PHYC40210 Problem Sheet 1, due 6 pm 1st March 2024

Q1 Suppose that we write the E_x and E_y components of a light wave generally as

$$\mathbf{E_x} = \mathbf{i}\, E_{0x} \cos(kz - \omega t) \; \textit{and} \; \mathbf{E_y} = \mathbf{j}\, E_{0y} \cos(kz - \omega t + \phi).$$

Show that at any instant E_x and E_y satisfy the ellipse equation on the E_y and E_x coordinate system:

$$\left(\frac{E_y}{E_{0y}}\right)^2 + \left(\frac{E_x}{E_{0x}}\right)^2 - 2\left(\frac{E_y}{E_{0y}}\right)\left(\frac{E_x}{E_{0x}}\right)\cos(\phi) = \sin^2(\phi) \quad (1)$$

Sketch schematically what this ellipse looks like assuming $E_{0x} = 2E_{0y}$.

Show when ellipse (1) would form:

(a) an ellipse with its major axis on the x-axis,

(b) linearly polarized light at 45°,

(c) right or left circularly polarized light?

Q2. Calculate the coherence length for a KDP crystal used to frequency double ruby laser light. (you can just use the ordinary indices (n_o) for this part).

Find also the phase matching angle given that
n_e (λ = 694 nm) = 1.465, n_e (λ = 347 nm) = 1.487,
n_o (λ = 694 nm) = 1.505, n_o (λ = 347 nm) = 1.534

Q3. Suppose that instead of angle matching we decided to use quasi-phase matching of the ordinary waves in KDP. From the data in Q2 above, estimate the grating period that would be required for frequency doubling of a ruby laser beam.

Q4. By drawing diagrams of index surfaces, show how angle matching is achieved for SHG using both positive and negative, Type I, uniaxial crystals.