This time

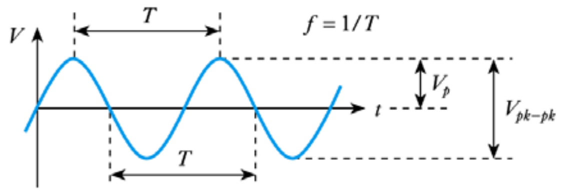
- Review Power in AC circuits (Chapter 7)
- Frequency Characteristics of AC Circuits (Chapter 8)

Definitions: Sine Waves

Video 2A

2.2

- The instantaneous voltage and current v, and i.
  - For example: $v(t) = V_p \sin(\omega \cdot t + \phi)$
- Where V_p and I_p are the Peak Voltage and Current
- And $V_{pk\text{-}pk} = 2 \cdot V_p$



- **r.m.s. value of a sine wave**
  - the instantaneous power (p) in a resistor is given by

$$p = \frac{v^2}{R}$$

  - therefore the average power is given by

$$P_{av} = \frac{[\text{average (or mean) of } v^2]}{R} = \frac{\overline{v^2}}{R}$$

  - where $\overline{v^2}$ is the **mean-square voltage**

- While the mean-square voltage is useful, more often we use the square root of this quantity, namely the root-mean-square voltage V_{rms}
  - where $V_{rms} = \sqrt{\overline{v^2}}$
  - we can also define $I_{rms} = \sqrt{\overline{i^2}}$
  - it is relatively easy to show that (see text for analysis)

$$V_{rms} = \frac{1}{\sqrt{2}} \times V_p = 0.707 \times V_p \qquad I_{rms} = \frac{1}{\sqrt{2}} \times I_p = 0.707 \times I_p$$

Text writes V_{rms} and I_{rms} as V and I

- r.m.s. values are useful because their relationship to average power is similar to the corresponding DC values

$$P_{av} = V_{rms} \cdot I_{rms} = VI = \frac{V_p}{\sqrt{2}} \cdot \frac{I_p}{\sqrt{2}}$$

$$P_{av} = \frac{V_{rms}^2}{R} = \frac{V^2}{R} = \frac{V_p^2}{2 \cdot R}$$

$$P_{av} = I_{rms}^2 R$$

Power in Resistive Components

- Suppose a voltage $v = V_p \sin \omega t$ is applied across a resistance R. The resultant current i will be

$$i = \frac{v}{R} = \frac{V_P \sin \omega t}{R} = I_P \sin \omega t$$

- The result power p will be:

$$p = vi = V_P I_P (\sin^2 \omega t) = V_P I_P \left(\frac{1 - \cos 2\omega t}{2}\right)$$

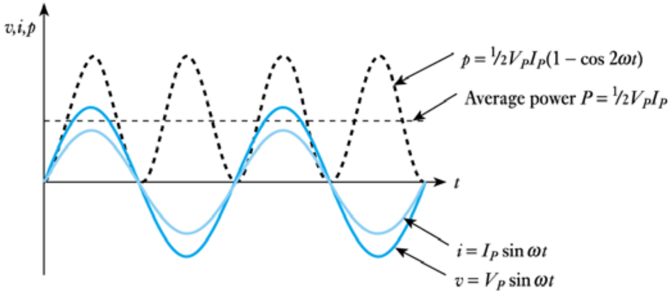
$$\text{and Average Power } P = \frac{1}{2} V_P I_P$$

$$= \frac{V_P}{\sqrt{2}} \times \frac{I_P}{\sqrt{2}}$$

$$= VI$$

Positive power ⇒ power delivered to the circuit

- **Relationship between *v*, *i* and *p* in a resistor**



Power in Capacitors

- From our discussion of capacitors we know that the current leads the voltage by 90°. Therefore, if a voltage $v = V_p \sin \omega t$ is applied across a capacitance C, the current will be given by $i = I_p \cos \omega t$
- Then

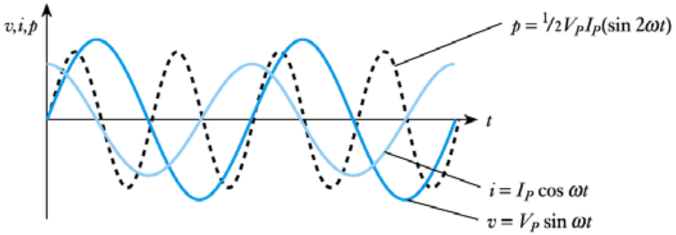
$$p = vi$$
$$= V_P \sin \omega t \times I_P \cos \omega t$$
$$= V_P I_P (\sin \omega t \times \cos \omega t)$$
$$= V_P I_P \left(\frac{\sin 2\omega t}{2}\right)$$

- The average power is zero

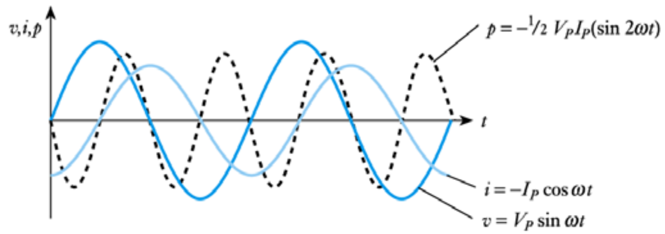
Positive power $\Rightarrow$ power delivered to the circuit

Negative power $\Rightarrow$ power the circuit delivers back

- **Relationship between *v*, *i* and *p* in a capacitor**



- **Relationship between *v*, *i* and *p* in an inductor**



Trig relation :

$$\{\cos x - \cos(y)\} = -2 \cdot \sin\left(\frac{x+y}{2}\right) \cdot \sin\left(\frac{x-y}{2}\right)$$

Circuit with Resistance and Reactance

7.5

- When a sinusoidal voltage $v = V_p \sin \omega t$ is applied across a circuit with resistance *and* reactance, the current will be of the general form $i = I_p \sin(\omega t - \phi)$
- Therefore, the instantaneous power, *p* is given by

$$p = vi$$

$$= V_P \sin \omega t \times I_P \sin(\omega t - \phi)$$

$$= \frac{1}{2} V_P I_P \{\cos \phi - \cos(2\omega t - \phi)\}$$

$$p = \frac{1}{2} V_P I_P \cos \phi - \frac{1}{2} V_P I_P \cos(2\omega t - \phi)$$

$$p = \frac{1}{2} V_P I_P \cos \phi - \frac{1}{2} V_P I_P \cos(2\omega t - \phi)$$

- The expression for *p* has two components
- The second part oscillates at 2ω and has an average value of zero over a complete cycle
  - this is the power that is stored in the reactive elements and then returned to the circuit within each cycle
- The first part represents the power dissipated in resistive components. **Average power dissipation** is

$$P = \frac{1}{2} V_P I_P (\cos \phi) = \frac{V_P}{\sqrt{2}} \times \frac{I_P}{\sqrt{2}} \times (\cos \phi) = VI \cos \phi$$

- The average power dissipation given by

$$P = \frac{1}{2} V_P I_P (\cos\phi) = VI\cos\phi$$

is termed the **active power** in the circuit and is measured in watts (W)

- The product of the r.m.s. voltage and current *VI* is termed the **apparent power**, ***S***. To avoid confusion this is given the units of volt amperes (VA)

- From the above discussion it is clear that

$$P = VI\cos\phi$$
$$= S\cos\phi$$

- In other words, the active power is the apparent power times the cosine of the phase angle.
- This cosine is referred to as the **power factor**

$$\frac{\text{Active power (in watts)}}{\text{Apparent power (in volt amperes)}} = \text{Power factor}$$

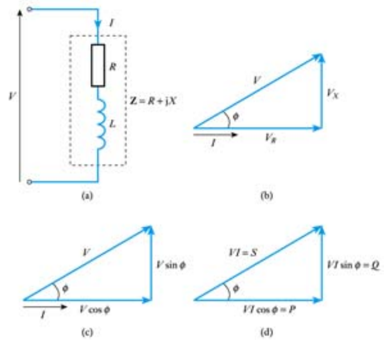
$$\text{Power factor} = \frac{P}{S} = \cos\phi$$

Active and Reactive Power

7.6

- When a circuit has resistive and reactive parts, the resultant power has 2 parts:
  - The first is *dissipated* in the resistive element. This is the **active power, *P***
  - The second is *stored* and *returned* by the reactive element. This is the **reactive power**, ***Q*** , which has units of **volt amperes reactive** or **var**
- While reactive power is not dissipated it does have an effect on the system
  - for example, it increases the current that must be supplied and increases losses with cables

- Consider an RL circuit
  - the relationship between the various forms of power can be illustrated using a power triangle



Applied voltage V
If V_R=Vcos(φ) then
V_L=Vsin(φ) since there
is a 90° phase shift
between V_R and V_L

- Therefore

Active Power	$P = VI \cos \phi$	watts
Reactive Power	$Q = VI \sin \phi$	var
Apparent Power	$S = VI$	VA
	$S^2 = P^2 + Q^2$	

Power Factor Correction

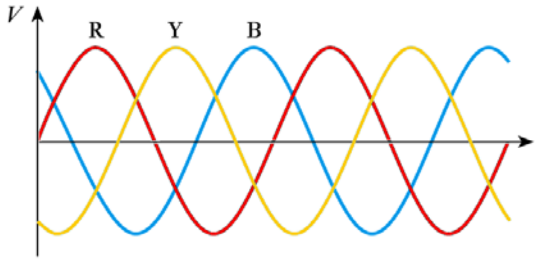
- **Power factor** (Active/Apparent power) is particularly important in high-power applications
- Inductive loads have a *lagging* power factor
- Capacitive loads have a *leading* power factor
- Many high-power devices are inductive
  - a typical AC motor has a power factor of 0.9 lagging
  - the total load on the national grid is 0.8-0.9 lagging
  - this leads to major inefficiencies
  - power companies therefore penalize industrial users who introduce a poor power factor

- The problem of poor power factor is tackled by adding additional components to bring the power factor back closer to unity
  - a capacitor of an appropriate size in parallel with a lagging load can 'cancel out' the inductive element
  - this is **power factor correction**
  - a capacitor can also be used in series but this is less common (since this alters the load voltage)
  - for examples of power factor correction see **Examples 7.2 and 7.3** in the course text

Three-Phase Systems

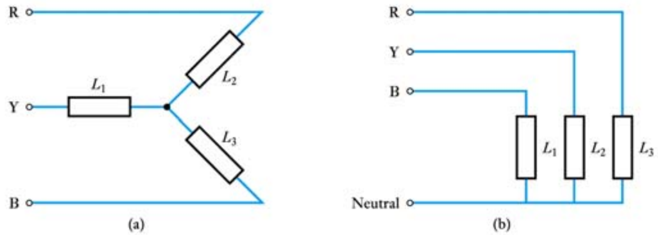
- So far, our discussion of AC systems has been restricted to **single-phase** arrangement
  - as in conventional domestic supplies
- In high-power industrial applications we often use **three-phase** arrangements
  - these have three supplies, differing in phase by 120°
  - phases are labeled **red**, **yellow** and **blue** (**R**, **Y** & **B**)

- Relationship between the phases in a three-phase arrangement



7.21

- Three-phase arrangements may use either 3 or 4 conductors



7.22

Power Measurement

7.9

- When using AC, *power* is determined not only by the r.m.s. values of the **voltage** and **current**, but also by the **phase angle** (which determines the **power factor**)
  - consequently, you cannot determine the power from independent measurements of current and voltage
- In **single-phase systems** power is normally measured using an **electrodynamic wattmeter**
  - measures power directly using a single meter which effectively multiplies instantaneous current and voltage

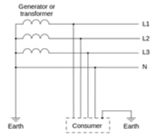
7.23

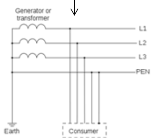
- In **three-phase systems** we need to sum the power taken from the various phases
  - in three-wire arrangements we can deduce the total power from measurements using 2 wattmeters
  - in a four-wire system it may be necessary to use 3 wattmeters
  - in balanced systems (systems that take equal power from each phase) a single wattmeter can be used, its reading being multiplied by 3 to get the total power

7.24

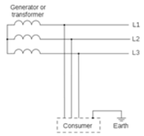
Who cares?

- In Europe, the standard is TT or TN-C earthing





- In Norway (and perhaps Albania) it is IT earthing



Buy equipment with TT or TN-C

You will need an additional transformer to run it in Norway

Further Study

Video 7B Further Study

- The Further Study section at the end of Chapter 7 is concerned with power factor correction for a high-power motor.
- Your task is to calculate the size of capacitor needed to achieve a specific power factor.
- Do the sums and then watch the video.

Key Points

- In resistive circuits the average power is equal to *VI*, where *V* and *I* are r.m.s. values
- In a capacitor the current *leads* the voltage by 90° and the average power is zero
- In an inductor the current *lags* the voltage by 90° and the average power is zero
- In circuits with both resistive and reactive elements, the average power is *VI* cos ϕ
- The term cos ϕ is called the power factor
- Power factor correction is important in high-power systems
- High-power systems often use three-phase arrangements

Frequency Characteristics of AC Circuits

Chapter 8

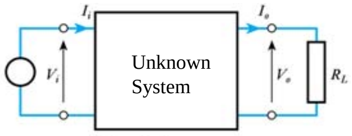
- Introduction
- Two-port Networks
- The Decibel (dB)
- Frequency Response
- A High-Pass *RC* Network
- A Low-Pass *RC* Network
- A Low-Pass *RL* Network
- A High-Pass *RL* Network
- A Comparison of *RC* and *RL* Networks
- Bode Diagrams
- Combining the Effects of Several Stages
- *RLC* Circuits and Resonance
- Filters
- Stray Capacitance and Inductance

Introduction

8.1

- Having now looked at the AC behaviour of simple components, we can consider their effects on the frequency characteristics of simple circuits
- While the properties of a pure *resistance* are not affected by the frequency of the signal concerned, this is not true of *reactive* components
- We will start with a few basic concepts and then look at the characteristics of simple combinations of resistors, capacitors and inductors

- We then define voltages and currents at the input and output



(b) A typical arrangement

- Then

$$\text{voltage gain } (A_v) = \frac{V_0}{V_i}$$

$$\text{current gain } (A_i) = \frac{I_0}{I_i}$$

$$\text{power gain } (A_p) = \frac{P_0}{P_i}$$

The Decibel (dB)

8.3

- The power gain of modern electronic amplifiers is often very high, gains of 10^6 or 10^7 being common
- With such large numbers it is often convenient to use a logarithmic expression for gain
- This is often done using **decibels**
- The decibel is a dimensionless figure for power gain

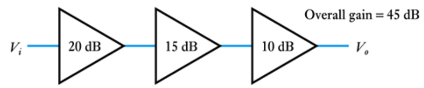
$$\text{Power gain (dB)} = 10 \log_{10} \frac{P_2}{P_1}$$

- Sample gains expressed in dBs

Power gain	Decibels (dBs)
100	20
10	10
1	0

Power gain	Decibels (dBs)
0.5	-3
0.1	-10
0.01	-20

- Using dBs simplifies calculation in cascaded circuits



- Power gain is related to voltage gain

$$\text{Power gain (dB)} = 10\log_{10}\frac{P_2}{P_1} = 10\log_{10}\frac{V_2^2 / R_2}{V_1^2 / R_1}$$

- If $R_1 = R_2$

$$\text{Power gain (dB)} = 10\log_{10}\frac{V_2^2}{V_1^2} = 20\log_{10}\frac{V_2}{V_1}$$

$$\text{Power gain (dB)} = 20\log_{10}(\text{Voltage gain})$$

- This expression is almost always used even when $R_1 \neq R_2$
  – see **Example 8.4** in the course text

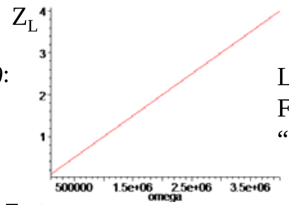
Frequency response

8.4

- Since the characteristics of reactive components change with frequency, the behaviour of circuits using these components will also change
- The way in which the gain of a circuit changes with frequency is termed its **frequency response**
- These variations take the form of variations in the *magnitude* of the gain and in the *phase response*

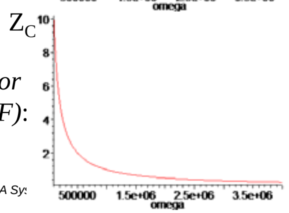
Impedance vs. Frequency

Magnitude of Inductor impedance ($L=1x10^{-6}H$):



Low Frequencies "Shorted Out"

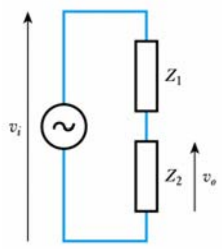
Magnitude of Capacitor impedance ($C=1x10^{-6}F$):



High Frequencies "Shorted Out"

Resistor Impedance?

- We will start by considering very simple circuits
- Consider the potential divider shown here



  – from our earlier consideration of the circuit

$$v_o = v_i \times \frac{\mathbf{Z_2}}{\mathbf{Z_1}+\mathbf{Z_2}}$$

  – rearranging, the gain of the circuit is

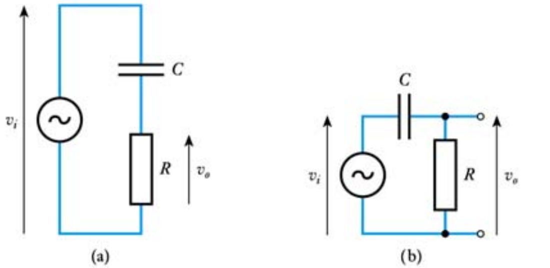
$$\frac{v_o}{v_i} = \frac{\mathbf{Z_2}}{\mathbf{Z_1}+\mathbf{Z_2}}$$

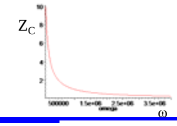
  – this is also called the **transfer function** of the circuit

A High-Pass *RC* Network

8.5

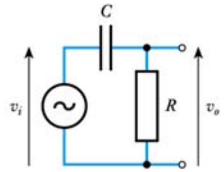
- Consider the following circuit
  - which is shown re-drawn in a more usual form





- Clearly the transfer function is

$$\frac{v_o}{v_i} = \frac{\mathbf{Z_R}}{\mathbf{Z_R} + \mathbf{Z_C}} = \frac{R}{R - j\frac{1}{\omega C}} = \frac{1}{1 - j\frac{1}{\omega CR}}$$



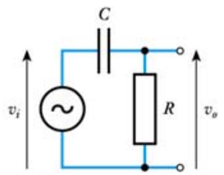
- At high frequencies
  - ω is large, voltage gain ≈ 1

    (capacitor looks like a wire⇒nearly all the voltage drops across resistor)
- At low frequencies
  - ω is small, voltage gain $\rightarrow 0$

    (capacitor looks like a huge resistor⇒hardly any voltage drop across resistor)

- Since the denominator has real and imaginary parts, the *magnitude* of the voltage gain is

$$|\text{Voltage gain}| = \frac{1}{\sqrt{1^2 + \left(\frac{1}{\omega CR}\right)^2}}$$



- When $1/(\omega CR) = 1$

$$|\text{Voltage gain}| = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}} = 0.707$$

- This is a halving of power, or a fall in gain of 3 dB

- The half power point is the **cut-off frequency** of the circuit
  - the angular frequency ω_C at which this occurs is given by

$$\frac{1}{\omega_C CR} = 1$$

$$\omega_C = \frac{1}{CR} = \frac{1}{\mathrm{T}} \text{ rad/s}$$

  - where T is the time constant of the *CR* network. Also

$$f_C = \frac{\omega_C}{2\pi} = \frac{1}{2\pi CR} \text{ Hz}$$

- Substituting $\omega = 2\pi f$ and $CR = 1/\,2\pi f_C$ in the earlier equation gives

$$\frac{v_o}{v_i} = \frac{1}{1 - j\dfrac{1}{\omega CR}} = \frac{1}{1 - j\dfrac{1}{(2\pi f)\left(\dfrac{1}{2\pi f_C}\right)}} = \frac{1}{1 - j\dfrac{f_C}{f}}$$

- This is the general form of the gain of the circuit
- It is clear that both the *magnitude* of the gain and the *phase angle* vary with frequency

7.41

- Consider the behaviour of the circuit at different frequencies:
- **When $f \gg f_c$**
  - $f_c/f \ll 1$, the voltage gain ≈ 1
- **When $f = f_c$**

$$\frac{v_o}{v_i} = \frac{1}{1 - j\dfrac{f_C}{f}} = \frac{1}{1-j} = \frac{1 \times (1+j)}{(1-j)\times(1+j)} = \frac{(1+j)}{2} = 0.5 + 0.5j$$

Output leads input by 45°

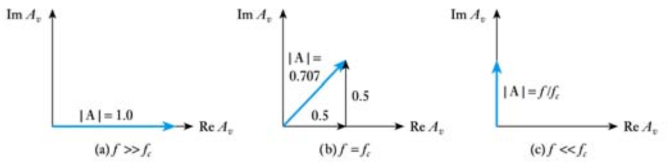
- **When $f \ll f_c$**

$$\frac{v_o}{v_i} = \frac{1}{1 - j\dfrac{f_C}{f}} \approx \frac{1}{-j\dfrac{f_C}{f}} = j\frac{f}{f_C}$$

Output leads input by 90°

7.42

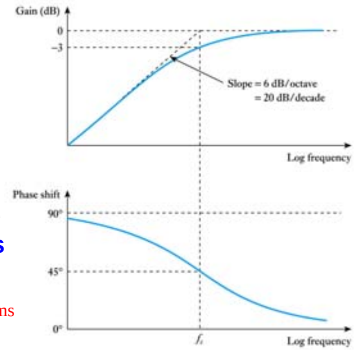
- The behaviour in these three regions can be illustrated using phasor diagrams



- At *low* frequencies the gain is linearly related to frequency. ½ the frequency ⇒ ½ the voltage gain
  Voltage gain falls at -6dB/octave (-20dB/decade)

7.43

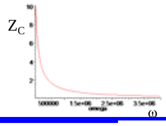
- Frequency response of the high-pass network
  - the gain response has two *asymptotes* that meet at the cut-off frequency
  - figures of this form are called **Bode diagrams**



Phase shifts important in control systems

7.44

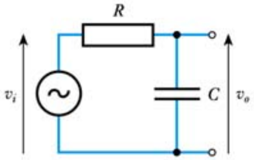
A Low-Pass *RC* Network



8.6

- Transposing the *C* and *R* gives

$$\frac{v_o}{v_i}=\frac{\mathbf{Z_C}}{\mathbf{Z_R}+\mathbf{Z_C}}=\frac{-\mathrm{j}\dfrac{1}{\omega C}}{R-\mathrm{j}\dfrac{1}{\omega C}}=\frac{1}{1+\mathrm{j}\omega CR}$$



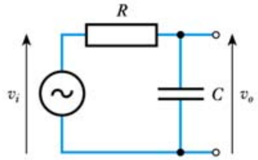
- At high frequencies
  - ω is large, voltage gain $\rightarrow 0$

    (capacitor looks like a wire⇒nearly all the voltage drops across resistor)
- At low frequencies
  - ω is small, voltage gain ≈ 1

    (capacitor looks like a huge resistor⇒hardly any voltage drop across resistor)

- A similar analysis to before gives

$$|\text{Voltage gain}|=\frac{1}{\sqrt{1+(\omega CR)^2}}$$



- Therefore when, when $\omega CR = 1$

$$|\text{Voltage gain}|=\frac{1}{\sqrt{1+1}}=\frac{1}{\sqrt{2}}=0.707$$

- Which is the cut-off frequency

- Therefore
  - the angular frequency ω_C at which this occurs is given by

$$\omega_C CR = 1$$

$$\omega_C=\frac{1}{CR}=\frac{1}{\mathrm{T}}\ \text{rad/s}$$

  - where T is the time constant of the *CR* network, and as before

$$f_C=\frac{\omega_C}{2\pi}=\frac{1}{2\pi CR}\ \text{Hz}$$

- Substituting $\omega = 2\pi f$ and $CR = 1/\,2\pi f_C$ in the earlier equation gives

$$\frac{v_o}{v_i}=\frac{1}{1+\mathrm{j}\omega CR}=\frac{1}{1+\mathrm{j}\dfrac{\omega}{\omega_C}}=\frac{1}{1+\mathrm{j}\dfrac{f}{f_C}}$$

- This is similar, but not the same, as the transfer function for the high-pass network

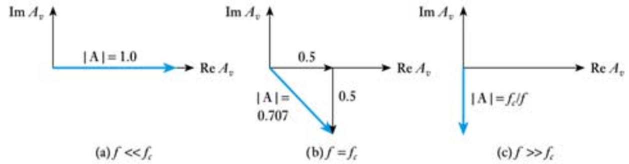
- Consider the behaviour of this circuit at different frequencies:
- **When $f \ll f_c$**
  - $f/f_c \ll 1$, the voltage gain ≈ 1
- **When $f = f_c$**

$$\frac{v_o}{v_i} = \frac{1}{1 + j\frac{f}{f_c}} = \frac{(1-j)(1+j)}{(1+j)} = \frac{(1-j)}{2} = 0.5 - 0.5j$$

- **When $f \gg f_c$**

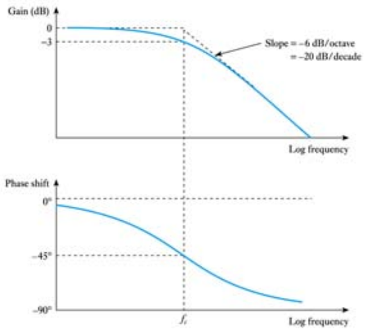
$$\frac{v_o}{v_i} = \frac{1}{1 + j\frac{f}{f_c}} \approx \frac{1}{j\frac{f}{f_c}} = -j\frac{f_c}{f}$$

- The behaviour in these three regions can again be illustrated using phasor diagrams

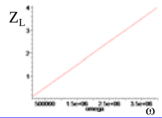


- At *high* frequencies the gain is linearly related to frequency. It falls at 6dB/octave (20dB/decade)

- Frequency response of the low-pass network
  - the gain response has two *asymptotes* that meet at the cut-off frequency
  - you might like to compare this with the Bode Diagram for a high-pass network

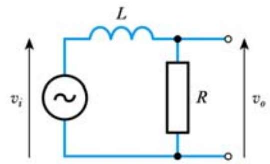


A Low-Pass *RL* Network



8.7

- Low-pass networks can also be produced using *RL* circuits
  - these behave similarly to the corresponding *CR* circuit
  - the voltage gain is



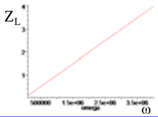
$$\frac{v_o}{v_i} = \frac{\mathbf{Z_R}}{\mathbf{Z_R} + \mathbf{Z_L}} = \frac{R}{R + j\omega L} = \frac{1}{1 + j\omega\frac{L}{R}}$$

At Low frequencies
L looks like a wire
At High frequencies
L looks like a big resistor

  - the cut-off frequency is

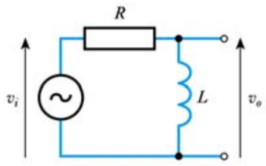
$$\omega_c = \frac{R}{L} = \frac{1}{T}\,\text{rad/s} \qquad f_c = \frac{\omega_c}{2\pi} = \frac{R}{2\pi L}\,\text{Hz}$$

A High-Pass *RL* Network



8.8

- High-pass networks can also be produced using *RL* circuits
  - these behave similarly to the corresponding *CR* circuit
  - the voltage gain is



$$\frac{v_o}{v_i} = \frac{\mathbf{Z_L}}{\mathbf{Z_R} + \mathbf{Z_L}} = \frac{\mathrm{j}\omega L}{R + \mathrm{j}\omega L} = \frac{1}{1 + \frac{R}{\mathrm{j}\omega L}} = \frac{1}{1 - \mathrm{j}\frac{R}{\omega L}}$$

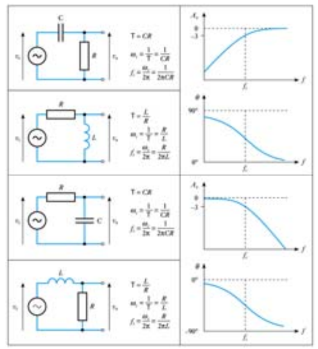
  - the cut-off frequency is

$$\omega_c = \frac{R}{L} = \frac{1}{\mathrm{T}} \text{rad/s} \qquad f_c = \frac{\omega_c}{2\pi} = \frac{R}{2\pi L} \text{Hz}$$

A Comparison of *RC* and *RL* Networks

8.9

- Circuits using RC and RL techniques have similar characteristics
  - see **Figure 8.12** in the course text

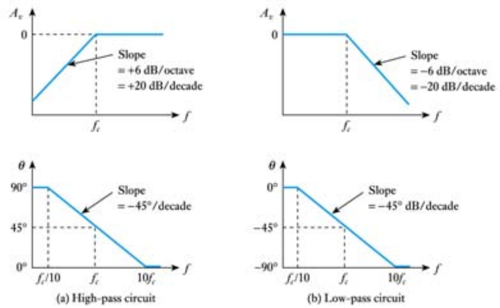


Bode Diagrams

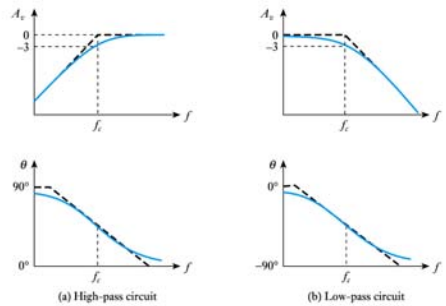
Video 8A

8.10

- Straight-line approximations



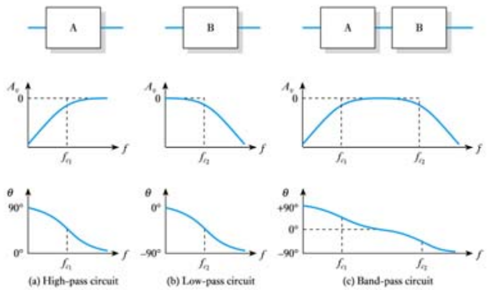
- Creating more detailed Bode diagrams



Combining the Effects of Several Stages

8.11

- The effects of several stages ‘add’ in bode diagrams



- Multiple high- and low-pass elements may also be combined
  - this is illustrated in **Figure 8.16** in the course text