FYS3120 Classical mechanics and electrodynamics
Final exam – Spring term 2020

May 20, 2020

Important information:

- Your answers are to be submitted electronically as pdf-files in Inspera, either generated from LaTeX or scanned, at the latest June 10 at 09:00 local Oslo time.
- This deadline is absolute, but you may submit multiple times. Only your last submission will be evaluated.
- This final exam accounts for roughly 80% of the total grade in FYS3120.
- As this is a home-exam you are free to use any sources of information you may want, and you may collaborate with other students on solving the problems. However, the text of the submitted answers must be your own, and the usual rules of plagiarism apply. (We may check answers for similarities.)
- The best possible score on this exam is 80 points. Up to four points will be given for clear, concise and well presented answers, including appropriate figures and/or diagrams.
- You may give your answers either in English or Norwegian.
- Good luck!

Question 1 Mirror, mirror

Assume that a source at rest in reference frame S sends a light signal with frequency ν towards a mirror with mass m and velocity v in the positive z-direction. The light is reflected back to the source.

a) Draw a sketch of the set-up including the reference frames involved. [3 points]

b) Find the relationship between the frequency of the emitted and reflected light observed in S. In this question only you can assume that in the rest frame of the mirror the frequency of the incoming and reflected light is the same. [6 points]

c) Given that light consists of photons, write down the four-vectors for an emitted photon, p_g^μ, a reflected photon, $p_g'^\mu$, and the mirror, p_m^μ, in terms of their respective relativistic momenta, $\vec{p}_g$, $\vec{p}_g'$, and $\vec{p}_m$. [5 points]

d) Find $p_g^\mu + p_m^\mu$ and $p_g p_m$. [5 points]

e) Show that the following relationship holds between the four-momenta

$$(p_g + p_m)p_g' = p_g p_m. \tag{1}$$

Assume that the mass of the mirror does not change in the interaction. [6 points]

f) Find the momentum of the reflected photon in terms of the momentum of the emitted photon. [6 points]

g) Assuming that the momentum of the photon is very small compared to the momentum of the mirror,[1] compare this to the result you found in **a)**. [4 points]

Question 2 Scattering

We start by considering a plane electromagnetic wave given by

$$\vec{E}(\vec{r}, t) = \vec{E}_0 \cos(\vec{k} \cdot \vec{r} - \omega t), \tag{2}$$

where $\vec{E}_0$ is a constant electric field amplitude and the wave propagates in the direction $\hat{n} = \vec{k}/k$. This wave impinges upon a free electron initially at rest.

a) Find the energy current density of the plane wave in terms of the electric field. [5 points]

[1] That would otherwise be a bloody impressive photon!

b) Assuming that the velocity v of the electron is always much less than the speed of light, $v \ll c$, find the motion, $\vec{r}(t)$, of the electron. [8 points]

c) Find the criterion we need to impose on the field for $v \ll c$ to be true. [2 points]

d) Find the power P radiated by the electron. Express your answer in terms of the **classical electron radius**

$$r_0 = \frac{e^2}{4\pi\epsilon_0 m_e c^2} = 2.8179 \cdot 10^{-15}\,\mathrm{m}. \tag{3}$$

[4 points]

In classical physics we can define the scattering cross section σ of a particle as the effective area of the incoming (electromagnetic) energy density that the particle eats up and spreads, *i.e.*

$$\langle S \rangle \sigma = \langle P \rangle, \tag{4}$$

where $\langle S \rangle$ is time-averaged magnitude of Poynting's vector (the energy current density) and $\langle P \rangle$ is the time-averaged dispersed power.

e) Find the unit of the scattering cross section. [2 points]

f) Find the scattering cross section of a free electron, also called the **Thomson cross section**, and evaluate it numerically. [5 points]

g) Now, let us consider an electron bound in some kind of matter. We model the binding by a harmonic oscillator restoring force plus a dampening force to represent heat loss in matter:

$$\vec{F} = -m_e\omega_0^2\vec{r} - m_e\Gamma\dot{\vec{r}}. \tag{5}$$

Here ω_0 represents the strength of the bond and Γ is the decay constant of the electron oscillations due to the heat loss. Show that the corresponding cross section for a bound electron is

$$\sigma = \frac{8\pi}{3} r_0^2 \frac{\omega^4}{(\omega^2 - \omega_0^2)^2 + \omega^2\Gamma^2}. \tag{6}$$

Hint: $\cos\omega t = \mathrm{Re}(e^{i\omega t})$. [8 points]

h) Make a plot of the cross section of both the free and the bound electron as a function of the frequency of the incoming wave. Assume that the binding potential is parametrised by $\omega_0 = 2.0 \cdot 10^{16}\,\mathrm{s}^{-1}$, and that the value of the decay constant for the bound electron is $\Gamma = 6.0 \cdot 10^{15}\,\mathrm{s}^{-1}$. [3 points]

i) Use the above to explain why the sky is blue. [4 points]