EEEN40690 Quantum Computing

Homework 2

Homework Problem Set for Topic 4: Measurement of Density Operators

Instructions

- This is homework set 2 of 8. This homework set accounts for 5% of the marks for this module.
- In your report, please provide answers to the questions of this homework set. Explain clearly how the answers are obtained and what are their meaning or interpretation. Include relevant intermediate steps of the solution and explain your approach.
- Make sure that the report is readable, and the graphs (if any) are presented according to scientific/engineering standards.
- Some of the questions of the homework sets and the projects in this module may be open-ended and include a research component. Please formulate clearly your hypothesis and explain what will prove (or disprove) your hypothesis. Make sure that you provide sufficient evidence (analytical results, numerical results, modelling and simulations, evidence from the literature) to support your answer to open-ended or research problems.
- The report must be submitted online through UCD Brightspace:

  My Brightspace → EEEN40690 → Assessment → Assignments → Homework 2 (Homework for Topic 4: Measurement of Density Operators)
- Late submissions will be accepted but a penalty will apply. In the case of late submissions, this module applies the standard UCD policy.
- Plagiarism and copying are offences under the terms of the Student Code, and you should be aware of the possible consequences.

Aim

The aim of this homework assignment is to enhance the understanding of the density operator (matrix) and connect it to measurement on a quantum system (qubit):

- Expected value of the operator based on the density matrix
- Partial trace
- State of the qubit after a measurement

Problem Set

1. Figure 1 shows a quantum circuit. This quantum circuit is known as the *qubit encoding stage* used, in particular, in repetition quantum error correction codes to mitigate phase flipping errors. Assume that the input state of the data qubit is $\frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$. Consider the circuit to be ideal.

   (a) What is the density matrix of the 3-qubit state at the output of the circuit?

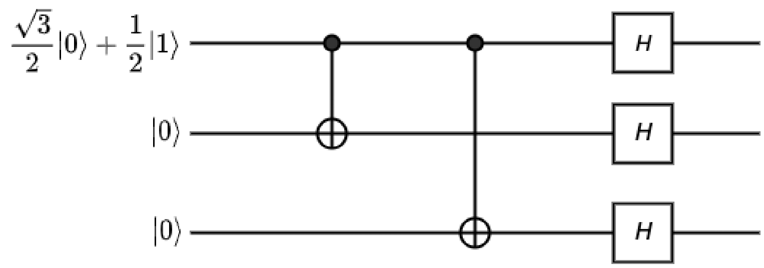

Figure 1: Quantum circuit from Problem 1, H is the Hadamard gate, the two-qubit gate is the CNOT.

(b) What is the expected value of the observable operator Z (measuring the qubit in the calculation basis, along the z-axis of the Bloch sphere) of the first qubit.

(c) What is the state of qubit 1 if you cannot measure qubits 2 and 3?

(d) What is the state of qubit 3 if you cannot measure qubits 1 and 2?

(e) Suppose that we applied the measurement from part (b) of this problem on the first qubit at the output. What is the state of the system after the measurement? What can you say about the system after the measurement?

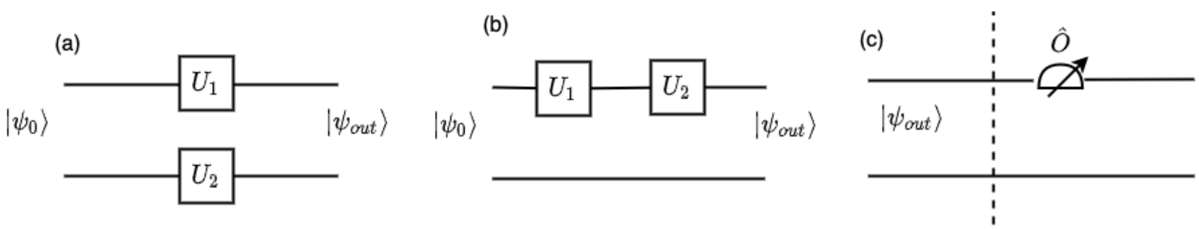

Figure 2: Quantum circuit from Problem 1, H is the Hadamard gate, the two-qubit gate is the CNOT.

Hint: A three-qubit circuit, similar to the one presented in this homework assignment, can still be analysed analytically. However it might be more convenient for you to write (or use already written) a python script to construct the density matrix, measurement operators, etc. Please feel free to use any numerical tool to solve this problem.

As a reminder, we note that quantum gates in a multi-qubit circuit can be combined.

The circuit in fig. 2(a) can be summarised as follows:

$$|\psi_{out}\rangle = (U_1 \otimes U_2)\,|\psi_0\rangle$$

where we used the Kronecker product for the gates acting on two different qubit lines. The state vectors $|\psi_{out}\rangle$ and $|\psi_0\rangle$ are multi-qubit state vectors.

The circuit in fig. 2(b) can be summarised as follows:

$$|\psi_{out}\rangle = (U_2 \otimes I)(U_1 \otimes I)\,|\psi_0\rangle$$

where we note that if there is no operation acting on the second qubit, we imply the identity operator. The gates acting sequentially on the same qubit line – we use the regular matrix product.

The measurement operation in fig. 2(c) can be summarised as follows:

$$M = \mathrm{O} \otimes I$$

where we note that the measurement can also act only on a selected qubit (in this case, on the first one). The measurement on the other qubit is the "identity" operation.