Prosjekt 1 FYS45555 Partikkelfysikk

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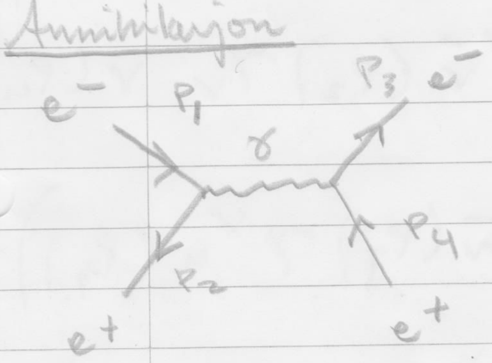
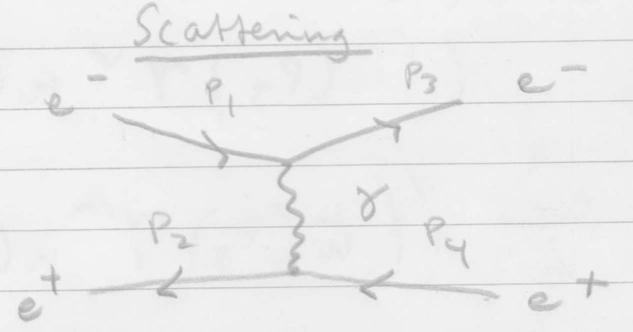
Oppgave 1 Først tegner vi de to laveste orden Feynman-diagrammer for prosessen:

$$e^+e^- \rightarrow e^+e^-$$

Også kjent som Bhabha scattering. Vi får her bidrag fra både scattering og annihilasjon. Derfor tegner vi begge disse.





Vi bruker så QED Feynmans regler. Vi legger i hvert fall først til at $M = M_A + M_S$. Basert på side 124 i Thomson har vi at:

$$M_A = \left[\bar{v}(p_2)\, ie\gamma^\nu u(p_1)\right] \cdot \frac{-ig_{\mu\nu}}{q^2} \cdot \left[\bar{u}(p_3)\, i\gamma_\nu e\, v(p_4)\right]$$

$$= e^2\left(\bar{v}(p_2)\, \gamma^\nu u(p_1)\right) \frac{-ig_{\mu\nu}}{q^2}\left(\bar{u}(p_3)\gamma_\nu v(p_4)\right)$$

hvor da $q^2 = (p_1+p_2)^2 = (p_3+p_4)^2 = s$

$$M_S = \left[\bar{u}(p_3)\, ie\gamma^\mu u(p_1)\right] \frac{-ig_{\mu\nu}}{q'^2}\left(\bar{v}(p_2)\, ie\gamma_\mu v(p_4)\right)$$

$$M_S = e^2\left(\bar{u}(p_3)\gamma^\mu u(p_1)\right) \frac{-ig_{\mu\nu}}{q'^2}\left(\bar{v}(p_2)\gamma_\mu v(p_4)\right)$$

$\mathcal{M}_{fi} = \mathcal{M}_t + \mathcal{M}_s$ gir da:

$$\mathcal{M}_{fi} = +e^2\left(\bar{v}(p_2)\gamma^\nu u(p_1)\right)\frac{-ig_{\mu\nu}}{s}\left(\bar{w}(p_3)\gamma_\nu v(p_4)\right)$$
$$- e^2\left(\bar{w}(p_3)\gamma^\mu u(p_1)\right)\frac{-ig_{\mu\nu}}{t}\left(\bar{v}(p_2)\gamma_\mu v(p_4)\right)$$

<u>Oppgave 2</u> Vi har Mandelstam-variablene

$$q^2 = (p_1+p_2)^2 = (p_3+p_4)^2 = s \quad \text{og}$$
$$q'^2 = (p_1-p_3)^2 = (p_2-p_4)^2 = t$$

Samtidig har vi regelen:

$$\langle |\mathcal{M}_{fi}|^2 \rangle = \frac{1}{4}\sum_{\text{spins}} |\mathcal{M}_{fi}|^2$$

Vi bruker da regelen for $(a+b)^2 = a^2+b^2+2ab$ som gir oss:

$$|\mathcal{M}_{fi}|^2 = \frac{e^4}{s^2}\left(\left(\bar{v}(p_2)\gamma^\nu u(p_1)\right)\left(\bar{w}(p_3)\gamma_\nu v(p_4)\right)\right)^2$$
$$+ \frac{e^4}{t^2}\left(\left(\bar{w}(p_3)\gamma^\mu u(p_1)\right)\left(\bar{v}(p_2)\gamma_\mu v(p_4)\right)\right)^2$$
$$-2\left(\frac{e^4}{st}\left(\bar{v}(p_2)\gamma^\nu u(p_1)\right)\left(\bar{w}(p_3)\gamma_\nu v(p_4)\right)\right.$$
$$\left(\bar{w}(p_3)\gamma^\mu u(p_1)\right)\left(\bar{v}(p_2)\gamma_\mu v(p_4)\right)$$

Da har vi at s i nevner gir annihilasjon, t gir scattering, mens st i nevner gir interferens.

Vi løser disse hver for seg før vi setter dem sammen

$$\frac{e^4}{t^2}\left(\left(\bar{u}(p_3)\gamma^\mu u(p_1)\right)\left(\bar{v}(p_2)\gamma_\mu v(p_4)\right)\right.$$
$$\left.\left(\left(\bar{u}(p_3)\gamma^\nu u(p_1)\right)^\dagger\left(\bar{v}(p_2)\gamma_\nu v(p_4)\right)\right)^*\right)$$

$$= \frac{e^4}{t^2}\left(\left(\bar{u}(p_3)\gamma^\mu u(p_1)\right)\left(\bar{u}(p_1)\gamma^\nu u(p_3)\right)\right.$$

$$\left(\bar{v}(p_2)\gamma_\mu v(p_4)\right)\left(\bar{v}(p_4)\gamma_\nu v(p_2)\right)$$

som en følge av kompleks konjugat $|M_{fi}|^2 = M_{fi} * M_{fi}^\dagger$.

Vi setter da som summen av to ortogonale spinntilstander:

$$\sum_{s=1}^{2} u_s(p)\,\bar{u}_s(p) = \left(\gamma^\mu p_\mu + mI\right) = \not{p} + m$$

Dette gir oss da:

$$\frac{e^4}{t^2}\left(\left(\bar{u}^{s'}(p_3)\gamma^\mu u^s(p_1)\right)\left(\bar{v}^r(p_2)\gamma_\mu v^{r'}(p_4)\right)\right.$$

$$\left(\bar{u}^{s'}(p_3)\gamma^\nu u^s(p_1)\right)\left(\bar{v}^r(p_2)\gamma_\nu v^{r'}(p_4)\right)$$

Tracen an dessd bein

$$= \mathrm{Tr}\left(\bar{u}(p_3)\gamma^{\mu} u(p_1)\right)\left(\bar{u}(p_1)\gamma^{\nu} u(p_3)\right)$$

$$\times \mathrm{Tr}\left(\bar{v}(p_2)\gamma_{\mu} v(p_4)\right)\left(\bar{v}(p_4)\gamma_{\nu} v(p_2)\right)$$

$$\mathrm{Tr}\left(u(p_3)\bar{u}(p_3)\gamma^{\mu} u(p_1)\bar{u}(p_1)\gamma^{\nu}\right)$$

$$\times \mathrm{Tr}\left(v(p_2)\bar{v}(p_2)\gamma_{\mu} v(p_4)\bar{v}(p_4)\gamma_{\nu}\right)$$

$$\mathrm{Tr}\left(\not{p}_3\gamma^{\mu}\right)\left(\not{p}_1\gamma^{\nu}\right) \times \mathrm{Tr}\left(\not{p}_2\gamma_{\mu}\right)\left(\not{p}_4\gamma_{\nu}\right)$$

$$\not{p}_3 = p_{3\sigma}\gamma^{\sigma}, \quad \not{p}_1 = p_{1a}\gamma^{a}, \quad \not{p}_2 = p_{2b}\gamma^{b}$$

$$\not{p}_4\ c\gamma^{c}$$

$$= e^4\left(\frac{(p_1\cdot p_4)(p_3\cdot p_2) + (p_3\cdot p_1)(p_2\cdot p_4)}{t^2}\right)$$

Så ser vi på annihilasjonsleddet:

$$\frac{e^4}{s^2}\left(\left(\bar{v}(p_2)\gamma^{\nu}u(p_1)\right)\left(\bar{w}(p_3)\gamma_{\nu}v(p_4)\right)\right)$$

$$\left(\bar{v}(p_2)\gamma^{\mu}u(p_1)\right)^{+}\left(\bar{w}(p_3)\gamma_{\mu}v(p_4)\right)^{+}$$

~~Vi~~ ~~bruker~~ da har vi identiteten:

$$\left[\bar{\psi}\Gamma\phi\right]^{+}=\bar{\phi}\Gamma\psi \quad (\text{side } 147)$$

Og det gir oss da:

$$\frac{e^4}{s^2}\left(\left(\bar{v}(p_2)\gamma^{\nu}u(p_1)\right)\left(\bar{w}(p_1)\gamma^{\mu}v(p_2)\right)\right)$$

$$\left(\bar{w}(p_3)\gamma_{\nu}v(p_4)\right)\left(\bar{v}(p_4)\gamma_{\mu}u(p_3)\right)$$

som da gir oss trace

$$= Tr\left(\bar{v}(p_2)\gamma^{\nu}u(p_1)\right)\left(\bar{w}(p_1)\gamma^{\mu}v(p_2)\right)\times$$

$$Tr\left(\bar{w}(p_3)\gamma_{\nu}v(p_4)\right)\left(\bar{v}(p_4)\gamma_{\mu}u(p_3)\right)$$

$$= Tr\left(v(p_2)\bar{v}(p_2)\gamma^{\nu}u(p_1)\bar{w}(p_1)\gamma^{\mu}\right)\times$$

$$Tr\left(u(p_3)\bar{w}(p_3)\gamma_{\nu}v(p_4)\bar{v}(p_4)\gamma_{\mu}\right)$$

som igjen blir til

$$\mathrm{Tr}\left[(\not{p}_2\gamma^\nu)(\not{p}_1\gamma^\mu)\right]\times \mathrm{Tr}(\not{p}_3\gamma_\nu \not{p}_4\gamma_\mu)$$

$$= \left(p_2^\nu p_1^\mu - g^{\mu\nu}(p_1\cdot p_2) + p_2^\mu p_1^\nu\right)\times$$

$$\left(p_{3\nu}p_{4\mu} - g_{\mu\nu}(p_3\cdot p_4) + p_{3\mu}p_{4\nu}\right)$$

$$\left.\begin{aligned} g^{\mu\nu}g_{\mu\nu} &= 4\\ p_2^\mu p_1^\nu g_{\mu\nu} &= p_1\cdot p_2\\ p_2^\mu p_1^\nu p_{3\mu}p_{4\nu} &= (p_2\cdot p_3)(p_1\cdot p_4)\end{aligned}\right\}\ \text{Kontraksjon Identiteter}$$

Så da regner vi ut (og ignorer alle ledd med m)

$$\Big(p_2^\nu p_1^\mu p_{3\nu}p_{4\mu} - g_{\mu\nu}(p_3\cdot p_4)(p_2^\nu p_1^\mu) + p_2^\nu p_1^\mu p_{3\mu}p_{4\nu}$$

$$- p_2^\nu p_1^\mu g^{\mu\nu} p_{3\nu}p_{4\mu} + g^{\mu\nu}g_{\mu\nu}(p_1\cdot p_2)(p_3\cdot p_4) - g^{\mu\nu}_{(p_1\cdot p_2)}p_{3\mu}p_{4\nu}$$

$$+ \Big(+ p_2^\mu p_1^\nu p_{3\nu}p_{4\mu} - p_2^\mu p_1^\nu g_{\mu\nu}(p_3\cdot p_4) + p_2^\mu p_1^\nu p_{3\mu}p_{4\nu}$$

$$= (p_2\cdot p_3)(p_1\cdot p_4) - (p_1\cdot p_2)(p_3\cdot p_4) + (p_1\cdot p_3)$$

$$(p_2\cdot p_4) - (p_1\cdot p_2)(p_3\cdot p_4) + 4(p_1\cdot p_2)(p_3\cdot p_4)$$

$$- (p_1\cdot p_2)(p_3\cdot p_4) + (p_2\cdot p_4)(p_1\cdot p_3) - (p_1\cdot p_2)$$

$$(p_3\cdot p_4) + (p_1\cdot p_4)(p_2\cdot p_3)$$

$$= 2(p_2\cdot p_3)(p_1\cdot p_4) + 2(p_2\cdot p_4)(p_1\cdot p_3)$$

(Tok dette i hodet)

Da får vi for annihilasjonsleddet:

$$\frac{e^4}{s^2}\left(2(p_2\cdot p_3)(p_1\cdot p_4)+2(p_2\cdot p_4)(p_1\cdot p_3)\right)$$

Så ser vi på interferensleddet som på forhånd skulle være så vanskelig. Dette var da:

$$-2\left(\frac{e^4}{st}\left(\bar v(p_2)\gamma^\nu u(p_1)\right)\left(\bar u(p_3)\gamma_\nu v(p_4)\right)\left(\bar u(p_3)\gamma^\mu u(p_1)\right)\left(\bar v(p_2)\gamma_\mu v(p_4)\right)\right)$$

$$=-2\left(\frac{e^4}{st}\left(\bar v(p_2)\gamma^\nu u(p_1)\right)\left(\bar u(p_3)\gamma^\mu u(p_1)\right)^\dagger\left(\bar u(p_3)\gamma_\nu v(p_4)\right)\left(\bar v(p_2)\gamma_\mu v(p_4)\right)^\dagger\right.$$

Bruker kompleks konjugasjon til å få

$$=-\frac{2e^4}{st}\left(\bar v(p_2)\gamma^\nu u(p_1)\right)\left(u(p_3)\gamma^\mu \bar u(p_1)\right)\left(\bar u(p_3)\gamma_\nu v(p_4)\right)\left(v(p_2)\gamma_\mu \bar v(p_4)\right)$$

Her setter vi da $\bar v(p_4)v(p_4)=\not p_4$, $u(p_3)\bar u(p_3)=\not p_3$ og tilslutt da $u(p_1)\bar u(p_1)=\not p_1$

Dette fører til at vi kan sette trace på hele side $\rightarrow$

Vi får først da

$$-\frac{ze^4}{st}\left(\bar{v}(p_2)\gamma_\mu \not{p}_1 \gamma^\nu \not{p}_3 \gamma^\mu \not{p}_4 \gamma_\nu v(p_2)\right)$$

Etter så trace:

$$= \mathrm{Tr}\left(\gamma_\mu \not{p}_1 \gamma^\nu \not{p}_3 \gamma^\mu \not{p}_4 \gamma_\nu v(p_2) \bar{v}(p_2)\right)$$

som gir oss

$$\mathrm{Tr}\left(\gamma_\mu \not{p}_1 \gamma^\nu \not{p}_3 \gamma^\mu \not{p}_4 \gamma_\nu \not{p}_2\right) =$$

som da siden den er sylisk blir

$$\mathrm{Tr}\left(\gamma^\nu \gamma^\mu \gamma_\mu \gamma_\nu \not{p}_1 \not{p}_3 \not{p}_4 \not{p}_2 \gamma^1 \gamma^2 \gamma^3 \gamma^4\right)$$

etter at vi bruker kontraksjonsidentiten (Peskin s. 805)

$$\gamma^\mu \gamma^\nu \gamma_\mu = -2\gamma^\nu \quad \text{, får vi}$$

$$-2\,\mathrm{Tr}\left(\gamma^\nu \gamma_\nu \not{p}_{1a} \not{p}_{3\rho} \not{p}_{4\lambda} \not{p}_{2\delta} \gamma^a \gamma^\rho \gamma^\lambda \gamma^\delta\right)$$

Så bruker vi identiteten

$$\gamma^\mu \gamma^\nu \gamma^\rho \gamma_\mu = 4g^{\nu\rho}$$

så får vi

$$-32\,\mathrm{Tr}\left(\not{p}_{1a} \not{p}_{3\rho} \not{p}_{4\lambda} \not{p}_{2\delta} \gamma^a \gamma^\lambda\right) =$$

$$-32\left((p_1 \cdot p_4)(p_3 \cdot p_2)\right)$$

(Håper dette er rett)

Til sammen blir dette da:

$$\frac{e^4}{t^2}\left((p_1\cdot p_4)(p_3\cdot p_2)+(p_3\cdot p_1)(p_2\cdot p_4)\right)$$

$$+\frac{e^4}{s^2}\left(2(p_2\cdot p_3)(p_1\cdot p_4)+2(p_2\cdot p_4)(p_1\cdot p_3)\right)$$

$$-\frac{8e^4}{st}\left((p_1\cdot p_4)(p_3\cdot p_2)\right)$$