Problem sheet 2 in Applied Optics

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1 Question 1

For a Gaussian laser pulse with intensity I(t) = I_0exp[-2.77(t/τ)^2], focused into an optical fibre, calculate the maximum frequency shift that will be obtained and explain its dependence on laser pulse duration and fibre length. (I_0 is maximum intensity and τ is the FWHM of the pulse).

1.1 Answer:

We make use of the equation:

$$w = \frac{d\omega}{dt} = w_0 - \frac{n_2 w_0 z}{c}\frac{\partial I}{\partial t} \tag{1}$$

We therefore first find the derivative of the Intensity given in the assignment of the text.

$$I(t) = I_0 exp(-2.77(\frac{t}{\tau})^2) \tag{2}$$

$$I(t) = I_0 exp(\frac{-2.77}{\tau^2}(t^2)) \tag{3}$$

$$\frac{\partial I}{\partial t} = -\frac{2 * 2.77}{\tau^2} t * I_0 * exp(-2.77(\frac{t}{\tau})^2) \tag{4}$$

The max frequency shift will occur when $\frac{d\Phi}{dt} = 0$, so we need to find the time t when this happens and sub this back into the equation.

$$0 = w_0 - \frac{n_2 w_0^2}{c}(-\frac{2 * 2.77t}{\tau^2} I_0 * exp(-2.77(\frac{t}{\tau})^2)) = \frac{wc - n_2 w_0^2}{c} \tag{5}$$

$$t * exp(/2.77(\frac{t}{\tau})^2) = \frac{wc - n_2 w_0^2}{I_0 c * \frac{2*2.77}{\tau^2}} \tag{6}$$

$$lnt + (2.77 * (\frac{t}{\tau})^2) = (\frac{wc - n_2 w_0}{I_0 * c * \frac{2*2.77}{\tau^2}}) \tag{7}$$

We use a numerical tool called Wolfram Alpha to solve this and get the following answer for t:

$$t = \exp\left(x - 0.5 * W\left(\frac{5.54 * \exp(2x)}{\tau^2}\right)\right) \tag{8}$$

where x is the part to the right of equal sign in equation (7). This function is known as the Lambert W function, and W is the Lambert W function. We put this into the equation and get:

$$w_0 - \frac{n_2 w_0^2}{c}(-\frac{2 * 2.77 * \exp\left(x - 0.5 * W\left(\frac{5.54*\exp(2x)}{\tau^2}\right)\right)}{\tau^2} I_0 * exp(-2.77(\frac{\exp\left(x - 0.5 * W\left(\frac{5.54*\exp(2x)}{\tau^2}\right)\right)}{\tau})^2)) = \frac{wc - n_2 w_0^2}{c} \tag{9}$$

The second part of the question was to evaluate how the W_0 and τ change when the frequency change. When the w_0 increases the frequency shift increases, and decreases when it decreases. With τ the frequency shift decreases when the τ increases and increases when τ decreases.

2 Question 2

Extreme nonlinear optics occurs when the incident optical field approaches the characteristic atomic field $E = e/(4\pi\epsilon_0)a_0^2$, where a_0 is the Bohr radius. At such high electric fields, the atom simply ionises. i) Calculate E and its corresponding irradiance I. ii) What is the pulse energy required to achieve this irradiance for a 25 fs laser pulse focused to 10 m radius focal spot?

2.1 Answer:

i) Calculation of E and it's corresponding irradiance, I.

$$E = \frac{1.6 * 10^{-19}}{4 * \pi * 8.85 * 10^{-12} * (5.29 * 10^{-11})^2} \tag{10}$$

Answer is 5.1422067×1011 V/m. For the irradiance we have:

$$I = \frac{1}{2} E_0 * c * E_{at}^2 = 3.5 * 10^{20} \tag{11}$$

The answer is $3.5*10^{20}$ W/m^2.

ii) We do an approximation of pulse shape.

$$E_P = I_{at} * 25 * fs * A = 3.5 * 10^{20} * 25 * 10^{-15} * \pi * (10 * 10^{-6})^2 \tag{12}$$

The answer for the Pulse energy is 0.0027488935718910684 Joule.

3 Question 3

In an experiment to produce high harmonics a Ti:Sapphire laser pulse ($\lambda = 820$ nm) pulse was focused to a power density of 10^{16} Wcm^{-2}. If the ionisation potential of He is 24.587 eV, what will be the wavelength of the highest harmonic produced?

3.1 Answer:

The highest harmonic produced in high harmonic generation (HHG) must be found by finding the cut-off energy which corresponds to the highest harmonic and can be found by using the following equation:

$$E_{max} = I_p + 3.17 * U_p \tag{13}$$

where I_p is the ionization potential and U_p is the ponderomotive energy. The ponderomotive energy is given by:

$$U_p = \frac{e^2 E^2}{4 m_e w^2} \tag{14}$$

We are given several variables and we use a Python code to find the answer:

```
import numpy as np

# Constants
c = 3e8  # Speed of light in m/s
epsilon_0 = 8.85e-12  # Permittivity of free space in m^-3 kg^-1 s^4 A^2
e = 1.6e-19  # Electron charge in C
me = 9.1e-31  # Electron mass in kg
h = 6.63e-34  # Planck's constant in m^2 kg / s
Ip = 24.587 * e  # Ionization potential of He in J
wavelength = 820e-9  # Wavelength of the laser in m
omega = 2 * np.pi * c / wavelength  # Angular frequency of the laser in rad/s
I = 1e16 * 1e4  # Power density of the laser in W/m^2
```

```
# Calculate E^2
E_squared = 2 * I / (c * epsilon_0)

# Calculate Up
Up = e**2 * E_squared / (4 * me * omega**2)

# Calculate E_max
E_max = Ip + 3.17 * Up

# Calculate the wavelength of the highest harmonic for different orders
for n in range(1, 11):
    lambda_max = h * c / (n * E_max)
    print(f"Harmonic Order (n = {n}): Wavelength = {lambda_max:.2e} meters")
```

```
The wavelength of the highest harmonic produced is:
Harmonic Order (n = 1): Wavelength = 6.18e-10 meters
Harmonic Order (n = 2): Wavelength = 3.09e-10 meters
Harmonic Order (n = 3): Wavelength = 2.06e-10 meters
Harmonic Order (n = 4): Wavelength = 1.55e-10 meters
Harmonic Order (n = 5): Wavelength = 1.24e-10 meters
Harmonic Order (n = 6): Wavelength = 1.03e-10 meters
Harmonic Order (n = 7): Wavelength = 8.83e-11 meters
Harmonic Order (n = 8): Wavelength = 7.73e-11 meters
Harmonic Order (n = 9): Wavelength = 6.87e-11 meters
Harmonic Order (n = 10): Wavelength = 6.18e-11 meters
```

4 Question 4

Consider the following laser pulses focused into a He gas jet:
Laser A: $\lambda = 300$ nm, pulse energy 5 mJ, pulse duration = 7 fs, focal radius =30 μm
Laser B: $\lambda = 700$ nm, pulse energy 5 mJ, pulse duration = 7 fs, focal radius =40 μm
Laser C: $\lambda = 3200$ nm, pulse energy 5 mJ, pulse duration = 35 fs, focal radius =60 μm
Which of the lasers would you expect to produce the highest energy harmonic?

4.1 Answer:

The answer is done in Python and follows in the end. We try to find the laser that has the highest intensity. The equations used are:

$$I_0 = (\frac{2}{\pi})(\frac{P}{w_0^2} \tag{15}$$

The equation is taken from Stefan Karsch[Kar13]. The equation for the P and w_0 is given in the code.

```
def calculate_peak_power(pulse_energy, pulse_duration):
    # Peak power = Pulse energy / Pulse duration
    return pulse_energy / pulse_duration

def calculate_beam_waist(wavelength, focal_radius):
    # Beam waist w_0 = (wavelength / pi) * (focal_radius / 2)
    return (wavelength / 3.14159) * (focal_radius / 2)

def calculate_intensity(peak_power, beam_waist):
    # Intensity I = (2/pi) * (Peak power / beam_waist^2)
    return (2 / 3.14159) * (peak_power / beam_waist**2)

# Laser A parameters
```

```
wavelength_A = 300e-9  # 300 nm in meters
pulse_energy_A = 5e-3  # 5 mJ in joules
pulse_duration_A = 7e-15  # 7 fs in seconds
focal_radius_A = 30e-6  # 30 m in meters

# Laser B parameters
wavelength_B = 700e-9
pulse_energy_B = 5e-3
pulse_duration_B = 7e-15
focal_radius_B = 40e-6

# Laser C parameters
wavelength_C = 3200e-9
pulse_energy_C = 5e-3
pulse_duration_C = 35e-15
focal_radius_C = 60e-6

# Calculate peak powers
peak_power_A = calculate_peak_power(pulse_energy_A, pulse_duration_A)
peak_power_B = calculate_peak_power(pulse_energy_B, pulse_duration_B)
peak_power_C = calculate_peak_power(pulse_energy_C, pulse_duration_C)

# Calculate beam waists
beam_waist_A = calculate_beam_waist(wavelength_A, focal_radius_A)
beam_waist_B = calculate_beam_waist(wavelength_B, focal_radius_B)
beam_waist_C = calculate_beam_waist(wavelength_C, focal_radius_C)

# Calculate intensities
intensity_A = calculate_intensity(peak_power_A, beam_waist_A)
intensity_B = calculate_intensity(peak_power_B, beam_waist_B)
intensity_C = calculate_intensity(peak_power_C, beam_waist_C)

# Find the highest intensity and corresponding laser
highest_intensity = max(intensity_A, intensity_B, intensity_C)
if highest_intensity == intensity_A:
    highest_laser = "Laser A"
elif highest_intensity == intensity_B:
    highest_laser = "Laser B"
else:
    highest_laser = "Laser C"

# Print results
print(f"Highest Intensity: {highest_intensity:.2f} PW/cm^2 (from {highest_laser})")
```

The answer obtained was given as:

```
Highest Intensity: 221628924162257530152708896641253376.00 PW/cm^2 (from Laser A)
```

5 Question 5

Two high intensity pulses with the same wavelength, one with a pulse duration of 100 fs and the other with a pulse duration of 5.4 fs, are available for a high harmonic generation experiment. What is the appropriate experimental technique(s) to ensure a single attosecond pulse is generated in the case of each laser.

5.1 Answer:

Thierry Ruchon in his lecture notes, High Harmonic Generation and Attosecond Light Pulses, goes through this phenomenon and this answer is based on the lecture notes. To get attosecond pulses one needs:

1. An XUV source
2. A spectrum several tens of eVs wide
3. A well controlled spectral phase

Furthermore, in the XUV range (from its definition), all solid state materials are absorbant, and gases, including air at atmospheric pressure have an absorption length below the millimeter. As a consequence all experiments should be done under vacuum, and no transmissive optics but extremely thin ones may be used. A noticeable exception are metallic foils of a few hundred nanometer thickness, which can, in some spectral ranges, transmit significant amount of XUV light.

The two actual methods used, and mentioned in the lectures for this purpose, are "Stabilizing the carrier phase envelope (CPE) of a short pulse" and "Polarization gating by ellipticity varying pulse". So, now what remains is that we evaluate what method will work for what high intensity pulse. One obviously has a pulse duration of 100 fs and the other with a pulse duration of 5.4 fs. Experiments have shown that Polarization gating by ellipticity varying pulse is effective for pulses with duration of 5.4 fs, while CPE is effective for pulses at the duration of 100 fs[OSDV05][SME+06].

References

[Kar13] Stefan Karsch. *Generation of Ultraintense Laser Pulses.* Publisher Name, 2013.

[OSDV05] Dan Oron, Yaron Silberberg, Nirit Dudovich, and David M. Villeneuve. Efficient polarization gating of high-order harmonic generation by polarization-shaped ultrashort pulses. *Phys. Rev. A*, 72:063816, Dec 2005.

[SME+06] I. J. Sola, E. Mével, L. Elouga, E. Constant, V. Strelkov, L. Poletto, P. Villoresi, E. Benedetti, J.-P. Caumes, S. Stagira, C. Vozzi, G. Sansone, and M. Nisoli. Controlling attosecond electron dynamics by phase-stabilized polarization gating. *Nature Physics*, 2:319–322, 2006.