PROBLEM SHEET 1 IN APPLIED OPTICS

PHYC40210

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1 Question 1

We want to show that Ex and Ey satisfy the ellipse equation in an Ex-Ey coordinate system. In the following we will try to satisfy the Jones vector criteria for representation of an elliptic polarized light between linear and circular. And this satisfies that at any instance Ex and Ey satisfy the Fresnel equation for an ellipse. We begin by just inserting into the equation, as is customary.

$$\frac{E_{0x}^2 E_x^2}{E_{0x}^2} + \frac{E_{0y}^2 E_y^2}{E_{0y}^2} - \frac{-2E_{0x}E_{0y}E_xE_y}{E_{0x}E_{0y}} cos(\theta) = sin^2(\theta) \tag{1}$$

After some arithmetic with trigonometric identities, we get:

$$E_x^2 + E_y^2 - 2E_xE_y cos(\theta) = \frac{1}{2}(1 - cos(\theta)) \tag{2}$$

$$E_x^2 + E_y^2 - 2E_xE_y cos(\theta) = \frac{1}{2}(1 - \frac{1 - tan^2(\theta)}{1 + tan^2(\theta)}) \tag{3}$$

We continue to make use of trigonometric identities like $sec^2(\theta) = 1 + tan^2(\theta)$ and $sin^2(\theta)$ = 1 - $cos^2(\theta)$ to obtain:

$$1 - \frac{1 - tan^2(\theta)}{1 + tan^2(\theta)} = 1 - \frac{1}{sec^2(\theta)} + \frac{tan^2(\theta)}{sec^2(\theta)} \tag{4}$$

From this (4) we get:

$$1 - \frac{1}{1 + tan^2(\theta)} + \frac{tan^2(\theta)}{1 + tan^2(\theta)} \tag{5}$$

$$1 - \frac{1 + tan^2(\theta) - tan^2(\theta)}{1 + tan^2(\theta)} = 1 - \frac{1}{1 + tan^2(\theta)} \tag{6}$$

We then continue to make use of some more identities to get:

$$\frac{1}{1 + tan^2(\theta)} = 1 - \frac{cos^2(\theta)}{sin^2(\theta) + cos^2(\theta)} \tag{7}$$

giving us

$$1 - \frac{cos^2(\theta)}{1} = 1 - cos^2(\theta) \tag{8}$$

Then recall that

$$cos^2(\theta) = 2cos^2(\theta) - 1 = cos(2\theta) \tag{9}$$

And we get

$$= tan(2\theta) \tag{10}$$

which we of course use to get the following equation:

$$a = \frac{1}{2} arctan(\frac{2E_xE_y cos(\theta)}{E_x^2 - E_y^2}) \tag{11}$$

This angle, a, represents one of the axes of the ellipse and occurs at the angle with respect to the x-axis (see P6.8). This angle sometimes corresponds to the minor axis and sometimes to the major axis of the ellipse.

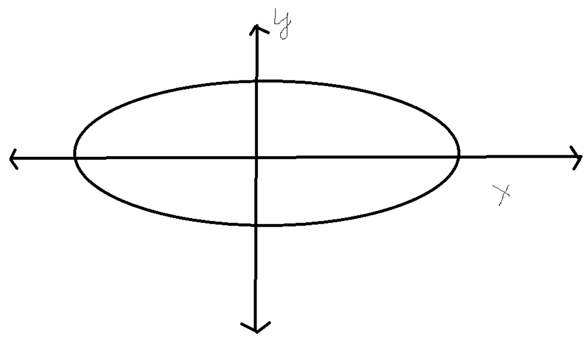

Figure 1: A schematic of an ellipse, a bit rudimentary but the intention was to show how it would look

1.1 a)

The question asks us to show an ellipse with the major Axis on the x-axis When the major axis is on the x-axis, the ellipse will look elongated horizontally, as is seen on Figure 1. . The electric field vectors oscillate predominantly in the x-direction.

1.2 b)

Second question asks us to show linearly polarized light at 45°, in this event we have equal contribution from Ex and Ey and we get a line 45 degrees from x-axis or origo.

1.3 c)

We are in the third question asked for a circular right. Here we have a 180 degrees light or zero degrees (or gone several revolutions).

2 Question 2

In this second question we are asked to Calculate the coherence length for a KDP crystal used to frequency double ruby laser light. The wavelength of a ruby light is 694 nm, it's refractive index is given at 1.505 and wavelength of the second harmonic (2): $\lambda' = \lambda/2 =$ 347 nm (0.347 m). The equation for coherence length is given according to:

$$L = \frac{\lambda^2}{n\Delta\lambda} \tag{12}$$

where the term $\Delta\lambda$ represents the spectral width of the light source and is given by:

$$\Delta\lambda = \frac{\lambda^2 \Delta v}{c} \tag{13}$$

Here the frequency (v), wavelength (λ) and speed of light (c) for ruby are already readily accessible from literature and inserted in the code snippet below. By making use of the following Python code we get:

```
e = 694*10e-9
c = 3*10^8
f = (c/e)
n = 1.505

Delta_lambda = ((e**2)*f)/c

Coherence_length = ((e**2)/(n*Delta_lambda))

print(Coherence_length)
```

The code gave us the answer of the coherence length for a KDP crystal is 4.611295681063123e-06 meters. Which based on the coherence length for visible light being around 15e-06 meters should be correct.

The second part of the question posed asks us to also find the phase matching angle given that:
ne (λ = 694 nm) = 1.465, ne (λ = 347 nm) = 1.487, no (λ = 694 nm) = 1.505, no (λ = 347 nm) = 1.534. The equation used to obtain the phase matching angle was, with values inserted:

$$(1/(1.505)^2) = ((sin^2(\theta))/(1.465^2)) + ((cos^2(\theta))/(1.534^2)) \tag{14}$$

This then became, due to theta being real:

$$cos(2\theta) = 0.192885 \tag{15}$$

$$2\theta = arccos(0.192885) \tag{16}$$

$$\theta = 39.4^\circ \tag{17}$$

3 Question 3

In this question we use quasi-phase matching of the ordinary waves in KDP instead of angle matching. We estimate the grating period that would be required for frequency doubling of a ruby laser beam.

$$\Delta k = 2\pi \left(\frac{n_o(\lambda_2)}{\lambda_2} - \frac{n_o(\lambda_1)}{\lambda_1} \right) \tag{18}$$

Where Δ k is the phase mismatch defined as Δ k = k_{2w} - $2k_w$. We insert the values and get:

$$\Delta k = 2\pi \left(\frac{1.534}{347e-9} - \frac{1.505}{694e-9} \right) \tag{19}$$

$$\Delta k \approx 1.4150710^7 \tag{20}$$

The grating period can be related to the phase mismatch as following:

$$\Delta k = \frac{\pi}{\Lambda} \tag{21}$$

$$\Lambda \approx \frac{\pi}{1.4150710^7} \tag{22}$$

This then gives us the final answer for the grating as:

$$\Lambda = 2.2201010e - 7[meter] \tag{23}$$

Or more presicely, a grating period of 222 nm.

4 Question 4

We are asked to by drawing diagrams of index surfaces, show how angle matching is achieved for SHG using both positive and negative, Type I, uniaxial crystals. This is done in the figure below.

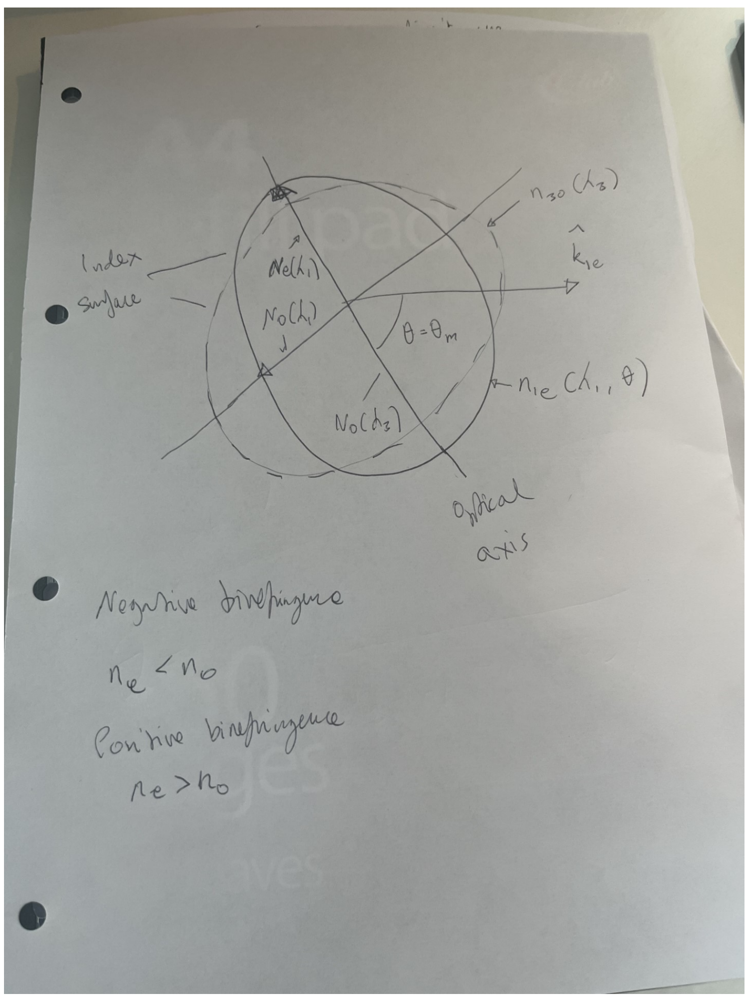

Figure 2: Figure shows an index surface with type 1 phase matching