The final exam (67%) of this course has been converted into a project. Deadline for the delivery of the project is May, 26 (the day of the planned exam). Three students at maximum can form a team. Submit a ranked list of favorite projects until April, 17. You may suggest also a suitable topic on your own – I'll see if it fits. Proposed projects:

1. Monte Carlo simulation for QCD:
   Study chapter 18.2 "DGLAP equations". Derive the two missing splitting functions in QCD. Develop a Monte Carlo program which solves the DGLAP evolution for a time-like gluon shower starting at a given $s = Q^2$ down to the cut-off scale Q_0^2 and derive the number of produced gluons as function of s. For simplicity use only gluons.

2. Higgs decays:
   Look up the Feynman rules relevant for Higgs decays in the SM, reading e.g. chapter 14 of the script. Calculate the decay widths; consider additional to the tree-level decays also the decays into gluons and photons. Plot the branching ratios as function of the Higgs mass.

3. Gravity as neutrino exchange:
   Feynman speculated that gravity is caused by the exchange of neutrino pairs. Calculate the corresponding loop diagram in the Fermi theory, and derive the resulting potential between static fermion sources.

4. Axial anomaly:
   Study the chapter 17.1 on the axial anomaly. Repeat the calculation of the anomaly including the $\mathrm{U}_V(1)$ transformation.

5. Partial-wave unitarity:
   Calculate $\sigma(e^-\nu_\mu \to \mu^-\nu_e)$ in the high-energy limit, both a) in the Fermi theory and b) accounting for the propagator of the W. Read the section about perturbative unitarity in chapter 14.3. Show that

   $$\frac{d\sigma}{d\Omega} = \frac{1}{s}\left|\sum_{l=0}^{\infty}\left(\frac{4\pi}{2l+1}\right)^{1/2} T_l Y_{l0}(\phi)\right|^2$$

   where T_l is a partial-wave amplitude and Y_{lm} a spherical harmonics. Find the values of s above which in the two cases unitarity is violated. What is your interpretation of the result?

6. Dark Matter;
   A popular dark matter candidate is the lightest neutralino $\tilde{\chi}_1$, a Majorana fermion which is a mixture of the supersymmetric partners of the photon, the Z boson and two neutral higgs bosons. Calculate the matrix element and cross section for $\tilde{\chi}f \to \tilde{\chi}f$ scattering, assuming that the Z exchange dominates. In the unitary gauge,

   $$\mathcal{L}_{Z\chi_1\chi_1} = \frac{g}{2c_W}(N_{13}^2 - N_{14}^2)\bar{\chi}_1\gamma^\mu\gamma^5\chi_1 Z_\mu\,,$$

with $N_{13}^2 - N_{14}^2 \in [0 : 1]$. Give a rough estimate of the numerical value of the cross section on a nucleon. Use crossing-symmetry to estimate the low-energy limit of the neutralino annihilation cross section into fermions, $\chi\chi \to \bar{f}f$. Explain why annihilations into light fermions are suppressed.

7. Gravity:
   Consider the scattering of a scalar particle (the Sun) and a photon via graviton exchange. Calculate the scattering amplitude and the cross section in the static limit $p^\mu = (M_\odot, \vec{0})$ for small-angle scattering ($k^2 = \theta = 0$ in the nominator) and show that it agrees with Einstein's prediction for light deflection by the Sun.

8. Brackets:
   Study chapter 9.3.2 "helicity methods". Apply this method to the process $\bar{q}q \to gg$.

9. Cosmological constant:
   Calculate the stress tensor $T^{\mu\nu} = (\rho, P, P, P)$ of the vacuum of real scalar field, using a) a momentum cutoff and b) DR. Derive the E.o.S. $w = P/\rho$ for the two cases. Check also the relation between the propagator at coincident points $i\Delta_F(0)$ and the vacuum energy density ρ derived in sec. 2.7.2 for the cases. Your interpretation of the results? [You can derive the expression for the pressure P using that

$$n = \frac{g}{(2\pi)^3} \int d^3p \, f(p) \tag{1}$$

$$\rho = \frac{g}{(2\pi)^3} \int d^3p \, E f(p) \tag{2}$$

$$P = \frac{g}{(2\pi)^3} \int d^3p \, \frac{p^2}{3E} \, f(p) \tag{3}$$