.71
LWOA	10.72	11.47	11.75	11.31
WOA	10.47	9.43	9.50	8.81
RDWOA	12.14	12.36	13.04	12.95
OLPSO	3.97	3.97	3.04	2.97
OLBSO	9.00	7.66	7.41	6.28
BMWOA	15.17	15.53	15.61	15.28
EWOA	6.83	5.78	7.13	7.98
ACWOA	3.07	2.88	4.59	4.02
CCMWOA	5.64	6.40	6.61	6.43
HWGO	12.24	13.31	13.61	13.84
WOAmGWO	8.53	8.67	8.71	8.14
BWOA	3.43	4.88	5.70	7.22
HWOAG	11.28	11.02	7.11	7.19
MCCWOA	**1.09**	**1.34**	**1.57**	**2.24**
Crit. Diff $\alpha=0.1$	3.39	3.39	3.39	3.39
Crit. Diff $\alpha=0.05$	3.67	3.67	3.67	3.67

positive ranks, and $R-$ is the number of negative ranks. '$+/-/=$' respectively expressed as below. MCCWOA is superior to the comparison algorithms on the value of '$+$' functions, inferior to the comparison algorithms on '$-$' functions, and equal on '$=$' functions. If the p-value is less than α, there are significant differences in the pairwise algorithms. When the p-value is greater than or equal to α, it means that the two algorithms have no significant difference or that one is inferior to another.

From Tables 11 to 14, it is found that all p-values are less than the α on 30-dimensional and 50-dimensional functions. Therefore, the MCCWOA is significantly better than the other 15 algorithms on the two kinds of dimensions. Although significant differences are not observed between MCCWOA, OLPSO, and ACWOA on 10-dimensional and 100-dimensional functions, the number of MCCWOA superior to OLPSO and ACWOA is 16 and 19, respectively. Hence, the MCCWOA obtains better solutions than OLPSO and ACWOA in the above cases. In general, the test results on the 29 functions show that MCCWOA is significantly better than any of its comparison algorithms on the 10-dimensional, 30-dimensional, 50-dimensional, and 100-dimensional functions.

In summary, MCCWOA is significantly better than any comparison algorithm in solving complex functions. Compared with the comparison algorithms, it has the following features: (1) better optimization performance and scalability; (2) stronger universality; (3) stronger stability; (4) higher efficiency.

5. Application to the engineering problems

The performance of the proposed MCCWOA is investigated to tackle three well-known problems, including two continuous optimization problems (gear train engineering design (GTD) and

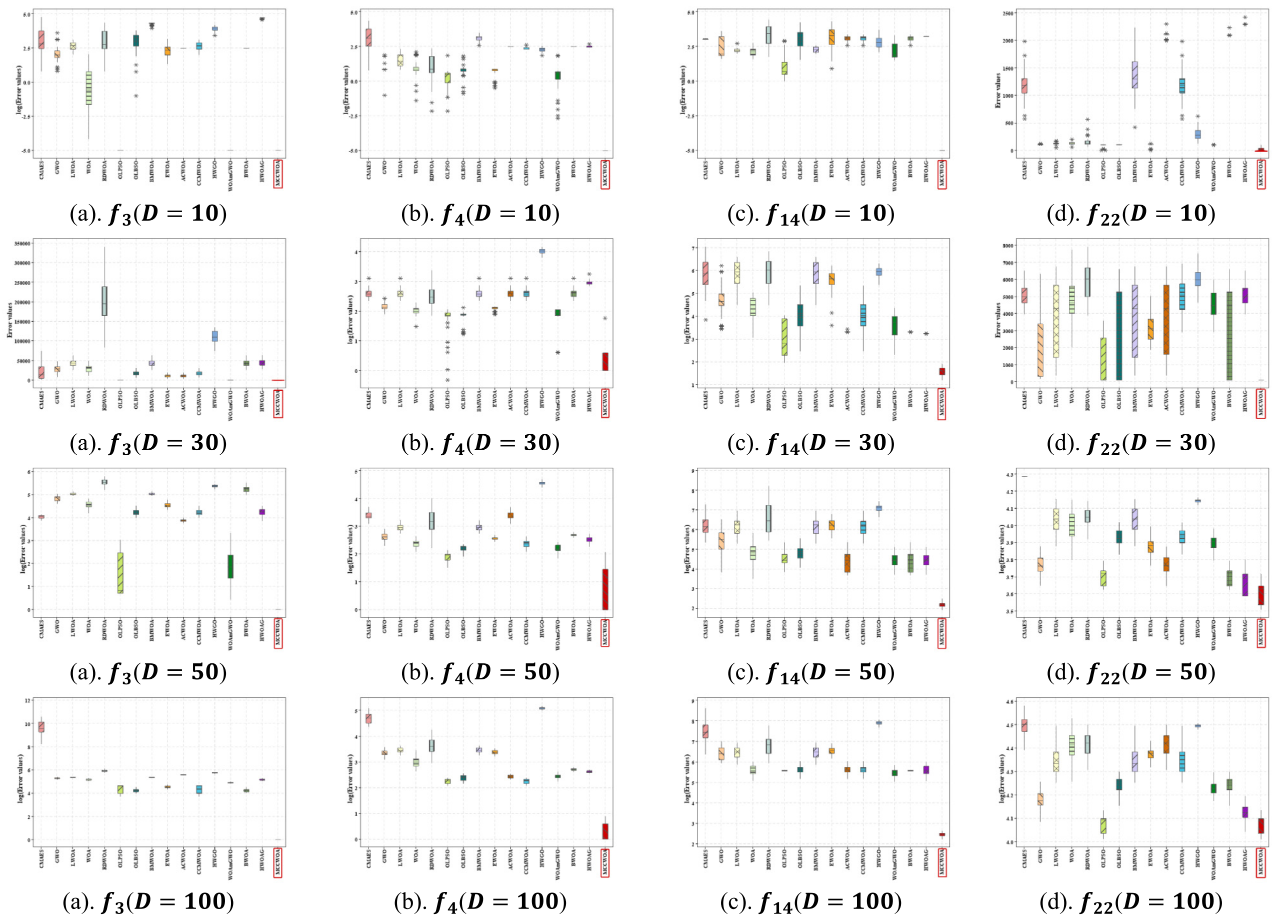


Fig. 11. The box plots of all algorithms on $D =$ 10,30,50,100.

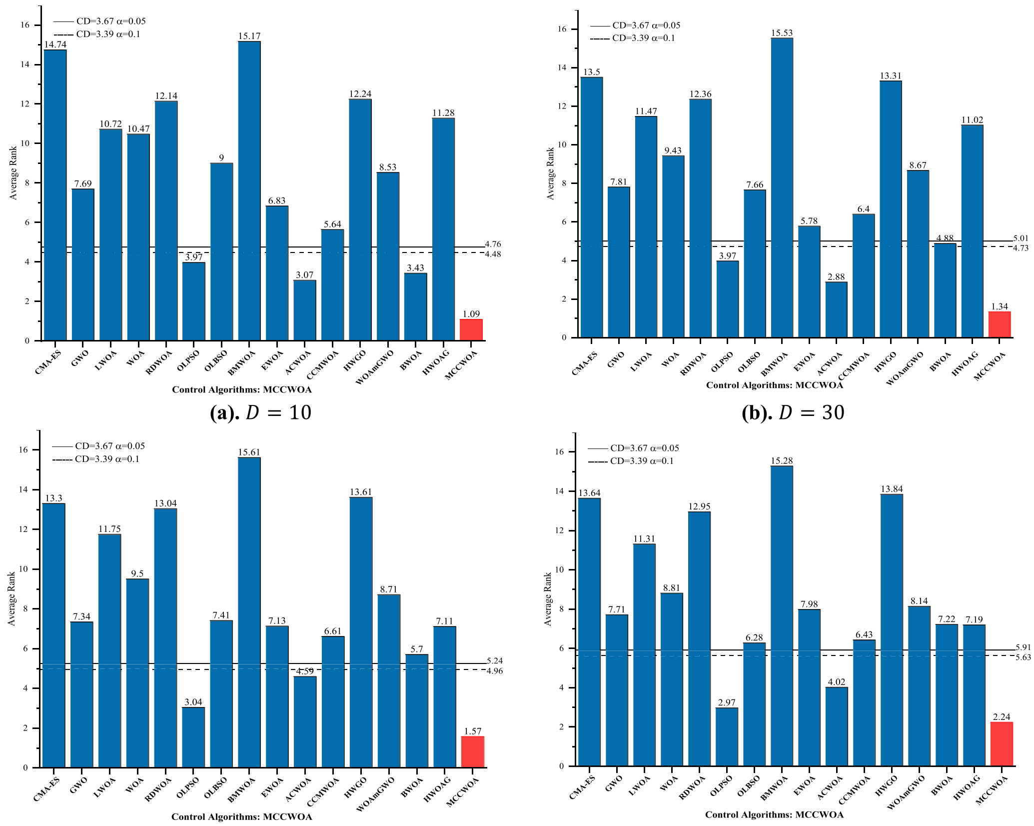
Table 11
The results of Wilcoxon's rank-sum test on $D = 10$.

MCCWOA *vs.*	+	-	=	$R+$	$R-$	Z	p−value	$\alpha = 0.05$	$\alpha = 0.1$
CMA-ES	28	1	0	432	3	−4.638	1.38E−06	Yes	Yes
GWO	27	2	0	422	13	−4.422	8.32E−06	Yes	Yes
LWOA	29	0	0	435	0	−4.703	2.00E−07	Yes	Yes
WOA	28	1	0	431	4	−4.617	1.38E−06	Yes	Yes
RDWOA	29	0	0	435	0	−4.703	2.00E−07	Yes	Yes
OLPSO	16	13	0	261	174	−0.941	7.10E−01	No	No
OLBSO	27	2	0	424	11	−4.465	8.32E−06	Yes	Yes
BMWOA	19	10	0	348	87	−4.703	2.00E−07	Yes	Yes
EWOA	28	1	0	430	5	−4.595	1.38E−06	Yes	Yes
ACWOA	19	10	0	348	87	−2.822	1.37E−01	No	No
CCMWOA	26	3	0	404	31	−4.033	4.40E−05	Yes	Yes
HWGO	28	1	0	432	3	−4.638	1.38E−06	Yes	Yes
WOAmGWO	29	0	0	435	0	−4.703	2.00E−07	Yes	Yes
BWOA	29	0	0	435	0	−4.703	2.00E−07	Yes	Yes
HWOAG	27	2	0	407	28	−4.098	8.32E−06	Yes	Yes

Table 12
The results of Wilcoxon's rank-sum test on $D = 30$.

MCCWOA *vs.*	+	-	=	$R+$	$R-$	Z	p−value	$\alpha = 0.05$	$\alpha = 0.1$
CMA-ES	27	1	1	405	1	−4.725	2.31E−06	Yes	Yes
GWO	29	0	0	435	0	−5.199	2.00E−07	Yes	Yes
LWOA	29	0	0	435	0	−5.199	2.00E−07	Yes	Yes
WOA	28	1	0	434	1	−4.828	1.38E−06	Yes	Yes
RDWOA	29	0	0	435	0	−5.199	2.00E−07	Yes	Yes
OLPSO	29	0	0	435	0	−5.199	2.00E−07	Yes	Yes
OLBSO	28	1	0	433	2	−4.828	1.38E−06	Yes	Yes
BMWOA	29	0	0	435	0	−5.199	2.00E−07	Yes	Yes
EWOA	27	2	0	431	4	−4.457	8.32E−06	Yes	Yes
ACWOA	27	2	0	429	6	−4.457	8.32E−06	Yes	Yes
CCMWOA	29	0	0	435	0	−5.199	2.00E−07	Yes	Yes
HWGO	28	1	0	434	1	−4.828	1.38E−06	Yes	Yes
WOAmGWO	29	0	0	435	0	−5.199	2.00E−07	Yes	Yes
BWOA	27	1	1	404	2	−4.725	2.31E−06	Yes	Yes
HWOAG	29	0	0	435	0	−5.199	2.00E−07	Yes	Yes

pressure vessel design (PVD)) and discrete optimization problem (blocking flow-shop scheduling problem (BFSP)). In addition, the performance of the MCCWOA is compared with canonical methods to analyze the advantages of the MCCWOA in addressing the two kinds of engineering optimization problems respectively.



(a). $D = 10$ **(b).** $D = 30$

(a). $D = 50$ **(b).** $D = 100$

Fig. 12. The results of the Friedman-test.

Table 13
The results of Wilcoxon's rank-sum test on $D = 50$.

MCCWOA *vs.*	+	-	=	$R+$	$R-$	Z	p−value	$\alpha = 0.05$	$\alpha = 0.1$
CMA-ES	27	1	1	403	3	−4.725	2.31E−06	Yes	Yes
GWO	29	0	0	435	0	−5.199	2.00E−07	Yes	Yes
LWOA	29	0	0	435	0	−5.199	2.00E−07	Yes	Yes
WOA	27	2	0	406	29	−4.457	8.32E−06	Yes	Yes
RDWOA	29	0	0	435	0	−5.199	2.00E−07	Yes	Yes
OLPSO	21	7	1	360	46	−2.457	1.40E−02	Yes	Yes
OLBSO	26	2	1	399	7	−4.347	1.38E−05	Yes	Yes
BMWOA	29	0	0	435	0	−5.199	2.00E−07	Yes	Yes
EWOA	28	1	0	429	6	−4.828	1.38E−06	Yes	Yes
ACWOA	29	0	0	435	0	−5.199	2.00E−07	Yes	Yes
CCMWOA	29	0	0	435	0	−5.199	2.00E−07	Yes	Yes
HWGO	28	1	0	432	3	−4.828	1.38E−06	Yes	Yes
WOAmGWO	29	0	0	435	0	−5.199	2.00E−07	Yes	Yes
BWOA	28	1	0	420	15	−4.828	1.38E−06	Yes	Yes
HWOAG	29	0	0	435	0	−5.199	2.00E−07	Yes	Yes

Table 14
The results of Wilcoxon's rank-sum test on $D = 100$.

MCCWOA *vs.*	+	-	=	$R+$	$R-$	Z	p−value	$\alpha = 0.05$	$\alpha = 0.1$
CMA-ES	28	1	0	432	3	−4.828	1.38E−06	Yes	Yes
GWO	27	2	0	422	13	−4.457	8.32E−06	Yes	Yes
LWOA	29	0	0	435	0	−5.199	2.00E−07	Yes	Yes
WOA	28	1	0	431	4	−4.828	1.38E−06	Yes	Yes
RDWOA	29	0	0	435	0	−5.199	2.00E−07	Yes	Yes
OLPSO	16	13	0	261	174	−0.371	7.10E−01	No	No
OLBSO	27	2	0	424	11	−4.457	8.32E−06	Yes	Yes
BMWOA	29	0	0	435	0	−5.199	2.00E−07	Yes	Yes
EWOA	28	1	0	430	5	−4.828	1.38E−06	Yes	Yes
ACWOA	19	10	0	348	87	−1.486	1.37E−01	No	No
CCMWOA	26	3	0	404	31	−4.085	4.40E−05	Yes	Yes
HWGO	28	1	0	432	3	−4.828	1.38E−06	Yes	Yes
WOAmGWO	29	0	0	435	0	−5.199	2.00E−07	Yes	Yes
BWOA	29	0	0	435	0	−5.199	2.00E−07	Yes	Yes
HWOAG	27	2	0	407	28	−4.457	8.32E−06	Yes	Yes

5.1. GTD problem

The GTD problem [156] is utilized to verify the performance of the MCCWOA in addressing engineering problems. GTD aims to minimize the gear ratio of the gear train as shown in Fig. 13. There are four types of parameters in the gear train engineering design problem. The details of the mathematical model about this problem are described as follows. More details can be found in [156]. The model is explained as follows:

Decision variable:

$$\vec{g} = [g_1, g_2, g_3, g_4] = [M_A, M_B, M_C, M_D] \tag{23}$$

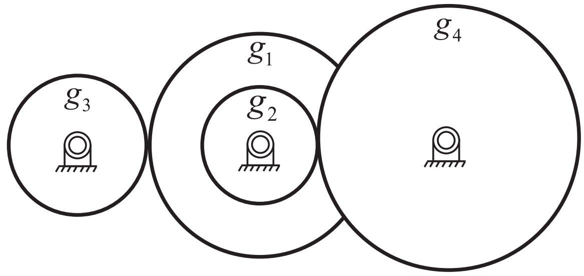


Fig. 13. Geat train design problem.

Table 15
The results of the GTD problem.

Algorithms	Sandgren	GeneAS	ABC	WOA	MCCWOA
g_1	6.00E+01	5.00E+01	4.90E+01	5.11E+01	6.00E+01
g_2	4.50E+01	3.00E+01	1.60E+01	1.90E+01	4.33E+01
g_3	2.20E+01	1.40E+01	1.90E+01	2.75E+01	1.20E+01
g_4	1.80E+01	1.70E+01	4.30E+01	7.10E+01	6.00E+01
$f(\vec{g})_{min}$	1.47E−01	1.44E−01	2.70E−12	3.92E−18	**2.92E−19**

Objective :

$$f(\vec{g})_{min} = (\frac{1}{6.931} - \frac{g_2 g_3}{g_1 g_4})^2 \tag{24}$$

Subject to:

$$12 \leq g_1, g_2, g_3, g_4 \leq 60 \tag{25}$$

The MCCWOA is run in 1000 fitness evaluations. The obtained statistical results for the GTD problem are compared in Table 15. More details about Sandgren, GeneAS, and ABC can be found in [156]. It is observed from Table 15 that the MCCWOA outperforms other algorithms.

5.2. PVD problem

The PVD problem [157] is intended as reducing the cost of the cylindrical pressure vessel. Such variables are described in [157] to understand the relation and impact of each rule in the final objective. The constraints make the feature space more restricted when obtaining the costs of design, and constraints try to rule the optimizer to do not violate some limits of the variables. Therefore, the variables are inner radius (R), the thickness of the head (T_h), thickness of the shell (T_s) and length of cylindrical section of the vessel, not including the head (L). The model is explained as follows:

Decision variable:

$$\vec{x} = [x_1, x_2, x_3, x_4] = [T_s, T_h, R, L], x_1, x_2 \in [0, 99], x_3, x_4 \in [10, 200] \tag{26}$$

Objective :

$$f(\vec{x})_{min} = 0.6224x_1x_3x_4 + 1.7781x_2{x_3}^2 + 3.1661x_1^2x_4 + 19.84{x_1}^2x_3 \tag{27}$$

Subject to:

$$g_1(\vec{x}) = -x_1 + 0.0193x_3 \leq 0 \tag{28}$$

$$g_2(\vec{x}) = -x_2 + 0.00954x_3 \leq 0 \tag{29}$$

$$g_3(\vec{x}) = -\pi {x_3}^2 x_4 - \frac{4}{3}\pi {x_3}^3 + 1296000 \leq 0 \tag{30}$$

$$g_4(\vec{x}) = x_4 - 240 \leq 0 \tag{31}$$

The obtained statistical results for the PVD problem are listed in Table 16. The results are compared with those of IHS [158], PSO, GA, Lagrange multiplier [159], and branch-and-bound (B&b) [160]. It is observed from Table 16 that the MCCWOA outperforms other algorithms.

5.3. The BFSP

The blocking flow shop scheduling problem (BFSP) [161] which is one of the most important scheduling types, is widespread in modern industries. In the BFSP there are no buffers between machines, the job remains in the current machine until the next machine is available for processing. With the increase of the scheduling scale, the difficulty and computation time of solving the problem will increase exponentially. Existing experiments and literature have shown that BFSP with more than two machines is a typical NP-hard problem [48,49].

The definitions of the variables and parameters mentioned in this section are recorded in Table 17. The details of the mathematical model of the BFSP are described as follows.

Decision variable :

$$x_{i,k} \in \{0, 1\}, (i, k = 1, 2, \ldots, n) \tag{32}$$

Objective :

$$\text{Min } C_{max} = \max_{k=1,2,\ldots,n} \left(D_{k,m}\right) \tag{33}$$

Subject to:

$$\sum_{k=1}^{n} x_{i,k} = 1, i \in \{1, 2, \ldots, n\} \tag{34}$$

$$\sum_{i=1}^{n} x_{i,k} = 1, k \in \{1, 2, \ldots, n\} \tag{35}$$

$$D_{1,0} \geq 0 \tag{36}$$

$$D_{k,0} \geq D_{k-1,1}, (k = 2, 3, \ldots, n) \tag{37}$$

$$D_{k,j} \geq D_{k,j-1} + \sum_{i=1}^{n} x_{i,k} \cdot P_{i,j}, (k = 1, 2, \ldots, n), (j = 1, 2, \ldots, m) \tag{38}$$

$$D_{k,j} \geq D_{k-1,j+1}, (k = 2, 3, \ldots, n), (j = 1, 2, \ldots, m-1) \tag{39}$$

5.3.1. Mathematical model of the BFSP

In this study, the objective is to minimize the makespan criterion. The makespan of the schedule π is $C_{max} = D_{n,m}$. The computation complexity of this task is $O(n * m)$.

$$[P_{i,j}]_{3\times 3} = \begin{bmatrix} P_{1,1} & P_{1,2} & P_{1,3} \\ P_{2,1} & P_{2,2} & P_{2,3} \\ P_{3,1} & P_{3,2} & P_{3,3} \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 3 & 2 \\ 2 & 1 & 3 \end{bmatrix} \tag{40}$$

Here, an example is presented to show how the decision variables reflect the solution by considering a problem with three jobs ($n = 3$) and three machines ($m = 3$). The processing times are given in Eq. (40). The scheduling Gantt is shown in Fig. 14. The makespan of schedule sets $\pi_1 = \{J_1, J_2, J_3\}$ in the BFSP is 13.

5.3.2. Experimental settings and analysis

The traditional WOA algorithm and its variants cannot be directly utilized to solve the combinational optimization problem with discrete characteristics. Therefore, the coding scheme and decoding rule is utilized to help the algorithms work directly in the discrete domain by representing individuals as discrete job permutations. In this paper, the LOV rule is utilized to represent individuals as discrete job permutations. More details about LOV rule can be found in [162]. The average relative percentage deviation (ARPD) index is utilized to measure the results, the calculate method is shown in Eq. (41), where R is the number of runs, C_i is the solution generated by a specific algorithm in the ith experiment for a given instance. C_{opt} is the minimum

Table 16
The results of the PVD problem.

Algorithms	IHS	GA	Lagrange multiplier	B&b	MCCWOA
$T_s(x_1)$	1.125	0.93750	1.12500	1.12500	0.8755
$T_h(x_2)$	0.62500	0.50000	0.62500	0.62500	0.50000
$R(x_3)$	58.29015	48.32900	58.29100	47.70000	42.0939
$L(x_4)$	43.69268	112.6790	43.69000	117.7100	177.0805
$g_1(\vec{x})$	0.00000	−0.004750	0.000016	−0.204390	−0.00309
$g_2(\vec{x})$	−0.06891	−0.038941	−0.068904	−0.169942	−0.09842
$g_3(\vec{x})$	−2.01500	−3652.877	−21.220104	54.226012	−2119.15
$g_4(\vec{x})$	−196.307	−127.3120	−196.3100	−122.299000	−62.9195
$f(\vec{x})_{min}$	7197.730	6410.381	7198.043	8129.104	**6206.957**

Table 17
The definitions of the notations.

Notations	Definition
n	Number of jobs.
m	Number of machines.
i	The index of the job, $i = 0, 1, 2, \ldots, n$. 0 is a virtual job.
j	The index of the machine, $j = 1, 2, \ldots, m$.
π	A feasible scheduling job sequence, $\pi = \{\pi(1), \pi(2), \ldots, \pi(n)\}$.
$P_{i,j}$	Processing time of job i on machine j.
k	The index of a certain job in the job sequence, $k \in \{1, 2, \ldots, n\}$.
$D_{k,0}$	Start processing time of the kth job on the first machine.
$D_{k,j}$	Departure time of the kth job on machine j.
$x_{i,k}$	The binary variable that takes value 1 if the kth job of π is i, and 0 otherwise.
C_{max}	The maximum assembly completion time of the schedule π.
$\{J_1, J_2, \ldots, J_n\}$	Job sequence.
$\{M_1, M_2, \ldots, M_m\}$	Machine sequence.

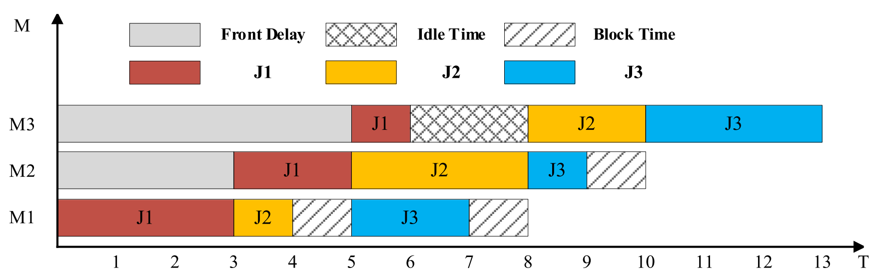


Fig. 14. Gantt Chart for a solution to the example problem.

Table 18
Parameter settings of L-SHADE and LSHADE-cnEpsin.

Algorithms	Authors	Parameter settings
L-SHADE	Tanable, et al. [164]	$N^{init} = 18 \cdot D, H = 6, \mu F = 0.5, \mu Cr = 0.5, N^{min} = 4$
LSHADE-cnEpsin	Awad, et al. [146]	$N^{init} = 18 \cdot D, H = 5, \mu F = 0.5, \mu Cr = 0.5, N^{min} = 4, freq = 0.5$

makespan found by all algorithms. The algorithm with minimum ARPD outperforms other algorithms.

$$ARPD = \frac{1}{R} \times \sum_{i=1}^{R} \frac{C_i - C_{opt}}{C_{opt}} \times 100\% \tag{41}$$

The well-known standard benchmark set of Taillard [163] is used for evaluating the performance of MCCWOA. This benchmark is composed of 120 different problem instances. The instances are categorized into 12 subsets of different combinations of n (number of jobs) and m (number of machines). These combinations range from 20 jobs and 5 machines up to 500 jobs and 20 machines. The results of WOA, LWOA, RDWOA, CMA-ES, L-SHADE [164], LSHADE-cnEpsin [146], and MCCWOA are shown in Table 19. Each algorithm is run in $10M \times N$ milliseconds (ms). The parameters of the comparison algorithms (L-SHADE and LSHADE-cnEpsin) are shown in Table 18, and more details can be found in [51,52]. The parameters of the other comparison algorithms are consistent with the previous ones. L-SHADE and LSHADE-cnEpsin are excellent variants of the mainstream SI algorithm DE. Especially, LSHADE-cnEpsin is the winning algorithm for the bound-constrained continuous optimization problems in CEC2017.

The mean plot with a 95% confidence interval for instances of MCCWOA and the compared algorithm is shown in Fig. 15. The numbers 1 to 7 of horizontal represent the different algorithms. The vertical axis was the ARPD values for the instances. The results obtained by the MCCWOA are better than the other compared algorithms in all groups and the ARPD achieved by the MCCWOA on average are 0.1407, 0.1151, 0.0760, 0.1617, 0.1347, and 0.1043, respectively. It is seen that the mean plot of the MCCWOA is under the plots of the other compared algorithms and the values of ARPD for the MCCWOA again illustrated the performance of the MCCWOA. The number of jobs and machines have a significant impact on all the compared algorithms from Table 19. The experiment results show the excellent performance of the MCCWOA for solving the BFSP. The effectiveness of MCCWOA is that coevolution combined the advantages of WOA and comprehensive learning from the OL and characteristic subpopulations. Specifically, the search direction is guided more purposefully by the two-stage OL to improve the searching efficiency. The information of characteristic subpopulations is better utilized to generate more potential scheduling solutions by the multipopulation cooperative strategy. Therefore, MCCWOA is a competitive algorithm for solving BFSP.

In summary, the optimization algorithm not only can be effectively applied to these kinds of engineering optimization problems but also has important significance in management. Massive systems require performing a series of decision-making processes [165,166]. With the rapid development of the economy, the days

Table 19
The ARPD of all compared algorithms.

$n*m$	WOA		LWOA		RDWOA		CMA-ES		L-SHADE		LSHADE-cnEpsin		MCCWOA	
	ARPD	SD	ARPD	SD	ARPD	SD	ARPD	SD	ARPD	SD	ARPD	SD	ARPD	SD
20 × 5	0.659	0.099	0.509	0.081	0.633	0.089	1.101	0.133	1.218	0.142	0.355	0.073	**0.140**	0.042
20 × 10	0.659	0.110	0.504	0.063	0.618	0.113	1.051	0.135	1.093	0.144	0.332	0.089	**0.115**	0.032
20 × 20	0.529	0.082	0.324	0.064	0.503	0.101	0.807	0.125	0.775	0.125	0.252	0.070	**0.076**	0.022
50 × 5	0.638	0.121	0.467	0.115	0.258	0.085	1.056	0.118	1.103	0.121	0.832	0.119	**0.161**	0.077
50 × 10	0.558	0.121	0.374	0.111	0.317	0.070	0.917	0.123	0.931	0.129	0.327	0.087	**0.134**	0.075
50 × 20	0.548	0.080	0.350	0.074	0.330	0.061	0.856	0.084	0.834	0.080	0.130	0.041	**0.104**	0.034

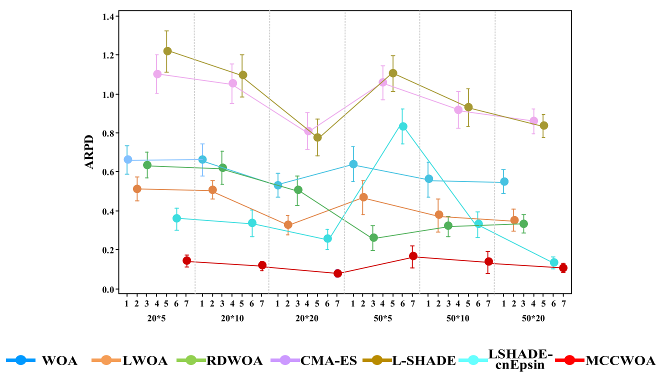


Fig. 15. Mean plot with 95% confidence interval for all compared algorithms.

of relying only on experience toward challenges have gone forever. In terms of a managerial perspective, especially in the field of operational research optimization, there is an urgent need for a series of efficient optimization algorithms. On the one hand, the combinatorial optimization problems in production management are addressed via optimization algorithms to improve the utilization of production resources and reduce the cost of inventory and logistics. On the other hand, effective decision support for management is provided via optimization algorithms [165,167]. Additionally, Certain methods applied in the field of the supply chain are also an important direction to solve this kind of optimization problem. For deep reviews of other solution methods, consider the below Refs. [165–184].

6. Conclusions and future research

A two-stage OL framework with the mechanism of multi-population cooperative coevolution is proposed in this paper to improve WOA. A prediction model of the neighborhood structure is established to guide the search direction of the offspring population via the first OL in the convergence operation. Another OL mechanism is introduced to quickly and effectively jump out of the optimal position by the auxiliary vector pool when the individual sank into a stagnant state. The self-learning and self-organizing capability of the proposed algorithm are strengthened by the cooperation of the multipopulation cooperative strategy and the OL mechanisms.

From the results for the CEC2017 benchmarks, the MCCWOA indeed obtains significant improvements over the other 15 state-of-the-art algorithms. In particular, the proposed MCCWOA significantly outperforms the other comparison algorithms when solving 10-dimensional and 30-dimensional test instances, obtaining better results in terms of the mean on 28 out of 29 instances. The performance of this result on 50-dimensional instances and 100-dimensional instances is 22/29 and 15/29, respectively. Therefore, the cooperative strategy and OL mechanisms are regarded as an effective direction for improving the performance of the basic WOA. MCCWOA is an effective algorithm to solve continued optimization problems including the GTD problem and the PVD problem. On the other hand, the proposed MCCWOA is applied to solve BFSP effectively. The results for the benchmark set of Taillard demonstrate that the proposed algorithm has good performance.

Despite the excellent performance, the following issues need to be solved in future work: (1) the two-stage OL strategy can flexibly switch to make the OL strategies adapt to different search stages; (2) the characteristic populations are divided more reasonably (e.g., dynamic adjustment according to the evolutionary state of the population); (3) the actual production optimization problems in management should be addressed via the improved WOA. Search strategies with problem-specific knowledge in the framework of WOA should be designed for actual industrial application scenarios.

CRediT authorship contribution statement

Fuqing Zhao: Funding acquisition, Investigation, Supervision. **Haizhu Bao:** Investigation, Software, Original draft, Experiments of the algorithms. **Ling Wang:** Methodology, Resources. **Jie Cao:** Project administration, Review. **Jianxin Tang:** Conceptualization, Formal analysis. **Jonrinaldi:** Visualization, Editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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