

# Problem sheet 3 in Applied Optics

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## 1 Question 1

If the velocity of sound in TeO<sub>2</sub> is  $3.1 \times 10^3$  ms<sup>-1</sup>, what is the minimum acoustic wave frequency needed for successful operation of a TeO<sub>2</sub> AOM with a laser beam diameter of 0.5 mm and an operating wavelength of 633 nm? What is the corresponding Bragg angle?

### 1.1 Answer:

The frequency can be found by the equation:

$$\nu_s > \frac{2}{\pi D} * v_s \quad (1)$$

By inserting the values into equation (1) we get:

$$\nu_s = \frac{2}{\pi * (0.0005)} * 3.1 * 10^3 \quad (2)$$

This gives a frequency,  $\nu_s$ , of 3947042.6 Hz.

Second part of the question posed to us asks us to find the corresponding Bragg angle. We make us of the following equation for that task:

$$2\theta_B = \frac{\lambda}{\Lambda} \quad (3)$$

where  $\Lambda$  is the sound wave wavelength (as perhaps a bit unfamiliar term for an undergraduate physicist). We insert the values from the text of the assignment and get:

$$\theta_B = \frac{(\frac{\lambda}{\Lambda})}{2} \approx 1.0 * 10^{-10} \text{ rad} \quad (4)$$

## 2 Question 2

For the above set up, what will be the fractional change in wavelength of the first order diffracted beam?

### 2.1 Answer:

We use the following equation:

$$\frac{\Delta v}{v} = 2 * \nu_s \frac{\lambda/2\Lambda}{c} \quad (5)$$

Inserting the values gives us:

$$\frac{\Delta v}{v} = 2 * (3947042.6) \frac{633 * 10^{-9}/2 * (3.1 * 10^3)}{3 * 10^8} \approx 2.68 * 10^{-12} \quad (6)$$

### 3 Question 3

For the above set up, what variation in the acoustic wave frequency is required to produce 100 resolvable spots?

#### 3.1 Answer:

The answer is given in the code below:

```
import numpy as np

# Given values
D = 0.0005 # Diameter of the laser beam in meters
v_s = 3.1*(10**3) # Velocity of sound in TeO2 in m/s
N = 100 # Number of resolvable spots

# Calculate the variation in the acoustic wave frequency
delta_v_s = (N * 2 * v_s) / (np.pi * D)

# Format the answer to two significant figures
delta_v_s_sig_figs = "{:.1e}".format(delta_v_s)

print('Answer in Hz:', delta_v_s_sig_figs)
```

The answer was:

```
Answer in Hz: 3.9e+08
```

The answer is therefore  $3.9 \times 10^8$  Hz.

### 4 Question 4

What voltage is required to produce 100 resolvable spots with a laser beam of diameter 1.0 mm in an electro-optic deflector consisting of a cube of KD\*P of side 2 cm given that  $r_{63} = 9.810^{-12} \text{ mV}^{-1}$  and  $n_o = 1.510$  at  $\lambda = 540 \text{ nm}$ ?

#### 4.1 Answer:

Here we make use of the following equations:

$$\theta = \frac{1}{D} n_o^3 r_{63} E_z \quad (7)$$

$$\theta_B = \frac{\lambda}{\pi w} \quad (8)$$

$$N = \frac{\theta}{\theta_B} \quad (9)$$

With

$$V = - \int E \cdot dl \quad (10)$$

Because we have already been given the values, we insert them into the equations to get numerically:

```
import numpy as np

#constants
l = 0.02
D = 0.02
```

```

lamda = 540*10**-9
w = 0.01
r63 = 9.8*10**-12
n0 = 1.510
N = 100

#the equations

theta_B = lamda/(np.pi*w)

theta = N*theta_B

E = theta/((1/D)*(n0**3)*r63)

print(E)

```

$E_z$  is 50943287.72. With the equation for voltage we get that  $V \approx -1.02 \times 10^6$  volts.

## 5 Question 5

A Faraday isolator is to be made from a glass slab 50 mm thick. What size of magnetic field needs to be applied assuming that the glass has a Verdet constant of  $520 \text{ min T}^{-1}\text{cm}^{-1}$ ? If the modulator has a length of 5 cm and a coil of 200 turns is wrapped around it, what will the corresponding current be?

### 5.1 Answer:

We make use of the equation:

$$\theta = V * B * L \quad (11)$$

Where  $\theta$  = is the angle of rotation (in radians),  $V$  = Verdet constant (in  $\text{rad T}^{-1}\text{m}^{-1}$ ),  $B$  is the magnetic field strength (in Tesla) and  $L$  is the length of the path (in meters). In the equation we have that for Faraday insulators we normally have a  $\theta = \pi/4$  [BRV13]. So, we have to change some input values to make use of the equation. Min refers to minutes of arc, and we need to change this to rad.

$$V = 520 * \frac{\pi}{180} * 100 * \frac{1}{60} = 15.13 \quad (12)$$

This becomes for the Verdet constant =  $906.92 \text{ T}^{-1}\text{m}^{-1}$ . And the thickness of the slab is 0.05 m. With this we get:

$$B = \frac{\pi/4}{15.13 * 0.05} = 1.04 \quad (13)$$

The answer is 1.04 Tesla.

Second part of the question asks us to find the corresponding current given several conditions. We make use of Ampere's law and get:

$$I = \frac{B * L}{\mu_0 * N} \quad (14)$$

here  $N$  is the number of turns the coil is wrapped around and  $\mu_0$  is the permeability in vacuum given as  $= 4\pi * 10^{-7}$ . We insert these values into the equation to get the current,  $I$ .

$$I = \frac{1.04 * 0.05}{200 * 4\pi * 10^{-7}} \quad (15)$$

The answer is 206.90 A.

## 6 Question 6

The electro-optic coefficient  $r_{63}$  for KD\*P is  $9.8 \times 10^{-12} \text{ m/V}$  for  $\lambda = 540 \text{ nm}$ . What voltage must be applied to a KD\*P crystal in order to produce half wave modulation? What is the polarisation state of the output if this voltage is halved? (The ordinary refractive index of KD\*P at 540 nm is 1.510).

## 6.1 Answer:

The equation for the half wave voltage is given as<sup>[[Lnr](#)]</sup>:

$$V = \frac{\lambda}{2 * r_{63} * n_0^3} \quad (16)$$

We insert values into the equation and get:

$$V = \frac{540 * 10^{-9}}{2 * (9.8 * 10^{-12} * (1.510^3))} \approx 8000 \text{ V} \quad (17)$$

The voltage necessary to produce half wave modulation is 8000 volts.

The second part of the question was to consider what would happen to the polarisation state of the output if the voltage applied to obtain half wave modulation is halved. If the applied voltage is halved to  $V_{\pi/2}$ , the phase shift will also be halved. The half wave phase shift ( $\pi$ ) will become a quarter wave phase shift ( $\pi/2$ )<sup>[[Sch](#)]</sup>. Half wave retarders can rotate the polarization of linearly polarized light to twice the angle between the retarder fast axis and the plane of polarization.

## References

- [BRV13] Michał Berent, Andon A. Rangelov, and Nikolay V. Vitanov. Broadband faraday isolator. *J. Opt. Soc. Am. A*, 30(1):149–153, Jan 2013.
- [Inr] Inrad Optics. Electro-Optic Effect in Nonlinear Optical Crystals. [https://inradoptics.com/pdfs/Inrad\\_WP\\_ElectroOptic.pdf](https://inradoptics.com/pdfs/Inrad_WP_ElectroOptic.pdf). [Online; accessed 16.05.2024].
- [Sch] Matthew Schwartz. Lecture 14: Polarization. <https://scholar.harvard.edu/files/schwartz/files/lecture14-polarization.pdf>. Accessed 19.04.2024.