# CSE419 – Artificial Intelligence and Machine Learning 2018

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https://github.com/FurkanGozukara/CSE419 2018

# Lecture 3 Decision Trees

Based on Asst. Prof. Dr. David Kauchak (Pomona College) Lecture Slides

# Representing examples

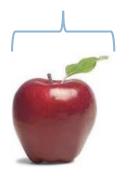
#### examples



What is an example? How is it represented?

### Features

#### examples









#### features

$$f_1, f_2, f_3, ..., f_n$$

How our algorithms actually "view" the data

Features are the questions we can ask about the examples

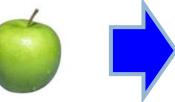
#### Features

#### examples

#### features



red, round, leaf, 3oz, ...



green, round, no leaf, 4oz, .

yellow, curved, no leaf, 4oz,

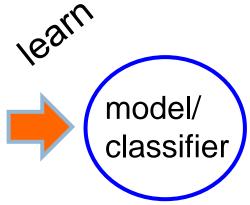
green, curved, no leaf, 5oz, ...

How our algorithms actually "view" the data

Features are the questions we can ask about the examples

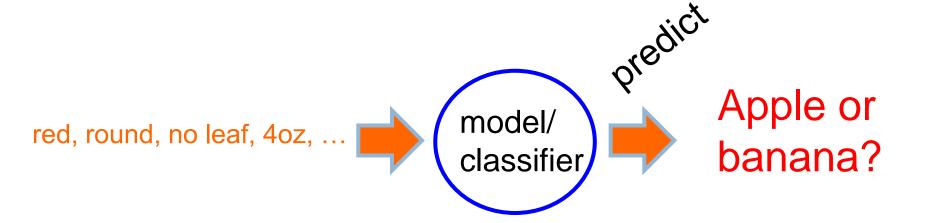
examples label red, round, leaf, 3oz, ... apple green, round, no leaf, 4oz, ... apple

yellow, curved, no leaf, 4oz, ...banana

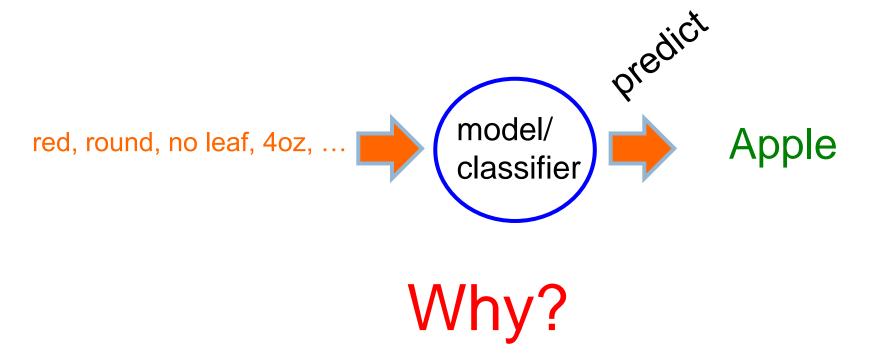


green, curved, no leaf, 5oz, ...banana

During learning/training/induction, learn a model of what distinguishes apples and bananas based on the features



The model can then classify a new example based on the features



The model can then classify a new example based on the features

Training data

Test set

examples

label

red, round, leaf, 3oz, ...

apple

green, round, no leaf, 4oz, ... apple

red, round, no leaf, 4oz, ...?

yellow, curved, no leaf, 4oz, ...banana

green, curved, no leaf, 5oz, ...banana

Training data

Test set

examples

label

red, round, leaf, 3oz, ...

apple

green, round, no leaf, 4oz, ... apple

red, round, no leaf, 4oz, ...?

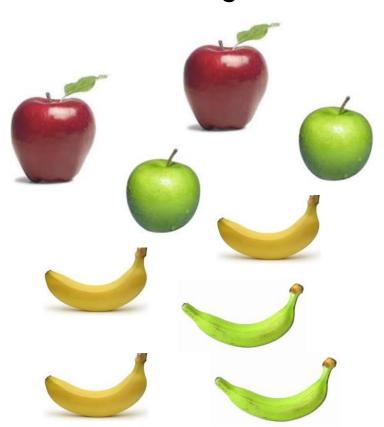
yellow, curved, no leaf, 4oz, ...banana

green, curved, no leaf, 5oz, ...banana

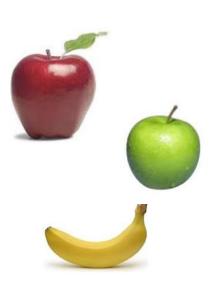
Learning is about generalizing from the training data What does this assume about the training and test set?

## Past predicts future

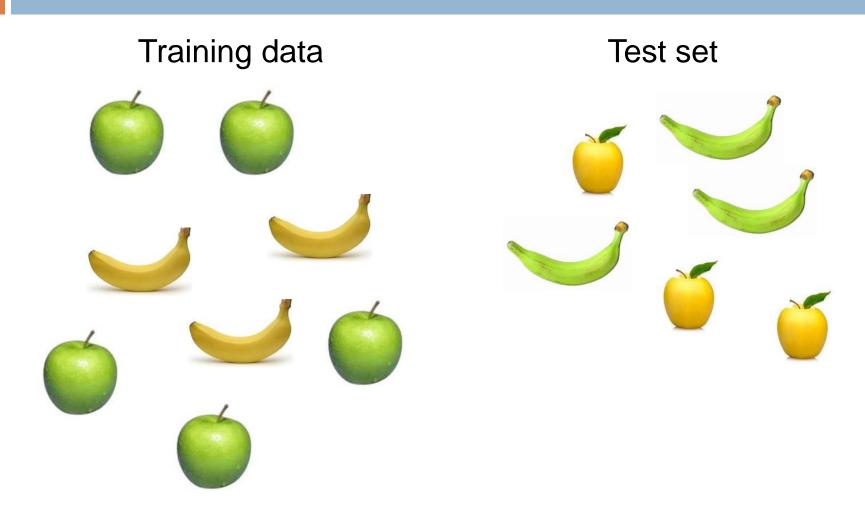
Training data



Test set

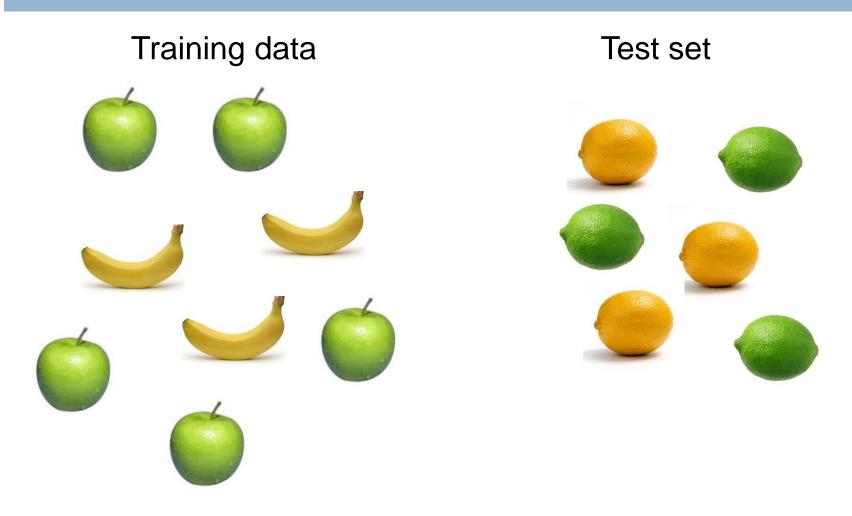


# Past predicts future



Not always the case, but we'll often assume it is!

### Past predicts future



Not always the case, but we'll often assume it is!

# More technically...

We are going to use the *probabilistic model* of learning

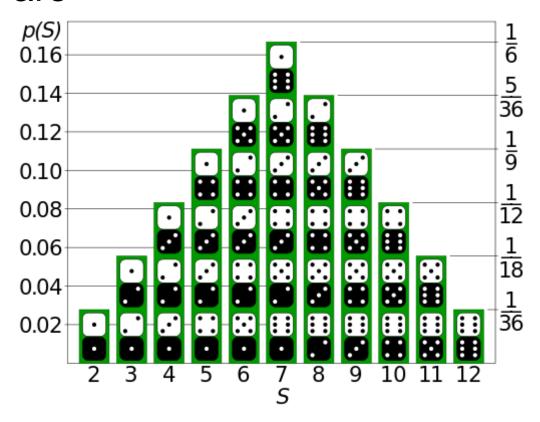
There is some probability distribution over example/label pairs called the *data generating distribution* 

**Both** the training data **and** the test set are generated based on this distribution

What is a probability distribution?

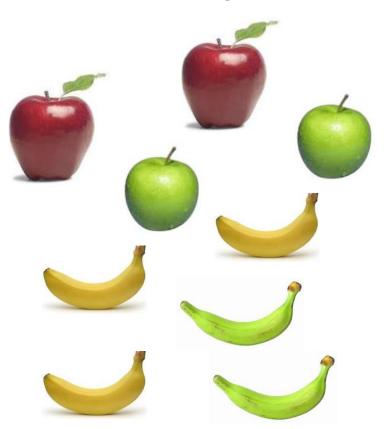
### Probability distribution

Describes how likely (i.e. probable) certain events are



### Probability distribution

#### Training data



#### High probability

round apples

curved bananas

apples with leaves

. . .

#### Low probability

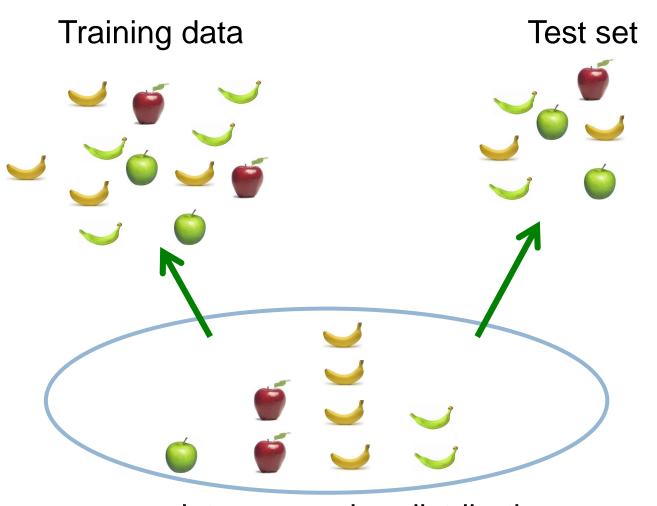
curved apples

red bananas

yellow apples

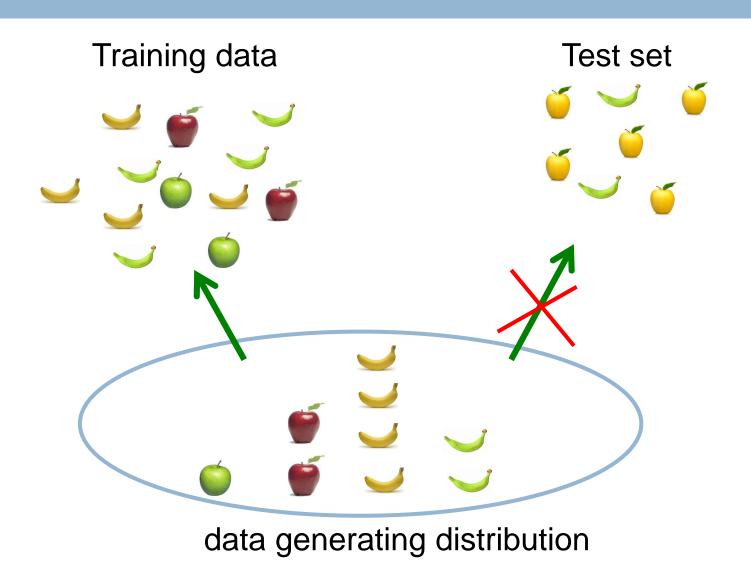
. . .

# data generating distribution

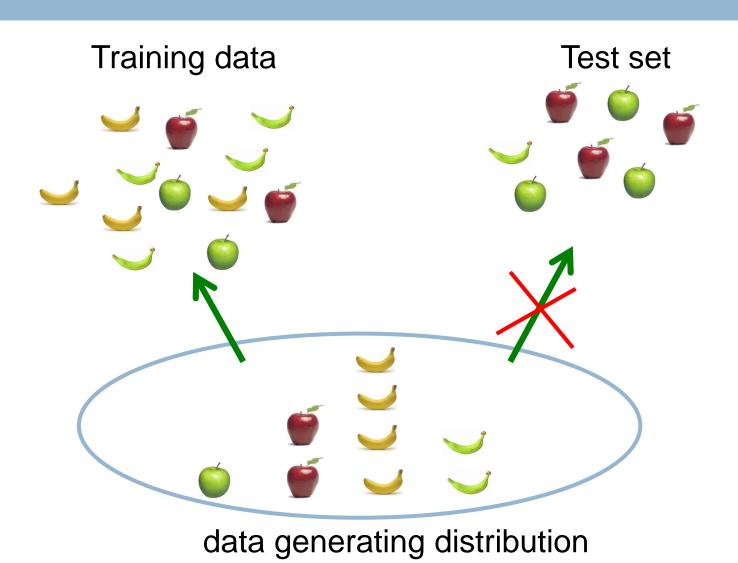


data generating distribution

# data generating distribution



# data generating distribution



# To ride or not to ride, that is the question...

Terrain	Unicycle- type	Weather	Go-For- Ride?
Trail	Normal	Rainy	NO
Road	Normal	Sunny	YES
Trail	Mountain	Sunny	YES
Road	Mountain	Rainy	YES
Trail	Normal	Snowy	NO
Road	Normal	Rainy	YES
Road	Mountain	Snowy	YES
Trail	Normal	Sunny	NO
Road	Normal	Snowy	NO
Trail	Mountain	Snowy	YES

Build a decision tree

### Recursive approach

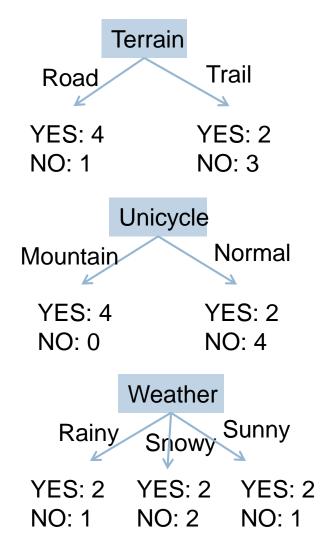
Base case: If all data belong to the same class, create a leaf node with that label

#### Otherwise:

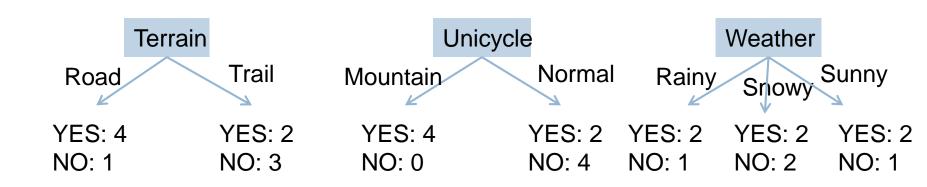
- calculate the "score" for each feature if we used it to split the data
- pick the feature with the highest score, partition the data based on that data value and call recursively

# Partitioning the data

Terrain	Unicycle- type	Weather	Go-For- Ride?
Trail	Normal	Rainy	NO
Road	Normal	Sunny	YES
Trail	Mountain	Sunny	YES
Road	Mountain	Rainy	YES
Trail	Normal	Snowy	NO
Road	Normal	Rainy	YES
Road	Mountain	Snowy	YES
Trail	Normal	Sunny	NO
Road	Normal	Snowy	NO
Trail	Mountain	Snowy	YES



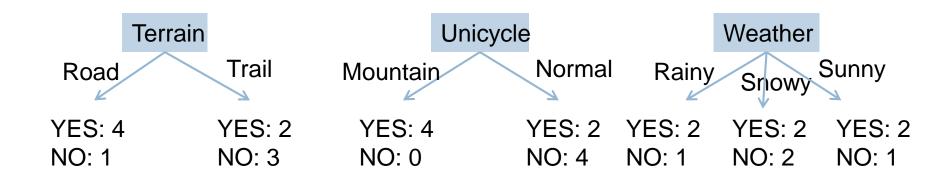
# Partitioning the data



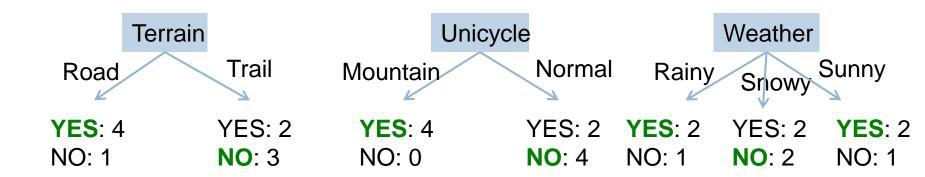
calculate the "score" for each feature if we used it to split the data

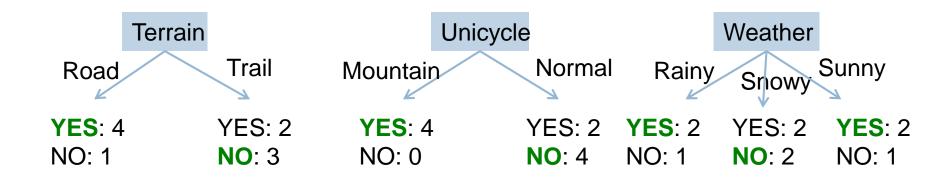
What score should we use?

If we just stopped here, which tree would be best? How could we make these into decision trees?



How could we make these into decision trees?

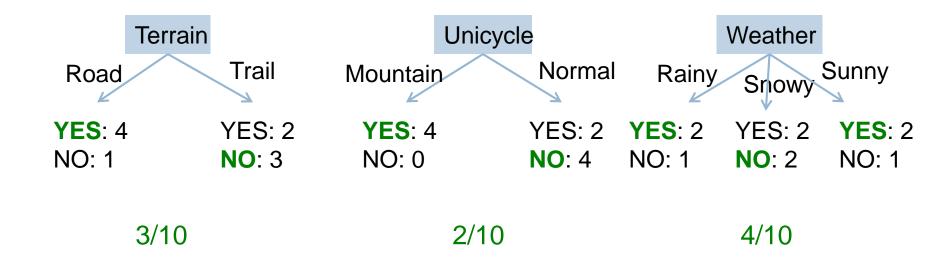




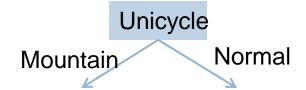
Training error: the average error over the training set

For classification, the most common "error" is the number of mistakes

Training error for each of these?



Training error: the average error over the training set



YES: 4

NO: 0

YES: 2

NO: 4

Terrain	Unicycle- type	Weather	Go-For- Ride?
Trail	Mountain	Sunny	YES
Road	Mountain	Rainy	YES
Road	Mountain	Snowy	YES
Trail	Mountain	Snowy	YES

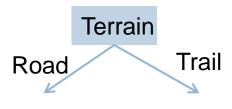
Terrain	Unicycle- type	Weather	Go-For- Ride?
Trail	Normal	Rainy	NO
Road	Normal	Sunny	YES
Trail	Normal	Snowy	NO
Road	Normal	Rainy	YES
Trail	Normal	Sunny	NO
Road	Normal	Snowy	NO

Unicycle Mountain Normal

**YES**: 4 YES: 2

NO: 0 NO: 4

Terrain	Unicycle- type	Weather	Go-For- Ride?
Trail	Normal	Rainy	NO
Road	Normal	Sunny	YES
Trail	Normal	Snowy	NO
Road	Normal	Rainy	YES
Trail	Normal	Sunny	NO
Road	Normal	Snowy	NO



YES: 2 YES: 0 NO: 1 NO: 3

Weather
Rainy Snowy Sunny

YES: 1 YES: 0 YES: 1

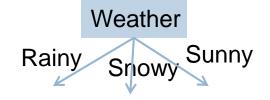
NO: 1 NO: 2 NO: 1

Unicycle Mountain Normal

**YES**: 4 YES: 2 NO: 4

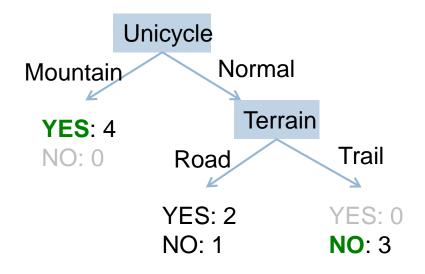
**Terrain** Unicycle-Weather Go-For-Ride? type Normal Trail Rainy NO **Normal** Road YES Sunny Trail Normal Snowy NO Road Normal Rainy YES Trail Normal Sunny NO Road Normal Snowy NO

Terrain
Road Trail
YES: 2
NO: 1
NO: 3

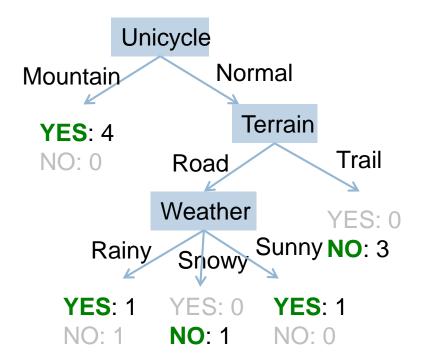


**YES**: 1 YES: 0 **YES**: 1 NO: 1 **NO**: 2 NO: 1

2/6



Terrain	Unicycle- type	Weather	Go-For- Ride?
Road	Normal	Sunny	YES
Road	Normal	Rainy	YES
Road	Normal	Snowy	NO



# Building decision trees

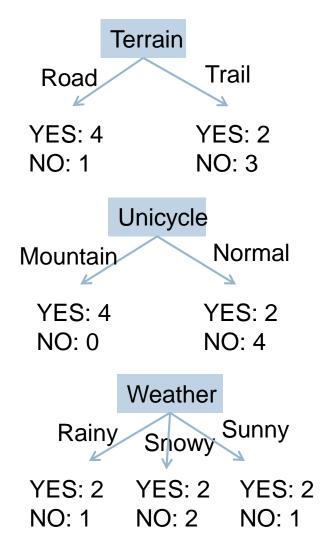
Base case: If all data belong to the same class, create a leaf node with that label

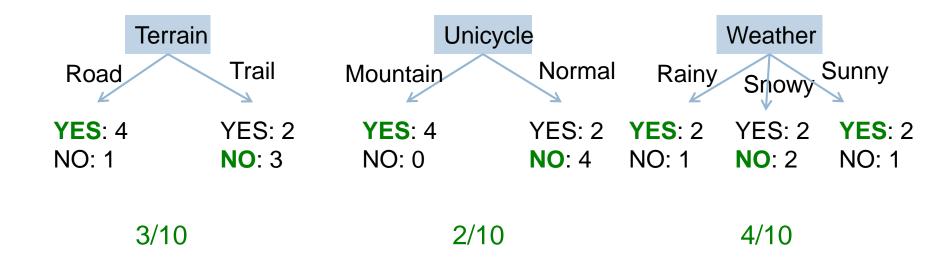
#### Otherwise:

- calculate the "score" for each feature if we used it to split the data
- pick the feature with the highest score, partition the data based on that data value and call recursively

# Partitioning the data

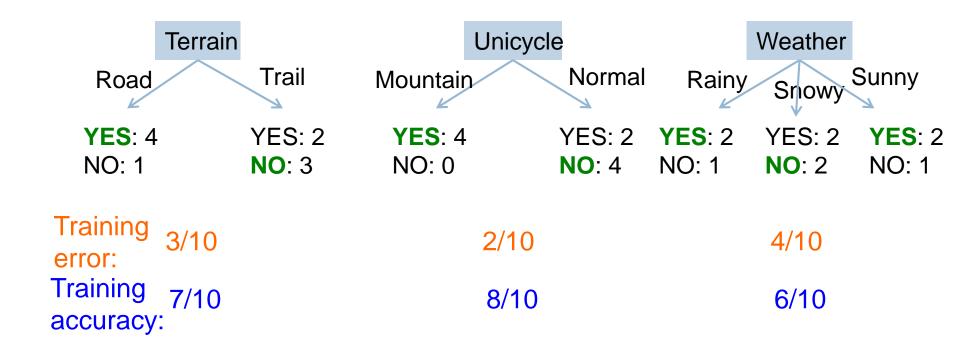
Terrain	Unicycle- type	Weather	Go-For- Ride?
Trail	Normal	Rainy	NO
Road	Normal	Sunny	YES
Trail	Mountain	Sunny	YES
Road	Mountain	Rainy	YES
Trail	Normal	Snowy	NO
Road	Normal	Rainy	YES
Road	Mountain	Snowy	YES
Trail	Normal	Sunny	NO
Road	Normal	Snowy	NO
Trail	Mountain	Snowy	YES





Training error: the average error over the training set

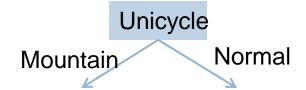
### Training error vs. accuracy



training error = 1-accuracy (and vice versa)

Training error: the average error over the training set

Training accuracy: the average percent correct over the training set



YES: 4

NO: 0

YES: 2

NO: 4

Terrain	Unicycle- type	Weather	Go-For- Ride?
Trail	Mountain	Sunny	YES
Road	Mountain	Rainy	YES
Road	Mountain	Snowy	YES
Trail	Mountain	Snowy	YES

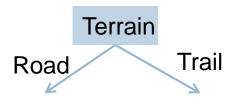
Terrain	Unicycle- type	Weather	Go-For- Ride?
Trail	Normal	Rainy	NO
Road	Normal	Sunny	YES
Trail	Normal	Snowy	NO
Road	Normal	Rainy	YES
Trail	Normal	Sunny	NO
Road	Normal	Snowy	NO

Unicycle Mountain Normal

**YES**: 4 YES: 2

NO: 0 NO: 4

Terrain	Unicycle- type	Weather	Go-For- Ride?
Trail	Normal	Rainy	NO
Road	Normal	Sunny	YES
Trail	Normal	Snowy	NO
Road	Normal	Rainy	YES
Trail	Normal	Sunny	NO
Road	Normal	Snowy	NO



YES: 2 YES: 0 NO: 1 NO: 3

Weather
Rainy Snowy Sunny

YES: 1 YES: 0 YES: 1

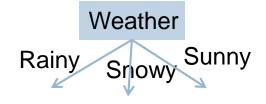
NO: 1 NO: 2 NO: 1

Unicycle Mountain Normal

**YES**: 4 YES: 2 NO: 4

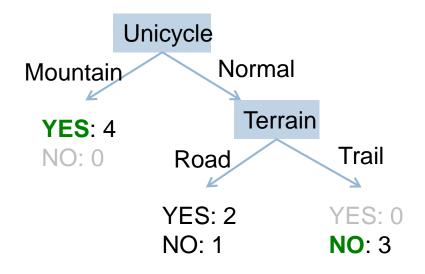
**Terrain** Unicycle-Weather Go-For-Ride? type Normal Trail Rainy NO **Normal** Road YES Sunny Trail Normal Snowy NO Road Normal Rainy YES Trail Normal Sunny NO Road Normal Snowy NO

Terrain
Road Trail
YES: 2
NO: 1
NO: 3

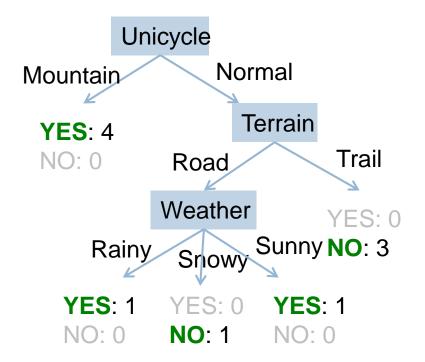


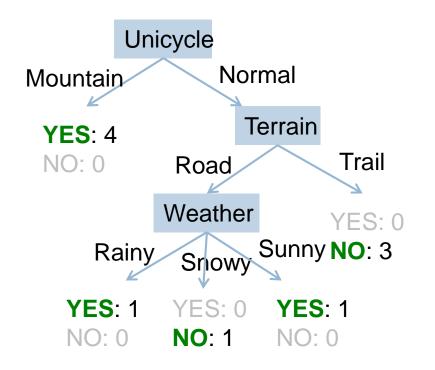
**YES**: 1 YES: 0 **YES**: 1 NO: 1 **NO**: 2 NO: 1

2/6



Terrain	Unicycle- type	Weather	Go-For- Ride?
Road	Normal	Sunny	YES
Road	Normal	Rainy	YES
Road	Normal	Snowy	NO





Terrain	Unicycle- type	Weather	Go-For- Ride?
Trail	Normal	Rainy	NO
Road	Normal	Sunny	YES
Trail	Mountain	Sunny	YES
Road	Mountain	Rainy	YES
Trail	Normal	Snowy	NO
Road	Normal	Rainy	YES
Road	Mountain	Snowy	YES
Trail	Normal	Sunny	NO
Road	Normal	Snowy	NO
Trail	Mountain	Snowy	YES

Are we always guaranteed to get a training error of 0?

Training error?

### Problematic data

Terrain	Unicycle- type	Weather	Go-For- Ride?
Trail	Normal	Rainy	NO
Road	Normal	Sunny	YES
Trail	Mountain	Sunny	YES
Road	Mountain	Snowy	NO
Trail	Normal	Snowy	NO
Road	Normal	Rainy	YES
Road	Mountain	Snowy	YES
Trail	Normal	Sunny	NO
Road	Normal	Snowy	NO
Trail	Mountain	Snowy	YES

When can this happen?

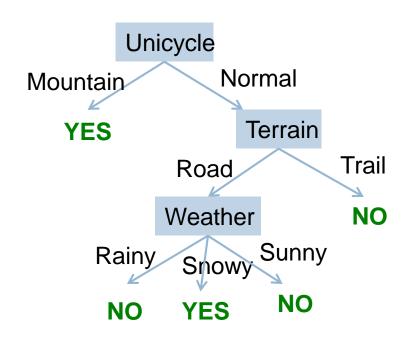
## Recursive approach

Base case: If all data belong to the same class, create a leaf node with that label *OR* all the data has the same feature values

Do we always want to go all the way to the bottom?

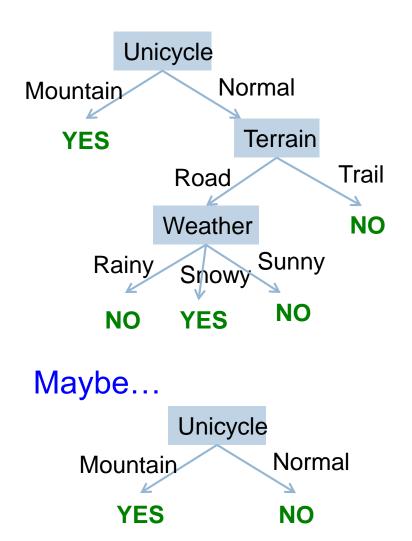
Terrain	Unicycle- type	Weather	Go-For- Ride?
Trail	Mountain	Rainy	YES
Trail	Mountain	Sunny	YES
Road	Mountain	Snowy	YES
Road	Mountain	Sunny	YES
Trail	Normal	Snowy	NO
Trail	Normal	Rainy	NO
Road	Normal	Snowy	YES
Road	Normal	Sunny	NO
Trail	Normal	Sunny	NO

Terrain	Unicycle- type	Weather	Go-For- Ride?
Trail	Mountain	Rainy	YES
Trail	Mountain	Sunny	YES
Road	Mountain	Snowy	YES
Road	Mountain	Sunny	YES
Trail	Normal	Snowy	NO
Trail	Normal	Rainy	NO
Road	Normal	Snowy	YES
Road	Normal	Sunny	NO
Trail	Normal	Sunny	NO



Is that what you would do?

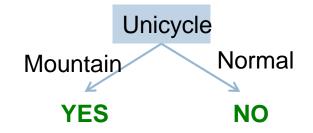
Terrain	Unicycle- type	Weather	Go-For- Ride?
Trail	Mountain	Rainy	YES
Trail	Mountain	Sunny	YES
Road	Mountain	Snowy	YES
Road	Mountain	Sunny	YES
Trail	Normal	Snowy	NO
Trail	Normal	Rainy	NO
Road	Normal	Snowy	YES
Road	Normal	Sunny	NO
Trail	Normal	Sunny	NO



Terrain	Unicycle-type	Weather	Jacket	ML grade	Go-For-Ride?
Trail	Mountain	Rainy	Heavy	D	YES
Trail	Mountain	Sunny	Light	C-	YES
Road	Mountain	Snowy	Light	В	YES
Road	Mountain	Sunny	Heavy	Α	YES
	Mountain				YES
Trail	Normal	Snowy	Light	D+	NO
Trail	Normal	Rainy	Heavy	B-	NO
Road	Normal	Snowy	Heavy	C+	YES
Road	Normal	Sunny	Light	A-	NO
Trail	Normal	Sunny	Heavy	B+	NO
Trail	Normal	Snowy	Light	F	NO
	Normal				NO
Trail	Normal	Rainy	Light	С	YES

# Overfitting

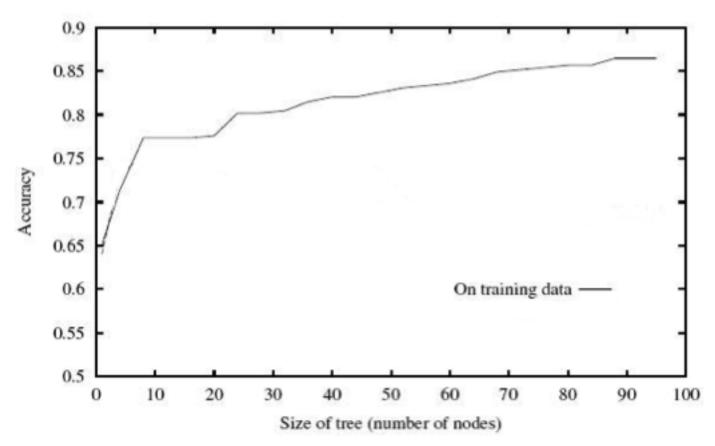
Terrain	Unicycle- type	Weather	Go-For- Ride?
Trail	Mountain	Rainy	YES
Trail	Mountain	Sunny	YES
Road	Mountain	Snowy	YES
Road	Mountain	Sunny	YES
Trail	Normal	Snowy	NO
Trail	Normal	Rainy	NO
Road	Normal	Snowy	YES
Road	Normal	Sunny	NO
Trail	Normal	Sunny	NO



Overfitting occurs when we bias our model too much towards the training data

Our goal is to learn a **general** model that will work on the training data as well as other data (i.e. test data)

# Overfitting

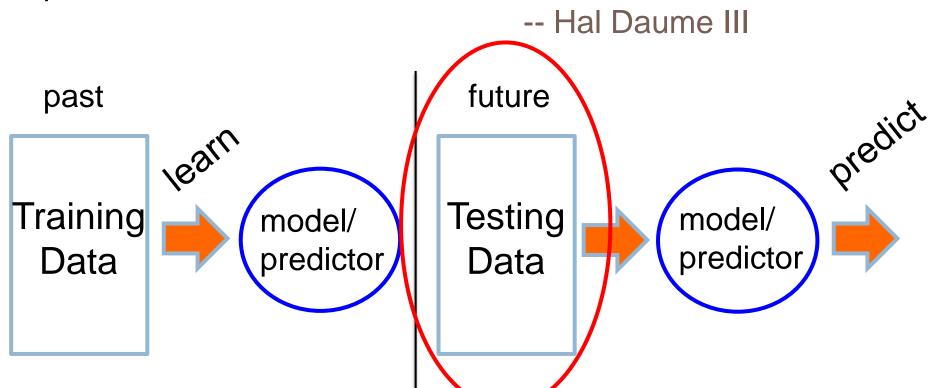


Our decision tree learning procedure always decreases training error

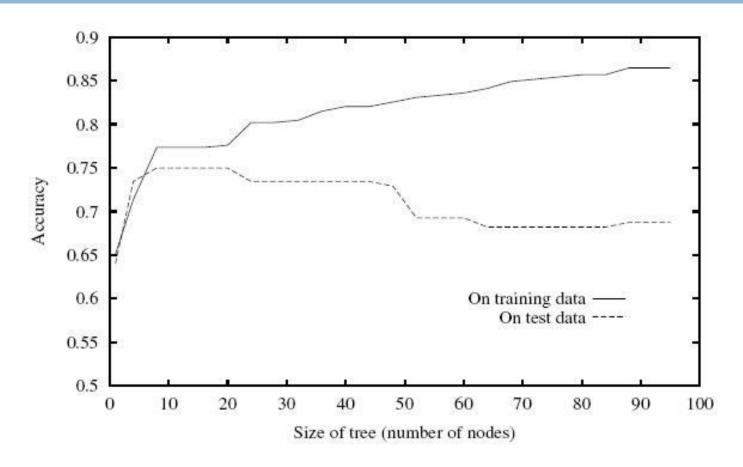
Is that what we want?

#### Test set error!

Machine learning is about predicting the future based on the past.



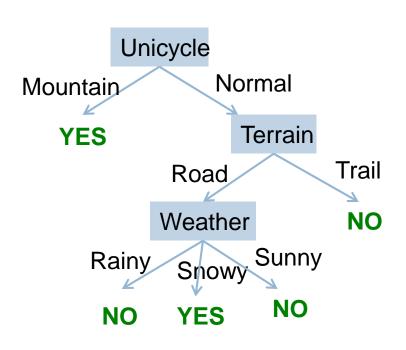
# Overfitting



Even though the training error is decreasing, the testing error can go up!

# Overfitting

Terrain	Unicycle- type	Weather	Go-For- Ride?
Trail	Mountain	Rainy	YES
Trail	Mountain	Sunny	YES
Road	Mountain	Snowy	YES
Road	Mountain	Sunny	YES
Trail	Normal	Snowy	NO
Trail	Normal	Rainy	NO
Road	Normal	Snowy	YES
Road	Normal	Sunny	NO
Trail	Normal	Sunny	NO



How do we prevent overfitting?

# Preventing overfitting

Base case: If all data belong to the same class, create a leaf node with that label *OR* all the data has the same feature values *OR* 

- We've reached a particular depth in the tree
- ?

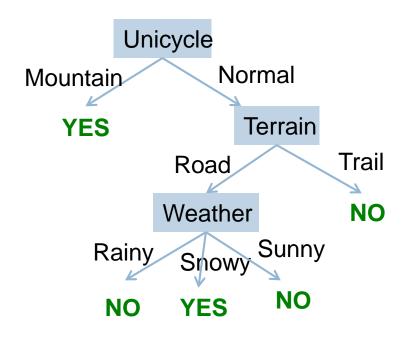
One idea: stop building the tree early

# Preventing overfitting

Base case: If all data belong to the same class, create a leaf node with that label *OR* all the data has the same feature values *OR* 

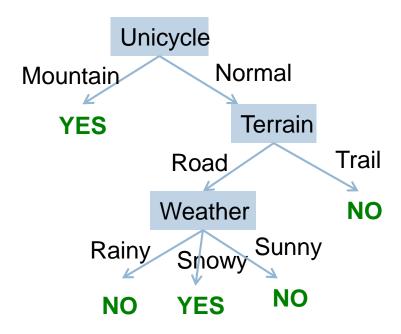
- We've reached a particular depth in the tree
- We only have a certain number/fraction of examples remaining
- We've reached a particular training error
- Use development data (more on this later)

- ...

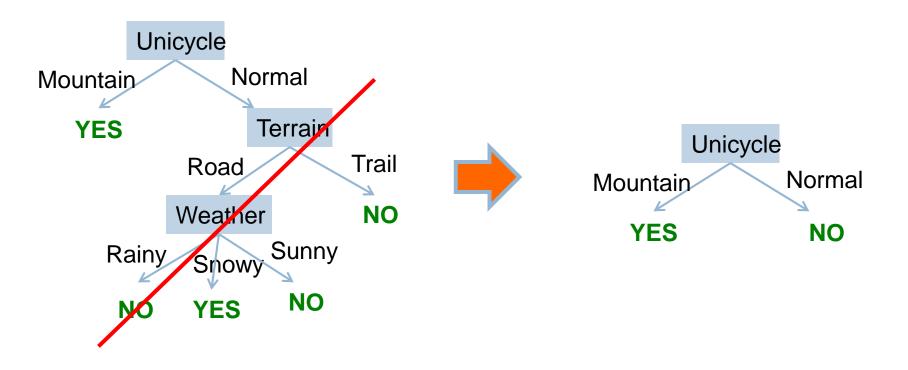


Pruning: after the tree is built, go back and "prune" the tree, i.e. remove some lower parts of the tree

Similar to stopping early, but done after the entire tree is built

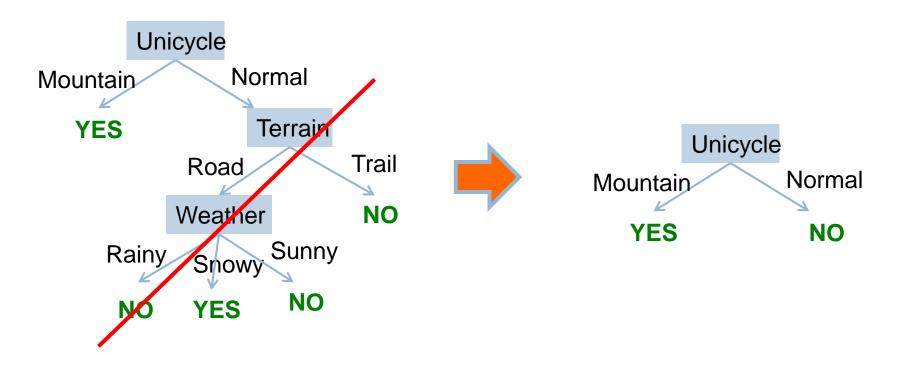


Build the full tree



Build the full tree

Prune back leaves that are too specific



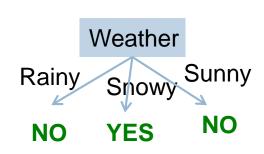
Pruning criterion?

# Handling non-binary attributes

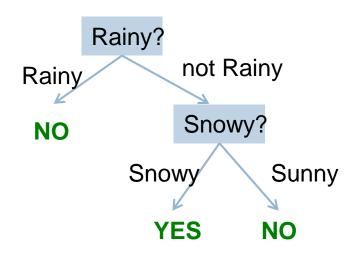
PassengerId	Polass	Sex	Age	SibSp	Parch	Ticket	Fare	Embarked	Survived
804	3			•					
					1				1
756	2	0	0.07		1	L 25064			1
470	3	1	0.75	5 2	1	L 266	5 19.2583	0	1
645	3	1	0.75	2	1	L 266	5 19.2583	0	1
79	2	0	0.83	0	2	24873	3 29	2	1
832	2	0	0.83	1	1	L 2910	5 18.75	2	1
306	1	0	0.92	. 1	2	11378	1 151.55	2	1
165	3	0	1	L 4	1	310129	39.6875	2	0
173	3	1	. 1	1	1	L 34774	2 11.1333	2	1
184	2	0	1	L 2	1	23013	5 39	2	1
382	3	1	. 1	L O	2	265	3 15.7417	0	1
387	3	0	1	L 5	2	2 214	46.9	2	0
789	3	0	1	1	2	2 231	20.575	2	1
828	2	0	1	L O	2	2079	37.0042	0	1
8	3	0	2	2 3	1	L 34990	9 21.075	2	0
17	3	0	2	2 4	1	l 38265	2 29.125	1	0
120	3	1	. 2	2 4	2	34708	2 31.275	2	0
206	3	1	. 2	2 0	1	L 34705	10.4625	2	0
298	1	1	. 2	2 1	2	11378	1 151.55	2	0
341	2	0	2	2 1	1	L 23008	26	2	1
480	3	1	. 2	2 0	1	310129	3 12.2875	2	1

What do we do with features that have multiple values? Real-values?

### Features with multiple values



Treat as an n-ary split

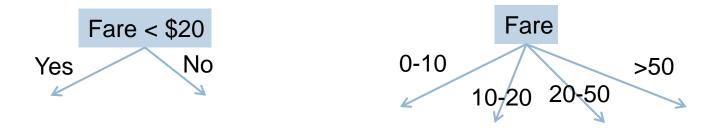


Treat as multiple binary splits

### Real-valued features

Use any comparison test (>, <, ≤, ≥) to split the data into two parts

Select a range filter, i.e. min < value < max



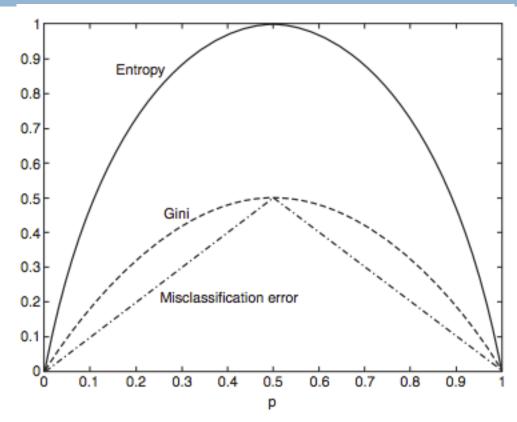
# Other splitting criterion

#### Otherwise:

- calculate the "score" for each feature if we used it to split the data
- pick the feature with the highest score, partition the data based on that data value and call recursively

We used training error for the score. Any other ideas?

# Other splitting criterion



- Entropy: how much uncertainty there is in the distribution over labels after the split
- Gini: sum of the square of the label proportions after split
- Training error = misclassification error

### Decision trees

#### Good? Bad?



### Decision trees: the good

Very intuitive and easy to interpret

Fast to run and fairly easy to implement

Historically, perform fairly well (especially with a few more tricks we'll see later on)

No prior assumptions about the data

#### Decision trees: the bad

#### Be careful with features with lots of values

Which feature would be at the top here?

ID	Terrain	Unicycle- type	Weather	Go-For- Ride?
1	Trail	Normal	Rainy	NO
2	Road	Normal	Sunny	YES
3	Trail	Mountain	Sunny	YES
4	Road	Mountain	Rainy	YES
5	Trail	Normal	Snowy	NO
6	Road	Normal	Rainy	YES
7	Road	Mountain	Snowy	YES
8	Trail	Normal	Sunny	NO
9	Road	Normal	Snowy	NO
10	Trail	Mountain	Snowy	YES

### Decision trees: the bad

Can be problematic (slow, bad performance) with large numbers of features

Can't learn some very simple data sets (e.g. some types of linearly separable data)

Pruning/tuning can be tricky to get right

# Final DT algorithm

#### Base cases:

- If all data belong to the same class, pick that label
- If all the data have the same feature values, pick majority label
- If we're out of features to examine, pick majority label
- If the we don't have any data left, pick majority label of *parent*
- 5. If some other stopping criteria exists to avoid overfitting, pick majority label

#### Otherwise:

- calculate the "score" for each feature if we used it to split the data
- pick the feature with the highest score, partition the data based on that data value and call recursively

# Alternative Score Calculation – Information Gain

- Why information gain is better over accuracy?
  - Decision trees are generally prone to over-fitting and accuracy doesn't generalize well to unseen data.
  - One advantage of information gain is that -- due to the factor ¬p\*log(p) in the entropy definition -- leafs with a small number of instances are assigned less weight (limp→0+p\*log(p)=0) and it favors dividing data into bigger but homogeneous groups.
  - This approach is usually more stable and also chooses the most impactful features close to the root of the tree.

#### **Decision Tree - Classification**

Decision tree builds classification or regression models in the form of a tree structure. It breaks down a dataset into smaller and smaller subsets while at the same time an associated decision tree is incrementally developed. The final result is a tree with **decision nodes** and **leaf nodes**. A decision node (e.g., Outlook) has two or more branches (e.g., Sunny, Overcast and Rainy). Leaf node (e.g., Play) represents a classification or decision. The topmost decision node in a tree which corresponds to the best predictor called **root node**. Decision trees can handle both categorical and numerical data.

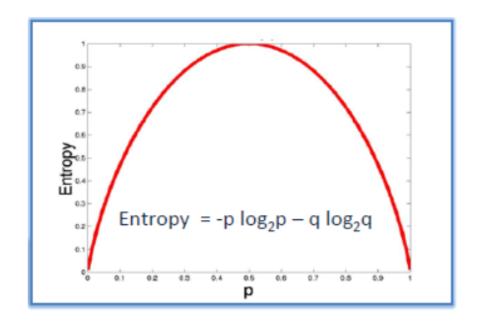
	Pre	dictors		Target		Decision Tre
Outlook	Temp.	Humidity	Windy	Play Golf	I	Outlook
Rainy	Hot	High	Falce	No	I	
Rainy	Hot	High	True	No	I	
Overoast	Hot	High	Falce	Yes	I	Sunny Overcast Rainy
Sunny	Mild	High	Falce	Yes	I	Overcast
Sunny	Cool	Normal	Falce	Yes	Ī	
Sunny	Cool	Normal	True	No	I	
Overoast	Cool	Normal	True	Yes		Windy Yes Humidity
Rainy	Mild	High	Falce	No		
Rainy	Cool	Normal	Falce	Yes	Ī	
Sunny	Mild	Normal	Falce	Yes	I	FALSE TRUE High Norma
Rainy	Mild	Normal	True	Yes	I	
Overoast	Mild	High	True	Yes	I	
Overoast	Hot	Normal	Falce	Yes	I	Yes No No Yes
Sunny	Mild	High	True	No	Ī	

#### Algorithm

The core algorithm for building decision trees called **ID3** by J. R. Quinlan which employs a top-down, greedy search through the space of possible branches with no backtracking. ID3 uses *Entropy* and *Information Gain* to construct a decision tree. In ZeroR model there is no predictor, in OneR model we try to find the single best predictor, naive Bayesian includes all predictors using Bayes' rule and the independence assumptions between predictors but decision tree includes all predictors with the dependence assumptions between predictors.

#### Entropy

A decision tree is built top-down from a root node and involves partitioning the data into subsets that contain instances with similar values (homogenous). ID3 algorithm uses entropy to calculate the homogeneity of a sample. If the sample is completely homogeneous the entropy is zero and if the sample is an equally divided it has entropy of one.



Entropy =  $-0.5 \log_2 0.5 - 0.5 \log_2 0.5 = 1$ 

To build a decision tree, we need to calculate two types of entropy using frequency tables as follows:

a) Entropy using the frequency table of one attribute:

$$E(S) = \sum_{i=1}^{c} -p_i \log_2 p_i$$

Yes No	Play	Golf
9 5	Yes	No
,	9	5

b) Entropy using the frequency table of two attributes:

$$E(T, X) = \sum_{c \in X} P(c)E(c)$$

		Play	Golf	
		Yes	No	
	Sunny	3	2	5
Outlook	Overcast	4	0	4
	Rainy	2	3	5
				14



$$\mathbf{E}(PlayGolf, Outlook) = \mathbf{P}(Sunny)*\mathbf{E}(3,2) + \mathbf{P}(Overcast)*\mathbf{E}(4,0) + \mathbf{P}(Rainy)*\mathbf{E}(2,3)$$

$$= (5/14)*0.971 + (4/14)*0.0 + (5/14)*0.971$$

$$= 0.693$$

#### Information Gain

The information gain is based on the decrease in entropy after a dataset is split on an attribute. Constructing a decision tree is all about finding attribute that returns the highest information gain (i.e., the most homogeneous branches).

Step 1: Calculate entropy of the target.

Step 2: The dataset is then split on the different attributes. The entropy for each branch is calculated. Then it is added proportionally, to get total entropy for the split. The resulting entropy is subtracted from the entropy before the split. The result is the Information Gain, or decrease in entropy.

		Play	Golf
		Yes	No
	Sunny	3	2
Outlook	Overcast	4	0
	Rainy	2	3
	Gain = 0	.247	

		Play	Golf
		Yes	No
	Hot	2	2
Temp.	Mild	4	2
	Cool	3	1
	Gain = 0	.029	

		Play	Golf
		Yes	No
Unmidien	High	3	4
Humidity	Normal	6	1
	Gain = 0	).152	

		Play	Golf
		Yes	No
Minde	False	6	2
Windy	True	3	3
	Gain = 0	.048	

$$Gain(T, X) = Entropy(T) - Entropy(T, X)$$

Step 3: Choose attribute with the largest information gain as the decision node, divide the dataset by its branches and repeat the same process on every branch.

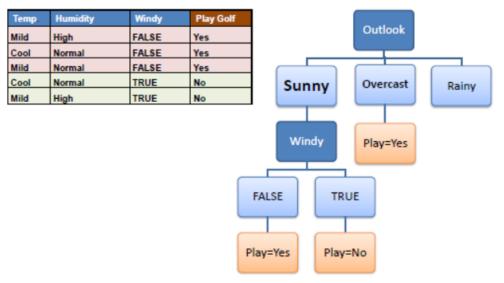
4		Play Golf		
7	~	Yes	No	
Outlook	Sunny	3	2	
	Overcast	4	0	
	Rainy	2	3	
	Gain = 0.	247		



Step 4a: A branch with entropy of 0 is a leaf node.

Temp	Humidity	Windy	Play Golf			
Hot	High	FALSE	Yes			
Cool	Normal	TRUE	Yes	1	Outlook	
Mild	High	TRUE	Yes	1	Outlook	
Hot	Normal	FALSE	Yes			
				Sunny	Overcast	D-
				Gailing	Overcast	Rai
				Sami,	Overcast	Ка

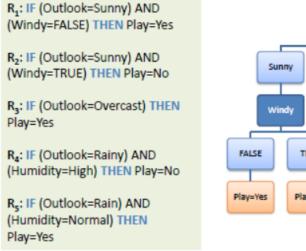
Step 4b: A branch with entropy more than 0 needs further splitting.

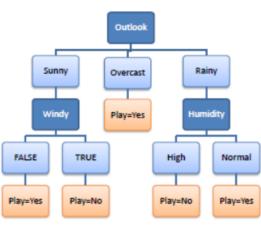


Step 5: The ID3 algorithm is run recursively on the non-leaf branches, until all data is classified.

#### **Decision Tree to Decision Rules**

A decision tree can easily be transformed to a set of rules by mapping from the root node to the leaf nodes one by one.





The source of above Information Gain example >

https://www.saedsayad.com/decision\_tree.htm