CSE419 – Artificial Intelligence and Machine Learning 2020

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https://github.com/FurkanGozukara/CSE419-Artificial-Intelligence-and-Machine-Learning-2020

Lecture 5 Perceptron Learning

Based on Asst. Prof. Dr. David Kauchak (Pomona College) Lecture Slides

Source: https://cs.pomona.edu/~dkauchak/classes/f13/cs451-f13/lectures/

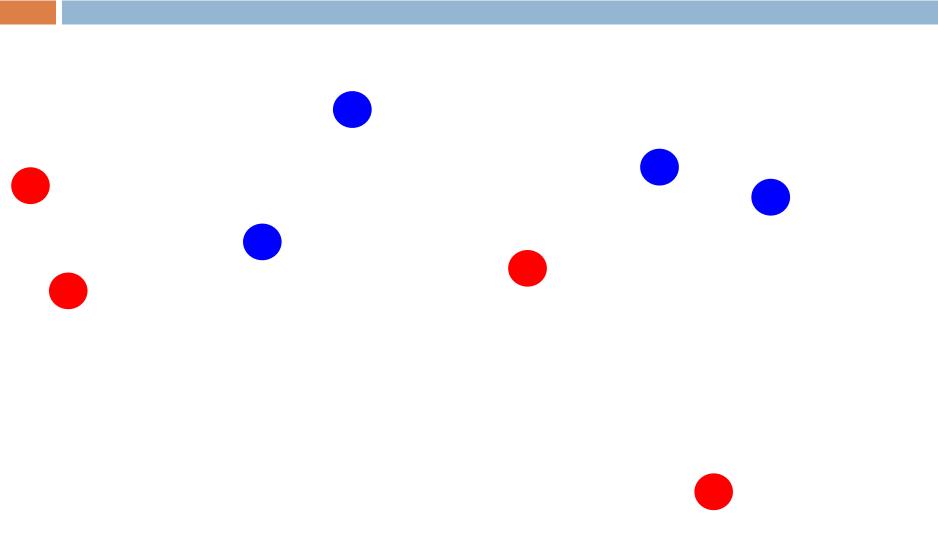
Machine learning models

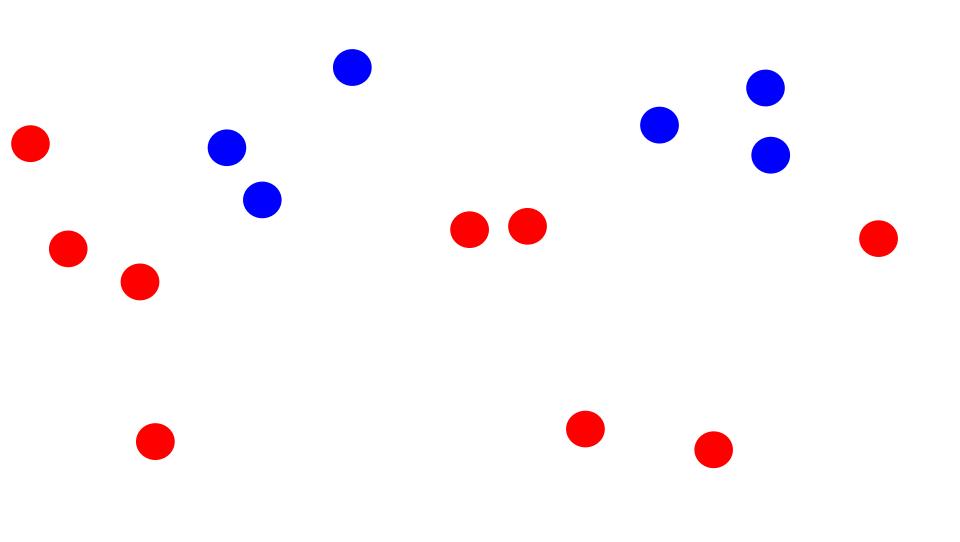
Some machine learning approaches make strong assumptions about the data:

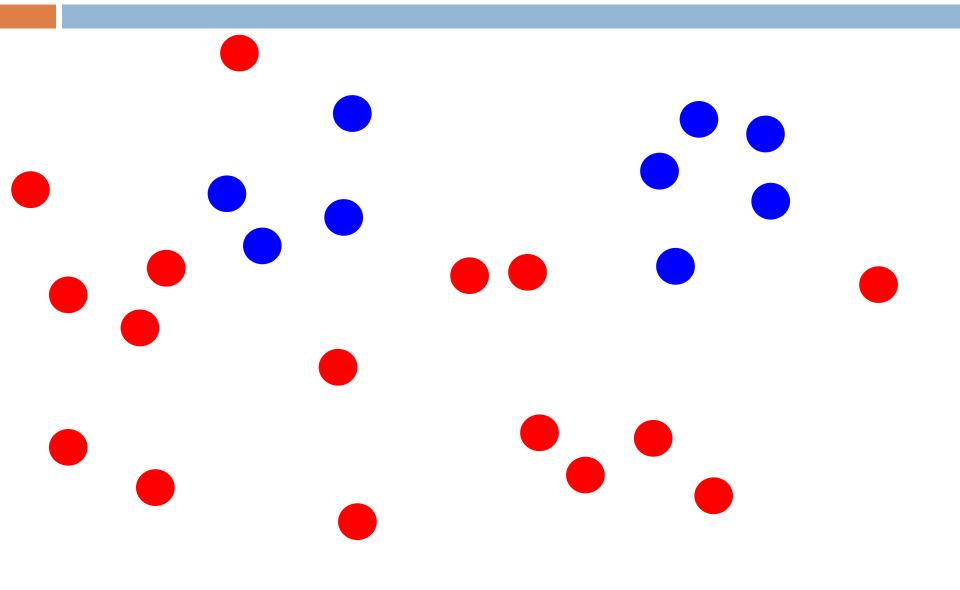
- If the assumptions are true this can often lead to better performance.
- If the assumptions aren't true, they can fail miserably.

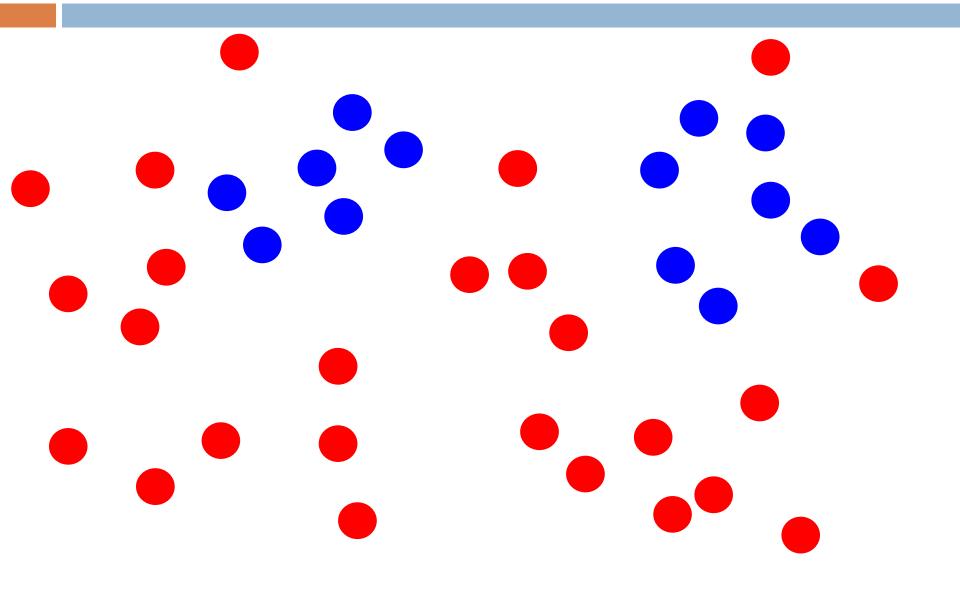
Other approaches don't make many assumptions about the data:

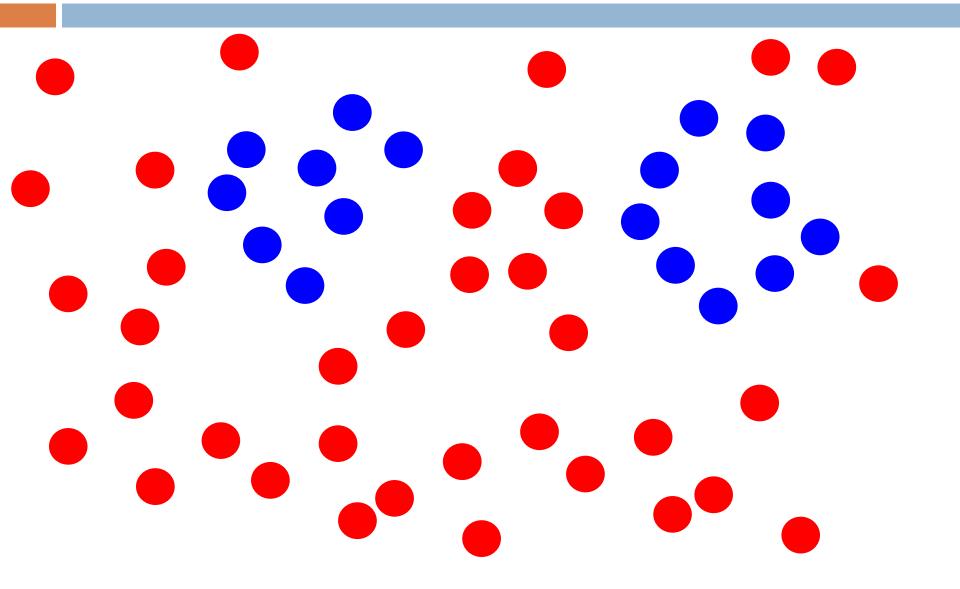
- This can allow us to learn from more varied data.
- But, they are more prone to overfitting.
- and generally require more training data.



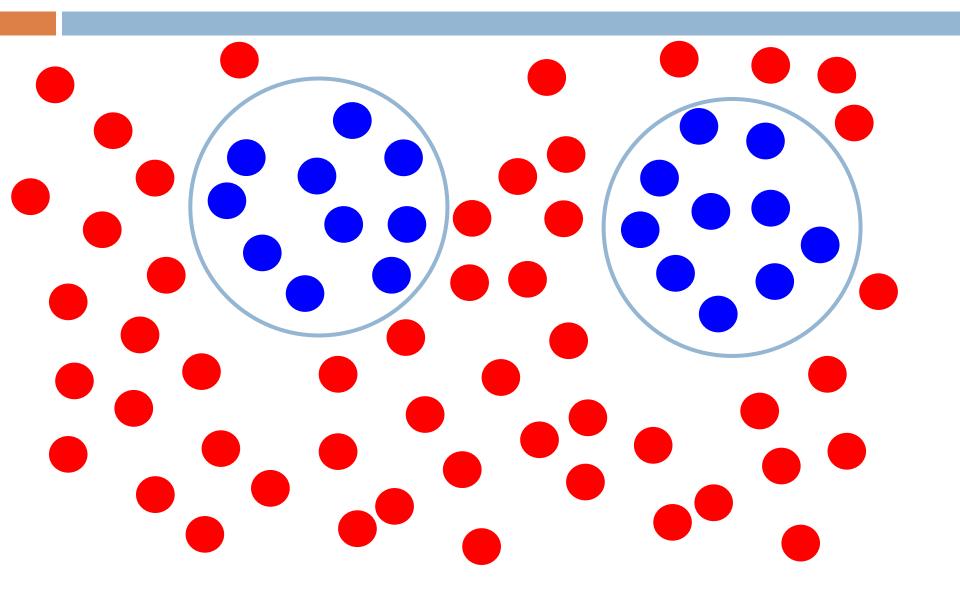








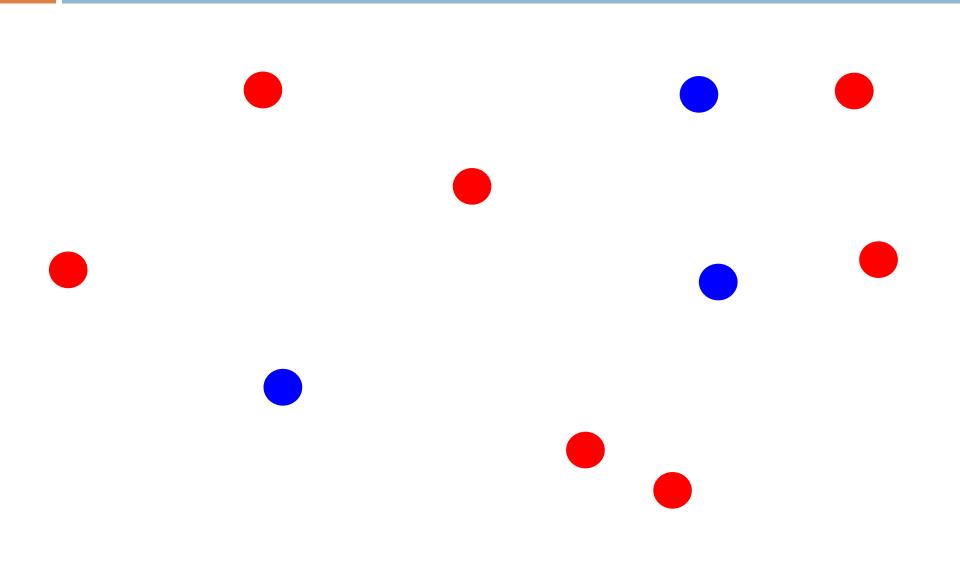
Actual model

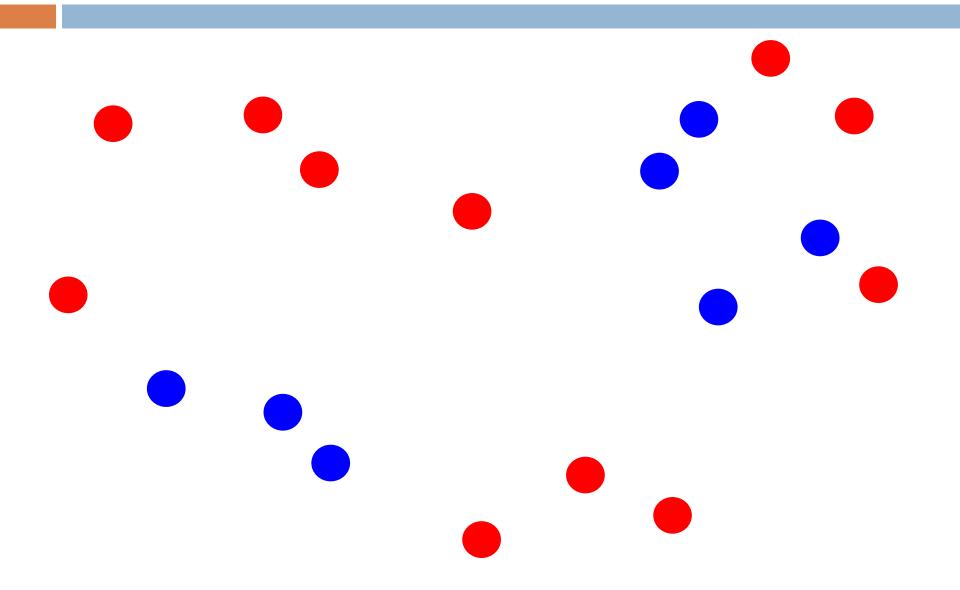


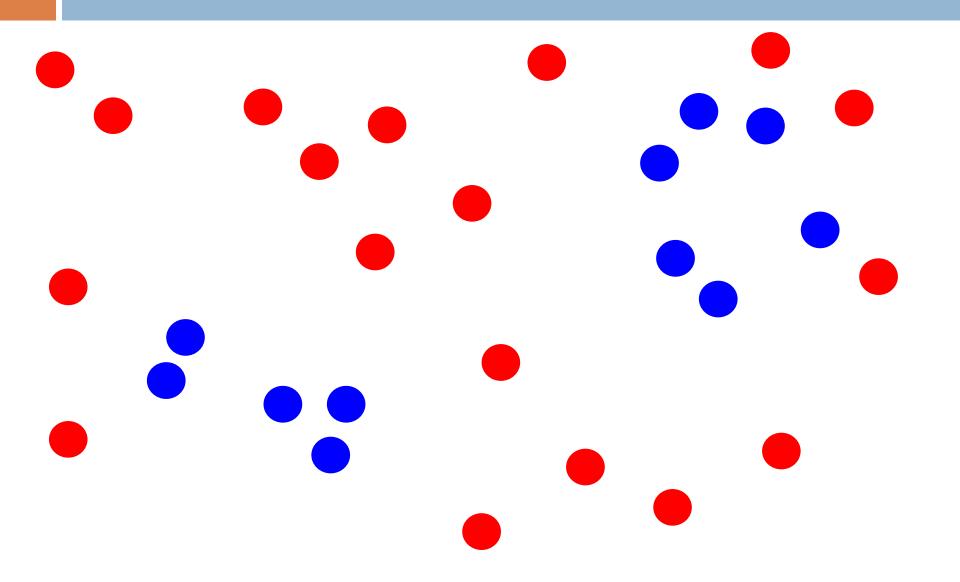
Model assumptions

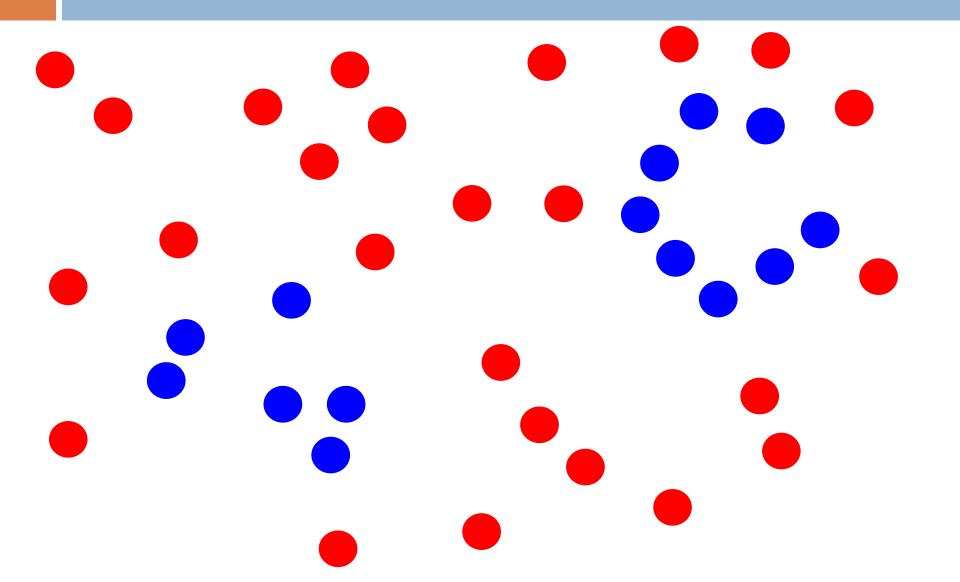
If you don't have strong assumptions about the model, it can take you a longer to learn.

Assume now that our model of the blue class is two circles.

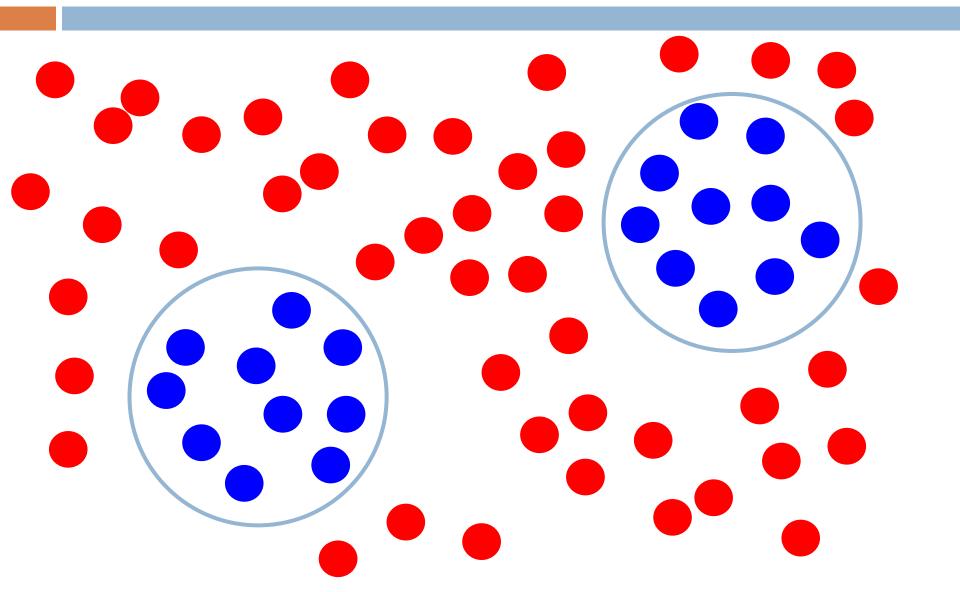


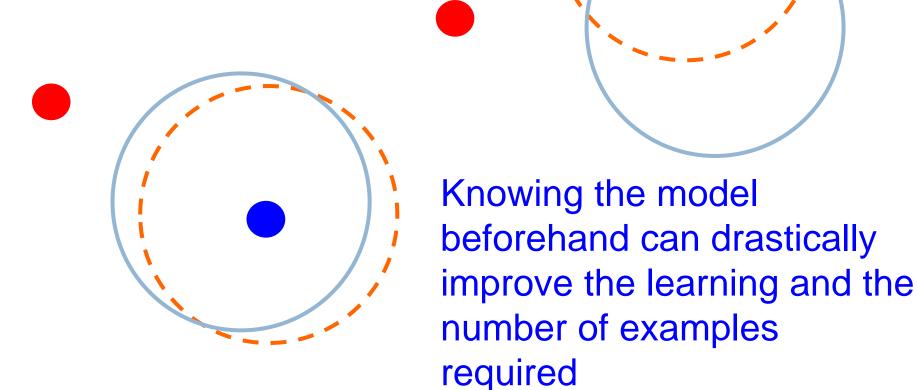


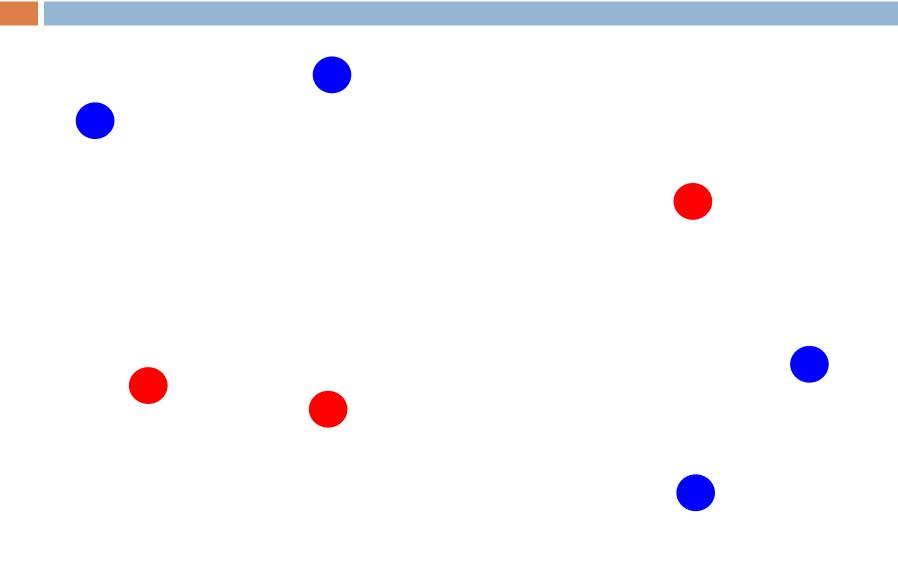




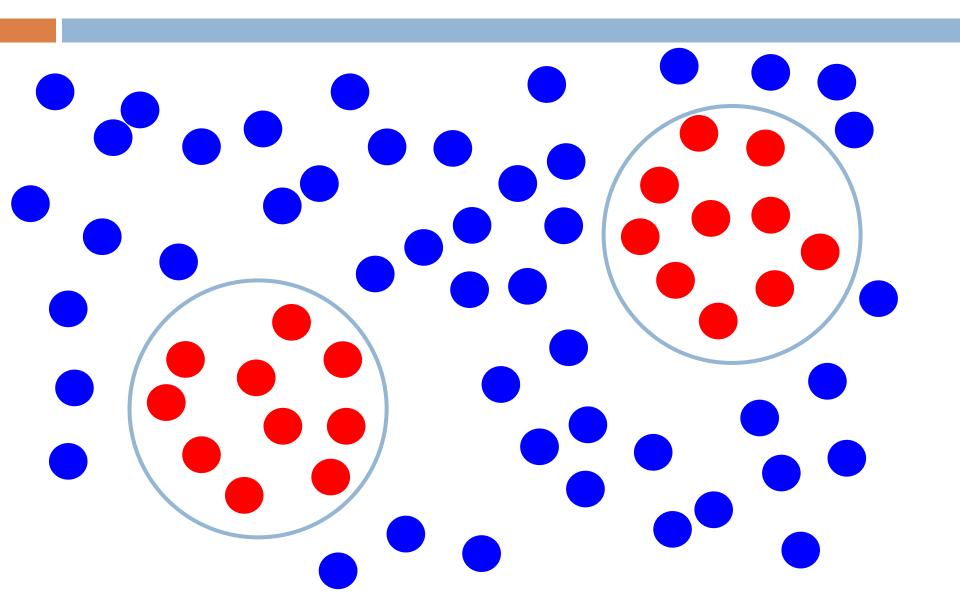
Actual model







Make sure your assumption is correct, though!



Machine learning models

- What were the model assumptions (if any) that k-NN and decision trees make about the data?
 - KNN is an non parametric lazy learning algorithm.
 - That is a pretty concise statement.
 - When you say a technique is non parametric, it means that it does not make any assumptions on the underlying data distribution.

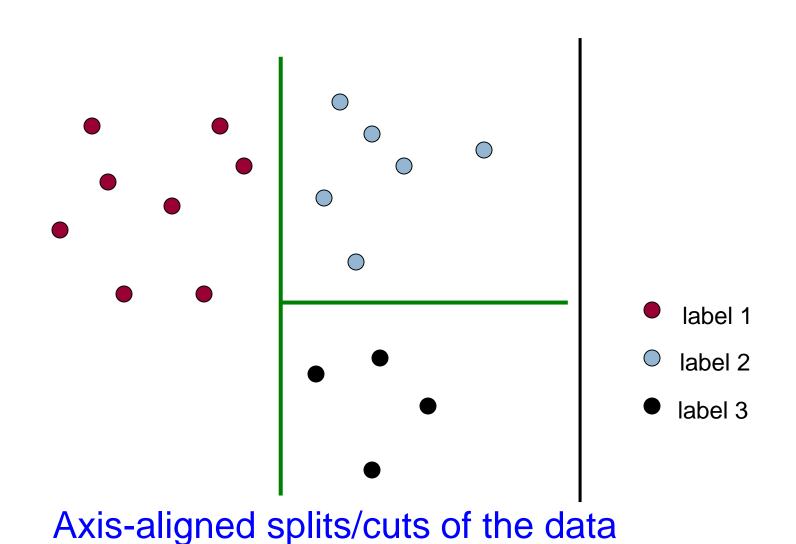
KNN-Pros

- The training phase of K-nearest neighbor classification is much faster compared to other classification algorithms.
- There is no need to train a model for generalization, that is why KNN is known as the simple and instance-based learning algorithm.
- KNN can be useful in case of nonlinear data.
- It can be used with the regression problem.
- Output value for the object is computed by the average of k closest neighbors value.

KNN-Cons

- The testing phase of K-nearest neighbor classification is slower and costlier in terms of time and memory.
- It requires large memory for storing the entire training dataset for prediction.
- KNN requires scaling of data because KNN uses the Euclidean distance between two data points to find nearest neighbors.
- Euclidean distance is sensitive to magnitudes.
- The features with high magnitudes will weight more than features with low magnitudes.
- KNN also not suitable for large dimensional data.

Decision tree model



Bias

The "bias" of a model is how strong the model assumptions are.

low-bias classifiers make minimal assumptions about the data (*k*-NN and DT are generally considered low bias.

high-bias classifiers make strong assumptions about the data.

Linear models

A strong high-bias assumption is *linear separability*:

- in 2 dimensions, can separate classes by a line.
- in higher dimensions, need hyperplanes.

A *linear model* is a model that assumes the data is linearly separable.

Hyperplanes

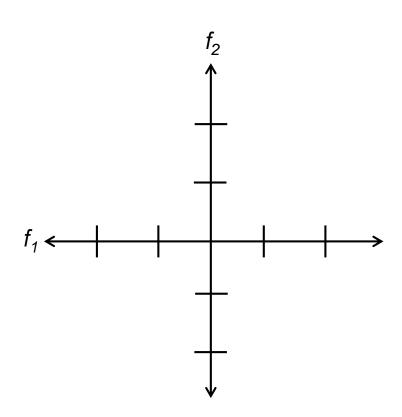
A hyperplane is line/plane in a high dimensional

space

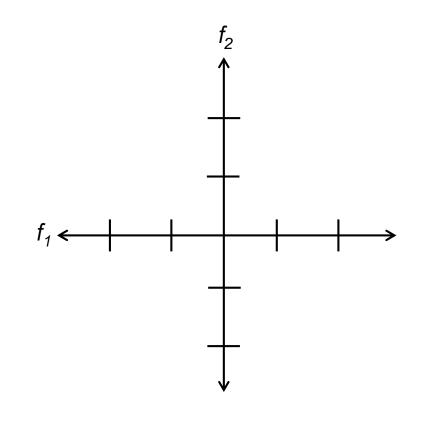


What defines a line? What defines a hyperplane?

$$0 = w_1 f_1 + w_2 f_2$$



$$0 = w_1 f_1 + w_2 f_2$$



$$0 = w_1 f_1 + w_2 f_2$$

$$0 = 1f_1 + 2f_2$$

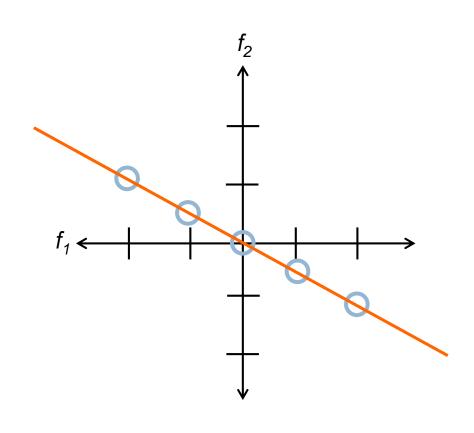
$$-2 \qquad 1$$

$$-1 \qquad 0.5$$

$$0 \qquad 0$$

$$1 \qquad -0.5$$

$$2 \qquad -1$$

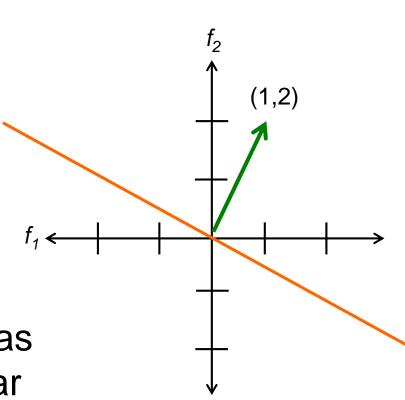


Any pair of values (w_1, w_2) defines a line through the origin:

$$0 = w_1 f_1 + w_2 f_2$$

$$0 = 1f_1 + 2f_2$$
w=(1,2)

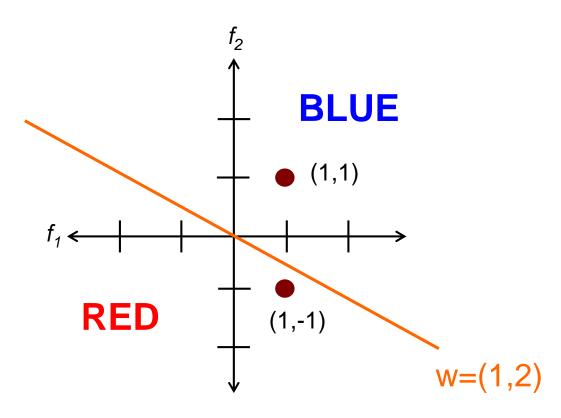
We can also view it as the line perpendicular to the weight vector



Classifying with a line

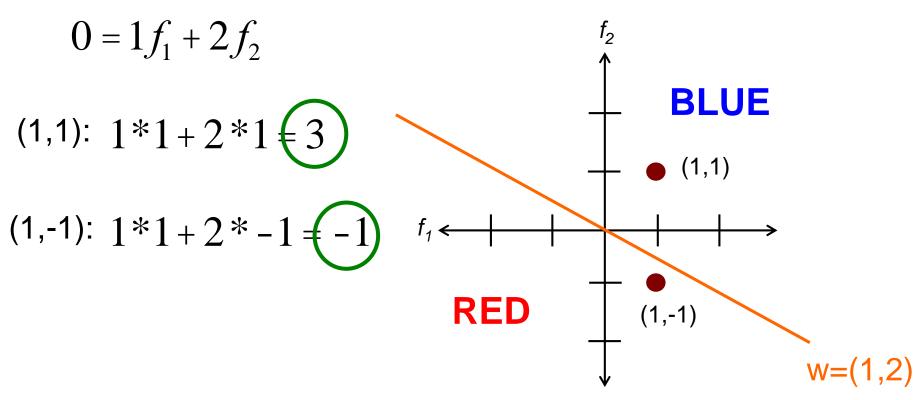
Mathematically, how can we classify points based on a line?

$$0 = 1f_1 + 2f_2$$



Classifying with a line

Mathematically, how can we classify points based on a line?

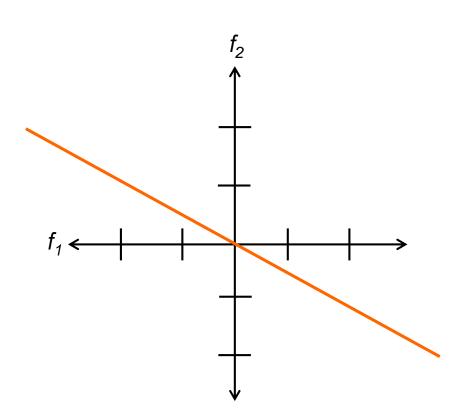


The sign indicates which side of the line

Any pair of values (w_1, w_2) defines a line through the origin:

$$0 = w_1 f_1 + w_2 f_2$$

$$0 = 1f_1 + 2f_2$$



How do we move the line off of the origin?

Any pair of values (w_1, w_2) defines a line through the origin:

$$-1 = 1f_1 + 2f_2$$

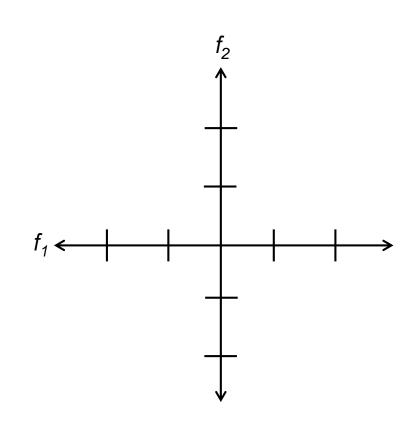
-2

-1

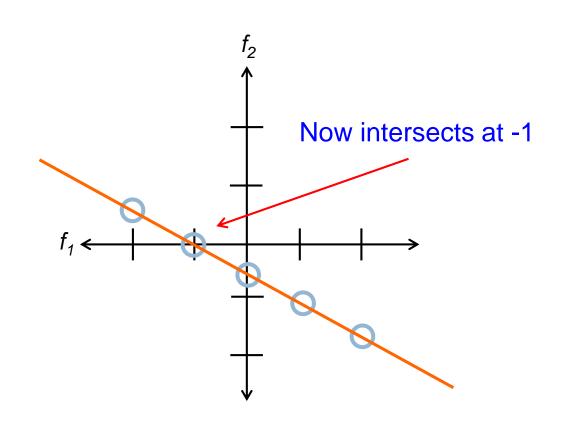
0

1

2



$$a = w_1 f_1 + w_2 f_2$$



Linear models

A linear model in *n*-dimensional space (i.e. *n* features) is define by *n*+1 weights:

In two dimensions, a line:

$$0 = w_1 f_1 + w_2 f_2 + b$$
 (where b = -a)

In three dimensions, a plane:

$$0 = w_1 f_1 + w_2 f_2 + w_3 f_3 + b$$

In *n*-dimensions, a *hyperplane*

$$0 = b + \mathop{\mathring{a}}\nolimits_{i=1}^n w_i f_i$$



Classifying with a linear model

We can classify with a linear model by checking the sign:

$$f_1, f_2, ..., f_n$$
 classifier

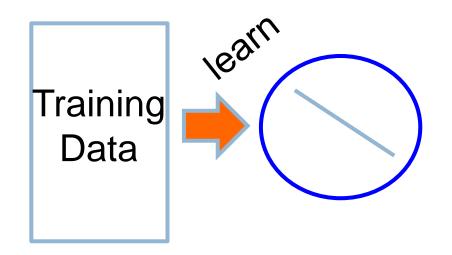
$$b + \mathring{a}_{i=1}^n w_i f_i > 0$$
 Positive example

$$b + \mathop{\mathring{a}}\nolimits_{i=1}^n w_i f_i < 0$$
 Negative example

Learning a linear model

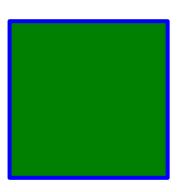
Geometrically, we know what a linear model represents.

Given a linear model (i.e. a set of weights and b) we can classify examples.

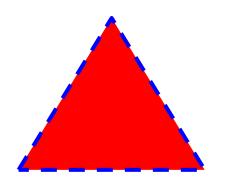


How do we learn a linear model?

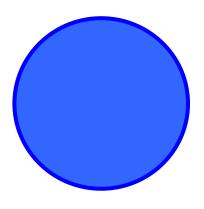
(data with labels)



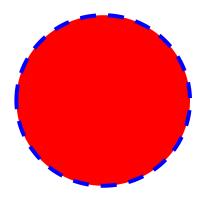
NEGATIVE



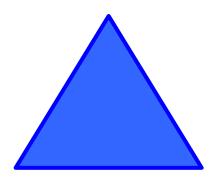
NEGATIVE



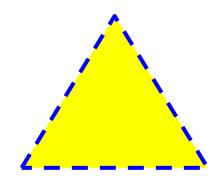
POSITIVE



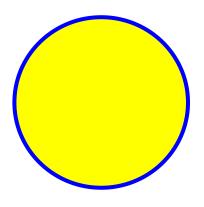
NEGATIVE



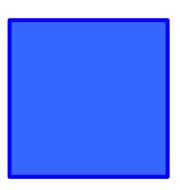
POSITIVE



POSITIVE



NEGATIVE



POSITIVE

A method to the madness

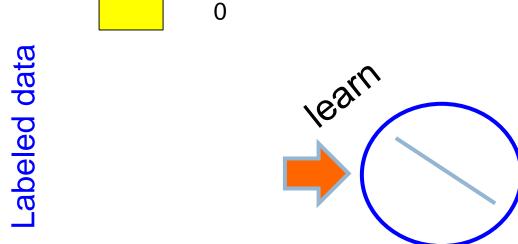
blue = positive

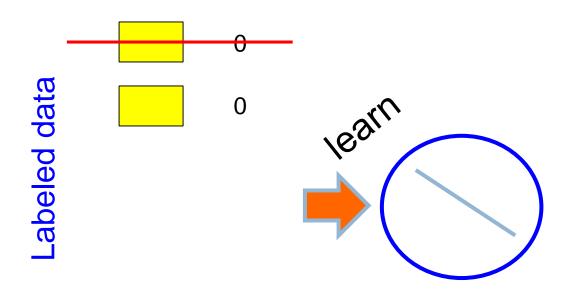
yellow triangles = positive

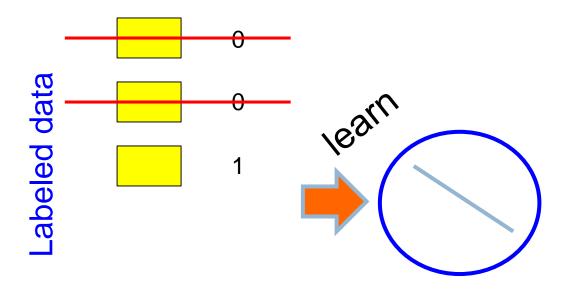
all others negative

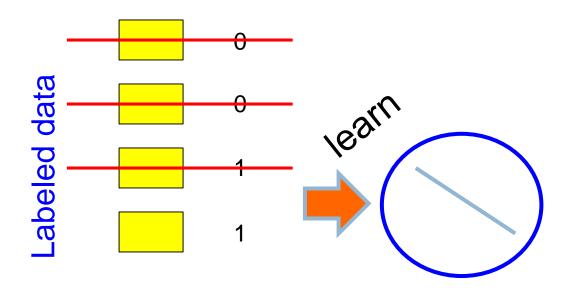
How is this learning setup different than the learning we've done before?

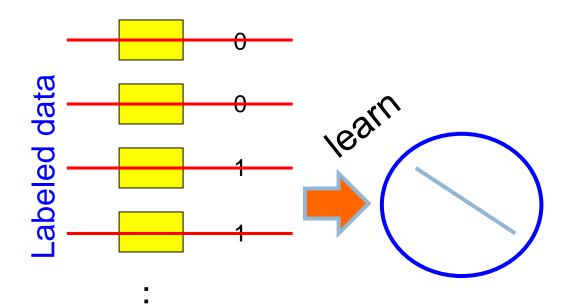
When might this arise?

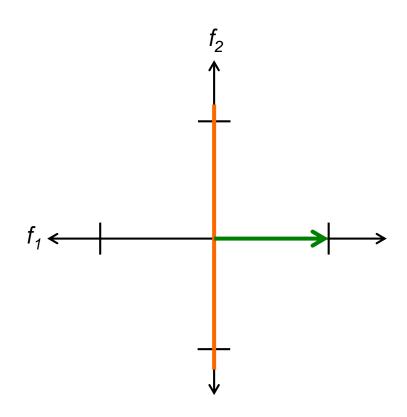






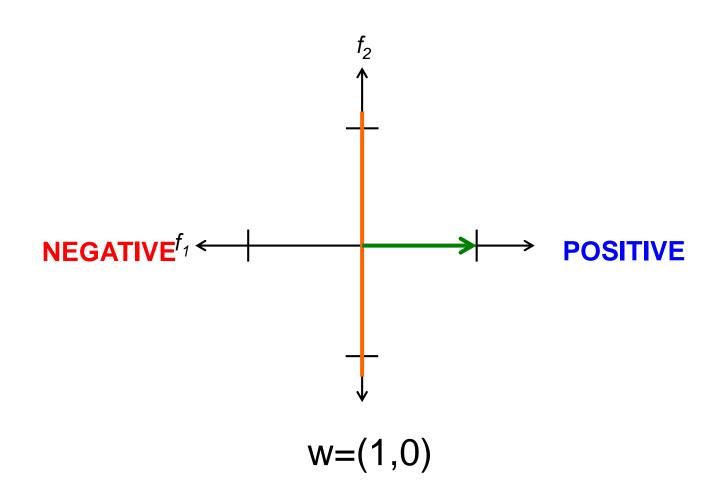




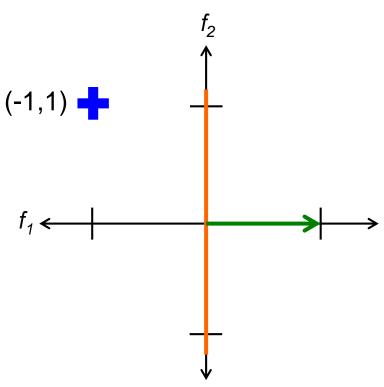


What does this model currently say?

$$w = (1,0)$$



$$0 = w_1 f_1 + w_2 f_2$$



Is our current guess: right or wrong?

$$W=(1,0)$$

$$0 = w_1 f_1 + w_2 f_2$$

$$1*f_1 + 0*f_2 = 1*-1+0*1 \neq -1$$

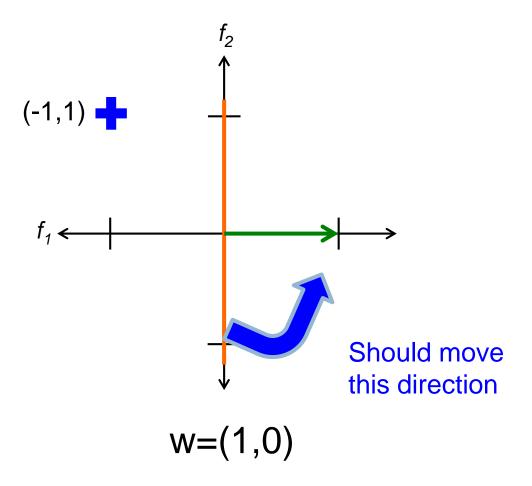
 $(-1,1) \stackrel{t_2}{\longleftarrow}$ $f_1 \longleftarrow \longrightarrow \longrightarrow$

predicts negative, wrong

How should we update the model? W=(1,0)

$$0 = w_1 f_1 + w_2 f_2$$

$$1*f_1 + 0*f_2 = 1*-1+0*1 \neq -1$$



$$W_1$$
 W_2 $1*f_1+0*f_2=$ W_2 $1*-1+0*1=-1$ position value.

(-1, 1, positive)

We'd like this value to be positive since it's a positive value

Which of these contributed to the mistake?

$$W_1$$
 W_2
 $1*f_1+0*f_2=$
 $1*-1+0*1=-1$

(-1, 1, positive)

We'd like this value to be positive since it's a positive value

contributed in the wrong direction could have contributed (positive feature), but didn't

How should we change the weights?

$$W_1$$
 W_2
 $1*f_1+0*f_2=$
 $1*-1+0*1=-1$

(-1, 1, positive)

We'd like this value to be positive since it's a positive value

contributed in the wrong direction decrease

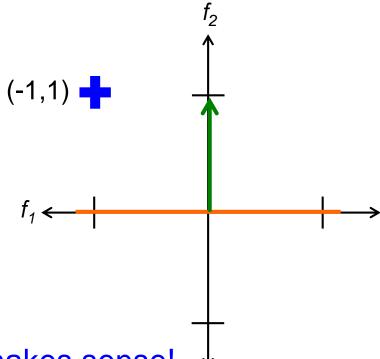
1 -> 0

could have contributed (positive feature), but didn't

increase

$$0 -> 1$$

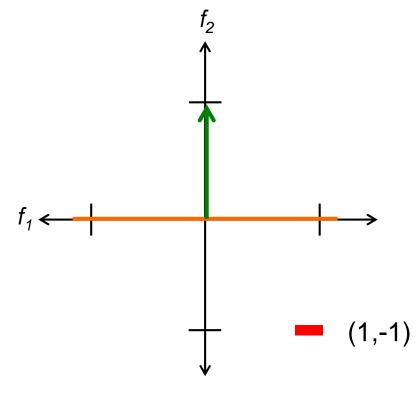
$$0 = w_1 f_1 + w_2 f_2$$



Graphically, this also makes sense!

$$W = (0,1)$$

$$0 = w_1 f_1 + w_2 f_2$$



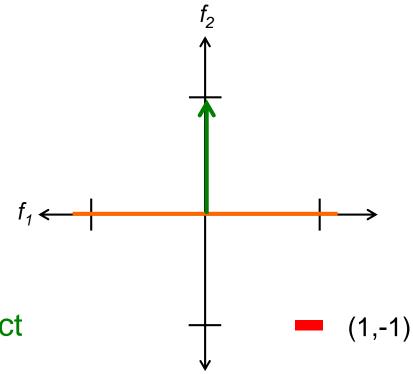
Is our current guess: right or wrong?

$$w = (0,1)$$

$$0 = w_1 f_1 + w_2 f_2$$

$$0*f_1+1*f_2= \\ 0*1+1*-1 \neq -1$$

predicts negative, correct

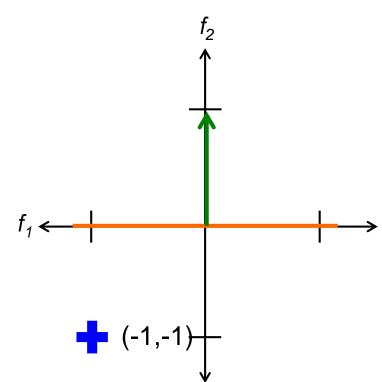


How should we update the model? W=(0,1)

$$0 = w_1 f_1 + w_2 f_2$$

$$0*f_1+1*f_2 = 0*1+1*-1 \neq -1$$
Already correct... don't change it!
$$w=(0,1)$$

$$0 = w_1 f_1 + w_2 f_2$$



Is our current guess: right or wrong?

$$w = (0,1)$$

$$0 = w_1 f_1 + w_2 f_2$$

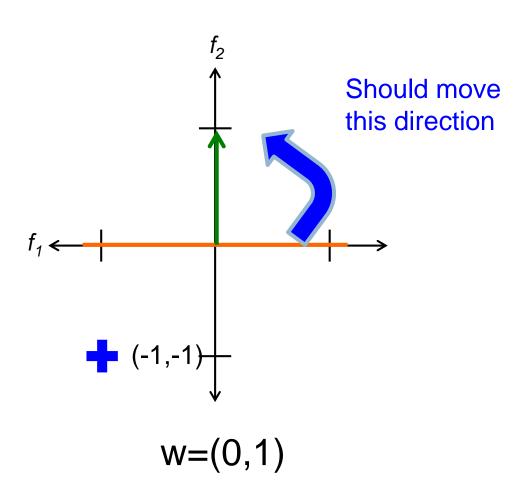
$$0*f_1+1*f_2 = 0*-1+1*-1 = -1$$

 $f_1 \longleftrightarrow$

predicts negative, wrong

How should we update the model? W=(0,1)

$$0 = w_1 f_1 + w_2 f_2$$



$$w_1$$
 w_2 (-1, -1, positive)
$$0*f_1+1*f_2=$$
 We'd like this value to be positive since it's a positive value

Which of these contributed to the mistake?

$$w_1$$
 w_2 (-1, -1, positive)
$$0*f_1+1*f_2=0*-1+1*-1=-1$$
 We'd like this value to be positive since it's a positive value
$$value$$
 didn't contribute, contributed in the wrong direction

How should we change the weights?

$$W_1 W_2$$

$$0*f_1+1*f_2=$$

$$0*-1+1*-1=-1$$

(-1, -1, positive)

We'd like this value to be positive since it's a positive value

didn't contribute, but could have

contributed in the wrong direction

decrease

$$0 -> -1$$

decrease

$$1 -> 0$$

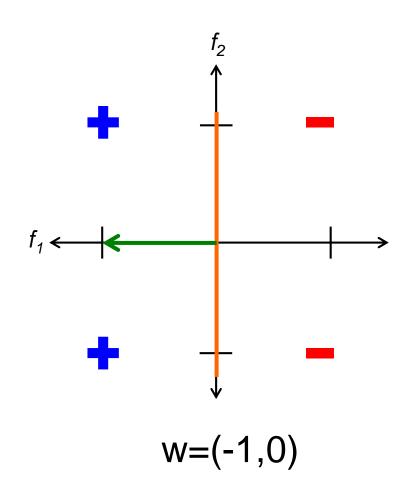
 f_1 , f_2 , label

-1,-1, positive

-1, 1, positive

1, 1, negative

1,-1, negative



Perceptron learning algorithm

```
repeat until convergence (or for some # of iterations):
 for each training example (f_1, f_2, ..., f_n, label):
   check if it's correct based on the current model
   if not correct, update all the weights:
                                               W_i f_i
     if label positive and feature positive:
       increase weight (increase weight = predict more positive)
     if label positive and feature negative:
       decrease weight (decrease weight = predict more positive)
     if label negative and feature positive:
       decrease weight (decrease weight = predict more negative)
     if label negative and negative weight:
       increase weight (increase weight = predict more negative)
```

A trick...

Let positive label = 1 and negative label = -1

if label positive and feature positive:

increase weight (increase weight = predict more positive)

if label positive and feature negative:

decrease weight (decrease weight = predict more positive)

if label negative and feature positive:

decrease weight (decrease weight = predict more negative)

if label negative and negative weight:

increase weight (increase weight = predict more negative)

label * f_i

1*1=1

1*-1=-1

-1*1=-1

-1*-1=1

A trick...

Let positive label = 1 and negative label = -1

if label positive and feature positive:

increase weight (increase weight = predict more
positive)

if label positive and feature negative:

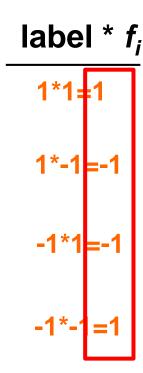
decrease weight (decrease weight = predict more
positive)

if label negative and feature positive:

decrease weight (decrease weight = predict more
negative)

if label negative and negative weight:

increase weight (increase weight = predict more
negative)



Perceptron learning algorithm

```
repeat until convergence (or for some # of iterations):
 for each training example (f_1, f_2, ..., f_n, label):
    check if it's correct based on the current model
    if not correct, update all the weights:
      for each w;
       W_i = W_i + f_i^* label
      b = b + label
```

How do we check if it's correct?

Perceptron learning algorithm

```
repeat until convergence (or for some # of iterations):
  for each training example (f_1, f_2, ..., f_n, label):
       prediction = b + \mathring{a}_{i-1}^n w_i f_i
    if prediction * label ≤ 0: // they don't agree
      for each w;
        W_i = W_i + f_i^* \text{label}
```

b = b + label

Perceptron learning algorithm

```
repeat until convergence (or for some # of
iterations):
  for each training example (f_1, f_2, ..., f_n, label):
     prediction = b + \mathring{\mathbf{A}}_{i-1}^{n} w_i f_i
     if prediction * label ≤ 0: // they don't agree
       for each w<sub>i</sub>:
         W_i = W_i + f_i^* \text{label}
       b = b + label
```

Would this work for non-binary features, i.e. real-valued?

repeat until convergence (or for some # of iterations):

for each training example $(f_1, f_2, ..., f_n, ..., f_n)$

label):

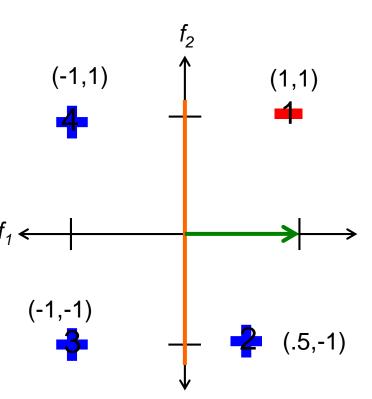
$$prediction = \mathring{\mathbf{a}}_{i=1}^{n} w_i f_i$$

if $prediction * label \le 0$: // they don't $f_1 \leftarrow$ agree

for each w_i:

$$W_i = W_i + f_i^*$$
label

- Repeat until convergence
- Keep track of w₁, w₂ as they change
- Redraw the line after each step



$$w = (1, 0)$$

repeat until convergence (or for some # of iterations):

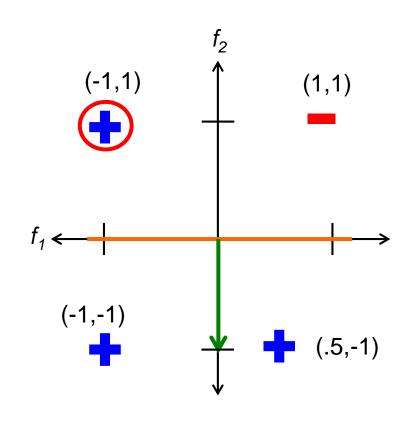
for each training example (f_1 , f_2 , ..., f_n , label):

$$prediction = \mathring{a}_{i=1}^{n} w_{i} f_{i}$$

if *prediction* * *label* ≤ 0: // they don't agree

for each wi:

$$W_i = W_i + f_i^* \text{label}$$



$$W = (0, -1)$$

repeat until convergence (or for some # of iterations):

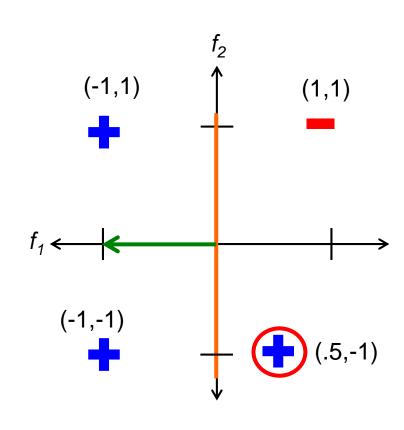
for each training example (f_1 , f_2 , ..., f_n , label):

$$prediction = \mathring{a}_{i=1}^{n} w_{i} f_{i}$$

if *prediction* * *label* ≤ 0: // they don't agree

for each w_i:

$$W_i = W_i + f_i^* \text{label}$$



$$w = (-1, 0)$$

repeat until convergence (or for some # of iterations):

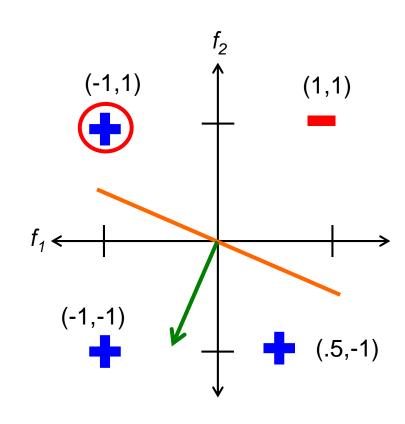
for each training example (f_1 , f_2 , ..., f_n , label):

$$prediction = \mathring{a}_{i=1}^{n} w_{i} f_{i}$$

if *prediction* * *label* ≤ 0: // they don't agree

for each wi:

$$W_i = W_i + f_i^* \text{label}$$



$$w = (-.5, -1)$$

repeat until convergence (or for some # of iterations):

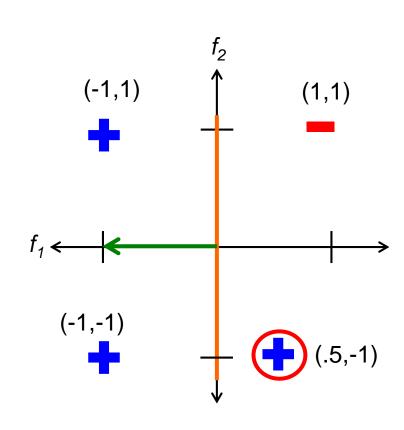
for each training example (f_1 , f_2 , ..., f_n , label):

$$prediction = \mathring{a}_{i=1}^{n} w_{i} f_{i}$$

if *prediction* * *label* ≤ 0: // they don't agree

for each w_i:

$$W_i = W_i + f_i^* \text{label}$$



$$w = (-1.5, 0)$$

repeat until convergence (or for some # of iterations):

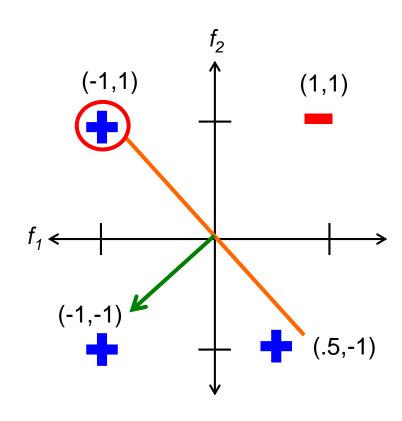
for each training example (f_1 , f_2 , ..., f_n , label):

$$prediction = \mathring{a}_{i=1}^{n} w_i f_i$$

if *prediction* * *label* ≤ 0: // they don't agree

for each wi:

$$W_i = W_i + f_i^* \text{label}$$



$$W = (-1, -1)$$

repeat until convergence (or for some # of iterations):

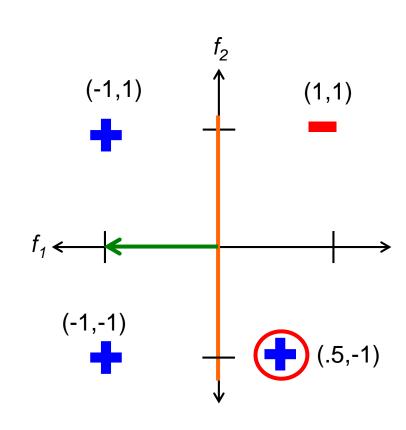
for each training example (f_1 , f_2 , ..., f_n , label):

$$prediction = \mathring{a}_{i=1}^{n} w_{i} f_{i}$$

if *prediction* * *label* ≤ 0: // they don't agree

for each w_i:

$$W_i = W_i + f_i^* \text{label}$$



$$w = (-2, 0)$$

repeat until convergence (or for some # of iterations):

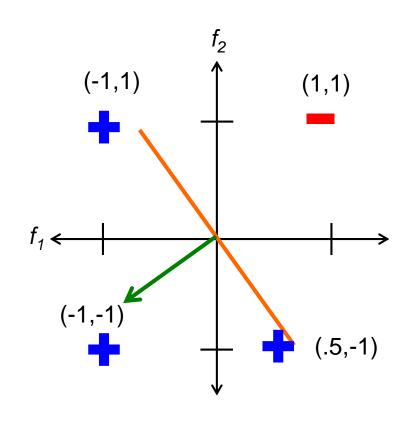
for each training example (f_1 , f_2 , ..., f_n , label):

$$prediction = \mathring{a}_{i=1}^{n} w_{i} f_{i}$$

if *prediction* * *label* ≤ 0: // they don't agree

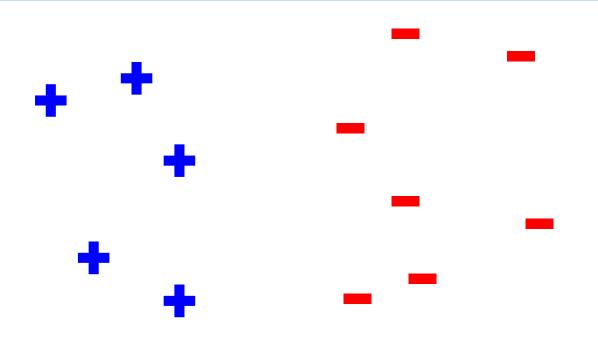
for each w_i:

$$W_i = W_i + f_i^* \text{label}$$

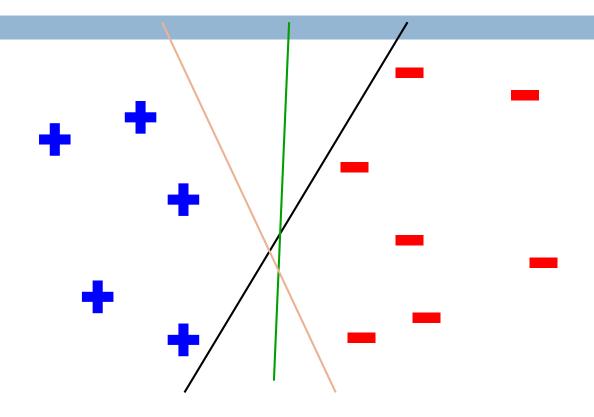


$$W = (-1.5, -1)$$

Which line will it find?



Which line will it find?



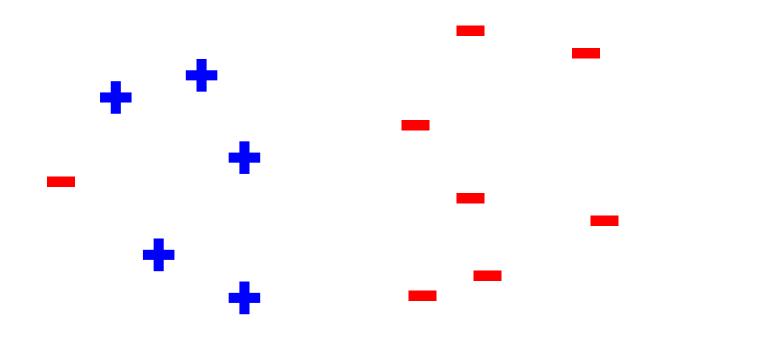
Only guaranteed to find **some** line that separates the data

Convergence

```
repeat until convergence (or for some # of iterations): for each training example (f_1, f_2, ..., f_n, label): prediction = b + \mathring{\triangle}_{i=1}^n w_i f_i if prediction * label \le 0: // they don't agree for each w_i: w_i = w_i + f_i^*label b = b + label
```

Why do we also have the "some # iterations" check?

Handling non-separable data



If we ran the algorithm on this it would never converge!

Convergence

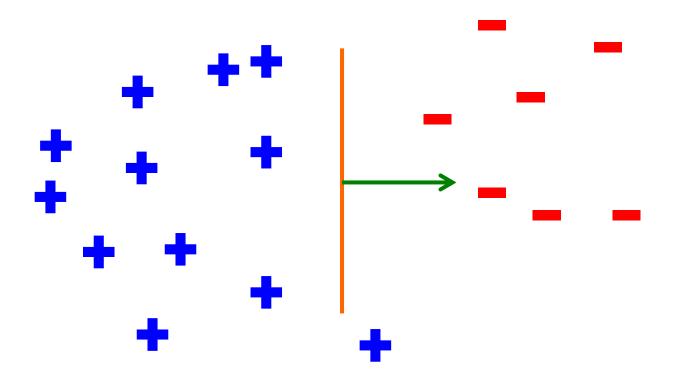
```
repeat until convergence (or for some # of iterations):
  for each training example (f_1, f_2, ..., f_n, label):
    prediction = b + \mathring{a}_{i=1}^{n} w_i f_i
    if prediction * label ≤ 0: // they don't agree
      for each w;
        W_i = W_i + f_i^*label
       b = b + label
```

Ordering

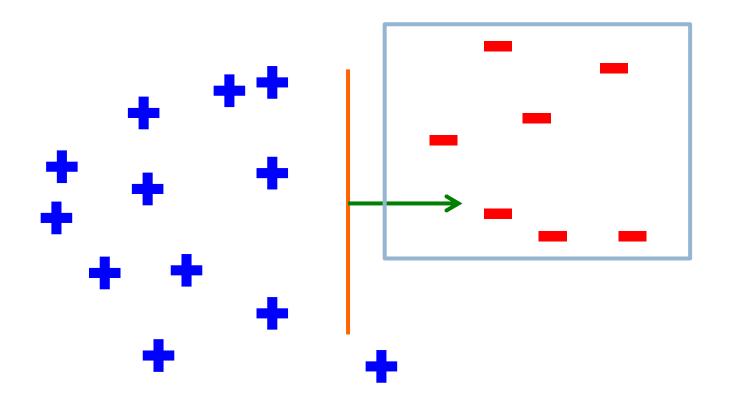
```
repeat until convergence (or for some # of iterations): for each training example (f_1, f_2, ..., f_n, label): prediction = b + \mathring{\triangle}_{i=1}^n w_i f_i if prediction * label \le 0: // they don't agree for each w_i: w_i = w_i + f_i^* label b = b + label
```

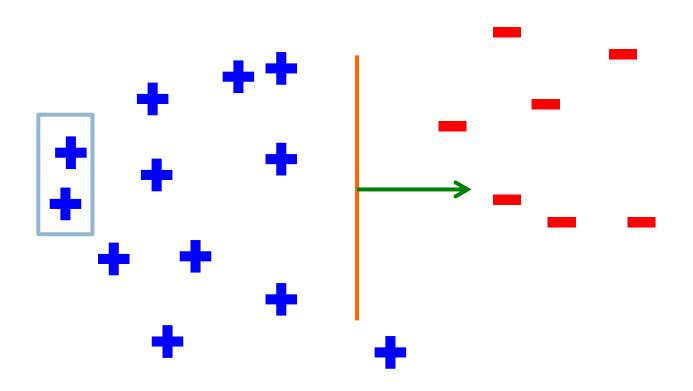
What order should we traverse the examples? Does it matter?

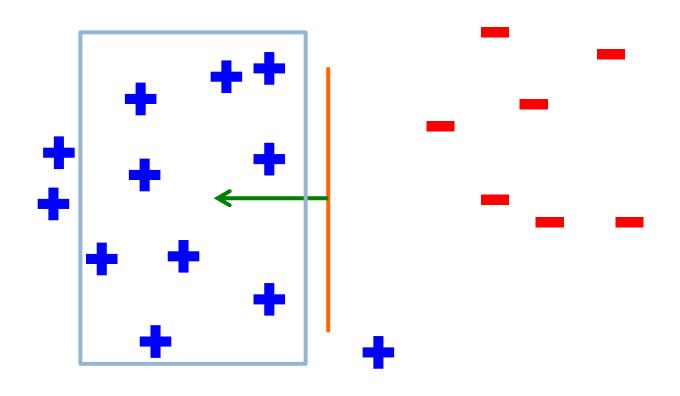
Order matters

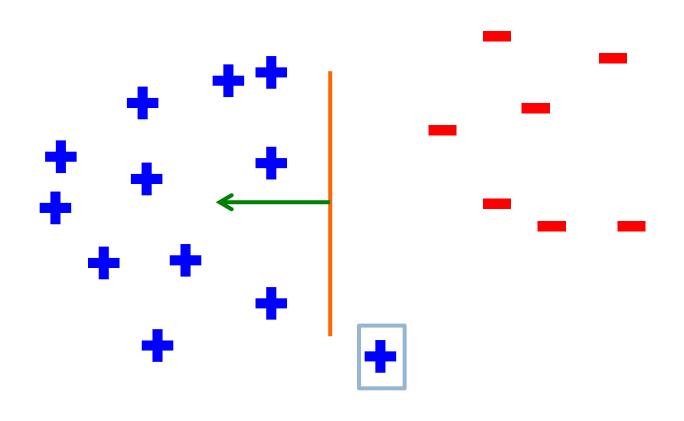


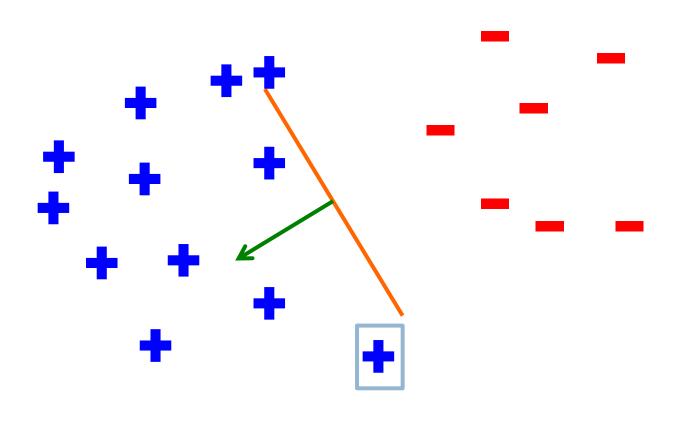
What would be a good/bad order?

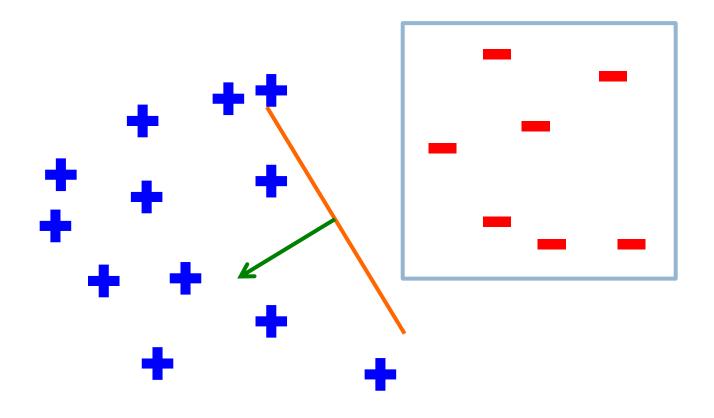










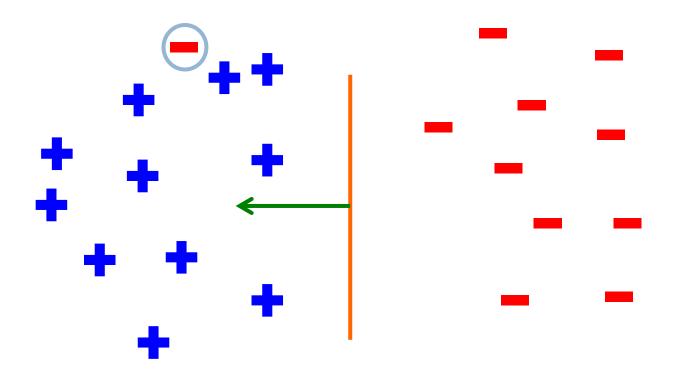


Solution?

Ordering

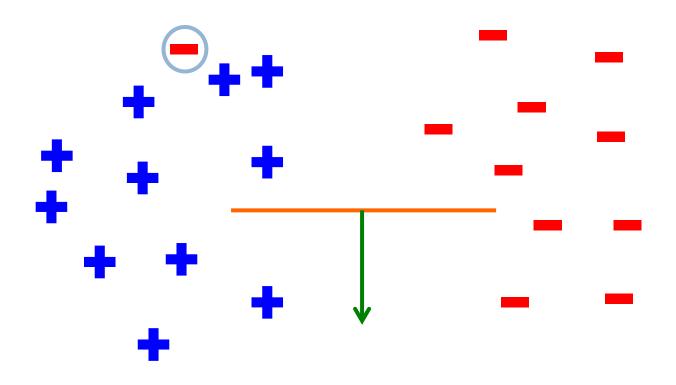
```
repeat until convergence (or for some # of iterations):
  randomize order or training examples
  for each training example (f_1, f_2, ..., f_n, label):
    prediction = b + \mathring{a}_{i=1}^{n} w_i f_i
    if prediction * label ≤ 0: // they don't agree
      for each w;
        W_i = W_i + f_i^*label
      b = b + label
```

Improvements



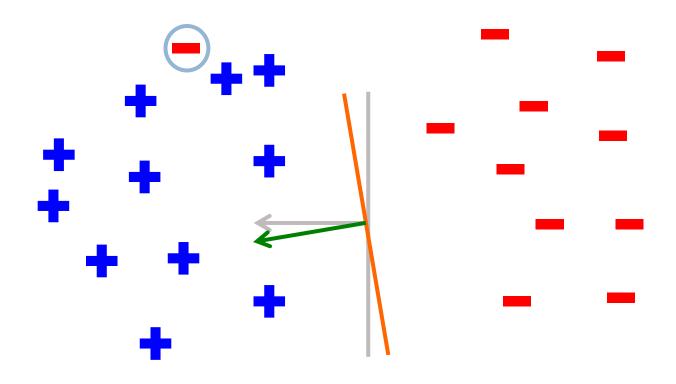
What will happen when we examine this example?

Improvements



Does this make sense? What if we had previously gone through ALL of the other examples correctly?

Improvements



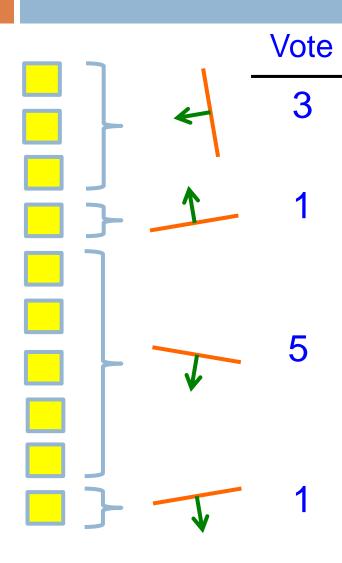
Maybe just move it slightly in the direction of correction

Training

- every time a mistake is made on an example:
 - store the weights (i.e. before changing for current example)
 - store the number of examples that set of weights got correct

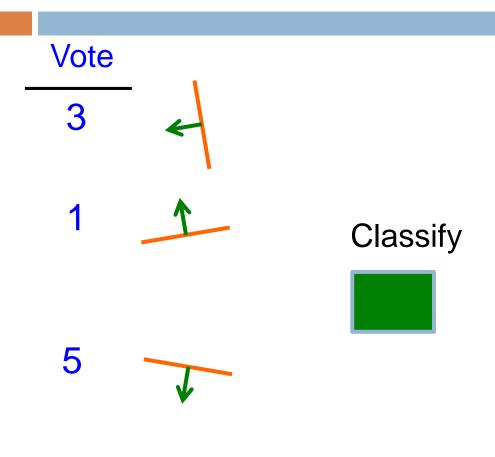
Classify

- calculate the prediction from ALL saved weights
- multiply each prediction by the number it got correct (i.e a weighted vote) and take the sum over all predictions
- said another way: pick whichever prediction has the most votes

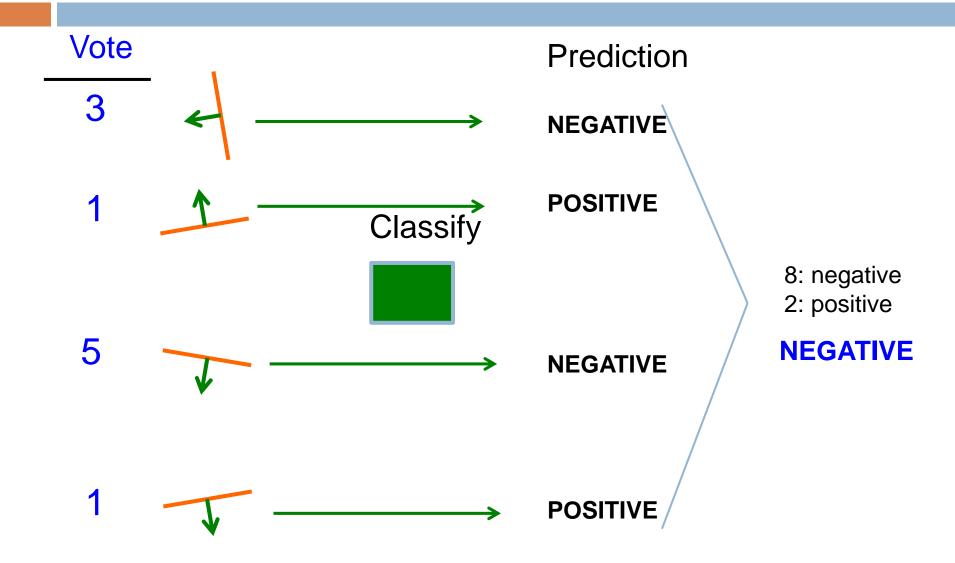


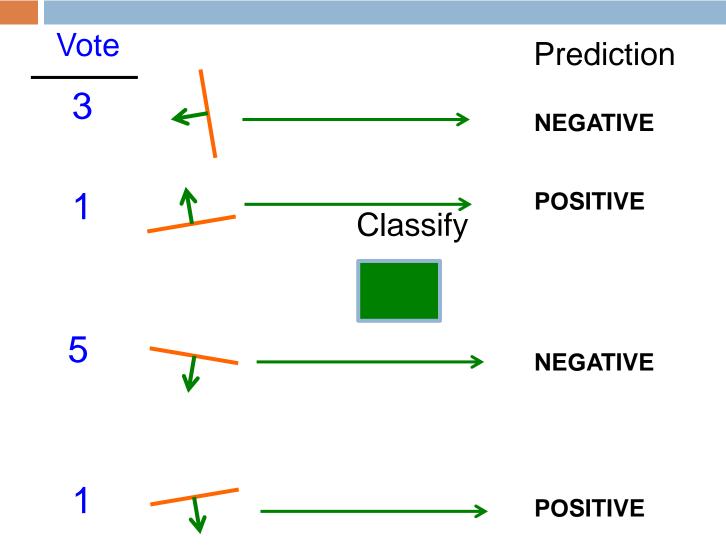
Training every time a mistake is made on an example:

- store the weights
- store the number of examples that set of weights got correct









Works much better in practice

Avoids overfitting, though it can still happen

Avoids big changes in the result by examples examined at the end of training

Voted perceptron learning

Training

- every time a mistake is made on an example:
 - store the weights (i.e. before changing for current example)
 - store the number of examples that set of weights got correct

Classify

- calculate the prediction from ALL saved weights
- multiply each prediction by the number it got correct (i.e a weighted vote) and take the sum over all predictions
- said another way: pick whichever prediction has the most votes

Any issues/concerns?

Voted perceptron learning

Training

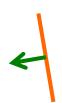
- every time a mistake is made on an example:
 - store the weights (i.e. before changing for current example)
 - store the number of examples that set of weights got correct

Classify

- calculate the prediction from ALL saved weights
- multiply each prediction by the number it got correct (i.e a weighted vote) and take the sum over all predictions
- said another way: pick whichever prediction has the most votes
 - 1. Can require a lot of storage
 - 2. Classifying becomes very, very expensive

Average perceptron

Vote



$$w_1^1, w_2^1, ..., w_n^1, b^1$$



$$w_1^2, w_2^2, ..., w_n^2, b^2$$



$$w_1^3, w_2^3, ..., w_n^3, b^3$$

1
$$w_1^4, w_2^4, ..., w_n^4, b^4$$

$$\frac{1}{w_i} = \frac{3w_i^1 + 1w_i^2 + 5w_i^3 + 1w_i^4}{10}$$

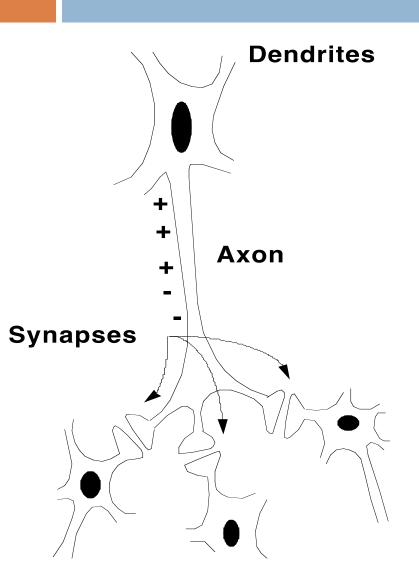
The final weights are the weighted average of the previous weights

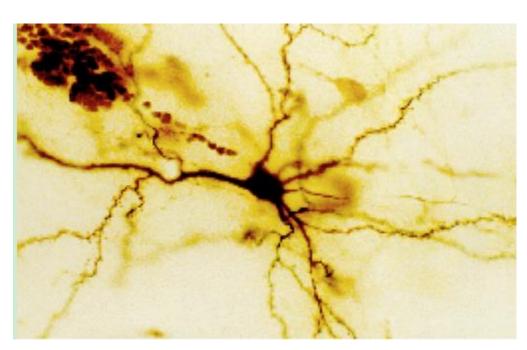
Perceptron learning algorithm

```
repeat until convergence (or for some # of iterations): for each training example (f_1, f_2, ..., f_n, label): prediction = b + \mathring{\triangle}_{i=1}^n w_i f_i if prediction * label \le 0: // they don't agree for each w_i: w_i = w_i + f_i^* label b = b + label
```

Why is it called the "perceptron" learning algorithm if what it learns is a line? Why not "line learning" algorithm?

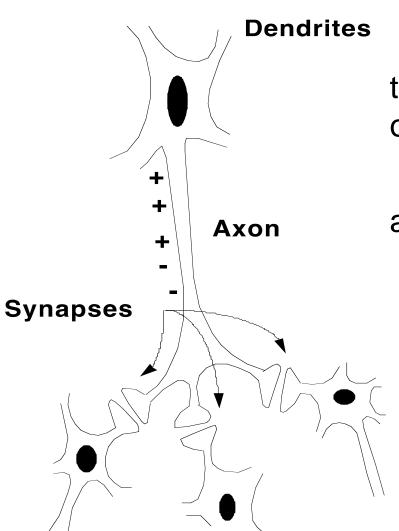
Our Nervous System





Neuron

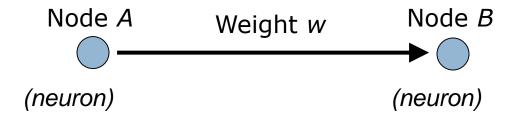
Our nervous system: *the computer science view*



the human brain is a large collection of interconnected neurons

a **NEURON** is a brain cell

- collect, process, and disseminate electrical signals
- Neurons are connected via synapses
- They FIRE depending on the conditions of the neighboring neurons



w is the strength of signal sent between A and B.

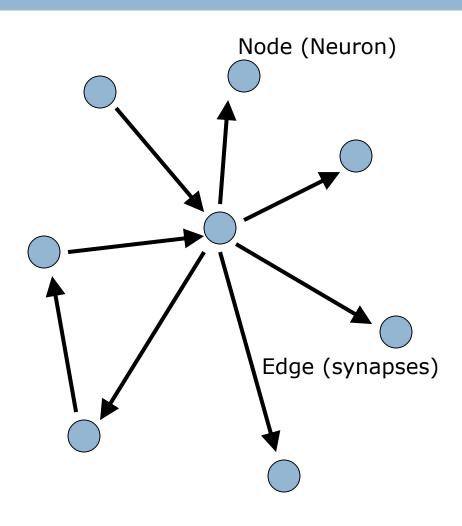
If A fires and w is positive, then A stimulates B.

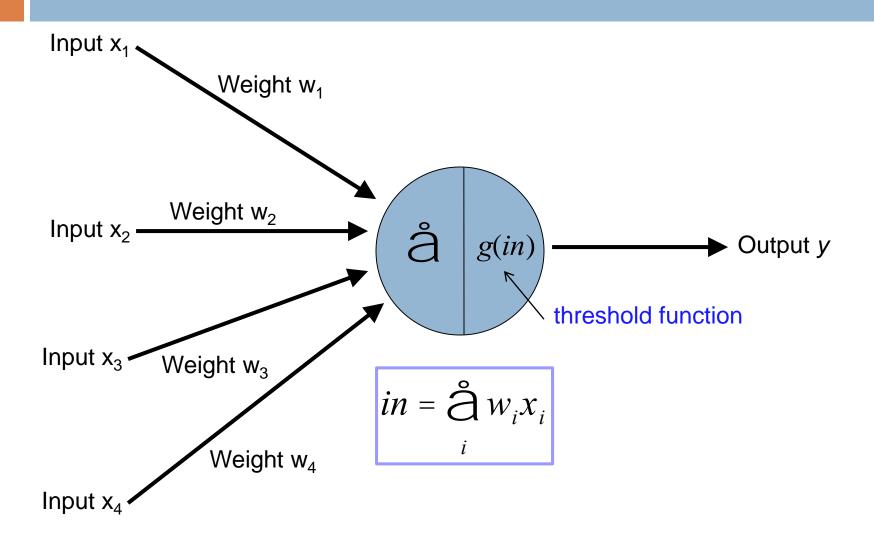
If A fires and w is negative, then A inhibits B.

If a node is stimulated enough, then it also fires.

How much stimulation is required is determined by its threshold.

Neural Networks

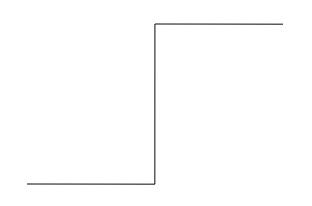




Possible threshold functions

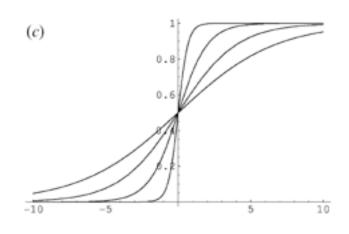
hard threshold:

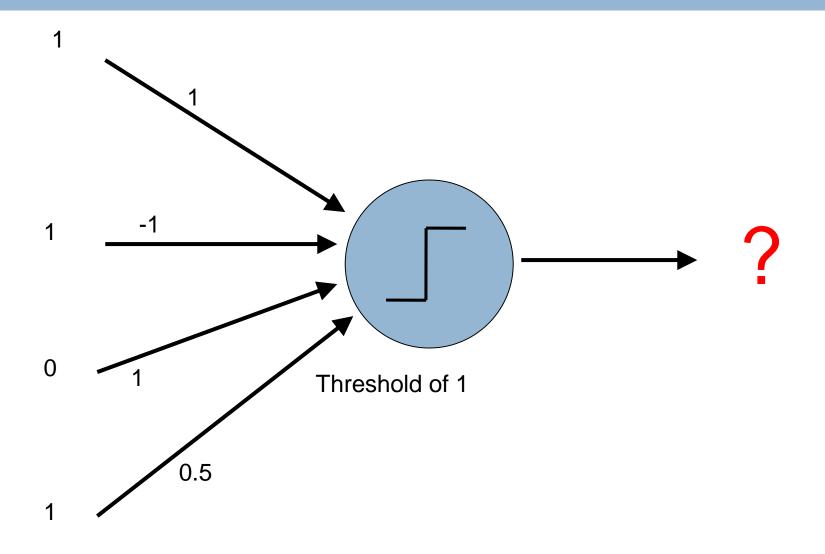
if *in* (the sum of weights) >= *threshold* 1, 0 otherwise

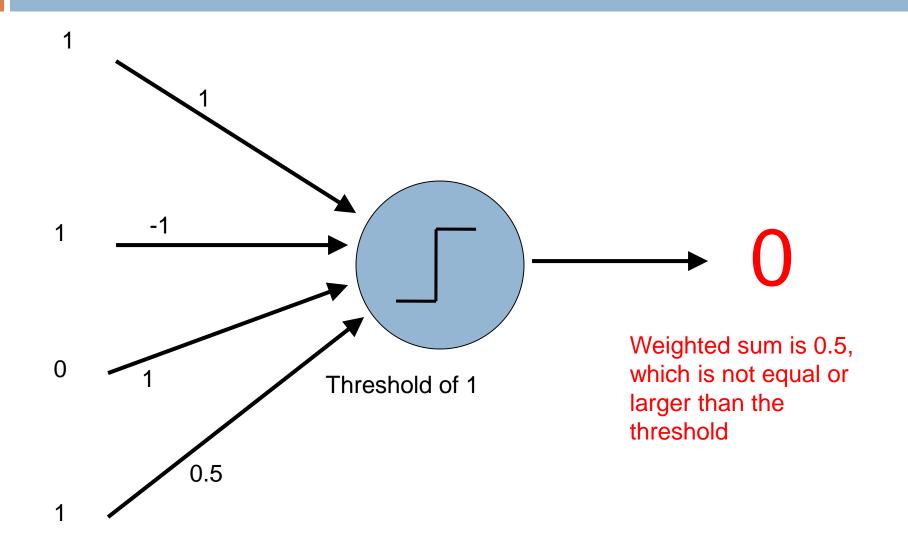


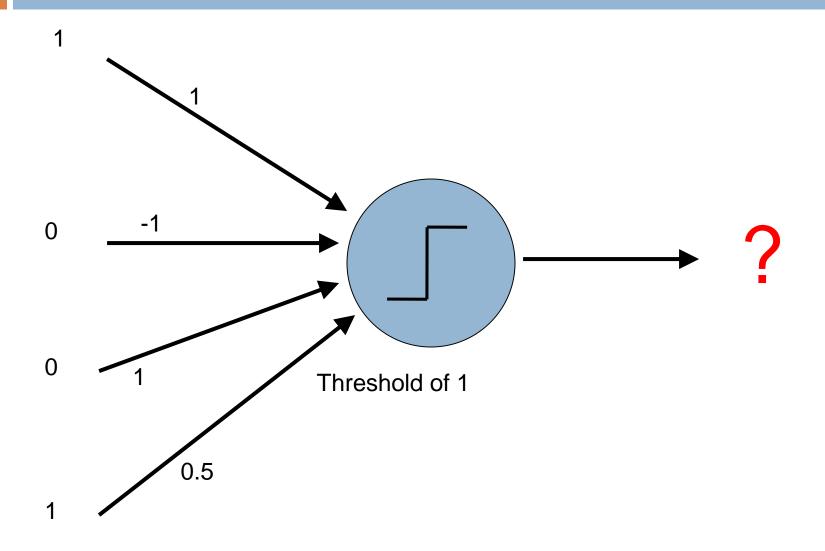
Sigmoid

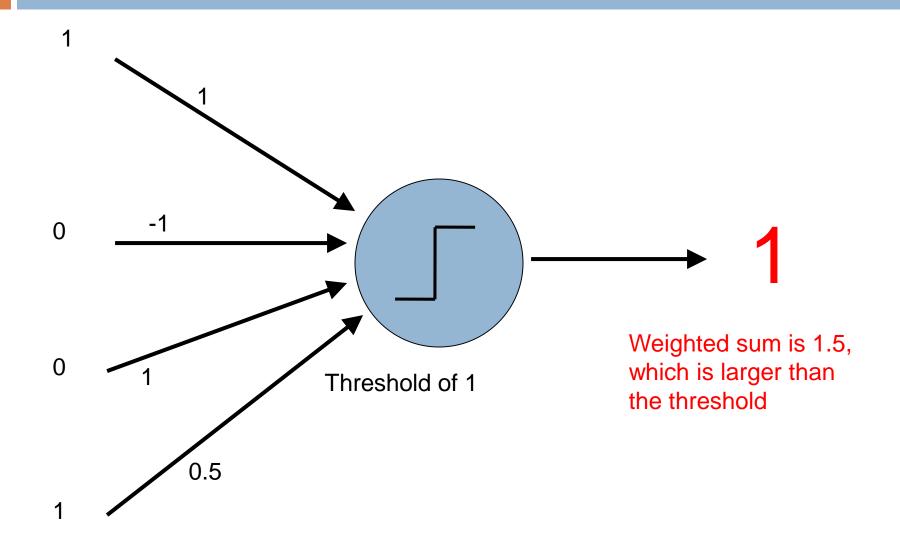
$$g(x) = \frac{1}{1 + e^{-ax}}$$

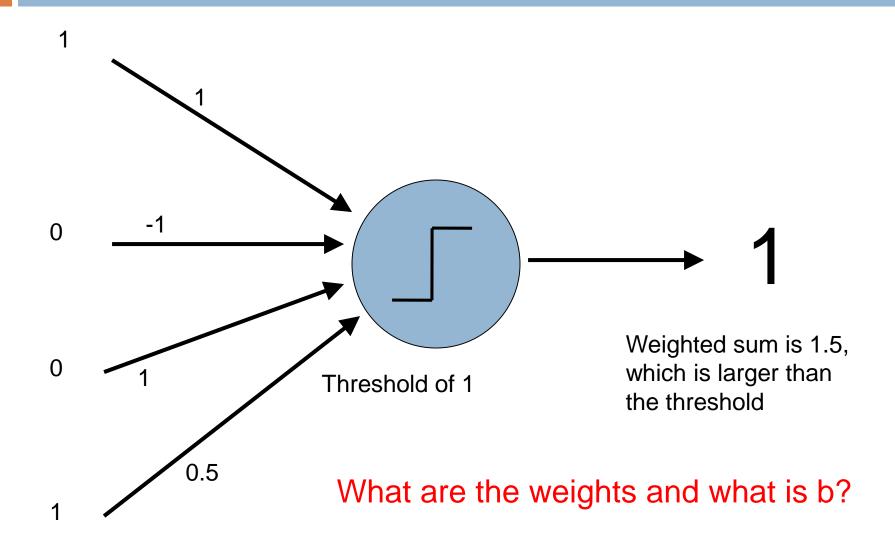












History of Neural Networks

McCulloch and Pitts (1943) – introduced model of artificial neurons and suggested they could learn

Hebb (1949) – Simple updating rule for learning

Rosenblatt (1962) - the *perceptron* model

Minsky and Papert (1969) – wrote *Perceptrons*

Bryson and Ho (1969, but largely ignored until 1980s) – invented back-propagation learning for multilayer networks

Must Watch Videos to Fully Understand

- Part 1 : https://www.youtube.com/watch?v=aircAruvnKk
- Part 2 :
 <u>https://www.youtube.com/watch?v=IHZwWFHWa-w</u>
- Part 3 : https://www.youtube.com/watch?v=Ilg3gGewQ5U
- Part 4: https://www.youtube.com/watch?v=tleHLnjs5U8