Emittance preservation techniques in a linear accelerator

Obligatory Exercise, FYS4565/9565, March 2019

Erik Adli
Department of Physics, University of Oslo

March 5, 2019

1 Description

In this project the students will simulate a small linear accelerator, with similar structure to linacs to be used in a future linear electron-positron collider. In order to generate enough luminosity, the beams in a linear collider must be very small at the interaction point. This implies that both the beam focusing (beta function) and the beam quality (emittance) must be very small. Preservation of very small beam emittance all the way to the interaction point is one of the key challenges in linear collider design.

In the lectures we have gone through some sources of emittance growth for what we call "perfect" or "ideal" machines, for which all accelerator components are assumed to have an ideal and error-free behaviour. Examples of such emittance growth sources are space charge, wake fields, chromatic effects, non-linear fields from sextupoles. All real machines have a number of imperfections that must be taken into account in the design phase. One such imperfection is that the accelerators components (magnets, instrumentation) can not be perfectly aligned. In this exercise the students will investigate the deterioration of beam emittance arising from misaligned lattice elements. The students will apply two different beam trajectory steering techniques, and investigate how the emittance preservation is improved by applying these techniques. The two steering techniques to be studied are 1-to-1 steering and dispersion-free steering.

For master students: the project can be performed in groups of one, two or three students. For PhD students the project is supposed to be done individually and the quality of the answers will count towards the grading.

2 Reading list

I "Estimates of emittance dilution and stability in high-energy linear accelerators", T.O. Raubenheimer (http://journals.aps.org/prstab/pdf/10.1103/PhysRevSTAB.3.121002)

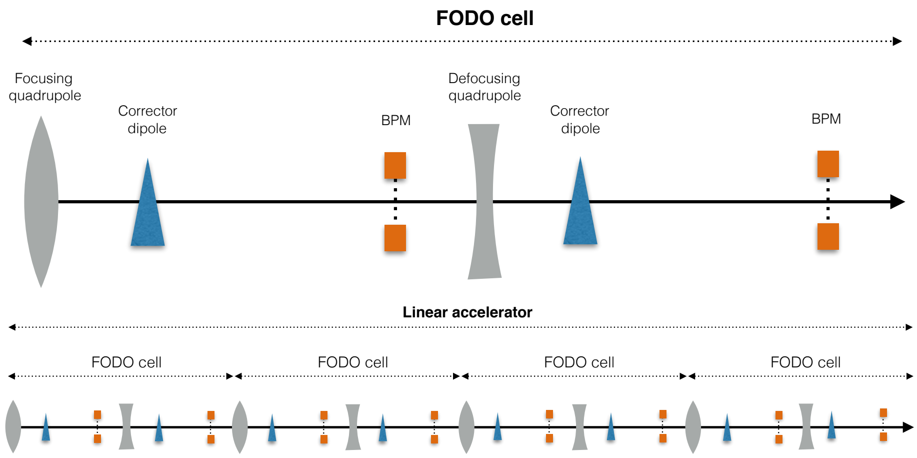

Figure 1: Schematic of the MADX accelerator lattice, consisting of 4 FODO-cells, each two meters long. There are 8 quadrupoles, 8 beam position monitors (BPM) and 8 corrector dipoles.

II ”A dispersion-free trajectory correction technique for linear colliders”, T.O. Raubenheimer & R.D. Ruth (http://www.sciencedirect.com/science/article/pii/016890029190403D)

Reference I introduces you in more detail to sources of emittance dilution on linear accelerators. Reference II describes the theory of the dispersion-free correction technique you will study in this exercise. The equations you will need to implement the steering techniques are given in the text below.

3 Simulation code and setup

Students must install the accelerator optics code MADX. The executable, for all common operative systems, can be downloaded from this page :

`http://madx.web.cern.ch/madx/`

To ease the use of MADX we have provided easy-to-understand auxiliary scripts in MATLAB format that wraps around the MADX input and output. The recommended language for analysis is therefore MATLAB. However, script languages similar to MATLAB, like Octave or Python may be used (some modification to the provided scripts will in this case be required). The function calls you should use are indicated in the exercises below [`using this font`].

4 Exercises

4.1 Theory

1. What is the relation between geometric emittance ϵ and normalized emittance ϵ_N? How do they change as the beam is accelerated?
2. What is the relation between the phase space variables $(x,\, x')$ and the Twiss parameters $(\beta_x,\, \alpha_x,\, \gamma_x)$?

3. Write down expressions for the Twiss parameters and the geometric emittance of a beam with a discrete number of particles N and phase space $\{x_n, x'_n\}$.

4.2 Emittance growth

In the rest of the exercise we will simulate a beam with the following parameters:

- 1 GeV electrons
- 10 μm rad normalized emittance in both planes
- 1% rms relative energy spread
- 1000 particles to represent the beam (or more for higher accuracy)
- We assume that the beam charge is so small that collective effects are insignificant. In this case the results do not depend on beam charge.
- It is sufficient to study one transverse plane only (e.g the horizontal), as the physics are the same for both planes.

The provided MADX script tracks this particle beam through the lattice shown in Figure 1. The lattice consists of four FODO cells, each two meters long. The particle beam is matched to the entrance of the FODO lattice. Note that MADX must be executed after every parameter change. You might also need to run `addpath('get','set')` to access the predefined MATLAB functions. You can change the input parameters for the simulation, including the number of particles, in the file [`params.tsf`]. This can be used for example to adjust simulation speed versus accuracy. The number of particles should be left high enough ($\geq$ 10000) to be able to distinguish physical emittance growth (excepted to be a few % to few 10% growth) from numerical fluctuations due to simulating a finite number of particles).

1. Get the scripts running and track a beam [`runMADX()`].
2. Load the initial beam with MATLAB [`getInitialBeam()`]. The beam is characterized by the 6D phase space distribution (x, x', y, y', z, $\Delta E/E$). The units are in SI-units.
3. Calculate the rms normalized emittance of the input beam from the phase space distribution. Verify that your number coincide with the input parameters (10 μm rad). Expect a relative error of order $1/\sqrt{N}$, where N is the particle number, due to statistical fluctuation. Note that we did not provide a script to calculate the emittance, you have to write this yourself.
4. Calculate the TWISS parameters (β_x, β_y, α_x, α_y) for the input beam. Verify that this is the same as the FODO-matched solution (as printed in the MADX output [`plot.ps`]). Again, expect a relative error around $1/\sqrt{N}$.
5. A beam position monitor (BPM) measures the centroid of the beam position ($\bar{x}$, $\bar{y}$). Plot the beam orbit ($\bar{x}$, $\bar{y}$ vs. s) as measured in the BPMs, using [`getBPMreadings()`].

6. Introduce quadrupole misalignments [`setQuadMisalignments()`], with a 1 mm rms offset. Plot the new BPM orbit.

7. How is the dispersion function of an accelerator beam line defined? Calculate the dispersion function by first tracking a beam with the nominal energy, then by tracking a beam with a slightly different energy. Compare the dispersion you have calculated with the dispersion as printed in the MADX output file (`plots.ps`).

8. Set the energy spread in the input beam to zero [`setEnergySpread()`]. Plot the relative emittance growth ($\Delta\epsilon/\epsilon$) at the end of the lattice as a function of quadrupole misalignment (ranging from 0 to 2 mm).

9. Introduce an energy spread (1% rms). Replot emittance growth as function of quadrupole misalignment.

10. Explain the observed emittance growth for 0% and 1% energy spreads.

11. Change the quad strength (k) of the FODO-cells [`setQuadStrength()`]. What effect does this have and why? Is there a change in emittance growth?

12. (Optional) Attempt to estimate the emittance growth due to dispersion analytically. First, estimate the emittance increase in a beam, with a given beta function, as function of the dispersion. Then, estimate the dispersion growth in a lattice for a given rms quadrupole misalignment. Section VIII b in Reference II has detailed derivation of how emittance growth due to misalignments may be calculated in a more rigorous manner (however, we do not expect you to derive the equations in Reference II as part of this exercise).

4.3 Beam-based correction

We will now implement beam-based steering in a MATLAB script.

4.3.1 1-to-1 correction

The growth of the centroid envelope due to quadrupole kicks might be strongly mitigated by steering the beam into the BPM centres. If a corrector is available for each BPM one can steer the beam into the centre of each BPM (1-to-1 correction). If fewer correctors than BPMs are available one can use a least-squares solutions to optimise the performance (few-to-few correction).

It is fruitful to describe the correction in a response matrix framework. The corrector-to-BPM response matrix, $\mathbf{R}$, describes the effect of each corrector change on each BPM reading. The matrix elements R_{ij} define the incremental BPM reading i, y_i, produced by an incremental change in corrector j, θ_j

$$R_{ij} = \frac{\partial y_i}{\partial \theta_j}.$$

1-to-1 correction can be then performed by sending a pulse, store the BPM readings as $\mathbf{y}$ and apply corrections according to

$$\Delta\boldsymbol{\theta} = -\mathbf{R}^{\dagger}\mathbf{y}, \tag{1}$$

where $\Delta\boldsymbol{\theta}$ is the vector of corrector adjustments and $\mathbf{R}^\dagger$ the pseudo-inverse of $\mathbf{R}$. Subsequent pulses will then yield zero BPM readings to within the BPM precision, assuming a linear system, one corrector per BPM and an $\mathbf{R}$ that is a perfect model of the system to be corrected. For non-linear systems, or if imperfect system models are used, the correction might still converge to yield zero BPM readings by applying Eq. (1) iteratively.

4.3.2 Dispersion-free steering

The idea of the correction scheme "dispersion-free steering" (DFS) is to reduce the energy dependence of the centroid trajectories (see Reference 2). In dispersion-free steering, the measured orbit of the nominal beam, denoted $\mathbf{y_0}$, is compared to the measured orbit of a "test-beam", denoted $\mathbf{y_1}$, in which the particles see a stronger or weaker focusing optics than in the nominal beam, and thus follow dispersive orbits. By adjusting the correctors to minimise the difference between the nominal and test-beam orbits, $\Sigma(y_{1,i} - y_{0,i})^2$, the harmful components of the quadrupole misalignment can be compensated for. The different optics for the test-beam can be achieved either by adjusting the lattice magnet strengths or by using test pulses with energy different from the nominal; in both cases the absolute beam optics will change. From operational and performance points of view, it may be preferred not to change the machine lattice when performing correction, and instead use test-pulses with different energies. In practice, minimising only the difference trajectory generally lead to unstable solutions in presence of noise, for example due to a finite BPM resolution. To account for this effect, the weighted sum of the difference trajectory and the reference trajectory itself may be minimized.

$$\chi^2 = w_0^2 \Sigma y_{0,i}^2 + w_1^2 \Sigma (y_{1,i} - y_{0,i})^2. \tag{2}$$

where w_0 is the relative weighting of the reference trajectory and w_1 the relative weighting of the difference orbit. The least squares solution with respect to the correctors is found by solving the resulting matrix equations:

$$\frac{\partial \chi^2}{\partial \boldsymbol{\theta}} = \frac{\partial}{\partial \boldsymbol{\theta}} \Big\{ w_0^2 (\mathbf{y}_0 + \mathbf{R}_0 \Delta\boldsymbol{\theta})^T (\mathbf{y}_0 + \mathbf{R}_0 \Delta\boldsymbol{\theta})$$
$$+ w_1^2 \big((\mathbf{y}_1 - \mathbf{y}_0) + (\mathbf{R}_0 - \mathbf{R}_1)\Delta\boldsymbol{\theta} \big)^T \big((\mathbf{y}_1 - \mathbf{y}_0) + (\mathbf{R}_0 - \mathbf{R}_1)\Delta\boldsymbol{\theta} \big) \Big\} = 0$$

$$\Downarrow$$

$$\begin{bmatrix} w_0 \mathbf{y}_0 \\ w_1(\mathbf{y}_1 - \mathbf{y}_0) \end{bmatrix} = - \begin{bmatrix} w_0 \mathbf{R}_0 \\ w_1(\mathbf{R}_1 - \mathbf{R}_0) \end{bmatrix} \Delta\boldsymbol{\theta}$$

$$\Downarrow$$

$$\Delta\boldsymbol{\theta} = - \begin{bmatrix} w_0 \mathbf{R}_0 \\ w_1(\mathbf{R}_1 - \mathbf{R}_0) \end{bmatrix}^\dagger \begin{bmatrix} w_0 \mathbf{y}_0 \\ w_1(\mathbf{y_1} - \mathbf{y}_0) \end{bmatrix}. \tag{3}$$

where $\mathbf{R}_0$ is the response-matrix seen by the nominal beam and $\mathbf{R}_1$ the response-matrix seen by the test beam. Compare Eq. (3) to Eq. (1) to see the conceptual similarities of dispersion-free steering and 1-to-1 correction.

4.3.3 Exercises

We will assume noise-free/ideal BPMs, implying that you may set $w_0 = 0$ in Eq. 3. We set the energy spread of the beam at 1% in the exercise. It is sufficient to study a single plane, e.g. x only, since the x-y motion in our example is not coupled.

1. Correct the beam orbit by adjusting the corrector magnets [`setKickers()`] according to 1-to-1 steering. Use `getResponseMatrix()` to calculate the response matrix. Plot uncorrected and 1-to-1 corrected BPM orbits.

2. Plot emittance growth vs. quadrupole misalignment for both uncorrected and 1-to-1 steered orbits.

3. Introduce BPM misalignments [`setBpmMisalignments()`] (1 mm rms). Replot uncorrected and 1-to-1 corrected BPM orbits.

4. Replot emittance growth vs. quadrupole misalignment for both uncorrected and 1-to-1 steered orbits. Explain the effect of BPM misalignments. The range of quadrupole misalignments can be set to 0 to 2 mm.

5. Dispersion-free steering (DFS) can be used to find an orbit that gives less emittance growth than steering through the centers of the BPMs. Correct the beam orbit by adjusting the corrector magnets according to dispersion-free steering (still with quadrupole and BPM misalignments). Plot uncorrected and DFS corrected BPM orbits.

6. Plot emittance growth vs. quadrupole misalignment for both uncorrected and DFS corrected orbits. The range of quadrupole misalignments can be set to 0 to 2 mm.

7. Plot emittance growth vs. BPM misalignment for uncorrected, 1-to-1 and DFS corrected orbits. Explain the observed behavior. The range of BPM misalignments can be set to 0 to 10 mm.

8. (Optional) The BPM signals in the MADX simulation have no noise, except the effects of representing a beam with only 1000 particles. Add simulated noise to the BPM signals, to represent a finite BPM resolution. You can do this either by using MADX commands, or "by hand" in your MATLAB scripts. Comment on how the BPM resolution influences the results of the dispersion free steering.