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ORIGINAL RESEARCH PAPER

Hybrid biogeography-based optimization with enhanced mutation and CMA-ES for global optimization problem

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Abstract
In recent years, scheduling problems have attracted enormous attentions from practitioners and researches in manufacturing systems, for instance, the scheduling of computing resource in cloud infrastructure and cloud services. The scheduling problems in cloud services, big data and other service-oriented computing problems are regarded as non-separable problems. In this paper, a hybrid biogeography-based optimization with the enhanced mutation operator and CMA-ES (HBBO-CMA) is proposed to enhance the ability of exploitation on non-separable problems and alleviate the rotational variance. In the migration operator, the rotationally invariant migration operator is designed to reduce the dependence of BBO on the coordinate system and control the diversity of population. In the mutation operator, an enhanced mutation operator, which is sampled from the mean value and stand deviation of the variables of population, is employed to effectively escape the local optimum. Furthermore, the CMA-ES, which has outstanding performance on the non-separable problem, is applied to extend the exploitation of HBBO-CMA. Experimental results on CEC-2017 demonstrated the effectiveness of the proposed HBBO-CMA.

Keywords Biogeography-based optimization · CMA-ES · Non-separable problems · Enhanced mutation operator · Rotational variance

1 Introduction

The optimization and allocation of resources have become significantly important as the challenge of limited land, labor force, goods and materials. The SOC is a new computing mode, which takes services as the basic components to support the application combination of fast, low-cost and simple distributed or even heterogeneous environments. In general, SOC problems can be regarded as combinatorial optimization problems. Thus, it is possible to transform all the combinational problems into numerical optimization problems with one or more objective functions. Single-objective optimization plays a vital role in optimization problems. The single-objective optimization problems are formulated as the following model. $\min f(x), x = (x_1, x_2, x_3, \ldots, x_D)$, where D is the dimension number of the variables optimized.

Biogeography-based optimization (BBO), which was proposed by Simon [1], is demonstrated as a simple and powerful meta-heuristic algorithm. In [1], BBO also has various features that are unique among biology-based optimization methods. The basic theory of BBO is that the migration behavior and mutation behavior of species in different habitats are emulated to solve global optimization problems. BBO has received increasing attention in the research field with the unique mechanism and the desirable ability of exploration. However, there are certain drawbacks in BBO, such as insufficient exploitation ability and rotationally variance. Therefore, various developments have been proposed to enhance the performance of BBO. The theoretical analysis of BBO, the improvements of

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BBO and the applications of BBO were introduced in [2]. The central developments of BBO are summarized as follows. (a) The primary theoretical analysis of BBO. (b) The research of the operational mechanism of BBO. (c) The combination of BBO and other meta-heuristic algorithms to overcome the weakness of BBO. (d) The solutions of the complex combinational problems with BBO.

In BBO, the superior candidates tend to share their information with the inferior candidates. However, the variables which have been shared to the inferior candidates still exist in the superior candidates. The phenomenon leads to the insufficient exploitation ability and the poor rotational invariant of BBO. In spite of the fact that various researches of BBO were studied in recent years, little literature has been paid attention to utilizing the information of population to enhance the mutation operator. Furthermore, as the performance of BBO on non-separable problems is weak, the research for improving the performance of BBO on non-separable problems remains a significant area to be explored. This paper aims to make a contribution in this regard.

In this paper, a hybrid biogeography-based optimization with enhanced mutation operator and CMA-ES (HBBO-CMA) is proposed to alleviate the rotational variance and enhance the performance of BBO on non-separable problems. The innovations of the HBBO-CMA are summarized as follows.

(1) A rotationally invariant migrate operator is designed to enhance the rotational invariance and extend the exploration of BBO.
(2) An enhanced mutation operator is presented to help the population effectively escape the local optima and increase the exploration of BBO.
(3) The CMA-ES is introduced to search from the current optimal candidates and extend the exploitation of BBO on non-separable problems.

The experimental results reveal that the HBBO-CMA algorithm is superior to the compared variants of BBO tested on the CEC-2017 benchmark.

The remainder of this paper is organized as follows. The literature review on BBO is summarized in Sect. 2. The brief description of BBO is given in Sect. 3. The proposed algorithm HBBO-CMA is elaborated in Sect. 4. The experimental studies are listed in Sect. 5. Finally, Sect. 6 concludes the paper and future work is suggested.

2 Literature review

Various modifications, which are aimed to reinforce the accuracy and convergence speed of the BBO, have been proposed. In this paper, the development of BBO with focus on fundamental theory research, operational mechanism, hybrid framework and application filed is summarized.

The basic theory research plays an essential role in the meta-heuristics. In recent years, there are various researchers who have paid attention to the primary theoretical analysis of the BBO. The influence of five migration models, which are inspired by the mathematical model of biogeography theory, was analyzed by Ma [3]. From the experimental results, the nonlinear models were superior to the linear models. The expected hitting time of BBO was investigated by Guo et al. [4] with drift analysis. The simulation results indicate that the migration model, which generates a high mutation rate, enhanced the performance of BBO effectively. From the perspective of statistical mechanics, the mathematical description of the dynamics of BBO was proposed by Ma et al. [5].

The research of the operational mechanism of algorithms is also important to increase the performance of the algorithms. In the past few years, the development of the operational mechanism for the BBO is widely employed to enhance the performance of the BBO. Three migration operators were designed by Guo et al. [6] to improve the exploration ability of BBO. Markov analysis is conducted to confirm the advantages of migration operators. Experimental results demonstrate that the three migration operators are feasible and effective. A two-stage differential biogeography-based optimization (TDBBO), which combines a two-stage migration model and a rotationally invariant arithmetic crossover operator, was proposed by Zhao et al. [7] to address the premature convergence and alleviate the rotational variance. The efficiency and effectiveness of TDBBO are demonstrated with the experimental results. A random ring topology, a modified mutation operator and a self-adaptive Powell's method are embedded into the PRBBO by Feng et al. [8]. The simulation demonstrates that the exploration and exploitation ability of PRBBO are balanced effectively.

The mechanism of algorithm combined with other algorithms attracts great efforts from researches and practitioners in the manufacturing system. The hybrid framework, which combines the BBO and other methods to overcome the weakness of BBO, is important and efficient for the improvement of the BBO algorithm. A biogeography-based learning particle swarm optimization (BLPSO), which combines the BBO and PSO, was proposed by Chen et al. [9] for solving global optimization problems. Experimental results indicate the efficiency and effectiveness of BLPSO on CEC-2014 benchmark functions. A hybrid IWO/BBO, which mixed the migration operator of BBO and invasive weed optimization, was proposed by Khademi et al. [10] for solving the numeric optimization problems. The hybrid IWO/BBO outperforms other state-of-the-art algorithms. The covariance matrix-based migration was designed by Chen et al. [11] to alleviate the rotational variance of BBO. The numeric simulation shows that the modification reduces the dependence of BBO on the coordinate system effectively. The MpBBO was designed by Al-Roomi et al. [12] to combine the exploitation ability of BBO and the exploration ability of simulated annealing (SA). The simulation demonstrates that MpBBO has strong immunity against being trapped into the local optimums.

The BBO algorithm and the modifications of BBO have been widely applied to various applications, for instance, intelligent optimization, global optimization, non-separable problems and intelligent simulation and scheduling. An effective hybrid discrete biogeography-based optimization (HDBBO) was proposed by Lin et al. [13] for solving the permutation flow-shop problem with the makespan criterion. Computational results show the effectiveness of the HDBBO. An improved biogeography-based optimization was proposed by Liu et al. [14] for solving the blocking flow-shop scheduling problem (BFSP) with maximum completion time criterion. The worst opposition learning and random-scaled differential mutation (WRBBO) was designed by Zhang et al. [15] to cluster optimization and medical image segmentation. The experimental results show that WRBBO outperforms state-of-the-art BBO variants and other related algorithms. A hybrid biogeography-based optimization with variable neighborhood search (HBV) was proposed by Zhao et al. [7] for solving the no-wait flow-shop problems (NWFSP) with the makespan criterion. The computational results show that the efficiency and performance of HBV for the NWFSP. Thus, for different kinds of service oriented, algorithms are used to solve different kinds of SOC problems quickly and cheaply.

3 Biogeography-based optimization

The BBO is an iterative optimization algorithm, which explores the candidate solutions by simulating the migration, mutation and annihilation of the species distributed in different habitats. Each candidate is considered as a habitat with a habitat suitability index (HSI). The suitability index variables (SIVs) denote the features of the candidate. In the BBO algorithm, the new candidates are generated by employing migration and mutation operators. The BBO algorithm is briefly described as follows.

The *SIVs* are mixed among the habitats based on the immigration rate λ and the emigration rate μ in the migration operator. The immigration rate λ_k and emigration rate μ_k of each solution are different. As the sinusoidal migration model, λ_k and μ_k are calculated as follows:

$$\lambda_k = \frac{I}{2} \cdot \left(1 + \cos\left(\frac{k\pi}{n}\right)\right) \tag{1}$$

$$\mu_k = \frac{E}{2} \cdot \left(1 - cos\left(\frac{k\pi}{n}\right)\right) \tag{2}$$

where I and E are the maximum immigration rate and emigration rate, respectively. The k denotes the species count of the kth habitat. n represents the maximum count of species. In this paper, the maximum species count is equal to the population size. The I and E are set to one.

The mutation operator is employed to modify the SIVs in the BBO algorithm. The mutation rate m_k of the kth candidate is dynamically calculated as the migration model. The mutation rate m_i is expressed as follow:

$$m_k = m_{\max} \cdot \left(\frac{1 - p_k}{p_{max}}\right) \tag{3}$$

where $m_{\max}$ denotes the maximum mutation probability. p_k is the probability that the single habitat just contains k species. $p_{max} = \max\{p_k, i = 1, 2, \dots n\}$. The standard BBO algorithm is given in Algorithm 1.

Algorithm 1. Biogeography-based Optimization

1. Initial the population H.
2. Evaluate *HSI* for each individual in H.
3. **while** the halting criterion is not satisfied **do**
4. Calculate immigrate rate λ_k and emmigrate rate m_k for each candidate H_k.
5. Perform the migration operator.
6. Perform the mutation operator.
7. Update the best solution.
8. **end while**

4 Hybrid biogeography-based optimization with CMA-ES

4.1 Modification of BBO

In spite of the effectiveness of the BBO for solving the multimodal problems, the search performance of BBO is undesirable in solving the non-separable problems. Therefore, the performance of BBO is possible to improve further by analyzing the BBO from the viewpoint of search space. In this paper, two modifications are made to enhance the migration operator and mutation operator, respectively. In the standard migration operator, the SIVs of the inferior candidate are replaced by the SIVs of excellent candidates directly. The standard migration operator improves the quality of the poor candidates and enhances the exploitation ability of algorithm effectively. However, the diversity of the population is reduced because the identical features exist in several candidate solutions. In this paper, a rotationally invariant migrate operator is designed to enhance the exploration ability of BBO and avoid the rotational variant. The rotationally invariant migrate operator is given as follow:

$$U_k(j) = H_{\text{emmigrate}}(j) + F \cdot \left(H_{r1}(j) - H_{r2}(j)\right) \tag{4}$$

where $H_{\text{emigrate}}(j)$ is the j th dimension of the emigrate candidate H_{emigrate}, $H_{r1}(j)$ and $H_{r2}(j)$ are the j th dimension of the random candidates, respectively. It is worth noting that $H_{r1} \neq H_{r2} \neq H_{\text{emigrate}}$. F denotes the scaling factor associated with the migration model. The value F is updated as follow:

$$F_k = \lambda_k \times \text{normrnd}(0, 1) \tag{5}$$

where λ_i is the immigrate rate of the i th candidate. The normrnd(0, 1) is a random number which obeying the standard normal distribution.

The mutation operator plays an essential role in the BBO algorithm. The SIVs of the candidate are replaced by the random number directly in the standard mutation operator. However, the standard mutation operator leads to the slow convergence speed of BBO. In this paper, an enhanced mutation operator, which is based on the mean value μ_d and the standard deviation σ_d, is designed to replace the standard mutation operator. The enhanced mutation operator is described as follow:

$$U_k(j) = \mu_d(j) + \sigma_d(j) \times \text{normrnd}(0, 1) \tag{6}$$

4.2 CMA-ES

The CMA-ES [16] algorithm is an optimization algorithm which sampling η candidates from a multivariate normal distribution, and then, several excellent candidate is selected from the η candidates to adaptively estimate the local covariance matrix of the problem to increase the probability of successful samples in the next iteration. In this paper, the CMA-ES is employed to perform a local search. The parameters of standard CMA-ES are adopted to maintain the effectiveness of CMA-ES for solving the unimodal problems. However, the parameter η, which is the parameter λ in standard CMA-ES, is revised as $\eta = 3 \ln D + 80$.

4.3 Steps of HBBO-CMA

Since the BBO algorithm and CMA-ES algorithm have advantages in solving multimodal problems and unimodal problems, respectively, the combination of BBO and CMA-ES is effective for solving independent unimodal or multimodal problems. In this paper, the HBBO-CMA is proposed for solving the non-separable problems.

There are four cascade steps in the HBBO-CMA. The first step is the rotationally invariant migration operator. Once the population is uniformly initialized within the search space, the migration operator is performed to share information among the population. The second step is the mutation operator. The mutation mechanism of BBO is different from the mutation mechanism of other EAs. The best candidates and the worst candidates of the population have the maximum mutation probability. The middle candidates have the minimum mutation probability because the middle candidates are stable in general. Therefore, the enhanced mutation operator leads to the worst candidates approaching the middle candidates quickly and helps the best candidates escape the local optimal. The third step is the greedy selection strategy. The greedy selection strategy accelerates the convergence speed of HBBO-CMA. The final step is the implementation of CMA-ES.

In this paper, the CMA-ES is performed to search from the global optimal candidate. The main reason is that an excellent initialization of CMA-ES makes the algorithm robust to various disturbances. The CMA-ES is performed after the current number of function evaluations (NFEs) more than half of the maximum number of function evaluations because the CMA-ES consumes various NFEs. The complete pseudocode of the modified BBO is shown in Algorithm 2.

Algorithm 2. HBBO-CMA

```
Initial the population H.
Evaluate HSI for each individual in H.
while the halting criterion is not satisfied do
    Calculate immigrate rate λ_k and emmigrate rate m_k for each candidate H_k.
    for each candidate k ∈ [1, H] do
        Select H_emigrate with probability ∝ μ.
        for each dimension j ∈ [1, D] do
            if rand<λ_k then
                Perform rotationally invariant migrate operator.
            endif
            if rand<m_k then
                Perform the enhanced mutation operator.
            endif
        end for
    end for
    Perform the greedy selection strategy.
    if current_NFEs>0.5*Max_NFEs then
        Perform the CMA-ES algorithm.
    end if
    Update the best solution.
end while
```

5 The experiments and comparisons

The performance of HBBO-CMA is evaluated on the CEC-2017 single-objective bound constrained problems benchmark. The compared algorithms are as follows: CMA-ES [16], BSA [17], IWO [18], BBO [1].

5.1 Benchmark functions

The 29 test functions of CEC-2017 benchmark include unimodal functions f_1 and f_3, multimodal functions $f_4 - f_{10}$, hybrid functions $f_{11} - f_{20}$ and composition functions $f_{21} - f_{30}$. The hybrid function means the different variables of candidate own different properties.

5.2 Experimental settings

To make a fair comparison, the maximum fitness of function evaluations is set to $\mathrm{D} \times 10,000$ based on the guidelines provided in the special session of CEC-2017 [19]. The contrast algorithms were carefully reimplemented in Matlab programming language and run on a PC with a 3.4 Ghz Intel(R) Core(TM) i7-6700 CPU, 8 GB of RAM and 64-bit OS.

5.3 Statistical results

The statistical results of HBBO-CMA and the compared algorithms in 10D and 30D are listed in Tables 1 and 2. The error values and standard deviations smaller than 1e-8 are zero. The convergence curves of HBBO-CMA and compared algorithms are demonstrated in Fig. 1. The box-plots of HBBO-CMA and other compared algorithms are shown in Fig. 2. The statistical analysis results of the Wilcoxon's test are listed in Table 3.

5.4 Parameter analysis and settings

The analysis of parameters for the HBBO-CMA is shown as follows. The combinations of parameters are listed in Table 4. The importance of the parameter is illustrated in Table 5. The orthogonal array of the parameters is shown in Table 6.

Table 1 The results of the compared algorithm for $D=10$

Func	CMA-ES Mean$_{Std.}$	BSA Mean$_{Std.}$	IWO Mean$_{Std.}$	BBO Mean$_{Std.}$	HBBO-CMA Mean$_{Std.}$
1	**0.00E+00**$_{\mathbf{0.00E+00}}$	**0.00E+00**$_{\mathbf{0.00E+00}}$	3.25E+03$_{3.41E+03}$	3.61E+04$_{2.40E+04}$	**0.00E+00**$_{\mathbf{0.00E+00}}$
3	**0.00E+00**$_{\mathbf{0.00E+00}}$	**0.00E+00**$_{\mathbf{0.00E+00}}$	3.28E−06$_{1.02E-06}$	4.50E+03$_{3.59E+03}$	**0.00E+00**$_{\mathbf{0.00E+00}}$
4	1.56E−01$_{7.82E-01}$	6.84E−01$_{3.99E-01}$	5.01E−02$_{2.93E-02}$	8.94E+00$_{1.48E+01}$	**0.00E+00**$_{\mathbf{0.00E+00}}$
5	1.33E+02$_{1.70E+01}$	5.07E+00$_{\mathbf{1.61E+00}}$	9.38E+01$_{2.45E+01}$	8.08E+00$_{3.33E+00}$	**4.72E+00**$_{1.97E+00}$
6	5.90E+01$_{5.51E+00}$	**0.00E+00**$_{\mathbf{0.00E+00}}$	2.93E+01$_{1.79E+01}$	1.25E−01$_{6.31E-02}$	**0.00E+00**$_{\mathbf{0.00E+00}}$
7	1.52E+02$_{3.47E+01}$	1.52E+01$_{\mathbf{1.70E+00}}$	1.86E+01$_{5.38E+00}$	2.26E+01$_{5.36E+00}$	**1.46E+01**$_{1.98E+00}$
8	4.19E+01$_{1.05E+01}$	4.83E+00$_{2.18E+00}$	6.86E+01$_{2.20E+01}$	1.04E+01$_{3.77E+00}$	**4.72E+00**$_{\mathbf{1.92E+00}}$
9	9.44E+02$_{2.41E+02}$	3.86E−02$_{1.60E-01}$	8.26E+02$_{6.22E+02}$	1.27E+00$_{2.97E+00}$	**0.00E+00**$_{\mathbf{0.00E+00}}$
10	1.97E+03$_{2.92E+02}$	3.28E+02$_{1.43E+02}$	1.06E+03$_{3.02E+02}$	3.20E+02$_{1.97E+02}$	**7.35E+01**$_{\mathbf{8.44E+01}}$
11	5.10E+01$_{2.86E+01}$	2.09E+00$_{1.75E+00}$	7.00E+01$_{3.56E+01}$	9.60E+00$_{6.30E+00}$	**1.13E+00**$_{\mathbf{8.44E-01}}$
12	4.39E+02$_{2.13E+02}$	4.34E+02$_{6.71E+02}$	1.37E+04$_{1.23E+04}$	1.43E+06$_{1.92E+06}$	**2.80E+01**$_{\mathbf{4.98E+01}}$
13	3.13E+02$_{1.69E+02}$	5.93E+00$_{3.04E+00}$	1.24E+04$_{1.28E+04}$	1.02E+04$_{8.83E+03}$	**3.86E+00**$_{\mathbf{2.70E+00}}$
14	8.06E+01$_{5.71E+01}$	1.55E+00$_{1.00E+00}$	1.97E+02$_{2.58E+02}$	7.26E+03$_{7.41E+03}$	**1.21E+00**$_{\mathbf{8.04E-01}}$
15	6.75E+01$_{5.00E+01}$	1.25E+00$_{1.10E+00}$	1.34E+03$_{1.37E+03}$	6.97E+03$_{7.70E+03}$	**5.01E−01**$_{\mathbf{6.25E-01}}$
16	6.00E+02$_{1.51E+02}$	2.22E+00$_{6.03E+00}$	4.43E+02$_{1.94E+02}$	1.80E+02$_{1.33E+02}$	**5.09E−01**$_{\mathbf{2.74E-01}}$
17	2.42E+02$_{1.77E+02}$	2.64E+00$_{2.56E+00}$	2.97E+02$_{1.67E+02}$	4.49E+01$_{5.38E+01}$	**7.68E−01**$_{\mathbf{6.52E-01}}$
18	1.09E+02$_{1.35E+02}$	9.73E−01$_{9.59E-01}$	1.21E+04$_{1.05E+04}$	9.15E+03$_{7.69E+03}$	**3.92E−01**$_{\mathbf{4.54E-01}}$
19	5.01E+01$_{5.32E+01}$	2.27E−01$_{4.24E-01}$	6.90E+02$_{1.22E+03}$	8.85E+03$_{8.18E+03}$	**1.85E−02**$_{\mathbf{1.79E-02}}$
20	4.43E+02$_{1.53E+02}$	6.73E−01$_{7.32E-01}$	2.50E+02$_{1.19E+02}$	6.36E+00$_{4.86E+00}$	**2.28E−01**$_{\mathbf{3.56E-01}}$
21	2.08E+02$_{4.68E+01}$	1.67E+02$_{5.01E+01}$	2.54E+02$_{7.26E+01}$	2.02E+02$_{\mathbf{3.61E+01}}$	**1.59E+02**$_{5.40E+01}$
22	1.15E+03$_{1.06E+03}$	**9.33E+01**$_{1.93E+01}$	6.12E+02$_{6.92E+02}$	1.03E+02$_{\mathbf{1.07E+00}}$	9.52E+01$_{1.44E+01}$
23	9.51E+02$_{1.47E+02}$	3.07E+02$_{\mathbf{2.45E+00}}$	4.58E+02$_{5.95E+01}$	3.16E+02$_{5.43E+00}$	**3.00E+02**$_{2.86E+00}$
24	**1.30E+02**$_{7.54E+01}$	3.21E+02$_{5.61E+01}$	4.00E+02$_{1.27E+02}$	3.36E+02$_{\mathbf{4.80E+01}}$	2.95E+02$_{9.14E+01}$
25	4.31E+02$_{\mathbf{2.04E+01}}$	4.19E+02$_{2.29E+01}$	**4.11E+02**$_{7.69E+01}$	4.31E+02$_{2.33E+01}$	4.23E+02$_{2.30E+01}$
26	1.06E+03$_{9.04E+02}$	**2.99E+02**$_{2.00E+01}$	1.36E+03$_{7.70E+02}$	4.24E+02$_{2.68E+02}$	3.00E+02$_{\mathbf{0.00E+00}}$
27	4.96E+02$_{2.05E+02}$	3.91E+02$_{2.71E+00}$	4.75E+02$_{5.77E+01}$	3.99E+02$_{6.21E+00}$	**3.90E+02**$_{\mathbf{1.68E+00}}$
28	3.93E+02$_{\mathbf{3.56E+01}}$	**3.15E+02**$_{3.64E+01}$	4.81E+02$_{1.59E+02}$	5.30E+02$_{1.25E+02}$	3.28E+02$_{8.70E+01}$
29	3.45E+02$_{8.48E+01}$	2.49E+02$_{\mathbf{6.95E+00}}$	5.10E+02$_{1.61E+02}$	2.80E+02$_{3.35E+01}$	**2.38E+02**$_{8.98E+00}$
30	**3.25E+02**$_{\mathbf{1.42E+02}}$	1.19E+03$_{9.55E+02}$	3.81E+05$_{6.40E+05}$	5.81E+05$_{6.80E+05}$	6.78E+02$_{3.35E+02}$

Minimum error value and standard deviation are shown in bold

Table 2 The results of the compared algorithm for $D = 30$

Func	CMA-ES Mean$_{Std.}$	BSA Mean$_{Std.}$	IWO Mean$_{Std.}$	BBO Mean$_{Std.}$	HBBO-CMA Mean$_{Std.}$
1	**0.00E+00**$_{\mathbf{0.00E+00}}$	1.71E+02$_{5.17E+02}$	4.55E+03$_{5.65E+03}$	1.37E+05$_{5.37E+04}$	**0.00E+00**$_{\mathbf{0.00E+00}}$
3	**0.00E+00**$_{\mathbf{0.00E+00}}$	5.04E+00$_{1.31E+01}$	7.76E−05$_{1.31E−05}$	7.43E+04$_{2.63E+04}$	**0.00E+00**$_{\mathbf{0.00E+00}}$
4	**5.47E−01**$_{\mathbf{1.39E+00}}$	8.24E+01$_{1.59E+01}$	6.92E+01$_{2.80E+01}$	9.85E+01$_{3.01E+01}$	2.11E+00$_{2.01E+00}$
5	3.13E+02$_{3.30E+01}$	7.21E+01$_{1.76E+01}$	3.11E+02$_{6.29E+01}$	5.71E+01$_{1.36E+01}$	**3.65E+01**$_{\mathbf{9.94E+00}}$
6	6.62E+01$_{3.32E+00}$	2.69E−01$_{3.85E−01}$	5.50E+01$_{1.23E+01}$	1.37E−01$_{3.43E−02}$	**8.68E−06**$_{\mathbf{4.35E−05}}$
7	7.45E+02$_{1.04E+02}$	1.26E+02$_{2.37E+01}$	7.69E+01$_{1.21E+01}$	1.09E+02$_{1.98E+01}$	**6.75E+01**$_{\mathbf{1.01E+01}}$
8	2.17E+02$_{2.68E+01}$	5.94E+01$_{1.39E+01}$	2.83E+02$_{4.59E+01}$	6.05E+01$_{1.18E+01}$	**3.68E+01**$_{\mathbf{9.21E+00}}$
9	4.75E+03$_{4.15E+02}$	2.71E+02$_{1.54E+02}$	8.04E+03$_{2.08E+03}$	3.36E+02$_{2.87E+02}$	**9.80E−02**$_{\mathbf{2.09E−01}}$
10	4.66E+03$_{5.84E+02}$	2.54E+03$_{4.94E+02}$	3.12E+03$_{5.40E+02}$	2.22E+03$_{5.19E+02}$	**2.07E+03**$_{\mathbf{4.32E+02}}$
11	1.14E+02$_{4.86E+01}$	6.87E+01$_{3.24E+01}$	1.73E+02$_{6.26E+01}$	2.16E+03$_{2.48E+03}$	**1.92E+01**$_{\mathbf{7.91E+00}}$
12	**1.56E+03**$_{\mathbf{4.17E+02}}$	2.25E+04$_{9.22E+03}$	8.78E+05$_{7.04E+05}$	3.09E+06$_{2.13E+06}$	2.14E+03$_{1.05E+03}$
13	1.74E+03$_{6.57E+02}$	1.10E+04$_{9.36E+03}$	1.18E+05$_{6.95E+04}$	5.14E+04$_{3.52E+04}$	**3.05E+02**$_{\mathbf{2.68E+02}}$
14	1.58E+02$_{6.04E+01}$	7.79E+01$_{9.46E+01}$	2.80E+03$_{2.87E+03}$	1.84E+06$_{1.92E+06}$	**2.15E+01**$_{\mathbf{1.02E+01}}$
15	2.05E+02$_{9.74E+01}$	2.84E+03$_{4.12E+03}$	8.04E+04$_{5.31E+04}$	2.82E+04$_{2.10E+04}$	**2.00E+01**$_{\mathbf{1.19E+01}}$
16	7.35E+02$_{2.45E+02}$	6.41E+02$_{2.32E+02}$	1.08E+03$_{3.00E+02}$	1.16E+03$_{3.24E+02}$	**4.44E+02**$_{\mathbf{2.21E+02}}$
17	3.80E+02$_{1.63E+02}$	1.73E+02$_{1.04E+02}$	8.35E+02$_{3.07E+02}$	4.85E+02$_{2.35E+02}$	**5.94E+01**$_{\mathbf{6.39E+01}}$
18	1.74E+02$_{8.31E+01}$	7.55E+03$_{6.52E+03}$	8.73E+04$_{3.99E+04}$	2.44E+06$_{2.18E+06}$	**8.07E+01**$_{\mathbf{3.45E+01}}$
19	1.39E+02$_{5.24E+01}$	1.13E+03$_{2.07E+03}$	7.80E+04$_{3.72E+04}$	2.16E+04$_{1.46E+04}$	**1.42E+01**$_{\mathbf{1.40E+01}}$
20	1.27E+03$_{2.51E+02}$	1.60E+02$_{1.11E+02}$	8.77E+02$_{2.20E+02}$	4.99E+02$_{2.30E+02}$	**1.20E+02**$_{\mathbf{8.27E+01}}$
21	3.22E+02$_{1.18E+02}$	2.56E+02$_{1.35E+01}$	4.86E+02$_{6.35E+01}$	2.69E+02$_{1.72E+01}$	**2.41E+02**$_{\mathbf{9.77E+00}}$
22	5.62E+03$_{9.01E+02}$	1.73E+03$_{1.48E+03}$	3.66E+03$_{7.44E+02}$	1.98E+03$_{1.46E+03}$	**1.00E+02**$_{\mathbf{0.00E+00}}$
23	2.23E+03$_{3.31E+02}$	4.16E+02$_{2.09E+01}$	7.90E+02$_{1.60E+02}$	4.22E+02$_{1.76E+01}$	**3.90E+02**$_{\mathbf{1.19E+01}}$
24	**4.56E+02**$_{8.49E+01}$	4.94E+02$_{2.16E+01}$	7.63E+02$_{8.55E+01}$	4.96E+02$_{2.48E+01}$	4.65E+02$_{\mathbf{1.20E+01}}$
25	3.83E+02$_{1.14E+01}$	3.96E+02$_{1.57E+01}$	3.88E+02$_{4.00E+00}$	3.93E+02$_{1.25E+01}$	**3.80E+02**$_{\mathbf{7.89E−01}}$
26	**2.75E+02**$_{\mathbf{4.40E+01}}$	1.85E+03$_{6.12E+02}$	3.26E+03$_{1.56E+03}$	1.93E+03$_{1.87E+02}$	1.45E+03$_{1.26E+02}$
27	2.06E+03$_{1.49E+03}$	5.34E+02$_{1.69E+01}$	5.99E+02$_{6.53E+01}$	5.35E+02$_{1.34E+01}$	**5.12E+02**$_{\mathbf{4.41E+00}}$
28	3.43E+02$_{6.01E+01}$	3.93E+02$_{3.45E+01}$	3.46E+02$_{6.43E+01}$	4.23E+02$_{\mathbf{2.06E+01}}$	**3.28E+02**$_{4.79E+01}$
29	8.44E+02$_{2.02E+02}$	6.18E+02$_{1.15E+02}$	1.31E+03$_{2.75E+02}$	8.37E+02$_{2.20E+02}$	**4.79E+02**$_{\mathbf{8.18E+01}}$
30	**8.85E+02**$_{\mathbf{6.93E+02}}$	3.82E+03$_{1.08E+03}$	3.54E+05$_{2.19E+05}$	1.92E+04$_{1.77E+04}$	3.73E+03$_{1.06E+03}$

Minimum error value and standard deviation are shown in bold

The change tendency of the parameters is explained in Fig. 3.

The parameters of HBBO-CMA are set as follows. The population size $ps = 50$. The maximum mutation probability $m_{\max} = 0.01$. The maximum immigrate rate I and the emigrate rate E are set to 1. The parameter $\eta = 3 \ln D + 80$.

5.5 Analysis and discussion

In this paper, the BBO, CMA-ES, BSA and IWO are carried out to compare with the HBBO-CMA and all experimental results demonstrated the effectiveness of HBBO-CMA. The mean error and standard deviation of the five algorithms with $D = 10$, 30 are shown in Tables 1 and 2. From Tables 1 and 2, the HBBO-CMA outperforms the CMA-ES, BSA, IWO and BBO on the 29 functions when $D = 10$. Furthermore, the HBBO-CMA obtains the global optimum on the functions f_1, f_3, f_4, f_6 and f_9. For $D = 30$, the HBBO-CMA outperforms the CMA-ES, BSA, IWO and BBO on the 29 functions. It is worth mentioning that the HBBO-CMA obtains the global optimum on the functions f_1 and f_3.

The Wilcoxon's test, which compares the algorithms in pairs, is carried out to detect the significant difference between HBBO-CMA and contrast algorithms. The statistical analysis results are listed in Table 3. From Table 3, the proposed HBBO-CMA significantly outperforms other compared algorithms with $\alpha = 0.05$ and $\alpha = 0.01$ when $D = 10$ and 30 on solving the CEC-2017 benchmark functions. The convergence curves of HBBO-CMA and compared algorithms are demonstrated in Fig. 1. From Fig. 1, the convergence speed of HBBO-CMA is faster than that of other compared algorithms. The reason is that the rotational invariance

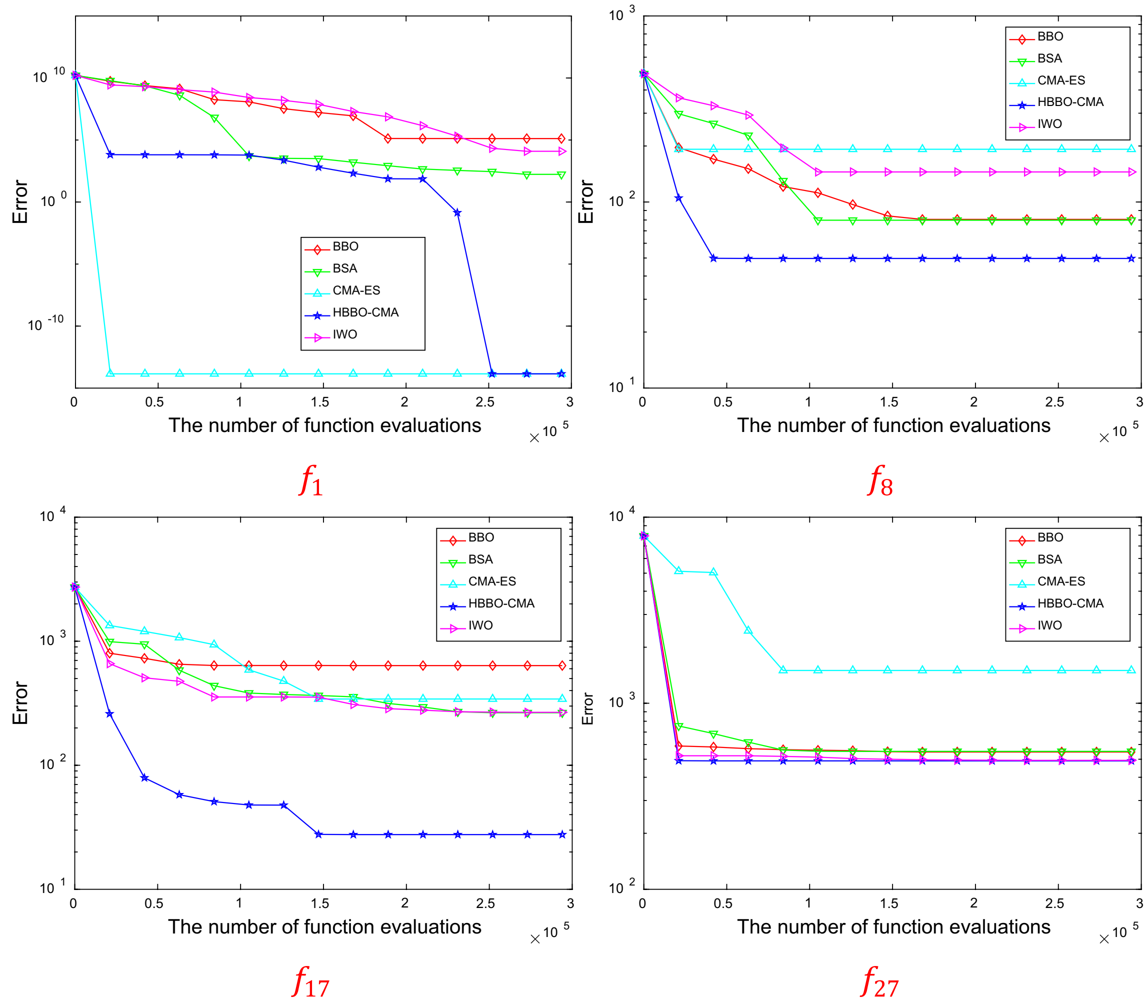

Fig. 1 The convergence curves of typical benchmark functions (30D)

is enhanced and the exploration of algorithm is improved by the rotationally invariant migrate operator which is proposed in this paper. The accuracy of HBBO-CMA is more precise than that of other compared algorithms. The cause is that the CMA-ES is introduced to search from the current optimal candidates and improve the exploitation of the algorithm. The box-plots of HBBO-CMA and other compared algorithms are shown in Fig. 2. The standard deviation of the HBBO-CMA is significantly smaller than that of the compared algorithms. The standard deviation of the HBBO-CMA compared with other algorithms shows that the HBBO-CMA has excellent performance on the stability. The combinations of parameters are listed in Table 4. The importance of the parameter is illustrated in Table 5. The orthogonal array of the parameters is shown in Table 6. In the experiments, different parameters values are combined to testify the importance of the parameters. The population size (ps) is the most important parameter of the four parameters according to Table 5. The change tendency of the parameters with different values is illustrated in Fig. 3 according to Table 5.

6 Conclusions and future research

In this paper, HBBO-CMA was proposed for solving the non-separable global optimization problems. An enhanced mutation operator was designed to replace the standard mutation operator. The CMA-ES was embedded into the framework to extend the performance of HBBO-CMA on non-separable global optimization problems. The statistical results demonstrate that the HBBO-CMA is significantly superior to the contrast algorithms with $\alpha = 0.05$ and $\alpha = 0.01$ when $D = 10$ and 30 on solving the CEC-2017

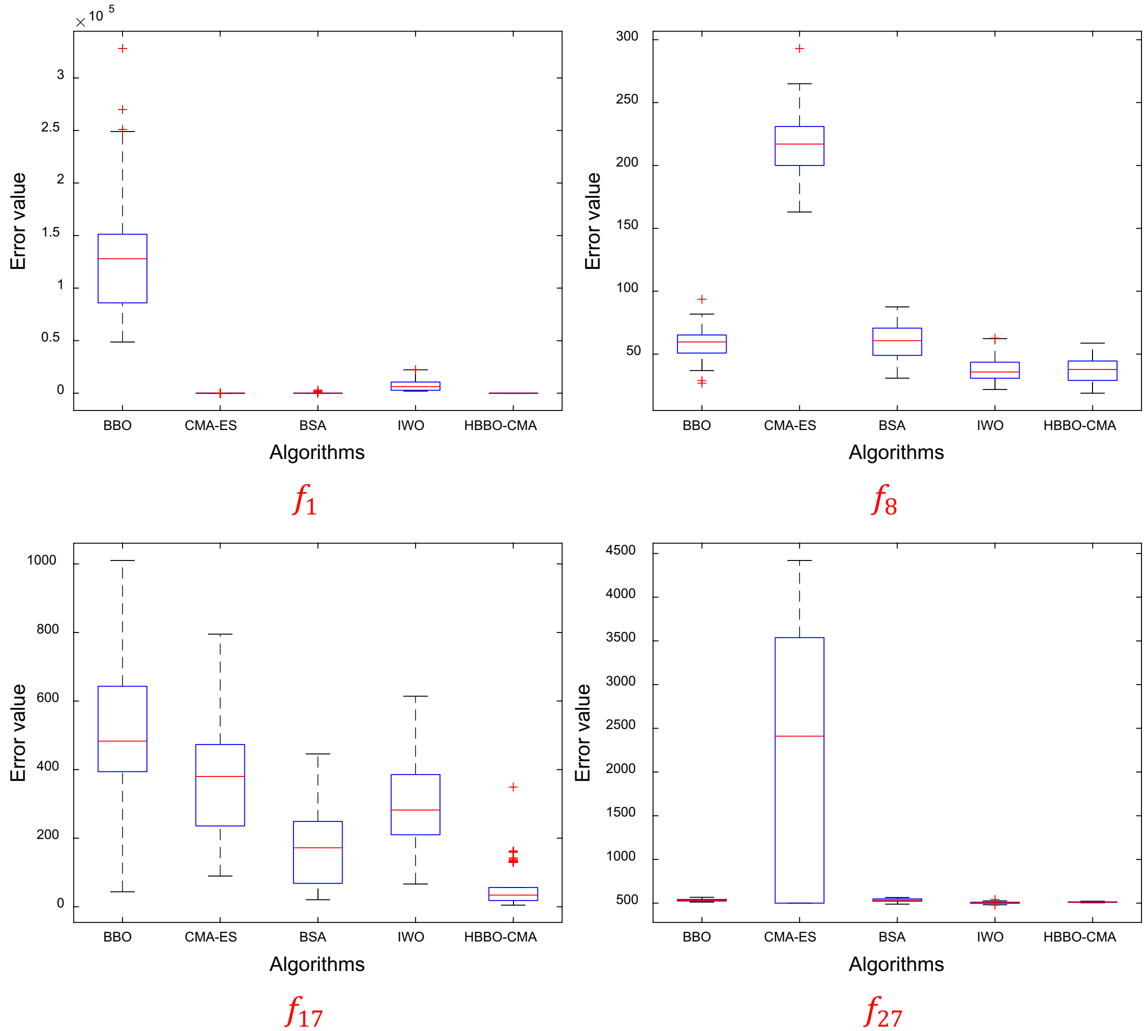

Fig. 2 The box-plots of typical benchmark functions (30D)

Table 3 Rankings obtained through Wilcoxon's test

Dimension	HBBO-CMA vs.	$R+$	$R-$	p value	$\alpha = 0.05$	$\alpha = 0.01$
10D	CMA-ES	343	35.00	2.157E−04	Yes	Yes
	BSA	261	89.50	2.894E−02	Yes	Yes
	IWO	431	4.00	3.902E−06	Yes	Yes
	BBO	435	0.00	2.561E−06	Yes	Yes
30D	CMA-ES	312	66	3.000E−03	Yes	Yes
	BSA	435	0.00	0.000E+00	Yes	Yes
	IWO	435	0.00	0.000E+00	Yes	Yes
	BBO	435	0.00	0.000E+00	Yes	Yes

benchmark functions. Therefore, the HBBO-CMA is efficient, effective and robust for solving SOC problems.

The future research is conducted in the following directions. Firstly, the primary theory and expected time analysis are important. Secondly, the HBBO-CMA is to apply in the cloud computing server scheduling problems such as specific SOC issues. Thirdly, the HBBO-CMA will also be embedded in the machine learning and other research fields.

Table 4 Combinations of parameters

Levels	ps	m_max	*I*	*E*
1	50	0.01	1	1
2	40	0.02	0.7	0.7
3	30	0.03	0.5	0.5

Table 5 Rank of parameters

Level	ps	m_max	I	E
1	1.05E+03	1.97E+03	1.07E+03	3.01E+03
2	3.85E+03	2.96E+03	2.94E+03	2.87E+03
3	2.06E+03	2.03E+03	2.95E+03	1.07E+03
Delta	2.79E+03	9.93E+02	1.88E+03	1.94E+03
Rank	1	4	3	2

Table 6 Orthogonal array

No.	Parameters				AVE
	ps	m_max	I	E	10D
1	1	1	1	1	1.43E+02
2	1	2	2	2	2.86E+03
3	1	3	3	3	1.51E+02
4	2	1	2	3	2.86E+03
5	2	2	3	1	5.81E+03
6	2	3	1	2	2.87E+03
7	3	1	3	2	2.89E+03
8	3	2	1	3	2.09E+02
9	3	3	2	1	3.09E+03

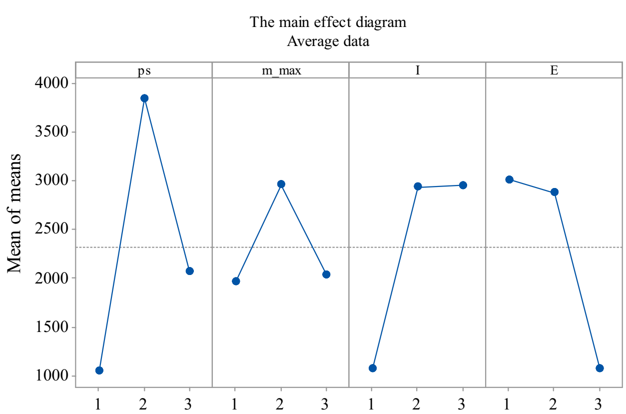

Fig. 3 Change tendency of the parameters

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