FYS4505/9505 - Unfolding-Gamma Spectrum

Autumn term 2018

Problem 1: Introduction

When a gamma ray interacts with matter it will interact in several different ways. For each process, the interaction probabilities depend on the detector material and the incident gamma ray energy E_γ. At a typical E_γ energy in nuclear physics of 5 MeV we may therefore find observe several peaks at different energies in the measured spectrum.

a) Explain the origin of the full energy peak, single escape peak, double escape peak, 511 keV peak, back-scatter peak and Compton background.

Problem 2: The response matrix

It is now quite common to derive the interaction probabilities from Monte-Carlo simulations of your detector system. Here we have simulated an experiment with a toy-model of OSCAR, the $LaBr_3$(Ce) scintillator array at the Oslo Cyclotron Laboratory, and we derive the number of counts in the different structures listed in Problem 1. The results are given in "Peaks.dat", which has following structure:

```
# Eg[MeV?] nEvents nCounts cntFE cntSE cntDE cnt511 cntRest cntRestNoThres
200        1e+06   302742  143660 0      0      0      159082 156932
[...]
```

where the first column lists the incident gamma-ray energy (attention, is is not sorted), then follows the total number of events simulated, and the number of counts in each peak. The integral number of the remaining spectrum, so mainly the Compton background, is given by the last column, *cntRestNoThres*. In this problem you should create the *response matrix* **A** of the detector system, which transform the true spectrum of gamma-rays **x** into the observed spectrum **y** by $\mathbf{y} = \mathbf{A}\mathbf{x}$. The entries A_{ij} are thus the probability to observe a count in bin i, if generated in bin j.

a) Plot the spectrum of gamma rays with an incident energy i) 800 keV and ii) 2200 keV. At first we neglect the Compton events. Comment on the spectrum and what it tells you about how the incident gamma deposits its energy in the detector.
Hint: Remember to take into account the detector resolution, which will smooth the spectrum. For this exercise you may assume a Gaussian smoothing with an energy dependent FWHM given by following equation:

$$FWHM(E) = \sqrt{p_0 + p_1 E + p_2 E^2}, \tag{1}$$

with p =([1.13e-03,3.99e-19,2.05e-04]).

b) Use the energy distribution of the scattered electrons derived from the Klein-Nishina formula (eq. (2.209) and (2.210) in Leo) to estimate the energy distribution of Compton scattering events in the detector. Create a new plot, where you have added the Compton background to the response. An example of a normalized response function generated from this recipe is given in Fig. 1.
Hint: I had to manually introduce a cutoff at the Compton edge to receive a "physical" shape of the Compton background. Remember to normalize the resulting Compton distribution to the number of counts specified in *cntRestNoThres*.

c) How would the response for a incident photon energy of 2100 KeV look like? Hint: The closes energies for which we provided the number of counts are 2000 and 2200 KeV. It is difficult to

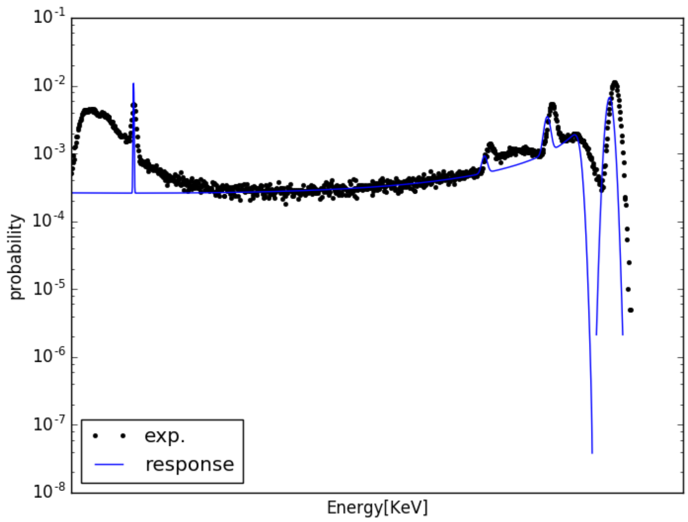

Figure 1: A typical normalized response function generated according to Problem 2 and some experimental data.

interpolate the spectrum of these neighbouring directly (think about why), however, you may interpolation the number of counts in each (peak) structure and then create a new spectrum from this.

d) Unfolding is most easily performed with a square response matrix. The spectrum that you will be given in Problem 3 has a bin width of 5 keV. Create a response matrix with 50 keV binning with the bin center of the first bin at 2.5 keV. Note that Monto-Carlo simulations of the detector response are computationally expensive, so we have calculated the responses only on a 200 keV grid.

e) Finally, if you did not already do this, make sure that the response to each incoming incoming gamma-ray is normalized to the efficiency to detect this gamma, $\epsilon = \frac{nCounts}{nEvents}$.

f) Looking at Fig. 1, how would you judge the quality of the response function we have generated? Do you have ideas on the sources of the discrepancy between the simulation and the experimental data?

Problem 3: Unfolding methods

We have now established a good overview of how the gamma-rays deposit their energy as they interact with the detector. We can now turn ourselves to the more challenging inverse problem, where we know the measured spectrum (also called raw spectrum) and we're interested in the ”true” physical distribution that created this spectrum. A common group of methods is called unfolding, where we try to obtain the ”true” physical distribution ”directly”. In this exercise we will look at the conceptually easiest method for unfolding, matrix inversion, and the challenges that come along with it . I strongly recommend reading the ”Data unfolding” presentation by Volker Blobel, that he gave at the Terascale Statistics Tools School in Spring 2010. You will find good examples in there.

In general the problem is to solve the Fredholm integral equation of first kind,

$$\int dx \boldsymbol{A}(y,x) f(x) = g(y)$$

where $f(x)$ is the true distribution, $g(y)$ is the measured distribution and $A(y,x)$ is the response function. By discretization the integral equation can be expressed as a linear matrix equation

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{y}$$

Where $\boldsymbol{x}$ is the histogram of the true variable, $\boldsymbol{y}$ is the histogram of the measured variable and $\boldsymbol{A}(\boldsymbol{y},\boldsymbol{x})$ is the response matrix. If we now multiply the equation with the inverse response matrix $\boldsymbol{A}^{-1}$ we get the true spectrum

$$\boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{y}$$

1. Let us try to get more familiar with the concept by a very simple example. Assume that you have only two bins, and the response matrix A is given as

$$\boldsymbol{A} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{pmatrix}., \tag{2}$$

so A_{12} is the probability to observe counts in bin 1 that originate from bin 2. The observed spectrum is given as

$$\boldsymbol{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 64 \\ 36 \end{pmatrix} \tag{3}$$

2. Try to to perform the same unfolding on the measured spectrum *lineXXkeV.dat*. What to you think was the energy of the incident gamma-ray(s)? Hint: If you did not complete the last task, you may use the response matrix provided as a pickled numpy array, *response.npy*.

3. Something went wrong in the previous subtask (at least hopefully.) Still, you may try to find the pseudo-inverse of the response matrix, eg. by its singular-value decomposition via *numpy.linalg.pinv*. Plot the observed spectrum for several cutoff parameter *rcond* of small singular values. Which one, if any, would you think works best? Can you think of any (ad-hoc) fixes that could improve the results?

4. If we have time left at the end of the semester, we will introduce an iterative unfolding procedure, that avoids many of the challenges that you have seen with matrix inversion here. If you are interested in it already now, you may want to read/try *Pyunfold*, which is based on G. D'Agostini. A multidimensional unfolding method based on bayes' theorem. *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, 362(2-3):487–498, aug 1995. Alternatively, if you're using the *Oslo Method*, you will probably perform unfold with another iterative method, M Guttormsen, T.S Tveter, L Bergholt, F Ingebretsen, and J Rekstad. The unfolding of continuum γ-ray spectra. *Nucl. Instrum. Methods Phys. Res. A*, 374(3):371–376, June 1996.