FYS4505 Radiation-matter interaction

By Furkan Kaya

Problem 1 a) Stopping power of α-particle

In equation (1) $-\frac{dE}{dx}$ is known as the stopping power. The range is defined as the distance a particle moves in a medium before all energy is lost. By considering the ~~most important~~ parameter's, we get:

$$-\frac{dE}{dx} = 0{,}3054 \cdot \frac{1}{2} \cdot \frac{2^2}{(0{,}0517)^2} \cdot$$

$$\left(\ln\left(\frac{2{,}739 \cdot 10^{-3}\ MeV}{10}\right) - 0{,}0517^2 \right) \cdot \rho$$

Leading us to for Silicon:

$$\frac{dE}{dx} = 1875{,}08 \cdot \rho = 1875{,}08 \cdot 2{,}329$$

$$\frac{dE}{dx} = 4367{,}06\ MeV/cm$$

For germanium, we get:

$$\frac{dE}{dx} = 1875{,}08 \cdot 5{,}323 = \underline{\underline{9981{,}05 \text{ MeV/cm}}}$$

b) <u>Bethe - Bloch formula for electrons:</u>

From the slides to the course it is given that the equation for electrons is:

$$-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{1}{\beta^2} \left(\ln \left(\frac{\tau^2 (\tau + 2)}{2 (I/m_e c^2)^2} \right) + F(\tau) - \delta - 2\frac{C}{Z} \right) \quad (3)$$

where τ is the kinetic energy of a particle in units of $m_e c^2$.

The difference between (2) and (3) is that the heavy charged particle M is not included in (3). This is due to the small energy of electrons (in the keV-scale compared to MeV for α-particles).

The $F(\tau)$ is also not seen in (2).
This term is different for e^- and e^+.

c) Stopping power of electron

For electrons with energy higher than 100 keV, the velocity is close to the speed of light, meaning $\beta \approx 1$. That gives us.

$$\frac{dE}{dx} = \rho \, (2 \text{ MeV cm}^2/\text{g})$$

For Silicon

$$\frac{dE}{dx} = 2 \cdot 2{,}329 = \underline{\underline{4{,}658}} \text{ MeV/cm}$$

For germanium

$$\frac{dE}{dx} = 2 \cdot 5{,}323 = \underline{\underline{10{,}646 \text{ MeV/cm}}}$$

Comparing with the α results we see a large difference between α-particle and electron. This is because the electron has a much higher velocity.

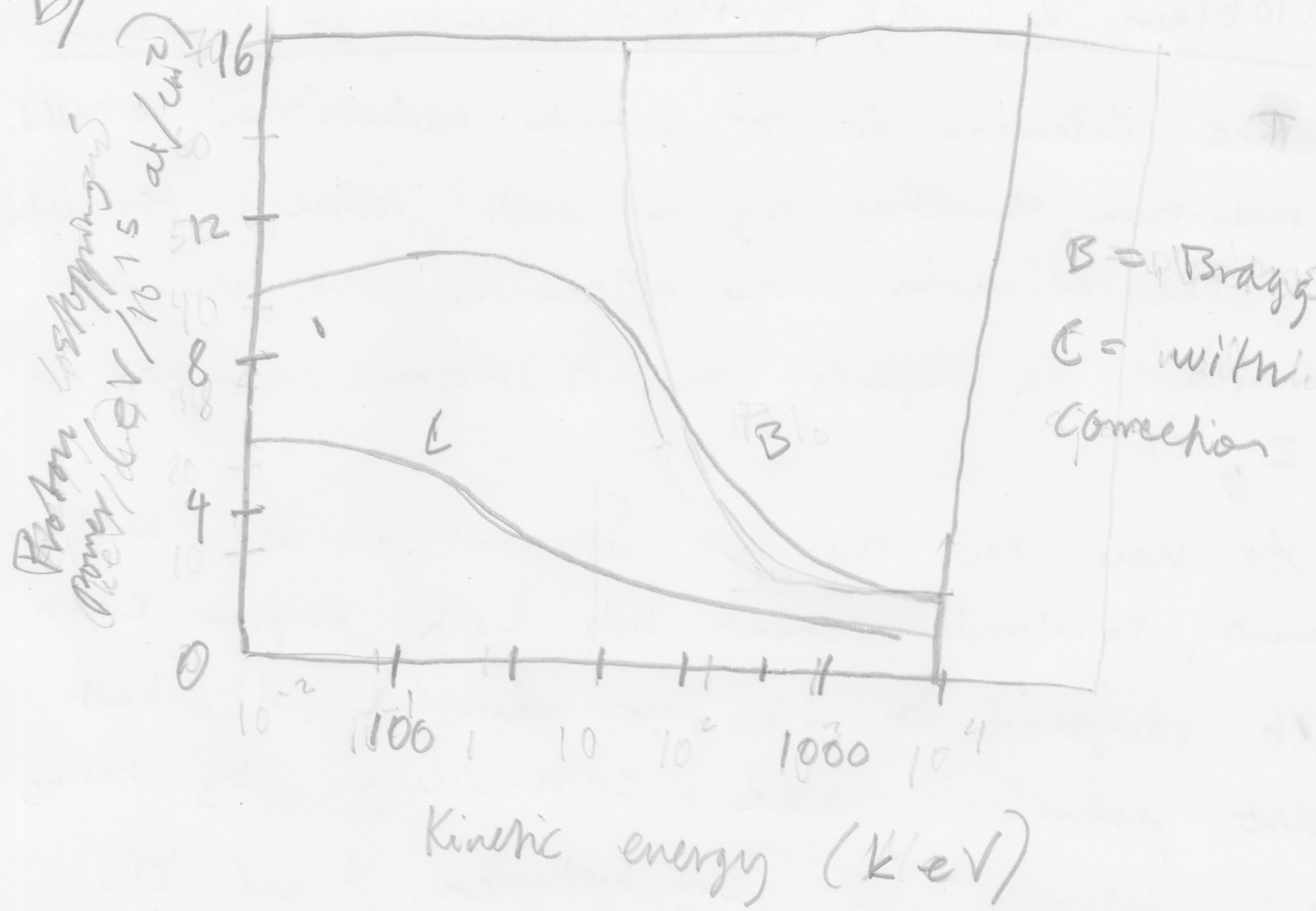
Problem 2 a) Portland cement or Aluminium

The intention is to avoid radiation. So-called radiation shielding by a wall reduces the intensity of the radiation. How effective the shielding material is depends on its atomic number, or Z.

As seen on the file attached in the assignment, Portland cement has both higher Z at 26 compared to 13 for elemental Al, but also provides graded Z-shielding by having an „interval" of Z between 1 and 26. This makes it the preferred material for a wall.

For the second part of the question; SRIM indicates that by adding more components increases the accuracy of the linear combination of the constituent. In Portland cement one can therefore safely assume that the excitation energy is correctly listed.

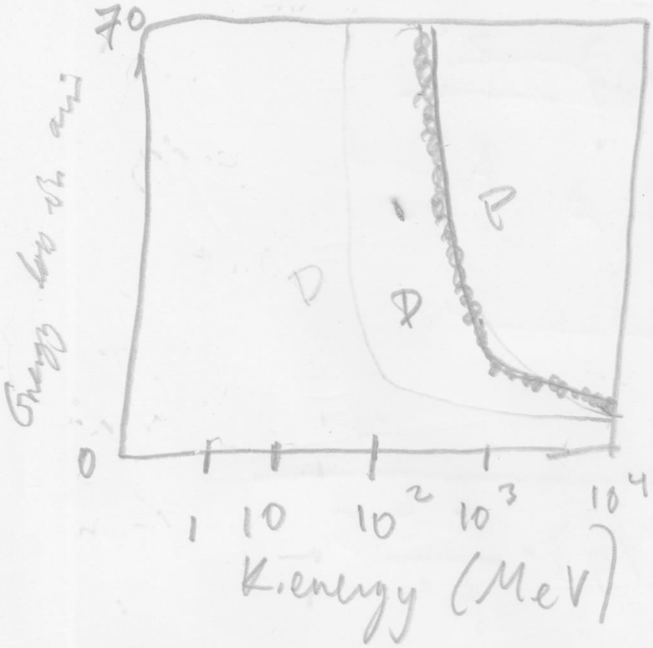
b)



Plot of energy loss for proton as a function of the incoming energy is seen above, exemplified by H^+-ions (proton) into styrene.

Braggs rule (8 C + 8 H) gives a stopping value that is too small. The correction is to add a C=C bond (increases the stopping power) and hydrogen in gas phase.

For the proton in air, we have a slightly different plot.

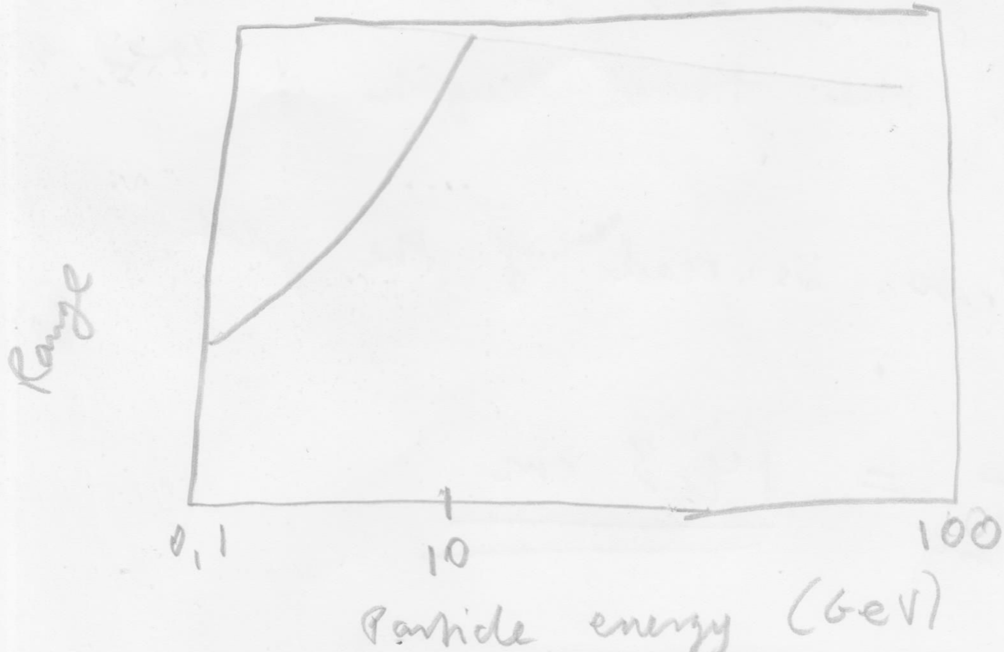


P = proton without density correction,

D = assume density correction

c) Plot of the particle range in material

we are to plot the particle range of proton in Portland cement.



Thickness of wall must be:

Relation for thickness is given by qualitative Bethe-Bloch

$$\frac{dE}{dx} \approx \rho \,(2\ \mathrm{MeV\,cm^2/g})\, \frac{z^2}{\beta^2}$$

$$dx = \frac{dE}{\rho\,(2\ \mathrm{MeV\,cm^2/g}) \cdot \frac{z^2}{\beta^2}}$$

$$dx = \frac{10 \cdot 10^9}{2{,}3 \cdot 2 \cdot 10^6 \cdot \frac{4}{0{,}0517^2}} = \underline{\underline{21{,}7453\ \mu\mathrm{cm}}}$$

d) Travel-length of escaped proton

I will use figure 2.3 in the syllabus to investigate the travel-length of the escaped proton.

70 keV/cm is read of the plot.

$$\frac{10\ \mathrm{MeV}}{70\ \mathrm{keV/cm}} = \underline{\underline{14{,}3\ \mathrm{cm}}}$$

Problem 3 a) Derivation of the exponential intensity relation for photons:

we have $I(x)$ radiation incident on the slab. After it has passed the slab the intensity is $I(x+\Delta x)$.

From quantum mechanic the probability of interaction is probabistic giving:

$$\text{Probability of interaction} = a(\Delta x)$$

The parameter a equals the probability for the photon to interact per unitlength.

Number of particles lost per area is:

$$I(x) - I(x+\Delta x) = a \Delta x \, I(x)$$

further becoming

$$\frac{I(x) - I(x+\Delta x)}{\Delta x} = a I(x)$$

The term at the left is the same as the differential.

$$\frac{dI(x)}{dx} = a I(x)$$

Solving the equation gives us

$$I(x) = I_0 e^{-\alpha x}$$

with I_0 = initial intensity.

b) Thickness of lead (Pb) shield

Based on figure 2.18 in syllabus, we have that α for 600 keV is roughly $\lambda = 10\ g/cm^2$ with $\lambda = \frac{1}{\alpha}$ leading to

$\underline{\alpha = 0{,}1}$. $I(x) = e$ and $I_0 = e^2$.

Giving us

$$\frac{1}{e} = e^{-0{,}1 \cdot x}$$

$$\ln\left(\frac{1}{e}\right) = -0{,}1 \cdot x$$

$$\underline{\underline{x = 10\ cm}}$$

c) Neutron and matter interactions

Neutron interactions are divided into high-energy neutrons, fast neutrons and slow neutrons, depending upon their energy.

- High-energy neutron is meant for neutrons with energy above 1 GeV. It behaves roughly like high-energy protons

- Fast neutrons term is meant for the energy-range [100 keV - 10 MeV]. The interaction is done mostly through elastic scattering on the nuclei of the medium. It undergoes elastic collisions so that their kinetic energy until the energy is equal to the thermal energy

- Slow neutrons means neutrons with energy less than 0,5 eV. Most probable interactions are elastic scattering and neutron capture.

Problem 4 a) Describe a cross-section

The cross-section of an object is the shape you get when you cut straight through an object. Shape of the cross-section depends on how it has been cut and can be circular, rectangular and elliptical for example. The equation for cross-section is:

$$(dW = d \times N \sigma)$$

b) Mechanism behind Cherenkov mechanism

A charged particle that moves inside a polarizable medium with molecules, excites these molecules to higher energylevels and exited states. By the process of relaxation, the molecules emit a photon. Huygens principle gives us how wavefronts are created, meaning that in this instance the emitted waves move out spherically at the phase velocity of the medium.

- When the motion of the particle is slow, there is no crossing of them. Cherenkov radiation occurs when particlemotion has high velocity.

The emitted waves add up by constructiv interference giving coherent radiation at angle θ,

c) Mechanism behind Bremsstrahlung

Bremsstrahlung is a process where an electron moving in a material has its movement slowed down by the forces of atoms in its way. If the movement is completely stopped, the electron has encountered a very large, nuclei-heavy material

When the electron is slowed down, it is so that the electron has lost energy. This energy, based on the law of conservation of energy, has therefore been absorbed by the material/atom

Problem 5 a) Derivation of the Compton-wavelength formula

(FYS2140)

I will in the derivation follow the compendium from A. Raklev, S. Viefers et. al.

We will look at the case where Compton-spread is an elastic collision between a photon and a free electron. We define energy and momentum before and after the collision.

	Before	After
Momentum		
γ	$P_\gamma = h/\lambda_0$	$P_\gamma' = \dfrac{h}{\lambda'}$
e^-	$P_e = 0$	P_e'
Energy		
γ	$E_\gamma = \dfrac{hc}{\lambda_0}$	$E_\gamma' = hc/\lambda'$
e^-	$E_e = m_e c^2$	$E_e' = \sqrt{(P_e' c)^2 + m_e^2 c^4}$

Energiconservation gives us

$$E_\gamma + E_e = E'_\gamma + E_e' \quad (1)$$ leading to

$$\frac{hc}{\lambda_0} + m_e c^2 = \frac{hc}{\lambda'} + \sqrt{(P_e' c)^2 + m_e^2 c^4} \quad (2)$$

Since the electron is at rest before the collision, we have:

$$P_\gamma + 0 = P'_\gamma + P'_e \qquad (3)$$

By squaring (2) and (3), we get (by multiplying with c^2).

$$(P_e' c)^2 = \left(\frac{hc}{\lambda_0}\right)^2 + \left(\frac{hc}{\lambda'}\right)^2 - 2\frac{h^2c^2}{\lambda_0 \lambda'}\cos\theta$$

By multiplying with $\frac{\lambda_0 \lambda'}{2hc}$, we get

$$-hc + m_e c^2(\lambda' - \lambda_0) = -hc \cos\theta \qquad (4)$$

$$\Delta\lambda = \lambda' - \lambda_0 = (1 - \cos\theta)\frac{h}{m_e c} \qquad (5)$$

b) <u>Energy of photon</u>

we will use the relation $E = \frac{hc}{\lambda}$ in this assignment.

<u>For 30 degrees:</u>

$$1 \cdot 10^6 = \frac{4{,}1356 \cdot 10^{-15} \cdot 3 \cdot 10^8}{\lambda} = \underline{1{,}295 \cdot 10^{-12}} \text{ m}$$

Inserted into (5) gives us

$$\lambda' = (1 - \cos(30°)) \cdot 2{,}43 \cdot 10^{-12} + 1{,}295 \cdot 10^{-12}$$

$$\lambda' = \underline{\underline{1{,}621 \cdot 10^{-12}\ m}}$$

<u>For 90 degrees:</u>

$$\lambda' = (1 - \cos(90°)) \cdot 2{,}43 \cdot 10^{-12} + 1{,}295 \cdot 10^{-12}$$

$$\lambda' = \underline{\underline{3{,}725 \cdot 10^{-12}\ m}}$$

b) When considering bremsstahlung, I would say that the maximum energy loss is ≈ 1 MeV due a material with long enough thickness to stop the electron completely.

c)

c) Amount of energy deposited in detector and how much does the energy lose

$\lambda = \underline{6,2034 \cdot 10^{-13} \ m}$

<u>for 30°:</u>

$\lambda' = \underline{9,459 \cdot 10^{-13} \ m}$

<u>for 60°:</u>

$\lambda' - (9,459 \cdot 10^{-13}) = (1 - \cos(60°)) \cdot 2,43 \cdot 10^{-12}$

$\lambda' = \underline{2,1609 \cdot 10^{-12} \ m}$

This is so converted to energy.

$$E = \frac{4,1356 \cdot 10^{-15} \cdot 3 \cdot 10^{8}}{2,1609 \cdot 10^{-12}} = \underline{\underline{0,574 \ MeV}}$$

leaves the detector.

$$2 \ MeV - 0,574 \ MeV = \underline{\underline{1,426 \ MeV}}$$

d) Energy of the Compton-edges

Compton-edge is the highest energy deposited in a detector.

$$E_{compton} = \frac{2E^2}{m_e c^2 + 2E}$$

for 60 keV:

$$E_{compton} = \frac{2 \cdot (60 \cdot 10^3)^2}{0,511 + 2 \cdot (60 \cdot 10^3)}$$

$$E_{compton} = \underline{\underline{59999 \text{ eV}}}$$

for 1,332 MeV:

$$E_{compton} = \frac{2 \cdot (1,332 \cdot 10^6)^2}{0,511 + 2 \cdot (1,332 \cdot 10^6)}$$

$$E_{compton} = \underline{\underline{1,331999 \text{ eV}}}$$

for 2,164 MeV:

$$E_{compton} = \frac{2\cdot(2{,}164\cdot10^{6})^{2}}{0{,}511+(2\cdot(2{,}164\cdot10^{6}))}$$

$$E_{compton} = \underline{\underline{2163999\ eV}}$$