Exercise sheet 5

Field-strength tensor and its dual.
Calculate $\tilde{F}_{\mu\nu}F^{\mu\nu}$, where $F^{\mu\nu}$ is the field-strength tensor of the electromagnetic field and $\tilde{F}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$ its dual. Show that this term can be written as $\tilde{F}_{\mu\nu}F^{\mu\nu} = \partial^\mu K_\mu$. What does this imply?

Casimir effect.
Repeat the calculation of the Casimir effect for a scalar field in 3+1 dimensions [Hint: If you use cutoff regularisation, convert the integration over transverse momenta into an integration over energy. However, it is more convenient to combine zeta function and dimensional regularisation, using then the definition of Euler's beta function $B(x,y) = 2\int_0^\infty \frac{t^{2x-1}}{(1+t^2)^{x+y}}\mathrm{d}t$.]

Feynman integral.
Study the appendix 4.A and familiarize yourself with the evaluation of Feynman integrals using DR. Check the relation $I_\mu(\omega,\alpha) = -p_\mu I(\omega,\alpha)$.

Renormalisation invariance of the propagator.
Derive analogously to Eqs. (4.83-85) an equation $\mathrm{d}m^2(\mu)/\mathrm{d}\mu = f(m^2)$ requiring that the propagator (4.33) is independent of the scale μ.

Solutions are discussed Monday, 18.02.19