Last time: Resistance and DC Circuits

Chapter 3

- Introduction
- Current and Charge
- Voltage Sources
- Current Sources
- Resistance and Ohm's Law
- Kirchhoff's Laws
- Resistors in Series and Parallel
- Thévenin's and Norton's Theorems
- Superposition
- Nodal Analysis
- Mesh Analysis
- Solving Simultaneous Circuit Equations
- Choice of Techniques

Resistance and Ohm's Law

3.5

- **Ohm's law**

$$V \propto I$$

– constant of proportionality is the resistance R

– hence

$$V = IR \qquad I = \frac{V}{R} \qquad R = \frac{V}{I}$$

– current through a resistor causes power dissipation

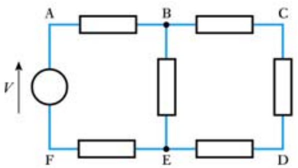
$$P = IV \qquad P = \frac{V^2}{R} \qquad P = I^2 R$$

Kirchhoff's Laws

Video 3A

3.7

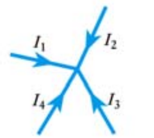
- **Node**
  – a point in a circuit where two or more circuit components are joined
- **Loop**
  – any closed path that passes through no node more than once
- **Mesh**
  – a loop that contains no other loop
- **Examples:**
  – A, B, C, D, E and F are *nodes*
  – the paths ABEFA, BCDEB and ABCDEFA are *loops*
  – ABEFA and BCDEB are *meshes*



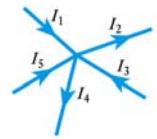
- **Current Law**

At any instant, the algebraic sum of all the currents flowing into any node in a circuit is zero

– if currents flowing *into* the node are positive, currents flowing *out of* the node are negative, then $\sum I = 0$



$I_1 + I_2 + I_3 + I_4 = 0$

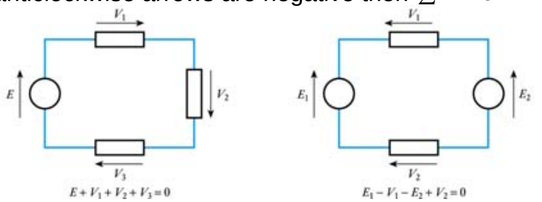


$I_1 - I_2 + I_3 - I_4 + I_5 = 0$

- **Voltage Law**

  At any instant the algebraic sum of all the voltages around any loop in a circuit is zero

  – if clockwise voltage arrows are positive and anticlockwise arrows are negative then $\sum V = 0$



Today: Resistance and DC Circuits

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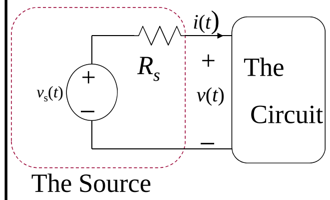
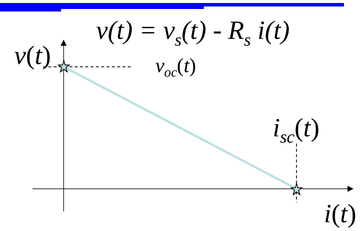
Thevenin's Theorem

3.8

- Any linear circuit with sources (dependent and/or independent) and resistors ***can be replaced by an equivalent circuit containing a single voltage source and a single resistor.***
- Thevenin's theorem implies that we can replace arbitrarily complicated networks with simple networks for purposes of analysis.
- If we have a single "load" resistor which is subject to change, we simplify circuit to a simple Thevenin equivalent with a single resistor and can recalculate easily for many different loads

A Realistic Source Model





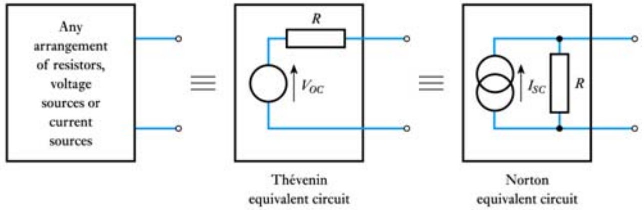
- Remove "The Circuit" $\Rightarrow$ source supplies maximum voltage at 0 current
- Short out "The Circuit" $\Rightarrow$ source supplies maximum current at 0 voltage
- Since the **open circuit voltage** and the **short circuit current** determine where the I-V line crosses both axes, they completely define the line.
- Any source circuit with the same I-V characteristics is equivalent.

Norton's Theory 2

- Any Thevenin equivalent circuit is in turn equivalent to a current source in parallel with a resistor [source transformation].
- A current source in parallel with a resistor is called a Norton equivalent circuit.
- Finding a Norton equivalent circuit requires essentially the same process as finding a Thevenin equivalent circuit.

An arbitrary, complicated circuit with independent sources

Can be transformed to a Thévenin or Norton equivalent circuit:

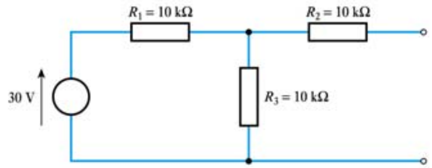


– from the Thévenin equivalent circuit $I_{SC} = \frac{V_{OC}}{R}$

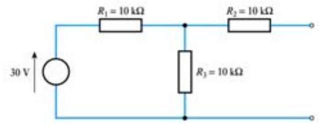
– hence for *either* circuit $R = \frac{V_{oc}}{I_{sc}}$

- **Example** – see **Example 3.3** from course text

  Determine Thévenin and Norton equivalent circuits of the following circuit



Example (continued)



– if nothing is connected across the output no current will flow in R_2 so there will be no voltage drop across it. Hence V_o is determined by the voltage source and the potential divider formed by R_1 and R_3. Hence

$$V_{OC} = \frac{30}{2} = 15\ \text{V}$$

– if the output is shorted to ground, R_2 is in parallel with R_3 and the current taken from the source is 30V/15 kΩ = 2 mA. This will divide equally between R_2 and R_3 so the output current, and so

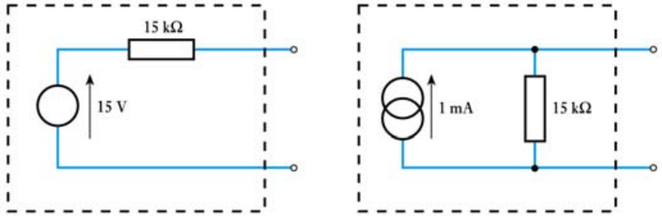
$$I_{sc} = 1\text{mA}$$

– the resistance in the equivalent circuit is therefore given by

$$R = \frac{V_{oc}}{I_{sc}} = \frac{15\ \text{V}}{1\text{mA}} = 15\ \text{k}\Omega$$

Example (continued)

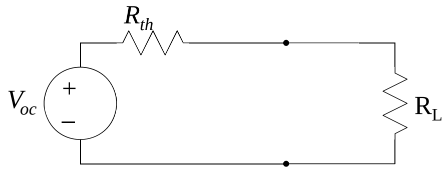
– hence equivalent circuits are:



Thévenin equivalent circuit

Norton equivalent circuit

Now compute the Thevenin equivalent circuit for calculations



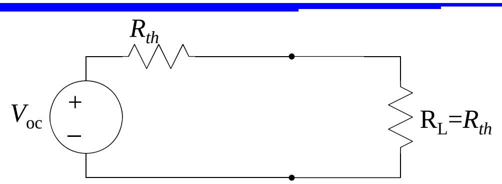
$I_L = V_{oc} /(R_{th}+R_L)$ and $V_L = R_L\ V_{oc} /(R_{th}+R_L)$

Note, Power dissipated in the load is :

$P_L=R_L I^2 =R_L\ [V_{oc}/(R_{th}+R_L)]^2$

Differentiate with respect to R_L to find that the maximum energy transfer is when $R_L=R_{th}$

Impedance Matched Circuit



This circuit will dissipate the maximum energy in the load

E.g. audio circuit or heating circuit

3.9

Circuit analysis techniques: Superposition

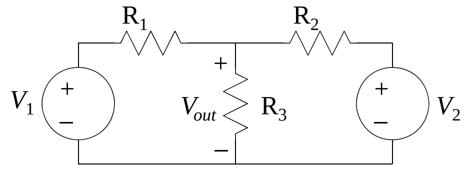
- **Principle of superposition**

  In any linear network of resistors, voltage sources and current sources, each voltage and current in the circuit is equal to the algebraic sum of the voltages or currents that would be present if each source were to be considered separately. When determining the effects of a single source the remaining sources are replaced by their internal resistance

How to Apply Superposition

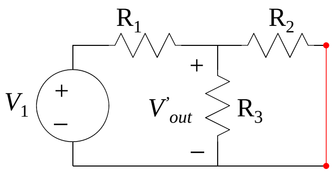
- To find the contribution due to an individual independent source, zero out the other independent sources in the circuit.
  - Voltage source ⇒ short circuit.
  - Current source ⇒ open circuit.
- Solve the resulting circuit using your favorite technique(s).

The Summing Circuit



For a given V_1 and V_2, what is the voltage V_{out} across R_3?

Superposition-Solve for V'_{out} from V_1 only



Short out the second voltage source, V_2.

$I' = V_1/(R_1+R_{2//3})$

$V'_{out} = I' R_{2//3}$

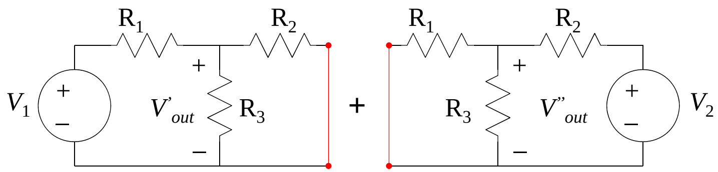
Where

$R_{2//3} = R_2R_3/(R_2+R_3)$

Note $R_{2//3} \Rightarrow R_2$ in parallel with R_3.

$R_{2//3} = (R_2 \cdot R_3) / (R_2+R_3)$

Superposition-Then get V''_{out} from V_2 only



$I' = V_1/(R_1+R_{2//3})$

$V'_{out} = I' R_{2//3}$

Where

$R_{2//3} = R_2R_3/(R_2+R_3)$

$I'' = V_2/ (R_2+R_{1//3})$

$V''_{out} = I\, R_{1//3}$

Where

$R_{1//3} = R_1R_3/(R_1+R_3)$

Use of Superposition

$$V_{out} = V'_{out} + V''_{out}$$

$$V'_{out} = \frac{V_1 \cdot R_2 \cdot R_3}{R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3} \qquad V''_{out} = \frac{V_2 \cdot R_1 \cdot R_3}{R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3}$$

If $R_1 = R_2 = R_3 = 1\text{k}\Omega$

then

$$V'_{out} = V_1 / 3$$

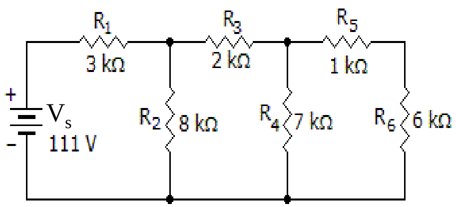
$$V''_{out} = V_2 / 3$$

$$V_{out} = V'_{out} + V''_{out} = V_1/3 + V_2/3$$

Other circuit analysis techniques

- **Brute force application of Ohm's law**
- **Nodal analysis**
- **Mesh analysis**

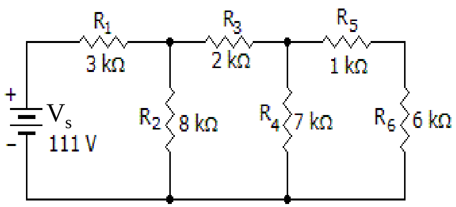
Want to know the currents through R_2, R_4 and R_6



Can be solved using:

1) Ohms law
2) Nodal analysis
3) Mesh analysis

Ohms law solution:



Total current is V_s across the total equivalent resistance

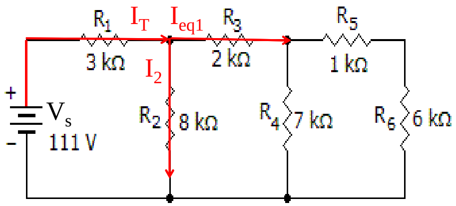
1) Equivalent resistance of R_3 through R_6:

$R_{eq1} = R_3 + R_4//(R_5+R_6) = 5500\ \Omega$

2) Total Equivalent Resistance R_1 through R_6:

$R_{eq} = R_1 + R_2//R_{eq1} = 6259.26\ \Omega$

Ohms law solution:



Total current is V_s across the total equivalent resistance

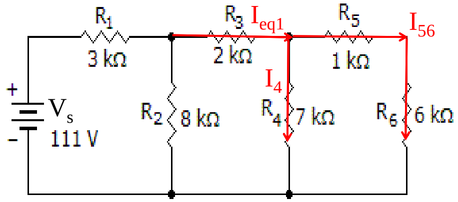
$I_T = V_s / R_{eq} = 111\text{ V} / (6259.26\ \Omega) = 17.73\text{ mA}$

This current is split into I_2 and I_{eq1} such that $I_T = I_2 + I_{eq1}$

At the node, $V_2 = V_{eq1}$, and Ohms law says $V = I{\cdot}R$, so

$I_2 \cdot R_2 = I_{eq1} \cdot R_{eq1} \Rightarrow I_2 = I_T/(1+R_2/R_{eq1}) = 7.22\text{ mA}$

Ohms law solution:



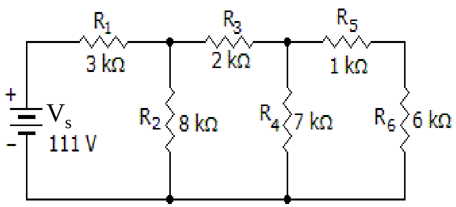
Similarly we can find that $I_{eq1} = 10.5\text{ mA}$

This current is split into I_4 and I_{56} such that $I_{eq1} = I_4 + I_{56}$

At the node, $V_4 = V_{56}$, and Ohms law says $V = I{\cdot}R$, so

$I_4 \cdot R_4 = I_{56} \cdot (R_5+R_6) \Rightarrow I_4 = 5.25\text{ mA}, \ \&\ \ I_{56} = 5.25\text{ mA}$

Want to know the currents through R_2, R_4 and R_6



Can be solved using:

1) Ohms law √
2) Nodal analysis
3) Mesh analysis

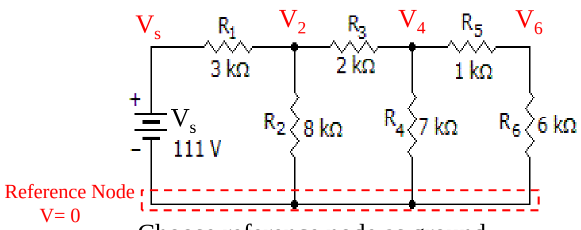
Nodal Analysis

Video 3B

3.10

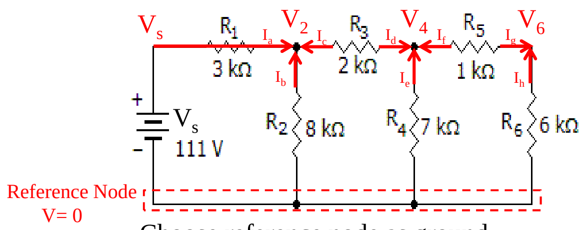
- Six steps:
  1. Chose one node as the reference node
  2. Label remaining nodes V_1, V_2, etc.
  3. Label any known voltages
  4. Apply Kirchhoff's current law to each unknown node
  5. Solve simultaneous equations to determine voltages
  6. If necessary calculate required currents

Want to know the currents through R_2, R_4 and R_6



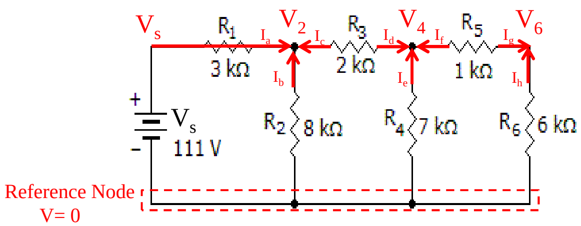
Choose reference node as ground
Label unknown nodal voltages V_2, V_4 and V_6
Label the known nodal voltage V_s = 111 V

Nodal Analysis



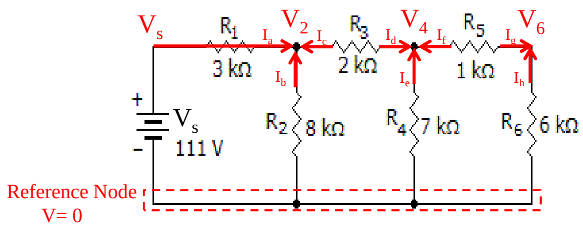
Choose reference node as ground
Label unknown nodal voltages V_2, V_4 and V_6
Label the known nodal voltage V_s = 111 V
Postulate currents into each unknown node

Nodal Analysis - Apply KCL at each node



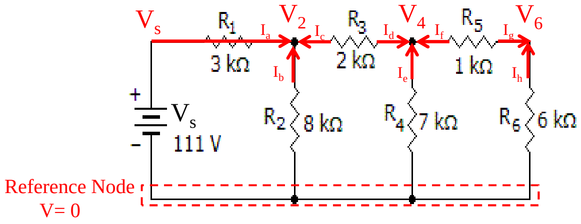
$I_a + I_b + I_c = 0$
$I_d + I_e + I_f = 0$
$I_g + I_h = 0$

Nodal Analysis -Apply Ohms law for each current



$I_a + I_b + I_c = 0 \quad \Rightarrow \quad (V_s - V_2) / R_1 + (0 - V_2) / R_2 + (V_4 - V_2) / R_3 = 0$
$I_d + I_e + I_f = 0 \quad \Rightarrow \quad (V_2 - V_4) / R_3 + (0 - V_4) / R_4 + (V_6 - V_4) / R_5 = 0$
$I_g + I_h = 0 \quad \Rightarrow \quad (V_4 - V_6) / R_5 + (0 - V_6) / R_6 + 0 = 0$

Nodal Analysis –Gather terms for 3 unknown V's



$V_2 \cdot (-1/R_1 -1/R_2 -1/R_3) + V_4 \cdot (1/R_3) + 0 = V_s /R_1$

$V_2 \cdot (1/ R_3) + V_4 \cdot (-1/R_3 -1/R_4 -1/R_5) + V_6 \cdot (1/ R_5) \quad = 0$

$0 + V_4 \cdot (1/R_5) + V_6 \cdot (-1/R_5 -1/R_6)= 0$

3.33

Solving Simultaneous Circuit Equations

– these equations can be expressed as

$$\begin{bmatrix} -(1/R_1+1/R_2+1/R_3) & (1/R_3) & 0 \\ (1/R_3) & -(1/R_2+1/R_3+1/R_4) & (1/R_5) \\ 0 & (1/R_5) & -(1/R_5+1/R_6) \end{bmatrix} \begin{bmatrix} V_2 \\ V_4 \\ V_6 \end{bmatrix} = \begin{bmatrix} -111V_s/R_1 \\ 0 \\ 0 \end{bmatrix}$$

– This yields V_2 = 57.8 V, V_4 = 36.8 V and V_6= 31.5 V

– **And Ohms law ⇒ I = V/R**

- **Current through $R_2 = V_2/R_2$ = (57.8 V)/(8000 Ω) = 7.22 mA**
- **Current through $R_4 = V_4/R_4$ = (36.8 V)/(7000 Ω) = 5.25 mA**
- **Current through $R_6 = V_6/R_6$ = (31.5 V)/(6000 Ω) = 5.25 mA**

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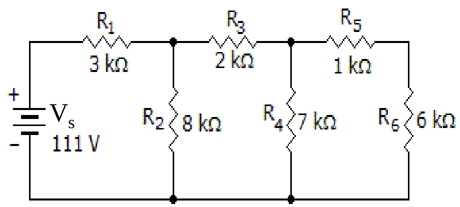
Want to know the currents through R_2, R_4 and R_6



Can be solved using:

1) Ohms law √
2) Nodal analysis √
3) Mesh analysis

3.35

Mesh Analysis

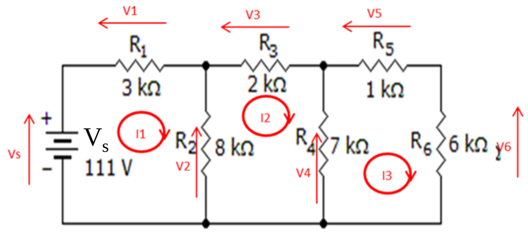
Video 3C

3.11

- Five steps:
  1. Identify the meshes and assign a clockwise-flowing current to each. Label these I_1, I_2, etc.
  2. Assign unknown voltages for each loop anti parallel to I_{loop}
  3. Apply Kirchhoff's voltage law to each mesh
  4. Solve the simultaneous equations to determine the currents I_1, I_2, etc.
  5. Use these values to obtain voltages if required

3.36

Mesh analysis



KVL around each mesh gives

$V_S - V1 - V2 = 0$

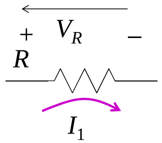
$V2 - V3 - V4 = 0$

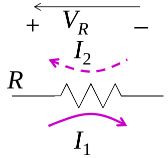
$V4 - V5 - V6 = 0$

And want to apply Ohms law to each voltage drop

3.37

Voltages from Mesh Currents





$$V_R = I_1 R \qquad V_R = (I_1 - I_2)\, R$$

Note Sign Convention: I_1 is anti-parallel to $V_R \Rightarrow V_{R1} = I_1 \cdot R$

I_2 parallel to $V_R \Rightarrow V_{R2} = -I_2 \cdot R$

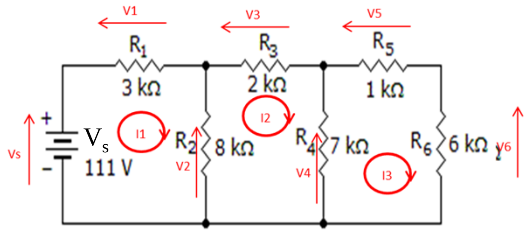
total voltage is the sum $V_R = (I_1 - I_2) \cdot R$

6.38

$V_S - V1 - V2 = 0$

$V2 - V3 - V4 = 0$

$V4 - V5 - V6 = 0$



Using Ohms law gives

$111V - R_1 \cdot I_1 - R_2 \cdot (I_1 - I_2) = 0$

$R_2 \cdot (I_1 - I_2) - R_3 \cdot I_2 - R_4 \cdot (I_2 - I_3) = 0$

$R_4 \cdot (I_2 - I_3) - R_5 \cdot I_3 - R_6 \cdot I_3 = 0$

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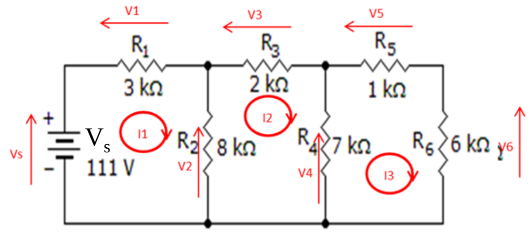
$V_S - V1 - V2 = 0$

$V2 - V3 - V4 = 0$

$V4 - V5 - V6 = 0$



Using KVL gives

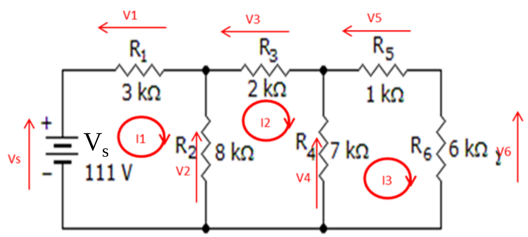
$111V - R_1 \cdot I_1 - R_2 \cdot (I_1 - I_2) = 0$

$R_2 \cdot (I_1 - I_2) - R_3 \cdot I_2 - R_4 \cdot (I_2 - I_3) = 0$

$R_4 \cdot (I_2 - I_3) - R_5 \cdot I_3 - R_6 \cdot I_3 = 0$

3.40

Mesh Analysis



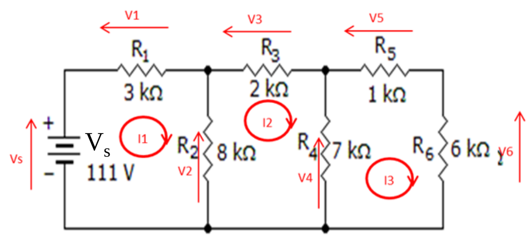
Using KVL gives

$$111V - R_1 \cdot I_1 - R_2 \cdot (I_1 - I_2) = 0$$

$$R_2 \cdot (I_1 - I_2) - R_3 \cdot I_2 - R_4 \cdot (I_2 - I_3) = 0$$

$$R_4 \cdot (I_2 - I_3) - R_5 \cdot I_3 - R_6 \cdot I_3 = 0$$

Mesh Analysis



Expand to:

$$111V - (R_1+R_2) \cdot I_1 + \quad (R_2) \quad \cdot I_2 \qquad 0 \qquad \cdot I_3 = 0$$

$$(R_2) \quad \cdot I_1 - (R_2+R_3+R_4) \cdot I_2 + \quad (R_4) \quad \cdot I_3 = 0$$

$$0 \quad \cdot I_1 \qquad (R_4) \quad \cdot I_2 - (R_4+R_5+R_6) \cdot I_3 = 0$$

Solving Simultaneous Circuit Equations

– these equations can be expressed as

$$\begin{bmatrix} -(R_1+R_2) & R_2 & 0 \\ R_2 & -(R_2+R_3+R_4) & R_4 \\ 0 & R_4 & -(R_4+R_5+R_6) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -111V \\ 0 \\ 0 \end{bmatrix}$$

– This yields I_1 = 17.73 mA, I_2 = 10.51 mA and I_3= 5.25 mA

– **But remember**

- **Net Current through $R_2 = I_1 - I_2$ = 7.22 mA**
- **Net Current through $R_4 = I_2 - I_3$ = 5.25 mA**
- **Net Current through $R_6 = I_3$ = 5.25 mA**

Advantages of Nodal Analysis

- Solves directly for node voltages.
- Current sources are easy.
- Voltage sources are either very easy or somewhat difficult.
- Works best for circuits with few nodes.
- Works for any circuit.

Advantages of Loop Analysis

- Solves directly for some currents.
- Voltage sources are easy.
- Current sources are either very easy or somewhat difficult.
- Works best for circuits with few loops.

Disadvantages of Loop Analysis

- Some currents must be computed from loop currents.
- Does not work with non-planar circuits.
- Choosing the right mesh may be difficult.

- FYI: PSpice uses a nodal analysis approach

Choice of Techniques

3.13

- How do we choose the right technique?
  - nodal and mesh analysis will work in a wide range of situations but are not necessarily the simplest methods
  - no simple rules
  - often involves looking at the circuit and seeing which technique seems appropriate

Further Study

Video 3D

Further Study

- The Further Study section at the end of Chapter 3 investigates the choice of circuit analysis techniques.
- A circuit is presented which could be analysed in a number of ways.
- Have a look and see which you think is best, then watch the video.

Key Points

- An electric current is a flow of charge
- A voltage source produces an e.m.f. which can cause a current to flow
- Current in a conductor is directly proportional to voltage
- At any instant the sum of the currents into a node is zero
- At any instant the sum of the voltages around a loop is zero
- Any two terminal network of resistors and energy sources can be replaced by a Thévenin or Norton equivalent circuit
- Nodal and mesh analysis provide systematic methods of applying Kirchhoff's laws

Next time: Alternating Voltages and Currents

Chapter 6

- Introduction
- Voltage and Current
- Reactance of Inductors and Capacitors
- Phasor Diagrams
- Impedance
- Complex Notation