FYS4505 - Radiation – matter interaction

Autumn term 2018

Problem 1: Stopping power

For a heavy charged particle with mass M the energy loss is given by the Bethe-Bloch formula

$$-\frac{dE}{dx} = \rho K \frac{Zz^2}{A\beta^2}\left[ln\left(\frac{2m_ec^2(\beta\gamma)^2T_{max}}{I^2}\right) - 2\beta^2\right] \tag{1}$$

where $K = 2\pi N_a r_e^2 m_e c^2$ and we have neglected the density and shell corrections. By use of relativistic kinematics, one can show that the maximum kinetic energy transfer is equal to

$$T_{max} = \frac{2m_ec^2(\beta\gamma)^2}{1 + \frac{2m_e}{M}\gamma + \frac{m_e^2}{M^2}} \tag{2}$$

a) Calculate the stopping power of a 5 MeV alpha particle in silicon and germanium.

b) Write down the Bethe-Bloch formula for electrons, and explain the difference from equation (2). And as in equation (2), you may drop the density and shell corrections.

c) Calculate the stopping power of a 5 MeV electron in silicon and germanium. Compare with the answer you got for the alpha particle, and comment on the result.

Problem 2: Shielding calculation using Bethe-Bloch formula

We want to build a particle accelerator, and since radiation can be harmful we need to consider shield people from the radiation. Therefore we need collect information about the interaction of charged particles with the material we want to shield with. Let us say we want to build a wall for shielding of the surroundings, and we use a beam of protons in the range 0.1 to 10 Gev.

a) There are two proposed materials for the wall. Elemental aluminum and Portland concrete. Discuss briefly which material you suggest and choose that material for the rest of the task. Hint: The composition of portland concrete is given in `https://physics.nist.gov/cgi-bin/Star/compos.pl?matno=144`. Verify whether the listed mean excitation energy I of 135.2 eV can be explained by the Bragg rule, which states that the stopping power of a compound can be estimated by the linear combination of it's constituents. See also `http://www.srim.org/SRIM/Compounds.htm`.

b) Plot the energy loss of the proton as a function of incoming energy. In the previous exercise we neglected the corrections in the Bethe-Bloch formula. Evaluate the need to include the corrections for this exercise, by plotting the energy loss with and without the density correction. Explain the features of the plot and give the minimum ionization energy.

c) Plot the particle range in the material as a function of the incoming energy. How thick a wall do you need to stop a proton with energy 10 GeV.

d) Assume now that your wall was too thin, and the proton comes out with an energy of 10 MeV. How far does the proton travel until it has lost all its energy?

Problem 3: Neutral particle interaction

The behaviour of neutral particles in matter is quite different from that of charged particles.

a) Derive the exponential intensity relation for photons

$$I(x) = I_0 e^{-\mu x} \tag{3}$$

Hint: Divide the material into small slices of thickness Δx, then you evaluate the change in intensity as the photons move through that slice. Use that from quantum mechanics the interaction of photons with the material is probabilistic, so (Probability of one interaction) $\propto \Delta x$. Which means that the change in intensity should also be proportional to Δx.

More information on the X-ray mass attenuation coefficients can be found from NIST, `https://www.nist.gov/pml/x-ray-mass-attenuation-coefficients`

b) How thick should a lead shield be in order to reduce the intensity of 600 keV photons by a factor of 1/e?

c) Depending on the energy, the interaction between neutrons and matter varies. Describe the different interactions and in which energy range they dominate.

Problem 4:

a) Describe what a cross section is.

b) Describe the mechanism behind Cherenkov radiation.

c) Describe the mechanism behind Bremsstrahlung.

Problem 5: Compton Scattering

Compton scattering is the scattering of photons on electrons. A common simplification is to assume that the electron is free and at rest before scattering.

a) Use relativistic kinematics to show that the Compton-wavelength formula is given by

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - cos\theta) \tag{4}$$

Hint: Choose a frame where the electron is at rest, and write down the four momenta of each particle. By energy and momentum conservation you should have: $P_\gamma^\mu + P_e^\mu = \bar{P}_\gamma^\mu + \bar{P}_e^\mu$. For the scattered electron use the Lorentz invariant quantity $(\bar{P}_e)^\mu(\bar{P}_e)_\mu = m_e^2$, and for photons we have the quantity $(P_\gamma)^\mu(P_\gamma)_\mu = 0$.

b) What is the energy of a 1 MeV photon after Compton scattering with scattering angle of 30 and 90 degrees? What is the maximum energy loss of the 1 MeV photon?

c) A gamma ray of 2 MeV is incident on a detector, undergoes two sequential Compton scatterings, and then escapes. If the scattering angles are respectively 30 and 60 degrees, how much energy is deposited in the detector, i.e how much energy does the photon loose?

d) Determine the energy of the Compton edges of 60 keV, 1.332 MeV and 2.164 MeV photons. Also find the back-scatter energies at those given photon energies.

Further References

- Kraan2015, Range verification methods in particle therapy: underlying physics and Monte Carlo modeling, https://doi.org/10.3389/fonc.2015.00150
- Particle Data Group. The Review of Particle Physics on the passage of particles through matter. Version of 2018: http://pdg.lbl.gov/2018/reviews/rpp2018-rev-passage-particles-matter.pdf