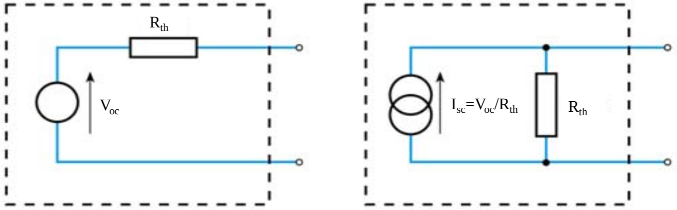
Last time

- Circuit analysis using:
  - Superposition
  - Ohms Law, Kirchoff's voltage law, Kirchoff's current law
  - Nodal analysis
  - Mesh analysis
- This time
  - AC circuits

Generalized Thevenin/Norton Analysis

1. Pick a good breaking point in the circuit (cannot split a dependent source and its control variable).
   Example of a good break point:
   Remove the load resistor and
   Compute the open circuit voltage, V_{OC}.
2. Short the load and
   Compute the short circuit current, I_{SC}.
3. Compute the Thevenin equivalent resistance, R_{Th}.
   $R_{Th} = (V_{OC}) / (I_{SC})$.

Replace the complex network with the equivalent circuit:



Thévenin equivalent circuit

Norton equivalent circuit

How to Apply Superposition

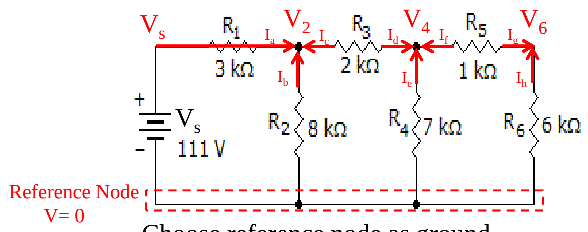
- To find the contribution due to an individual independent source, zero out all the other independent sources in the circuit.
  - Voltage source $\Rightarrow$ short circuit.
  - Current source $\Rightarrow$ open circuit.
- Solve the resulting circuit using your favorite technique(s).
- Add the contributions from the individual sources together.

Nodal Analysis

Video 3B 3.10

- Six steps:
  1. Chose one node as the reference node
  2. Label any nodes with known voltages
  3. Label remaining nodes V_1, V_2, etc. (you will solve for these)
  4. Postulate currents into each unknown node and apply Kirchhoff's current law to each unknown node
  5. Apply Ohms law to the current for each node & solve the resulting simultaneous equations for unknown voltages
  6. If necessary calculate required currents

Nodal Analysis



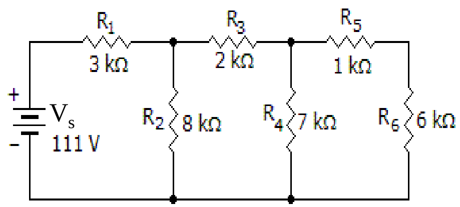
Choose reference node as ground

Label the known nodal voltage V_s = 111 V

Label unknown nodal voltages V_2, V_4 and V_6

Postulate currents into each unknown node

Want to know the currents through R_2, R_4 and R_6



Can be solved using:

1) Ohms law ⇒ I_2=7.22 mA, I_4=5.25 mA, I_6 = 5.25 mA√
2) Nodal analysis ⇒ I_2=7.22 mA, I_4=5.25 mA, I_6 = 5.25 mA √
3) Mesh analysis

This time

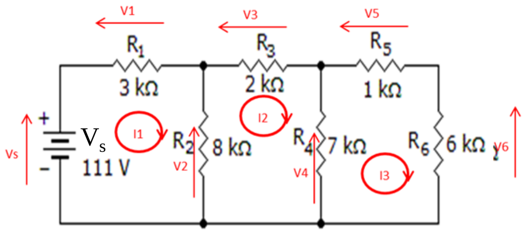
- Circuit analysis using:
  - Superposition
  - Ohms Law, Kirchoff's voltage law, Kirchoff's current law
  - Nodal analysis
  - Mesh analysis
- This time
  - AC circuits

Mesh Analysis

Video 3C 3.11

- Five steps:
  1. Identify the meshes and assign a clockwise-flowing current to each. Label these I_1, I_2, etc.
  2. Assign unknown voltages for each loop anti parallel to I_{loop}
  3. Apply Kirchhoff's voltage law to each mesh
  4. Solve the simultaneous equations to determine the currents I_1, I_2, etc.
  5. Use these values to obtain voltages if required

Mesh analysis



KVL around each mesh gives

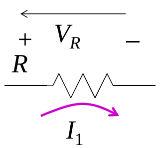
$V_S - V1 - V2 = 0$

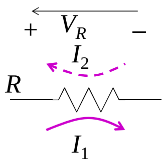
$V2 - V3 - V4 = 0$

$V4 - V5 - V6 = 0$

And want to apply Ohms law to each voltage drop

Voltages from Mesh Currents





$$V_R = I_1 R$$

$$V_R = (I_1 - I_2)\, R$$

Note Sign Convention: I_1 is anti-parallel to $V_R \Rightarrow V_{R1} = I_1 \cdot R$

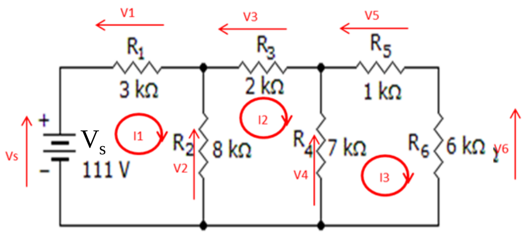
I_2 parallel to $V_R \Rightarrow V_{R2} = -I_2 \cdot R$

total voltage is the sum $V_R = (I_1 - I_2) \cdot R$

$V_S - V1 - V2 = 0$

$V2 - V3 - V4 = 0$

$V4 - V5 - V6 = 0$



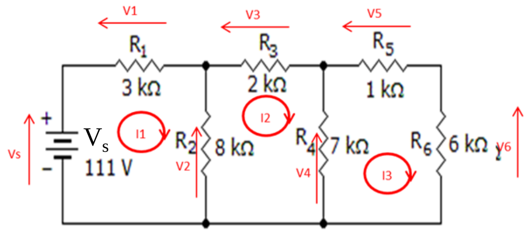
Using Ohms law gives

$111V - R_1 \cdot I_1 - R_2 \cdot (I_1 - I_2) = 0$

$R_2 \cdot (I_1 - I_2) - R_3 \cdot I_2 - R_4 \cdot (I_2 - I_3) = 0$

$R_4 \cdot (I_2 - I_3) - R_5 \cdot I_3 - R_6 \cdot I_3 = 0$

$V_S - V1 - V2 = 0$
$V2 - V3 - V4 = 0$
$V4 - V5 - V6 = 0$



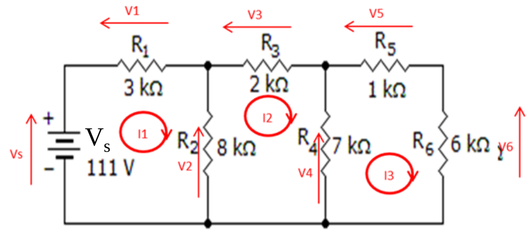
Using Ohms law gives

$111V - R_1 \cdot I_1 - R_2 \cdot (I_1 - I_2) = 0$

$R_2 \cdot (I_1 - I_2) - R_3 \cdot I_2 - R_4 \cdot (I_2 - I_3) = 0$

$R_4 \cdot (I_2 - I_3) - R_5 \cdot I_3 - R_6 \cdot I_3 = 0$

Mesh Analysis



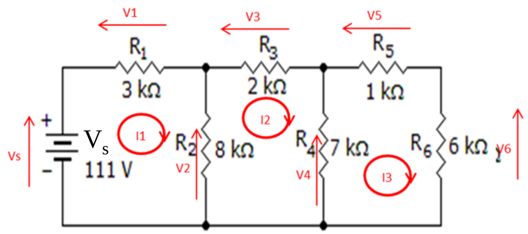
Using Ohms law gives

$111V - R_1 \cdot I_1 - R_2 \cdot (I_1 - I_2) = 0$

$R_2 \cdot (I_1 - I_2) - R_3 \cdot I_2 - R_4 \cdot (I_2 - I_3) = 0$

$R_4 \cdot (I_2 - I_3) - R_5 \cdot I_3 - R_6 \cdot I_3 = 0$

Mesh Analysis



Expand to:

$111V - (R_1+R_2) \cdot I_1 + (R_2) \cdot I_2 \quad 0 \cdot I_3 = 0$

$(R_2) \cdot I_1 - (R_2+R_3+R_4) \cdot I_2 + (R_4) \cdot I_3 = 0$

$0 \cdot I_1 \quad (R_4) \cdot I_2 - (R_4+R_5+R_6) \cdot I_3 = 0$

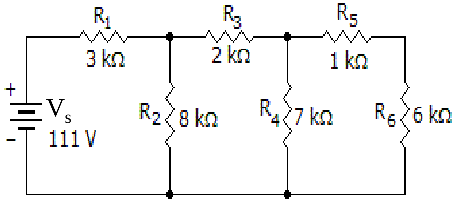
Solving Simultaneous Circuit Equations

- these equations can be expressed as

$$\begin{bmatrix} -(R_1+R_2) & R_2 & 0 \\ R_2 & -(R_2+R_3+R_4) & R_4 \\ 0 & R_4 & -(R_4+R_5+R_6) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -111V \\ 0 \\ 0 \end{bmatrix}$$

- This yields $I_1 = 17.73$ mA, $I_2 = 10.51$ mA and $I_3 = 5.25$ mA
- **But remember**
  - **Net Current through $R_2 = I_1 - I_2 = 7.22$ mA**
  - **Net Current through $R_4 = I_2 - I_3 = 5.25$ mA**
  - **Net Current through $R_6 = I_3 \quad = 5.25$ mA**

Want to know the currents through R_2, R_4 and R_6

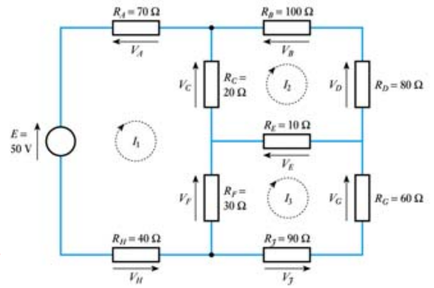


Can be solved using:

1) Ohms law ⇒ I_2=7.22 mA, I_4=5.25 mA, I_6 = 5.25 mA √
2) Nodal analysis ⇒ I_2=7.22 mA, I_4=5.25 mA, I_6 = 5.25 mA √
3) Mesh analysis ⇒ I_2=7.22 mA, I_4=5.25 mA, I_6 = 5.25 mA √

Example 3.8 from book (continued)

– first assign loops currents and label voltages for each current loop



Note, book has assigned voltages for I_1 first, then I_3 and finally I_2.

Advantages of Nodal Analysis

- Solves directly for node voltages.
- Current sources are easy to deal with.
- Voltage sources are either very easy or somewhat difficult.
- Works best for circuits with few nodes.
- Works for any circuit.

Advantages of Mesh Analysis

- Solves directly for some currents.
- Voltage sources are easy.
- Current sources are either very easy or somewhat difficult.
- Works best for circuits with few meshes.

Disadvantages of Mesh Analysis

- Some currents must be computed from loop currents.
- Does not work with non-planar circuits.
- Choosing the right mesh may be difficult.

- FYI: PSpice uses a nodal analysis approach

Choice of Techniques

3.13

- How do we choose the right technique?
  - nodal and mesh analysis will work in a wide range of situations but are not necessarily the simplest methods
  - no simple rules
  - often involves looking at the circuit and seeing which technique seems appropriate

Further Study

Video 3D Further Study

- The Further Study section at the end of Chapter 3 investigates the choice of circuit analysis techniques.
- A circuit is presented which could be analysed in a number of ways.
- Have a look and see which you think is best, then watch the video.

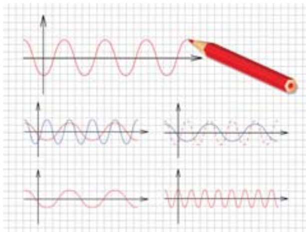
Key Points

- An electric current is a flow of charge
- A voltage source produces an e.m.f. which can cause a current to flow
- Current in a conductor is directly proportional to voltage
- At any instant the sum of the currents into a node is zero
- At any instant the sum of the voltages around a loop is zero
- Any two terminal network of resistors and energy sources can be replaced by a Thévenin or Norton equivalent circuit
- Nodal and mesh analysis provide systematic methods of applying Kirchhoff's laws

Alternating Voltages and Currents

Chapter 6

- Introduction
- Voltage and Current
- Reactance of Inductors and Capacitors
- Phasor Diagrams
- Impedance
- Complex Notation



6.25

Introduction

6.1

- From earlier knowledge (or reading revision chapters: we know that

$$v = V_p \sin(\omega t + \phi)$$

where V_p is the **peak voltage**

ω is the **angular frequency**

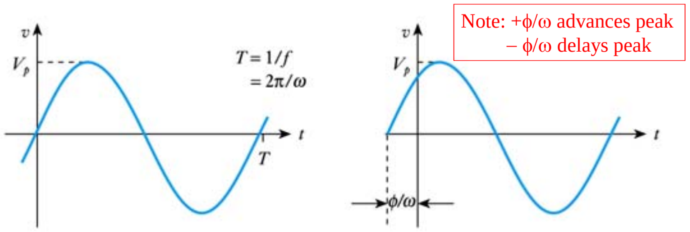
ϕ is the **phase angle**

- Since $\omega = 2\pi f$ it follows that the period T is given by

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

6.26

- If ϕ is in radians, then a time shift t is given by $\frac{\phi}{\omega}$ as shown below



(a) $v = V_p \sin(\omega t)$ (b) $v = V_p \sin(\omega t + \phi)$

6.27

Voltage and Current

Video 6A 6.2

- Consider the voltages across a resistor, an inductor and a capacitor, with a current of

$$i = I_P \sin(\omega t)$$

- **Resistors**
  - from Ohm's law we know

$$v_R = iR$$

  - therefore if $i = I_p \sin(\omega t)$

$$v_R = I_P R \sin(\omega t)$$

6.28

Voltage and Current

- **Inductors** - in an inductor $v_L = L\frac{di}{dt}$
  - therefore if $i = I_p\sin(\omega t)$

$$v_L = L\frac{d(I_P\sin(\omega t))}{dt} = \omega L \cdot I_P\cos(\omega t)$$

Voltage and Current

- **Inductors** - in an inductor $v_L = L\frac{di}{dt}$
  - therefore if $i = I_p\sin(\omega t)$

$$v_L = L\frac{d(I_P\sin(\omega t))}{dt} = \omega L \cdot I_P\cos(\omega t) = \omega L \cdot I_P\sin(\omega t + 90^o)$$

Voltage and Current

- **Inductors** - in an inductor $v_L = L\frac{di}{dt}$
  - therefore if $i = I_p\sin(\omega t)$

$$v_L = L\frac{d(I_P\sin(\omega t))}{dt} = \omega L \cdot I_P\cos(\omega t) = \omega L \cdot I_P\sin(\omega t + 90^o)$$

- **Capacitors** - in a capacitor $v_C = \frac{1}{C}\int i \cdot dt$
  - therefore if $i = I_p\sin(\omega t)$

$$v_C = \frac{1}{C}\int I_P\sin(\omega t) = -\frac{I_p}{\omega C}\cos(\omega t)$$

Voltage and Current

- **Inductors** - in an inductor $v_L = L\frac{di}{dt}$
  - therefore if $i = I_p\sin(\omega t)$

v_L peaks before i

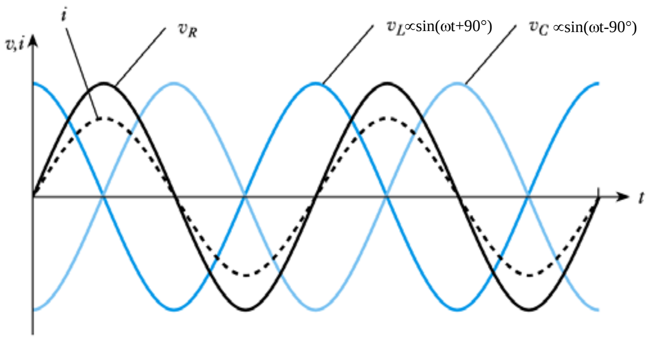
$$v_L = L\frac{d(I_P\sin(\omega t))}{dt} = \omega L \cdot I_P\cos(\omega t) = \omega L \cdot I_P\sin(\omega t + 90^o)$$

- **Capacitors** - in a capacitor $v_C = \frac{1}{C}\int i \cdot dt$
  - therefore if $i = I_p\sin(\omega t)$

v_C peaks after i

$$v_C = \frac{1}{C}\int I_P\sin(\omega t) = -\frac{I_p}{\omega C}\cos(\omega t) = \frac{I_p}{\omega C}\sin(\omega t - 90^o)$$

CIVIL (C⇒I leads V; L ⇐V lead I) or ELI the ICE man (E leads I in L, I leads E in C)



6.33

Reactance of Inductors and Capacitors

6.3

- Let us ignore, for the moment the phase angle and consider the magnitudes of the voltages and currents
- Let us compare the peak voltage and peak current
- **Resistance**

$$\frac{\text{Peak value of voltage}}{\text{Peak value of current}} = \frac{\text{Peak value of } (I_P R\sin(\omega t))}{\text{Peak value of } (I_P\sin(\omega t))} = \frac{I_P R}{I_P} = R$$

6.34

- **Inductance**

$$\frac{\text{Peak value of voltage}}{\text{Peak value of current}} = \frac{\text{Peak value of } (\omega L \cdot I_p\cos(\omega t))}{\text{Peak value of } (I_p\sin(\omega t))} = \frac{\omega L \cdot I_p}{I_p} = \omega L$$

- **Capacitance**

$$\frac{\text{Peak value of voltage}}{\text{Peak value of current}} = \frac{\text{Peak value of } (-\frac{I_p}{\omega C}\cos(\omega t))}{\text{Peak value of } (I_p\sin(\omega t))} = \frac{\frac{I_p}{\omega C}}{I_p} = \frac{1}{\omega C}$$

6.35

- The ratio of voltage to current is a measure of how the component opposes the flow of electricity
- In a resistor this is termed its *resistance*
- In inductors and capacitors it is termed its **reactance**
- Reactance is given the symbol X
- Therefore

$$\text{Reactance of an inductor, } X_L = \omega L$$

$$\text{Reactance of a capacitor, } X_C = \frac{1}{\omega C}$$

6.36

- Since reactance represents the ratio of voltage to current it has units of **ohms**
- The reactance of a component can be used in much the same way as resistance:
  - for an inductor

$$V = I X_L$$

  - for a capacitor

$$V = I X_C$$

6.37

- **Example** – see **Example 6.3** from course text

A sinusoidal voltage of 5 V peak and 100 Hz is applied across an inductor of 25 mH. What will be the peak current?

At this frequency, the reactance of the inductor is given by

$$X_L = \omega L = 2\pi f L = 2 \times \pi \times 100 \times 25 \times 10^{-3} = 15.7\ \Omega$$

Therefore

$$I_L = \frac{V_L}{X_L} = \frac{5}{15.7} = 318\ \text{mA peak}$$

6.38

Phasor Diagrams

6.4

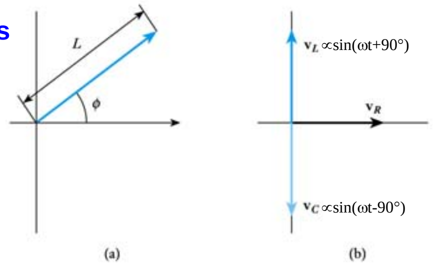
- Sinusoidal signals are characterised by their **magnitude**, their **frequency** and their **phase**
- In many circuits the frequency is fixed (perhaps at the frequency of the AC supply) and we are interested in only magnitude and phase
- In such cases we often use **phasor diagrams** which represent magnitude and phase within a single diagram

6.39

- Examples of **phasor diagrams**

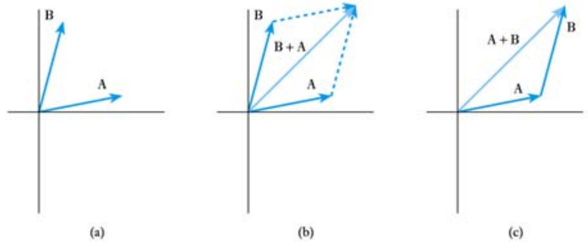
(a) here L represents the magnitude and ϕ the phase of a sinusoidal signal

(b) shows the voltages across a resistor, an inductor and a capacitor for the same sinusoidal current

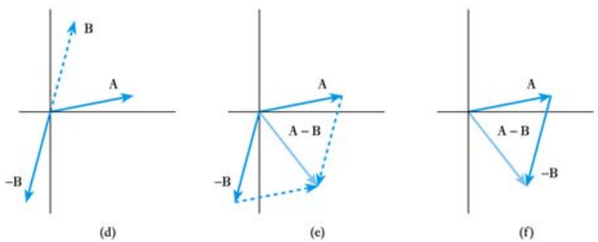


6.40

- Phasor diagrams can be used to represent the *addition* of signals. This gives both the magnitude and phase of the resultant signal



- Phasor diagrams can also be used to show the *subtraction* of signals

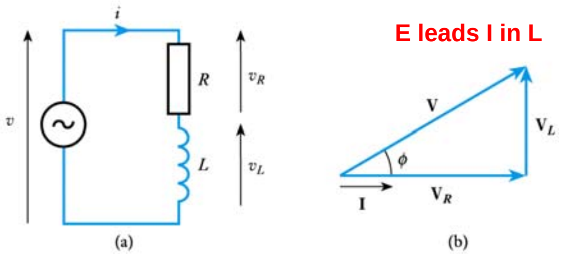


Phasor analysis of RL and RC <u>series</u> circuits

Current the same in both ∴ what is the voltage?

- **Phasor analysis of an *RL* circuit**

E leads I in L



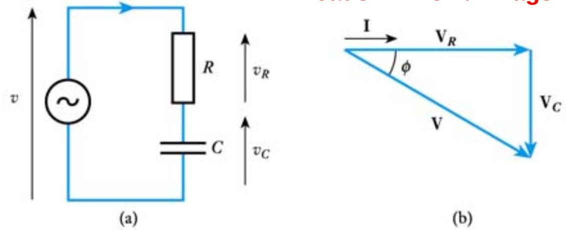
- See **Example 6.5** in the text for a numerical example

Phasor analysis of RL and RC <u>series</u> circuits

Current the same in both ∴ what is the voltage?

- **Phasor analysis of an *RC* circuit**

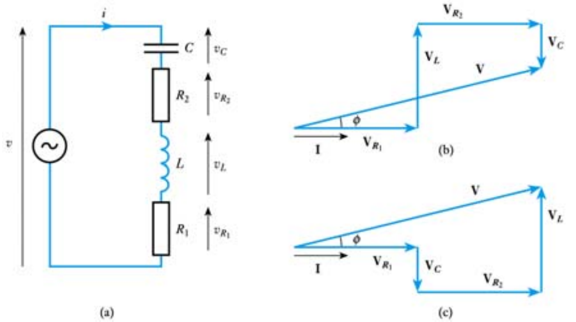
I leads E in C ⇒ E lags I in C



- See **Example 6.6** in the text for a numerical example

Phasor analysis of RLC <u>series</u> circuits
Current the same in both ∴ what is the voltage?

- **Phasor analysis of an *RLC* circuit**

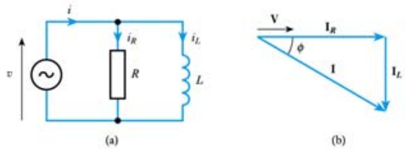


Phasor analysis of RL and RC <u>parallel</u> circuits
Voltage the same over both ∴ what is the current?

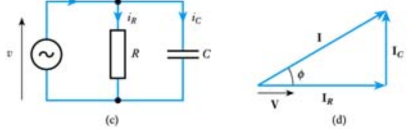
- **Phasor analysis of parallel circuits**

  in such circuits the voltage across each of the components is the same and it is the *currents* that are of interest

E leads I in L ⇒ I lags E in L



I leads E in C

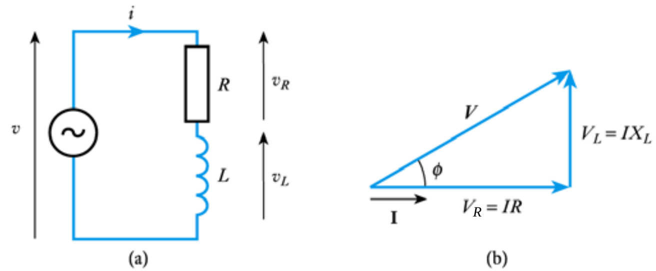


Impedance

6.5

- In circuits containing only resistive elements the current is related to the applied voltage by the *resistance* of the arrangement
- In circuits containing *reactive*, as well as *resistive* elements, the current is related to the applied voltage by the **impedance**, **Z** of the arrangement
  - this reflects not only the magnitude of the current but also its phase
  - impedance can be used in reactive circuits in a similar manner to the way resistance is used in resistive circuits

- Consider the following circuit and its phasor diagram



- From the phasor diagram it is clear that that the magnitude of the voltage across the arrangement V is

$$\begin{aligned} V &= \sqrt{V_R^2 + V_L^2} \\ &= \sqrt{(IR)^2 + (IX_L)^2} \\ &= I\sqrt{R^2 + X_L^2} \\ &= IZ \end{aligned}$$

where $Z = \sqrt{R^2 + X_L^2}$

- Z is the magnitude of the impedance, so $Z = |\mathbf{Z}|$

- From the phasor diagram the phase angle of the impedance is given by

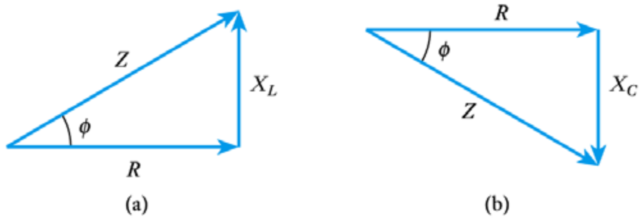
$$\phi = \tan^{-1}\frac{V_L}{V_R} = \tan^{-1}\frac{IX_L}{IR} = \tan^{-1}\frac{X_L}{R}$$

- This circuit contains an inductor but a similar analysis can be done for circuits containing capacitors
- In general

$$Z = \sqrt{R^2 + X^2} \quad \text{and} \quad \phi = \tan^{-1}\frac{X}{R}$$

Sign from diagram

- **A graphical representation of impedance**

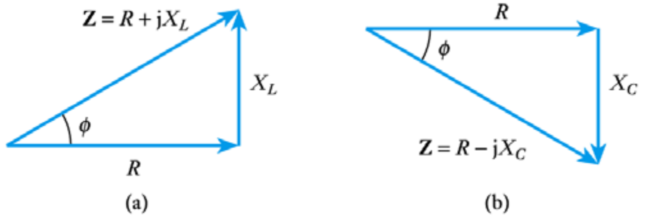


Complex Notation

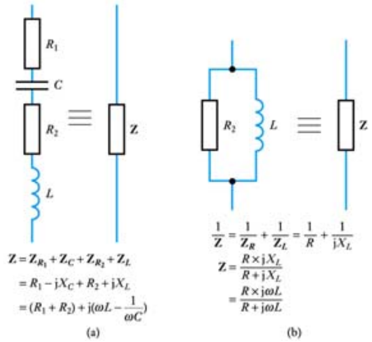
6.6

- Phasor diagrams are similar to **Argand Diagrams** used in complex mathematics
- We can also represent impedance using complex notation where
- Resistors: $\mathbf{Z_R} = R$
- Inductors: $\mathbf{Z_L} = jX_L = j\omega L$
- Capacitors: $\mathbf{Z_C} = -jX_C = -j\frac{1}{\omega C} = \frac{1}{j\omega C}$

- **Graphical representation of complex impedance**



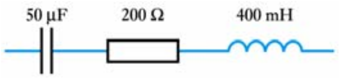
- **Series and parallel combinations of impedances**
  - impedances combine in the same way as resistors



- **Manipulating complex impedances**
  - complex impedances can be *added*, *subtracted*, *multiplied* and *divided* in the same way as other complex quantities
  - they can also be expressed in a range of forms such as the **rectangular**, **polar** and **exponential** forms
  - if you are unfamiliar with the manipulation of complex quantities (or would like a little revision on this topic) see **Appendix D** of the course text which gives a tutorial on this subject

- **Example** – see **Example 6.7** in the course text

Determine the complex impedance of this circuit at a frequency of 50 Hz



At 50Hz, the angular frequency $\omega = 2\pi f = 2 \times \pi \times 50 = 314$ rad/s

Therefore

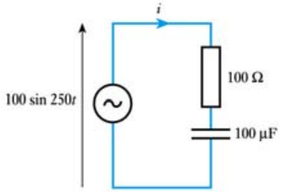
$$\mathbf{Z} = \mathbf{Z_C} + \mathbf{Z_R} + \mathbf{Z_L} = R + \mathrm{j}(X_L - X_C) = R + \mathrm{j}(\omega L - \frac{1}{\omega C})$$

$$= 200 + \mathrm{j}(314 \times 400 \times 10^{-3} - \frac{1}{314 \times 50 \times 10^{-6}})$$

$$= 200 + \mathrm{j}62 \text{ ohms}$$

- **Using complex impedance**
- **Example** – see **Section 6.6.4** in course text

Determine the current in this circuit



Since v = 100 sin 250t , then ω = 250

Therefore

$$\begin{aligned}\mathbf{Z} &= R - jX_C \\ &= R - j\frac{1}{\omega C} \\ &= 100 - j\frac{1}{250 \times 10^{-4}} \\ &= 100 - j40\end{aligned}$$

- **Example** (continued)

The current is given by v/**Z** and this is easier to compute in polar form

$$\mathbf{Z} = 100 - j40$$

$$|\mathbf{Z}| = \sqrt{100^2 + 40^2} = 107.7$$

$$\angle \mathbf{Z} = \tan^{-1}\frac{-40}{100} = -21.8^\circ$$

$$\mathbf{Z} = 107.7\angle -21.8^\circ$$

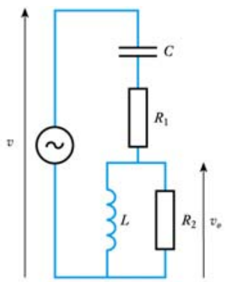
Therefore

$$i = \frac{v}{\mathbf{Z}} = \frac{100\angle 0}{107.7\angle -21.8} = 0.93\angle 21.8^\circ$$

- **A further example**

A more complex task is to find the output voltage of this circuit

The analysis of this circuit, and a numerical example based on it, are given in **Section 6.6.4** and **Example 6.8** of the course text



(a)

Further Study

Video 6B

Further Study

- The Further Study section at the end of Chapter 6 looks at the characteristics of a simple reactive circuit.
- See if you can model the behaviour of the circuit at a single frequency and then watch the video to check your analysis.

Key Points

- A sinusoidal voltage waveform can be described by the equation $v = V_p \sin(\omega t + \phi)$
- The voltage across a resistor is *in phase with* the current, the voltage across an inductor *leads* the current by 90°, and the voltage across a capacitor *lags* the current by 90°
- The reactance of an inductor $X_L = \omega L$
- The reactance of a capacitor $X_C = 1/\omega C$
- The relationship between current and voltage in circuits containing reactance can be described by its impedance
- The use of impedance is simplified by the use of complex notation