Exercise sheet 1

If some exercise is known to you, simply skip it.

1. Units.
a.) The four fundamental constants $\hbar$ (Planck's constant), c (velocity of light), G_N (gravitational constant) and k_B (Boltzmann constant) can be combined to obtain the dimension of a length, time, mass, energy and temperature. Find the four relations and calculate the numerical values of two of them.
b.) There exists three main variants of electromagnetic units, corresponding to different choice of k in the Coulomb law $F = kq_1q_2/r^2$. Which one do you suggest to use? Is an independent charge unit needed? Give k in three unit systems.
c.) Write the Thomson cross-section in SI, cgs and natural units; find the numerical value in cm^2.
d.) Assume that the density inside the core of a neutron star is $n = 0.1m_p^3$, where m_p is the proton mass. How big is number density, the energy denisty in cgs units?

2. Classical limit.
Sketch (without detailed calculation) why in the path integral the allowed paths dominate in the classical limit.

3. Step function.
Heaviside's step function $\vartheta(\tau)$ is defined by $\vartheta(\tau) = 0$ for $\tau < 0$ and $\vartheta(\tau) = 1$ for $\tau > 0$.
a.) Use Chauchy's residuum theorem to show that the integral representation

$$\vartheta(\tau) = -\frac{1}{2\pi \mathrm{i}} \lim_{\varepsilon \to 0} \int_{-\infty}^{\infty} \mathrm{d}\omega \frac{\mathrm{e}^{-\mathrm{i}\omega\tau}}{\omega + \mathrm{i}\varepsilon}$$

is valid.
b.) Show that $\mathrm{d}\vartheta(\tau)/d\tau = \delta(\tau)$.

4. Index gymnastics.
Splitt the arbitrary tensor $T^{\mu\nu}$ into its symmetric part $S^{\mu\nu}$ and its anti-symmetric part $A^{\mu\nu}$.
a.) Show that this splitting is invariant under Lorentz transformations, $x_\mu = \Lambda^\rho{}_\mu x_\rho$.
b.) Show that $S^{\mu\nu}A_{\mu\nu} = 0$.

5. Relativity of simultaneity.
Draw a space-time diagram (in $d = 2$) for two inertial frames connected by a boost with velocity β: What are the angles between the axes t and t', x and x'? Draw lines of constant t and t' and convince yourself that the time order of two space-like events is not invariant.

Solutions are discussed Monday, 21.01.19