Final exam in FY3464

Quantum Field Theory I

By

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Motivation:

The field of Dark Matter is something that has interested me for a long time. Ever since I searched for Dark Matter in an another course by using a mono-Z process leading to the final state l+l- and missing transverse energy MET, I've investigated the field for other potential Dark Matter candidates. For the other project, I hereby refer to my Github-profile.[1]

Introduction:

A large portion of the total matter in the Universe does not emit light. This form of matter is what is known as Dark Matter. Measurements by the Planck and WMAP demonstrate that nearly 85 percent of Universe's matter density is dark.[2] Sjnce we know so little of the nature of Dark Matter and the Standard Model of particle physics does not provide sufficient explanation of the phenomena makes, suggests that more research is needed. This project will look at one of them, the lightest neutralino $\widetilde{\chi_1}$. $\tilde{\chi}$ is a Majorana fermion which is a mixture of the supersymmetric partners of the photon, the Z boson and two neutral higgs boson.

Before answering the posed questions in the project, we will digress for a bit by explaining the neutralino $\widetilde{\chi_1}$, Majorana fermion and the two types of bosons. This is done so that we know what are working on. Then the project will try to calculate the matrix element and cross section of the scattering:

$$\tilde{\chi} f \rightarrow \tilde{\chi} f \quad (1)$$

It is also recommended that the reader carefully reads the appendix where a comparison between the cross section found here and those from the published literature is done.

Neutralino:

is a hypothetical particle. There are four types of neutralinos and they are classified according to their weight. The lightest one is denoted $\tilde{\chi}$. Their mass is below 300 GeV, electric charge = 0 and spin = ½. The minimal supersymmetric extension of the Standard Model from that contains four neutralino eigenstates: a bino, a wino and two higgsinos.[3] Because of collider constraints on superparticles, especially on the masses of heavier neutralinos, it is so that the mass eigenstates are pure states with little mixing.

Majorana fermion:

I will here try to explain Majorana fermions based on Steven Weinberg's The Quantum Theory of Fields.[4] It should be considered a minor digression and is not necessary for the project. The famous Dirac equation for a free particle of spin ½ is written as:

$$(\gamma^{\mu}\, \partial_{u} + m)\, \psi\, (x) = 0 \quad (2)$$

Weinberg notes that under a space inversion the particle annihilation and antiparticle annihilation undergo the following transformations:

$$P_{a}\, (p, \sigma) P^{-1} = \eta * a(-p, \sigma)$$

$$P_{a}^{c+}\, (p, \sigma)\, P^{-1} = \eta^{c}\, a^{c+}(-p, \sigma)$$

After a rather long derivation (not suited for this text format) of the spin sum we get the following expressions for it.

$$N(p) = \frac{1}{2p^{0}}\, \left[-ip^{\mu}\, \gamma_{\mu} + m\right]\beta$$

$$M(p) = \frac{1}{2p^{0}}\, \left[-ip^{\mu}\, \gamma_{\mu} - m\right]\beta$$

And the anticommutator becomes:

$$\left[\psi_{l}\, (x), \psi_{\bar{l}}^{+}\, (y)\right] = \{[-\gamma^{\mu}\, \partial_{u} + m]\beta\}_{l\bar{l}}\, \Delta(x - y)$$

Weinberg states that a requirement for space inversion is that the field $\psi\ (x)$ must transform into something proportional to $\psi\ (Px)$. This means that the intrinsic parities of particles and antiparticles must be related by:

$$\eta^{c} = \ -\eta *$$

Meaning that the intrinsic parities $\eta\eta^{c}$ if a state consisting of a spin ½ article and its antiparticle is odd. The transformation of the causal Dirac field under space inversion is then:

$$P\, \psi\, (x)\, P^{-1} = \eta * \beta\psi(Px)$$

Furthermore, Weinberg gives us the eigenvectors as $-\frac{ip^{\mu}\gamma_{\mu}}{m}$ with eigenvalues $\pm$ 1. And therefore becoming:

$$\left(ip^{\mu}\gamma_{\mu} + m\right) u\,(p,\sigma) = 0$$

$$\left(-ip^{\mu}\gamma_{\mu} + m\right) v(p,\sigma) = 0$$

This then satisfies equation (2). Weinberg has made the free-particle Dirac equation a Lorentz invariant by putting together two irreducible representations of the proper orthochronus Lorentz group to a field that transforms under space inversion.

In order for the field to transform under charge-conjugation into another field which it commutes at space-like separations, it is so that the charge conjugation parities of the particle and antiparticle are related through:

$$\xi^{c} = \xi *$$

Leading to a field transformation described according to:

$$C\psi\,(x)\,C^{-1} = \,-\xi * \beta\,\vartheta\,\psi\,(x)$$

So far, it has been assumed that particles and antiparticles are different. But if they are identical with spin ½ , we call them Majorana fermions. The Majorana fermions must have a field that satisfies the reality condition:

$$\psi\,(x) = \,-\beta\,\vartheta\,\psi\,(x)$$

Weinberg further specifies that for the Majorana fermions the intrinsic space inversion parity must be imaging $\eta = \,\pm i$, while the charge-conjugation parity must be real, $\xi = \pm 1$. This is in essence what we consider a Majorana fermion. The derivation was done by the eminent Steven Weinberg who famously won the Nobel Prize in Physics. For more information of Majorana fermions, I recommend Michael Kachelreiss on page 332-333.[5]

Bosons:

I will here try to introduce the bosons, Z boson and Higgs boson. Obviously it will only be a small introduction to them. For more information on the subject, I refer to good books like Michael Thomson's, Modern Particle Physics.[6]

From the quantum theory of many-particle systems, we have the term first and second quantization. We use the first quantization of get a theoretical explanation for bosons.

If the wavefunction of a system of N identical particles is $\psi\ (x_1, x_2, \dots, x_N)$, where $x_i = (r_i, s_i)$ denoted the position and spin coordinates of the particle i respectively, then quantum indistinguishability leads us to:

$$\left|\psi\ (x_1, \dots, x_j, \dots, x_k, \dots, x_N)\right|^2 = \left|\psi\ (x_1, \dots, x_j, \dots, x_k, \dots, x_N)\right|^2$$

This equation says that the probability is the same for two configurations that differ in the exchange of the coordinates of the particles j and k. In spatial 3 dimensions, we get two different possibilities when two particles are involved, as seen below:

$$\psi \rightarrow +\psi$$

$$\psi \rightarrow -\psi$$

In these two equations, the first one means that the wavefunction is symmetric under such exchange, while in the second one the wavefunction is anti-symmetric. The symmetric wavefunction represents particles are called bosons. They have an integer spin and follow Bose-Einstein statistics. The anti-symmetric wavefunction represents fermions. Fermions have a half-integer spin and follow Fermi-Dirac statistics, well known from Solid State Physics.

Having explained what a boson is, we now take a closer look at Z boson and Higgs boson. The Z boson mediates the weak charged-current interaction, closely related to the charged current. The mass of a Z boson is roughly 91.2 GeV. Higgs bosons were found rather late in 2012. They have a mass of 125 GeV, and a Higgs boson is different from other Standard Model particles. It has a spin 0 and it provides the mechanism by which all other elementary particles acquire mass. In Quantum Field Theory this particular boson can be viewed upon as an excitation of the Higgs field.

The project:

We will first draw the Feynman-diagram for the process seen in equation (1). This will serve as the starting point for the rest of the project and follows below in figure 1. The diagram was drawn in the program Krita.

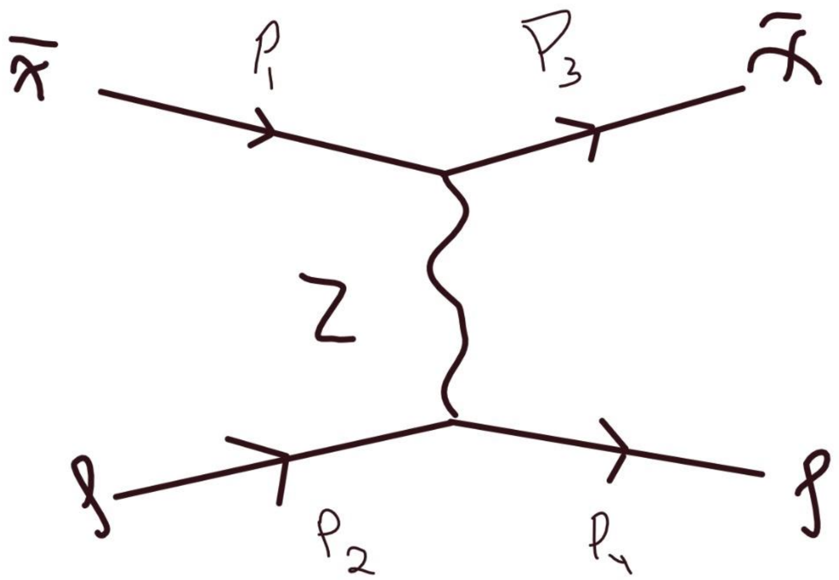

Figure 1: Schematic shows the Feynman-diagram for the process in equation (1). The Z boson is the mediator.

On figure 1, we see a Feynman-diagram of the process. The Feynman-diagram takes into account both scattering and the process. But we will primarily use it to define a matrix for the process. This is the next then, to create a matrix element for the process. In this we follow the procedure from Thomson, “Modern Particle Physics”. It is specified that we are only to find the matrix-element for the scattering process.

With a foundation in figure 1 and the Feynman-rules for diagrams as such, we get:

$$iM_{fi} = \left(\bar{u}\,(p_3)\left(-g_Z\gamma^\mu \frac{1}{2}\left(C_V^{\tilde{\chi}} - C_A^{\tilde{\chi}}\gamma^5\right)\right)u(p_1)\right)\left(\frac{-ig_{uv}}{q^2 - m_Z^2}\right)(\bar{u}(p_4)\left(-g_Z\,\gamma^v \frac{1}{2}\,(C_V^f - C_A^f\,\gamma^5))u(p_2)\right)$$

And this then becomes the final expression for the matrix-element for process (1).

$$M_{fi} = \left(-\frac{g_z^2}{q^2 - m_Z^2} g_{uv}\,\left[\bar{u}\,(p_3)\gamma^\mu \frac{1}{2}\left(C_V^{\tilde{\chi}} - C_A^{\tilde{\chi}}\gamma^5\right) u(p_1)\right]\left[\bar{u}\,(p_4)\gamma^v \frac{1}{2}\left(C_V^f - C_A^f\,\gamma^5\right) u(p_2)\right]\right) \quad (3)$$

This is the matrix-element. We use the notation from Thomson since the lecturer also teaches the course Particle Physics and recommends that book in particular. And C_V and C_A are notations for V and A currents respectively. V is the vector and A is Axial vector as proposed by Feynman and Gell-Mann.

In equation (3) the allowed interactions are vectorial (V), axial (A) and scalar. With the neutralino being a Majorana fermion (introduced earlier in the text) we do not include the vectorial interaction since it vanishes.[7] We then only have two interactions: scalar and axial. Axial is spin-dependent, while scalar is spin-independent. This then gives us the matrix seen in equation (4) below.

$$M_{fi} = \left(-\frac{g_z^2}{q^2 - m_Z^2} g_{uv}\,\left[\bar{u}\,(p_3)\gamma^\mu \frac{1}{2}\left(-C_A^{\tilde{\chi}}\gamma^5\right) u(p_1)\right]\left[\bar{u}\,(p_4)\gamma^v \frac{1}{2}\left(-C_A^f\,\gamma^5\right) u(p_2)\right]\right) \quad (4)$$

Having found the matrix-element, we will continue on to find the cross section for process (1). The total cross section is given by the following equation for the differential equation that is so integrated over a solid angle:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} < \left|M_{fi}\right|^2 > \quad (5)$$

But obviously we have to first find $< \left|M_{fi}\right|^2 >$. This is done below by using the relation $|M_{fi}|^2 = M * M^+$.

$$\left(\left(\frac{g_Z^2}{q^2-m_Z^2}\right) g_{\mu\nu} \left[\bar{u}\ (p_3)\gamma^{\mu}\frac{1}{2}\left(C_A^{\tilde{\chi}}\gamma^5\right) u(p_1)\right]\left[\bar{u}\ (p_4)\gamma^{\nu}\frac{1}{2}\left(C_A^{f}\ \gamma^5\right) u(p_2)\right]\right)$$
$$*\left(\left(\frac{g_Z^2}{q^2-m_Z^2}\right) g^{\mu\nu} \left[u(p3)\gamma_{\nu}\frac{1}{2}\ \left(C_A^{\tilde{\chi}}\gamma^5\right)\bar{u}\ (p1)\right]\left[u(p4)\gamma_{\mu}\frac{1}{2}\left(C_A^{f}\gamma^5\right)\bar{u}\ (p2)\right]^{+}\right)$$

We use trace on this equation and get:

$$Tr\ \left(\bar{u}\ (p_3)\gamma^{\mu}\frac{1}{2}\left(C_A^{\tilde{\chi}}\gamma^5\right) u(p_1)\right) * Tr\ \left(\bar{u}\ (p_3)\gamma^{\mu}\frac{1}{2}\left(C_A^{\tilde{\chi}}\gamma^5\right) u(p_1)\right)$$
$$* Tr\ \left(u(p3)\gamma_{\nu}\frac{1}{2}\ \left(C_A^{\tilde{\chi}}\gamma^5\right)\bar{u}\ (p1)\right) * Tr\ \left(u(p4)\gamma_{\mu}\frac{1}{2}\left(C_A^{f}\gamma^5\right)\bar{u}\ (p2)\right)$$

With the trace formalism we can both switch matrices of a product without changing the result and by having a trace that is invariant under cyclic permutations.[8] With this we can do the following below.

We can use the abovementioned properties from the previous paragraph to turn the expression into:

$$Tr\ \left(\bar{u}\ (p3)\gamma^{\mu}\ u(p3)\frac{1}{2}\ C_A^{\tilde{\chi}}\gamma^5\ \right) * Tr\ \left(u(p1)\gamma^{\nu}\bar{u}(p1)\frac{1}{2}C_A^{\tilde{\chi}}\gamma^5\right)$$
$$* Tr\ \left(\bar{u}\ (p4)\gamma_{\mu}\ u(p4)\frac{1}{2}\ C_A^{f}\gamma^5\right) * Tr\ \left(u(p2)\gamma_{\nu}\ \bar{u}(p2)\frac{1}{2}\ C_A^{f}\gamma^5\right)$$

With this expression and the Feynman slash notation this expression becomes:

$$Tr\ \left(\gamma^{\mu}\ (\not{p}3+m)\ \gamma^{\nu}\ (\not{p}1+m)\left(C_A^{\tilde{\chi}}\gamma^5\right)^2\right)\ Tr\left(\gamma_{\mu}\ (\not{p}4+m)\ \gamma_{\nu}\ (\not{p}2+m)\ \left(C_A^{f}\gamma^5\right)^2\right)$$

From one of the many references on the homework of this text we have the following contraction relations:

$$g^{\mu\nu}g_{\mu\nu} = 4$$

$$p_2^{\mu}\ p_1^{\nu}\ g_{\mu\nu} = p1 * p2$$

$$p_2^{\mu}\ p_1^{\nu}\ p_{3\mu}p_{4\nu} = (p2 * p3)(p1 * p4)$$

We ignore the mass m and get the following derivation. I should clarify that the notation is slightly different from the contraction relations above and the derivation below because I thought it might be difficult for the external reader to find them if I wrote them differently:

$$= (p3^{\mu}\, p1^{v} - g^{\mu v}\,(p3 * p1) + p3^{v}\, p1^{\mu})\,\big(p4_{\mu}\, p2_{v} - g_{\mu v}\,(p4 * p2) + p4_{v} p2_{\mu}\big)\frac{1}{4}\Big(C_A^{\tilde{\chi}} C_A^{f}\Big)^2 \quad (6)$$

$$= \big(p3^{\mu}\, p1^{v}\, p4_{\mu}\, p2_{v} - p3^{\mu}\, p1^{v}\, g_{\mu v}\,(p4 * p2) + p3^{\mu}\, p1^{v}\, p4_{v}\, p2_{\mu} - g^{\mu v}\,(p3 * p1)\, p4_{\mu}\, p2_{v} + g^{\mu v} g_{\mu v}\,(p3 * p1)(p4 * p2) - g^{\mu v}\,(p3 * p1)(p4 * p2) - p3^{v} p1^{\mu}\, p4_{\mu} p2_{v} - p3^{v} p1^{\mu} g_{\mu v}(p4 * p2) + p3^{v} p1^{\mu} p4_{v} p2_{\mu}\big)$$

In the first equation above we have used the cyclic property to move the $\Big(C_A^{\tilde{\chi}} C_A^{f}\Big)^2$ to the end of the expression. We will not include this until the latter end of the derivation. Having done this, we continue with our derivation. We use the contraction relations to get:

$$= (p3 * p4)\,(p1 * p2) - (p1 * p3)(p4 * p2) + (p3 * p2)(p1 * p4) - (p3 * p1)(p4 * p2) + 4(p3 * p1)(p4 * p2) - (p3 * p1)(p4 * p2) + (p1 * p4)(p3 * p2) - (p1 * p3)(p4 * p2) + (p1 * p2)(p3 * p4)$$

$$= -4(p1 * p3)(p4 * p2) + 2(p1 * p2)(p3 * p4) + 2(p3 * p2)(p1 * p4) + 4(p3 * p1)(p4 * p2)$$

Hereby giving us the final expression of:

$$2(p1 * p2)(p3 * p4) + 2(p3 * p2)(p1 * p4) \quad (7)$$

It should be said that in equation (6) we've used the relations $(\gamma^5)^2 = 1$. As earlier mentioned we've not included the current terms in equation (7). This was done out of overview and space constraints. So, we include them in the equation (8) below.

From Thomson we have the relations:

$$p1 = (E, 0, 0, E)$$

$$p2 = (E, 0, 0, -E)$$

$$p3 = (E, E\sin\theta, 0, E\cos\theta)$$

$$p4 = (E, -E\sin\theta, 0, -E\cos\theta)$$

And also we should include the relations (in order to understand the procedure to find the differential cross section):

$$p1 * p2 = p3 * p4, \;\; p1 * p4 = p2 * p3, \;\; p1 * p3 = p2 * p4$$

The relations in the CoM frame (reference page 137-138 in Thomson) gives us:

$$p1 * p2 = 2E^2$$

$$p1 * p3 = E^2\,(1 - \cos\theta)$$

$$p1 * p4 = E^2\,(1 + \cos\theta)$$

So, we then get the differential cross section:

$$\frac{d\sigma}{d\Omega} = -\frac{1}{64\pi^2 s}\frac{g_Z^4}{q^2 - m_Z^2}\frac{1}{4}\,(4E^4 + 2E^4\,(1 + \cos(\theta))^2\left(C_A^{\tilde{\chi}} C_A^f\right)^2 \;\; (8)$$

Since we've used Thomson's notation, we can set $s = 4E^2 = q^2$, we get the following expression for the differential cross section:

$$\frac{d\sigma}{d\Omega} = -\frac{1}{64\pi^2 s}\frac{g_Z^4}{s - m_Z^2}\frac{1}{4}\left(C_A^{\tilde{\chi}} C_A^f\right)^2\,(1 + \cos^2\theta + 2\cos\theta)$$

We integrate over the solid angle $d\Omega = d\o d\,(\cos\theta) = 2\pi d\,(\cos\theta)$ giving us:

$$\int_{-1}^{+1}(1 + \cos^2\theta)\,d(\cos\theta) = \int_{1}^{+1}\,(1 + x^2)\,dx = \frac{8}{3}$$

And $\int_{-1}^{+1}\cos\theta\,d\,(\cos\theta) = 0$. Where in the two last integrals the substitution rule and orthogonality-principle has been used respectively. Here we've included 3 flavours because of symmetry, meaning that we multiply the part that is integrated with 3 and 2π. So, this then gives us by only considering the t-channel and not including the s-terms:

$$\sigma = \frac{1}{4\pi}\frac{g_Z^4}{m_Z^2}\,\frac{1}{4}\left(C_A^{\tilde{\chi}} C_A^f\right)^2 \;\; (9)$$

We now proceed to the second part of the project. For more on the cross section, I refer to the appendix where the result found in this project is compared with other comparable works.

The Lagrangian field density, L, is in the unitary gauge given as:

$$L_{Z\chi1\chi1} = \frac{g}{2c_W} \left(N_{13}^2 - N_{14}^2\right) \overline{\chi_1}\, \gamma^\mu\, \gamma^5\, \chi_1\, Z_\mu$$

With $N_{13}^2 - N_{14}^2\, \epsilon\, [0 : 1]$. We are asked to give a rough estimate of the numerical value of the cross section on a nucleon. Unitary gauge is a particular choice of gauge fixing in a gauge theory with a spontaneous symmetry breaking. The scalar fields responsible for the Higgs mechanism are transformed into a basis in which their Goldstone boson components are zero. Introduced by the highly influential Steven Weinberg, the dominance will be by the Z-boson in the gauge.

One researcher who has worked extensively on finding the cross section for the neutralino dark matter is David G. Cerdeno.[17] He gives us a relation for finding the contribution from the Z boson exchange. It should be reminded that there is both a spin independent (SI) and spin dependent (SD) contribution to consider. With Majorana fermions however the SI scattering cross section vanishes and we only consider the spin dependent cross section. According to Cerdeno this is represented by the following equation:

$$\alpha_{2i}^Z = -\left(g^2/(4m_Z^2 \cos^2\theta_W)\right) [N_{13}]^2 - [N_{14}]^2 \frac{T_{3i}}{2} \quad (10)$$

First, we find the angle. Cerdeno suggests an angle given as for the lightest neutralinos:

$$\tan\theta \leq 40$$

$$\theta = \arctan(40) = 88.6\,°$$

The other variables are also given, T = $10^{-13}\ GeV, m_Z = 90\ GeV, g = 200\ GeV.$ In the last one we assume grand unification of the gauge couplings meaning that the upper bound on the mass of the lightest neutralino can be expressed in term of the gluino mass.[18] Inserting all of these values in equation (10) we get the following contribution:

$$\alpha_{2i}^Z = 1.034 * 10^{-10}\ GeV$$

By using the relation that 1 pb = $2.568 * 10^{-9}\ GeV^{-2}$, we get the numerical value for the cross section given as:

$$\sigma = 0.0402\ pb$$

This is in accordance with the value found by Greist and Cerdeno. Cerdeno gives an upper bound at 0.06 pb and Greist gives an approximate value at 0.03 pb.

However, it should be pointed out that the numerical value I found above is for the entire Z-exchange. For scattering off the nucleon, we need another equation. We use the Lagrangian to find the relation for elastic scattering cross section of a nucleus mass m_N:

$$\sigma_{el} = \frac{24 m_\chi^2 m_N^2\ G_F^2}{\left(\pi m_\chi m_N\right)^2} \left\{\frac{4}{3}\ \lambda^2 J\ (J+1)\ (A'\Delta q)^2 + \left(\frac{2m_N}{27m_W}\right)^2\ \left((b-c)x_q^2\ dq\right)\right\} \quad (11)$$

This term includes the photino, higgsino and zino contributions to the scattering. Here we have two terms. The second one is difficult, but we ignore the light quarks and find (by using that c^2 = the coupling for zino) that:

$$\frac{2\ m_N\ x_q^2}{27m_W}\left(\frac{2Z_{14}}{sin\beta} - \frac{Z_{13}}{\cos\beta}\right) \quad (12)$$

Here we've obviously suggested many terms that need an numerical value before we can actually get a final value. I just want to point out that Griest gives a photino contribution at 0.03 pb and I've decided to use that for the first term in equation (11). Below follows a table for the different values used (but one can also find them in any particle physics themed book).[19]

Variables	Value (all in GeV)
m_χ	300
m_W	80.38
G_F	1.166*10^{-5}
m_N	0.938
x_q	-0.129
$Z_{13}^2 - Z_{14}^2$	0.03

We put in the numerical values for the different variables suggested in the table above and get a value of: $3.36 * 10^{-6}\ GeV$ for the equation (12). Basically, we are asked for a rough estimation, do not necessary include everything, but just relate to the photino value. We therefore get a final value of:

$$9.007 * 10^{-10}\,(0.03 + 3.35 * 10^{-6})\,GeV = 2.702 * 10^{-11}\,GeV^2$$

But like in the other example we want to find the value in pb. So this value therefore must be transformed like:

$$\frac{2{,}792 * 10^{-11}}{2.568 * 10^{-9}\,\frac{1}{pb}}$$

And thereby becoming:

$$\sigma = 0.0105\,pb$$

This is the numerical value for elastic scattering on a nucleon. But it includes all “exchanges” and does not only look at the Z-scattering. This was not asked of either I believe, but we are somewhat close to the values obtained by several other researchers, as already mentioned. But if we were to only look at the Z-boson part, I think we would get a value approximated as:

$$1.174 * 10^{-6}\,pb$$

Here we’ve only looked at the second term that concerned with the Z boson and not included the photino. This is in line with what for example PICO-60 found for proton-scattering.[20] I also add that P. Fayet used a value of 0.0524 for the cross section of the photino, but I used 0.03 here instead.[21] And we remember that we were only asked for a rough numerical value.

Then we were tasked to use crossing symmetry to estimate the low-energy limit of the neutralino annihilation cross section into fermions, $\chi\chi \rightarrow f\bar{f}$. In my attempt to provide a reasonable explanation of the term crossing symmetry, I refer to Ronald Kleiss.[22] He explains crossing symmetry with an analogy: the production (absorption) of a particle is, in a sense, equivalent to the absorption (production) of its antiparticle. As a relation between various scattering amplitudes, this is called crossing symmetry. In order to understand this phenomenon better, we look at a generic 2 -> 2 scattering process:

$$a(p1) + b(p2) \rightarrow c(p3) + d(p4)$$

We use this to write an amplitude M (p1, p2, p3, p4). By moving particles from the initial to the final state, or the other way around, we are able to find the amplitudes for the crossing-related process. The other limitation we were given was to find the annihilation cross section in the low-energy limit. This means that we must presume that the fermions have a mass because in the high-energy limit the fermions are effectively massless.[23] And in the low-energy limit, the massive vector bosons disappear from the theory we've looked at so far. According to Klaus Winter only the light fermions, the photons and possibly the Higgs bosons remains.[24] But here we will omit this boson as well. Winter wrote his text in 1993, before its discovery and only knew of the theoretical Higgs. This of course also means that there are no interference terms to consider in the evaluation.

When we consider the lightest supersymmetric particle, we look at mostly the bino, $\tilde{\beta}$, in the equation:

$$\tilde{\chi} = Z_{11}\tilde{\beta} + Z_{12}\widetilde{W}^3 + Z_{13}\widetilde{H_1} + Z_{14}\widetilde{H_2} \quad (13)$$

Love and Bailin suggest that the annihilation proceeds mainly via t-channel sfermion exchange, so this is in line with what we've written so far in this section. Therefore, we proceed. We want to find the photino limit cross section. A photino is the fermion superpartner to photon (which is a boson) and we do it by drawing the t-channel Feynman diagram for the process. Then we use crossing symmetry, as explained by Kleiss, to find the cross section for the annihilation.

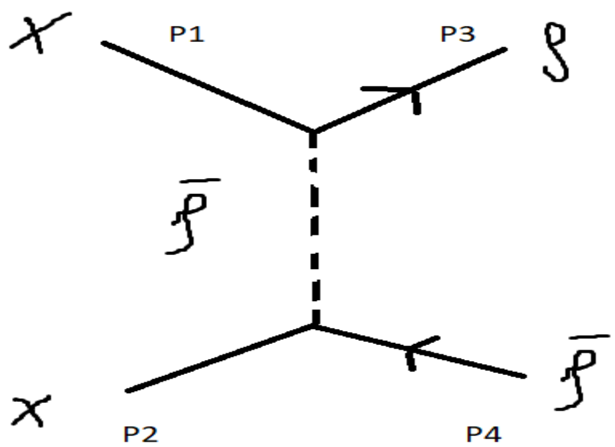

Figure 2: Schematic shows the t-channel sfermion exchange Feynman diagram of the annihilation

By comparing figure and figure 2 we see that the momentum parameters are named p1, p2…etc. on both. With this we can use cross symmetry to find the matrix and we use the following relation:

$$\sigma v = \frac{\left(\alpha^2 m_\chi^2\right)}{16\pi^3}\ |M|^2$$

To find the cross section. Without going through all the calculations again (as we did in scattering matrix) we find a cross section annihilation for the photino limit as:

$$\sigma v_{rel} = \frac{\alpha^4}{pi}\ m_\chi^2 \,|\sum_i\ Q_i^4 \frac{c_i}{M_f^2}\ (1 - \frac{I(x_i)}{x_i^2}|^2$$

Here $\alpha = \frac{Q^2}{4\pi}$ (fine structure constant), $x_i = \frac{m_\chi}{m_i},\ c_i$ = quark colour factor (3 for quark and 1 for lepton), Q_i = electronic charge, m_i = mass of fermions, M_f = sfermion mass and m_χ = mass of neutralinos v_{rel} = relative velocity. There is one term that needs a bit more explaining. This term was borrowed from Greist and Giudice and other researchers who use the effective lagrangian to come to the same cross section as we have in this text (I will elaborate a little bit more in the next paragraph, but first I must explain this term).

$$I(x_i) = (\arcsin(x)^2),\ \ |x| < 1$$

$$I(x_i) = (\frac{\pi}{2} + i\log\left(|x| + \sqrt{x^2+1}\,\right)),\qquad |x| > 1$$

The way the expression for the matrix was found has already been done in a previous calculation of cross section and is quite straightforward. Next equation shows the matrix for figure 2 with the crossing symmetry included.

$$M_{fi} = Q^2\ ((\bar{v}(p3)\,\gamma^\mu\ u(p4))\frac{ig_{\mu v}}{q^2}\left(u(p1)\gamma_\mu\ \bar{u}(p2)\right)$$

Here $q^2 = (p1+p2)^2 = (p3+p4)^2$, as is common in QED. By doing a crossing symmetry where the final fermions now become o3 and p1, while the initial fermions are p2 and o4. This is calculated as $\left|M_{fi}\right|^2$ and inserted into the cross section to obtain the photino limit. It should be added that in the photino limit we remove the contributions from the bosons, and this gives us the expressions we want.

The final part of the question was to explain why annihilations into light fermions are suppressed. Here we mean light in context of mass, and not in terms of photonics/photons. Basically, this is due to something called helicity constraints. We have already explained that Neutralinos are Majorana fermions, and in the limit of zero relative velocity, they are in a relative S-wave (meaning that we have null relative orbital angular momentum). This means that because of Fermi statistics they must have spins that are of opposite direction and its total angular momentum is therefore also null. So, this infers that the two fermions $f\bar{f}$ in the final state have opposite spins as well. The configuration introduces a helicity factor in the probability decay into $f\bar{f}$ which is proportional to fermion mass M_f. From that we see that the Neutralinos decay into the fermions with the highest mass possible. A preference for the "final product" (using a chemistry language here) annihilation is the heavy quarks b and t quarks and the τ lepton, while the lighter fermions like u, d, s and c quarks and e and μ leptons are, in comparison, less preferred (μ here means muon, and must be viewed separately from the rest of the text, also α means something else in two different parts of the text).

Finally, it should also be added that direct decay into massless particles is not allowed.

Conclusion:

In this project about the possible Dark Matter candidate Neutralino, we calculated a matrix element and cross section for the process, $\tilde{\chi}f \rightarrow \tilde{\chi}f$. We assumed a Z-exchange dominance. Furthermore, we provided a numerical value for the cross sections of the entire previously mentioned Z-exchange and the cross section on a nucleon (here proton was used). Cross symmetry was used to calculate the Neutralino annihilation, $\chi\chi \rightarrow f\bar{f}$, and we found the photino limit. In the appendix a comparison between the cross sections found here and relevant literature values can be found.

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Appendix:

From the course coordinator in FY3464, Professor Michael Kachelriess, I was given a link by Kim Griest which also had found the cross section of the neutralino-fermion process.[12] However, we were told to only find the scattering where the Z-boson dominates. As suggested by Torsten Bringmann we will only consider the t-channel because of this.[13] Here in this appendix I will try to compare my result with what Griest got.

So, I will begin with writing down the result Griest got and compare with mine. The assumptions done by Griest will also be included here flowing with the text as we go along. But the first assumption is already specified and that we only consider the t-channel and with dominance from the Z-boson (meaning that we omit the Higgs-part).

The scattering term for the s-, t- and u-channels is:

$$\sigma = \frac{8\, G_F^2\, m_X^2\, m_q^2}{\pi\left(m_X + m_q\right)^2} \left\{ 3\, \left[(2a^2 + b^2 + c^2)\overline{x_q^2} + \frac{1}{2}(C_R - C_L)\,(Z_{13}^2 - Z_{14}^2) \right]^2 \right.$$

$$\left. + 4\overline{x_q^4}\; a^2 (b - c)^2 \right\}$$

Here $\overline{x_q^2} = \frac{m_W^2}{M_q^2 - m_X^2}$, where M_q^2 is the squark mass. This we look past and do not include because it is the s-channel. We are also told that s = $\left(m_X + m_q\right)^2$. This is also not included because it is the s-channel. The terms a and b represent the coupling constants for higgsino and photino, but we only want c = zino coupling constant. This term can be found in the Fermi constant, G_F. And obviously we have that c = g_Z in our expression. We are then left with the $(8\ G_F^2\left(m_X^2\, m_q^2\right)/\pi)$ where $G_F = \left(\frac{\sqrt{2}}{8}\right)\frac{g_Z^2}{M_Z^2 c^4}$. The rest is the photino limit. All of this of course leaves us with the following expression from Greist:

$$\sigma = \frac{g_Z^4}{4\pi}\frac{1}{M_Z^2}\left(\frac{1}{2}\ (C_R - C_L)\,(Z_{13}^2 - Z_{14}^2)\right)^2$$

And we have also not included the c-term from the Fermi constant because Griest emphasizes that it is in the non-relativistic limit. The final important aspect to include is something referred as the Weinberg angle.[14] By using this angle we get the relation between the W and Z boson as:

$$M_Z = \frac{M_W}{\cos\theta_W}$$

This expression is further compared with the one from Nihei et al. and we see that we get the same expression there, but with slightly different notation.[15] Our intention was however to compare Greist's expression with the one we found in this project. We refer to the derivation that led to equation (9) and rewrite it here for overview-purposes:

$$\sigma = \frac{1}{4\pi}\frac{g_Z^4}{m_Z^2}\,\frac{1}{4}\left(C_A^{\tilde{\chi}} C_A^f\right)^2$$

First of all, we obviously set $m_Z = M_Z$. So, that leaves us with the expression in the parentheses. We set $C_A^{\tilde{\chi}} C_A^f = C_A^{\tilde{\chi} f}$. From Thomson[16], we have the following relation: $C_A = (C_L - C_R)$ and from Nihei, we have that:

$$C_A = \frac{g}{2\cos\theta_W}\,\left(O_{ij}^{\prime\prime L} + O_{ij}^{\prime\prime L*}\right)$$

Where

$$O_{ij}^{\prime\prime L} = \frac{1}{2}\,\left(-N_{i3}\,N_{j3}^* + N_{i4}N_{j4}^*\right)$$

And this gives us (with N = Z obviously) where they use $-\,C_A$, so we get the expression gives as Griest if we take that into account. From these changes in notations we see that our results are the same as Griest and Nihei.