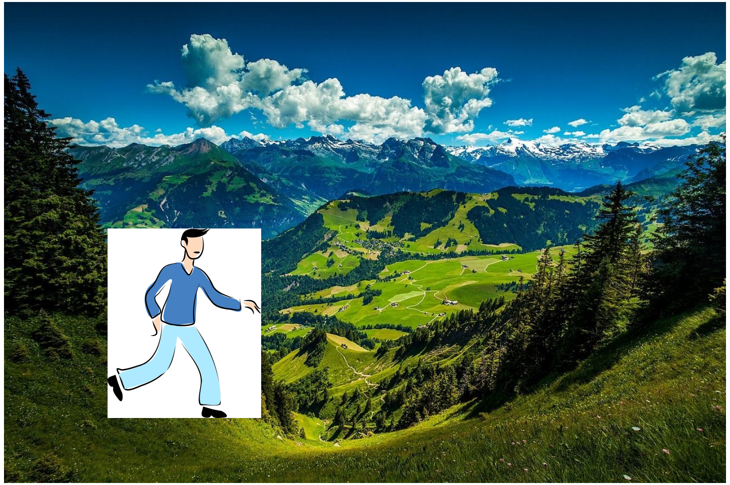
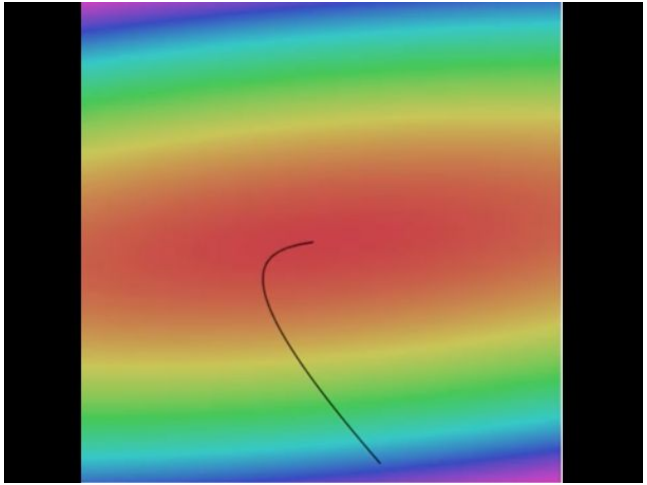
# Optimization





```
# Vanilla Gradient Descent

while True:
  weights_grad = evaluate_gradient(loss_fun, data, weights)
  weights += - step_size * weights_grad # perform parameter update
```

Landscape image is CC0 1.0 public domain
Walking man image is CC0 1.0 public domain

# Gradient descent

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

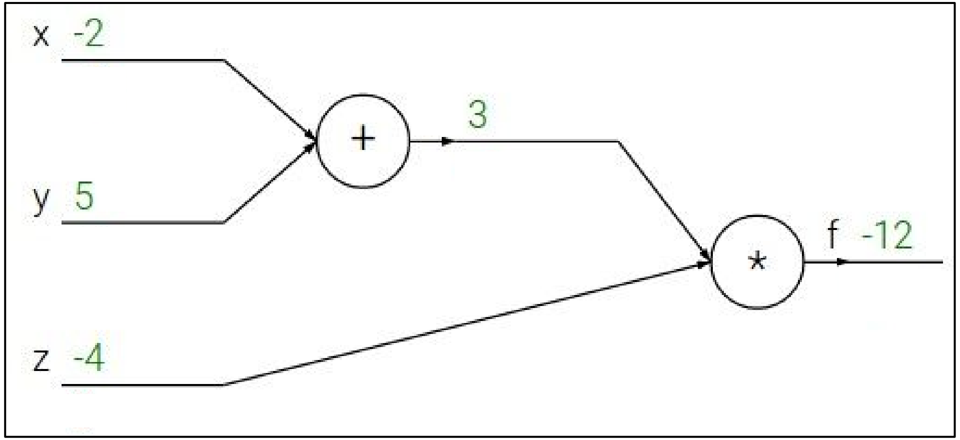
**Numerical gradient**: slow :(, approximate :(, easy to write :)
**Analytic gradient**: fast :), exact :), error-prone :(

In practice: Derive analytic gradient, check your implementation with numerical gradient

# Backpropagation: a simple example

$$f(x, y, z) = (x + y)z$$

e.g. x = -2, y = 5, z = -4



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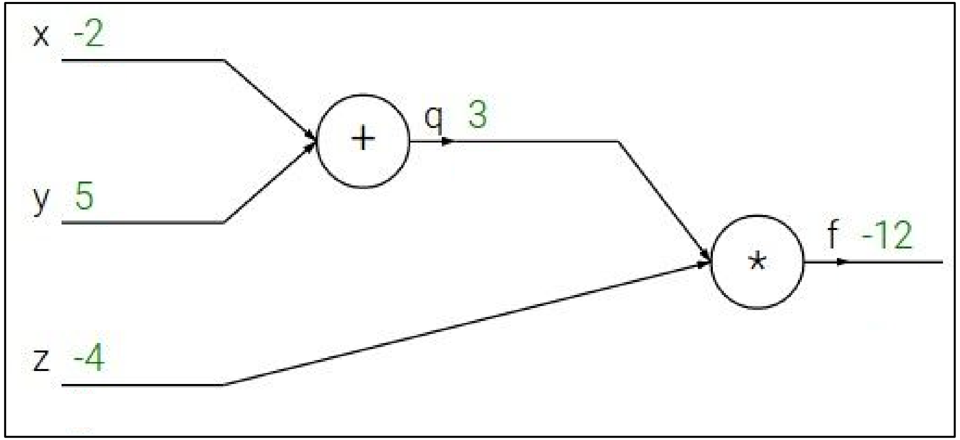
Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



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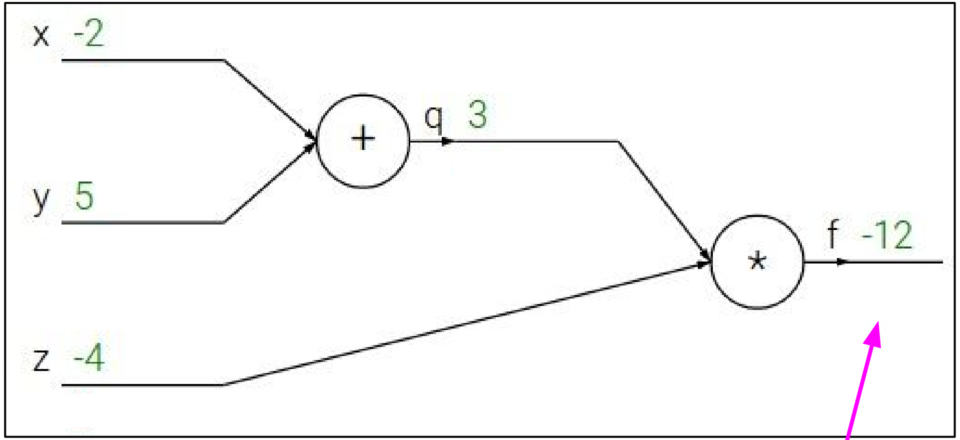


$$\frac{\partial f}{\partial f}$$

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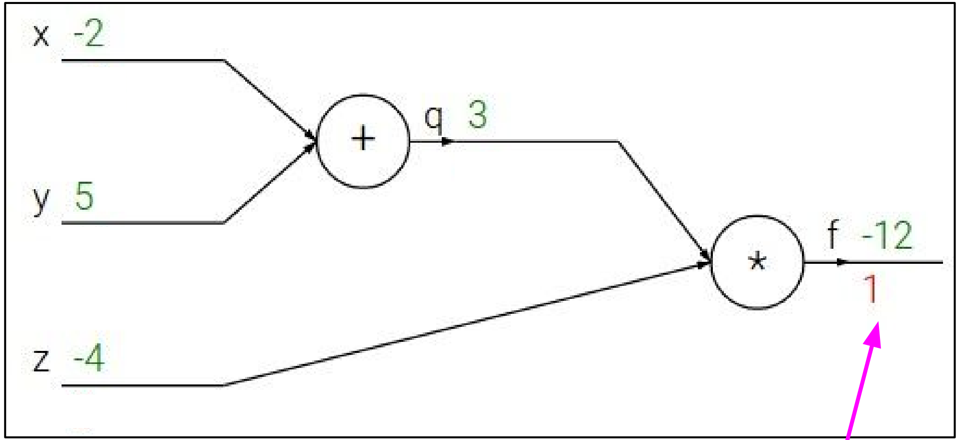


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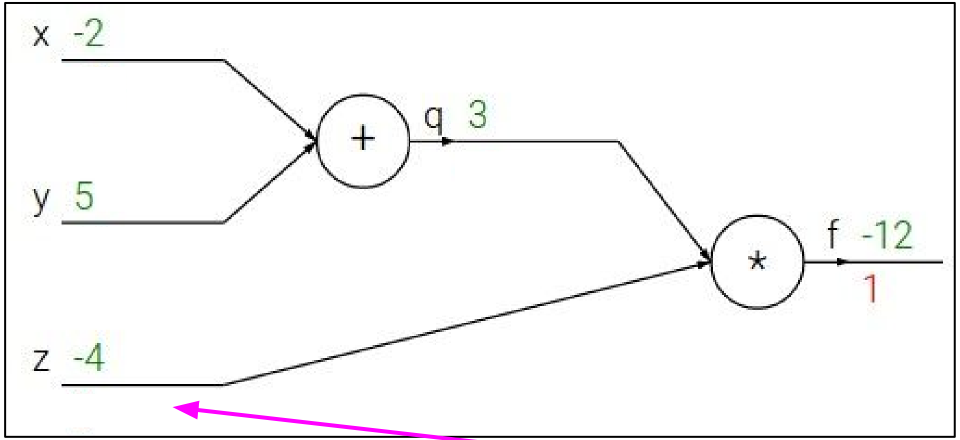


$$\boxed{\frac{\partial f}{\partial z}}$$

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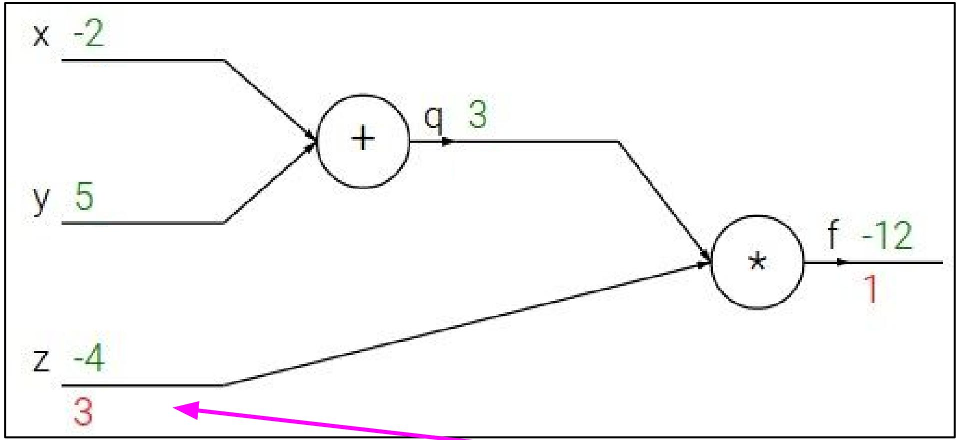


$$\frac{\partial f}{\partial z}$$

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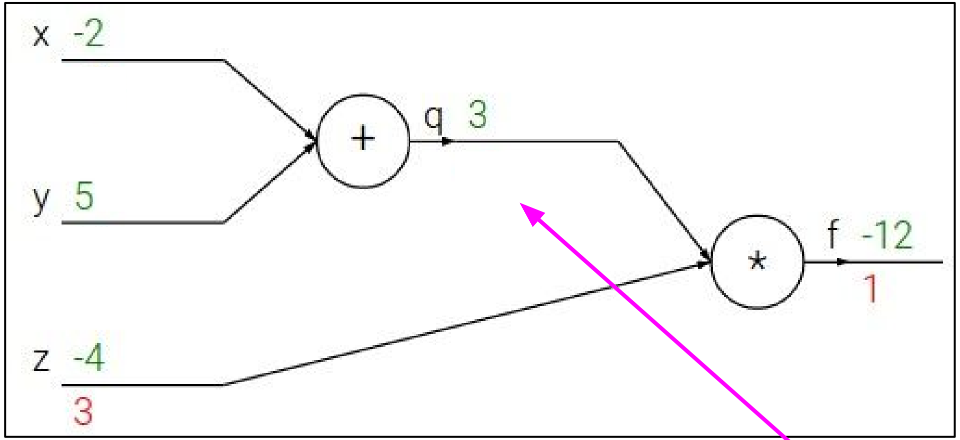


$$\frac{\partial f}{\partial q}$$

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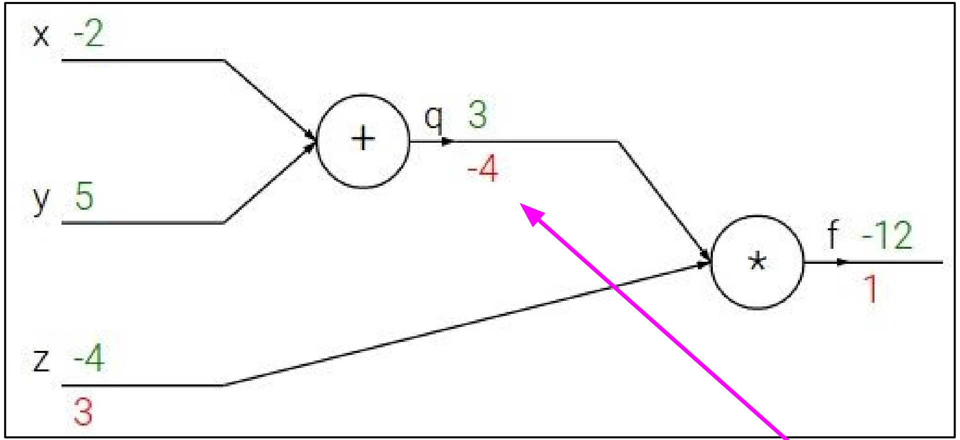


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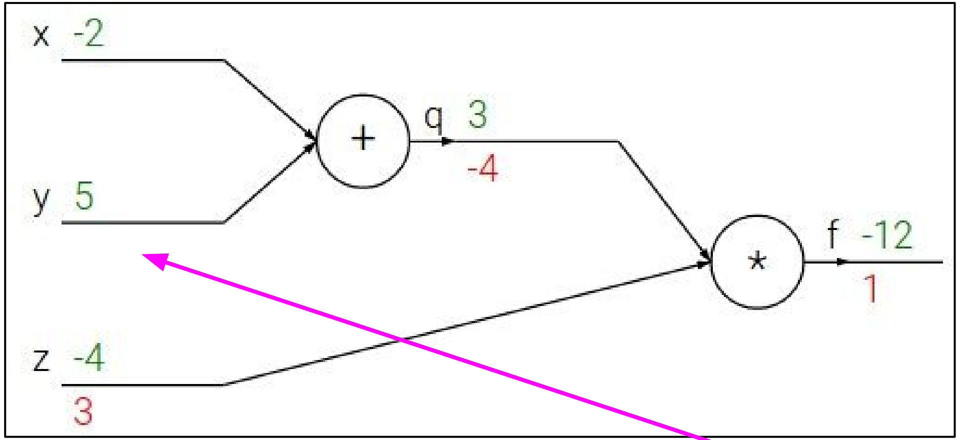


$$\frac{\partial f}{\partial y}$$

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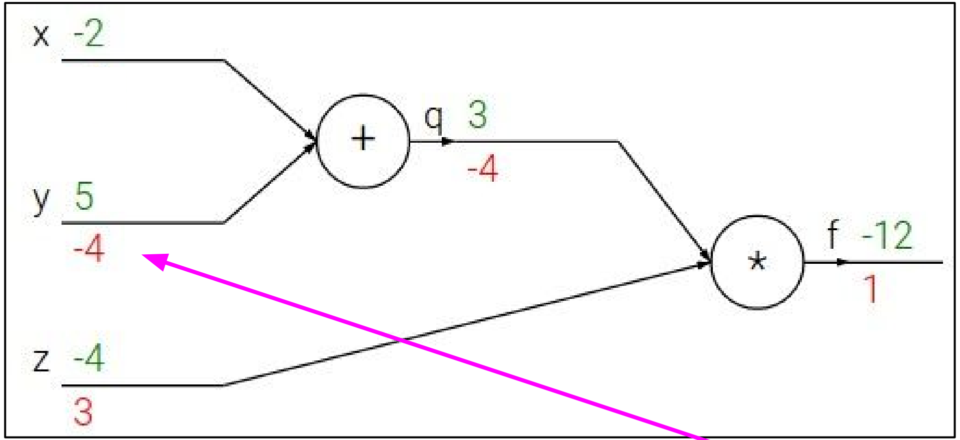


$$\frac{\partial f}{\partial y}$$

Chain rule:

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial y}$$

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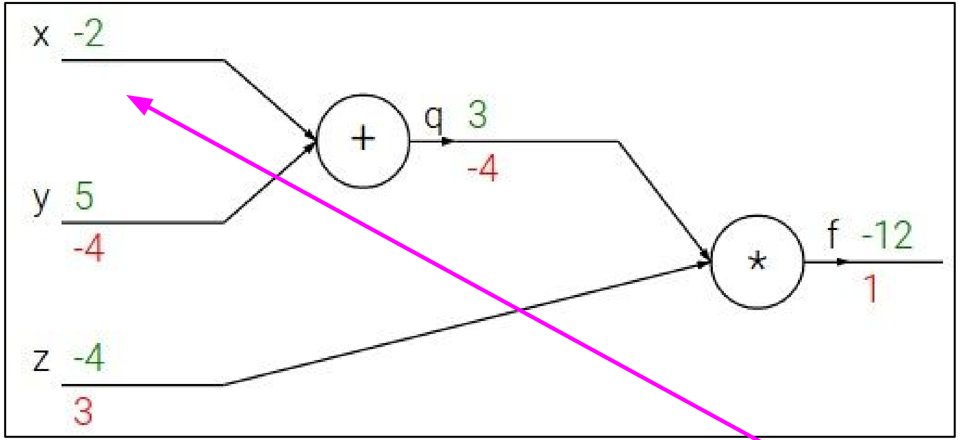


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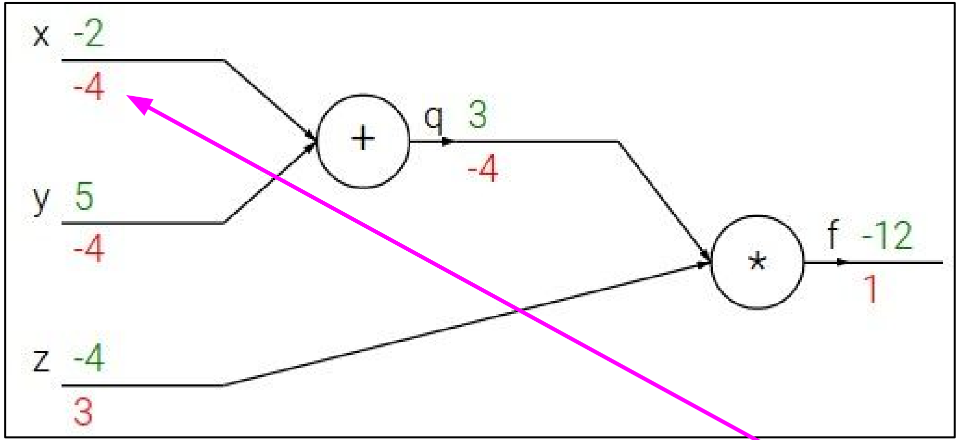


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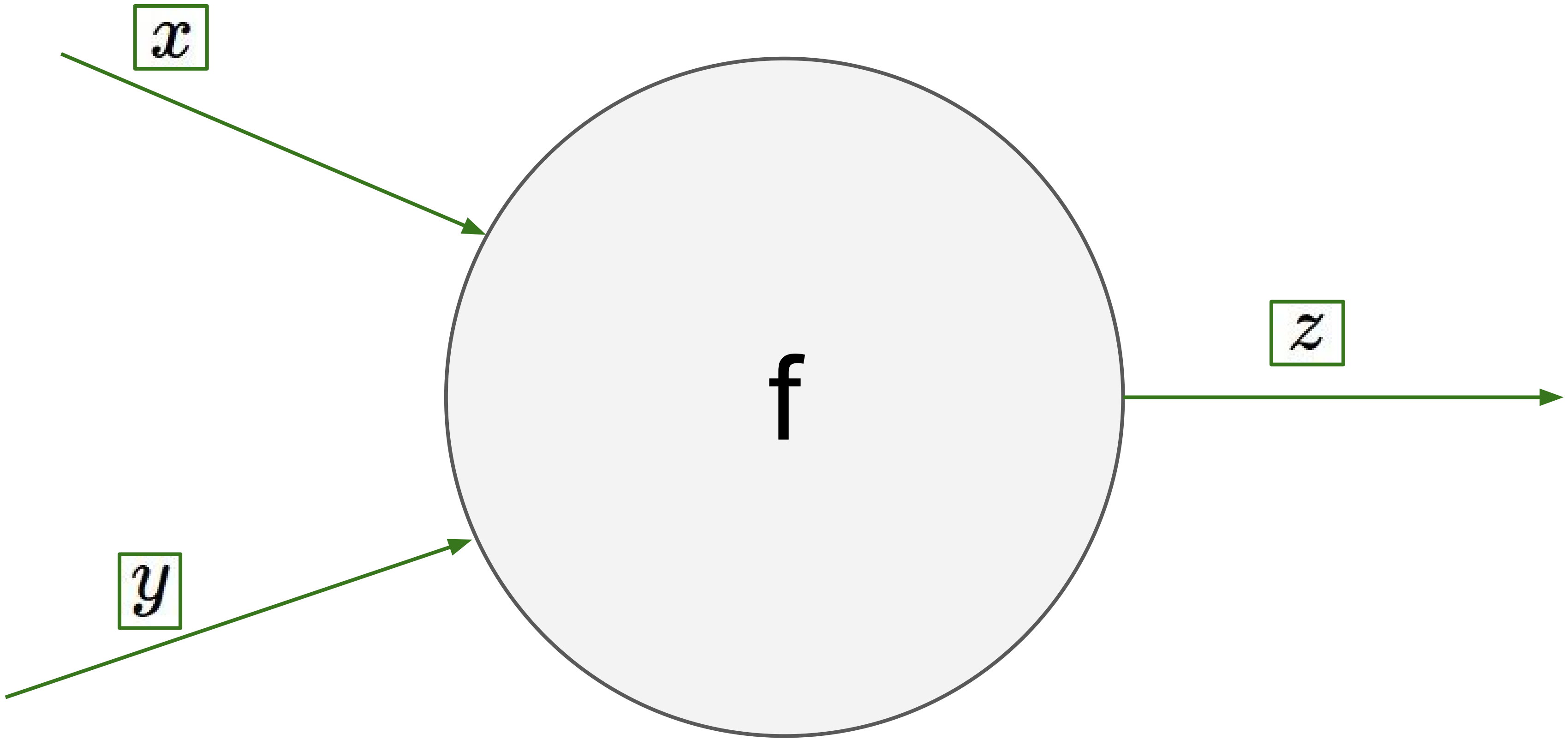
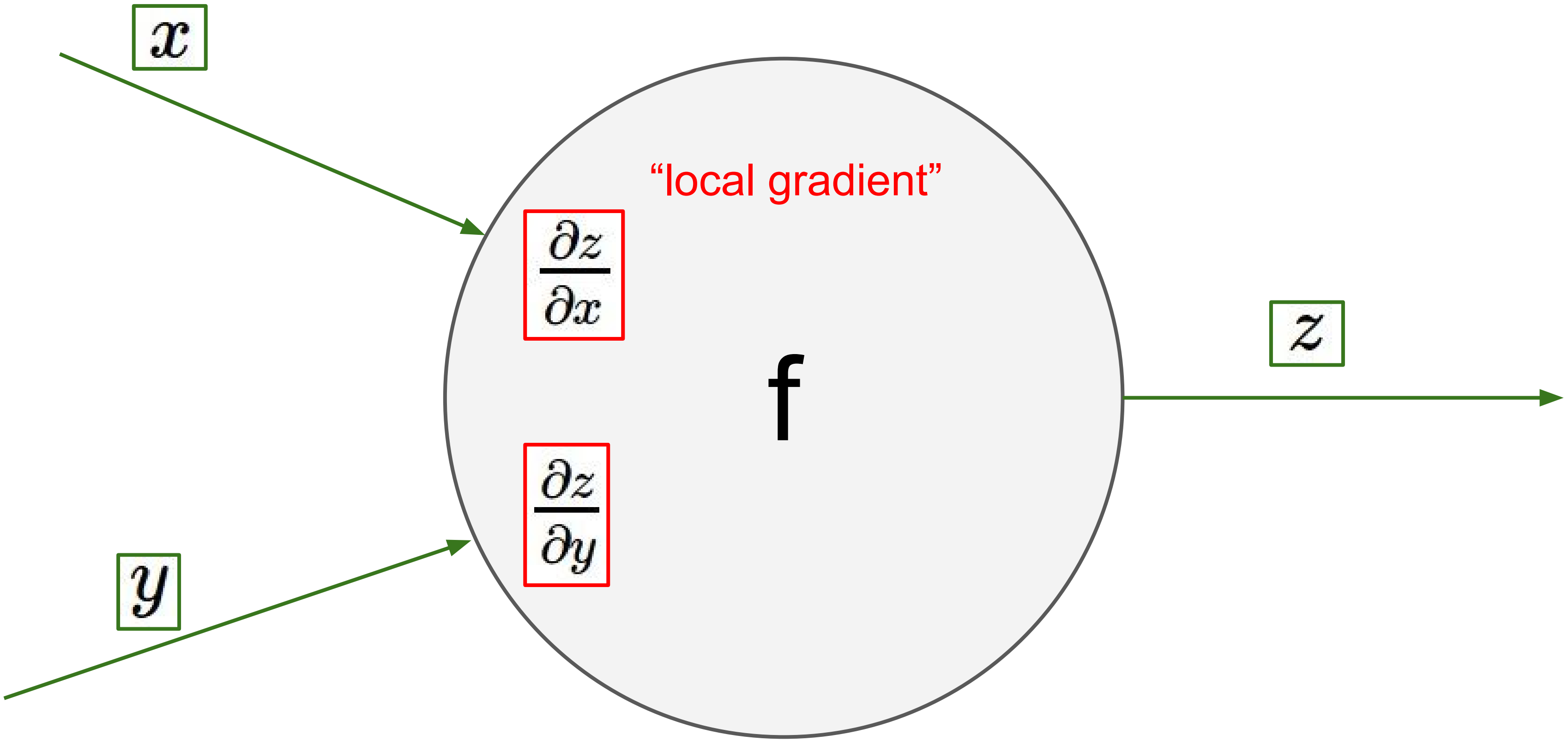
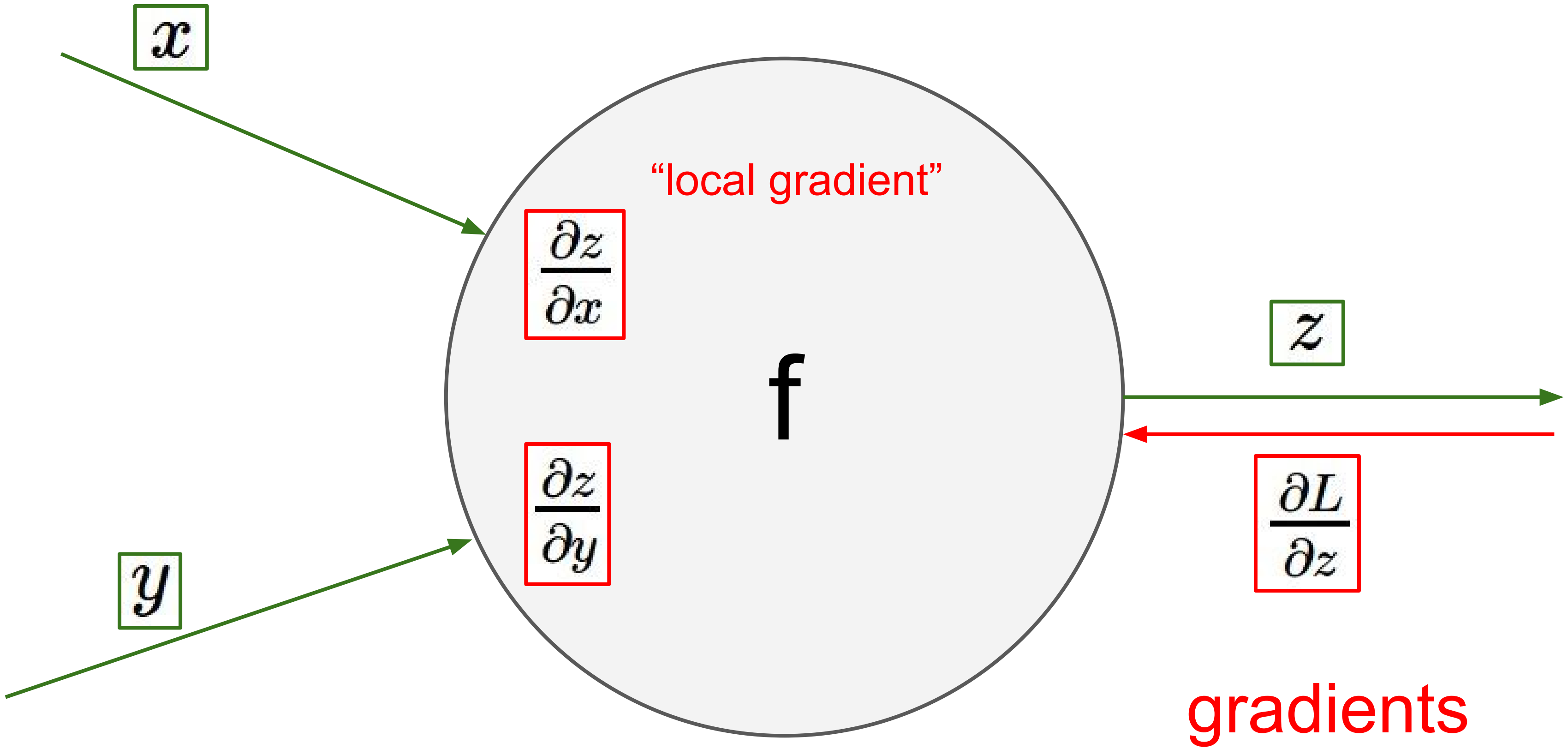
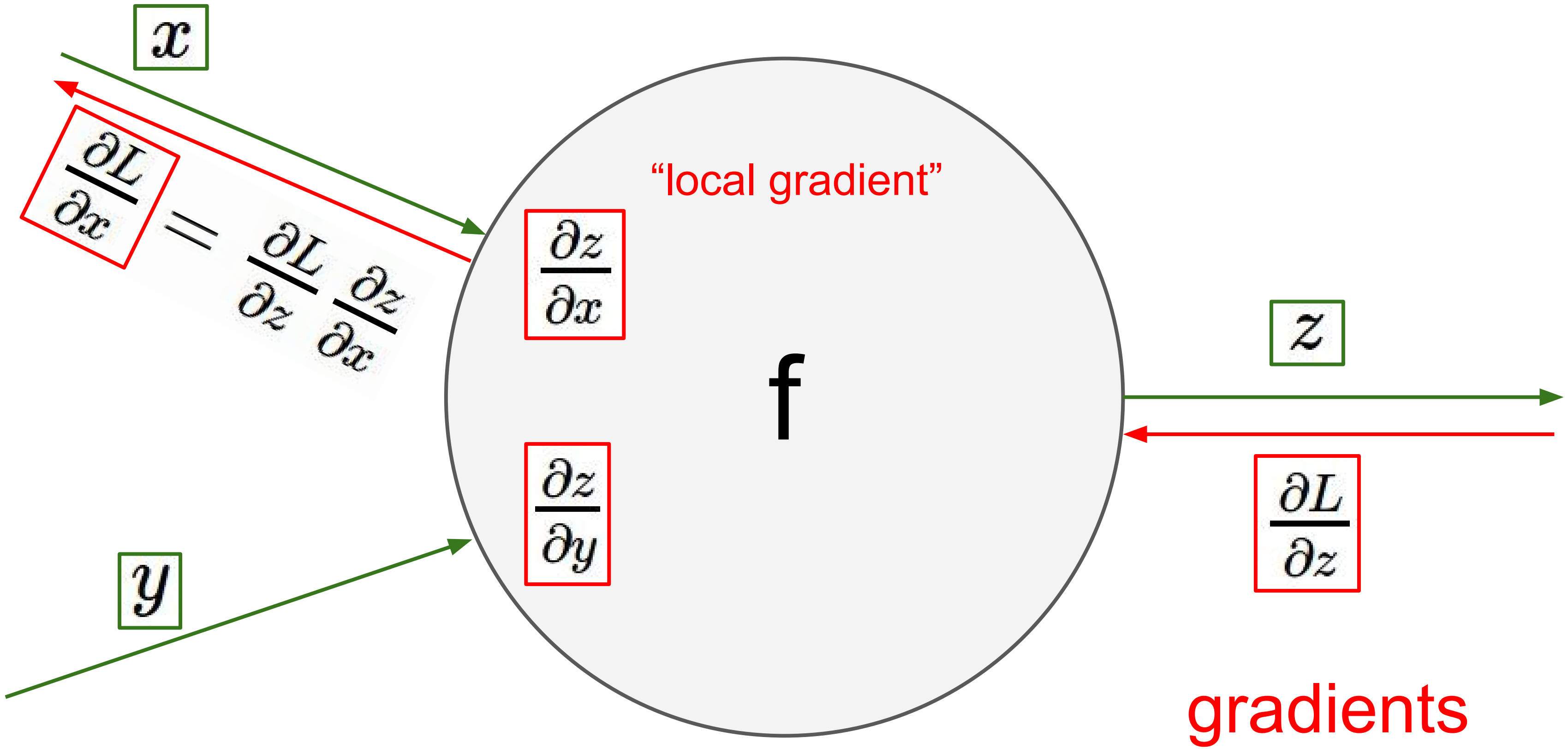

$x$
$y$
f
$z$



$x$
"local gradient"
$\frac{\partial z}{\partial x}$
f
$z$
$\frac{\partial z}{\partial y}$
$y$



$x$
$y$
"local gradient"
$\frac{\partial z}{\partial x}$
$\frac{\partial z}{\partial y}$
f
$z$
$\frac{\partial L}{\partial z}$
gradients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$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial x}$
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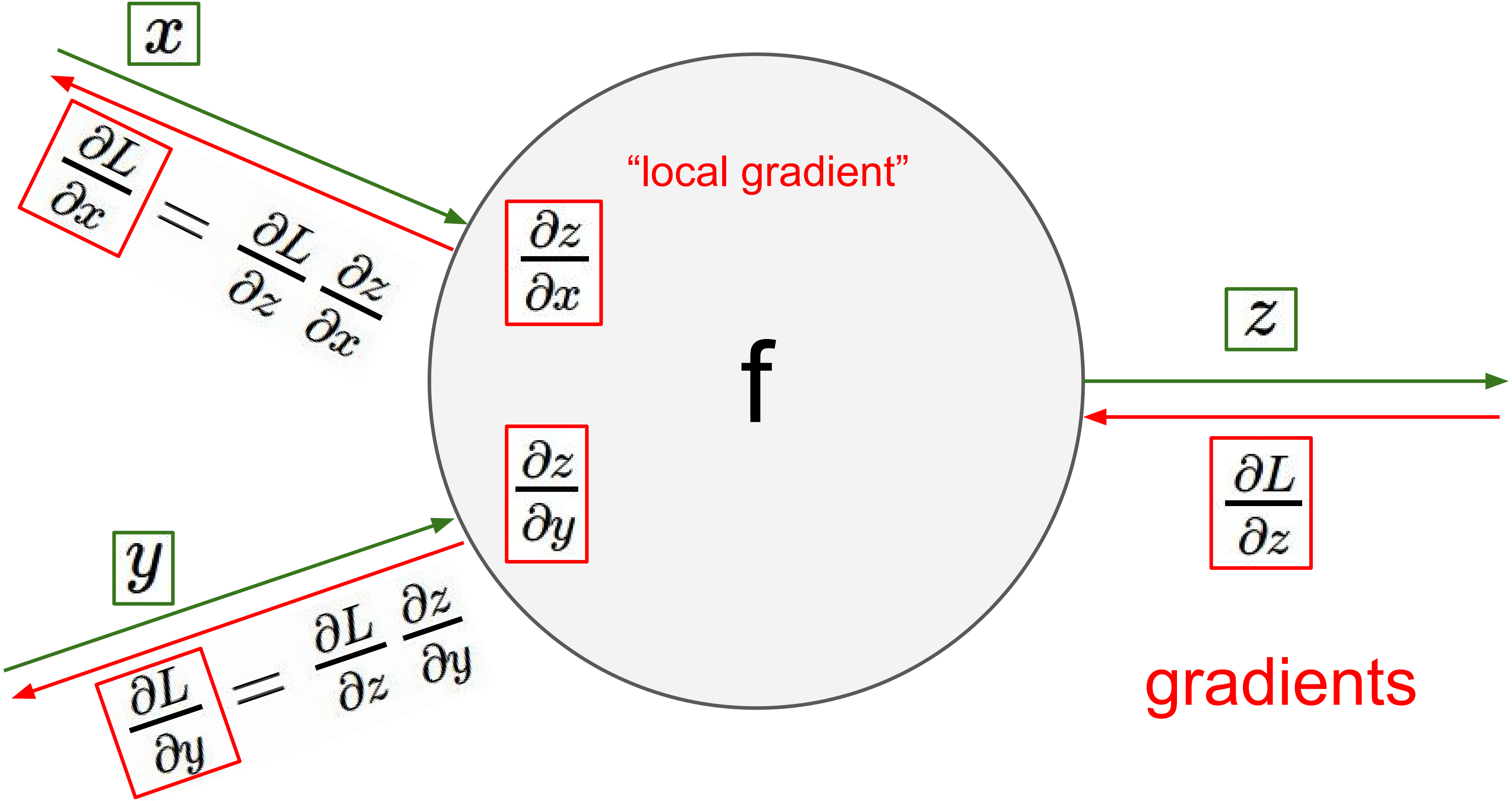
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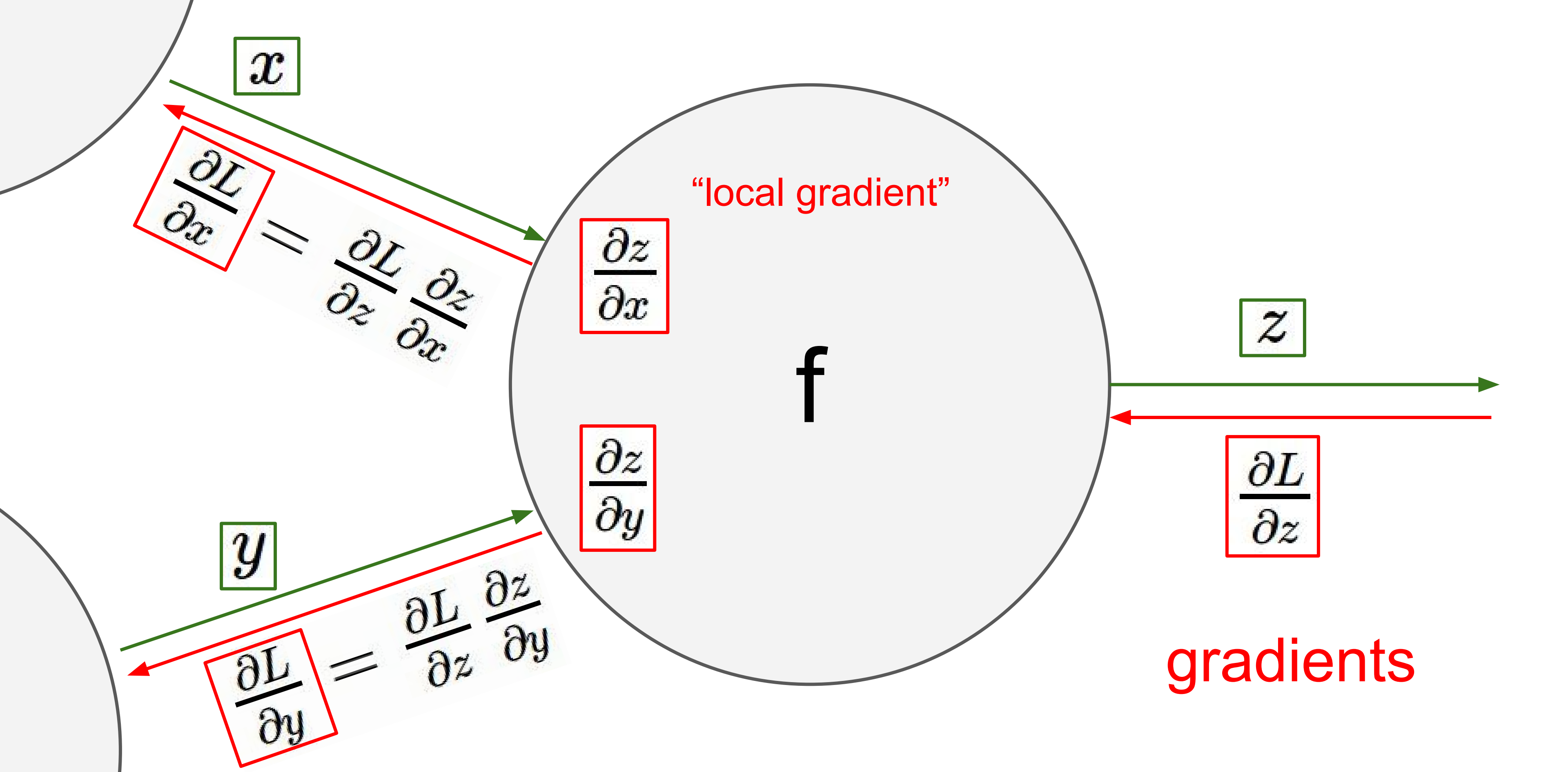
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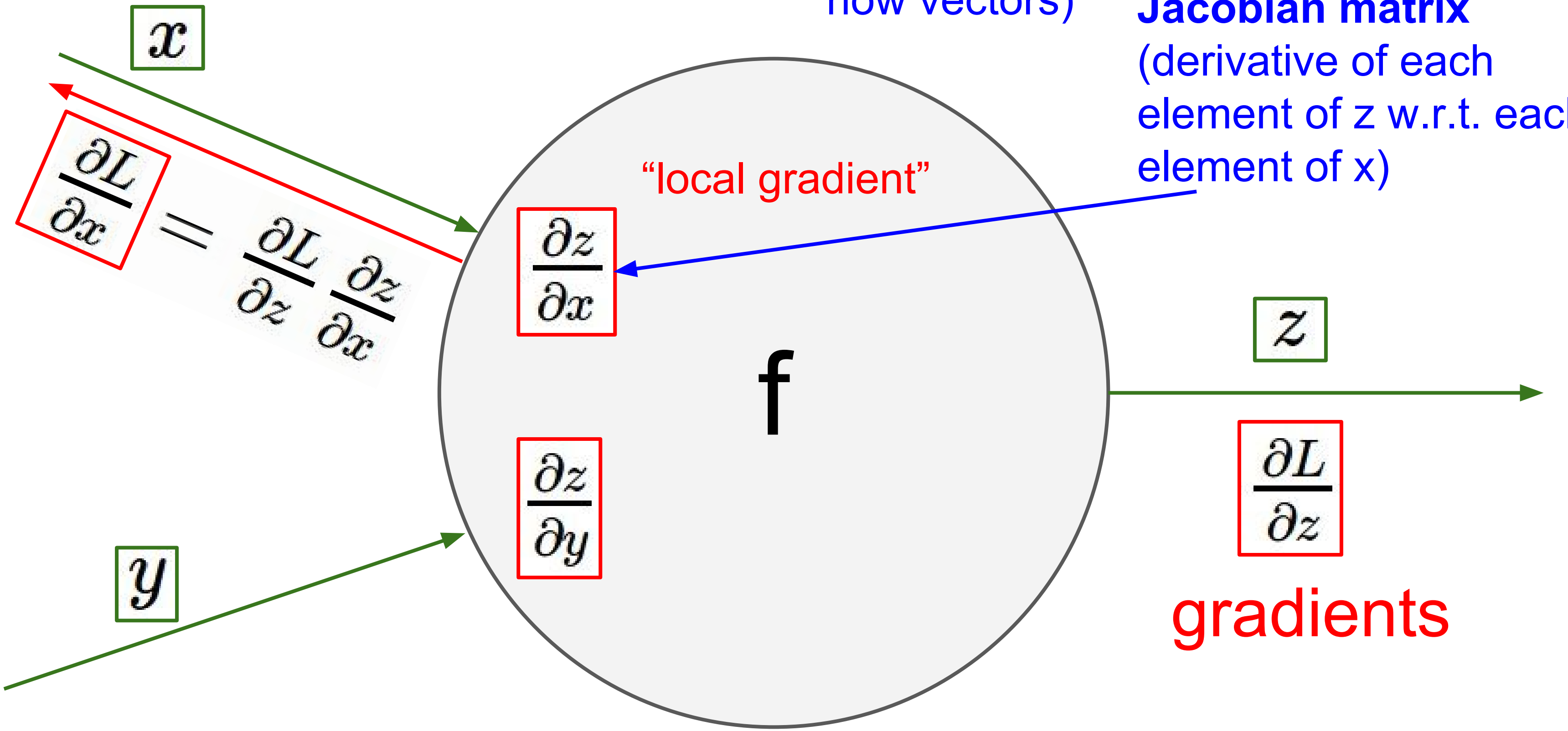
gradients

# Gradients for vectorized code

(x,y,z are now vectors)

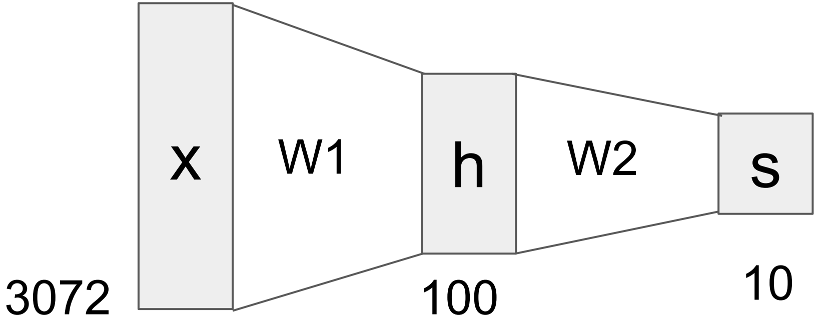
This is now the **Jacobian matrix** (derivative of each element of z w.r.t. each element of x)

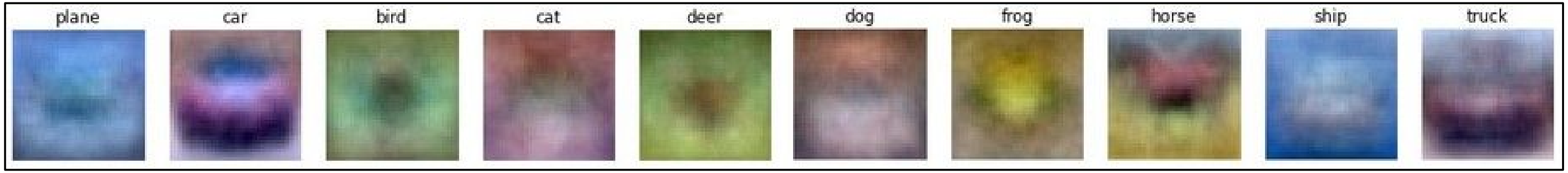


# Neural networks: without the brain stuff

(**Before**) Linear score function: $f = Wx$

(**Now**) 2-layer Neural Network $f = W_2 \max(0, W_1 x)$



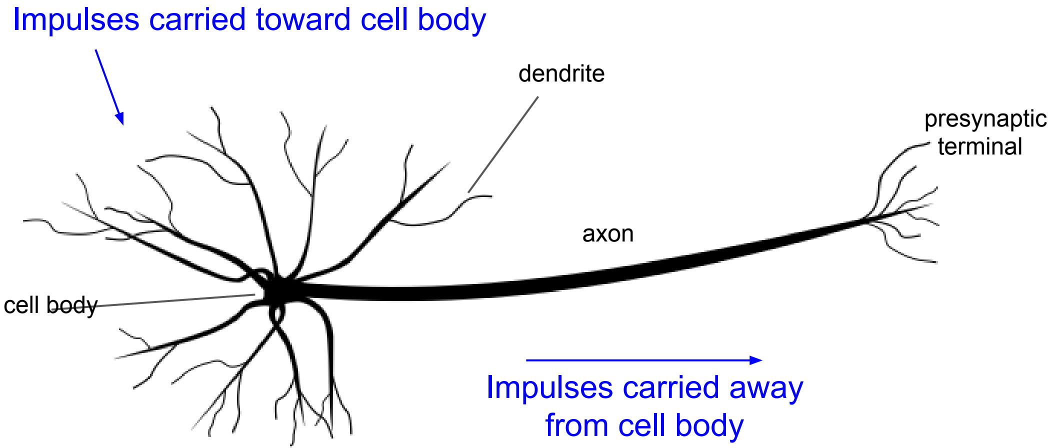



# Neural networks: without the brain stuff

(**Before**) Linear score function: $f = Wx$

(**Now**) 2-layer Neural Network $f = W_2 \max(0, W_1 x)$
or 3-layer Neural Network

$$f = W_3 \max(0, W_2 \max(0, W_1 x))$$

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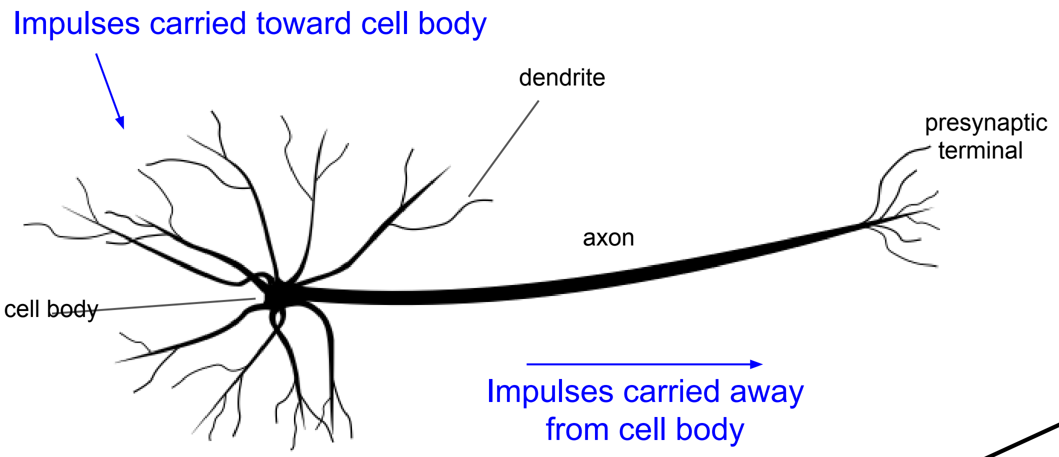


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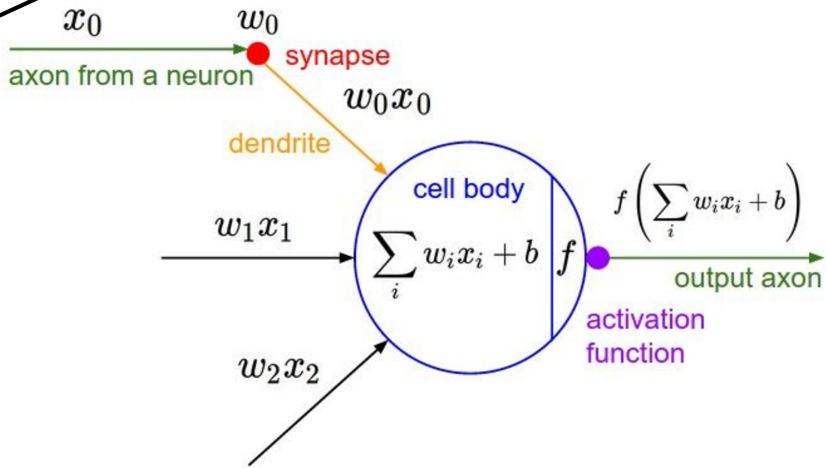

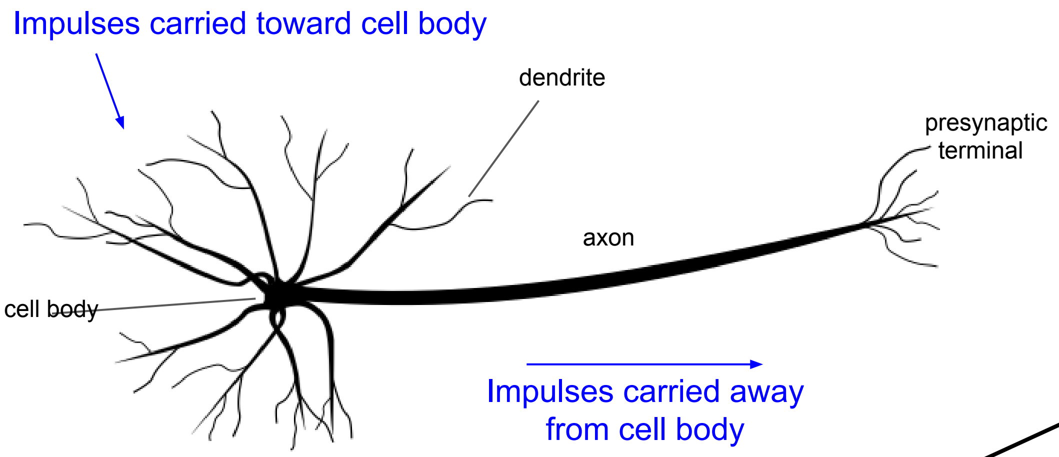


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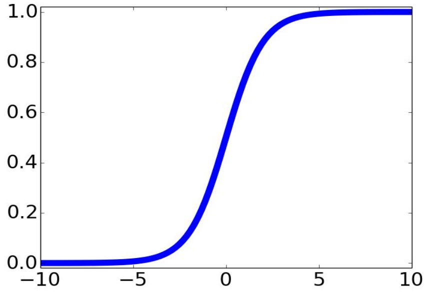
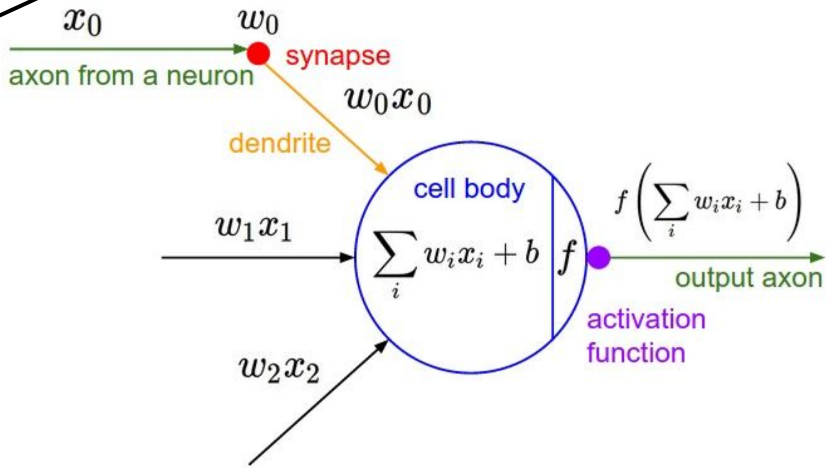



sigmoid activation function

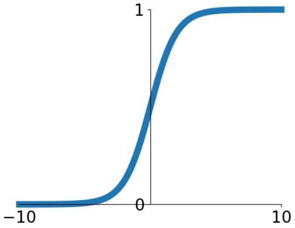
$$\frac{1}{1+e^{-x}}$$



# Activation functions

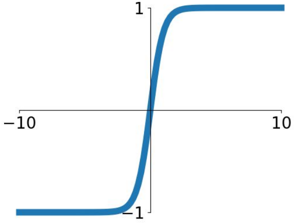
## Sigmoid

$\sigma(x) = \frac{1}{1+e^{-x}}$



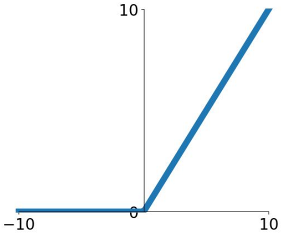

## tanh

$\tanh(x)$



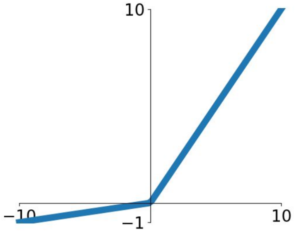

## ReLU

$\max(0, x)$




## Leaky ReLU

$\max(0.1x, x)$

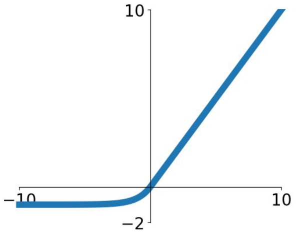



## Maxout

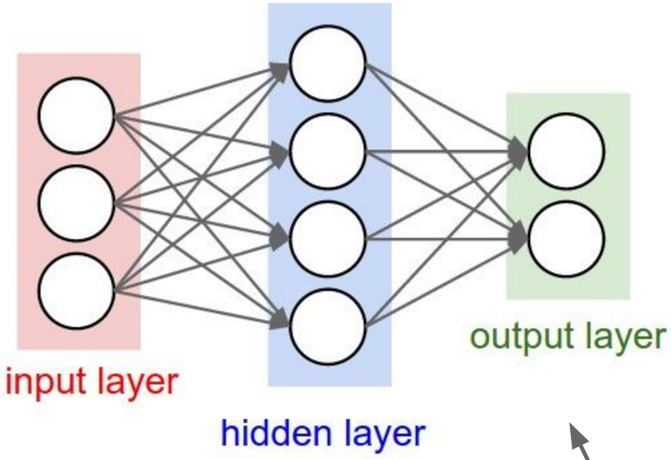
$\max(w_1^T x + b_1, w_2^T x + b_2)$

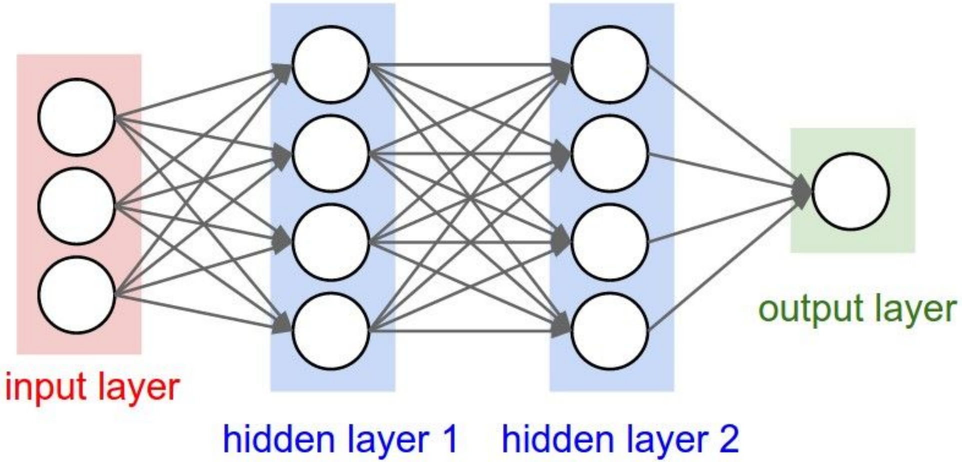
## ELU

$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



# Neural networks: Architectures




"2-layer Neural Net", or
"1-hidden-layer Neural Net"

"3-layer Neural Net", or
"2-hidden-layer Neural Net"

**"Fully-connected" layers**