Exercise sheet 6

1. Stress tensor for the electromagnetic field.
Determine the stress tensor $T^{\mu\nu}$ of the free Maxwell field. Symmetrize $T^{\mu\nu}$, if necessary. Confirm that T^{00} corresponds to the energy-density ρ. Find the trace of $T^{\mu\nu}$ and the Equation of State (EoS) defined by $w = P/\rho$.
[Possible ways to find $T^{\mu\nu}$: i) use Noether's theorem, ii) use Newton's law, iii) convert $\rho = (E^2 + B^2)/2$ into a tensor law, iv) use that the stress tensor is the source of the gravitational field.]

2. Scalar QED.
a.) Write down the Lagrangian of scalar QED, i.e. a complex scalar field coupled to the photon via $D_\mu = \partial_\mu + \mathrm{i}qA_\mu$. Derive the Noether current and the current to which the photon couples (defined by $\Box A^\mu = j^\mu$).
b.) Find the vertices of this theory. [Pay attention to the sign of the momentum of scalar particles.]
c.) Write down the matrix element for "scalar Compton scattering" $\phi\gamma \to \phi\gamma$ and show that it is gauge invariant: I.e. that after replacing $\varepsilon \to k$, the matrix element vanishes.

3. Local U(1) transformation.
Show that the transformation law for the classical Langrangian $\mathscr{L}$ of a complex scalar field under a *local* U(1) transformation $\phi(x) \to \tilde{\phi}(x) = \mathrm{e}^{\mathrm{i}\alpha(x)}\phi(x)$ can be expressed as

$$\delta\mathscr{L} = (\partial_\mu \alpha) j^\mu \,,$$

where j^μ is the Noether current.

Solutions are discussed Tuesday, 25.02.19