Journal of Computational Information Systems 9: 18 (2013) 7485–7492
Available at http://www.Jofcis.com

Repairable Queuing Model Analysis on a Manufacturing Cell ⋆

Fuqing ZHAO [1,2,∗], Qin ZHAO [1], Junbiao WANG [2]

[1] *School of Computer and Communication Technology, Lanzhou University of Technology, Lanzhou 730050, China*

[2] *Key Laboratory of Contemporary Design, Integrated Manufacturing Technology, Ministry of Education, Northwestern Polytechnical University, Xi'an 710072, China*

Abstract

In this paper, we consider a queuing model extension for a manufacturing cell composed of a machining center and several parallel downstream predestinations. In the machining center, there is an ample supply of raw material for a number of different part-types. For each type, parts are first processed in the machining center, and then in one of parallel production stations (one for each type). A queuing model is extended with the repairable queuing system to capture failures of production stations and disasters of the manufacturing cell. We present an exact solution for the steady-state probabilities of the proposed queuing model. We use the nonlinear matrix equation to solve the relevant parameters of this queuing system. Further, in order to illustrate the usefulness of the proposed methods, numerical examples are solved. On the basis of the results of these examples, some important conclusions are drawn.

Keywords: Queuing Theory; Manufacturing Cell; Matrix Equation; Performance Analysis

1 Introduction

Performance modeling and evaluation activities play an important role in the design and operation of manufacturing systems. Therefore, the search for new mathematical analysis methods for the performance evaluation of manufacturing systems has been an intensive research area [1, 2].

The system of interest is composed of a machining center (abbreviated to MC hereafter) and m parallel downstream production stations. A system with three parallel downstream stations is shown in Fig. 1 for further illustration. The MC can process raw materials for part-types. Raw material for a part-type (say, type- j) is processed in the MC. Upon the completion of the process in the MC, the part is moved to station- for further process. In the meantime, the MC starts processing the next part which may be or maybe not the same type as the previous one.

⋆Project supported by the National Nature Science Foundation of China (No. 61064011).

∗Corresponding author.
Email address: `fzhao2000@hotmail.com` (Fuqing ZHAO).

DOI: 10.12733/jcisP3245
September 15, 2013

Such a system with finite buffer sizes was originally studied by Seidmann and Schweitzer (1984) [3]. In their article, the sequence of different types processed in the MC was to be determined to minimize the expected shortage penalty per unit time, incurred by the stations. Seidmann and Tenenbaum (1994) considered the problem of maximizing the system throughput [4]. Those two researches modeled their problems as semi-Markov decision processes and used the value-iteration algorithm to find the optimal sequence. Thesen (1999) proposed a heuristic rule (called the rotation rule) to achieve the maximization of the system throughput [5]. The matrix-geometric method [6] was applied by Chen [2] to calculate the stationary probabilities of the queuing model. Negative customers remove positive customers in the queue and have been used to model random neural networks, task termination in speculative parallelism, faulty components in manufacturing systems and server breakdowns and a reaction network of interacting molecules [7, 8]. Queuing Models with negative customers can account for burstiness and correlation, but in addition the negative customers, with an appropriate killing discipline, can represent additional behavior such as breakdowns, killing signals, losses and load balancing [9, 10].

In the actual MC production systems, the buffer of systems can not be infinite and the machines always break down. In this paper we use an M/M/n repairable queuing system with one repairman and the failure rates of idle machines and busy machines are different to characterize the performance parameters. As we will see, modern manufacturers are very concerned about the production lead time. So we use the nonlinear matrix equation to compute the relevant parameters of this repairable queuing system and their relation among waiting time, failure rate and arrival rate as so on.

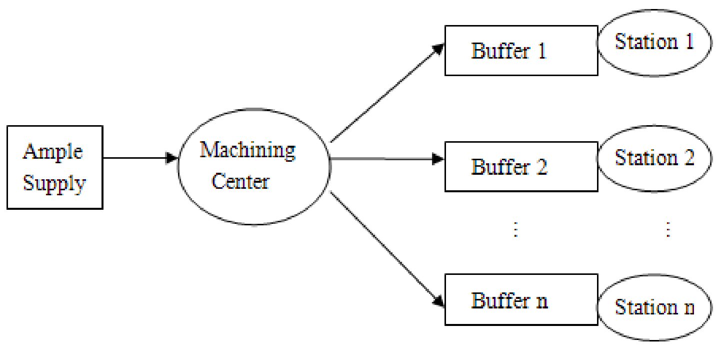

Fig. 1: Illustration for a system with n production stations

2 Modeling Description

In order to propose a feasible solution to determine the waiting time, we should first characterize the MC queuing model. Here, for simplicity, we assume a two-stage production system, as illustrated in Fig. 1. Each stage includes only one machine. The jobs from supply station into MC system and are processed in the MC. Upon the completion of the process in the MC, the jobs are moved to the buffer of stations for further process with different Poisson stream. The arrival rate of jobs is same before same stations. In this queuing model, we assume each station include only one machine. The performance of machines can be same in different stations. When one job arrives in the buffer, if the machine is idle, the job access to process. Otherwise, the jobs wait in the buffer in queue, and the jobs obey FCFS (First Come First Server). In this paper, the jobs are independent each other and the machines are independent mutual. The jobs leave one machine only when they are processed. In our model, we assume the stations can break down at any time, but manufacture cell does not break down all the time. In such a system, poor system throughput is mainly due to the blocking, occurred at the moment the MC completes a part. Since the MC is a shared resource for all downstream stations, the manufacturer is supposed to increase the buffer size such that the probability of occurring blocking is tiny (or even negligible). Hence, the buffer size in each station is assumed to be infinite in this paper. If machines break down, the repairman will repair it immediately. We assume the machines are same as new. Some notations are introduced, as follows:

λ	customer arrival rate follows a Poisson process with the parameter λ
$\xi_1 + o(\Delta t)$	the breakdown probability of idle machines in time $t + \Delta t$
$\xi_2 + o(\Delta t)$	the breakdown probability of busy machines in time $t + \Delta t$
$(\eta \Delta t) + o(\Delta t)$	the probability of being repaired in time $t + \Delta t$

Where ξ_1, ξ_2, η are non-negative constant.

3 Model Performance Analysis

In this paper, we use M/M/n repairable queuing system with [11] one repairman to analysis MC production model.

There are N identical machines, one repairman, and the arrivals of jobs obey parameter Poisson process $f(x) = \mu e^{-\mu t}, t \geq 0$ for λ and μ are nonnegative constant.

Let $X(t)$ as the number of effective machines and $Y(t)$ as the number of jobs in the system at time t. Then, the random process $\{X(t), Y(t); t \geq 0\}$ describes its instantaneous state at the moment. Obviously, this random process can be see two-dimensional Markov process.

The state of MC system at time t is (i, j), for $X(t) = i, Y(t) = j$. $P_{i,j}(t)$ express the instantaneous state probability of this system at moment t and it has $P_{i,j}(t) = P\{X(t) = i, Y(t) = j\}, i = 0, 1, \cdots, N; j = 0, 1, \cdots$. we use $P_{i,j}$ as the state probability at state (i, j), then,

$$f(x) = \lim_{t \to +\infty} P\{X(t) = i, Y(t = j)\}, i = 0, 1, \cdots, N; j = 0, 1, \cdots$$

Considering the state transitions of the system at time$(t, t + \Delta t)$, we can obtain their steady-state balance equation in different states of the system.

$$(\lambda + \eta)P_{0,0} = \xi_1 P_{1,0}, i = 0, j = 0$$
$$(\lambda + \eta)P_{0,j} = \xi_2 P_{1,j} + \lambda P_{0,j-1}, i = 0, j > 0$$
$$(i\xi_1 + \lambda + \eta)P_{i,0} = \eta P_{i-1,0} + (i + 1)\xi_1 P_{i+1,0} + \mu P_{i,1}, 0 < i < N, j = 0$$
$$(j\mu + (i - j)\xi_1 + j\xi_2 + \lambda + \eta)P_{i,j}$$
$$= \eta P_{i,j} + [(i + 1 - j)\xi_1 + j\xi_2]P_{i+1,j} + \lambda P_{i,j-1} + (j + 1)\mu P_{i,j+1}, 0 < i < N, j < i$$
$$(i\mu + i\xi_2 + \lambda + \eta)P_{i,i} = \eta P_{i-1,i} + (\xi_1 + i\xi_2)P_{i+1,i} + \lambda P_{i,i-1} + i\mu P_{i,i+1}, 0 < i < N, j = i$$
$$(i\mu + i\xi_2 + \lambda + \eta)P_{i,j} = \eta P_{i-1,j} + (i + j)\xi_2 P_{i+1,j} + \lambda P_{i,i-1} + i\mu P_{i,i+1}, 0 < i < N, j > i$$
$$(N\xi_1 + \lambda)P_{N-1,0} + \mu P_{N,1}, i = N, j = 0$$
$$(j\mu + (N - j)\xi_1 + j\xi_2 + \lambda)P_{N,j} = \eta P_{N-1,j} + \lambda P_{N,j-1} + (j + 1)\mu P_{N,j+1}, i = N, 0 < j < N$$

$$(N\mu + N\xi_2 + \lambda)P_{N,j} = \eta P_{N-1,j} + \lambda P_{N,j-1} + N\mu P_{N,j+1}, i = N, N \le j$$

Let $G_i(z) = \sum_{j=0}^{\infty} z^j P_{i,j}$, $G(z) = \sum_{i=0}^{N}$,$0 \le i \le N; |z| \le 1$. Then, $G_i(1) = \sum_{j=0}^{\infty} P_{i,j}$.

This is the steady-state probability of i machines when they are active. Therefore, we have $\sum_{i=0}^{N} G_i(1) = 1$. According to the same steady-state equations of state i and j, when $1 < k < N$ we obtain $N+1$ equations as follows:

$$\begin{cases} \eta G_0(1) = \xi_2 G_1(1) + (\xi_1 - \xi_2)P_{1,0}(k\xi_2\eta)G_k(1) + \sum\limits_{m=1}^{k} m(\xi_1 - \xi_2)P_{k,k-m} \\ = \eta G_{k-1}(1) + (k+1)\xi_2 G_{k+1}(1) + \sum\limits_{m=1}^{k+1} m(\xi_1 - \xi_2)P_{k+1,k+1-m} N\xi_2 G_N(1) \\ + \sum\limits_{m=1}^{N} (\xi_1 - \xi_2)P_{N,N-m} = \eta G_{N-1}(1). \end{cases} \tag{1}$$

From (1), it can be see that $\frac{N(N+1)}{2}$ probability value can be obtained by $P_{i,0}$,$i = 1, 2, \cdots, N$. If we obtained the N probability value, we also obtain steady-state distribution $G_i(1), i = 0, 1, 2, \cdots, N$ of the number of effective machines in this system. Further, the steady-state availability of the system can be obtained. If there are N machines, the steady-state availability can be expressed as $A = \sum_{k=1}^{N} G_k(1)$.

When the system has exist the steady-state availability, the N probability value $P_{i,0}, i = 1, 2, \cdots, N$ can be obtained.

When N=1, it can be written as follows: $G_0(1) = \frac{\xi_2 + (\xi_1 - \xi_2)P_{1,0}}{\xi_2 + \eta}$, $G_1(1) = \frac{\eta - (\xi_1 - \xi_2)P_{1,0}}{\xi_2 + \eta}$.

On both sides of the system balance equation multiply z^{i+1} for different i. Then, N+1 equations about $G_i(z)$ for $i = 0, 1, \cdots, N$ can be obtained by summing state j and the Probability generating function equations $(0 < i < N)$ can be given as follows:

$$\begin{cases} \eta[(\lambda + \eta)z - \lambda z^2]G_0(z)z = (\xi_1 - \xi_2)P_{1,0}z - \eta z G_{i-1}(z) + \\ [(i\mu + i\xi_2 + \lambda + \eta)z - \lambda z^2 - i\mu]G_i(z) - (i+1)z\xi_2 G_{i+1}z = \\ \sum\limits_{m=1}^{i} m(\mu + \xi_2 - \xi_1)P_{1,1-m}z^{i+1-m} + \sum\limits_{m=1}^{i+1} m(\xi_1 - \xi_2 P_{i+1,i+1-m})z^{i+2-m} \\ - \sum\limits_{m=1}^{i} m\mu P_{i,i-m}z^{i-m} - \eta z G_{N-1}(z) + [(N\mu + N\xi_2 + \lambda)z - \lambda z^2 - N\mu] \\ G_N(z) \sum\limits_{m=1}^{N} m(\mu + \xi_2 - \xi_1)P_{N,N-m}z^{N+1-m} - \sum\limits_{m-1}^{N} m\mu P_{N,N-m}z^{N-m}. \end{cases} \tag{2}$$

If it is exist that $f_i(z) = (i\mu + i\xi_2 + \lambda + \eta)z - \lambda z^2 - i\mu$ for $(i = 0, 1, \cdots, N-1)$, from (5) it can be written matrix form as follows:

$$\begin{vmatrix} f_0(z) & -\xi_2 z & 0 & 0 & \cdots & 0 & 0 & 0 \\ -\eta z & f_1(z) & -2\xi_2 & 0 & \cdots & 0 & 0 & 0 \\ 0 & -\eta z & f_2(z) & -3\xi_2 z & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & -\eta z & f_{N-1}(z) & -N\xi_2 z \\ 0 & 0 & 0 & 0 & \cdots & 0 & -\eta z & f_N(z) \end{vmatrix} \tag{3}$$

$$b_i(z)=\sum_{m=1}^{i} m(\mu+\xi_2-\xi_1)P_{1,i-m}z^{i+1-m}+\sum_{m=1}^{i+1} m(\xi_1-\xi_2)P_{i+1,i+1-m}-\sum_{m=1}^{i} m\mu P_{i,i-m}z^{i-m}$$
$$(0\le i\le N)$$

For all the z makes $A(z)$ as non-singular matrix, it can be obtained

$$|A(z)|G_i(z)=|A_i(z)|,(i=0,1,\cdots,N) \tag{4}$$

This can be written simply as follows: $|A_i(z)|=(z-1)D_i(z),\quad i=0,1,\cdots,N.$

When $z=1,\quad D(1)G_i(1)=D_i(1),\ i=0,1,\cdots,N.$

When $N=1,\quad D(1)G_0(1)=D_0(1),\ i=0,1,\cdots,N.$

Take it into $G_0(1)$, it has:

$$P_{1,0}=\frac{\eta\mu-\eta\lambda-\lambda\xi_2}{\mu(\xi_1+\eta)} \tag{5}$$

Take equation (5) into equation $G_0(1),G_1(1)$, the steady-state availability of system can be given as follows:

$$A=G_1(1)=\frac{\eta-(\xi_1-\xi_2)\frac{\eta\mu-\eta\lambda-\lambda\xi_2}{\mu(\xi_1+\eta)}}{\xi_2+\eta}$$

If we know steady-state's value $P_{i,0}$, for $i=1,2,\cdots,N$, according to the steady-state balance equation we can compute all steady-state value in equations (1). Then, from (4), we get $G_i(z)$, $i=1,2,\cdots,N$, further, the probability generating function of queue length can be expressed as $G(z)=\sum_{i=0}^{N}G_i(z)$. According to the nature of generating function, the expected average queue length as follows:

$$E(Y)=\frac{dG(z)}{dz}|_{z=1}.$$

When N=1, from (4) we get

$$G_0(z)=\frac{|A_0(z)|}{|A(z)|}=\frac{(\xi_1-\xi_2)(\mu-\lambda z)P_{1,0}+\xi_2\mu P_{1,0}}{(\lambda+\eta-\lambda z)(\mu-\lambda z)-\lambda\xi_2 z},\ G_0(z)=\frac{|A_0(z)|}{|A(z)|}=\frac{(\lambda+\eta-\lambda z)\mu P_{1,0}+\lambda(\xi_1-\xi_2)P_{1,0}}{(\lambda+\eta-\lambda z)(\mu-\lambda z)-\lambda\xi_2 z}.$$

Therefore, $G(z)=G_0(z)+G_1(z)=\frac{(\xi_1-\xi_2)(\mu-\lambda z)P_{1,0}+\xi_2\mu P_{1,0}}{(\lambda+\eta-\lambda z)(\mu-\lambda z)-\lambda\xi_2 z}$

By taking above equation into equation (5), we have $G(z)=\frac{[(\mu-\lambda)\eta-\xi_2\lambda][\xi_1+\lambda(1-z)+\eta]}{(\xi_1+\eta)[(\lambda(1-z)+\eta)(\mu-\lambda z)-\lambda\xi_2 z]}$

Therefore, the average queue length can be expressed as

$$E(Y)=\frac{dG(z)}{dz}|_{z=1}=\frac{\lambda[\lambda(\xi_2-\xi_1)+\mu\xi_1+(\xi_1+\eta)(\xi_2+\eta)]}{(\xi_1+\eta)[\eta(\mu-\lambda)-\lambda\xi_2]} \tag{6}$$

In addition, the average waiting time of jobs in system can be obtained as follows:

$$E(T)=\frac{E(Y)}{\lambda}=\frac{\lambda(\xi_2-\xi_1)+\mu\xi_1+(\xi_1+\eta)(\xi_2+\eta)}{(\xi_1+\eta)[\eta(\mu-\lambda)-\lambda\xi_2]} \tag{7}$$

In this paper, the main research is the function relationship between the waiting time of jobs in buffer and other target parameters. In case of the buffer unlimited, we can control the waiting time easily base on the exist function relations, also the jobs lead time.

4 Experiment Examples and Analysis

In this section, we present numerical examples to illustrate the function relation between waiting time of jobs in buffer and other target parameters. Relevant input parameters are as follows:

Fixed parameters: $\xi_1 = 0.1, \eta = 0.8$

Table 1: The relation between waiting time of jobs in buffer and other parameters

$\xi_1 = 0.1, \xi_2 = 0.2, \eta = 0.8$	–	–	–	–	–	–	–
μ	0.80	0.90	1.00	1.10	1.20	1.30	1.40
$\lambda = 0.4, E(t)$	4.722	3.576	2.889	2.431	2.355	1.858	1.667
$\lambda = 0.5, E(t)$	8.175	5.253	3.889	3.099	2.584	2.220	1.953
$\lambda = 0.6, E(t)$	28.889	9.720	5.889	4.246	3.333	2.753	2.350
$\xi_1 = 0.1, \xi_2 = 0.2, \eta = 0.8$	–	–	–	–	–	–	–
λ	0.10	0.20	0.30	0.40	0.50	0.60	0.70
$\mu = 1.0, E(t)$	1.603	1.889	2.289	2.889	3.889	5.889	11.889
$\mu = 1.5, E(t)$	1.071	1.188	1.333	1.514	1.746	2.055	2.488
$\mu = 2.0, E(t)$	0.822	0.889	0.966	1.056	1.162	1.289	1.444
$\xi_1 = 0.1, \xi_2 = 0.2, \lambda = 0.6$	–	–	–	–	–	–	–
η	0.50	0.60	0.70	0.75	0.80	0.85	0.90
$\mu = 1.0, E(t)$	12.083	8.572	6.875	6.324	5.889	5.550	5.250
$\mu = 1.2, E(t)$	5.556	4.405	3.750	3.529	3.333	3.266	3.047
$\mu = 1.5, E(t)$	3.182	2.619	2.279	2.162	2.055	1.971	1.898
$\xi_1 = 0.1, \lambda = 0.6, \eta = 0.8$	–	–	–	–	–	–	–
ξ_2	0.10	0.15	0.20	0.25	0.30	0.35	0.40
$\mu = 1.0, E(t)$	3.899	4.758	5.899	7.418	9.603	12.980	18.889
$\mu = 1.5, E(t)$	1.616	1.746	2.056	2.310	2.593	2.908	3.264
$\mu = 2.0, E(t)$	1.059	1.170	1.288	1.415	1.548	1.691	1.843

The waiting time of jobs in buffer is computed by means of expressions (7) for all circumstances are shown in Table 1.

Further, Figs. 2-5 illustrate the influence of every varied parameter on the waiting time, respectively. Some important conclusions drawn from Figs. 2-5 are as follows:

(1) From Fig. 2, we give there value $\lambda = 0.4$, $\lambda = 0.5$ and $\lambda = 0.6$, it can see that the average waiting time $E(t)$ of jobs in buffer is a strictly monotone decreasing function with respect to λ. This nature is obviously when $\lambda = 0.4$. This implies that larger the serve rate for keeping the machines on, smaller the average waiting time in buffer.

(2) From Fig. 3, we can see that the average waiting time $E(t)$ for jobs queuing in buffer is a strictly monotone increasing function with respect to μ. We also give there value about μ, they are $\mu = 1.0$, $\mu = 1.5$ and $\mu = 2.0$. When $\mu = 1.0$, $\lambda > 0.6$, $E(t)$ is increasing sharply. As we know,

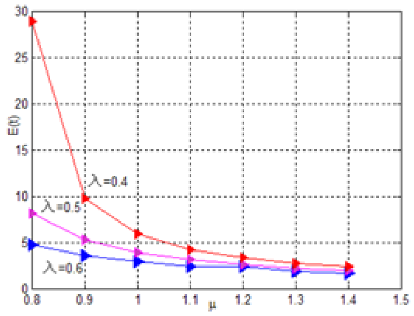

Fig. 2: The relation between $E(t)$ and μ

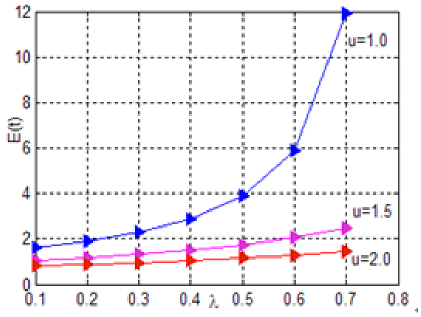

Fig. 3: The relation between $E(t)$ and λ

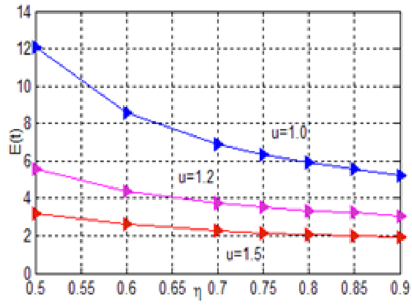

Fig. 4: The relation between $E(t)$ and η

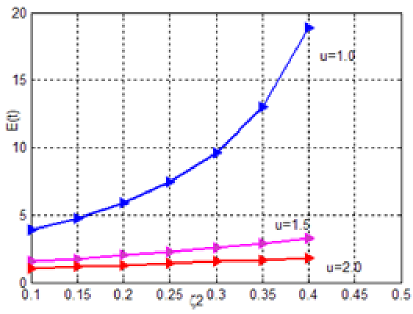

Fig. 5: The relation between $E(t)$ and ξ_2

in actual production, more jobs arrive in buffer, more time for jobs consuming in queue. But this can not mean smaller arrival rate of jobs is better, this may lead to next stage production stagnated, resulting in idle resources, all these will influence jobs lead time.

(3) It can be see that the impact of machines performance to waiting time from Fig. 4 and Fig. 5. It is obviously that the average waiting time $E(t)$ for jobs queuing in buffer is a monotone decreasing function with respect to the machines repair rate and the average waiting time is a monotone increasing function with respect to the machine failure rate ξ_2. Therefore, we can improve repairmans ability. But we also can be seeing that when μ is bigger, the influence is littler.

5 Conclusion

In this paper, we have extended a model for a manufacturing cell composed of a machining center and several parallel down stream production stations. In this system, we use M/M/n repairable queuing system to describe MC system. Base on a nonlinear matrix equation method, solution procedures are developed to compute the waiting time of jobs in buffer. The numerical examples provided in the final part of the paper demonstrated that function relation ship between waiting time of jobs and other parameter. Although the example in the paper includes only two stages, we can easily extend it to a multistage environment. The limited buffer, a multiproduct system and jobs sequence should be considered in the future search.

Acknowledgement

This work was financially supported by the National Natural Science Foundation of China under grant numbers 61064011. It was also supported by scientific research funds from Gansu University, the General and Special Program of the Postdoctoral Science Foundation of China, the Science Foundation for Excellent Youth Scholars of Lanzhou University of Technology under grant numbers 1114ZTC139, 2012M521802, 2013T60889 and 1014ZCX017, respectively.

References

[1] S. L. Chan, et al. A model for predicting customer value from perspectives of product attractiveness and marketing strategy, Expert Systems with Applications, 37, 2010, pp.1207-1215.

[2] L. De Boeck, N. Vandaele. Analytical analysis of a generic assembly system, International Journal of Production Economics, 131, 2011, pp.107-114.

[3] Seidmann A, Schweitzer P J. Part selection policy for a flexible manufacturing cell feeding several production lines. IIE transactions, 16, 1984, pp.355-362.

[4] A. Seidmann, A. Tenenbaum. Throughput maximization in flexible manufacturing systems, IIE transactions, 26, 1994, pp.90-100.

[5] T. Y. S. Lee. Analysis of infinite servers polling systems with correlated input process and state dependent vacations, European journal of operational research, 115, 1999, pp.392-412.

[6] M. Neuts. Matrix-Geometric Solutions in Stochastic Models: An Algorithmic Approach, Dover Publications, 1995.

[7] E. Gelenbe, S. Timotheou. Synchronized interactions in spiked neuronal networks, The Computer Journal, 51, 2008, pp.723-730.

[8] E. Gelenbe. Network of interacting synthetic molecules in steady state, Proceedings of the Royal Society A: Mathematical, Physical and Engineering Science, 464, 2008, pp.2219-2228.

[9] J. M. Fourneau, B. Plateau, W. J. Stewart. An algebraic condition for product form in stochastic automata networks without synchronizations, Performance Evaluation, 65, 2008, pp.854-868.

[10] E. Gelenbe. Fundamental concepts in computer science, Imperial College Press, 2009.

[11] J. H. Li, J. H. Wang. M/M/N repairable queuing system with impatient customers, Journal of Beijing Jiaotong University, 2006.