Home Exam 2020 - FYS3120: Classical Mechanics and Electrodynamics

Furkan Kaya

March 2020

1 Introduction

This is the home exam in Classical Mechanics and Electrodynamics course lectured at the University of Oslo done by Furkan Kaya. Its subject code is FYS3120. Due to the covid-19 virus pandemic all across the world, the work was done without any cooperation from classmates or supervisors. However, resources from Internet was used. And also Krita was used since I am sort of quarantined and I do not have the possibility to scan pages at my current location in a student dormitory.

2 Question 1: Coneheads

2.1 a)

The first assignment in this home exam has several question posed that needs to be answered. We must draw a figure of the set-up specified in the assignment, find the number of degrees of freedom, identify the appropriate generalised coordinated and relate these to the Cartesian coordinates.

Lets start with the drawing of the figure. This was done in the program called Krita and follows below in figure 1. I should also provide an explanation to figure 1. m represents the mass going in the path inside the hollow cone, α is the angle shown in the figure and required in the assignment. The particle's position is given by the polar coordinates (r, θ) of the projection of the position vector inside the x,y plane. The acceleration due to gravity is g in the negative z-direction.

Second task was to find the number of degrees of freedom. We will use the following equation in order to do so:

$$d = 3N - M \tag{1}$$

where d = degrees of freedom, 3N = number of variables and M = number of constraints. For more background, I refer to the syllabus by Leinaas. The particle can move in the x, y, z plane (standard Cartesian), but its movement is constrained by the cone. That gives us (by inserting into equation (1), the following:

$$d = 3 * 1 - 1 = 2 \tag{2}$$

2 degrees of freedom. This means that we can describe the system by standard polar coordinates, r and θ. In the drawing we had several parameters, they are as following. θ = around the cone, α = half-angle at the tip, r = the distance from the particle to the axis.

We have that if the particle's distance from the axis is r, then its height is r/(tan θ) and its distance along the cone is r/(sin θ). Having found the generalised coordinates above in the polar coordinates, we will therefore conclude this assignment by relating them to the Cartesian coordinates.

$$x = \frac{r}{sin\,(\theta)} \tag{3}$$

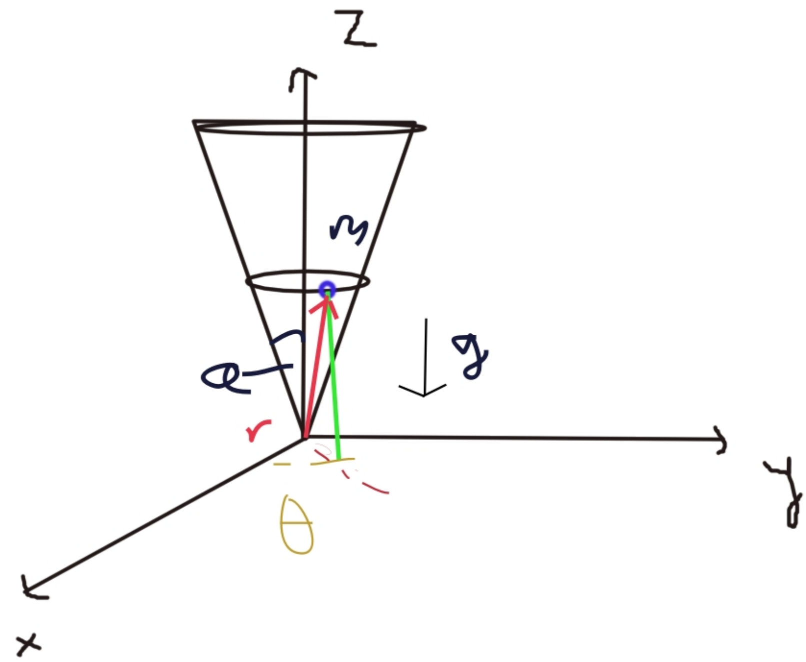

Figure 1: Schematic shows the figure of a hollow cone pointing downwards with all the parameters suggested in the text of the assignment. It includes the 3 axis, Polar coordinates, mass m, angle α and the gravity.

$$y = \frac{r}{cos\,(\theta)} \tag{4}$$

$$z = \frac{r}{tan\,(\theta)} \tag{5}$$

2.2 b)

Assignment b) wants us to find the Lagrangian. From Leinaas, we know that this is given by the relation:

$$L = T - V \tag{6}$$

representing that the Lagrangian (L) is given by the difference between kinetic (T) and potential (V) energy.

We know that the kinetic energy is given by $\frac{1}{2}mv^2$ and the potential energy in similar fashion is given by mgh. Breaking the velocity into components up long the cone and around the cone, we get for the velocity the following:

$$v^2 = \frac{\dot{r}^2}{sin^2\,(\alpha)} + r^2\dot{\theta}^2 \tag{7}$$

We have also found parts of V in the previous assignment by finding the relation between z and the polar coordinates, so this then leads us to the Lagrangian below:

$$L = \frac{1}{2}m(\frac{\dot{r^2}}{sin^2(\alpha)} + (\dot{r}\theta)^2) - mgr\frac{1}{tan\,(\alpha)} \tag{8}$$

Knowing from elementary mathematics that $cot\ x = \frac{1}{tan\,x}$, we find that the final version of the equation is written as:

$$L = \frac{1}{2}m(\dot{r^2}\,(1 + cot^2\,(\alpha)) + (r\dot{\theta})^2) - mgr\ cot\,(\alpha) \tag{9}$$

2.3 c)

The question says to find two conserved quantities in the system. Furthermore, we should show that they are conserved and what physical properties they represent. The two conserved quantities we find in the system are the angular momentum and forces. Angular momentum is the rotational equivalent of linear momentum. It is an important quantity physics because it is a conserved quantity, meaning that the total angular momentum of a closed system remains constant. We must then compare this notion with the figure we have drawn in figure 1. On that figure we can clearly see that the system is closed because we have constraints in the x,y,z-plane due to the cone. And the gravity pulls the particle down, so that it won't leave its path upwards and eject out of the system. The angular momentum is represented by θ in the equations above.

We also have conservation of forces in the system. This phenomena occurs when the mechanical system is in static equilibrium. Meaning that there is a balance between the forces that act on each part of the system, so that it has no movement. One particular aspect associated with this phenomena and relevant for our system, is that if we have a particle travelling in a closed loop, then the total work done by a conservative force is zero. As we can see on figure 1, the system is both closed and the movement is in a closed loop. And the forces are thereby conserved. Gravitation is an another example of a conservative force seen in the system.

After we've found the equations of motion in the next assignment, I will relate the conserved quantities quantitatively as well.

2.4 d)

The equation of motions are found for r and θ respectively. They follow Lagrange's equation given according to:

$$\frac{d}{dt}(\frac{\partial L}{\partial\dot{\theta}}) - \frac{\partial L}{\partial\theta} = 0 \tag{10}$$

Equation (10) is obviously only for θ, but r follows the same procedure.

We begin with θ and find the different parts in equation (10) separately.

$$\frac{\partial L}{\partial \theta} = 0 \tag{11}$$

$$\frac{\partial L}{\partial \dot{\theta}} = \frac{1}{2} m (r\dot{\theta})^2 = mr^2 \dot{\theta} \tag{12}$$

$$\frac{d}{dt} \frac{\partial L}{\partial \theta} = mr^2 \ddot{\theta} \tag{13}$$

that finally becomes in Lagrange equation setup from equation (10):

$$mr^2 \ddot{\theta} = 0 \tag{14}$$

This equation expresses the conservation of angular momentum from the previous assignment.

Second equation of motion considers the r parameter. We follow the same procedure as above. All variables are included below.

$$\frac{\partial L}{\partial r} = mr\dot{\theta}^2 - mg\, cot\,(\alpha) \tag{15}$$

$$\frac{\partial L}{\partial \dot{r}} = m\dot{r}\,(1 + cot\,(\alpha)) \tag{16}$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{r}} = m\ddot{r}\,(1 + cot\,(\alpha)) \tag{17}$$

leading to when inserted into the Lagrange equations from (10).

$$m\ddot{r}\,(1 + cot\,(\alpha)) - mr\dot{\theta}^2 - mg\, cot\,(\alpha) = 0 \tag{18}$$

This equation represents the F = ma statement for the x-direction. We basically divide the expression by sin α and get something along the lines of:

$$\ddot{x} = (r\dot{\theta}^2)\, sin\,(\alpha) - g\, cos\,(\alpha) \tag{19}$$

Which is the same as Newtons second law for the system.

2.5 e)

It is here demanded that we must plot the phase space of the system. This is done in MATLAB and both code and plots follow below.

3 Question 2: The pi-factory

3.1 a)

Draw a figure of the setup from the assignment and answer the question about the number of collisions when M = m. The figure is seen on figure 2. Before investigating the amount of collisions, we must briefly mention that the collisions are elastic due to there being no friction. Furthermore, the component with the mass m stands still and has no momentum, while the component with mass M has a momentum because it is moving forward. Theoretically, some of the momentum from M will transfer to m, so that m will move forward and collide with the wall. With momentum being conserved I would say that there will be infinite amount of collisions because there is no loss off momentum from the wall or the blocks.

3.2 b)

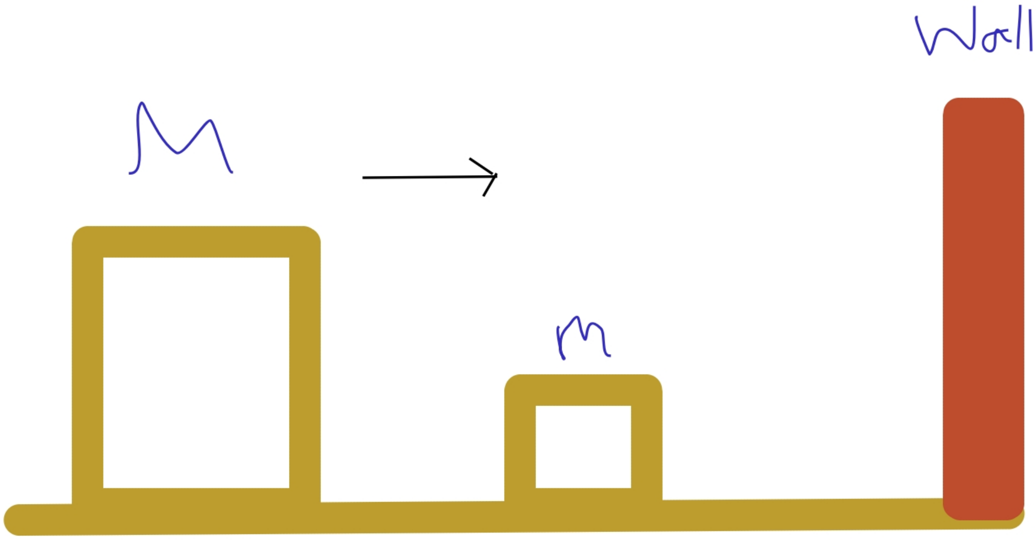

Figure 2: Schematic shows the setup as required in the text. We can see two components with masses M and m respectively. On the far-right there is a wall with infinite mass. Also worth noticing is the momentum for the component to the far-left.