Oblig 4: Trapping atoms

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1 Introduction

The Oblig is a term used for mandatory assignments at the University of Oslo. The project will investigate a magneto-optical trap (MOT) as seen on figure 2. The point is to introduce a model that can be justified by quantum-mechanical calculations and that captures the main features of the process.

Before actually beginning answering the questions in the Oblig, we take a closer look at the MOT in the subsection below. We also introduce some applications of the MOT despite it not really being necessary for the fulfillment of the task.

1.1 Magneto-Optical Trap (MOT) - theoretical working principle

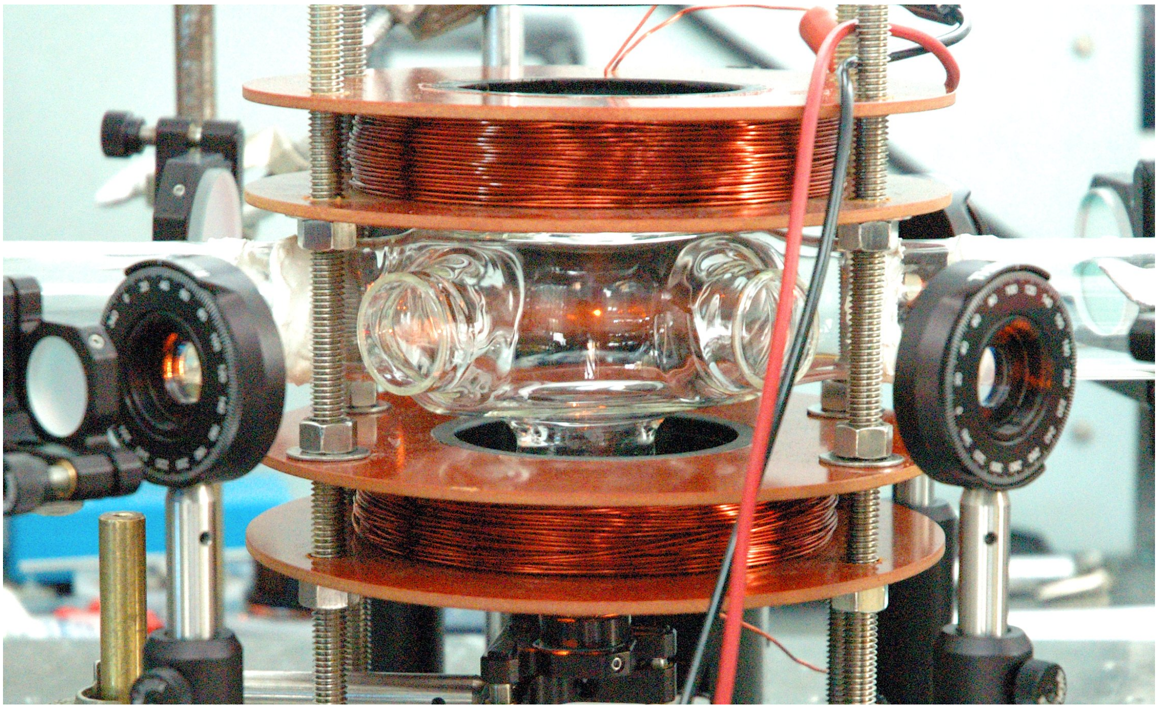

Figure 1: Illustration of a Magneto-Optical trap (MOT). The atoms are " in the center of the glass container. A laser enters from the sides, while a magnetic field is generated by the coils.

Our intention with this project is to use a simple model for how one trap and cools atoms. Cold atoms have longer interaction and observation times compared to atoms in room temperatures. This makes them easier to study. Today we have the capability to cool atoms down to -273,1 C (or 0 K). A MOT utilize techniques of laser-cooling with the influence of strong magnetic coils to collect samples of cooled and "trapped" atoms. A MOT reduces the speed of neutral atoms with a large amount before cooling them. They so go on to bring these atoms to a centralized location for use in experiments.

A typical MOT setup is built around an ultra-high vacuum chamber into which the neutral atom sample is contained. The magnetic coils are placed on opposing sides of the chamber, thus creating a centralized magnetic field through the the sample. In addition, laser light is reflected through the chamber and subjects the sample to absorb photons from the incident light. These absorptions cause the atoms to slowly lose momentum, as they spontaneously emit photons in all directions. The magnetic field then centralizes these cooled atoms. The result is a created cold atom cloud.

1.2 MOT applications

MOT can be used in a wide array of experiments. Among these are Bose-Einstein condensates, quantum computing and atomic clocks. A Bose-Einstein condensate is a group of atoms cooled to near absolute zero (meaning 0 Kelvin). At this point the atoms enter the same energy state and begin to behave as one single atom and a wave. One uses a MOT to reach the desired temperature for the atoms.

A quantum computer is a complex tool not to be reviewed here. But one common method in quantum computation is to use trapped neutral atoms as qubits (comparable to a normal computers two-bit system). The MOT is used to create cold atom cloud, where the atoms are then ready to be sent through quantum logic gates for computing purposes.
Atomic clocks are very precise measurement instruments meant to measure time. Modern optical atomic clocks utilize MOTs to trap single ions and neutral atoms to determine an extremely precise frequency measurement of electronic transition.

2 The Oblig

So, having explained what a MOT is, we will so go on to the answering the Oblig.

a) We will first make a sketch of U(x). U(x) is a potential with origin in the magnetic field and can be modelled as:

$$U(x) = U_0, \ |x| \geq x_0$$

$$U(x) = U_0 \frac{|x|}{x_0}, \ |x| < x_0$$

Other important relations worth remembering, from Electromagnetism, is that the potential is the integration of the electric field between two points. This gives us the figure 2 we want below. The sketch is done in Krita due to lack of scanning tools in the place I am currently quarantined at. As already said above, the

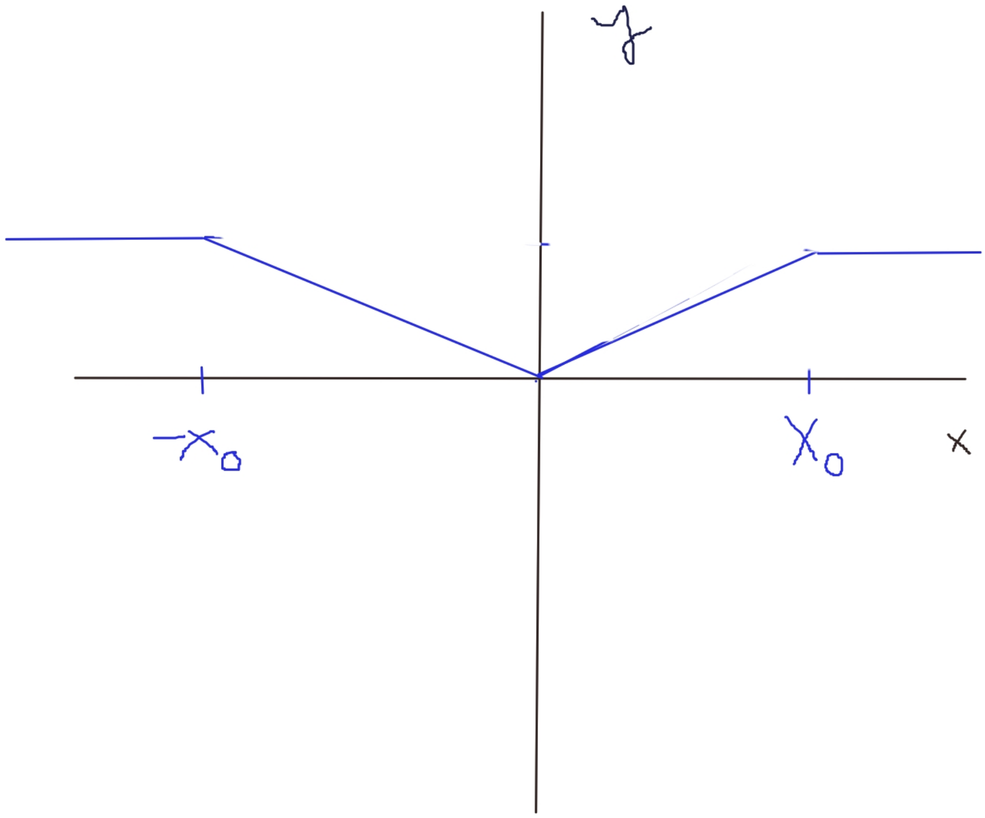

Figure 2: Schematic shows the sketch for the potential in the magnetic field

potential is the integral of the electrical field. When a charge is accelerated by an electrical field, then it is given kinetic energy. The kinetic energy is already seen as $\frac{1}{2}mv^2$. An integration then gives us the potential $U = mv$.

The equation then gives us a linear slope in the interval we are investigating. We know this is slope is positive when $|x| \geq x_0$ and negative when $|x| < x_0$, giving us the sketch seen on figure 2. While over x_0 there is a constant potential, so a straight line is drawn. The same can be said of course for the line for potential-values under x_0.

Equilibrium points are -x_0 and x_0, as well as x = 0. The stability at -x_0 and x_0 is low. These are in my view turning points (vendepunkter i Norwegian language) from mathematics. The stability at x = 0 is high because the equation has continuity there for the split function.

b) The relation for force and potential is given according to:

$$F(x) = -\frac{dU(x)}{dx} \tag{1}$$

So, introduced to our equation on page 3, we get the following relation for the force.

$$F(x) = U_0 \frac{1}{x_0},\ 0 < x \leq x_0$$
$$F(x) = 0,\ otherwise$$

For a force to be conservative, we must a force like, F = 0. This force is therefore not conservative while in the magnetic field, but conservative otherwise.

c) The problem wants us to find the velocity at x = $\frac{x_0}{2}$ and x = $2x_0$. On beforehand we are given a relation as: $v_0 = \sqrt(\frac{4U_0}{m})$. We use the law of conservation of energy and its form as a mathematical expression:

$$U(x_0) + K_0 = U(x_1) + K_1 \tag{2}$$

The actual derivation of the answer follows below.

$$-F_0 x_0 + \frac{1}{2} m v_0^2 = -F_0 x_1 + \frac{1}{2} m v_1^2 \tag{3}$$

$$0 + \frac{1}{2} m v_0^2 = U_0 \frac{x_1}{x_0} + \frac{1}{2} m v_1^2 \tag{4}$$

We set the initial force as zero because of the equation in the assignment preceding this one. It clearly specifies that the force should be zero.

$$\frac{1}{2} m (\frac{4U_0}{m}) = U_0 \frac{x_1}{x_0} + \frac{1}{2} m v_1^2 \tag{5}$$

$$2(\frac{2U_0 - U_0 \frac{x_1}{x_0}}{m}) = v_1^2 \tag{6}$$

We then proceed to insert the $x_1 = \frac{x_0}{2}$ giving us the final relation we want:

$$v_1 = \sqrt{\frac{3U_0}{m}} \tag{7}$$

And doing the same with $x_1 = 2x_0$ giving us:

$$v_1 = \sqrt{\frac{4U_0}{m} - \frac{4U_0}{m}} = 0 \tag{8}$$

A velocity = 0 means no potential increase or decrease, as can be confirmed by fig 2.

d) Now we have a similar expression as in the previous problem. But the sign before the expression is negative rather than positive. So we do the calculation again. We start with equation (6) here.

$$v_1 = \sqrt{-2\frac{(2U_0 - U_0\frac{x_1}{x_0})}{m}} \tag{9}$$

$$v_1 = \sqrt{\frac{-3U_0}{m}} \tag{10}$$

That is for $x_1 = -x_0/2$. For $x_1 = -2x_0$, we have the following result:

$$v_1 = 0 \tag{11}$$

e) We want to find the escape velocity here. The english language version of Wikipedia has a nice article concerning escape velocity and how to approach this type of problems. Therefore it is recommended that the reader also reads that article. It gives us a nice way to find the escape velocity.

$$-F_0x_0 = -F_0x_1 + \frac{1}{2}mv_1^2 \tag{12}$$

$$-F_0x_0 + F_0x_1 = \frac{1}{2}mv_1^2 \tag{13}$$

$$F_0(x_1 - x_0) = \frac{1}{2}mv_1^2 \tag{14}$$

$$F_0 = \frac{1}{2}\frac{mv_1^2}{(x_1 - x_0)} \tag{15}$$

Equation (15) is for when the kinetic energy is equal to zero. Second part demands that K = $\frac{U_0}{2}$ at x = 0. We then redo the calculation with this parameter.

$$-F_0x_0 + \frac{U_0}{2} = -F_0x_1 + \frac{1}{2}mv_1^2 \tag{16}$$

$$F_0(x_1 - x_0) = \frac{1}{2}mv_1^2\frac{U_0}{2} \tag{17}$$

$$F_0 = \frac{\frac{1}{2}mv_1^2 - \frac{U_0}{2}}{(x_1 - x_0)} \tag{18}$$

Obviously here the v_1^2 varies according to the assignment c) and d).

f) Problems f) wants us to find whether a force F given as:

$$F = -\alpha v \tag{19}$$

is conservative. This force is on an atom in the trap and is due to a continuous adsorption (and emission) of photons. The force acts in the range $-x_0$ < x < x_0. v is the velocity of the atom and α is a constant.

From the book used in course, "Elementary Mechanics using MATLAB", we have this following relevant statement: Non-conservative forces are typically forces that not only depend on position, but for example on velocity. Based on that statement from the syllabus and equation (19), we conclude that the force is not conservative.

g)

$$a = 0,\ for\ |x| \leq x_0$$
$$a = \frac{-\alpha v}{m},\ for\ x\ =\ 0$$
$$a = \frac{\frac{-U_0}{x_0} - \alpha v}{m},\ for\ 0\ <\ |x| <\ x_0$$

We know that the movement is constant for |x| > x_0 from the sketch in figure 2. At x = 0 it is only influenced by the force from the trap. While for 0 <|x|< x_0, we have the expression found in assignment b) plus the velocity. According to the second law of thermodynamics then the charge in (mechanical) energy is equal to the work done by non-conservative forces.

The initial conditions are velocity and position.

h) Below follows the MATLAB I made to answer this question.

```
U0 = 150;
m = 23;
x0 = 2;
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alpha = 39.48;

t = linspace(0,10);
dt = (t(2)-t(1));

x = zeros(length(t));
v = zeros(length(t));
a = zeros(length(t));

x(1) = -5;
v(1) = 8;
a(1) = 0;

for i = 1:(length(t)-1)
    f = -(alpha*v(i))./m;
    z = -((U0./m)./x0)*(abs(x(i))./x(i));
    a(i+1) = (f + z);
    v(i+1) = v(i) + dt*(a(i+1));
    x(i+1) = x(i) + dt*(v(i+1));
    switch x(i)
        case {0}
            a = f;
        case {linspace(0,x0)}
            a = f + z;
        otherwise
            a = 0;
    end
end

plot(t,x);
xlabel('t');
ylabel('x');
title('x vs t');
```

i) It says to find the motion with input parameters v_0 = 8.0 and x = -5. The plot follows below in figure 3 The figure shows that the initial speed is so low that the trap is able to catch the atom. The rest of the motion represents that.

j) Same as the previous assignment, but with different parameters. Desired plot can be found in figure 4 The figure shows that the initial speed is to high for the trap to catch the atom.

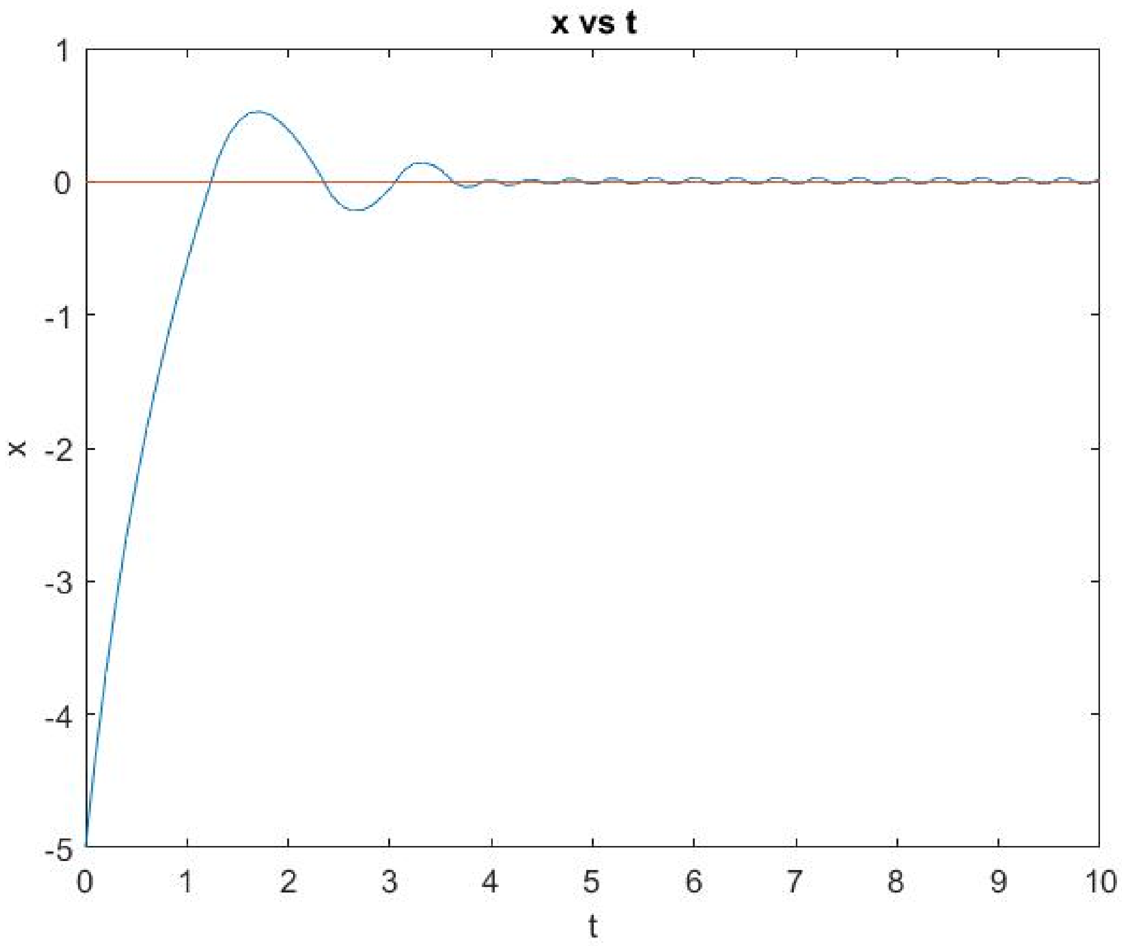

Figure 3: Plot shows the motion with input parameters form the text of the assignment

k) For the final answer to a problem in this Oblig, we must find the maximal initial velocity the atom may have and still be trapped. I do this analytically. We make use of the equation for distance:

$$v^2 = 2as - v_0^2 \tag{20}$$

From previous assignments we've already found the acceleration a, and v_0 is zero because it stands still. So that gives us after some calculations:

$$v^2 + 2 * (2 * 39.48/23)\,v - 2 * (150/23) = 0 \tag{21}$$

And the answer therefore becomes x = 8.41594.

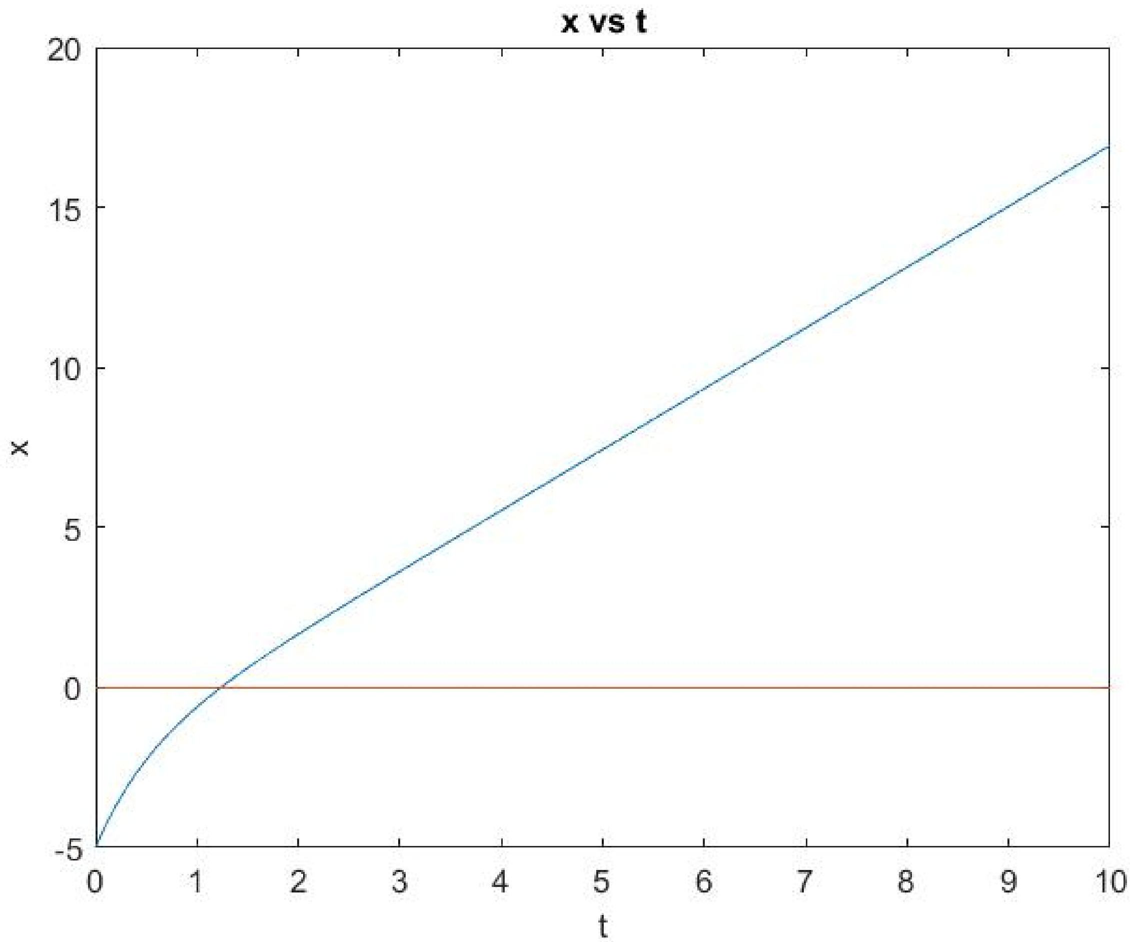

Figure 4: Plot shows the motion with input parameters from the text of the assignment