EEEN40690 Quantum Computing

Homework 1

Homework Problem Set for Topic 3: Introduction to the Density Operator

Instructions

- This is homework set 1 of 8. This homework set accounts for 5% of the marks for this module.
- In your report, please provide answers to the questions of this homework set. Explain clearly how the answers are obtained and what are their meaning or interpretation. Include relevant intermediate steps of the solution and explain your approach.
- Make sure that the report is readable, and the graphs (if any) are presented according to scientific/engineering standards.
- Some of the questions of the homework sets and the projects in this module may be open-ended and include a research component. Please formulate clearly your hypothesis and explain what will prove (or disprove) your hypothesis. Make sure that you provide sufficient evidence (analytical results, numerical results, modelling and simulations, evidence from the literature) to support your answer to open-ended or research problems.
- The report must be submitted online through UCD Brightspace:

  My Brightspace $\rightarrow$ EEEN40690 $\rightarrow$ Assessment $\rightarrow$ Assignments $\rightarrow$ Honework 1 (Homework for Topic 3: Introduction to the Density Operator)
- Late submissions will be accepted but a penalty will apply. In the case of late submissions, this module applies the standard UCD policy.
- Plagiarism and copying are offences under the terms of the Student Code, and you should be aware of the possible consequences.

Aim

The aim of this tutorial is to introduce the density operator (matrix)

- Density matrix definition
- What properties can be captured by the density matrix
- Obtaining density matrices for pure and mixed states
- Distinguishing pure and mixed states based on the density matrix
- Including finite temperature into the density matrix

Problem Set

1. Figure 1 shows a quantum circuit. This circuit contains single-qubit and two-qubit gates, which operate as ideal gates, and a non-ideal component that generates *classical uncertainty* just prior to measuring the output quantum state. As a result, the output state $|\psi_{out}\rangle$ is a mixed state. This non-ideal block in the quantum circuit gives the correct coherent state with 80% classical probability and adds 10%

probability to observe states $|00\rangle$ and $|11\rangle$ each even if these states are not the result of the operation of this quantum circuit.

(a) What is the density matrix of $|\psi_{out}\rangle$?

(b) Verify that this is a valid mixed-state density matrix.

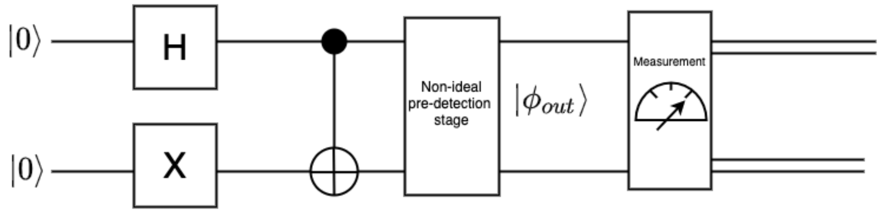

Figure 1: Quantum circuit from Problem 1, H is the Hadamard gate, X is the Pauli-X gate, the two-qubit gate is the CNOT.

2. A quantum mechanical system has two energy levels and we would like to use it as a qubit. We plan to operate this system at finite temperature of 500 mK. (All realistic qubits operate at finite temperatures). To check the feasibility of such a qubit, we would like to prepare its quantum state at $|0\rangle$ but we expect that due to the finite temperature we are not able to do so with very high accuracy. The energies of the qubit levels are $E_0 = h\nu_0$ and $E_1 = h\nu_1$ where $h = 6.62 \times 10^{-34}$ J.s is the Planck constant, $\nu_1 = 2 \times 10^9$ Hz and $\nu_2 = 14 \times 10^9$ Hz. The Boltzmann constant is $k_B = 1.38 \times 10^{-23}$ J/K.

   (a) What is the probability to measure state $|1\rangle$ at 500 mK even though we prepare the qubit in $|0\rangle$? Is the result satisfactory in your opinion?

   (b) Will the accuracy of state preparation improve if we cool down the qubit to 200 mK?

   (c) At what temperature should we operate this qubit in order to be able to initialise the ground state $|0\rangle$ with accuracy exceeding 99%?

   Hint: to answer this questions, you can calculate the density matrix of the system as a mixed state and use its diagonal elements as the populations of the levels. The classical uncertainty in this example is due to temperature, so you can use Example 3.2 from the lecture notes (Lecture 3) on thermal equilibrium density matrices.