PHYSICS AT A MUON COLLIDER

FYS5555 - Researchbased Particle Physics

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1 Introduction

The intention of the project is to study the process of producing a pair of bottom quarks $b\bar{b}$ in $\mu^+\mu^-$ collisions at a potential future muon collider. There has been a lot of recent interest in developing a muon collider, as the collisions of this elementary particle has not been done before due to the short lifespan of the muon. When at rest, it decays after only 2.2 microseconds into an electron and two kinds of neutrinos, however its lifetime increases with energy.

The process is given by the equation:

$$\mu^+\mu^- = b\bar{b} \tag{1}$$

And we will try to find transition amplitude by the way of the Feynman diagrams of the process, followed by the differential section, the forward-backward symmetry as well as the total cross section. These results will then be compared to the results obtained with a CompHEP. The final part of the project is to evaluate the prospects of new physics produced with a muon collider.

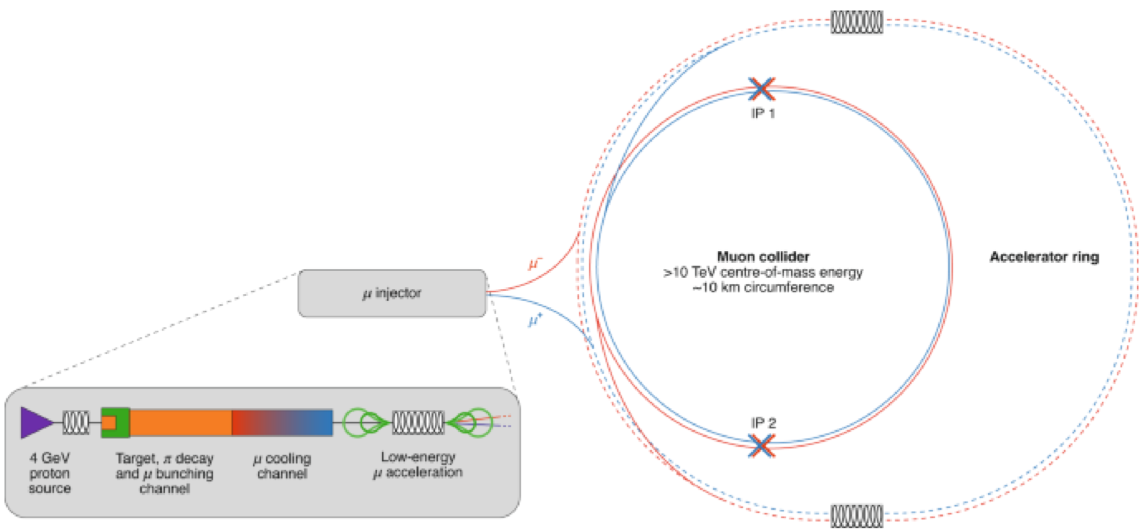

Figure 1: Figure shows a schematic of a potential muon collider.

2 The Project

In 2.1 we will find the transition amplitude by drawing the Feynman diagrams. 2.2 will concern the differential cross section, while in 2.3 we have to find the forward-backward symmetry. In 2.4 we will find the total cross section, before comparing our work with the CompHep in 2.5. The final part of the project related to evaluating the prospect of new physics.

2.1 Transition Amplitudes

We begin with drawing all the Feynman diagrams of the process given in equation (1). These can be found in figure 2. Having done that we continue to make the transition amplitudes of

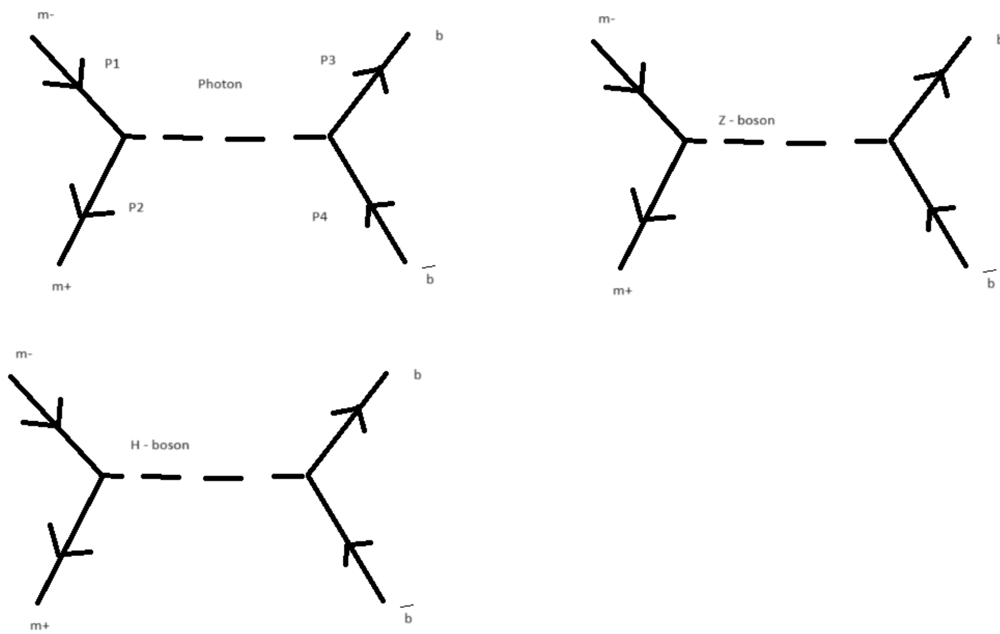

Figure 2: Schematic shows the Feynman diagrams for equation (1)

the three Feynman diagrams. We use the rules defined in the curriculum book of Thomson on page 124 and get the following amplitude:

$$M = \frac{-eQ}{s}(\bar{u}(p3)\gamma^{v}v(p4))(\bar{v}(p2)\gamma_{v}u(p1)) \tag{2}$$

where s = q^2 = $(p1 + p2)^2$ = $(p1' + p2')^2$ = $E^2_{c.m}$. And the quark-photon vertex is proportional to the quark's charge Q.

In the process we also have a Z exchange. We therefore make a new amplitude where we change the 1/s propagator of the photon with the propagator for the Z exchange.

$$\frac{1}{s - M_Z^2 c^4} \tag{3}$$

where M_Z is the mass of the boson and c is speed of light. Making the substitution we get the following amplitude.

$$M = \frac{-eQ}{s - M_Z^2 c^4}(\bar{u}(p3)\gamma^{v}v(p4))(\bar{v}(p2)\gamma_{v}u(p1)) \tag{4}$$

2.2 Differential Cross Section

We are asked to apply the Dirac γ-matrix trace formalism and the completeness relations for Dirac spinors to calculate the differential cross section (s),

$$\frac{d\sigma}{dcos(\theta)} \tag{5}$$

and furthermore, to draw the differential cross section(s) as a function of cos θ. We introduce the abbreviation u_1 = u(p1) to simplify things. This gives us that (2) becomes:

$$M = \frac{-eQ}{(p_1+p_2)^2}(\bar{u_3}\gamma^v v_4)(\bar{v_2}\gamma_v u_1) \tag{6}$$

We have to sum over the polarizations (spins) of the final particles and average over the polarizations (spins) of the initial ones, leading us to get:

$$\frac{1}{4}|M|^2 \tag{7}$$

The 1/4 factor is because the initial fermion has two spin states. In finding —M—^{2}, we need to find the complex conjugate and for that we need to make use of the following identity:

$$[\bar{u}\gamma^v v]* = [\bar{v}\gamma^v u] \tag{8}$$

By using this in the complex conjugate setup we get:

$$|\bar{M}|^2 = \frac{1}{4}\frac{(-eQ)^2}{(p_1+p_2)^4}(\bar{v_4}\gamma^u u_3)(\bar{u_1}\gamma_u v_2)(\bar{u_3}\gamma^v v_4)(\bar{v_2}\gamma_v u_1) \tag{9}$$

In the next part we want to get rid of all the spinors, and we use Casimir's trick to do so. By using the Casimir's trick we get the following equation (10).

$$|\bar{M}|^2 = \frac{1}{4}\frac{(-eQ)^2}{(p_1+p_2)^4}Tr[(p_1' - m)\gamma^v(p_2' + m)\gamma^u]Tr[(p_3' - m)\gamma_u(p_4' + m)\gamma_v] \tag{10}$$

We are in the relativistic limit and can therefore neglect the mass. We write:
p1 = (E, 0, 0, E)
p2 = (E, 0, 0, -E)
p3 = (E, E sin θ, 0, E cos θ)
p4 = (E, -E sin θ, 0, -E cos θ)

We refer to curriculum Handout 4 (Mark Thomson) when we try to find the possible combinations for initial value spinors that give a non-zero value. Leading us to two get four different combinations that would lead to non-zero values. Furthermore, we make use of the Feynman rules where the couplings at the respective vertices are given by a mass and total decay width. With the four helicity combinations, and q^2 = s = 4E = m_2, we get:

$$|M|^2 = M = (\frac{m_\mu m_b m_H}{v^2\Gamma_H})^2 \tag{11}$$

The differential cross section for the three Feynman diagrams can now be found according to the following formula:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} < |M|^2 > \tag{12}$$

We already found the matrix in Equation (11), The differential cross section for the Feynman diagram with the Higgs boson is:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \frac{m_\mu m_b}{v^2 \Gamma_H} \tag{13}$$

While the differential cross section for the QED process (with photon instead of Higgs boson) is:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4\sqrt{m_H}}(1 + cos^2(\theta))Q_b^2 \tag{14}$$

It is important to emphasize that the x-axis in the differential cross section plot is identified as cos (θ) and it's range is between -1 and 1. So, in our instance we can go for domain of either [0, pi] or [pi, 2pi]. The code used to plot the differential cross section was:

```
import numpy as np
import matplotlib.pyplot as plt

# The scattering angle (in radians) between -1 and 1
theta = np.linspace(np.pi, 2*np.pi)

# The differential cross section formula
def differential_cross_section(theta):
    # Constants
    alpha = 1/128  # Fine-structure constant
    Mh = 126

    # The equation
    Equitan = ((alpha**2) / (4 * np.sqrt(Mh))) * 2*((1+np.cos(theta))**2)
    return Equitan

# Plot the result
plt.figure(figsize=(8, 6))
plt.plot(theta, differential_cross_section(theta), label="Differential Cross Section")
plt.xlabel("Scattering Angle (radians)")
plt.ylabel("Cross Section (m^2/steradian)")
plt.title("Differential Cross Section")
plt.grid(True)
plt.legend()
plt.show()
```

And the subsequent plot is:

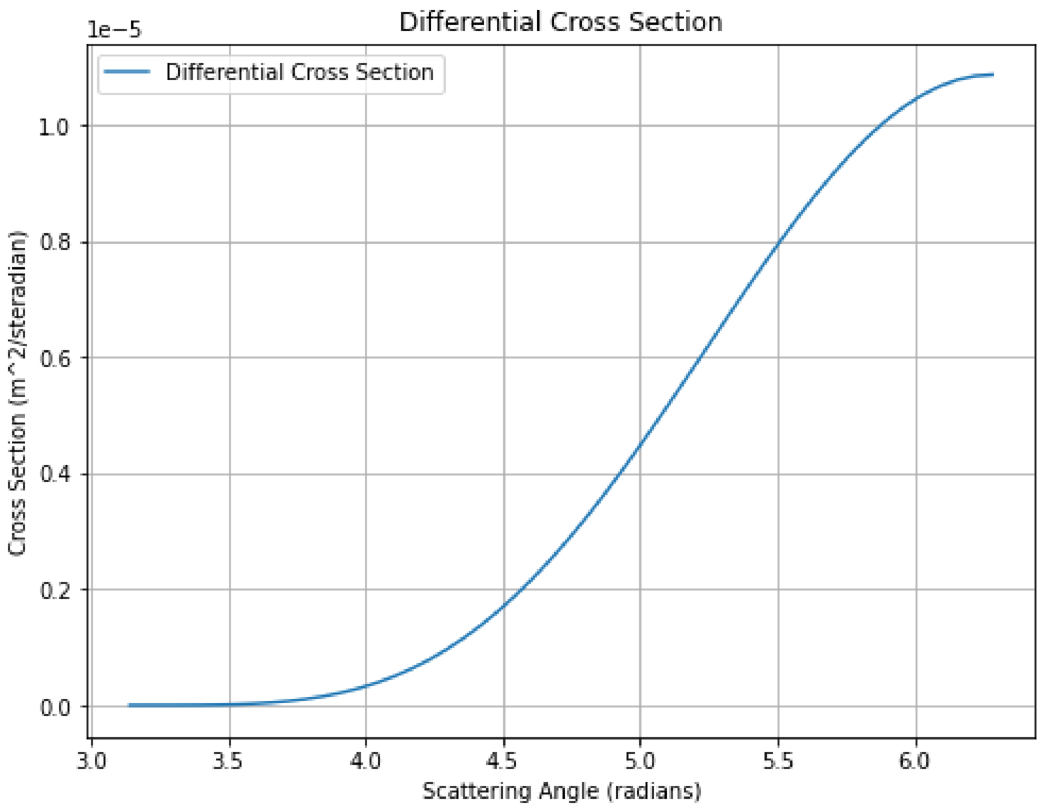

Figure 3: Plot of the differential cross section

In the ultra-relativistic limit, the helicity eigenstates are the chiral eigenstates, meaning the RR and LL for scattering processes and LR and RL for annihilation processes. Therefore we only include the M_{RR} and M_{LL}.

2.3 Forward-backward Asymmetry

In this question we find the forward-backward asymmetry by adding a quark term and integrate between the domain defined in the assignment. This gives us:

$$\int_0^1 dcos(\theta) \int_0^1 \frac{d\sigma}{d\cos^2(\theta)} \tag{15}$$

We make the substitution x = cos (θ) and get:

$$\int_0^1 dx \int_0^1 (1+x^2) = \frac{4}{3} \tag{16}$$

We do the same for the other domain and get:

$$\int_{-1}^0 dx \int_{-1}^0 (1+x^2) = -\frac{2}{3} \tag{17}$$

We insert these numbers into the equation given to us at the start and get:

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{\frac{4}{3} + \frac{2}{3}}{\frac{4}{3} - \frac{2}{3}} = 3 \tag{18}$$

The next part of the question is to plot the forward-backward asymmetry as a function of $\sqrt{s}$.

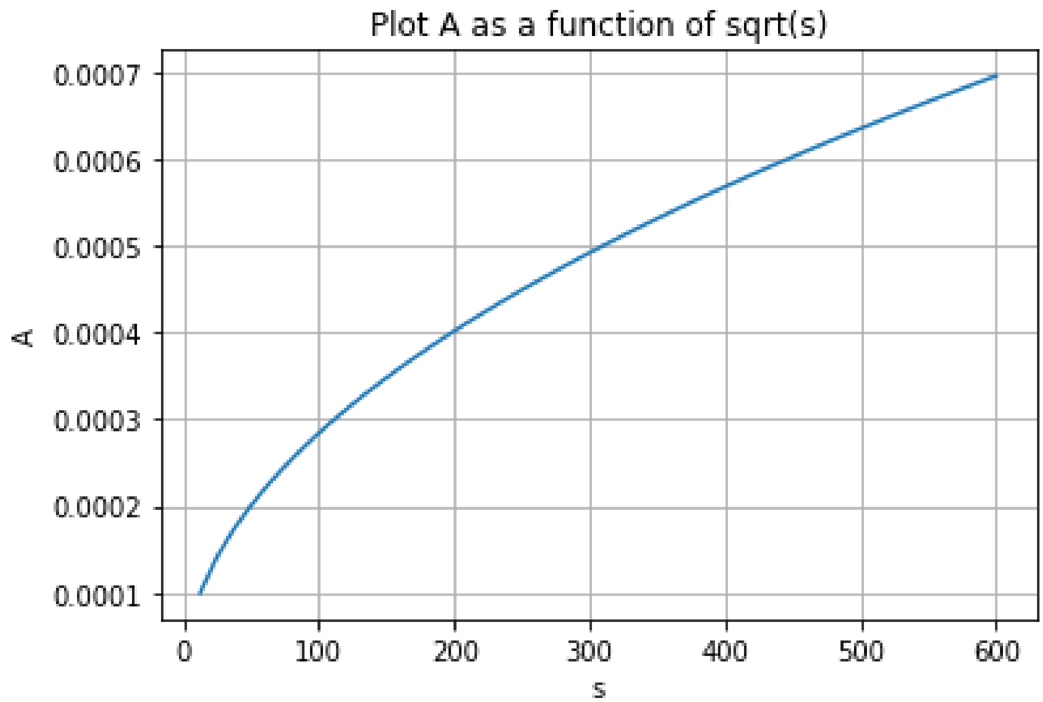

Figure 4: Plot shows forward-backward asymmetry as a function of the square root of s

When we compare with other processes, we see that for the c quark pair the plot is very similar in rate of increase and slope, as well as shape. However, the c quark pair has a deeper valley and steeper slope (Inoue et al.). For the muon pair to electron pair asymmetry, it is emphasized in the curriculum (Thomson) that although it seems A$_{FB}$ seems to be zero, it is in fact not. There is very small asymmetry. In plot shape we see a straight line through origo (0,0) on an x-y plane. For the last process muon to muon, A$_{FB}$ is zero and the slope is a straight line.

2.4 Total Cross Section

The question asks us to calculate the total cross section and to plot it. Furthermore, we are asked to discuss the result. The total cross section is calculated to be:

$$f \int_0^1 (1 + cos^2(\theta)) dcos(\theta) = \frac{4}{3} f \tag{19}$$

Where f is the constants. This gives us the following cross section:

$$\frac{4}{3}\frac{\alpha^2}{s} * Q_b^2 = \frac{16\pi^2}{27m_H^2} \tag{20}$$

We are asked to plot it as a function of $\sqrt{(}s)$ and the plot follows as:

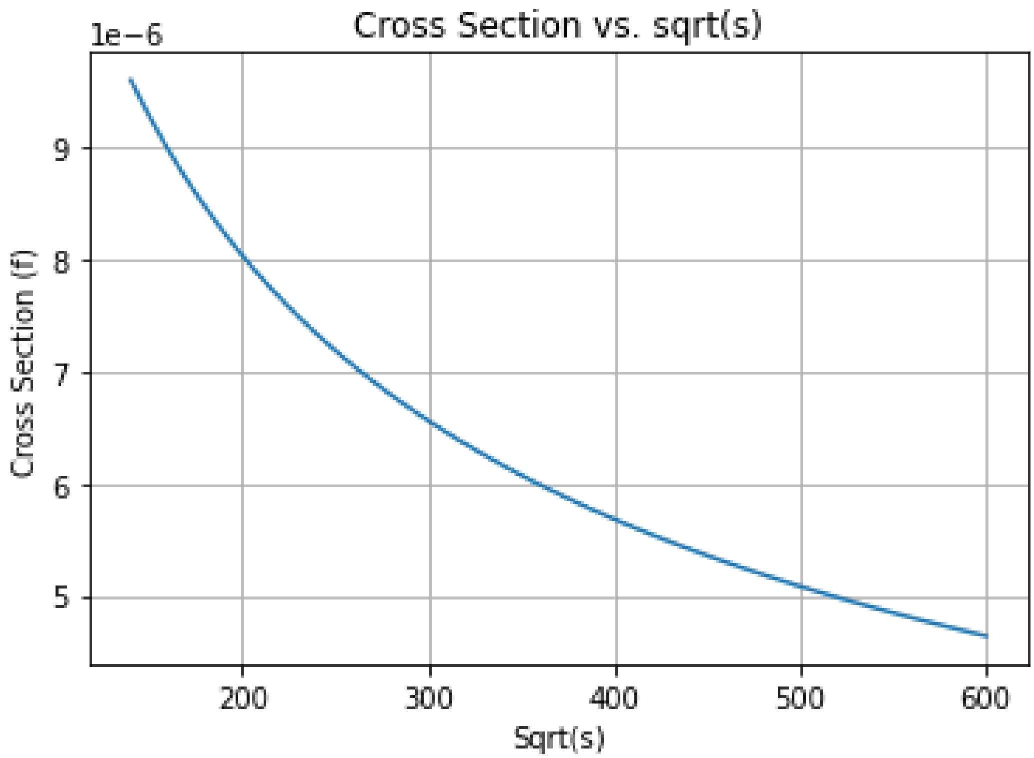

Figure 5: Plot of cross section as a function of $\sqrt{(}s)$

The cross section is a measure of the probability that a specific process occurs. It represents the overall probability of the interaction process, considering all possible outcomes. We see that as the $\sqrt{s}$ is increasing the probability of of the process is decreasing. The most likely reason for this is that other interactions that the process of muon pair to bottom quark pair becomes more important. Bogdanov et al have shown that various regions of dominance of different muon interaction processes exist based on the energy levels. For moderate muon energies, at any energy transfer (up to the limit defined by kinematics of-e scattering) the most important process is the-electron production. At higher energy transfer the process of muon bremsstrahlung dominates. To transfer to our process, we see that the bottom quark pair production occurs at low energy levels.

2.5 Comparison with computational tool

The tool used for comparison was the CompHEP. First comparison is made for the differential cross section. A plot was done on CompHEP and is compared with Figure 3.

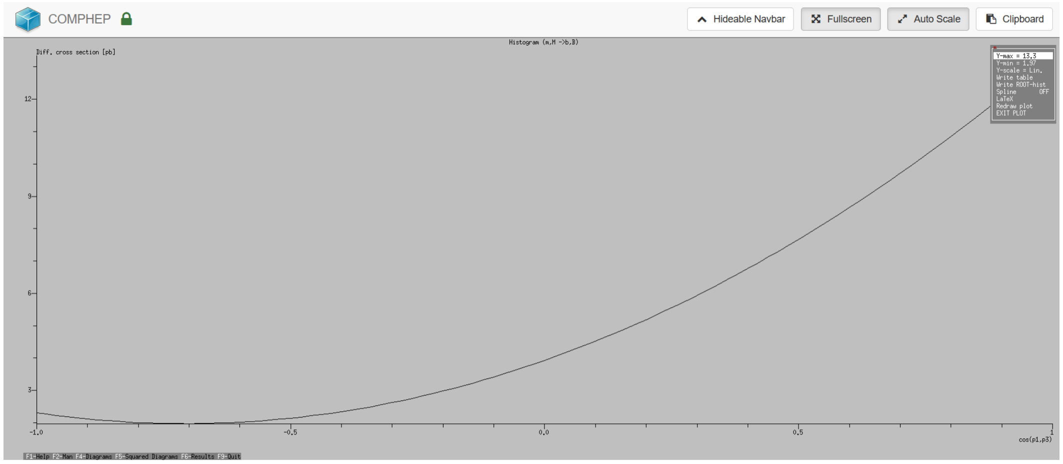

Figure 6: Plot of the differential cross section done on CompHEP

We see that they correlate well. The next part to investigate was the forward-backward asymmetry.

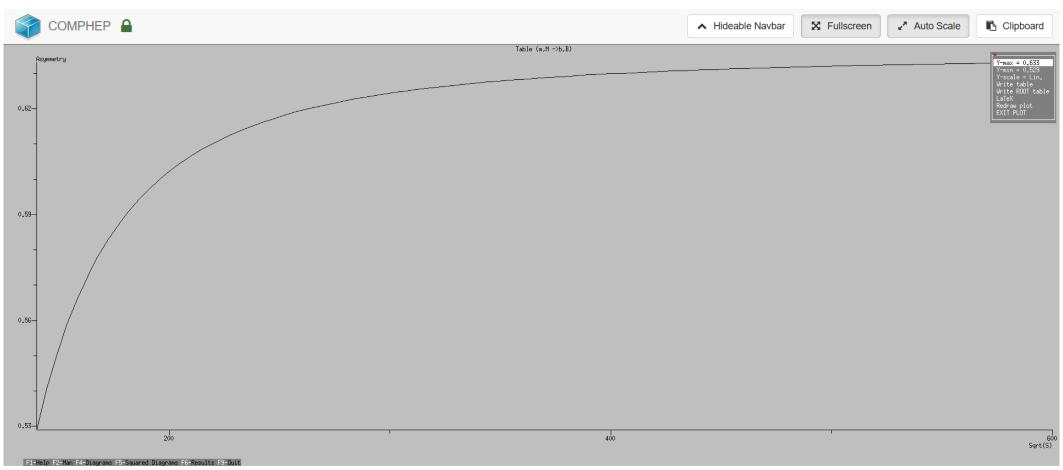

Figure 7: Plot of forward-backward asymmetry done on CompHEP

The figure obtained with CompHEP also looks to coordinate well with the one done manually. The manually obtained figure is Figure 4. Final figure for comparison was for total cross section.

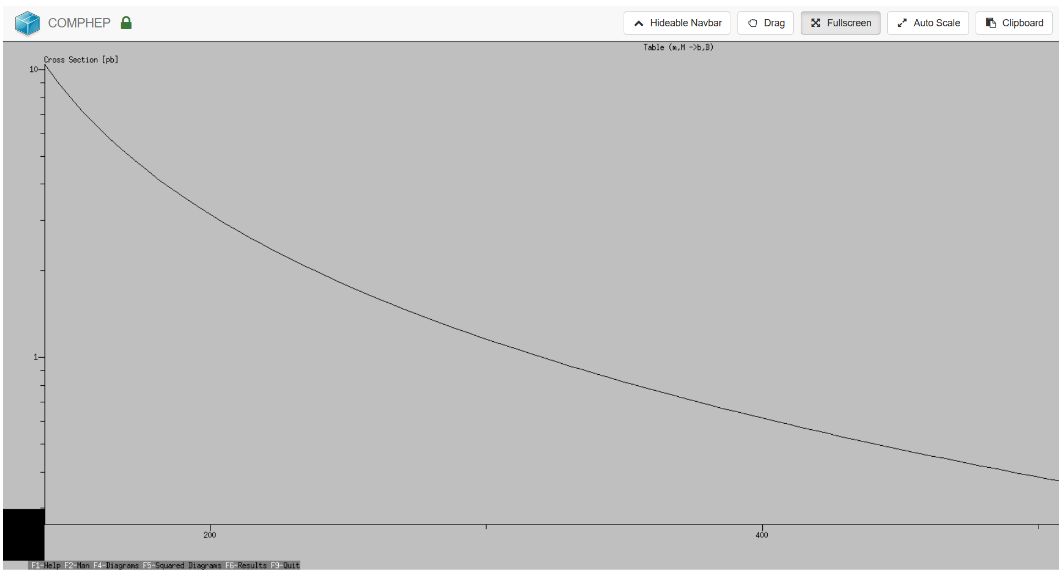

Figure 8: Plot of total cross section done on CompHEP

The final figure obtained with CompHEP also comes up good compared with the manually created figure seen in Figure 5.

2.6 New Physics

A new gauge boson, Z', mediates a new hypothetical weak interaction. The particle is implemented into CompHEP and several plots are drawn. The mass of the new Z' was put as 500 GeV/c^2 and it's width was increased from 2 to 12. Otherwise it has the same properties as the ordinary Z boson. The intention was to compare with the results already obtained in this project and to see how size affects the reactions. The reaction used to observe was muon pair to bottom quark pair, as already mentioned. The first result we will investigate is the differential cross section.

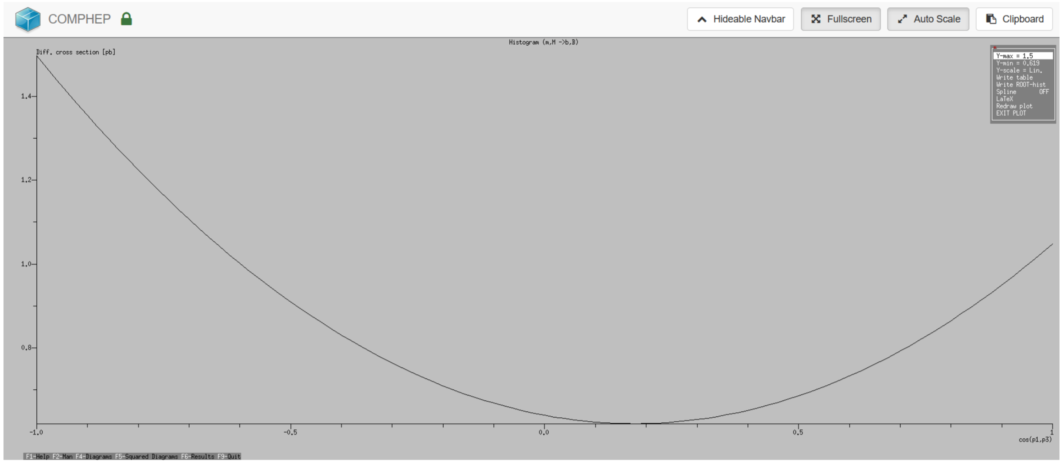

Figure 9: Plot of differential cross section of the new Z' boson done on CompHEP

Compared with the ordinary process done with Z and photon seen on Figure 6 shows a very stark difference. The probability of an interaction to occur increases when the angle is small for the new Z' boson, while it was the other way for the Z boson. We proceed to the forward-backward asymmetry.

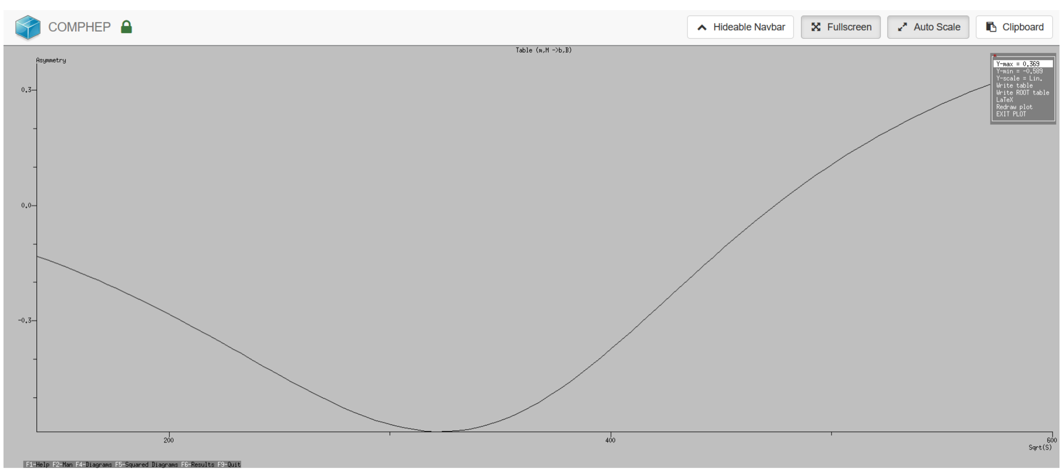

Figure 10: Plot of forward-backward asymmetry of the new Z' boson done on CompHEP

On Figure 7 it can be seen that as s increases the asymmetry follows in an increase. For the Z' we see that it both decreases and increases during the same variable parameters.

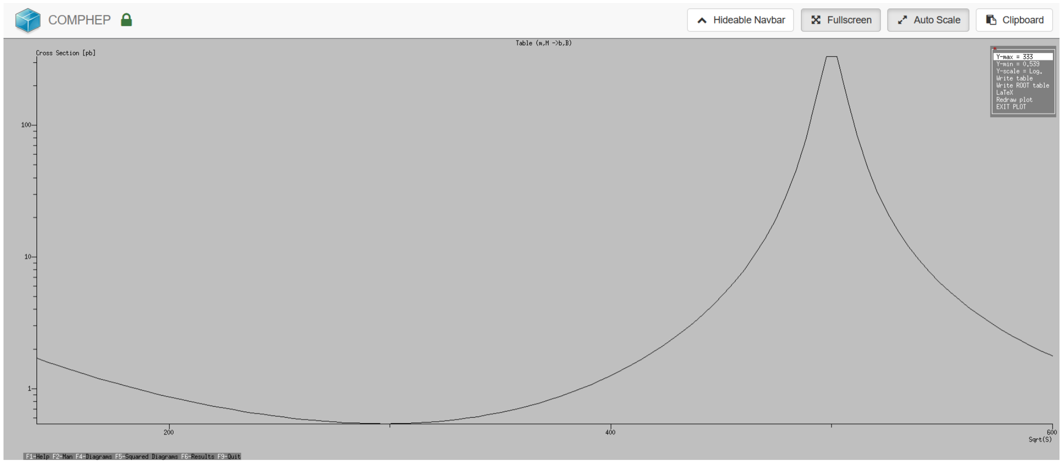

Figure 11: Plot of total cross section of the new Z' boson done on CompHEP

The total cross section shows that the probability of an interaction, when all variables considered, is low with low energies. However, a plateau is seen between 400 to 500. We recall that the mass of the Z' was put as 500 GeV/c^2. Naturally, there is no probability of an interaction for the Z' before the mass is gained. The small amount of action are photons.

3 Conclusion

Project shows a walkthrough of the muon pair to bottom quark pair interaction. The cross sections, differential sections and forward-backward asymmetry were found, both manually and through a computer simulation tool called CompHEP. A comparison shows that the results were comparable and seems to be true. Furthermore, a new Z' boson was created meant to look for new physics. The Z' boson was similar to the original Z boson, but with differences in mass and width. This lead to great changes in all results when compared to Z in the same process.