EEEN40690 Quantum Computing

Homework Problem Set for Topic 10: Error Correction Techniques Part 2

Instructions

- This is homework set 7 of 8. This homework set accounts for 5% of the marks for this module.
- In your report, please provide answers to the questions of this homework set. Explain clearly how the answers are obtained and what are their meaning or interpretation. Include relevant intermediate steps of the solution and explain your approach.
- Make sure that the report is readable, and the graphs (if any) are presented according to scientific/engineering standards.
- Some of the questions of the homework sets and the projects in this module may be open-ended and include a research component. Please formulate clearly your hypothesis and explain what will prove (or disprove) your hypothesis. Make sure that you provide sufficient evidence (analytical results, numerical results, modelling and simulations, evidence from the literature) to support your answer to open-ended or research problems.
- The report must be submitted online through UCD Brightspace:

  My Brightspace → EEEN40690 → Assessment → Assignments → Homework 7 (Homework for Topic 10: Error Correction Techniques Part 2)
- Late submissions will be accepted but a penalty will apply. In the case of late submissions, this module applies the standard UCD policy.
- Plagiarism and copying are offences under the terms of the Student Code, and you should be aware of the possible consequences.

Aim

The aim of this homework assignment is to introduce basic error correction techniques and understand some basic considerations behind error correction techniques.

- Repetition error correction
- Kraus operators
- Error channel models combined with repetion codes

Problem Set

Consider the quantum circuit presented in Fig. 1. The data qubit (qubit 2) and two auxiliary qubits are used in the three-qubit repetition code for phase flip errors. After the encoding block, the three qubits experience a phase-flip (Z-operator) error channel. In this channel, qubits 1 and qubit 3 are unchanged, while qubit 2 experiences a Pauli-Z error with probability p_z. At the end of the channel (after snapshot 1), the three qubits decoded and measured (at snapshot 2).

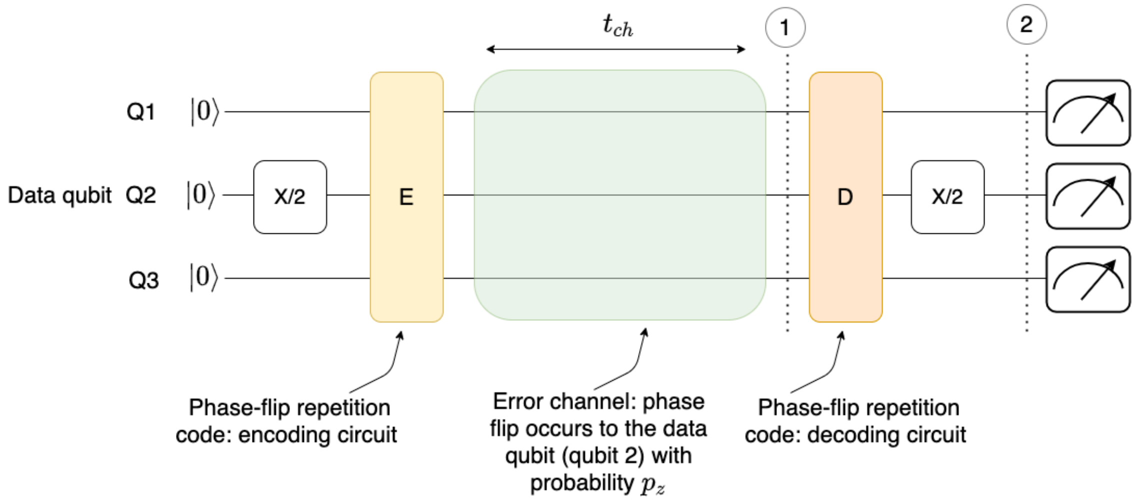

Figure 1: Three-qubit repetition code and phase-flip error channel.

Plot the probability of state $|0\rangle$ (or $|1\rangle$) for all the three qubits before and after the decoding block (snapshot 1 and snapshot 2 of the circuit) as a function of time t_{ch} spent by the qubits in the channel.

To connect the change in the probability (errors) with time, we have to use some error channel model. Assume that qubit 1 and qubit 3 are unchanged. The probability p_z of the phase-flip Pauli-Z error *only* on the second qubit is connected to the time t_{ch} spent by the qubits in the error channel as suggested in the following table:

Time sample	t_{ch}, μs	p_z
1	1	0.00987
2	6	0.0558
3	11	0.0974
4	16	0.135
5	21	0.169
6	26	0.199
7	31	0.227
8	36	0.252
9	41	0.275
10	46	0.295
11	50	0.309

Hint: Use the density matrix. The states of the qubits before the encoding and decoding blocks can be calculated since we know the quantum gates that comprise these blocks. For the evolution in the error channel, use the Kraus model on the density matrix. While the model of the channel itself is time-independent, we know that the probability of error p_z increases with time as shown in the table. As a result, you obtain $P_{|0\rangle}(n)$ where n is the time instance from the table and represents the state of the error channel as a function of time.

1 References

We have encountered this problem in earlier assignments and tutorials. The system is experimentally implemented in following reference:

[1] K. Takeda et al, "Quantum error correction with silicon spin qubits", Nature, Vol. 608, p. 682, 2022, https://doi.org/10.1038/s41586-022-04986-6