PROJECT 1

IN FYS5555

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[Revised edition]

1.

We are to draw all possible lowest order Feynman diagrams for the process in question. Which is:

$$\mu^+ \, \mu^- \rightarrow b \, \bar{b} \ \ (1)$$

The equation clearly shows muon to bottom quark process. By looking at (1) we see that we have on the left side of the arrow, a muon and an anti-muon. On the right side of the “reaction arrow” we have a bottom quark and an anti-bottom quark. So basically, this suggests that a reaction between muon and anti-muon gives a photon since no charge is seen and the product is the creation of bottom quarks. This gives us the following Feynman diagrams:

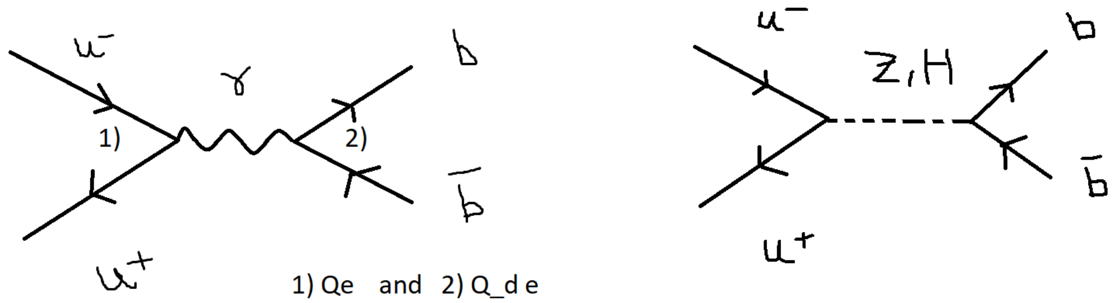

Figure 1: Schematic of the two Feynman diagrams for (1). There are three different diagram in the schematic: photon, Higgs contribution (H) and weak interaction Z.

When we are creating the transition amplitude, we make use of the notation from Mandl and Shaw’s work: Quantum Field Theory. Following simplifications are also done:

$$Qe = ie\gamma u$$

$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137.04}$$

$$q^2 = s = (p1 + p2)^2 = (p3 + p4)^2$$

Where p1, p2 etc represents the arrows in the Feynman-diagram placed as p1 = u^- $and\ p2 =$ u^+, p3 = b etc. in the first diagram of Figure 1. α is the fine structure constant.

This gives us the following transition amplitudes for the two (three) diagrams in the schematic on figure 1.

$$M = \frac{1}{q^2}\ Q_d\sqrt{\alpha}\,\sqrt{\alpha}$$

We can also write the amplitude as (with only the notation from Mandl and Shaw):

$$-iM = \ \bar{u}_{ar}(p3)v_{ar}(p4)u_r(p1)\overline{v_r}(p2)(ie\gamma^{1)}ie\gamma^{2)})\left(-\frac{g_{1)2)}}{(p1+p2)^2}\right)$$

Amplitude for the second diagram is found by just changing p1 and p2, and p3 and p4 in the respective parentheses.

In the last equation we have taken into account that the quark propagator is identical to the fermion propagator for free fields. Also, the distance for photon line is given as p1 + p2.

2.

So above, we have found M. We rewrite this.

$$iM = \frac{e^2}{q^2}\ g_{12}(u_1\gamma^1\overline{v_2})(\overline{u_3}\gamma^2 v_4)$$

The g_12 (where 12 is equal to 1)2), but changed to make it easier for student) allows us to change γ^2 into γ^1 and then get the following Feynman amplitude:

$$iM = \frac{e^2}{q^2}\ (u_1\,\gamma^1\overline{v_2})(\overline{u_3}\gamma^1 v^4)$$

For the unpolarized matrix element we must average over the initial spin states and sum over the final spin states.

$$|M|^2 = \frac{e^4}{q^4}\left(\frac{1}{(2S1+1)(2S3+1)}\right)\sum_{S2,S4}(u_1\gamma^1\overline{v_2})(u_1\gamma^2\overline{v_2})^*(\overline{u_3}\gamma^1 v_4)(\overline{u_3}\gamma^2 v_4)^*$$

We separate out the muon and quark states to get:

$$= \frac{e^4}{q^4}\left(\frac{1}{2S1+1}\right)\sum_{S2}(u_1\gamma^1\overline{v_2})\ (u_1\gamma^2\overline{v_2})\ \left(\frac{1}{2S3+1}\right)\sum_{S4}(\overline{u_3}\gamma^1 v^4)\left(\overline{u_3\gamma^2}v_4\right)$$

$$= \frac{e^4}{q^4}L_\mu L_b$$

Where L_μ *and* L_b are for muon and the bottom quark, and obviously given as above.

According to the text of the assignment we are to use the trace theorems to find the differential cross-section. Before doing that, I will just briefly mention that the spin is S1 = ½ for the muon and S3 = ½ for the bottom quark. While the charge is -1 for the muon and -1/3 for the bottom quark. The theorems used are:

$$Tr(\gamma^1\gamma^2) = 4g^{12}$$

$$Tr\left(\gamma_1 \overleftarrow{P1}\overleftarrow{P3}\,\gamma^1\right) = 4(p1 * p3)$$

$$\overleftarrow{A} = \lambda^\alpha\, A_\alpha$$

$$Tr\left(\overleftarrow{A}\overleftarrow{B}\overleftarrow{C}\overleftarrow{D}\right) = 4\,[(AB)(CD) - (AC)(BD) + (AD)(BC)]$$

Where the backward arrows represent slashed letters (sadly I could not find the slashed sign on the keyboard). All the theorems are from Mandl and Shaw page 453.

Based on those, we take a look at L_μ *and* L_b.

$$L_\mu^{12} = 2((p1^1p3^2) + (p1^2p3^1) - \left((p1 * p3) - m_\mu^2\right) g^{12})$$

$$L_b^{12} = \frac{4}{9}\left((p2^1p4^2) + (p2^2p1^1) - (p2 * p4 - m_b^2)g^{12}\right)$$

In the last equation we have the charge q = 1/3 and therefore q^2 = 1/9 taken into account. For high energy scattering the mass of muon and quark can be neglected. That leads to the matrix:

$$|M|^2 = \frac{4}{9}\left((p1p4)(p2p3) + (p1p3)(p2p4)\right) = \frac{1}{9}\frac{e^4(u^2 + t^2)}{s^2}$$

The result is given in Mandelstam-variables. They can be found on the following link:

https://en.wikipedia.org/wiki/Mandelstam_variables

At high energies annihilating fermion/antifermion have opposite helicities. That gives us four possible spin combinations:

$$e^2\,(1 + \cos\theta)\ and\ e^2\,(1 - \cos\theta)$$

Multiplied with 1/9, we get:

$$|M|^2 = \frac{1}{9}\, e^4\, (1 + \cos^2\theta^*)$$

The differential cross section is given by an equation (abbreviated by me due to space constraint):

$$\frac{d\sigma}{d\Omega} = \frac{\hbar^2 c^2}{64\pi^2 s}\, |M|^2$$

Inserting the amplitude gives us the following equation for the differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{36s}(1 + \cos^2\theta)$$

Next part of the assignment demanded that the differential section was presented in a plot. The plot is presented before a code is added to the text. The latter is done to provide confirmation.

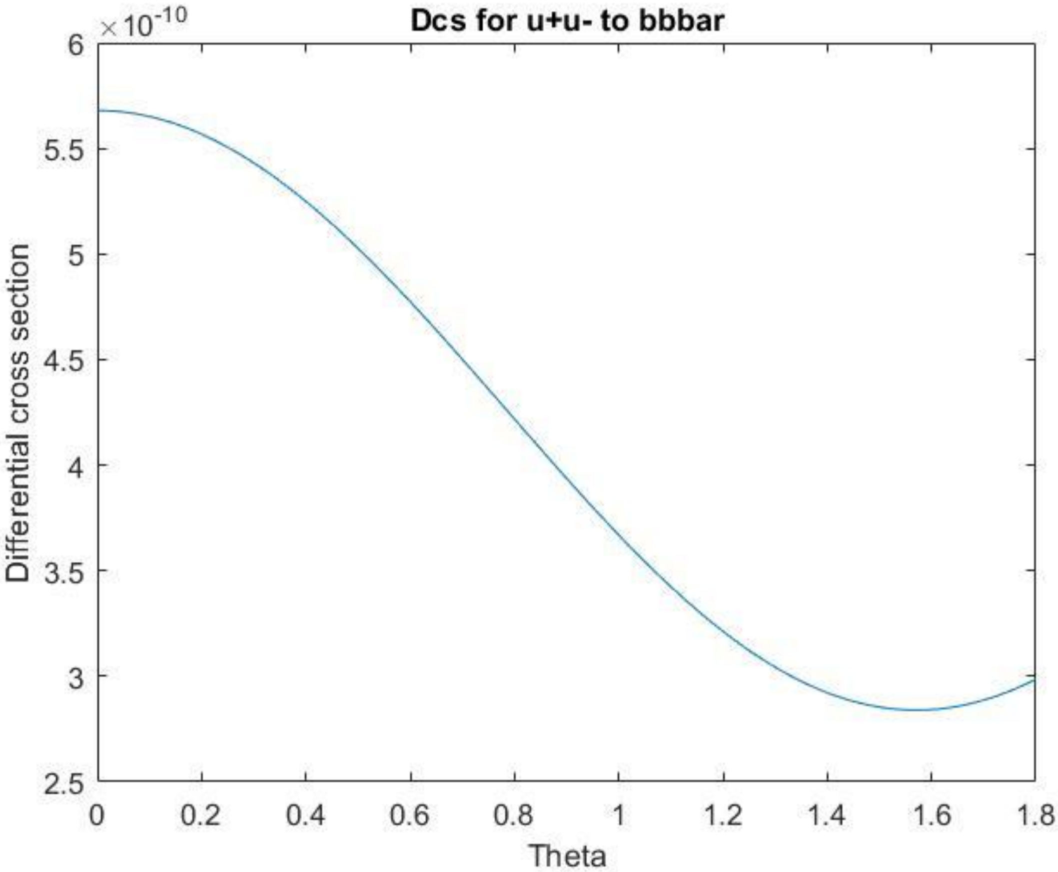

Figure 1: Schematic shows differential cross section for u+u- to bbbar

```
alpha = ((1./(137.04)).^2);
theta = linspace(0,1.8);
s = sqrt(13);  %nominal value since its the same for all
functions
diff2 = (((alpha.^2)./36*s)* (1 + (cos(theta)).^2));

plot(theta, diff2);
xlabel('Theta');
ylabel('Differential cross section');
title('Dcs for u+u- to bbbar');
```

The final part of assignment 2 was to derive the forward-backward asymmetry denoted as A_{FB}.

Q1 =0

Q2 =0

With the following code:

```
alpha = ((exp(2))./(4*pi));
syms theta;
s = 1;

diff1 = ((alpha.^2)./4*s)*(1 + (cos(theta)).^2);
diff2 = (((alpha.^2)./36*s)* (1 + (cos(theta)).^2));

q1 = int(diff2,[pi,2*pi]);
q2 = int(diff2,[0,pi]);
q3 = int(diff1,[pi,2*pi]);
q4 = int(diff1,[0,pi]);


Q1 = (q3 - q4)./(q3 + q4)
Q2 = (q1 - q2)./(q1 + q2)
```

Inferred from that is that there is no asymmetry seen in the two annihilation involving muons. An asymmetry is seen in Bhabha scattering.

When asked to draw the symmetry as a function of sqrt(s), the implication is that theta is a constant. And s is introduced as a variable. The plot follows after the MATLAB-code.

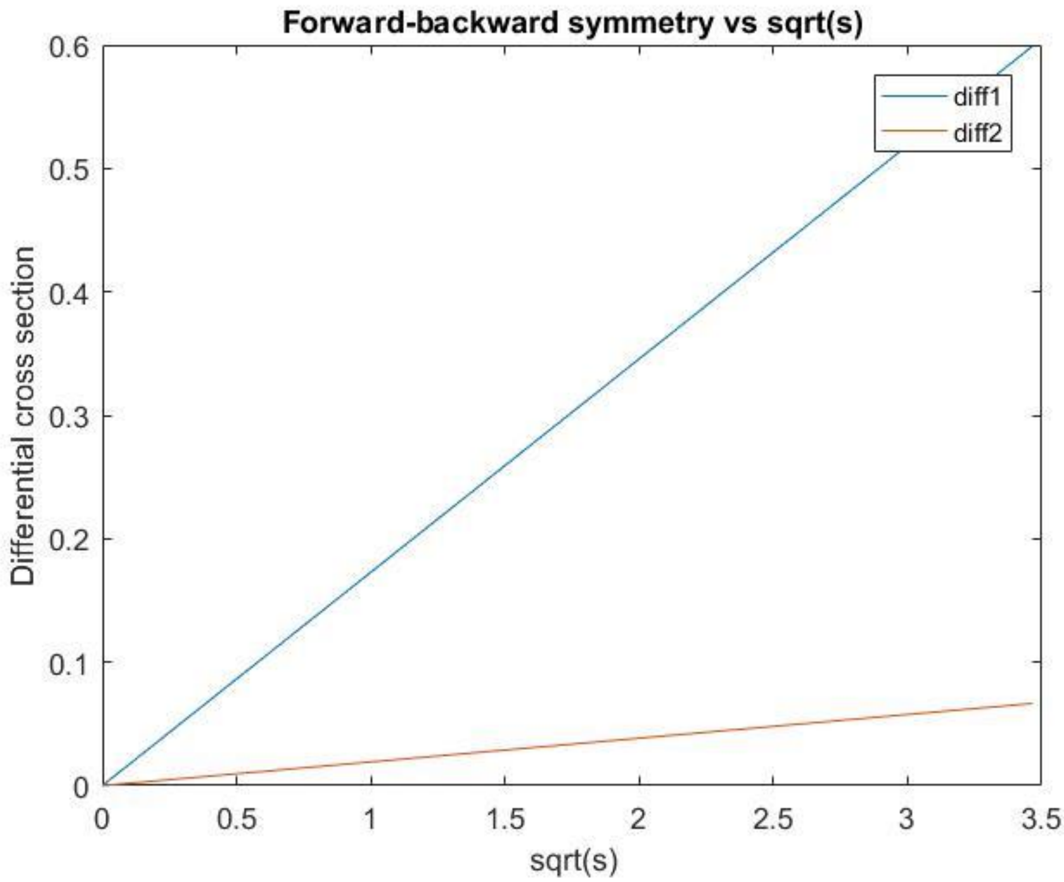

Figure 2: Forward-backward symmetry vs sqrt(s)

3.

We are to calculate and plot the total cross section, σ, as a function of $\sqrt{s}$. To get the total cross section, we integrate over the solid angle, $d\cos\theta\ d\phi$.

$$\sigma = \int \frac{d\sigma}{d\Omega}\, d\Omega = \frac{\alpha^2}{38\pi s} \int (1 + \cos^2\theta)\ d\ cos\theta\ d\phi$$

$$= \frac{\alpha^2}{36\pi s}\ [\phi]_{-pi}^{pi}\ [\cos\theta + \frac{1}{3}\cos^3\theta]$$

Integration limits being $\cos\theta = 1\ and \cos\theta = -1$. That leads to:

$$= \frac{\alpha^2}{18s}\left[\left(1 + \frac{1}{3}\right) - \left(-1 - \frac{1}{3}\right)\right]$$

$$\sigma = \frac{\alpha^2}{18s}\left(\frac{8}{3}\right) = \frac{4\alpha^2}{27s}$$

The MATLAB-code for the plot is:

```
alpha = ((exp(2))./(4*pi));
s = linspace(0,100);

sigma = ((4*(alpha.^2))./(27*sqrt(s)));

plot(s, sigma);

xlabel('sqrt(s)');
ylabel('sigma');
title('sigma vs sqrt(s)');
```

And the actual plot is given as:

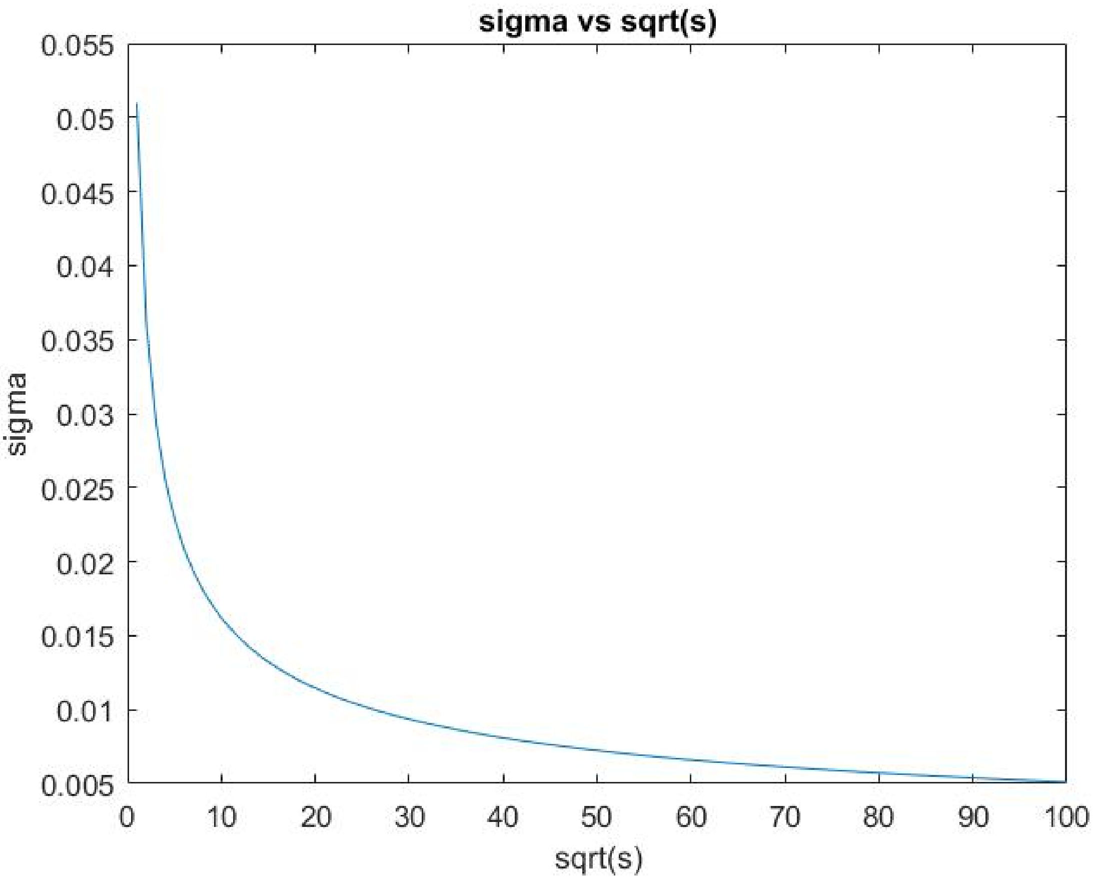

Figure 3: Schematic of sigma vs sqrt(s), with sigma given in nb and s given in GeV.

The cross section can be thought of as the effective cross-sectional area of the target particles for the interaction to occur. If two particles interact the cross section is the area transverse to their relative motion within which they must meet in order to scatter from each other. We can see on figure 3 that as the s increases in GeV, the cross-sectional area decreases. The plot has the same shape as the ((1)/(ln x))-plot famous from basic mathematics.

We also have that the probability for any given reaction to occur is in proportion to its cross section. We can infer from that and the plot, that the probability for the reaction decreases as the s increases. Remembering again that s is given as s = q^2 = (p1 + p2)^2 representing the distance in the Feynman diagrams between the muons and bottom quarks.

4.

In the fourth assignment we are to compare our results with the ones obtained in the numerical program CompHEP. The entire work was also done on VirtualBox. We start with the Feynman diagram, first the parameters followed by the actual diagram.

Standard model -> Feynman gauge -> Scattering process

mu+ = 45.5 GeV, mu- = 45.5 GeV -> b,B

Exclusion of zero diagrams, keeping all diagrams (for comparison in case something is forgotten).

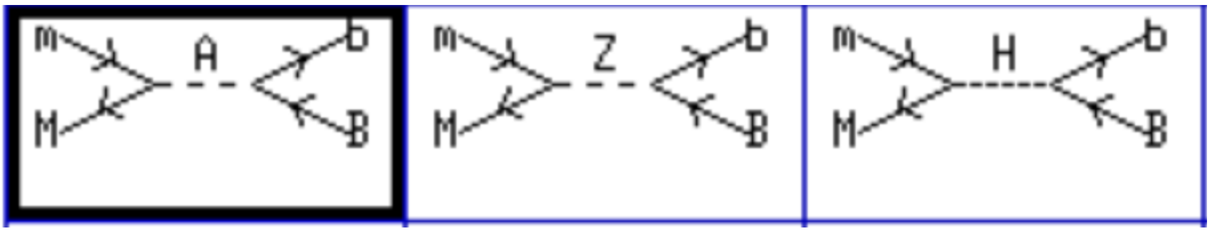

Figure 4: Schematic made in CompHEP that shows the reaction with the parameters given above

We can see on figure 4 that a gamma-ray is created through annihilation of the two muons and a b and B is created. Just as in figure 1. On the other diagrams a Higgs boson and Z-boson is created. The Higgs boson has spin zero and no electric and colour charge. A Higgs fields is created when two neutral and two electrically charged components that form a complex doublet of SU(2) symmetry. Basically our reaction of m, M and b, B. And where there is a Higgs field, there are also excitations of it called the Higgs bosons.

With the setup we have suggested there is clearly electric neutrality. The Z-boson is electrically neutral and its own anti-particle. Whenever an electron (or muon) moves with a kinetic energy, it is assumed to be as a result of a neutrino interacting with the muon. This interaction must therefore be mediated by a Z-boson (electromagnetically neutral).

With regards to the other results found and to be compared, it should be said that the muon energy was set at 45 GeV as already indicated. We start off with differential cross section.

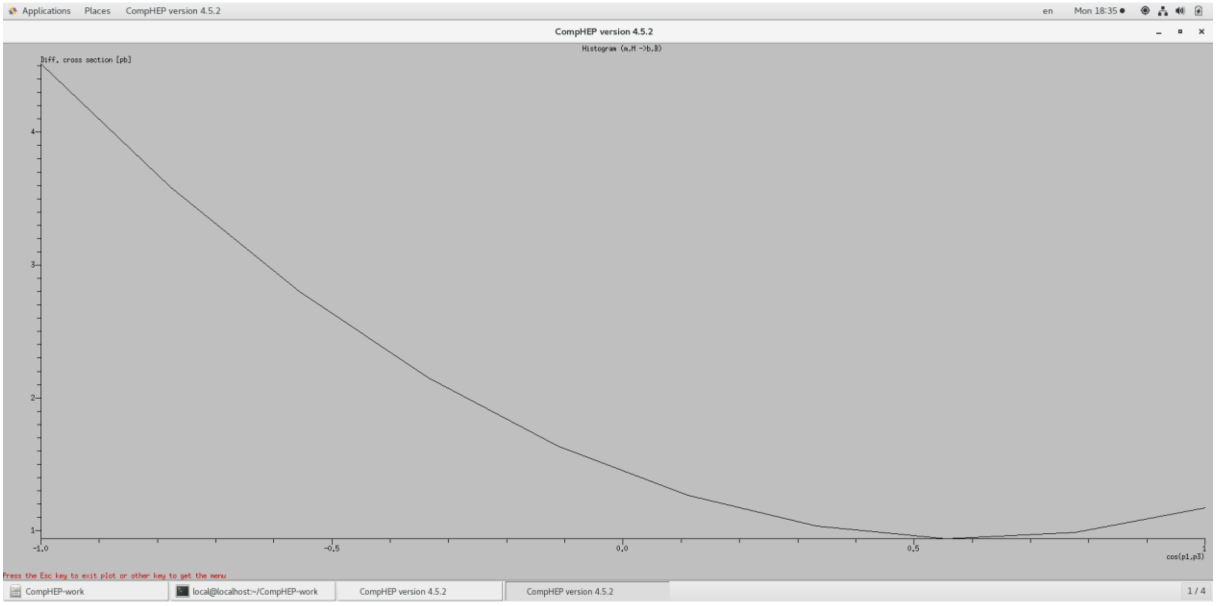

Figure 5: Schematic of the differential cross section vs angular dependence

Compared with figure 1, we see that trajectory is reaching the top when it crosses origo. The angle seen on the x-axis is also like the one on figure 1. Based on that, we can see that the differential cross section is quite close to what it should be.

On figure 6, the asymmetry is seen. We see that the asymmetry is more linear on figure 2 compared to figure 6. This indicates that the plot done on MATLAB is wrong. Based on discussions with Dr. Farid this was further confirmed. However, due to time constraint nothing was done with this mistake.

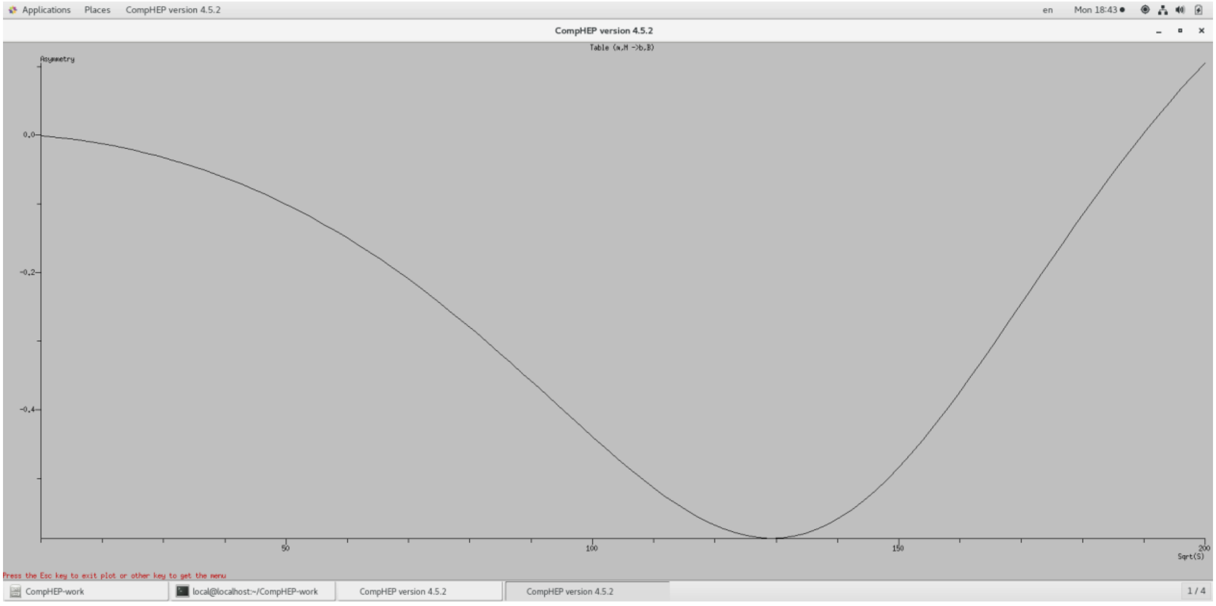

Figure 6: Schematic of the asymmetry

The final comparison done in CompHEP is the total cross section.

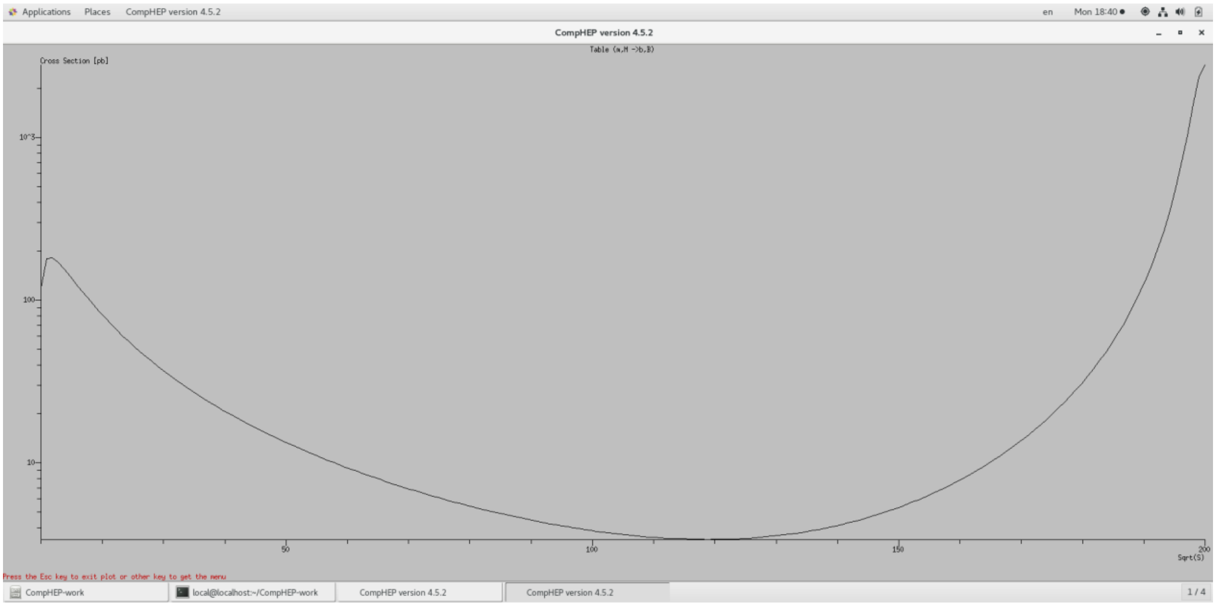

Figure 7: Schematic shows the total cross section vs sqrt(s) done in CompHEP

Again, we compare with the results obtained manually. This time with the plot on figure 3. We see the ((1)/(ln x)) shape in this plot as well. This clearly indicates that total cross section is correct. This cross section was also done for 45.5 GeV and the same tendency was seen there as well.

5.

The Z’-gauge boson refer to hypothetical gauge bosons that arise from extensions of the electroweak symmetry of SM. The SM interactions are mediated by spin-1 gauge bosons. While SU(2) group is connected to W’-bosons, the Z’ models with a U(1) meant for QED. So, we need to change the symmetry of the Z boson already in the CompHEP-system.

The way we choose to do that in this text is by looking at the mass of the Z-boson.

$$W_Z = \frac{1}{2} v \sqrt{g^2 + g'^2}$$

Where g^2 is the SU(2) gauge coupling and g’^2 is the U(1) coupling. v is the Higgs vacuum expectation value. Based on that and what we evaluated in the paragraph above, we put g = 0. g’^2 is normally used for photons and we therefore give it a very small value. But in CompHEP, it is better to use unitary gauge instead of the Feynman gauge. With n=1. We will use the process as in (1), but change the gauge and then evaluate the Z. We will begin with the appropriate Feynman diagram before venturing on to the differential cross section.

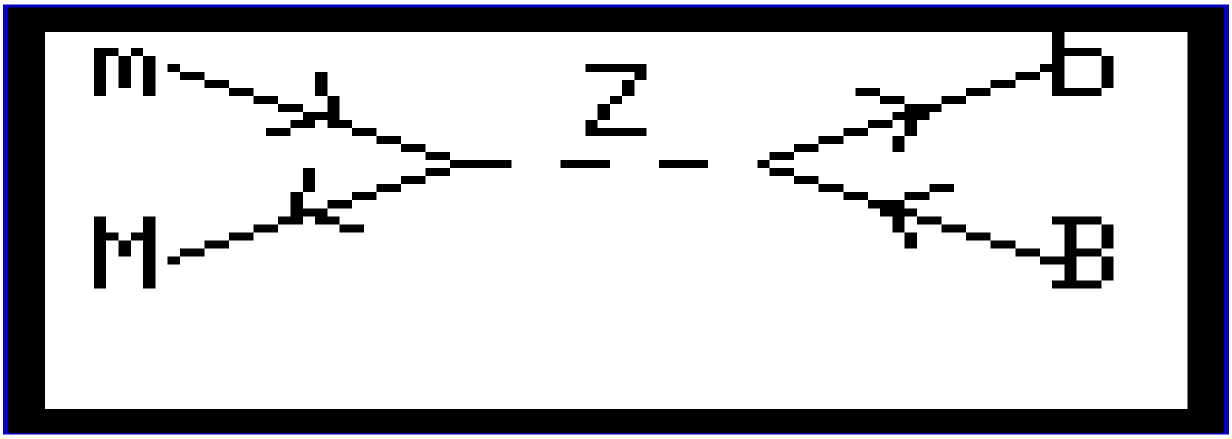

Figure 8: shows the Feynman-diagram for the Z'-gauge boson

Figure 9 shows the differential cross section for this Z-‘gauge boson based on the new parameters we have introduced into CompHEP.

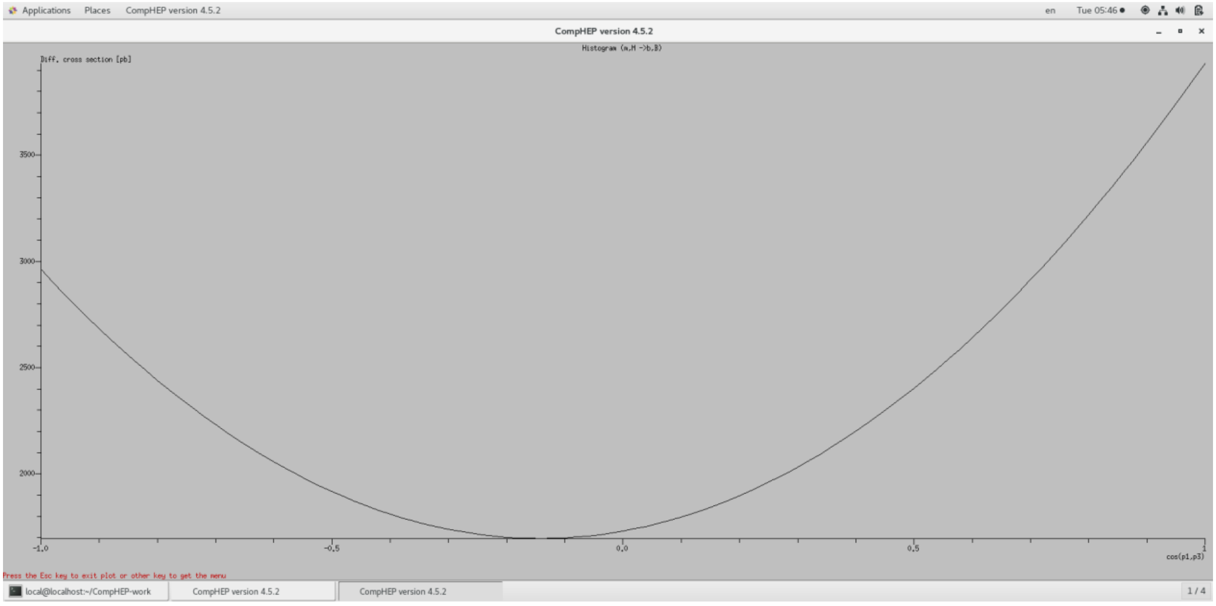

Figure 9: Schematic shows the differential cross section for the Z'-gauge boson done in CompHEP

Figure 10 is a schematic of the asymmetry of the new Z’gauge boson.

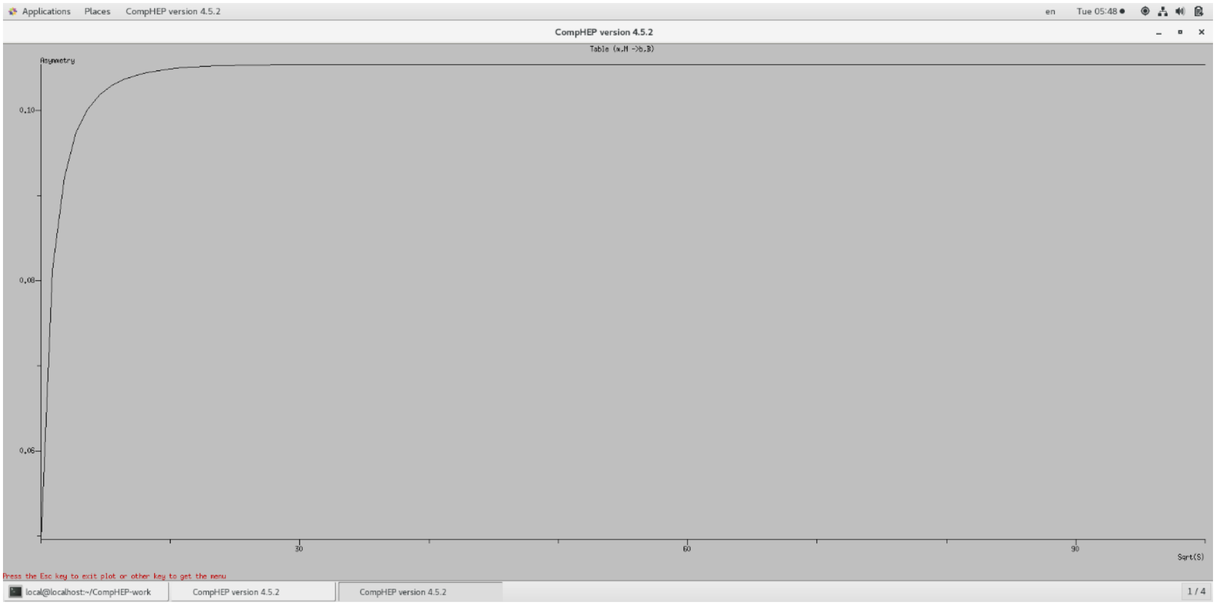

Figure 10: Schematic of the assymetry of the Z'-gauge boson

It must be said that the author of this report is uncertain of whether the asymmetry for the Z’-gauge boson is correct. The final figure is of total cross section and can be seen in figure 11.

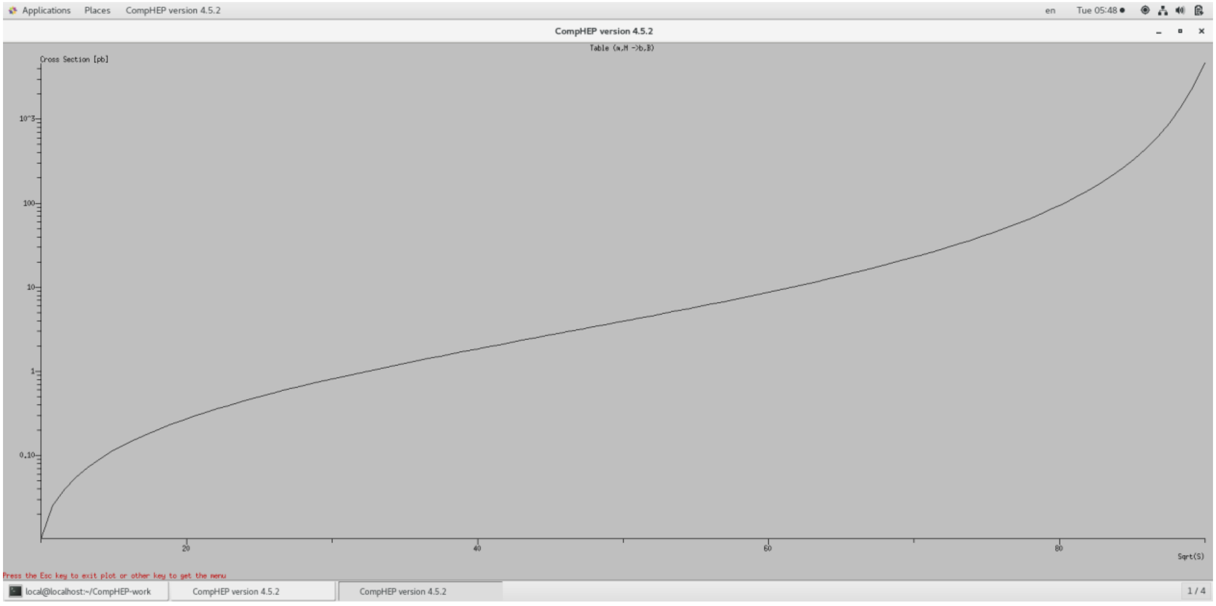

Figure 11: Schematic shows the total cross section of the Z'-gauge boson

6.

Not done since it was optional for FYS5555 due to time constraint due to my trip to Turkey to visit a sick family member.

7.

The first part of the assignment is to analyse some ATLAS data. We chose the Z’ -> $t\bar{t}$ for $m_{Z'}$ = 400 GeV, 500 GeV, 750 GeV, 1000 GeV, 1250 GeV, 1500 GeV, 1750 GeV, 2000 GeV, 2250 GeV, 2500 GeV and 3000 GeV. We have chosen to evaluate the primary vertices of this reaction. Primary vertices are defined as the point in space where proton-proton (pp) interactions have occurred. In this case it is where the interactions are. Our intention is to evaluate the number of primary vertices with respect to mass of Z’.

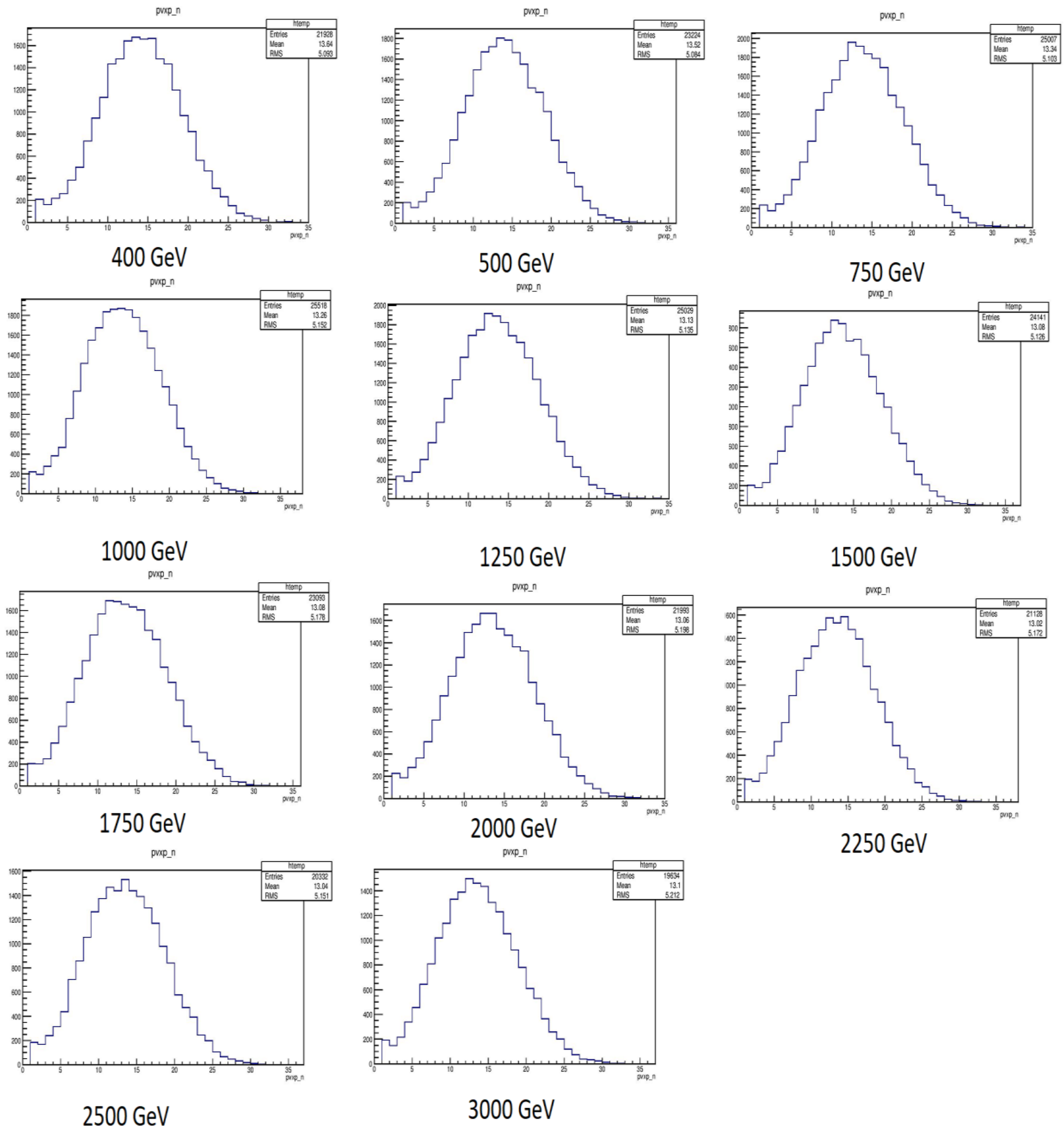

Figure 12: Shows the primary vertices for each energy level done in Root for Z' -> ttbar

We were also to plot a distribution of the invariant mass distribution of two leptons. This was done in Jupyter Notebook based on a code given beforehand. The plot is below and code follows at the end of this assignment.

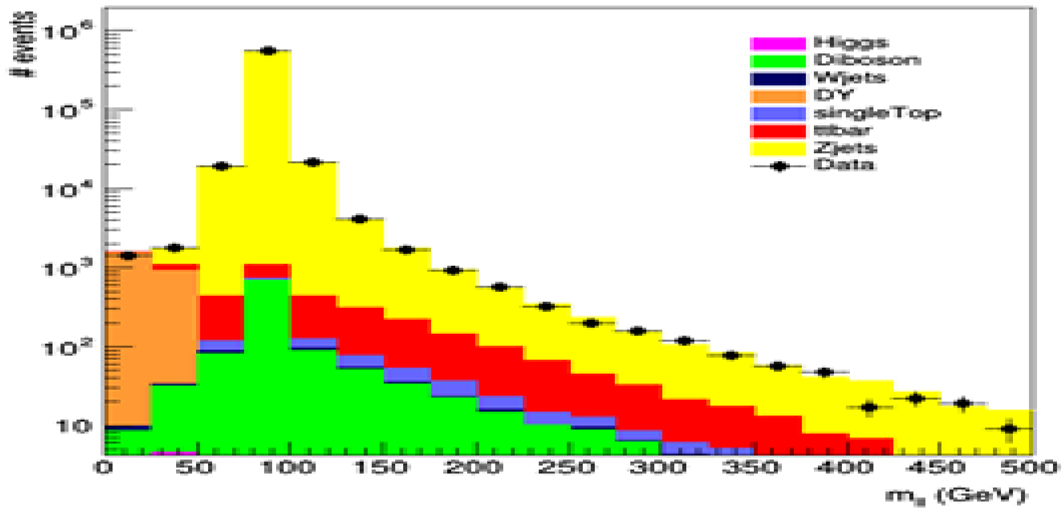

Figure 13: Schematic shows the mum-lepton distribution done in ATLAS

We are so to compare this to the Standard Model (SM). First, it should be said that the SM has been rigorously experimentally tested and proven to be scientifically sound. Although there are some aspects within particle physics that it is incapable of explaining. On figure 14 we have events on y-axis and mass on the x-axis. The Z boson in yellow obviously generates more events than any other elementary particle. Including the Higgs boson. It follows the data because it is an intermediate in every process in the experiment.

Compared to the SM, the plots in figure 14 follow the projected trajections as they should. The predicted tendency is seen and that shows that the figure is qualitatively good to do further analysis with.

I am not sure what the questions means with the four processes. But a reaction is considered to be:

$$H \rightarrow W^+ W^- \rightarrow l^+ l^- \nu \bar{\nu}$$

Higgs boson to W-boson to lepton and neutrino. The neutrino comes about due to movement of leptons with kinetic energy. Those are four processes.

Besides that, we have:

$$H \to W^+W^-, WW, t\bar{t}\ and\ Z$$

Code for Jupyter Notebook:

```
#include <iostream>
#include <string>
#include <stdio.h>
```

```
%jsroot on
TChain *dataset = new TChain("mini");
//dataset->Reset(); // You can reset the chain if you want to "start over".
dataset->Add("http://opendata.atlas.cern/release/samples/MC/mc_147771.Zmumu
.root");
```

```
const int vs = 5;

Int_t lepton_n = -1, lepton_charge[vs], lepton_type[vs];

Float_t lepton_pt[vs], lepton_E[vs], lepton_phi[vs], lepton_eta[vs];

dataset->SetBranchAddress("lep_n",       &lepton_n);
dataset->SetBranchAddress("lep_charge", &lepton_charge);
dataset->SetBranchAddress("lep_type",    &lepton_type);
dataset->SetBranchAddress("lep_pt",      &lepton_pt);
dataset->SetBranchAddress("lep_eta",     &lepton_eta);
dataset->SetBranchAddress("lep_phi",     &lepton_phi);
dataset->SetBranchAddress("lep_E",       &lepton_E);
```

```
TH1F *hist_mll = new TH1F("mll", "Invariant mass", 20, 0, 200);

TH1F *hist_E = new TH1F("lep_E", "Lepton energy", 20, 0, 1000);

TH1F *hist_pT = new TH1F("lep_pT", "Lepton pT", 20, 0, 1000);
```

```
int nentries, i;
nentries = (Int_t)dataset->GetEntries(); // Get number of entries (i.e. num
ber of events) in your data set.

fraction_events = 1; // Specify the fraction of events you want to run over
.
events_to_run = nentries*fraction_events;

std::cout << "Total # events = "  << nentries
          << ". Events to run = " << events_to_run
          << " corresponding to " << fraction_events*100
          << "% of total events!" << std::endl;

TLorentzVector l1; TLorentzVector l2; TLorentzVector dileptons;
```

```
hist_mll->Reset(); hist_E->Reset(); hist_pT->Reset(); // Reset your his
tograms, in case you have filled them before

for (i = 0; i < nentries; i++)
{
    dataset->GetEntry(i); // We "pull out" the i'th entry in the chain.
The variables are now
                          // available through the names we have given
them.

    // Cut #1: At least 2 leptons
    if(lepton_n == 2)
    {
        // Cut #2: Leptons with opposite charge
        if(lepton_charge[0] != lepton_charge[1])
        {
            // Cut #3: Leptons of the same family (2 electrons or 2 muo
ns)
            if(lepton_type[0] == lepton_type[1])
            {
                l1.SetPtEtaPhiE(lepton_pt[0]/1000., lepton_eta[0], lept
on_phi[0], lepton_E[0]/1000.);
                l2.SetPtEtaPhiE(lepton_pt[1]/1000., lepton_eta[1], lept
on_phi[1], lepton_E[1]/1000.);
                // Variables are stored in the TTree with unit MeV, so
we need to divide by 1000
                // to get GeV, which is a more practical unit.

                dileptons = l1 + l2;
                hist_mll->Fill(dileptons.M());
                hist_E->Fill(l1.E());
                hist_E->Fill(l2.E());
                hist_pT->Fill(l1.Pt());
                hist_pT->Fill(l2.Pt());
            }
        }
    }
}
```

```
TCanvas *c = new TCanvas("c", "c", 10, 10, 700, 700);
```

```
hist_mll->GetYaxis()->SetTitle("# events");
hist_mll->GetXaxis()->SetTitle("m_{ll} (GeV)");
hist_mll->GetXaxis()->SetTitleOffset(1.3);
hist_mll->SetFillColor(kBlue);
```

```
hist_mll->Draw();
c->Draw();
```