EEEN40690 Quantum Computing

Homework 3
Homework Problem Set for Topic 5: Time Evolution of Density Operators

Instructions

- This is homework set 3 of 8. This homework set accounts for 5% of the marks for this module.
- In your report, please provide answers to the questions of this homework set. Explain clearly how the answers are obtained and what are their meaning or interpretation. Include relevant intermediate steps of the solution and explain your approach.
- Make sure that the report is readable, and the graphs (if any) are presented according to scientific/engineering standards.
- Some of the questions of the homework sets and the projects in this module may be open-ended and include a research component. Please formulate clearly your hypothesis and explain what will prove (or disprove) your hypothesis. Make sure that you provide sufficient evidence (analytical results, numerical results, modelling and simulations, evidence from the literature) to support your answer to open-ended or research problems.
- The report must be submitted online through UCD Brightspace:

  My Brightspace → EEEN40690 → Assessment → Assignments → Homework 3 (Homework for Topic 5: Time Evolution of Density Operators)
- Late submissions will be accepted but a penalty will apply. In the case of late submissions, this module applies the standard UCD policy.
- Plagiarism and copying are offences under the terms of the Student Code, and you should be aware of the possible consequences.

Aim

The aim of this homework assignment is to investigate how density operators change under the action of evolution operators which could be unitary and non-unitary.

- Time evolution of the density operator
- Von Neumann and Lindblad Equation
- Rabi oscillations, relaxation and dephasing

Problem

This homework assignment contains a research aspect. Use the script solving the Lindblad equation provided to support this week's topic. The script generates numerical solutions of the Lindblad equation. You can change the Hamiltonian matrix, jump operators, relaxation and dephasing rates and the initial state of the qubit. Attach the results of "numerical" experiments and comments you consider necessary to support your answers to the questions below.

The qubit parameters are as follows (as in the script) : $E_0 = -0.0005 \times \mu_B$ and $E_1 = 0.0005 \times \mu_B$. The suggested decay rate is $\gamma = 1/(111)$ ns (can be changed).

Questions:

1. How long does it take for an isolated system to undergo relaxation when starting in the excited state?
2. How should we modify the Hamiltonian to observe Rabi oscillations in an isolated system? What influences the frequency of Rabi oscillations of the isolated system?
3. How would you describe the qubit's dynamics in an open quantum system allowing for energy loss from the system to the environment? Discuss this quantitatively using simulations with the suggested value of the decay rate γ? How do the dynamics change if the decay rate is varied to $2 * \gamma$ and $0.2 * \gamma$? Ensure you discuss the effect on the diagonal and off-diagonal elements of the density matrix.
4. Consider a qubit coupled to an external noisy environment, such that it undergoes dephasing. What is the form of the Lindblad equation in this case? How do the diagonal and off-diagonal elements of the density matrix evolve over time. You should use the decay rate from the previous questions as your dephasing rate.