Last time
Frequency Characteristics of AC Circuits

Chapter 8

- Introduction
- Two-port Networks
- The Decibel (dB)
- Frequency Response
- A High-Pass *RC* Network
- A Low-Pass *RC* Network
- A Low-Pass *RL* Network
- A High-Pass *RL* Network
- A Comparison of *RC* and *RL* Networks
- Bode Diagrams
- Combining the Effects of Several Stages
- *RLC* Circuits and Resonance
- Filters
- Stray Capacitance and Inductance

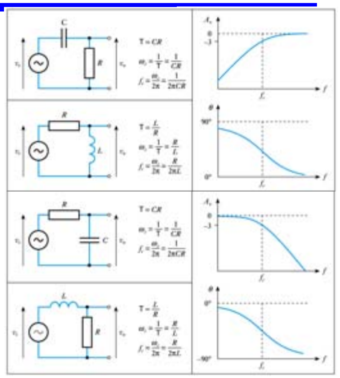
Key Points

- The reactance of capacitors and inductors is dependent on frequency
- Single *RC* or *RL* networks can produce an arrangement with a single upper or lower cut-off frequency
- In each case the angular cut-off frequency ω_o is given by the reciprocal of the time constant T
- For an *RC* circuit $\mathrm{T} = CR$, for an *RL* circuit $\mathrm{T} = L/R$
- Simple *RC* or *RL* networks represent single-pole filters

A Comparison of *RC* and *RL* Networks

8.9

- Circuits using RC and RL techniques have similar characteristics
  - see **Figure 8.12** in the course text

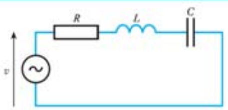
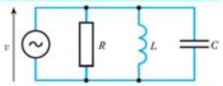
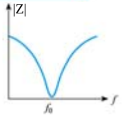
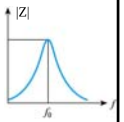


Filters

8.13

- ***RC* Filters**
- The *RC* networks considered earlier are **first-order** or **single-pole** filters
  - these have a maximum roll-off of 6 dB/octave
  - they also produce a maximum of 90° phase shift
- Combining multiple stages can produce filters with a greater ultimate roll-off rates (12 dB, 18 dB, etc.) but such filters have a very soft 'knee'

RLC Circuits and Resonance

	Series resonant circuit	Parallel resonant circuit
Circuit		
Impedance, Z	$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$ 	$Z = \frac{1}{\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)}$ 
Resonant frequency, f_0	$f_0 = \frac{1}{2\pi\sqrt{LC}}$	$f_0 = \frac{1}{2\pi\sqrt{LC}}$
Quality factor, Q	$Q = \frac{1}{R}\sqrt{\left(\frac{L}{C}\right)}$	$Q = R\sqrt{\left(\frac{C}{L}\right)}$
Bandwidth, B	$B = \frac{R}{2\pi L}$ Hz	$B = \frac{1}{2\pi RC}$ Hz

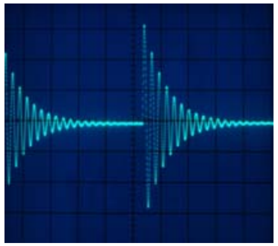
Key Points

- The reactance of capacitors and inductors is dependent on frequency
- Single *RC* or *RL* networks can produce an arrangement with a single upper or lower cut-off frequency
- In each case the angular cut-off frequency ω_o is given by the reciprocal of the time constant T
- For an *RC* circuit T = *CR*, for an *RL* circuit T = *L/R*
- Simple *RC* or *RL* networks represent single-pole filters
- Resonance occurs when the reactance of the capacitive element cancels that of the inductive element
- RLC circuits in resonance will have maximum impedance, R, in a parallel configuration and minimum impedance,R, when in series
- 2nd order circuits (RLC) can produce sharper filters
- Active filters produce high performance without inductors
- Stray capacitance and inductance are found in all circuits

Transient Behaviour

Chapter 9

- Introduction
- Charging Capacitors and Energising Inductors
- Discharging Capacitors and De-energising Inductors
- Response of First-Order Systems
- Second-Order Systems
- Higher-Order Systems



Introduction

9.1

- So far we have looked at the behaviour of systems in response to:
  - fixed DC signals
  - constant AC signals
- We now turn our attention to the operation of circuits before they reach steady-state conditions
  - this is referred to as the **transient response**
- We will begin by looking at simple *RC* and *RL* circuits

Charging Capacitors and Energising Inductors

Capacitor Charging

- Consider the circuit shown here
  - Applying Kirchhoff's voltage law

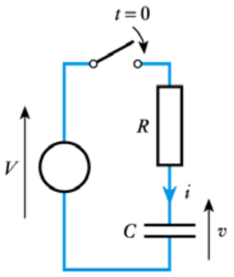
$$iR + v = V$$

  - Now, in a capacitor

$$i = C\frac{dv}{dt}$$

  - which substituting gives

$$CR\frac{dv}{dt} + v = V$$



- The above is a first-order differential equation with constant coefficients
- Assuming V_C = 0 at t = 0, this can be solved to give

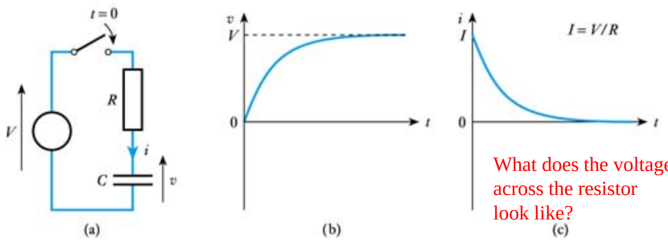
$$v = V(1 - e^{-\frac{t}{CR}}) = V(1 - e^{-\frac{t}{T}})$$

  - see **Section 9.2.1** of the course text for this analysis
- Since i = $C dv/dt$ this gives (assuming V_C = 0 at t = 0)

$$i = Ie^{-\frac{t}{CR}} = Ie^{-\frac{t}{T}}$$

  - where $I = V/R$

- Thus both the voltage and current have an exponential form



What does the voltage across the resistor look like?

Inductor energising

KVL ⇒ iR + v = V, and v=L(di/dt)

- A similar analysis of this circuit gives

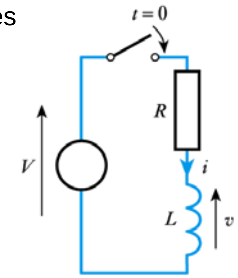
$$iR + L\frac{di}{dt} = V$$

$$i = I(1 - e^{-\frac{Rt}{L}}) = I(1 - e^{-\frac{t}{T}})$$

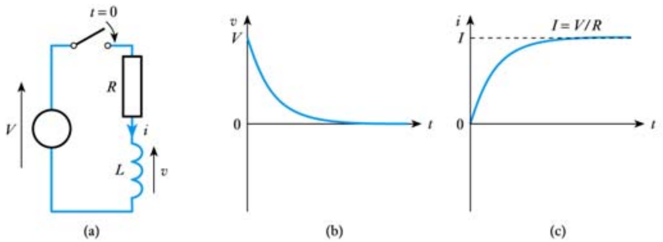
$$v = Ve^{-\frac{Rt}{L}} = Ve^{-\frac{t}{T}}$$

where $I = V/R$

  - see **Section 9.2.2** for this analysis



- Thus, again, both the voltage and current have an exponential form



Discharging Capacitors and De-energising Inductors

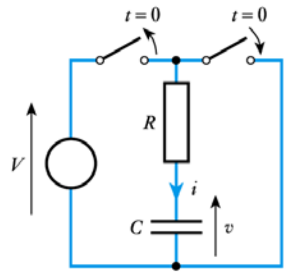
Capacitor discharging

- Consider this circuit for discharging a capacitor
  - At $t = 0$, $V_C = V$
  - From Kirchhoff's voltage law

$$iR + v = 0$$

  - giving

$$CR\frac{dv}{dt} + v = 0$$



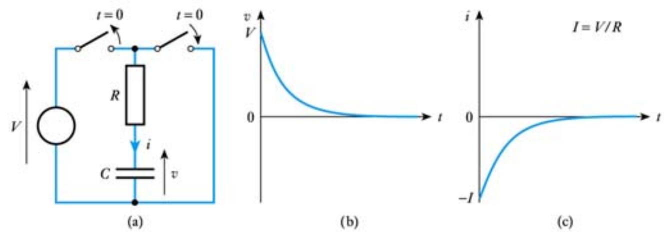
- Solving this as before gives

$$v = Ve^{-\frac{t}{CR}} = Ve^{-\frac{t}{T}}$$

$$i = -Ie^{-\frac{t}{CR}} = -Ie^{-\frac{t}{T}}$$

  - where $I = V/R$
  - see **Section 9.3.1** for this analysis

- In this case, both the voltage and the current take the form of decaying exponentials



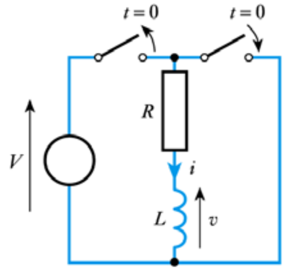
Inductor de-energising

- A similar analysis of this circuit gives

$$iR + L\frac{di}{dt} = 0$$

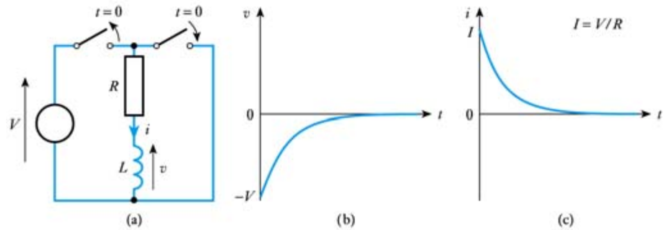
$$i = Ie^{-\frac{Rt}{L}} = Ie^{-\frac{t}{T}}$$

$$v = -Ve^{-\frac{Rt}{L}} = -Ve^{-\frac{t}{T}}$$

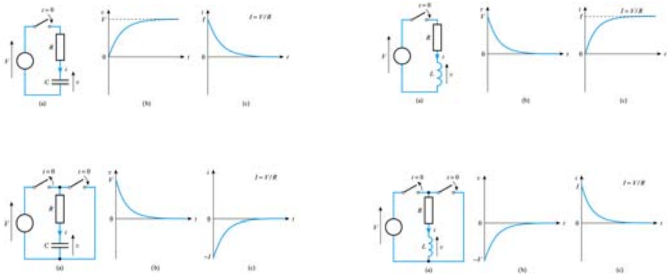


  – where $I = V/R$
  – see **Section 9.3.1** for this analysis

- And once again, both the voltage and the current take the form of decaying exponentials



- A comparison of the four circuits



Response of First-Order Systems

9.4

- **Initial and final value formulae**
  – increasing or decreasing exponential waveforms (for either voltage or current) are given by:

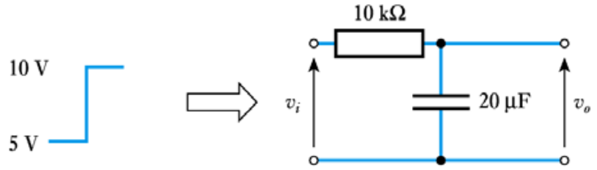
$$v = V_f + (V_i - V_f)e^{-t/T}$$

$$i = I_f + (I_i - I_f)e^{-t/T}$$

  – where V_i and I_i are the *initial* values of the voltage and current
  – where V_f and I_f are the *final* values of the voltage and current
  – the first term in each case is the **steady-state response**
  – the second term represents the **transient response**
  – the combination gives the **total response** of the arrangement

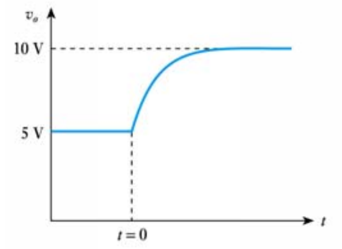
Example – see Example 9.3 from course text

The input voltage to the following *CR* network undergoes a step change from 5 V to 10 V at time t = 0. Derive an expression for the resulting output voltage



Here the initial value is 5 V and the final value is 10 V. The time constant of the circuit equals $CR = 10 \times 10^3 \times 20 \times 10^{-6} = 0.2$s. Therefore, from above, for $t \geq 0$

$$
\begin{aligned}
v &= V_f + (V_i - V_f)e^{-t/T} \\
&= 10 + (5 - 10)e^{-t/0.2} \\
&= 10 - 5e^{-t/0.2} \text{ volts}
\end{aligned}
$$



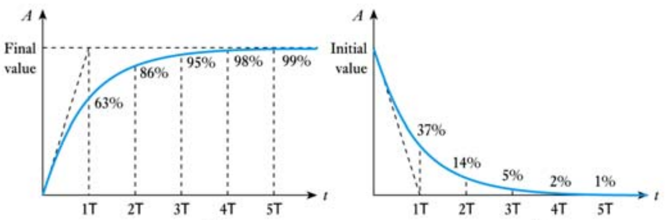
And the current?

$i = C(dv/dt)$

$i = (V_f - V_i)/R \cdot e^{-t/T}$

$I_i = (V_f - V_i)/R$, $I_f = 0$

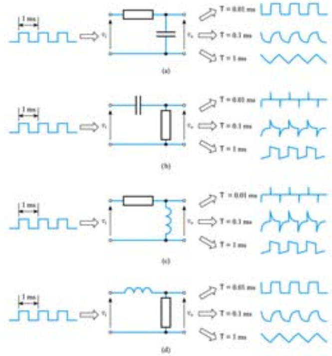
The nature of exponential curves



(a) $A(1 - e^{-t/T})$ (b) $A\,e^{-t/T}$

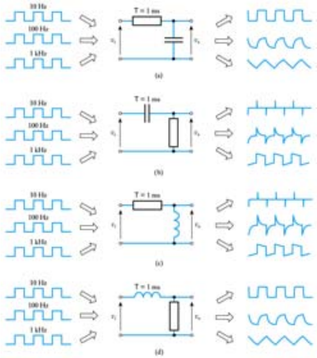
Response of first-order systems to a square waveform

– see Section 9.4.3



Response of first-order systems to a square waveform of different frequencies

– see **Section 9.4.3**



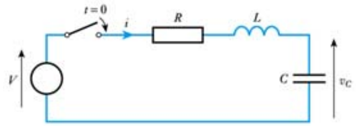
Second-Order Systems

9.5

- Circuits containing both capacitance and inductance are normally described by second-order differential equations. These are termed **second-order systems**
  – for example, this circuit is described by the equation

$v_R + v_L + v_C = V$ and $v_L = L(di/dt)$, and $v_R = iR$
but $i = C(dv_C/dt)$. So

$$LC\frac{d^2v_C}{dt^2} + RC\frac{dv_C}{dt} + v_C = V$$



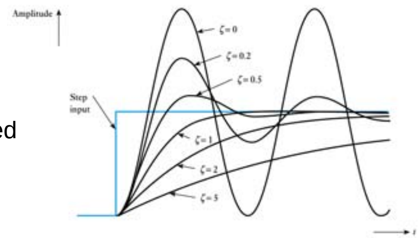
- When a step input is applied to a second-order system, the form of the resultant transient depends on the relative magnitudes of the coefficients of its differential equation. The general form of the response is

$$\frac{1}{\omega_n^2}\frac{d^2y}{dt^2} + \frac{2\zeta}{\omega_n}\frac{dy}{dt} + y = x$$

– where ω_n is the **undamped natural frequency** in rad/s and ζ (Greek Zeta) is the **damping factor**

Response of second-order systems

ζ =0 undamped
ζ <1 under damped
ζ =1 critically damped
ζ >1 over damped



Higher-Order Systems

9.6

- Higher-order systems are those that are described by third-order or higher-order equations
- These often have a transient response similar to that of the second-order systems described earlier
- Because of the complexity of the mathematics involved, they will not be discussed further here

Further Study

Video 9B Further Study

- The Further Study section at the end of Chapter 9 considers the problem of determining the time constant of a circuit, so that the initial and final value theorems can be applied.

- Two sample circuits are given so that you can test your understanding.
- Calculate the time constants of the circuits and then check your results by looking at the video.

Key Points

- The charging or discharging of a capacitor, and the energising and de-energising of an inductor, are each associated with exponential voltage and current waveforms
- Circuits that contain resistance, and either capacitance or inductance, are termed first-order systems
- The increasing or decreasing exponential waveforms of first-order systems can be described by the initial and final value formulae
- Circuits that contain both capacitance and inductance are usually second-order systems. These are characterised by their undamped natural frequency and their damping factor

Today: Amplification

Chapter 14

- Introduction
- Electronic amplifiers
- Sources and loads
- Equivalent circuit of an amplifier
- Output power
- Power gain
- Frequency response and bandwidth
- Differential amplifiers
- Simple amplifiers

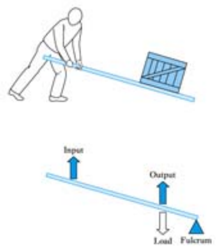
Introduction

14.1

- Amplification is one of the most common processing functions
- **Amplification** means making things bigger
- **Attenuation** means making things smaller
- There are many non-electronic forms of amplification

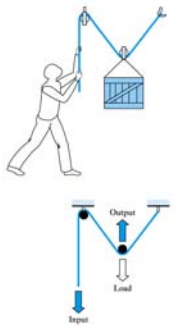
Amplifiers

- **Non-electronic amplifiers**
  - **Levers**
    - Example shown on the right is a force *amplifier*, but a displacement *attenuator*
    - Reversing the position of the input and output would produce a force *attenuator* but a displacement *amplifier*
    - This is an example of a **non-inverting amplifier** (since the input and output are in the same direction)



A lever arrangement

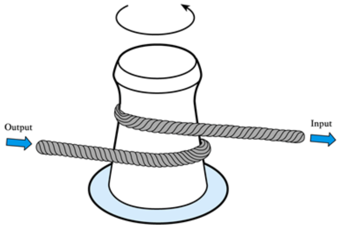
- **Non-electronic amplifiers**
  - **Pulleys**
    - Example shown right is a force *amplifier*, but a displacement *attenuator*
    - This is an example of an **inverting amplifier** (since the input and output are in opposite directions) but other pulley arrangements can be non-inverting



A pulley arrangement

- **Passive and active amplifiers**
  - Levers and pulleys are examples of **passive amplifiers** since they have no external energy source
    - In such amplifiers the power delivered at the output must be less than (or equal to) that absorbed at the input
  - Some amplifiers are not passive but are **active amplifiers** in that they have an external source of power
    - In such amplifiers the output can deliver more power than is absorbed at the input

- **Non-electronic active amplifiers**
  - an example is the torque amplifier shown here



Small force here Increase friction and proportionally get the motor's power at output

Big motor turning the winch

9.37

Electronic amplifiers

14.2

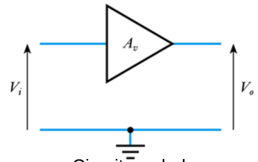
- Can be passive (e.g. a transformer) but most are active
- We will concentrate on *active* electronic amplifiers
  - take power from a power supply
  - amplification described by gain

What about in dB?

$$\text{Voltage Gain } (A_v) = \frac{V_o}{V_i}$$

$$\text{Current Gain } (A_i) = \frac{I_o}{I_i}$$

$$\text{Power Gain } (A_p) = \frac{P_o}{P_i}$$

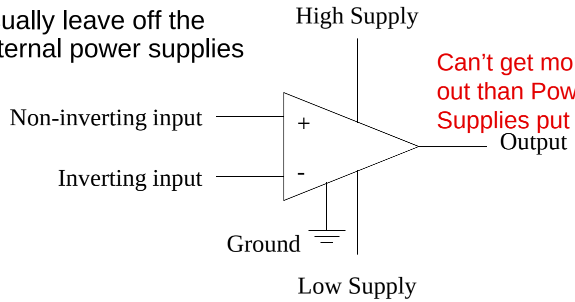


Circuit symbol

9.38

Electronic amplifiers

- Circuit Symbol
- Usually leave off the external power supplies

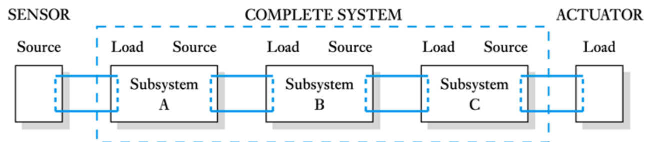


Can't get more out than Power Supplies put in

Sources and loads

14.3

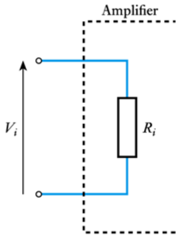
- An *ideal* voltage amplifier would produce an output determined only by the input voltage and its gain.
  - irrespective of the nature of the source and the load
  - in real amplifiers this is not the case
  - the output voltage is affected by **loading**



9.40

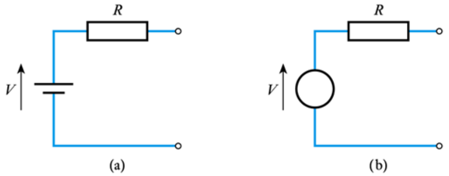
Modelling sources and loads

- **Modelling the input of an amplifier**
  - the input can often be adequately modelled by a simple resistor
  - the **input resistance**



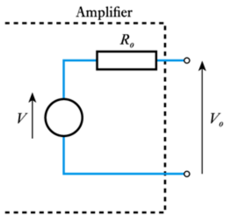
9.41

- **Modelling the output of a circuit**
  - all real voltage sources have an output resistance
  - for example, a battery can be represented by an ideal voltage source and a series resistance representing its **output resistance**



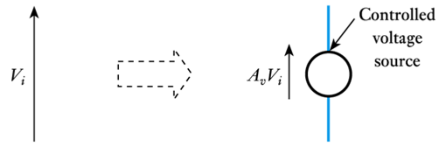
9.42

- **Modelling the output of an amplifier**
  - Similarly, the output of an amplifier can be modelled by an ideal voltage source and an output resistance.
  - This is an example of a **Thévenin equivalent circuit**



9.43

- **Modelling the gain of an amplifier**
  - can be modelled by a controlled voltage source
  - the voltage produced by the source is determined by the input voltage to the circuit



9.44

Equivalent circuit of an amplifier

Video 14A

14.4

- Having modelled the input, the output and the gain, we can now model the entire amplifier