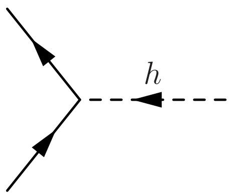
Formalities.
Solutions should be emailed latest Friday 27.03, at 15.00.

Executive summary.
Write a 3–4 page executive summary of the lectures (ending [and including] with spin 1/2). Omit derivations.

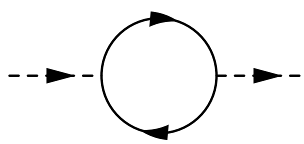
Higgs decay into fermions and the optical theorem.
In the Standard Model, the Higgs particle h is a scalar particle that interacts with all fermions via a Yukawa coupling y proportional to the fermion mass m, $y = \frac{1}{2}gm/m_W$,



$-\frac{1}{2}\,\mathrm{i}g\,\frac{m}{m_W}$

a.) Calculate the decay width $\Gamma(h \to \bar{f}f)$ of a Higgs particle with mass M into a antifermion-fermion pair (at tree-level).
b.) Show that a fermion loop leads to an additional minus sign in the Feynman amplitude.
c.) Consider the following contribution of fermions to the self-energy $\Sigma(p^2)$ of the Higgs,

$\mathrm{i}\Sigma^{\bar{f}f} =$



Use dimensional regularisation to calculate $\Sigma^{\bar{f}f}$ and show that

$$\Sigma^{\bar{f}f} = \frac{A}{\varepsilon} + B\left[C + \int_0^1 \mathrm{d}z a^2 \ln(a^2/\mu^2)\right]$$

with $a^2 = m^2 - p^2 z(1-z) - \mathrm{i}\varepsilon$. Note: In d spacetime dimensions, the Clifford algebra becomes $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} I_d$ with I_d as the d-dimensional unit matrix. Thus contractions change to

$$\gamma^\mu \gamma_\mu = dI_d, \qquad \gamma^\mu \not{a} \gamma_\mu = (2-d)\not{a}, \qquad \gamma^\mu \not{a}\not{b} \gamma_\mu = 4a\cdot b I_d - (d-4)\not{a}\not{b}. \tag{1}$$

However, it is standard to define $\mathrm{tr}(I_d) = 4$, which has the advantage that trace relations like $\mathrm{tr}[\not{a}\not{b}] = 4a\cdot b$ are unchanged.
d.) Determine the imaginary part $\Im\Sigma^{\bar{f}f}$ of the self-energy and show that the optical theorem holds, i.e. that $\Im\Sigma^{\bar{f}f} = M\Gamma(h \to \bar{f}f)$ for $p^2 = M^2$.
e.) Obtain $\Im\Sigma^{\bar{f}f}$ directly by "cutting the self-energy": Consider

$$\mathrm{i}\Sigma^{\bar{f}f}(p^2) = \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \cdots$$

for $p = (M, \mathbf{0})$; find the poles and apply the identity

$$\frac{1}{x \pm \mathrm{i}\varepsilon} = P\left(\frac{1}{x}\right) \mp \mathrm{i}\pi\delta(x)$$

to the q^0 integral in order to obtain the imaginary part.

Good luck!