# EEEN40690 Quantum Computing

### Homework 5

### Homework Problem Set for Topic 8: Quantum Information Entropies

## Instructions

- This is homework set 5 of 8. This homework set accounts for 5% of the marks for this module.
- In your report, please provide answers to the questions of this homework set. Explain clearly how the answers are obtained and what are their meaning or interpretation. Include relevant intermediate steps of the solution and explain your approach.
- Make sure that the report is readable, and the graphs (if any) are presented according to scientific/engineering standards.
- Some of the questions of the homework sets and the projects in this module may be open-ended and include a research component. Please formulate clearly your hypothesis and explain what will prove (or disprove) your hypothesis. Make sure that you provide sufficient evidence (analytical results, numerical results, modelling and simulations, evidence from the literature) to support your answer to open-ended or research problems.
- The report must be submitted online through UCD Brightspace:

  My Brightspace → EEEN40690 → Assessment → Assignments → Homework 5 (Homework for Topic 8: Quantum Information Entropies)
- Late submissions will be accepted but a penalty will apply. In the case of late submissions, this module applies the standard UCD policy.
- Plagiarism and copying are offences under the terms of the Student Code, and you should be aware of the possible consequences.

## Aim

The aim of this homework assignment is to reinforce the knowledge on measures of entanglement, particularly quantum information entropies.

- Classical and Quantum information metrics
- Von Neumann entropy
- Bipartite Entanglement
- Entanglement entropy
- Quantum mutual information

## Problem Set

The total Hamiltonian for two qubits with interaction is given as follows:

$$H_{\text{tot,int}} = H_a \otimes I + I \otimes H_b + H_{\text{int}}$$

where $H_a, H_b$ are the independent Hamiltonians for the qubits $a$ and $b$, and $H_{\text{int}}$ describes the interaction between the two qubits. This leads to evolution that is not simply the tensor product of the evolution operators of the individual qubits. Consider the following Hamiltonians as an example, the form of which could describe a semiconductor spin qubit:

$$H_{a,b} = \begin{pmatrix} 0 & -i0.75 \\ i0.75 & 0 \end{pmatrix}, \quad H_{\text{int}} = \begin{pmatrix} 0.2 & 0 & 0 & 0 \\ 0 & -0.35 & 0 & 0 \\ 0 & 0 & -0.35 & 0 \\ 0 & 0 & 0 & 0.2 \end{pmatrix}.$$

Consider both qubits starting from the initial states $|0\rangle$.

- Find the time evolution of the system: plot the probabilities of the states $|0_a\rangle$, $|1_a\rangle$, $|0_b\rangle$ and $|1_b\rangle$ as functions of time.
- Plot the entanglement entropy and quantum mutual information of qubit a and qubit b as functions of time. What is the relationship between the two entropies?
- Using this system, we would like to build a quantum gate that prepares a Bell state for these two qubits. What is the time of evolution we should allow for this system to achieve a maximally entangled state? Explain your answer.
- What is the state vector of the system at the time instance from the previous question? Is it exactly a Bell state? If not, what should be done to transform it to a Bell state?
- What is the density matrix of the system at the time instance from the previous question?