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A brain storm optimization algorithm with feature information knowledge and learning mechanism

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Abstract

Various optimization problems with multiple decision variables and complex constraints, which exist widely in the real world, are difficult to be solved by traditional methods. Brain storm optimization (BSO) algorithm, an advanced swarm intelligence optimization method, has high efficiency and flexibility in solving large-scale problems independent of problem characteristics. The essence of swarm intelligence optimization algorithm is that a population iteratively searches for the optimal solution in the solution space, and the process has randomness and blindness. To enhance the searching ability of BSO and strengthen the theoretical guidance of the algorithm, a brain storm optimization algorithm with feature information knowledge and learning mechanism (FIBSO) is proposed in this paper. In the process of BSO iteration, information interaction exists between individuals of each generation. The new individual is generated from the old individual, and the dominant individual contributes to the new individual. Theoretically, using the knowledge of characteristic information between individuals guides the evolution of the population in the dominant direction. Moreover, three search strategies guided by global and local optimal individuals are presented to balance the global and local search capabilities of the algorithm. The results of FIBSO and several comparison algorithms on the CEC2017 test suite indicate that FIBSO has superior performance to the state-of-the arts algorithms. The FIBSO is introduced to the no-wait flow shop scheduling problem, and the results show that FIBSO has the significant ability to solve practical engineering problems.

Keywords Brain storm optimization · Feature information · Interactive learning · Search strategies · Flow shop scheduling problem

1 Introduction

The real value optimization problem, one of the important problems in the real world, is often abstracted from complex engineering applications. Therefore, finding the solution methods for such real-value optimization problems is necessary to solve these large-scale complex problems. Due to the limitations of traditional methods [1] to solve complex and large-scale problems, an intelligent optimization algorithm emerges at the historical moment [2, 3]. Meta-heuristic method, which is a method to solve optimization problems, uses neighborhood knowledge to transit from one neighborhood to another for investigating new solutions. When solving large-scale problems with multiple decision variables and multiple constraints that are difficult to model, the meta-heuristic method is independent of the problem characteristics and has the ability to find satisfactory solutions with a quality close to the optimal solution. Commonly used algorithms to effectively

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solve such complex problems [4, 5] include: particle swarm optimization (PSO) [6], estimation distribution algorithm (EDA), genetic algorithm (GA) [7], differential evolution (DE) [8, 9], backtracking search algorithm (BSA) [10], ant colony optimization (ACO) [11], moth flame optimization (MFO), artificial bee colony (ABC) [12] and water wave optimization (WWO) [13], etc.

Brain storm optimization (BSO) [14, 15], which was first proposed in 2011, is a new intelligent algorithm and is easy to be implemented [16]. Convergence and divergence are the two core operations in BSO, in which k-means clustering is the divergence operation and the later individual generation is the convergence operation. Each cluster is a locally optimal area, and high-quality information is transferred between clusters. The introduction of the learning mechanism of clustering improves the ability of the BSO algorithm to obtain the optimal solution. Clustering is used to search for the local optimum, and the global optimum is achieved by competition between local optimums. The idea of mutation increases the diversity of the algorithm and avoids falling into the local optimum. The cooperative process of clustering and divergence effectively helps BSO find the optimal solution. According to the no free lunch theorem, the BSO algorithm with a simple model, fast convergence speed and few parameters, is very suitable for multi-modal and high-dimensional function problems that are difficult to be solved by classical optimization algorithms.

Various learning mechanisms are introduced to improve the performance of the BSO owing to the randomness of the search process and the lack of theoretical guidance. The following studies have been conducted to improve the self-learning ability of the BSO in finding the optimal solution. An orthogonal learning brain storm optimization algorithm (OLBSO) [17] is proposed to enhance the learning ability of BSO. Two orthogonal design engines are introduced to find and utilize useful search experience to improve the evolutionary ability of the population. The orthogonal decision mechanism is utilized to enhance the global and local search ability of the algorithm. The theory strength of the algorithm is high, but the effect of the algorithm is not apparent. Qu et al. [18] proposes a new brain storm optimization algorithm with a cooperative learning strategy (BSO-CLS) algorithm and adaptive learning strategy, which updates individuals through adaptive search, simulation, and learning. The solving capability of BSO-CLS algorithm is better than that of the comparison algorithms in terms of the evolution speed of individuals and the degree of group aggregation. The micro-relaxation selection and solution integration technology based on multi-population are proposed by Sun et al. [19]. The original BSO framework is embedded into heterogeneous and complementary multi-population solution integration to boost the solving ability of algorithm. The micro-relaxation selection mechanism is proposed to produce a variety of the individuals. The experiment shows that the proposed algorithm is superior to the most advanced current algorithms. A new BSO algorithm with an adaptive learning strategy (BSO-AL) is proposed in [20]. The mentioned algorithm considers the evolution speed factor of the individuals and generates new individuals through adaptive exploration, imitation or learning. Experimental results present that the capability of the BSO-AL algorithm is better than that of the classical algorithm, which proves the effectiveness of the learning strategy.

In order to reduce the computation cost of the algorithm, some other clustering methods are proposed to replace the k-means clustering method. For this reason, a random grouping brain storm optimization algorithm (RGBSO) is proposed in [21]. Random grouping strategy and dynamic step size are used to optimize the algorithm, which balances the exploration and development abilities under different search generations. A simple grouping method (SGM) in the modified brain storm optimization (MBSO) [22] is used to replace the clustering method for reducing the computational effort of the algorithm. A novel idea difference strategy (IDS) is used to avoid thinking trapped by local optimum, and creates better new ideas in the process of solving the problem. A new efficient brain storm optimization for real-parameter numerical optimization (BSO20) is proposed in [23] to improve the clustering and mutation methods. The designed new clustering method is combination of the best clustering and random grouping methods. In addition, instead of the original strategy, an improved strategy is used to generate new individuals is used to promote information sharing within and between clusters. The BSO20 is superior to several advanced comparison algorithms.

Each individual is expected to evolve in a different direction, which improves the ability of the algorithm to find the global optimal solution. Moreover, different updating strategies help to create appropriate exploratory behaviors for individuals. Therefore, a variety of mutation strategies to improve the ability of BSO are as follows. A multi-policy dynamic parameter adjustment BSO (MSBSO) is proposed in [24]. In MSBSO, four competitive strategies are designed to accommodate different search areas and obtain superior individuals. In [25], the learning method of one cluster and two clusters is combined to make the algorithm active learning. In addition, the algorithm adopts a dynamically changing clustering period and one-step clustering to reduce the cost of the clustering method. The proposed algorithm is an excellent method for optimizing complex and high-dimensional functions compared to the other well-known algorithms. A multi-information interaction BSO (MIIBSO) algorithm is proposed in [26]. The paper puts forward the strategy of multi-information interaction (MII) with three new models of the information industry models by considering all kinds of information interaction among individuals.

Through the individual stagnation feedback mechanism, the collaboration of the above three modes is established to enhance the exploitation of MIIBSO. Furthermore, the dynamic differential step-size (DDS) algorithm is designed.

The mechanism of adding other algorithms to the BSO is an effective optimization method. This mechanism helps researchers use the advantages of different algorithms and combine them to reach an appropriate one. For instance, a brain storm optimization algorithm with estimation of distribution (EDBSO) has good robustness [27]. Adding chaotic local search (CLS) into BSO algorithm [28] enhances the local search ability of the BSO. When the algorithm cannot find a better solution, the dynamic mechanism of CLS disperses individuals in the population and increases the distribution and diversity of individuals in the population. Nelder-Mead simplex (NMS) method [29] as an excellent local solution method based on direct search achieves a balance between global and local detection by combining the detection capability of BSO with that of NMS.

Researchers have also worked on the BSO algorithm characteristic. A population interaction network is proposed in [30] to construct the relationship between individuals. This network improves the performance of the BSO algorithm. The power-law distribution of the algorithm is studied from the population evolution perspective. In addition, the interaction among parameters is analyzed. Eventually, the parameter combinations that have the most significant influence on the BSO are realized through experiments. The power-law distribution effectively enhances the interaction of the population by comparing the BSO, DE, and PSO algorithms. In [31], population diversity is proposed to measure the change of solution distribution. The exploration and exploitation ability of the algorithm is measured based on the variation of population diversity. Two local re-initialization strategies are adopted to increase the diversity among individuals. Both two strategies improve the performance of the BSO algorithm. The BSO algorithm is used to solve the optimization classification model, and also the optimal weight vectors under different structures are searched [32]. This method reduces computational complexity and improves learning efficiency. The BSO effectively improves classification performance in classification problems. An adaptive latent factor analysis model based on the brain storm optimization algorithm is presented in [33] to improve the convergence speed and solving efficiency of the recommendation system. In order to enhance effective search, the particle retention technique is designed. An adaptive multidimensional learning method is developed to effectively recommend valuable data for users. Experimental results prove that the designed model based on the BSO has high convergence rate and computational efficiency.

The BSO algorithm has great potential for solving real-world engineering problems [34, 35] and scheduling problems. Li et al. [36] propose an improved brain storm optimization (IBSO) algorithm to solve the distributed hybrid flow shop scheduling problem, aiming at minimizing the fuzzy completion time between factories. For this purpose, two realistic constraints are set, including the fuzzy processing time and the setting time in an uncertain environment. According to the characteristics of the problem, several heuristic methods of local search are proposed to improve the local search ability. Experiments confirm that the proposed algorithm effectively solves the distributed hybrid flow shop scheduling problem compared with other effective algorithms. A new multi-objective BSO algorithm [37] is proposed by Fu et al. to solve the multi-objective distributed permutation flow shop scheduling problem.

As an evolutionary algorithm, the BSO obtains the optimal or the satisfactory suboptimal solutions in the solution space through iterative search. The search process of the evolutionary algorithm is usually blind. Thus, it is necessary to design the search operations based on solution characteristics to improve the quality of the population. The characteristic information of individuals is considered in the FIBSO and is used in the search process. Three search strategies are designed to search the whole solution space comprehensively. In addition, the FIBSO has proved its ability to solve practical problems in the no-wait flow shop scheduling problem. The paper has the following contributions.

- The feature information learning mechanism fully learns the feature information between individuals to help the algorithm find the optimal solution.
- Three search strategies, in which the global optimum and local optimum individuals are used to enhance the search ability of the population, are proposed to increase the rotation invariance of individuals in the solution space search process.
- The FIBSO algorithm is tested on the no-wait flow shop scheduling problem, and the ability of FIBSO to solve practical application problems is proved.

The structure of this article is as follows. The basic BSO algorithm is described in Section 2. The mentioned FIBSO is introduced in Section 3. Section 4 verifies the effectiveness of the algorithm and tests the FIBSO in the no-wait flow shop scheduling problem. Finally, Section 5 gives the conclusion.

2 The brain storm optimization algorithm

The BSO simulates human brainstorming behavior to generate new individuals, whose core is the learning mechanism of clustering. The basic BSO steps are as follows:

Step 1: Randomly initialize the initial solution and get the fitness value.

Step 2: N individuals are grouped into M clusters using the k-means method, and the best individual in each cluster is selected as the cluster center.

Step 3: Take a perturbation. Randomly select a cluster center, and replace it with a newly generated individual.

Step 4: Create new individuals. Two methods are utilized to generate new individuals. The first one is generating a new solution by one old individual, shown in (1). The second one is generating a new solution by two old individuals, shown in (2). The first method is selected with the probability of $P = 0.8$, and the second method is selected with the probability of $P = 0.2$.

$$X_i = \begin{cases} X_{rand} & if\ P1 < 0.4 \\ X_{center} & otherwise \end{cases} \tag{1}$$

where, X_i is the new individual, X_{rand} is a random individual in the current population, and X_{center} is the center in a cluster in the current population. $P1$ is a random number between 0 and 1.

$$X_i = \begin{cases} c{*}X_{rand1} + (1-c){*}X_{rand2} & if\ P2 < 0.5 \\ c{*}X_{center1} + (1-c){*}X_{center2} & otherwise \end{cases} \tag{2}$$

where, X_i is the new individual, X_{rand1} and X_{rand2} are two random individuals in two clusters in the current population, and $X_{center1}$ and $X_{center2}$ are the centers in two clusters in the current population. $P2$ and c are random numbers between 0 and 1.

Every new individual is generated in (3).

$$X_{new} = X_i + \xi{*}G(\mu, \sigma) \tag{3}$$

where, X_{new} is the new solution. ξ is the step size, and $G(\mu, \sigma)$ is a Gaussian random function.

Step 5: Select the individual with a small fitness value as the initial solution for the next generation.

$$X_i^{t+1} = \begin{cases} X_{new}^t & if\ f\left(X_{new}^t\right) < f\left(X_i^t\right) \\ X_i^t & otherwise \end{cases} \tag{4}$$

where, X_i^{t+1} is the final solution. X_{new}^t is the newly generated solution in Step 4, and X_i^t is the old individual in the current population.

3 The FIBSO algorithm

To enhance the learning and optimization capabilities of BSO, the FIBSO algorithm is presented in this section. This section mainly introduces the learning mechanism of feature information and the population updating strategy and finally gives the framework of the FIBSO.

3.1 The feature information learning

As an evolutionary algorithm, the BSO conducts a distributed search in solution space to find the superior solution. Individuals generate various useful information in the search

Fig. 1 The feature information learning mechanism

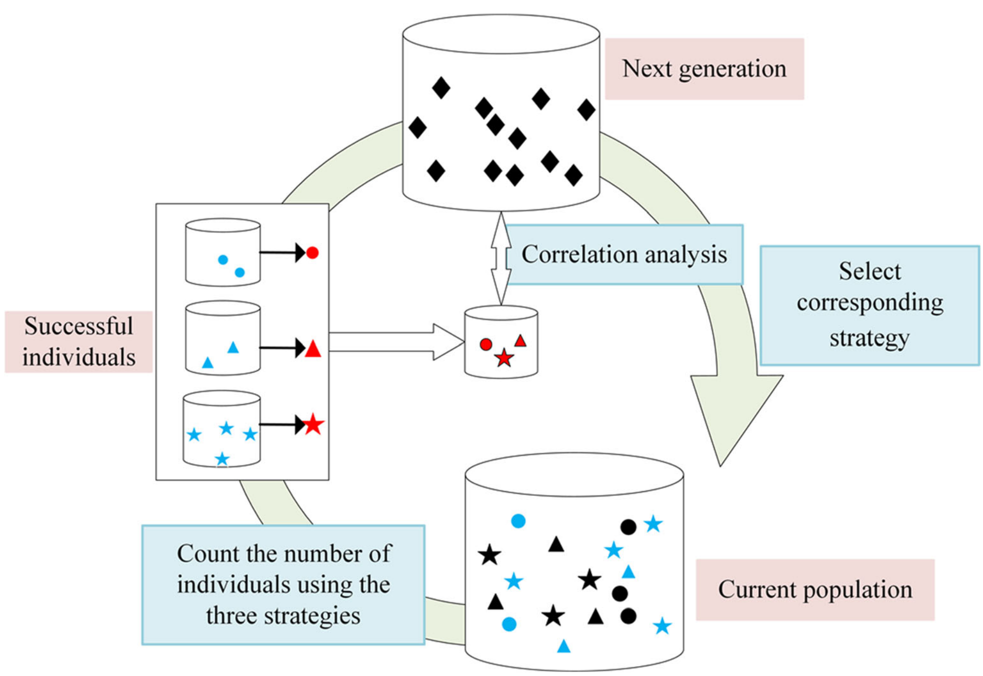


Table 1 ANOVA results of FIBSO

Source	*Sum Sq*	*d. f.*	*Mean Sq.*	*F−ratio*	*p−value*
Main effects					
N	77.1489	3	25.7163	166.95	**0**
k	0.3043	1	0.3043	1.98	0.1934
P	0.5781	3	0.1927	1.25	0.3479
Interactions					
$N*k$	1.1144	3	0.3715	2.41	0.1341
$N*P$	3.1565	9	0.3507	2.28	0.1181
$k*P$	0.1018	3	0.0339	0.22	0.8799
Error	1.3863	9	0.154		
Total	83.7903	31			

process. For instance, the search with large step size is helpful to jump out of local optimum, and the search with small step size is useful to enhance the exploitation ability of the algorithm, and searching along a certain direction contributes to finding a superior solution. Moreover, the population evolution along the search direction of the dominant individual helps to find a better solution. It is worth noting that the information exists between individuals in one generation and between different generations.

Finding and using the information help the population to evolve in a dominant direction. Each new generation is based on the previous historical generation, and there is a large correlation between the two generations of individuals. The feature information learning mechanism uses the feature information between each generation to guide the generation towards a new generation of solutions. The historical generation information of successful individuals is abstracted as characteristic knowledge, and stored. Then, the feature information knowledge is applied to the related individuals to the next generation to enhance the population improvement. The steps are as follows.

Step 1: Use the three mutation strategies to renew the individuals after the initial population. Count the total number of individuals using the three mutation strategies after the first iteration and choose each strategy in the current population are counted.

Step 2: Count the successful individuals in each of the three mutation strategies. That is to get individuals with fitness value decreases after each variation strategy.

Step 3: Select three individuals from the successful individuals of the three strategies as the optimal individual for the corresponding strategy. The three individuals has the smallest fitness value.

Fig. 2 Main effects plot of parameters

Table 2 The results of FIBSO and eight comparison algorithms (10-dimensional benchmark functions)

Fun	MDBSO		BSO		ALBSO		SBSA		IMFMBO		ABCNG		OLBSO		HOA		FIBSO	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
1	1.41E+10	4.35E+09	5.17E+02	7.30E+02	**8.66E+06**	4.33E+07	7.66E+07	9.76E+07	3.83E+07	1.46E+07	6.71E+03	1.11E+04	**0.00E+00**	**0.00E+00**	1.69E+10	5.44E+09	1.80E+04	8.03E+04
3	1.97E+04	2.95E+03	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	1.92E+03	**1.59E+03**	1.10E+04	3.37E+03	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	1.22E+05	2.10E+05	1.62E+01	7.20E+01
4	1.30E+03	4.91E+02	3.80E+00	1.50E+00	1.65E+00	1.31E+00	1.27E+01	1.33E+01	9.79E+00	1.63E+00	**4.83E-01**	**1.28E+00**	1.02E+00	1.48E+00	1.75E+03	7.04E+02	4.55E+00	1.53E+01
5	1.18E+02	1.95E+01	4.98E+01	9.85E+00	3.55E+01	1.34E+01	1.71E+01	5.04E+00	1.82E+01	**3.67E+00**	1.70E+01	6.25E+00	1.66E+01	4.29E+00	1.42E+02	2.04E+01	**1.65E+01**	1.23E+01
6	6.47E+01	1.29E+01	2.87E+01	1.05E+01	2.89E+01	9.64E+00	1.03E+00	1.02E+00	3.92E+00	8.14E-01	1.49E-01	4.54E-01	**5.40E-03**	**1.25E-02**	8.60E+01	1.38E+01	8.94E+00	6.82E+00
7	1.87E+02	**5.05E-05**	9.11E+01	7.41E+00	6.23E+01	2.02E+01	3.58E+01	6.64E+00	4.35E+01	4.69E+00	2.94E+01	6.14E+00	2.91E+01	5.85E+00	4.22E+02	7.22E+01	**2.90E+01**	1.50E+01
8	9.42E+01	4.51E+00	1.64E+01	1.48E+01	2.45E+01	9.02E+00	2.15E+01	3.66E+00	1.95E+01	**3.16E+00**	1.81E+01	8.35E+00	**9.68E+00**	3.64E+00	1.29E+02	1.90E+01	1.37E+01	9.18E+00
9	1.62E+03	1.96E+02	2.84E+02	7.96E+01	3.18E+02	1.84E+02	1.76E+01	1.59E+01	2.55E+01	1.09E+01	5.82E+00	9.27E+00	**1.25E-01**	**5.74E-01**	3.67E+03	1.10E+03	3.96E+01	6.13E+01
10	1.87E+03	**1.64E+02**	8.66E+02	**1.31E+02**	1.07E+03	2.60E+02	**3.16E+02**	1.35E+02	5.32E+02	1.44E+02	5.16E+02	2.68E+02	6.13E+02	2.22E+02	2.57E+03	2.34E+02	1.35E+03	4.85E+02
11	2.61E+03	2.10E+03	4.20E+01	**2.97E+00**	6.30E+01	3.54E+01	3.17E+01	2.60E+01	6.39E+01	7.63E+01	6.47E+00	5.36E+00	**4.03E+00**	3.16E+00	1.34E+04	1.42E+04	3.17E+01	2.39E+01
12	8.30E+08	5.14E+08	7.18E+04	1.80E+04	1.60E+04	3.73E+04	2.82E+05	5.82E+05	1.54E+06	1.11E+06	1.69E+04	1.63E+04	**5.93E+02**	**2.24E+03**	1.62E+09	8.89E+08	1.26E+03	2.56E+03
13	9.16E+06	1.33E+07	1.03E+04	2.63E+03	2.25E+03	3.04E+03	2.09E+03	2.64E+03	2.50E+04	1.93E+04	8.38E+03	9.91E+03	**6.23E+00**	**2.47E+00**	1.17E+08	1.33E+08	1.92E+02	3.17E+02
14	2.12E+02	9.39E+01	2.59E+02	3.04E+02	9.01E+01	8.51E+01	2.85E+01	1.53E+01	6.68E+02	5.16E+02	1.84E+02	3.23E+02	2.66E+01	**3.55E+00**	1.79E+06	2.47E+06	**2.64E+01**	**1.53E+01**
15	4.01E+03	2.77E+03	1.58E+03	3.32E+01	**1.34E+02**	1.64E+02	1.16E+02	3.05E+02	1.87E+03	2.72E+03	2.74E+01	**1.56E+01**	2.60E+01	9.92E+01	2.82E+06	4.92E+06	2.59E+01	4.33E+01
16	6.62E+02	1.23E+02	1.97E+02	7.48E+01	2.58E+02	1.11E+02	**4.11E+01**	5.61E+01	5.73E+01	**5.46E+01**	1.48E+02	9.65E+01	9.32E+01	9.03E+01	1.03E+03	1.94E+02	9.31E+01	9.11E+01
17	3.09E+02	7.99E+01	2.42E+01	**2.05E+00**	6.91E+01	3.79E+01	1.81E+01	1.06E+01	2.86E+01	1.17E+01	6.36E+01	5.18E+01	**1.79E+01**	1.83E+01	5.96E+02	1.66E+02	4.50E+01	2.72E+01
18	1.76E+07	2.50E+07	2.51E+03	1.66E+03	2.44E+03	3.18E+03	2.04E+03	2.43E+03	2.40E+04	1.81E+04	5.96E+03	1.29E+04	6.92E+01	**5.28E+00**	3.57E+08	3.06E+08	**6.31E+01**	1.01E+02
19	6.85E+03	7.79E+03	4.36E+03	4.59E+03	2.95E+02	1.81E+03	4.80E+02	8.36E+02	3.37E+03	4.36E+03	7.53E+00	9.84E+00	**1.04E+00**	**8.39E-01**	2.16E+07	4.68E+07	1.69E+01	4.05E+01
20	2.66E+02	5.71E+01	5.51E+01	7.63E+00	1.29E+02	6.09E+01	**1.35E+01**	9.34E+00	2.52E+01	**5.74E+00**	4.83E+01	4.72E+01	2.18E+01	1.71E+01	4.67E+02	8.82E+01	9.68E+01	7.05E+01
21	2.83E+02	**4.33E+01**	1.54E+02	7.71E+01	2.00E+02	5.68E+01	1.73E+02	4.81E+01	1.34E+02	4.08E+01	2.05E+02	4.19E+01	1.24E+02	3.64E+01	3.24E+02	4.32E+01	**1.23E+02**	4.76E+01
22	1.35E+03	3.64E+02	**9.52E+01**	7.85E+00	1.06E+02	**5.12E+00**	1.13E+02	2.45E+01	9.74E+01	2.95E+01	1.29E+02	1.66E+02	1.81E+02	2.61E+02	1.62E+03	4.17E+02	1.10E+02	1.45E+01
23	4.30E+02	2.05E+01	3.68E+02	6.67E+00	3.98E+02	3.53E+01	3.22E+02	6.25E+00	3.22E+02	**4.20E+00**	3.22E+02	8.30E+00	3.25E+02	5.23E+00	5.12E+02	4.13E+01	**3.21E+02**	1.04E+01
24	5.40E+02	4.99E+01	4.11E+02	2.61E+01	3.72E+02	1.36E+02	3.16E+02	7.75E+01	3.61E+02	9.90E+01	3.44E+02	5.06E+01	3.46E+02	**5.72E+00**	5.57E+02	5.14E+01	**3.15E+02**	7.67E+01
25	1.40E+03	3.62E+02	4.43E+02	**6.51E-03**	4.29E+02	5.22E+01	4.32E+02	2.58E+01	4.29E+02	1.54E+01	4.33E+02	2.94E+01	4.29E+02	2.36E+01	1.59E+03	5.56E+02	**4.28E+02**	2.29E+01
26	1.91E+03	3.67E+02	3.71E+02	2.37E+02	**8.13E+02**	3.80E+02	4.05E+02	1.59E+02	3.49E+02	**6.92E+01**	4.22E+02	1.74E+02	4.04E+02	1.32E+02	2.12E+03	4.02E+02	**3.47E+02**	1.56E+02
27	5.41E+02	4.78E+01	4.71E+02	5.62E+01	4.91E+02	5.44E+01	4.02E+02	5.86E+00	3.79E+02	2.50E+00	4.10E+02	5.35E+01	3.82E+02	3.00E+01	6.24E+02	6.81E+01	**3.74E+02**	**1.55E+00**
28	5.00E+02	**3.95E-03**	4.38E+02	7.55E+01	4.13E+02	8.29E+01	4.38E+02	8.91E+01	**3.73E+02**	3.32E+01	4.51E+02	5.77E+01	4.76E+02	2.22E+01	1.23E+03	2.03E+02	4.24E+02	6.39E+01
29	6.70E+02	1.11E+02	3.71E+02	5.43E+01	3.65E+02	8.58E+01	2.74E+02	1.98E+01	2.85E+02	**1.75E+01**	3.32E+02	6.30E+01	**2.58E+02**	2.74E+01	9.78E+02	1.69E+02	2.82E+02	5.45E+01
30	2.02E+07	1.44E+07	4.17E+04	1.88E+04	2.50E+03	3.03E+03	6.95E+04	8.69E+04	3.12E+04	5.02E+04	4.07E+02	7.30E+02	2.15E+02	2.80E+01	7.81E+07	5.12E+07	**2.03E+02**	**1.83E+00**

The mean and standard deviation of the optimal algorithm are shown in bold

Table 3 The results of FIBSO and eight comparison algorithms (30-dimensional benchmark functions)

Fun	MDBSO		BSO		ALBSO		SBSA		IMFMBO		ABCNG		OLBSO		HOA		FIBSO	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
1	8.02E+10	3.29E+09	2.63E+03	2.65E+03	7.47E+07	3.05E+08	6.16E+09	2.64E+09	2.34E+09	4.07E+08	1.18E+04	1.63E+04	**2.78E-01**	**1.37E+00**	1.12E+11	1.71E+10	2.57E+10	8.32E+09
3	9.40E+04	1.59E+02	**4.19E+01**	3.04E+01	4.58E+03	2.22E+04	4.32E+04	9.43E+03	1.36E+05	2.16E+04	5.12E+02	8.61E+02	1.29E+03	1.84E+03	6.32E+07	1.69E+08	4.45E+04	**1.36E+01**
4	2.28E+04	1.39E+03	7.45E+01	**2.77E+00**	8.78E+01	6.84E+01	6.50E+02	4.34E+02	3.53E+02	5.83E+01	**7.25E+00**	1.28E+01	3.76E+01	2.70E+01	4.13E+04	9.51E+03	3.73E+03	1.56E+03
5	5.07E+02	**1.08E-06**	2.15E+02	6.76E+01	1.92E+02	3.44E+01	1.68E+02	3.06E+01	1.64E+02	1.55E+01	9.82E+01	2.77E+01	**6.96E+01**	2.46E+01	6.41E+02	5.65E+01	3.73E+02	5.82E+01
6	1.09E+02	3.45E+00	5.59E+01	2.30E+01	5.53E+01	6.33E+00	2.87E+01	6.30E+00	1.60E+01	1.59E+00	4.34E+00	3.49E+00	**8.78E-01**	**9.94E-01**	1.35E+02	1.22E+01	9.46E+01	**1.36E+01**
7	8.87E+02	**5.75E-06**	5.43E+02	3.11E+01	5.70E+02	1.29E+02	3.86E+02	7.46E+01	3.14E+02	2.46E+01	1.44E+02	3.51E+01	**9.37E+01**	1.85E+01	2.51E+03	2.66E+02	6.09E+02	9.61E+01
8	4.37E+02	**3.07E-07**	1.23E+02	2.54E+01	1.47E+02	3.50E+01	1.34E+02	2.12E+01	1.61E+02	1.64E+01	9.66E+01	2.11E+01	**7.06E+01**	2.03E+01	5.64E+02	4.60E+01	3.26E+02	5.90E+01
9	1.66E+04	**1.44E-02**	2.70E+03	2.47E+02	**3.54E+03**	7.93E+02	2.29E+03	5.64E+02	1.85E+03	4.63E+02	4.78E+02	3.71E+02	**1.53E+02**	2.03E+02	3.26E+04	5.32E+03	8.61E+03	3.35E+03
10	8.63E+03	**2.96E+02**	4.40E+03	7.52E+02	4.47E+03	6.01E+02	**3.54E+03**	4.86E+02	4.49E+03	3.09E+02	4.83E+03	2.17E+03	3.28E+03	4.90E+02	9.44E+03	4.70E+02	**3.00E+03**	5.88E+02
11	1.44E+04	1.30E+03	1.22E+02	2.53E+01	1.48E+02	5.09E+01	1.67E+03	8.56E+02	2.57E+03	1.50E+03	5.88E+01	3.43E+01	**2.59E+01**	**1.20E+01**	4.15E+04	2.90E+04	1.15E+03	8.03E+02
12	2.17E+10	4.32E+09	4.95E+06	1.24E+06	1.00E+07	5.00E+07	3.53E+08	2.64E+08	1.29E+08	4.01E+07	1.44E+05	1.56E+05	**1.02E+05**	**1.04E+05**	2.27E+10	5.23E+09	8.63E+08	8.04E+08
13	1.77E+10	5.78E+09	4.85E+04	9.32E+03	9.60E+06	5.51E+07	1.70E+08	2.73E+08	6.93E+07	2.76E+07	2.43E+04	3.95E+04	**1.99E+03**	**7.66E+03**	2.03E+10	7.29E+09	2.77E+05	4.11E+05
14	9.09E+06	5.88E+06	8.72E+03	8.98E+03	5.39E+03	1.84E+04	3.01E+05	3.28E+05	5.69E+05	4.28E+05	9.76E+03	8.52E+03	1.54E+03	**4.58E+02**	4.36E+07	3.41E+07	**1.14E+03**	1.79E+03
15	3.37E+09	1.24E+09	2.13E+04	3.31E+04	6.53E+04	1.69E+05	1.53E+06	3.73E+06	1.25E+07	8.96E+06	1.36E+04	1.99E+04	**8.73E+02**	**2.38E+03**	5.43E+09	1.95E+09	2.03E+04	3.16E+04
16	5.09E+03	9.91E+02	1.88E+03	**9.93E+01**	1.57E+03	2.94E+02	1.08E+03	2.33E+02	1.28E+03	2.29E+02	9.67E+02	3.02E+02	9.56E+02	3.00E+02	6.72E+03	1.53E+03	**9.55E+02**	5.32E+02
17	5.92E+03	6.84E+03	6.24E+02	**8.81E+01**	7.38E+02	2.59E+02	**4.20E+02**	1.24E+02	5.28E+02	1.61E+02	5.18E+02	2.28E+02	5.01E+02	1.80E+02	1.38E+04	1.75E+04	**1.23E+03**	3.27E+02
18	1.63E+08	1.17E+08	9.23E+04	6.50E+04	1.12E+05	1.46E+05	9.64E+05	1.14E+06	2.09E+06	1.65E+06	8.50E+04	6.52E+04	4.83E+04	**2.08E+04**	6.03E+08	3.15E+08	**4.68E+04**	6.42E+04
19	3.28E+09	6.71E+08	1.29E+05	4.16E+04	1.03E+05	4.58E+04	1.98E+06	3.39E+06	1.38E+07	8.08E+06	1.05E+04	1.44E+04	**1.95E+01**	**9.79E+00**	6.59E+09	2.99E+09	1.27E+05	3.77E+05
20	1.36E+03	**1.44E+02**	7.25E+02	**2.09E+02**	7.27E+02	**2.09E+02**	3.92E+02	2.00E+02	5.52E+02	**1.43E+02**	**3.86E+02**	1.96E+02	4.84E+02	1.49E+02	1.73E+03	2.10E+02	9.63E+02	1.95E+02
21	7.68E+02	5.91E+01	3.89E+02	**1.81E+00**	4.11E+02	3.97E+01	3.51E+02	2.77E+01	3.62E+02	1.37E+01	3.95E+02	2.53E+01	3.69E+02	1.87E+01	7.96E+02	5.56E+01	**3.50E+02**	7.25E+01
22	8.80E+03	3.59E+02	**4.84E+03**	**1.58E+02**	4.66E+03	1.09E+03	1.86E+03	1.06E+03	**1.38E+03**	1.77E+03	3.76E+03	2.88E+03	3.37E+03	6.43E+02	9.73E+03	6.81E+02	6.02E+03	5.94E+02
23	1.40E+03	**1.44E+02**	**4.84E+03**	4.12E+01	1.05E+03	1.21E+02	8.81E+02	4.91E+01	5.26E+02	2.20E+01	4.85E+02	4.07E+01	**4.27E+02**	**1.94E+01**	1.59E+03	1.75E+02	8.49E+02	9.02E+01
24	1.56E+03	1.07E+02	1.11E+03	**1.74E+01**	1.13E+03	1.96E+02	8.59E+02	5.19E+01	6.50E+02	1.78E+01	5.64E+02	4.23E+01	**4.93E+02**	2.21E+01	1.73E+03	2.34E+02	8.55E+02	7.35E+01
25	4.71E+03	**2.20E-06**	3.88E+02	1.43E+01	3.96E+02	2.38E+01	5.77E+02	6.29E+01	5.58E+02	3.69E+01	3.80E+02	5.87E+00	**3.79E+02**	7.56E-01	1.36E+04	3.48E+03	**1.18E+03**	3.44E+02
26	1.14E+04	1.11E+03	4.19E+03	1.37E+03	5.75E+03	1.34E+03	3.86E+03	8.48E+02	3.54E+03	8.38E+02	3.52E+03	4.62E+02	3.54E+03	**3.05E+02**	1.28E+04	1.65E+03	**3.49E+03**	8.06E+02
27	5.51E+02	2.64E+02	1.34E+03	3.50E+02	**1.23E+03**	2.75E+02	6.68E+02	4.31E+01	5.01E+02	1.59E+01	5.00E+02	3.27E-04	5.00E+02	2.43E-04	2.51E+03	4.48E+02	**5.00E+02**	**1.67E-04**
28	6.11E+02	7.96E+02	4.36E+02	3.79E+01	**4.14E+02**	2.02E+01	7.88E+02	1.66E+02	6.60E+02	4.33E+01	4.86E+02	4.22E+01	5.00E+02	**2.16E+00**	8.97E+03	1.69E+03	5.34E+02	1.43E+02
29	9.50E+03	6.83E+03	**1.18E+03**	3.67E+02	1.61E+03	3.61E+02	1.41E+03	2.38E+02	9.76E+02	**1.54E+02**	8.44E+02	2.58E+02	**6.00E+02**	1.86E+02	3.56E+04	7.07E+04	**1.14E+03**	5.19E+02
30	2.76E+09	1.34E+09	9.37E+05	1.94E+06	2.16E+06	1.16E+07	4.92E+06	1.05E+07	8.31E+06	4.69E+06	7.36E+03	1.03E+04	**3.24E+02**	**4.26E+02**	3.37E+09	1.34E+09	9.11E+05	1.62E+06

The mean and standard deviation of the optimal algorithm are shown in bold

Table 4 The results of FIBSO and eight comparison algorithms (50-dimensional benchmark functions)

Fun	MDBSO		BSO		ALBSO		SBSA		IMFMBO		ABCNG		OLBSO		HOA		FIBSO	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
1	1.35E+11	3.87E-01	**1.87E+03**	1.78E+02	6.42E+07	4.58E+08	2.18E+10	4.85E+09	1.23E+10	1.62E+09	6.67E+03	7.40E+03	5.09E+03	9.62E+03	2.37E+11	2.41E+10	7.39E+10	1.14E+10
3	2.81E+05	5.90E+03	6.83E+03	5.19E+03	**1.30E+01**	3.76E+00	1.25E+05	2.36E+04	2.79E+05	3.57E+04	9.62E+04	6.34E+04	4.28E+04	1.91E+04	6.13E+08	2.64E+09	1.45E+05	7.78E+01
4	5.62E+04	1.86E-06	1.74E+02	5.49E+01	1.90E+02	1.45E+02	2.91E+03	1.22E+03	1.46E+03	2.82E+02	**2.01E+01**	1.98E+01	6.54E+01	3.29E+01	9.52E+04	1.79E+04	1.63E+04	4.89E+03
5	8.15E+02	2.15E-07	**3.28E+02**	1.27E+01	3.23E+02	3.80E+01	3.69E+02	3.90E+01	3.78E+02	2.33E+01	2.19E+02	4.83E+01	**1.52E+02**	2.84E+01	1.16E+03	6.82E+01	6.26E+02	7.34E+01
6	1.08E+02	2.79E-07	5.10E+01	1.05E+01	6.06E+01	5.66E+00	4.62E+01	6.53E+00	2.68E+01	2.44E+00	1.12E+01	6.02E+00	**7.12E+00**	3.50E+00	1.48E+02	8.38E+00	1.10E+02	1.13E+01
7	1.50E+03	3.99E-06	1.01E+03	1.07E+02	1.20E+03	1.96E+02	1.03E+03	1.29E+02	1.09E+03	5.01E+01	1.06E+03	1.02E+02	1.04E+03	4.61E+01	5.02E+03	3.73E+02	**1.00E+03**	8.06E+01
8	8.13E+02	4.21E+01	3.70E+02	5.50E+01	3.34E+02	3.74E+01	3.68E+02	4.00E+01	3.72E+02	2.48E+01	3.30E+02	5.16E+01	3.54E+02	3.02E+01	1.15E+03	7.00E+01	**3.28E+02**	7.78E+01
9	5.31E+04	3.18E+02	9.46E+03	3.35E+02	1.13E+04	2.39E+03	1.06E+04	1.93E+03	9.88E+03	2.36E+03	3.70E+03	2.93E+03	**3.69E+03**	2.74E+03	9.38E+04	1.08E+04	3.61E+04	9.25E+03
10	1.54E+04	5.04E+02	7.71E+03	4.64E+02	7.42E+03	8.26E+02	7.60E+03	5.80E+02	8.83E+03	3.68E+02	9.89E+03	4.00E+03	**5.64E+03**	8.06E+02	1.65E+04	5.93E+02	1.14E+04	9.81E+02
11	5.66E+04	4.82E+03	1.92E+02	4.24E+01	2.32E+02	5.29E+01	9.99E+03	3.67E+03	1.22E+04	4.81E+03	1.28E+02	5.88E+01	**7.61E+01**	2.70E+01	1.39E+05	1.77E+05	7.41E+03	3.19E+03
12	1.23E+11	1.76E+10	1.14E+07	2.28E+06	4.93E+07	1.89E+08	7.80E+09	4.04E+09	2.14E+09	5.45E+08	1.18E+06	1.40E+06	**6.04E+05**	3.24E+05	1.34E+11	2.02E+10	1.74E+10	7.14E+09
13	6.94E+10	1.57E+10	5.86E+04	2.05E+04	1.44E+07	8.36E+07	2.40E+09	1.51E+09	7.56E+08	2.77E+08	1.65E+04	2.10E+04	**8.49E+03**	1.13E+04	7.36E+10	1.62E+10	1.18E+09	1.23E+09
14	1.50E+08	8.38E+07	2.14E+04	1.62E+04	1.19E+05	5.58E+05	3.81E+06	2.76E+06	4.81E+06	3.20E+06	3.24E+04	3.69E+04	2.54E+04	2.36E+04	2.93E+08	1.53E+08	**5.93E+03**	8.85E+03
15	1.92E+10	4.92E+09	2.91E+05	3.57E+03	1.01E+06	7.00E+06	2.59E+08	2.99E+08	2.49E+08	1.43E+08	3.06E+04	3.50E+04	**6.08E+03**	8.43E+03	2.76E+10	8.32E+09	2.73E+05	4.92E+05
16	**1.07E+04**	1.61E+03	**2.26E+03**	1.07E+03	**2.26E+03**	4.95E+02	2.34E+03	3.31E+02	2.69E+03	3.30E+02	2.68E+03	3.77E+02	2.04E+03	4.56E+02	1.18E+04	2.10E+03	**2.03E+03**	3.14E+02
17	1.48E+05	4.87E+04	2.23E+03	8.26E+00	1.99E+03	4.38E+02	**1.80E+03**	3.39E+02	2.03E+03	2.85E+02	1.76E+03	3.53E+02	1.77E+03	3.47E+02	8.47E+05	8.67E+05	**1.72E+03**	1.02E+03
18	6.02E+08	2.41E+08	3.47E+05	5.82E+04	6.43E+05	2.83E+06	2.37E+07	1.56E+07	1.52E+07	9.09E+06	3.52E+05	1.22E+05	3.52E+05	1.48E+05	1.06E+09	5.76E+08	**3.33E+05**	3.78E+05
19	9.20E+09	1.75E+09	5.39E+05	2.45E+05	6.38E+05	6.47E+05	8.74E+07	8.75E+07	6.68E+07	3.21E+07	2.49E+04	4.43E+04	**1.19E+03**	3.02E+03	1.12E+10	2.81E+09	2.56E+05	2.72E+06
20	2.79E+03	1.43E+02	1.61E+03	4.73E+02	1.43E+03	3.02E+02	1.42E+03	2.16E+02	1.53E+03	2.21E+02	1.48E+03	3.54E+02	1.44E+03	2.75E+02	3.42E+03	2.72E+02	**1.41E+03**	3.44E+02
21	**1.34E+03**	8.42E+01	6.20E+02	1.52E+00	6.47E+02	6.89E+01	5.62E+02	4.32E+01	5.82E+02	2.47E+01	5.31E+02	5.46E+01	5.65E+02	3.66E+01	1.39E+03	9.23E+01	**5.29E+02**	1.23E+02
22	1.57E+04	5.68E+02	8.82E+03	7.72E+02	8.28E+03	1.10E+03	8.36E+03	7.27E+02	9.03E+03	1.83E+03	1.09E+04	4.23E+03	7.62E+03	9.24E+02	1.70E+04	6.79E+02	**7.39E+03**	9.64E+02
23	2.30E+03	1.28E+02	1.74E+03	5.09E+02	1.79E+03	2.05E+02	1.44E+03	7.93E+01	8.61E+02	2.57E+01	7.29E+02	1.01E+02	**5.77E+02**	5.98E+01	2.79E+03	3.09E+02	1.38E+03	1.15E+02
24	2.90E+03	1.97E+02	1.60E+03	1.54E+02	**1.80E+03**	1.10E+02	1.12E+03	1.03E+02	1.05E+03	3.57E+01	7.98E+02	6.36E+01	**6.59E+02**	3.83E+01	3.21E+03	3.78E+02	1.08E+03	1.02E+02
25	1.69E+04	6.32E-07	4.58E+02	2.78E+01	4.96E+02	4.48E+01	2.15E+03	6.24E+02	1.61E+03	2.13E+02	**4.51E+02**	2.84E+01	4.57E+02	2.75E+01	5.17E+04	7.73E+03	**1.34E+03**	5.77E+02
26	1.68E+04	3.55E-06	**1.07E+04**	5.91E+02	1.08E+04	1.53E+03	9.10E+03	1.23E+03	5.52E+03	7.00E+02	3.86E+03	1.29E+03	**2.89E+03**	5.99E+02	2.62E+04	2.73E+03	9.01E+03	1.94E+03
27	**1.87E+03**	1.94E+03	2.80E+03	6.51E+02	3.02E+03	6.16E+02	1.52E+03	1.93E+02	7.20E+02	2.56E+02	5.00E+02	3.02E-04	5.00E+02	2.67E-04	5.95E+03	8.36E+02	**5.00E+02**	2.25E-04
28	**5.00E+02**	9.48E-08	4.93E+02	4.15E+00	4.95E+02	2.67E+01	**2.44E+03**	5.35E+02	1.86E+03	3.47E+02	5.00E+02	2.95E+00	5.00E+02	2.09E-04	1.97E+04	2.91E+03	**4.57E+02**	5.01E+01
29	5.49E+05	5.86E+05	**2.44E+03**	5.13E+02	3.11E+03	4.59E+02	3.15E+03	6.30E+02	2.06E+03	2.48E+02	1.55E+03	4.08E+02	**1.31E+03**	3.35E+02	1.02E+06	8.90E+05	3.80E+03	4.70E+02
30	1.51E+10	4.35E+09	1.72E+07	2.15E+06	2.40E+07	4.99E+07	3.31E+08	3.16E+08	1.28E+08	6.25E+07	8.19E+03	7.44E+03	**1.01E+03**	1.87E+03	1.63E+10	4.07E+09	1.09E+07	1.14E+06

The mean and standard deviation of the optimal algorithm are shown in bold

Table 5 The results of FIBSO and eight comparison algorithms (100-dimensional benchmark functions)

Fun	MDBSO		BSO		ALBSO		SBSA		IMFMBO		ABCNG		OLBSO		HOA		FIBSO	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
1	2.97E+11	**1.04E+00**	1.94E+06	6.79E+05	7.87E+08	2.74E+09	1.01E+11	1.27E+10	8.38E+10	9.05E+09	9.87E+03	1.40E+04	**5.70E+03**	9.42E+03	5.93E+11	3.04E+10	2.28E+11	2.01E+10
3	3.66E+05	**1.30E-02**	1.88E+04	1.38E+04	**2.59E+02**	4.67E+02	3.54E+05	3.05E+04	6.84E+05	5.60E+04	9.95E+05	4.44E+05	7.84E+05	4.52E+05	9.59E+10	3.97E+11	3.30E+05	**1.34E+02**
4	1.60E+05	**4.77E-07**	2.70E+02	3.69E+01	3.70E+02	4.31E+02	1.67E+04	3.39E+03	1.17E+04	1.96E+03	**1.17E+02**	3.44E+01	1.88E+02	6.78E+01	2.65E+05	3.72E+04	5.74E+04	1.44E+04
5	1.81E+03	**5.07E-08**	8.87E+02	6.99E+01	8.04E+02	6.24E+01	1.06E+03	6.26E+01	1.12E+03	4.26E+01	6.21E+02	1.10E+02	**4.90E+02**	8.78E+01	2.56E+03	1.16E+02	1.54E+03	**1.34E+02**
6	1.20E+02	**5.69E-09**	6.81E+01	1.34E+00	6.80E+01	5.04E+00	6.70E+01	4.71E+00	5.12E+01	2.47E+00	2.90E+01	6.75E+00	**2.84E+01**	5.91E+00	1.61E+02	7.23E+00	1.12E+02	8.19E+00
7	**3.53E+03**	**1.33E-07**	3.25E+03	6.15E+02	3.23E+03	3.31E+02	3.26E+03	2.30E+02	3.17E+03	1.66E+02	3.19E+03	2.47E+02	3.27E+03	2.00E+02	1.23E+04	5.75E+02	**3.17E+03**	**1.16E+02**
8	1.94E+03	**1.31E-06**	8.26E+02	2.53E+00	9.33E+02	9.63E+01	1.14E+03	8.27E+01	1.14E+03	4.84E+01	6.37E+02	9.33E+01	**5.03E+02**	1.03E+02	2.72E+03	1.21E+02	1.66E+03	**1.16E+02**
9	1.03E+05	**3.93E-04**	2.43E+04	1.79E+03	2.96E+04	6.04E+03	**3.44E+04**	3.74E+03	5.51E+04	7.16E+03	3.33E+04	1.97E+04	**1.51E+04**	7.24E+03	2.20E+05	1.58E+04	6.09E+04	8.63E+03
10	3.33E+04	6.29E+02	**1.57E+04**	**5.00E+02**	**1.57E+04**	1.11E+03	2.00E+04	1.21E+03	2.24E+04	6.46E+02	2.91E+04	6.17E+03	1.46E+04	1.49E+03	3.49E+04	9.18E+02	**1.33E+04**	1.81E+03
11	3.97E+05	5.61E+04	1.18E+03	**5.45E+01**	1.96E+03	2.37E+03	1.12E+05	2.19E+04	1.38E+05	3.11E+04	7.57E+02	6.79E+02	**7.40E+02**	2.95E+02	4.50E+07	1.16E+08	3.13E+04	1.18E+04
12	2.61E+11	**6.55E-01**	1.05E+08	3.13E+06	6.95E+08	1.26E+09	3.89E+10	1.23E+10	2.37E+10	3.81E+09	5.75E+06	3.30E+06	**2.93E+06**	1.73E+06	3.39E+11	3.54E+10	4.75E+09	2.85E+09
13	6.51E+10	5.54E+08	3.51E+04	8.57E+03	3.79E+04	1.17E+04	7.58E+09	2.61E+09	3.84E+09	8.55E+08	8.13E+03	1.02E+04	**6.98E+03**	**7.34E+03**	8.68E+10	9.97E+09	3.32E+08	2.27E+09
14	2.96E+08	4.70E+07	3.65E+05	1.61E+05	4.15E+05	2.30E+05	1.66E+07	6.47E+06	4.11E+07	1.67E+07	2.22E+05	1.01E+05	2.09E+05	**7.64E+04**	6.38E+08	2.15E+08	**2.04E+05**	1.17E+05
15	3.94E+10	3.09E+09	4.24E+04	**1.76E+03**	3.94E+06	2.50E+07	2.54E+09	1.71E+09	1.43E+09	4.43E+08	1.25E+04	2.15E+04	**5.57E+03**	6.56E+03	4.50E+10	9.08E+09	3.79E+07	2.49E+08
16	3.23E+04	4.42E+03	5.83E+03	**3.01E+02**	5.62E+03	7.38E+02	7.97E+03	9.35E+02	8.11E+03	5.43E+02	5.42E+03	1.60E+03	5.42E+03	5.97E+02	3.52E+04	5.20E+03	**5.38E+03**	1.99E+03
17	4.26E+07	2.66E+07	**4.24E+03**	**4.13E+02**	6.04E+03	9.21E+03	2.71E+04	2.14E+04	1.11E+04	6.66E+03	5.51E+03	1.50E+03	**3.35E+03**	5.81E+02	5.88E+07	3.91E+07	**4.24E+03**	1.05E+03
18	1.09E+09	3.73E+08	4.71E+05	**1.20E+05**	6.28E+05	2.19E+05	2.74E+07	1.42E+07	3.28E+07	1.21E+07	5.18E+05	3.06E+05	5.64E+05	3.39E+05	1.35E+09	4.34E+08	**4.56E+05**	5.88E+05
19	4.01E+10	2.01E+09	2.28E+06	1.82E+06	9.46E+06	2.77E+07	3.25E+09	1.35E+09	1.46E+09	4.17E+08	**5.62E+03**	**6.33E+03**	6.48E+03	8.01E+03	4.35E+10	8.10E+09	1.06E+08	2.18E+08
20	6.76E+03	**2.96E+02**	3.81E+03	3.98E+02	3.74E+03	5.27E+02	3.80E+03	4.70E+02	4.55E+03	3.91E+02	3.46E+03	1.08E+03	3.92E+03	5.72E+02	7.57E+03	3.90E+02	**3.08E+03**	5.28E+02
21	3.07E+03	1.50E+02	1.79E+03	**4.50E+01**	1.89E+03	**2.16E+02**	1.46E+03	1.03E+02	1.38E+03	5.04E+01	1.18E+03	1.22E+02	1.24E+03	1.02E+02	3.26E+03	1.77E+02	**1.12E+03**	2.53E+02
22	**3.44E+04**	7.42E+02	1.66E+04	2.50E+03	1.73E+04	**1.39E+03**	2.16E+04	9.53E+02	2.36E+04	**7.10E+02**	2.86E+04	7.35E+03	1.61E+04	1.74E+03	3.65E+04	7.64E+02	**1.27E+04**	1.71E+03
23	4.93E+03	2.49E+02	**2.91E+03**	5.11E+02	3.41E+03	3.11E+02	**2.91E+03**	2.37E+02	1.69E+03	**5.23E+01**	1.46E+03	1.56E+02	**1.18E+03**	1.03E+02	5.75E+03	4.61E+02	**2.70E+03**	**2.16E+02**
24	8.67E+03	4.15E+02	4.11E+03	8.53E+02	4.05E+03	6.36E+02	**3.53E+03**	3.17E+02	2.40E+03	**7.46E+01**	2.12E+03	2.39E+02	**1.75E+03**	2.04E+02	1.10E+04	1.02E+03	**3.56E+03**	2.76E+02
25	3.26E+04	**1.82E-07**	**7.50E+02**	4.65E+01	8.41E+02	3.59E+01	9.05E+03	**1.76E+03**	9.96E+03	1.23E+03	7.64E+02	5.75E+01	7.73E+02	6.49E+01	1.26E+05	1.36E+04	**3.56E+03**	9.70E+02
26	5.63E+04	**1.37E+02**	2.74E+04	1.33E+03	2.89E+04	1.72E+03	3.08E+04	2.53E+03	1.84E+04	7.51E+02	1.63E+04	3.03E+03	**1.10E+04**	1.59E+03	8.74E+04	8.92E+03	2.24E+04	6.20E+03
27	2.57E+03	4.48E+03	3.30E+03	1.01E+03	5.39E+03	1.32E+03	3.48E+03	4.59E+02	1.76E+03	9.75E+01	5.00E+02	3.41E-04	5.00E+02	3.33E-04	1.39E+04	1.34E+03	**5.00E+02**	**1.87E-04**
28	8.22E+03	1.33E+04	5.94E+02	5.58E+01	5.95E+02	4.32E+01	1.23E+04	**1.39E+03**	1.22E+04	1.88E+03	**4.99E+02**	1.53E+01	5.00E+02	**2.74E-04**	6.82E+04	6.41E+03	**2.70E+03**	2.20E+03
29	6.95E+06	2.44E+06	6.46E+03	**1.67E+02**	7.46E+03	8.92E+02	1.51E+04	8.47E+03	9.48E+03	1.62E+03	4.01E+03	6.66E+02	**3.41E+03**	5.67E+02	1.23E+07	6.36E+06	6.33E+03	1.48E+03
30	5.87E+10	4.54E+09	8.17E+06	1.64E+06	5.30E+07	1.18E+08	5.77E+09	2.00E+09	2.13E+09	5.15E+08	1.38E+04	1.83E+04	**2.41E+03**	**4.30E+03**	6.88E+10	1.24E+10	4.33E+08	5.55E+08

The mean and standard deviation of the optimal algorithm are shown in bold

Step 4: Start from the second iteration, correlation analysis is conducted between the updated individuals and the optimal individuals of the three strategies. If it is linearly correlated with an optimal individual, the corresponding strategy is selected for updating. Otherwise, randomly select policy updates.

Step 5: Repeat the operations from Step 2 to Step 4 and constantly update the optimal individual of each mutation strategy. This step continues until the stop criterion of the algorithm is met.

The feature information learning mechanism is shown in Fig. 1. In the first generation, individuals in the population randomly select strategies to update. Some of the updated individuals are improved, and the others are not improved or even worse. Different shapes represent the individuals using different mutation strategies, and the blue shape represents individuals whose fitness is worth improving. Then, the improved individual in each strategy is calculated, and the individual with the greatest improvement degree is selected as the optimal individual in this strategy. The red dots represent the three best individuals who improved using their strategies. In the next generation, the correlation analysis is carried out between the individuals in the population and the optimal individuals represented by the three red dots. If the updated individuals are linearly correlated with one of the individuals, the strategy evolution of the optimal individuals is selected. In this process, the updated results of the previous generation are fed back to the next generation as the characteristic information knowledge between populations, so that the population iterates in a self-learning way.

3.2 The search strategies

Three mutation strategies are designed to balance the local search and global search capabilities of the FIBSO algorithm. In the three mutation strategies, X^t_{new} is the new individual, and X^t_i is the current individual. Generally, problems in the

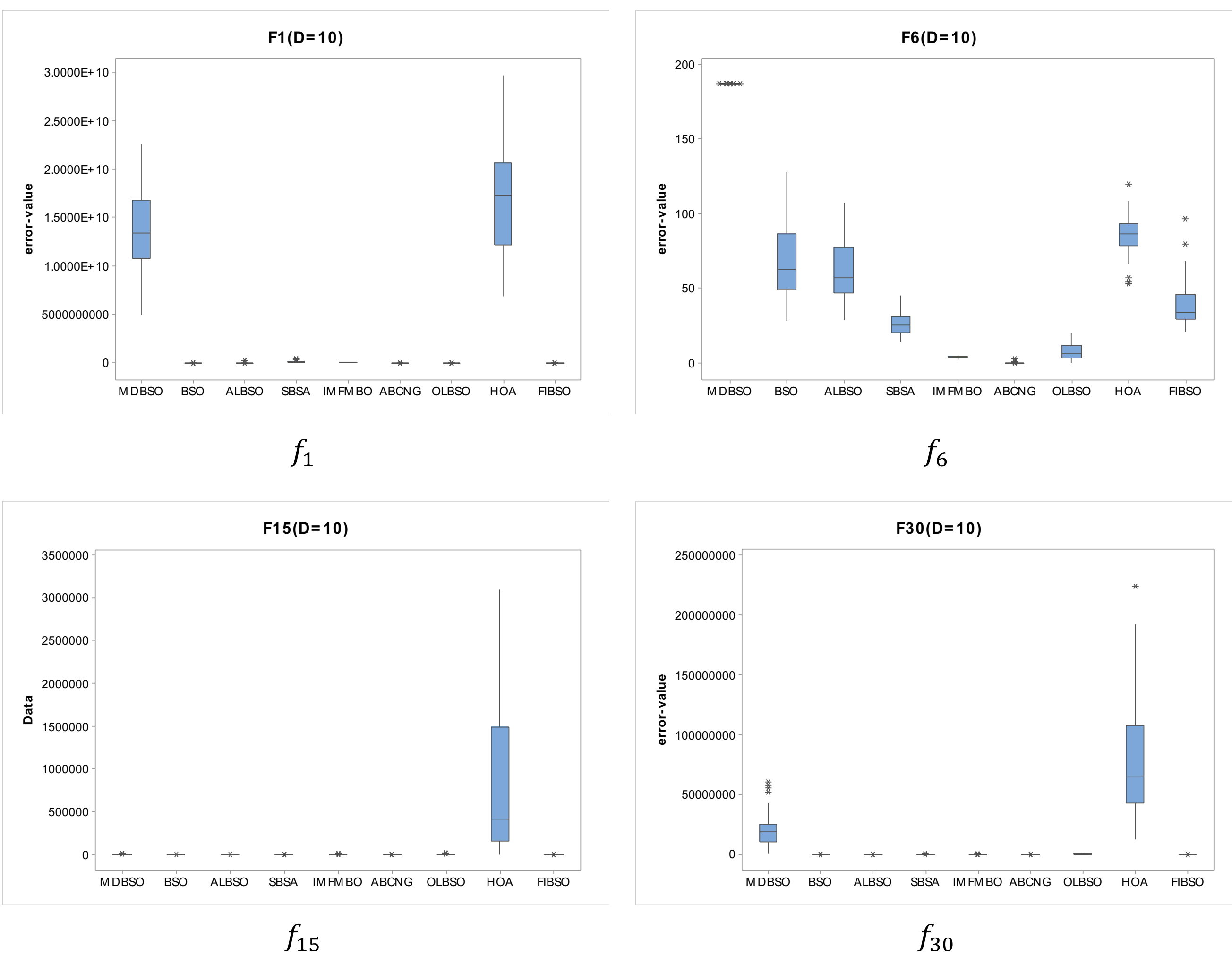


Fig. 3 Box plots of four typical benchmark functions (10D)

real world are characterized by multiple decision variables and multiple constraints. Compared with other test suites, the CEC2017 test suite is closer to the characteristics of complex problems. The FIBSO has effectively integrated three mutation strategies. The first strategy contains the best and worst individuals and has strong competitiveness in solving unimodal problems. The second and third strategies contain random individual difference terms and effectively solve problems with rotation invariance. The three strategies have their own advantages and effectively solve unimodal, multi-modal, hybrid and composition functions in the test suite.

The global best and worst individuals are combined in the first strategy to improve the diversity of the population, and elite individuals are used as perturbations.

$$X_{new}^{t} = w\left(xElitist^{t} + X_{worst}^{t}\right) - xElite^{t} * rand() \tag{5}$$

where, $xElitist^t$ is the best individual, and X_{worst}^t is the worst individual in the current iteration. $xElite^t$ is one of the top ten individuals, $rand()$ is a random number between 0 and 1.

The local optimal individual is introduced into the difference term of two individuals in the second strategy to enhance the local search ability of the individuals.

$$X_{new}^{t} = xElite^{t} + w\left(X_{i1}^{t} - X_{i2}^{t}\right) \tag{6}$$

where, $xElite^t$ is one of the top ten individuals, and X_{i1}^t and X_{i2}^t are two random individuals from two random clusters.

In the third strategy, two kinds of different items are added based on the current individual, which alleviates the problem of falling into local optimal in guiding the individual to find the global optimal direction.

$$X_{new}^{t} = X_{i}^{t} + w\left(xElitist^{t} - X_{i}^{t}\right) + w\left(X_{i1}^{t} - X_{i2}^{t}\right) \tag{7}$$

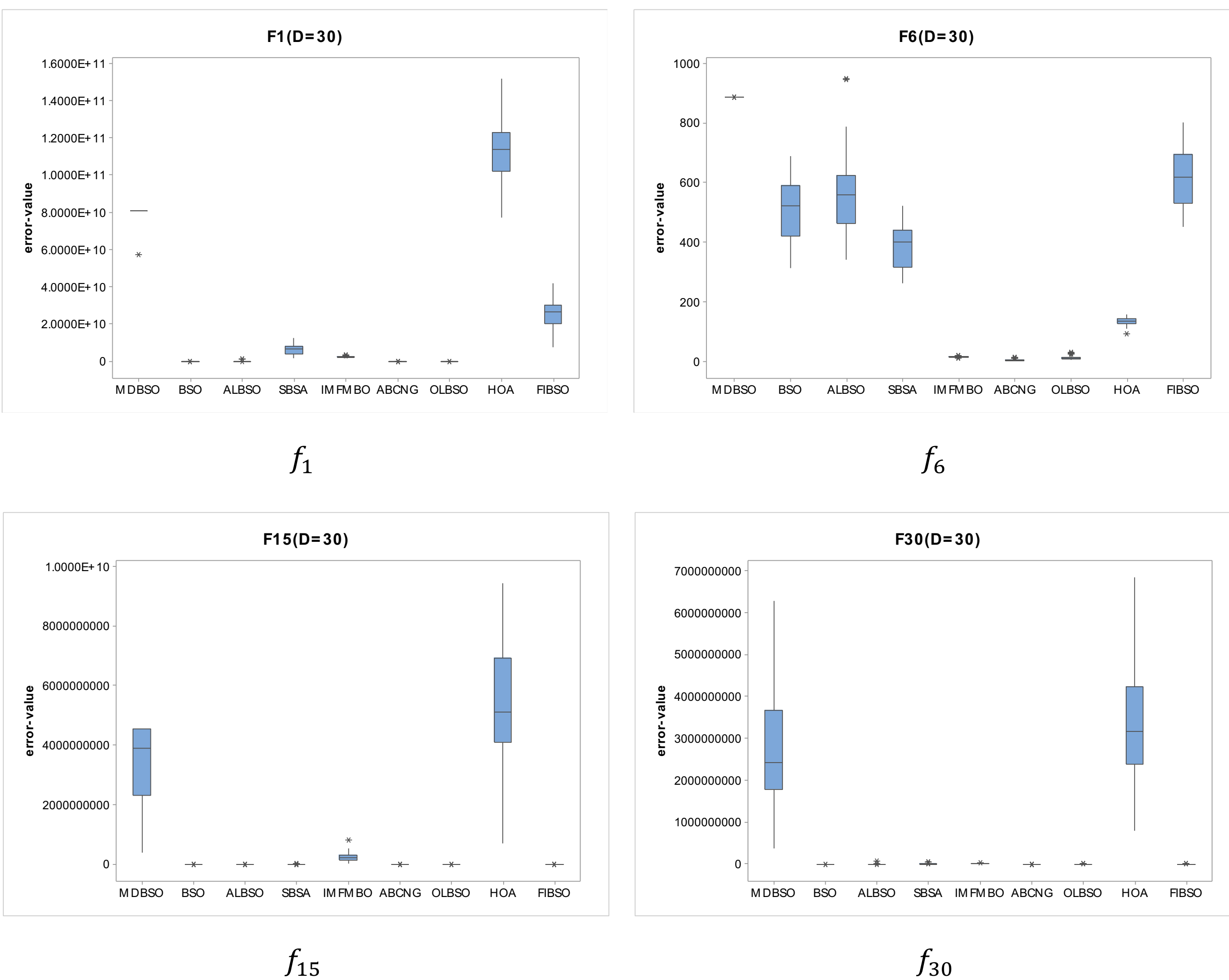


Fig. 4 Box plots of four typical benchmark functions (30D)

where, w is a weight coefficient. T is the maximum number of iterations, and t is the current iteration.

$$w = e^{\left(1-\frac{T}{T-t+1}\right)} \tag{8}$$

3.3 Cluster center disturbance

The randomly selected cluster center is replaced with a randomly generated individual in the cluster center perturbation step in the original BSO. Since randomly generated individuals are not dominant individuals and do not have guidance, the cluster centers generated by this method have no ability to guide the evolution of the population. Therefore, the global best individual and the elite individual are selected in this study to provide a good evolution direction for the cluster center.

$$X_{new}^{t} = (1-w)X_i^t + w\left(xElitist^t - X_{i1}^t\right) + w\left(xElite^t - X_{i2}^t\right) \tag{9}$$

where X_i^t is the current individual, and X_{i1}^t and X_{i2}^t are two random individuals.

3.4 The framework of FIBSO

The solving process is the same as that of meta-heuristic algorithm, which is divided into 4 steps:

- initial solution generation,
- fitness value calculation,
- new solution generation,
- and selection of a solution for the next generation.

Algorithm 1 The FIBSO algorithm

```
Require: N, population size; M, clusters number; T, max iteration
randomly initialize solution X_i
evaluate the fitness of X_i
t = 1
while t < T do
    cluster N individuals into M clusters
    find the cluster center
    if t = 1
        randomly use three mutation strategies to update the population
        calculate each individual feature information and store it
    else
        disturb the cluster centers
        for i = 1 to N do
            use feature information of the population to update individual i
        end for
        calculate each individual feature information and store it
    end if
    t = t+1
end while
```

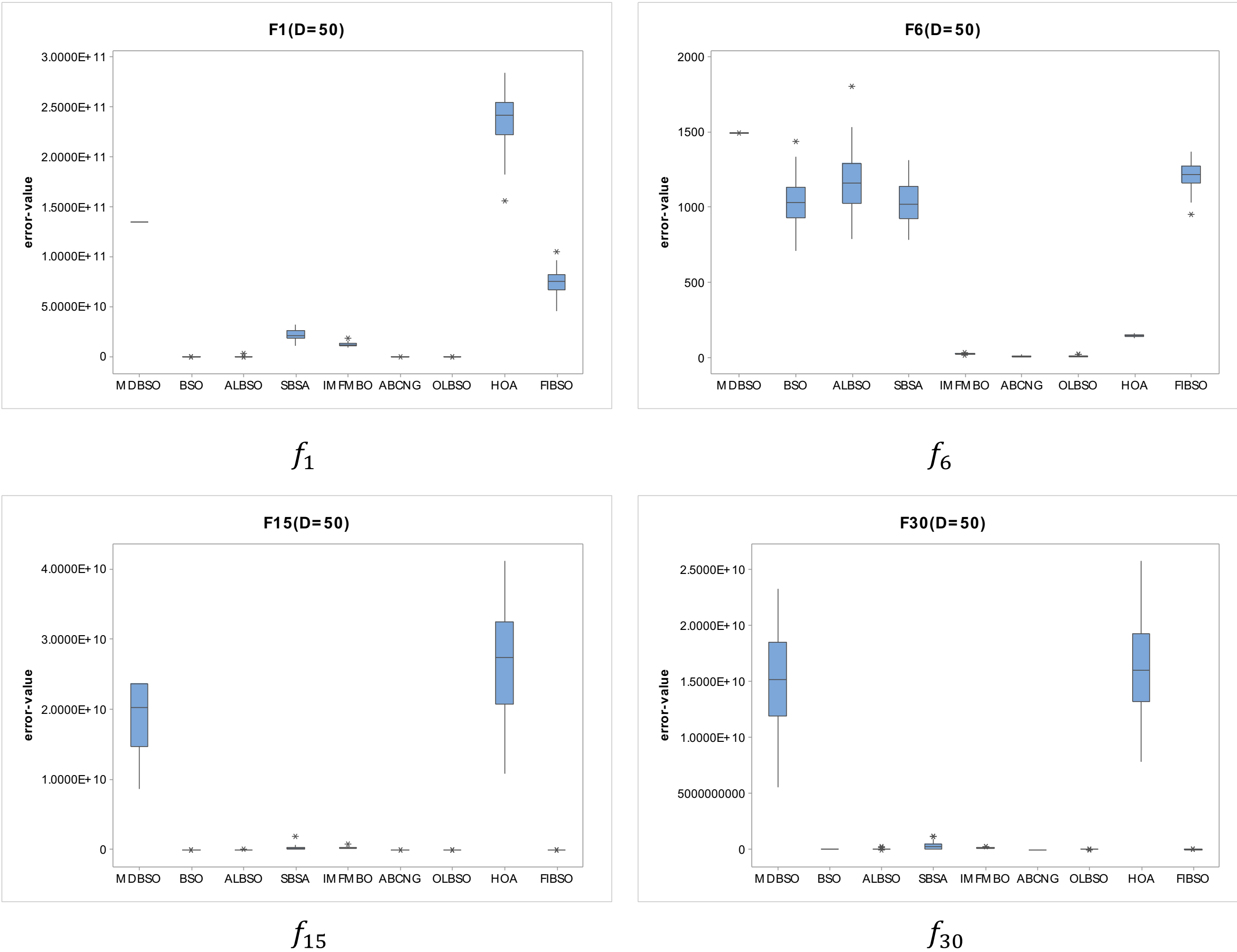


Fig. 5 Box plots of four typical benchmark functions (50D)

The pseudocode of the FIBSO algorithm is illustrated in Algorithm 1. First, the population is initialized randomly and produces N initial solutions, and these N solutions are evaluated. Then, the algorithm enters the iterative solution stage. In the iterative process, N individuals are clustered into M clusters, and the center of each cluster is calculated. In the first generation, individuals in the population randomly choose three strategies to evolve. The improvement of the population is calculated after adopting the evolutionary strategy, and then the corresponding characteristic information of individuals is recorded. From the second generation onwards, the centers of different clusters are disturbed to increase the population diversity as iterations go on. According to the characteristic information obtained from the evolution of the previous generation, each individual chooses to update its own advantageous strategy. Finally, the individual evolution trend is recorded.

4 Experiments and analysis

To verify its effectiveness, in this section, the FIBSO is compared with BSO, multiple diversity-driven brain storm optimization (MDBSO) [38], brain storm optimization algorithm with adaptive learning strategy (ALBSO) [20], parameter identification of chaotic systems using a shuffled backtracking search optimization algorithm (SBSA) [39], PSO-based improved multi-flocks migrating birds optimization (IMFMBO) [40], artificial bee colony algorithm based on adaptive neighborhood search and gaussian perturbation (ABCNG), the feature information learning mechanism [12], OLBSO [17], and horse herd optimization algorithm (HOA) [41]. Moreover, the FIBSO is applied to solve the no-wait flow shop scheduling problem. The parameters of the comparison algorithm are as follows.

Algorithm	Parameter Setting
BSO	$N = 100,\ M = 5,\ P_{5a} = 0.2,\ P = 0.8,\ P_{global} = 0.2,\ P_{local} = 0.5,\ k = 20$
MDBSO	$N = 100,\ M = 5$
ALBSO	$N = 100,\ M = 5,\ P_{5a} = 0.2,\ P = 0.8,\ P_{global} = 0.2,\ P_{local} = 0.5,\ k = 20$
OLBSO	$N = 100,\ M = 5,\ P_{5a} = 0.2,\ P = 0.8,\ P_{global} = 0.7,\ P_{local} = 0.7,\ k = 20$
SBSA	$N = 100,\ M = 5,\ F = rand(0,2),\ mixrate = 1$
IMFMBO	$sub\_pop = 100,\ Neighbor = 3,\ c_1 = 1,\ c_2 = 1,\ Flock = 15,\ x = 1,\ m = 20$
ABCNG	$N = 50$
HOA	$N = 50,\ w = 0.95,\ h_\beta = 0.9,\ h_\gamma = 0.5,\ s_\beta = 0.2,\ s_\gamma = 0.1,\ i_\gamma = 0.3,\ d_\alpha = 0.5,$ $d_\beta = 0.2,\ d_\gamma = 0.1,\ r_\sigma = 0.1,\ r_\gamma = 0.05$

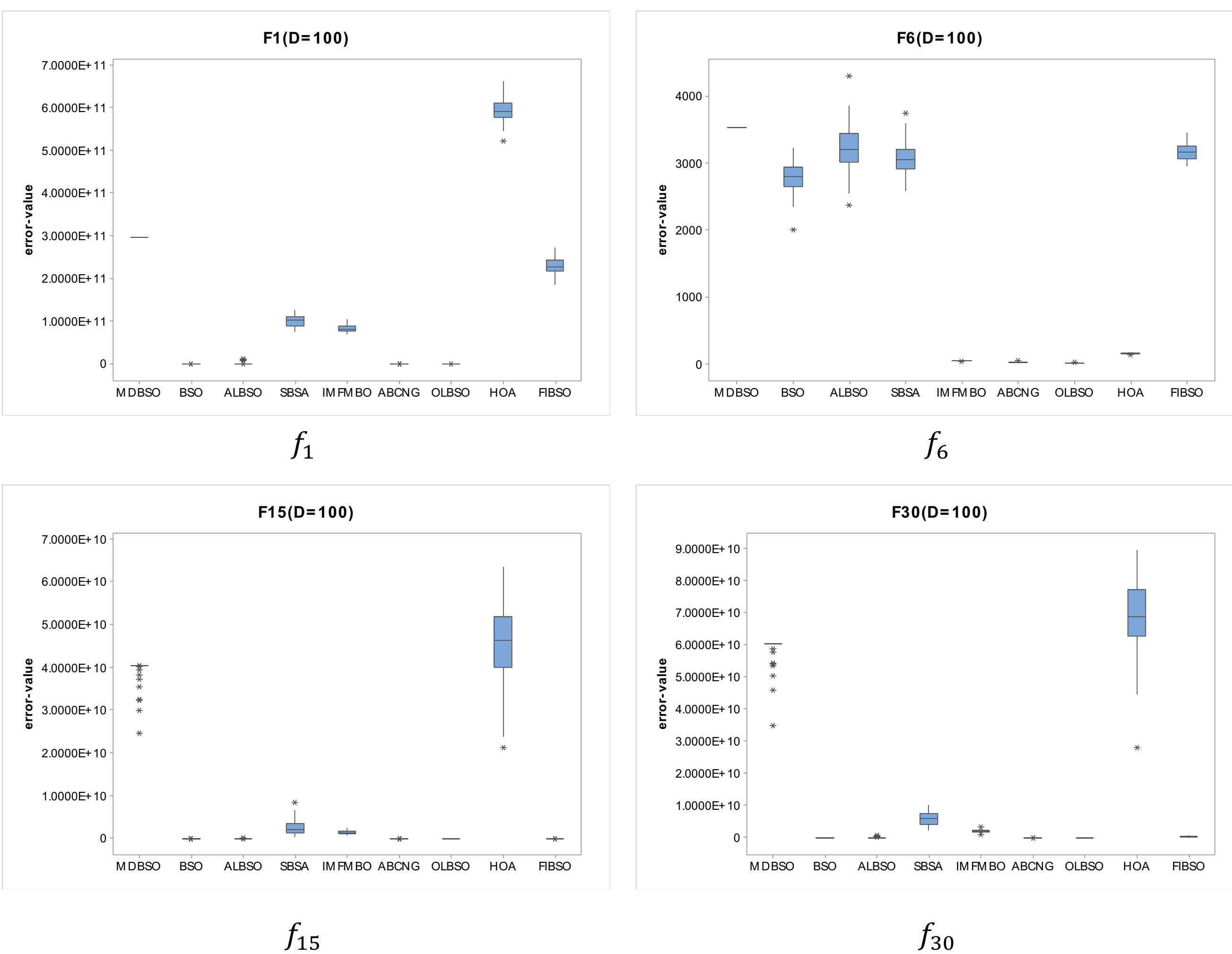


Fig. 6 Box plots of four typical benchmark functions (100D)

4.1 Parameter setting

Various parameter settings have significantly different effects on the algorithm. The orthogonal experiments design (OED) experiments are carried out on the involved parameters in the algorithm to obtain a reasonable combination of parameters. The main parameters are: population size $N = \{25,50,100,200\}$, slope in new individual generation $k = \{20, 25\}$, and probability of cluster center disturbance $P = \{0.2,0.4,0.6,0.8\}$. The 32 combinations of the parameters are tested in the FIBSO. The experimental results are analyzed by multivariate analysis of variance (ANOVA) in Table 1.

It is considered that there is a significant difference between the two variables, if $p - value$ is less than 0.05. The population size N has the most significant effect on the algorithm from Table 1, and $N = 100$ is determined according to the main effect diagram in Fig. 2. Furthermore, there are considerable differences among the other groups of variables, because the $p - value$ of other variables are all greater than 0.05. $k = 20$ and $P = 0.4$ are determined from the main effect diagram.

4.2 The benchmark test

The FIBSO and the eight comparison algorithms are compared by using the CEC2017 test suite. The nine algorithms

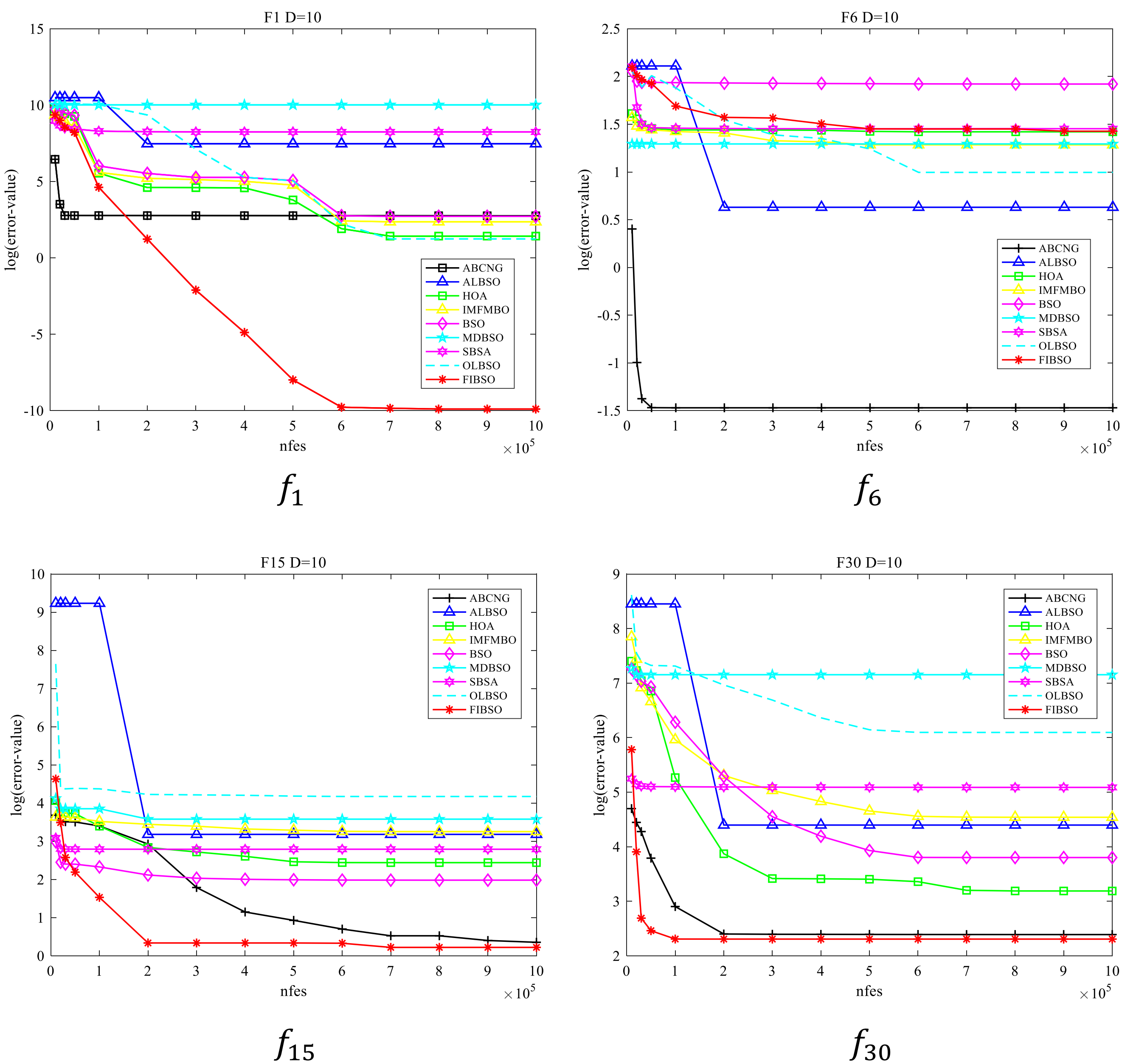


Fig. 7 Convergence curves of four typical benchmark functions (10D)

are run 51 times independently on 29 functions in the test suite with $D = 10$, $D = 30$, $D = 50$, $D = 100$. D represents the dimension of the functions. The algorithms are run on a PC with a 3.4 GHz Intel (R), Core (TM) i7–6700 CPU, 8GB of RAM and 64-bit OS. It can be seen from Tables 2, 3, 4 and 5 that the impact of the FIBSO is better than other comparison algorithms, especially in hybrid and composition functions.

The mean values of FIBSO in 17 functions are better than that of the comparison algorithms in 10D. This mean values for the FIBSO in 30D are also better than that of the comparison algorithms in 13 functions. In 50D, the results of 17 functions show that FIBSO has a better performance compared to the other algorithms. The mean values of 14 functions prove the significant performance of the FIBSO in 100D.

The f_1 function in the unimodal functions ($f_1 - f_3$), the f_6 function in the multi-modal functions ($f_4 - f_{10}$), the f_{15} function in the hybrid functions ($f_{11} - f_{20}$), and the f_{30} in the composition functions ($f_{21} - f_{30}$) are selected for further analysis of the algorithm. In Figs. 3, 4, 5 and 6, the FIBSO has higher stability in the hybrid and composition functions than the four comparison algorithms. The horizontal axis represents the different algorithms. In addition, Figs. 7, 8, 9, and 10 show that the convergence speed and solving accuracy of

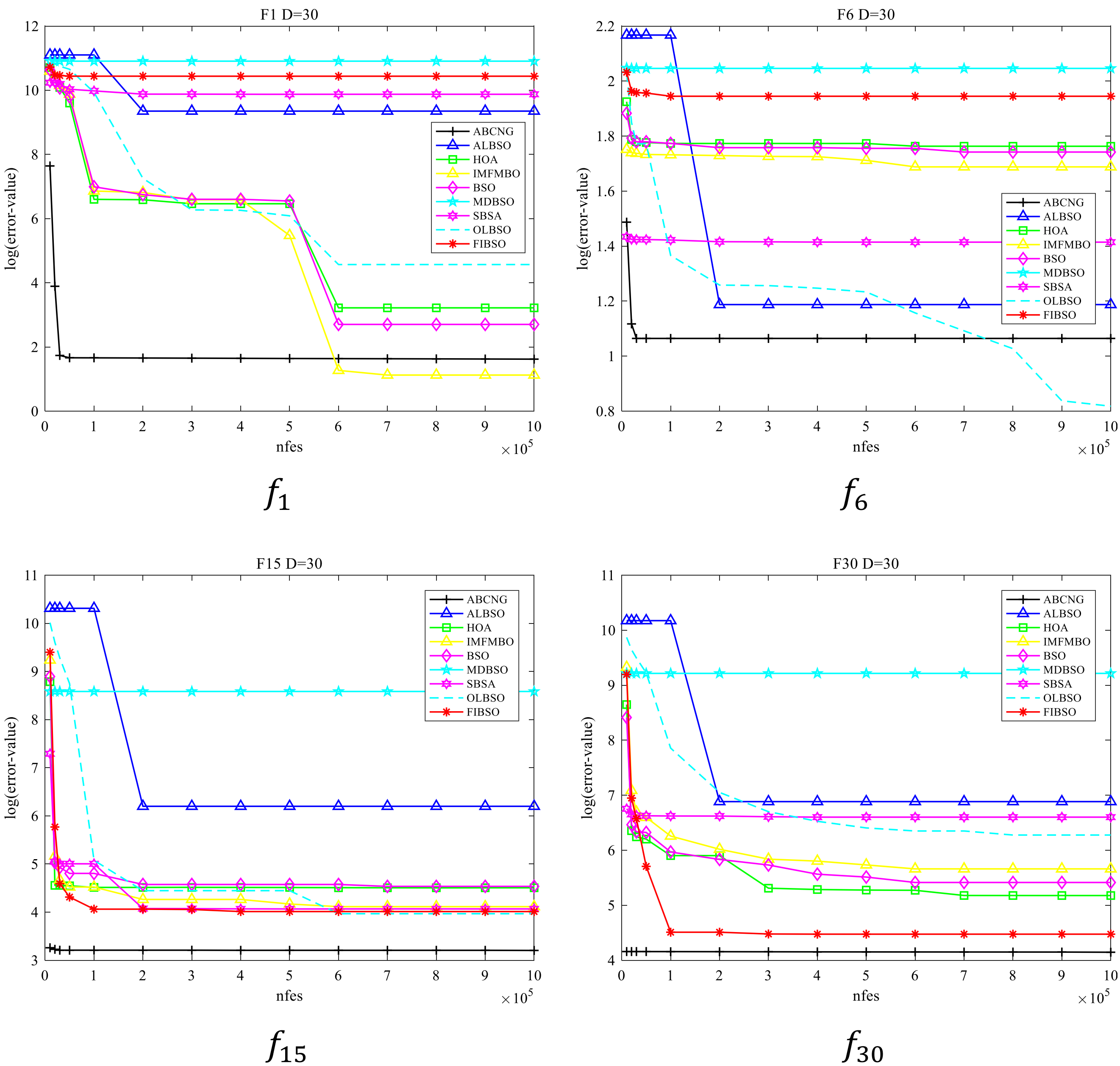


Fig. 8 Convergence curves of four typical benchmark functions (30D)

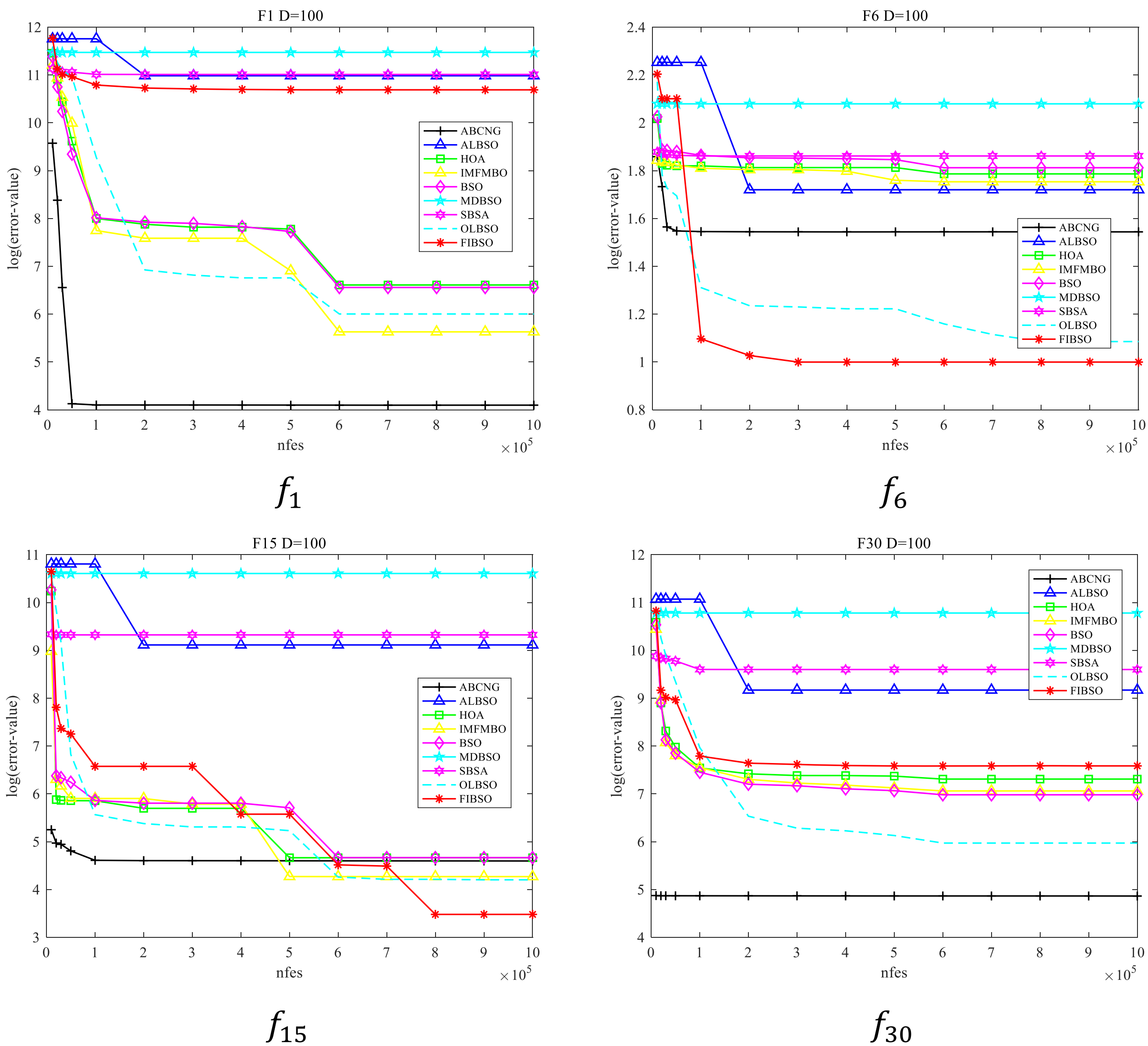


Fig. 9 Convergence curves of four typical benchmark functions (50D)

the FIBSO are better than those four comparison algorithms in hybrid and composition functions.

4.3 The results of Friedman test and the Wilcoxon symbolic rank test

The Friedman test is used to analyze the significant differences between the algorithms, and the results are shown in Fig. 11. The horizontal axis represents the algorithms, and the vertical axis represents the average ranking of each algorithm. The average rank of FIBSO is smaller than other comparison algorithms in $D = 10, D = 30, D = 50, D = 100$. The FIBSO has no significant difference from SBSA, but is significantly different from other algorithms in 10*D*. The FIBSO has a significant difference from MDBSO in 30*D* and 50*D*. In 100*D*, FIBSO is significantly different from MDBSO and SBSA. The Wilcoxon test is used to analyze the significant difference between the two pairs of algorithms. Tables 6, 7, 8, and 9 shows the differences between FIBSO and other algorithms in various dimensions, which are measured by $p-value$. The results indicate that the FIBSO is significantly different from the comparison algorithms when the confidence degree is $\alpha = 0.05$ and $\alpha = 0.1$.

4.4 Effects of FIBSO components

Three operations are utilized in the FIBSO, namely mutation operation, learning operation based on information features,

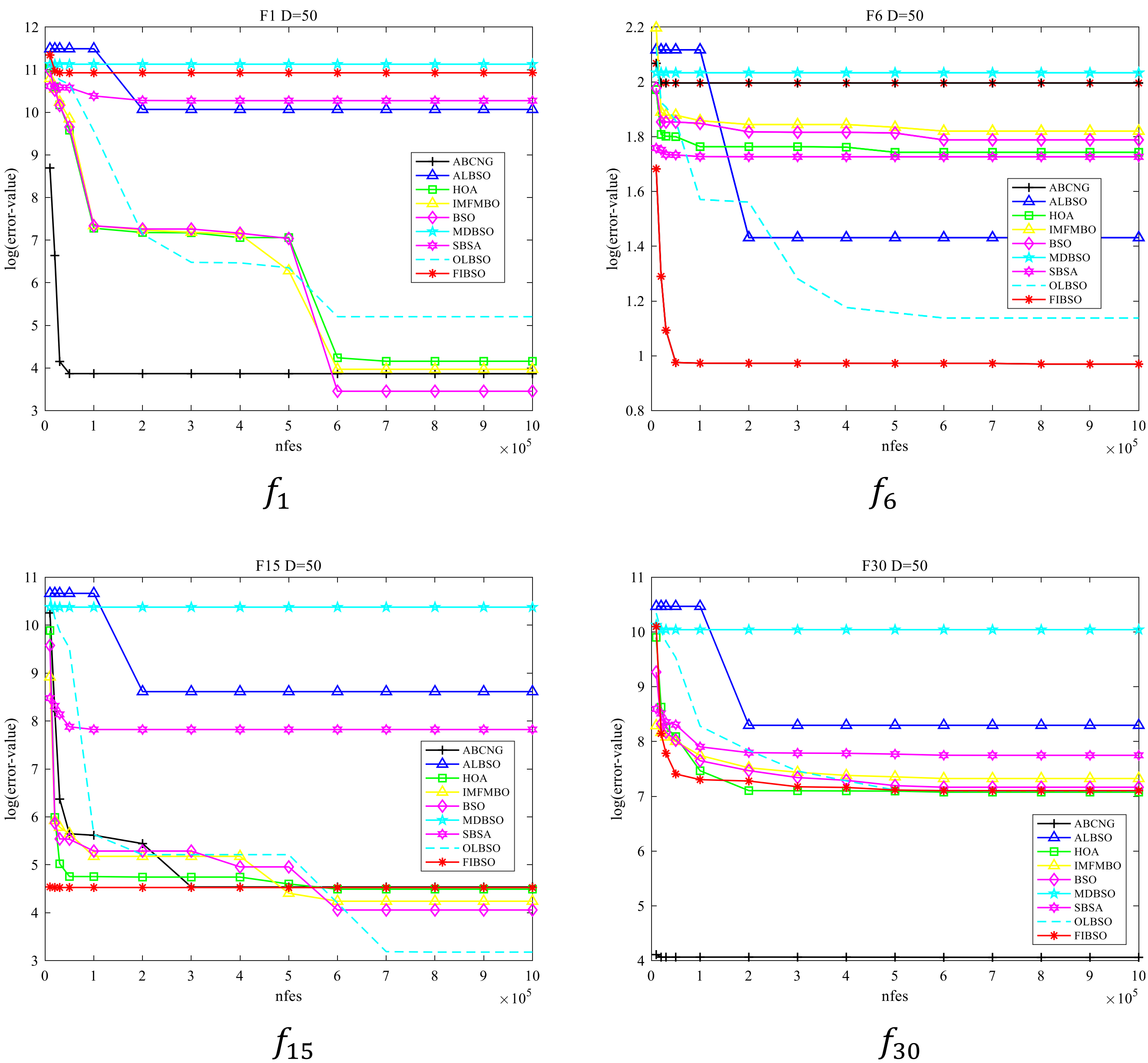
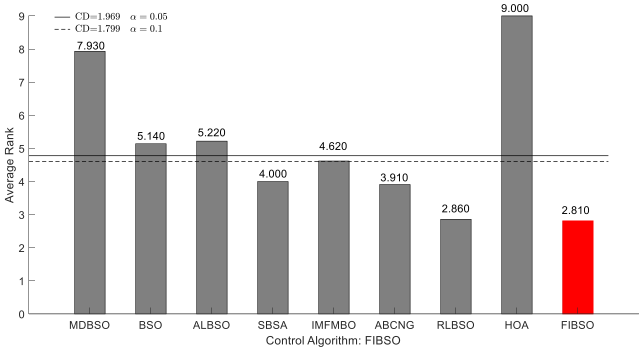


Fig. 10 Convergence curves of four typical benchmark functions (100D)

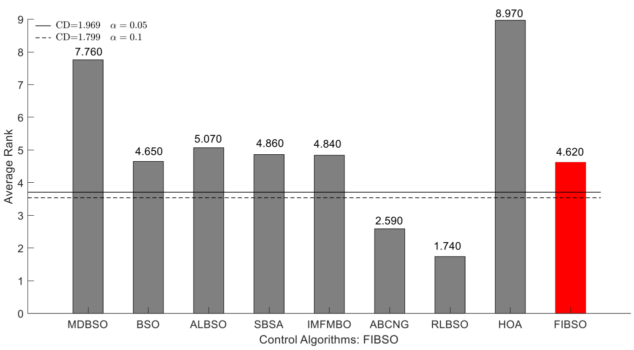
and cluster center disturbance operation. Three groups of experiments are designed to verify the effectiveness of these three operations. The algorithm that removes cluster center disturbance is marked as no-disturb, and the algorithm that removes feature information learning is indicated as no-learn. The algorithm that changes the update strategy to the individual update strategy in the original BSO is denoted as no-strategy. The experiment results of the three algorithms and the FIBSO on four types of functions are shown in Table 10. The mean values and standard deviation of FIBSO are better than that of the other three algorithms. The results of FIBSO in nine functions are generally better than that of the other three algorithms from the point plot in Fig. 12.

4.5 Results analysis and discussion

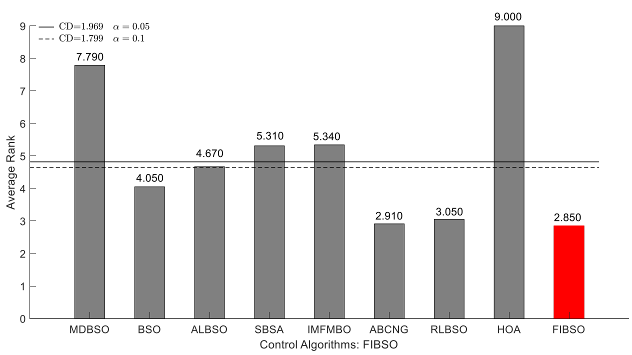
The results of the CEC2017 test suite and the experimental results of statistical analysis show that the performance of FIBSO is better than that of the comparison algorithms. The solution of the evolutionary algorithm is to find the optimal solution iteratively. The individual conducts a random search in the solution space in a blind process. In this paper, the characteristic information between individuals is considered to find the correlation between individuals. Hence, individuals no longer search blindly in the process of evolution, and the effective guidance speeds up the algorithm to find the optimal solution. The topography of the function is rough and complex in high-



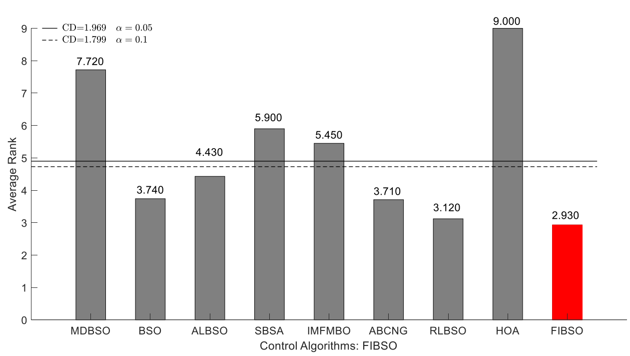
10 *D*



30 *D*



50 *D*



100 *D*

Fig. 11 Friedman test of nine algorithms on four dimensions

dimensional and large-scale problems. The algorithm has a significant advantage in the performance of hybrid and composition functions owing to the characteristic information learning.

Each individual has a different evolutionary direction, which effectively improves the probability of the algorithm in finding the optimal solution. The three operations use the global best individual guidance and enhance the ability of the algorithm to discover the global optimal solution. The reverse learning of the global best and worst individual composition increases the diversity of the population, which boosts the coarse search ability of the algorithm. The three operations make the algorithm better than the comparison algorithms in solving speed and accuracy.

The global best and local best individuals are used to perturbate the cluster center. When the search is stalled, the guidance of global best and local best helps the algorithm effectively jump out of the local best solution. The further operation gives the FIBSO an advantage in hybrid and composition functions.

4.6 The test on no-wait flow shop scheduling problem

In this section, FIBSO is applied to the no-wait flow shop scheduling problem [42] and compared with DPSO [43] and DWWO [44]. The no-wait flow shop scheduling problem is defined as follows. There are n jobs processed on m machines, and all the jobs have the same machining path on the machines. The constraint conditions are as follows: one job is processed on one machine at a certain time. A machine processes one job at a time. There is no wait time between each program of the job. The object is to arrange the processing order of the jobs reasonably to make the processing time of the job minimum.

The three algorithms are tested on Ta1-Ta60 of the Taillard benchmark test suite, and each algorithm is run 10 times independently. The termination time is: $n/2 \, * \, m \, * \, 10/1000$. The average relative percentage deviation (ARPD) is used to demonstrate the performance of FIBSO versus the two comparison algorithms.

$$ARPD = \frac{1}{run} \sum_{r=1}^{run} \frac{C - C^*}{C^*} * 100 \qquad (10)$$

where, C is the makespan of each algorithm on different instance, and C^* is the best value of the three algorithms. Parameter run is the maximum run number of the algorithm.

Table 11 presents the ARPD values of the three algorithms on 60 instances, in which FIBSO performs well, especially in the first 30 instances. Table 12 is the mean value of the three algorithms in 60 instances. Figure 13 is a time sequence diagram of the ARPD values of the three algorithms on 60 instances. FIBSO benefits over the other mentioned algorithms in solving 60 instance problems.

Table 6 Rankings obtained through Wilcoxon test (10D)

Dimension	FIBSO vs	R^+	R^-	Z	$p-value$	α=0.05	α=0.1
10D	MDBSO	0	435	−4.70E+00	3.00E-06	yes	yes
	BSO	84	351	−2.89E+00	3.89E-03	yes	yes
	ALBSO	39	396	−3.86E+00	1.13E-04	yes	yes
	SBSA	104	302	−2.25E+00	2.42E-02	yes	yes
	IMFMBO	118	317	−2.15E+00	3.14E-02	yes	yes
	ABCNG	136	299	−1.76E+00	7.80E-02	yes	yes
	OLBSO	281	154	−1.37E+00	1.70E-01	no	no
	HOA	0	435	−4.70E+00	3.00E-06	yes	yes

Table 7 Rankings obtained through Wilcoxon test (30D)

Dimension	FIBSO vs	R^+	R^-	Z	$p-value$	α=0.05	α=0.1
30D	MDBSO	0	435	−4.70E+00	3.00E-06	yes	yes
	BSO	237	198	−4.22E-01	6.73E-01	no	no
	ALBSO	224	211	−1.41E-01	8.88E-01	no	no
	SBSA	211	224	−1.41E-01	8.88E-01	no	no
	IMFMBO	211	224	−1.41E-01	8.88E-01	no	no
	ABCNG	341	65	−3.14E+00	1.68E-03	yes	yes
	OLBSO	364	42	−3.67E+00	2.46E-04	yes	yes
	HOA	0	435	−4.70E+00	3.00E-06	yes	yes

Table 8 Rankings obtained through Wilcoxon test (50D)

Dimension	FIBSO vs	R^+	R^-	Z	$p-value$	α=0.05	α=0.1
50D	MDBSO	1	434	−4.68E+00	3.00E-06	yes	yes
	BSO	224	211	−1.41E-01	8.88E-01	no	no
	ALBSO	217	218	−1.08E-02	9.91E-01	no	no
	SBSA	164	271	−1.16E+00	2.47E-01	no	no
	IMFMBO	195	240	−4.87E-01	6.27E-01	no	no
	ABCNG	321	85	−2.69E+00	7.21E-03	yes	yes
	OLBSO	330	76	−2.89E+00	3.83E-03	yes	yes
	HOA	0	435	−4.70E+00	3.00E-06	yes	yes

Table 9 Rankings obtained through Wilcoxon test (100D)

Dimension	FIBSO vs	R^+	R^-	Z	$p-value$	α=0.05	α=0.1
100D	MDBSO	0	435	−4.70E+00	3.00E-06	yes	yes
	BSO	275	131	−1.64E+00	1.01E-01	no	no
	ALBSO	281	154	−1.37E+00	1.70E-01	no	no
	SBSA	84	351	−2.89E+00	3.89E-03	yes	yes
	IMFMBO	88	318	−2.62E+00	8.83E-03	yes	yes
	ABCNG	295	111	−2.09E+00	3.62E-02	yes	yes
	OLBSO	310	96	−2.44E+00	1.48E-02	yes	yes
	HOA	0	435	−4.70E+00	3.00E-06	yes	yes

Table 10 Results of 9 typical functions

Fun	no-disturb		no-learn		no-strategy		FIBSO	
	mean	std	mean	std	mean	std	mean	std
1	1.68E+06	2.42E+06	1.22E+09	1.17E+09	7.16E+09	2.66E+09	**1.80E+04**	**8.03E+04**
5	3.97E+01	2.07E+01	4.77E+01	1.30E+01	1.24E+02	4.59E+01	**1.31E+01**	**5.04E+00**
9	1.53E+02	1.14E+02	4.48E+02	1.65E+02	1.04E+03	4.40E+02	**1.76E+01**	**1.59E+01**
12	1.53E+05	2.34E+05	**6.74E+04**	**5.36E+04**	1.10E+08	8.61E+07	2.82E+05	5.82E+05
15	1.27E+02	**2.72E+01**	4.66E+02	2.69E+02	1.48E+03	8.90E+02	**1.16E+02**	3.05E+02
18	**3.01E+02**	**1.40E+02**	1.49E+03	1.16E+03	2.84E+05	6.04E+05	2.04E+03	2.43E+03
22	1.15E+02	**6.46E+00**	2.07E+02	8.24E+01	1.19E+03	8.40E+02	**1.13E+02**	2.45E+01
25	4.39E+02	2.67E+01	4.95E+02	5.28E+01	7.54E+02	1.78E+02	**4.32E+02**	**2.58E+01**
29	3.43E+02	7.28E+01	3.73E+02	5.65E+01	7.23E+02	2.38E+02	**2.74E+02**	**1.98E+01**

The mean and standard deviation of the optimal algorithm are shown in bold

Fig. 12 Point plot of 9 functions

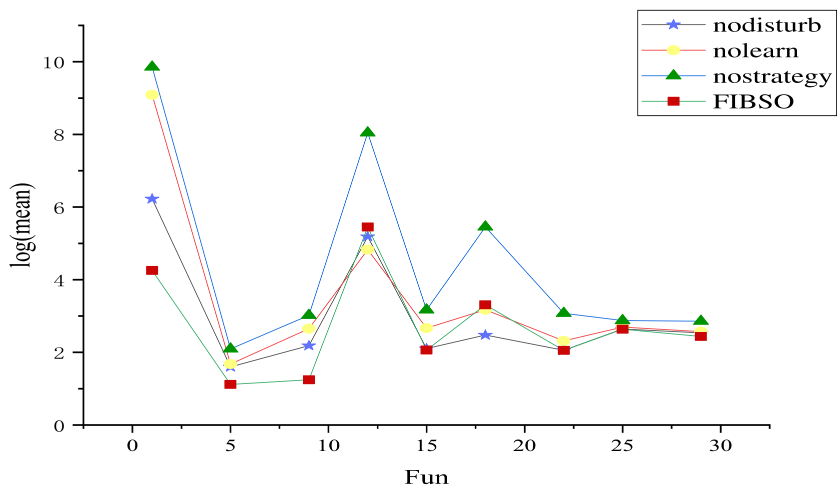


Table 11 The ARPD of the Taillard benchmark

Ta	FIBSO	DWWO	DPSO	Ta	FIBSO	DWWO	DPSO	Ta	FIBSO	DWWO	DPSO
1	0.00	0.00	0.07	21	0.00	0.00	0.03	41	0.28	0.00	0.56
2	0.00	0.00	0.20	22	0.00	0.00	0.00	42	0.00	0.00	0.69
3	0.00	0.00	0.00	23	0.03	0.00	0.00	43	0.00	0.07	1.02
4	0.00	0.13	0.06	24	0.00	0.00	0.07	44	7.79	0.00	7.91
5	0.00	0.00	0.07	25	0.00	0.00	0.00	45	5.58	0.00	6.14
6	0.00	0.07	0.81	26	0.00	0.03	0.07	46	4.97	0.00	5.53
7	0.00	0.13	0.07	27	0.00	0.00	0.00	47	7.84	0.00	8.88
8	0.00	0.13	0.13	28	0.00	0.00	0.18	48	5.75	0.00	6.02
9	0.00	0.00	0.00	29	0.00	0.00	0.00	49	0.00	0.14	0.41
10	0.00	0.15	0.15	30	0.00	0.00	0.10	50	0.00	0.07	0.33
11	0.00	0.05	0.05	31	0.00	0.25	1.23	51	0.00	0.03	0.75
12	0.00	0.00	0.09	32	0.00	0.03	0.90	52	0.00	0.03	0.78
13	0.00	0.05	0.10	33	0.00	0.19	0.74	53	2.33	0.00	2.83
14	0.00	0.00	0.00	34	0.00	0.06	0.74	54	0.00	0.91	2.49
15	0.00	0.16	0.05	35	1.24	0.00	0.92	55	0.00	0.59	3.93
16	0.00	0.21	0.26	36	0.00	0.06	0.98	56	0.00	1.05	3.82
17	0.00	0.10	0.00	37	0.00	0.03	0.80	57	4.20	0.00	4.55
18	0.00	0.15	0.00	38	0.74	0.00	0.92	58	0.00	0.02	1.08
19	0.00	0.00	0.00	39	0.74	0.00	0.61	59	0.00	0.03	0.42
20	0.00	0.00	0.00	40	0.00	0.09	0.83	60	0.00	0.05	0.28

Table 12 The result of the Taillard benchmark

Ta	FIBSO	DWWO	DPSO	Ta	FIBSO	DWWO	DPSO	Ta	FIBSO	DWWO	DPSO
1	1487	1487	1488	21	2973	2973	2974	41	4315	4303	4327
2	1529	1529	1532	22	2852	2852	2852	42	4202	4202	4231
3	1460	1460	1460	23	3020	3019	3019	43	4119	4122	4161
4	1588	1590	1589	24	3001	3001	3003	44	4443	4122	4448
5	1449	1449	1450	25	3003	3003	3003	45	4352	4122	4375
6	1481	1482	1493	26	2998	2999	3000	46	4327	4122	4350
7	1483	1485	1484	27	3052	3052	3052	47	4445	4122	4488
8	1482	1484	1484	28	2839	2839	2844	48	4359	4122	4370
9	1469	1469	1469	29	3009	3009	3009	49	4172	4178	4189
10	1377	1379	1379	30	2979	2979	2982	50	4296	4299	4310
11	2044	2045	2045	31	3182	3190	3221	51	6160	6162	6206
12	2166	2166	2168	32	3455	3456	3486	52	5760	5762	5805
13	1940	1941	1942	33	3243	3249	3267	53	5896	5762	5925
14	1811	1811	1811	34	3358	3360	3383	54	5710	5762	5852
15	1933	1936	1934	35	3419	3377	3408	55	5728	5762	5953
16	1892	1896	1897	36	3359	3361	3392	56	5702	5762	5920
17	1963	1965	1963	37	3253	3254	3279	57	6004	5762	6024
18	2059	2062	2059	38	3283	3259	3289	58	5945	5946	6009
19	1973	1973	1973	39	3126	3103	3122	59	5895	5897	5920
20	2051	2051	2051	40	3354	3357	3382	60	5971	5974	5988

5 Conclusion

In this paper, a FIBSO algorithm is proposed. Feature information between individuals, which is learned in the process of evolution, theoretically guides the population to evolve in a good direction. Global and local optimal individuals in the three strategies are introduced to enhance the ability of the algorithm to find the optimal solution. The FIBSO and the four comparison algorithms are tested on the CEC2017 test suite. The performance of FIBSO is better than that of the comparison

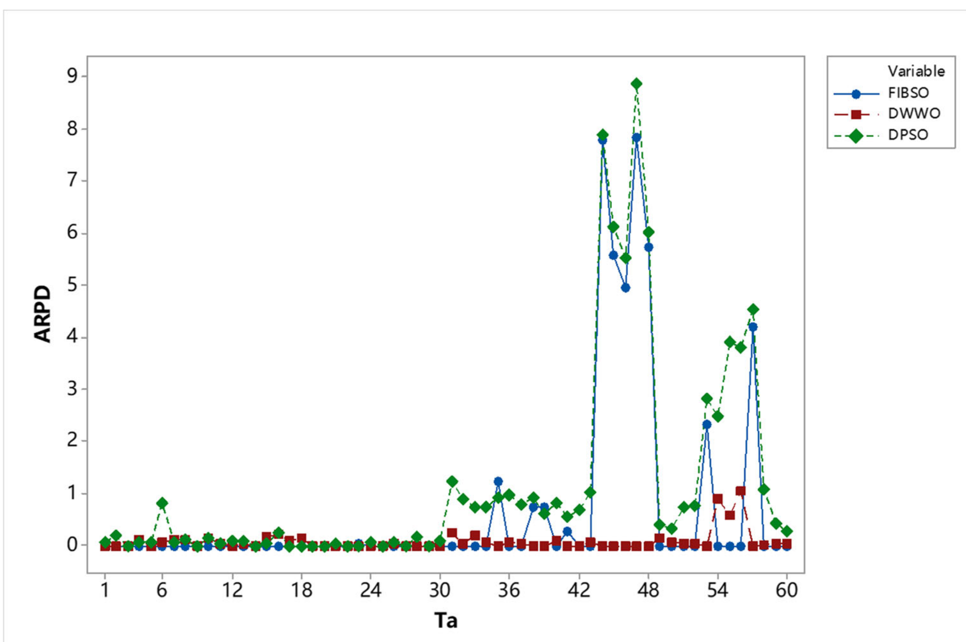


Fig. 13 The ARPD values of three algorithms on 60 instances

algorithms, especially in large-scale and complex problems. The FIBSO has been tested on four typical problems, and the results show that the convergence speed and solving accuracy of the FIBSO are better than the other four algorithms. The stability of the algorithm is tested, and it is proved that the algorithm has good robustness. The test results on no-wait flow shop scheduling problem show that the proposed algorithm has the ability to solve practical engineering problems. In the future, the application of the algorithm will be studied further in terms of distributed no-wait flow shop scheduling problem.

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Data availability The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

Declarations

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

Conflict of interest All authors certify that they have no affiliations with or involvement in any organization or entity with any financial interest or non-financial interest in the subject matter or materials discussed in this manuscript.

Informed consent Informed consent was obtained from all individual participants included in the study.

References

1. Fan L, Pan K, Guan Y (2019) A strengthened mixed-integer linear programming formulation for combined-cycle units. Eur J Oper Res 275:865–881. https://doi.org/10.1016/j.ejor.2018.12.020
2. Zhao X, Fang Y, Liu L, Li J, Xu M (2020) An improved moth-flame optimization algorithm with orthogonal opposition-based learning and modified position updating mechanism of moths for global optimization problems. Appl Intell 50:4434–4458. https://doi.org/10.1007/s10489-020-01793-2
3. Luo J, Liu Z (2020) Novel grey wolf optimization based on modified differential evolution for numerical function optimization. Appl Intell 50:468–486. https://doi.org/10.1007/s10489-019-01521-5
4. Zhao F, Zhao L, Wang L, Song H (2020) An ensemble discrete differential evolution for the distributed blocking flowshop scheduling with minimizing makespan criterion. Expert Syst Appl 160: 113678
5. Zhao F, He X, Wang L (2020) A two-stage cooperative evolutionary algorithm with problem-specific knowledge for energy-efficient scheduling of no-wait flow-shop problem. IEEE Trans Cybern 51: 5291–5303. https://doi.org/10.1109/tcyb.2020.3025662
6. Wang D, Tan D, Lei L (2018) Particle swarm optimization algorithm: an overview. Soft Comput 22:387–408
7. Holland JH (2019) Adaptation in natural and artificial systems. Adapt Nat Artif Syst. https://doi.org/10.7551/mitpress/1090.001.0001
8. Zt A, Kl A, Yi WB (2021) Differential evolution with adaptive mutation strategy based on fitness landscape analysis. Inf Sci (Ny) 549:142–163
9. Tan Z, Li K (2021) Differential evolution with mixed mutation strategy based on deep reinforcement learning. Appl Soft Comput 111:107678
10. Zhao F, Zhao J, Wang L et al (2021) A hierarchical knowledge guided backtracking search algorithm with self-learning strategy. Eng Appl Artif Intell:102. https://doi.org/10.1016/j.engappai.2021.104268
11. Gao S, Wang Y, Cheng J, Inazumi Y, Tang Z (2016) Ant colony optimization with clustering for solving the dynamic location routing problem. Appl Math Comput 285:149–173
12. Xiao S, Wang H, Wang W, Huang Z, Zhou X, Xu M (2021) Artificial bee colony algorithm based on adaptive neighborhood search and Gaussian perturbation. Appl Soft Comput 100:106955
13. Zhao F, Zhang L, Zhang Y et al (2019) An improved water wave optimisation algorithm enhanced by CMA-ES and opposition-based learning. Conn Sci:1–30
14. Shi Y (2011) An optimization algorithm based on brainstorming process. Int J Swarm Intell Res 2:35–62
15. Shi YH (2011) Brain storm optimization algorithm. In: Tan Y, Shi Y, Chai Y, Wang G (eds) Advances in swarm intelligence, Pt I, pp 303–309
16. Cervantes-Castillo A, Mezura-Montes E (2020) A modified brain storm optimization algorithm with a special operator to solve constrained optimization problems. Appl Intell 50:4145–4161. https://doi.org/10.1007/s10489-020-01763-8
17. Ma L, Cheng S, Shi Y (2020) Enhancing learning efficiency of brain storm optimization via orthogonal learning design. IEEE Trans Syst Man Cybern Syst:1–20. https://doi.org/10.1109/tsmc.2020.2963943
18. Qu L, Duan Q, Yang J et al (2020) BSO-CLS: Brain Storm Optimization Algorithm with Cooperative Learning Strategy. Lect Notes Comput Sci (including Subser Lect Notes Artif Intell Lect Notes Bioinformatics) 12145 LNCS:243–250. https://doi.org/10.1007/978-3-030-53956-6_22
19. Sun Y, Wei J, Wu T, Xiao K, Bao J, Jin Y (2020) Brain storm optimization using a slight relaxation selection and multi-population based creating ideas ensemble. Appl Intell 50:3137–3161. https://doi.org/10.1007/s10489-020-01690-8
20. Shen Y, Yang J, Cheng S, Shi Y (2020) BSO-AL: brain storm optimization algorithm with adaptive learning strategy. In: 2020 IEEE congress on evolutionary computation, CEC 2020 - conference proceedings
21. Cao Z, Shi Y, Rong X et al (2015) Random grouping brain storm optimization algorithm with a new dynamically changing step size. In: Lecture notes in computer science (including subseries lecture notes in artificial intelligence and lecture notes in bioinformatics), pp 357–364
22. Zhan ZH, Zhang J, Shi YH, Liu HL (2012) A modified brain storm optimization. In: 2012 IEEE congress on evolutionary computation, CEC 2012
23. Xu P, Luo W, Lin X, Cheng S, Shi Y (2021) BSO20: efficient brain storm optimization for real-parameter numerical optimization. Complex Intell Syst 7:1–22
24. Liu J, Peng H, Wu Z, Chen J, Deng C (2020) Multi-strategy brain storm optimization algorithm with dynamic parameters adjustment. Appl Intell 50:1289–1315. https://doi.org/10.1007/s10489-019-01600-7

25. Cao Z, Wang L (2019) An active learning brain storm optimization algorithm with a dynamically changing cluster cycle for global optimization. Cluster Comput 22:1413–1429. https://doi.org/10.1007/s10586-019-02918-0
26. Li C, Song Z, Fan J et al (2018) A brain storm optimization with multiinformation interactions for global optimization problems. IEEE Access, pp 1
27. Luo J, Zhang R, Weng J et al (2020) Brain storm optimization algorithm with estimation of distribution[M]//Communications in Computer and Information Science: 1159 CCIS
28. Yu Y, Gao S, Cheng S et al (2018) CBSO: a memetic brain storm optimization with chaotic local search[J]. Memetic Comput 10: 353–367
29. Chen W, Cao YY, Cheng S et al (2017) Simplex search based brain storm optimization. arXiv: Neural and Evolutionary Computing. https://doi.org/10.48550/arXiv.1712.03166
30. Wang Y, Gao S, Yu Y, Xu Z (2019) The discovery of population interaction with a power law distribution in brain storm optimization. Memetic Comput 11:65–87. https://doi.org/10.1007/s12293-017-0248-z
31. Cheng S, Shi Y, Qin Q et al (2014) Maintaining population diversity in brain storm optimization algorithm. In: Proceedings of the 2014 IEEE congress on evolutionary computation, CEC 2014, pp 3230–3237
32. Xue Y, Zhao Y (2021) Structure and weights search for classification with feature selection based on brain storm optimization algorithm. Appl Intell 1–10
33. Qla B, Ms B (2021) BALFA: a brain storm optimization-based adaptive latent factor analysis model. Inf Sci (Ny) 578:913–929
34. Jiang Y, Chen X, Zheng F, et al (2021) Brain storm optimization-based edge caching in fog radio access networks. IEEE Trans Veh Technol 1
35. Zheng M, Fukuyama Y, El-Abd M et al (2020) Overall optimization of Smart City by multi-population global-best brain storm optimization using cooperative coevolution. In: 2020 IEEE congress on evolutionary computation, CEC 2020 - conference proceedings
36. Li JQ, Yu H, Chen X et al (2020) An improved brain storm optimization algorithm for fuzzy distributed hybrid flowshop scheduling with setup time. In: GECCO 2020 companion - proceedings of the 2020 genetic and evolutionary computation conference companion, pp 275–276
37. Fu Y, Tian G, Fathollahi-Fard AM, Ahmadi A, Zhang C (2019) Stochastic multi-objective modelling and optimization of an energy-conscious distributed permutation flow shop scheduling problem with the total tardiness constraint. J Clean Prod 226:515–525. https://doi.org/10.1016/j.jclepro.2019.04.046
38. Yu Y, Gao S, Wang Y, Lei Z, Cheng J, Todo Y (2019) A multiple diversity-driven brain storm optimization algorithm with adaptive parameters. IEEE Access 7:126871–126888. https://doi.org/10.1109/ACCESS.2019.2939353
39. Ahandani MA, Ghiasi AR, Kharrati H (2018) Parameter identification of chaotic systems using a shuffled backtracking search optimization algorithm. Soft Comput 22:8317–8339. https://doi.org/10.1007/s00500-017-2779-0
40. Tongur V, ülker E (2018) PSO-based improved multi-flocks migrating birds optimization (IMFMBO) algorithm for solution of discrete problems. Soft Comput 23:5469–5484
41. Miarnaeimi F, Azizyan G, Rashki M (2020) Horse herd optimization algorithm: A nature-inspired algorithm for high-dimensional optimization problems. Knowledge-Based Syst 213
42. Wang J, Wang L, Wu C, Shen J (2018) A Cooperative Algorithm for Energy-efficient Scheduling of Distributed No-wait Flowshop. 2017 IEEE Symp Ser Comput Intell SSCI 2017 - Proc 2018-Janua: 1–8. https://doi.org/10.1109/SSCI.2017.8280956
43. Pan QK, Tasgetiren MF, Liang YC (2008) A discrete particle swarm optimization algorithm for the no-wait flowshop scheduling problem. Comput Oper Res 35:2807–2839
44. Zhao F, Liu H, Zhang Y, Ma W, Zhang C (2017) A discrete water wave optimization algorithm for no-wait flow shop scheduling problem. Expert Syst Appl 91:347–363

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