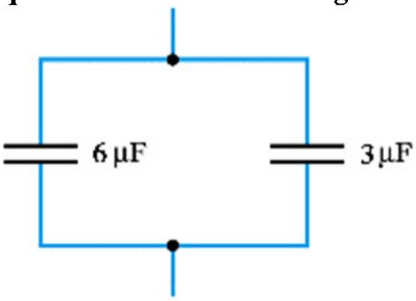
Problem set 2 TFY4185 Måleteknikk Issued 14 September 2015

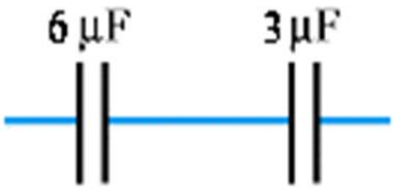
1) **What is the effective capacitance of the following arrangement?**



a) 0.5 Mf b) 2 μF c) 4.5 μF <u>**d) 9 μF**</u>

Each capacitor has the same voltage across it, V, and the charge stored on each capacitor, I, is $Q_i = V\cdot C_i$. A single equivalent capacitor must store the same total amount of charge, $Q=Q_1+Q_2$, with the same voltage, V, across it. Thus, $Q=V\cdot C = V\cdot C_1. + V\cdot C_2$, and the voltage drops out giving $C = 6x10^{-6}\ F + 3x10^{-6}F = 9x10^{-6}\ F = 9\mu F$.

2) **What is the effective capacitance of the following arrangement?**

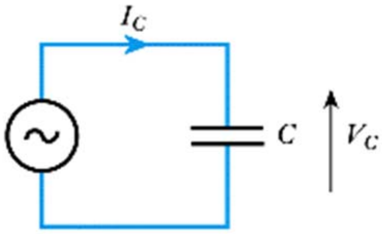


a) 0.5 μF <u>**b) 2 μF**</u> c) 4.5 μF d) 9 μF

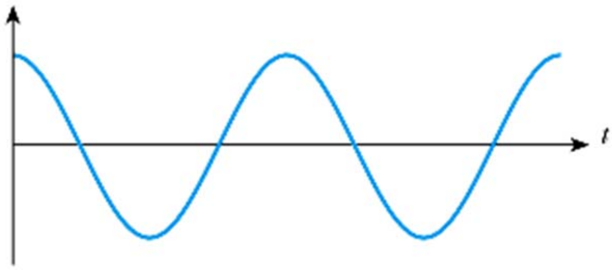
Here the charge on the lower plate of the 6μF capacitor must be the same as that on the upper plate of the 3μF capacitor. That is, they must all have the same charge, Q. Now the total voltage drop across the pair of capacitors, $V = V_1+V_2$, the sum of the two individual voltage drops. Given that the voltage across a capacitor is V=Q/C, An equivalent capacitor of capacitance C that delivers the same charge Q over this total voltage V would be $Q/C = Q/C_1 + Q/C_2$. Now the Q's drop out and we have:

$C = (1/6\mu F + 1/3\mu F)^{-1} = 2\ \mu F.$

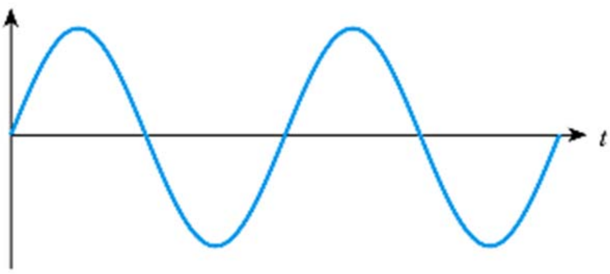
3) The circuit in (a) below shows an arrangement that applies a sinusoidal voltage across a capacitor. Given the relationship between the voltage and the current in a capacitor, which of the following statements is correct?



(a)



(b)



(c)

a) (b) represents the voltage V_C and (c) represents the current I_C.

b) (b) represents the current I_C and (c) represents the voltage V_C.

One can remember either the ELI the ICE man memnonic, or CIVIL. Bacially, E leads I in and inductor (L), and I leads E in a capacitor (C). Here curve (b) has already reached its peak and is starting down as curve (c) is just starting up. So, since I leads E in a capacitor, statement b) is correct.

4) The current in a 25 mH inductor changes at a constant rate of 7 A/s. What voltage is induced across this coil?

a) 3.57 mV   **b) 175 mV**   c) 350 mV   d) 1.75 V

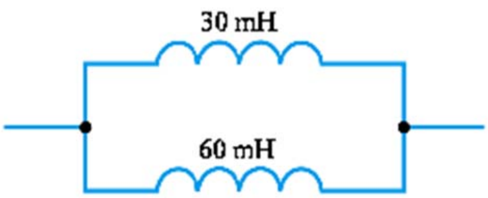
We know that the voltage in an inductor, $V = L\cdot(dI/dt)$, and we are given $L=25\text{x}10^{-3}$ H and $dI/dt = 7$ A/s. Thus $V = 7\cdot 25\text{x}10^{-3}$ V = 175 mV

5) Calculate the inductance of this arrangement.

a) 2mH   b) 20 mH   **c) 90 mH**   d) 120 mH

In this case, the current through the two inductors is the same, and so is the time rate of change of the current (dI/dt). The total voltage drop across the 2 inductors is the sum of the voltage drops across the individual inductors, so $V = V_1+V_2$, where the voltage drop across each inductor is $V_i=L_i\cdot(dI/dt)$. Thus, an equivalent inductance, L, that has the same voltage drop, V would be $L\cdot(dI/dt) = L_1\cdot(dI/dt) + L_2\cdot(dI/dt)$. The current terms cancel out and we have that L = 20mH + 60 mH = 90 mH.

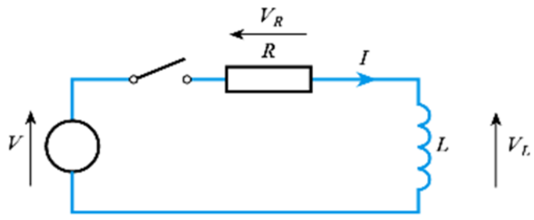
6) Calculate the inductance of this arrangement.



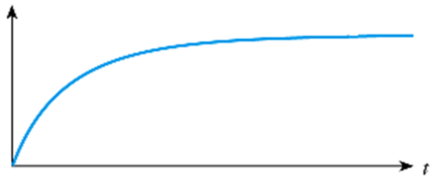
a) 2mH   **b) 20 mH**   c) 90 mH   d) 120 mH

Now it is the voltage, V, that is constant over the two inductors, and the total current is equal to the sum of the current through each inductor, $I = I_1+I_2$.
Thus, $dI/dt = dI_1/dt + dI_2/dt$, and since $V/L= (dI/dt)$, an equivalent inductor across this voltage V that has the same time rate of change would give $V/L = V/L_2 + V/L_2$, and the voltages can be divided out to give $L = (1/60\text{mH} + 1/30\text{ mH})^{-1}$ = 20 mH.

7) The circuit in (a) below shows an arrangement that applies a step voltage across a combination of a resistor and an initially unexcited inductor. What quantity is shown plotted against time in the graph in (b)?



(a)



(b)

a) The inductor voltage V_L <u>**b) The current I**</u>

An inductor stabilizes the current passing through it, and therefore the current will not change instantly (otherwise dI/dt = ∞). Thus, when the switch is closed there will be an instantaneous voltage across the inductor but no current will flow in the system immediately. The current will gradually build up, which means that answer (b), the current is the correct answer. Note, this would also be the voltage across the resistor (∝I), but this is not given as a choice.

8) Which one of the following statements is correct in relation to alternating waveforms?

a) In a capacitor, the voltage leads the current.
b) In an inductor, the voltage lags the current.
<u>**c) In a capacitor, the current leads the voltage.**</u>
d) In an inductor, the current leads the voltage.

One can remember either the ELI the ICE man mnemonic, or CIVIL. Basically, E leads I in and inductor (L), and I leads E in a capacitor (C). Thus choice c) is correct.

9) Calculate the reactance of an inductor of 15 mH at a frequency of 60 Hz.

a) 0.9 Ω b) 2.7 Ω <u>**c) 5.7 Ω**</u> d) 6.3 Ω

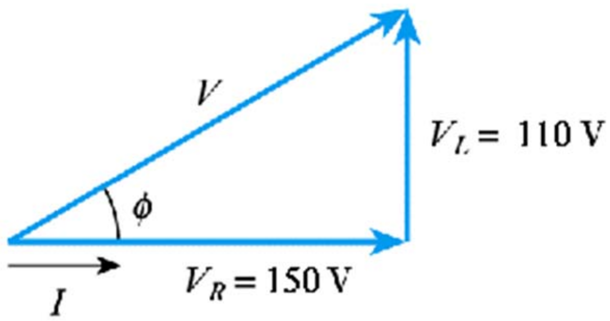
The reactance of an inductor is given by $X_L = \omega \cdot L$. *If the frequency is f=60 Hz, then the angular frequency,* $\omega = 2 \cdot \pi \cdot f = 2 \cdot \pi \cdot 60$ *rad/s = 377 rad/s. Thus the reactance of a 15 mH inductor would be* 15×10^{-3} *H ·377 rad/s= 5.7Ω.*

10) **Calculate the reactance of a capacitor of 470 µF at an angular frequency of 150 rad/s**

a) 14.2 Ω b) 56.1 Ω c) 89 Ω d) 130 Ω

The reactance of an inductor is given by $X_C = 1/(\omega \cdot C)$*. Since the frequency is given as an angular frequency,* ω*, we can calculate directly that the reactance is:*
$1/(150\ rad/s \cdot 470 x 10^{-6}\ F) = 1/(0.071) = 14.2\ \Omega$.

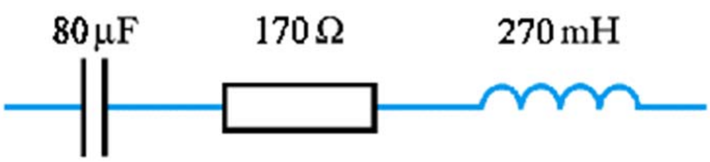
11) **The diagram below shows a phasor representation of the voltage *V* across a combination of a resistor and an inductor. Calculate the magnitude and phase of the voltage *V*.**



a) The magnitude is 186 V and the phase angle is 36 degrees
b) The magnitude is 168 V and the phase angle is 54 degrees
c) The magnitude is 168 V and the phase angle is 36 degrees
d) The magnitude is 186 V and the phase angle is 54 degrees

Here we can calculate the magnitude as $V = (V_L^2 + V_R^2)^{½}$*. This gives 186 V. The phase angle* $\phi = \tan^{-1}(V_L/V_R) = 36.25°$.

12) **Determine the complex impedance of the following series arrangement at a frequency of 60 Hz.**



a) 239 + j69 Ω b) 239 + j135 Ω c) 170 + j135 Ω **d) 170 + j69 Ω**

The frequency $f = 60$ *Hz gives an angular frequency* $\omega = 2 \cdot \pi \cdot f = 377$ *rad/s.* $Z_R = R$; $Z_L = j(\omega L)$; $Z_C = -j/(\omega C)$; *For series* $Z_T = Z_R + Z_L + Z_C$*. Thus,* $Z_T = 170\Omega + j(377 \cdot 0.270\ \Omega - 1/(377 \cdot 80 x 10^{-6}\ \Omega)$*. So* $Z_T = 170 + j(102 - 33)\ \Omega = 170 + j69\Omega$

13) Which of the following combinations of components represents an impedance of 110 + j 314 Ω at a frequency of 100 Hz?

a) A resistor of 100 Ω in series with a capacitor of 5 μH
b) An inductor of 50 mH in series with a capacitor of 5 μH
c) A resistor of 314 Ω in series with an inductor of 5 mH
d) **A resistor of 110 Ω in series with an inductor of 500 mH**

First, the angular frequency is $\omega = 2\cdot\pi\cdot f = 628$ rad/s. A resistor has a purely real impedance while capacitors and inductors have purely imaginary impedances, so the real part has to be a resistor of 110Ω. Our choice is a capacitor, which would have a negative, complex impedance (I leading V by 90°) so the only way to get a positive imaginary result is to have an inductor. The impedance of an inductor is $Z_L = j(\omega\cdot L)$ which must equal 314Ω at this frequency.
Thus, the inductor is 314Ω/628 rad/s = 0.500 H, or 500 mH in series with 110Ω resistor.

14) If a sinusoidal voltage $v = V_p \sin \omega t$ is applied across a capacitor, C, what is the average value of the power dissipated in the capacitor?

a) 0 b) CV_p^2 c) V_p^2 / C d) $2CV_p^2$

While there will be energy within the capacitor, it stores energy for one part of the cycle and returns it to the circuit during the other part of the cycle. Thus, the average power dissipated in the capacitor (or inductor) is zero.

15) The voltage across a component is measured as 80 V r.m.s. and the current through it is 4 A r.m.s. If the current leads the voltage by 20° what is the apparent power in the component?

a) 109 VA b) 116 VA c) 301 VA **d) 320 VA**

The apparent power is the product of the rms voltage and current without consideration of the phase angle, ϕ. Here that would be 80V·4A = 320 VA. The active power is the product of the rms voltage and current multiplied by the cosine of the phase angle.

16) The voltage across a component is measured as 80 V r.m.s. and the current through it is 4 A r.m.s. If the current leads the voltage by 20° what is the active power in the component?

a) 109 W b) 116 W **c) 301 W** d) 320 W

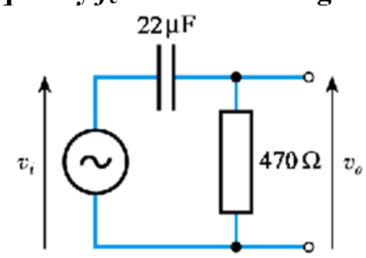
The active power is the product of the rms voltage and current multiplied by the cosine of the phase angle. Thus the active power is 80V·4A·cos(20°) = 301 W.

17) An amplifier has an output impedance Z_o of 70 + j 35 Ω. What value of load impedance will permit maximum power transfer?

a) 70 Ω + **j** 35 Ω   **b) 70 Ω - j 35 Ω**   c) -70 Ω - **j** 35 Ω   d) 70 Ω + **j** 35 Ω

We saw before that the power transfer to the load was a maximum when the output resistance and the load resistance were equal. For ac circuits, we want the magnitude of the load impedance equal to the magnitude of the output impedance. However, the power in an ac circuit will have both resistive and reactive (imaginary) parts. Only the real, resistive power that we call the active power, will be dissipated in the resistive element of the load. The reactive power, the imaginary part of the power will be stored and returned by the reactive components. However, we have to supply this reactive power to the circuit from the power supply in the first place, but it is just not appearing as power dissipated in the load (because the full V and the full I are never present at the same time).
The way to maximize the power transfer is to both match the impedance and to maximize the active power = $V \cdot I \cdot \cos(\phi)$, where V and I are the rms values. This maximizes when the phase angle, $\phi = 0$, or when the combination of the output impedance and load is purely real. The load in series with the output impedance that minimizes the imaginary part of the impedance while keeping magnitude of the load impedance equal to the magnitude of the output impedance is choice b.

18) Calculate the cut-off frequency f_c of the following circuit.



a) 15.4 Hz   b) 15.4 rad/s   c) 96.7 Hz   d) 96.7 rad/s

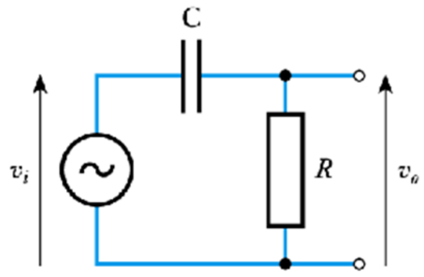
The cut-off (angular) frequency $\omega = 1/(R \cdot C)$ in rad/s. Here this is:
$\omega = 1/(22 \times 10^{-6}\ F \cdot 470\ \Omega) = 26.7$ rad/s. However, the frequency in Hz is:
$f = \omega/(2 \cdot \pi) = 26.7/(2 \cdot \pi)$ Hz = 15.4 Hz.

19) Which of the following statements is *not* correct?

a) Two octaves above 5 Hz is 20 Hz.
b) Three octaves below 64 Hz is 8 Hz.
c) Two decades below 10 MHz is 10 kHz.
d) Three decades above 470 Hz is 470 kHz.

Two octaves ⇒ double (or half) the frequency twice. That is, divide or multiply by 4.
Two decades ⇒ multiply or divide by 100.
Answer c) should be 10×10^4 Hz, or 100 kHz.

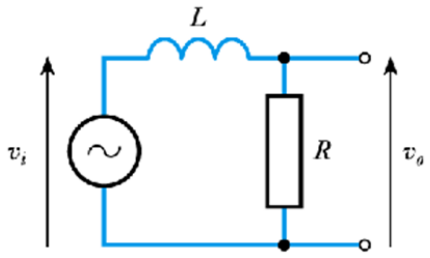
20) What are the characteristics of the following circuit?



a) A high-pass network b) A low-pass network

This is a voltage divider between the resistor and the capacitor. The reactance of the capacitor is $X_C = 1/(\omega \cdot C)$. This will be very large at low frequencies, so most of the voltage will be dropped over the capacitor and very little over the resistor. However, at high frequencies, the reactance of the capacitor is very small and most of the voltage in the divider will be across the resistor. Thus, we will get a higher voltage at high frequencies, so this is a high-pass filter.

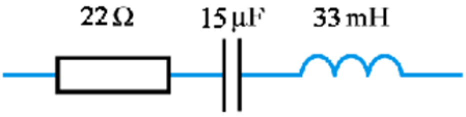
21) What are the characteristics of the following circuit?



a) A high-pass network **b) A low-pass network**

This is a voltage divider between the resistor and the inductor. The reactance of the inductor is $X_L = \omega \cdot L$. This will be very large at high frequencies, so most of the voltage will be dropped over the capacitor and very little over the resistor. However, at low frequencies, the reactance of the inductor is very small and most of the voltage in the divider will be across the resistor. Thus, we will get a higher voltage at low frequencies, so this is a low-pass filter.

22) Calculate the resonant frequency of the following arrangement.

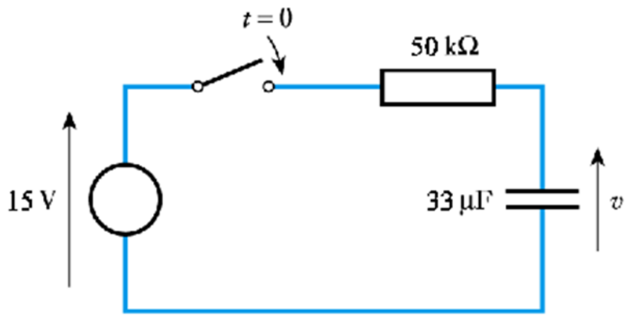


a) 53 Hz   b) 128 Hz   **c) 226 Hz**   d) 1421 Hz

The resonant frequency of a R-L-C series circuit occurs when the impedances of the capacitor and inductor are equal. When $\omega \cdot L = 1/(\omega \cdot C)$, the total complex impedance, $Z = R + j(\omega \cdot L - 1/(\omega \cdot C))$ is at a minimum and purely resistive. This minimum of impedance means that the current maximizes in the circuit at this frequency. Since there is a maximum of current, there will be large voltages in the capacitor and inductor, but they will be 180° out of phase and cancel, meaning that the only voltage drop will be across the resistor.

The angular frequency for which this occurs is (solving for ω above) is
$\omega_o = (LC)^{-½}$, and the resonant frequency in Hz is
$f_o = \omega_o /2\pi = (33x10^{-3}\ H \cdot 15x10^{-6}\ F)^{-½}/6.28 = 226$ Hz

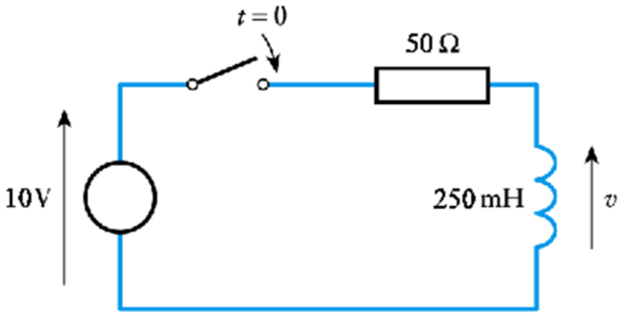
23) The switch in the following circuit closes at *t = 0*. If the capacitor is initially discharged, calculate the voltage on the capacitor at *t = 3* s.



a) 2.43 V   **b) 12.6 V**   c) 13.8 V   d) 14.2 V

Initially when the switch is closed, there is no charge on the capacitor so that the voltage across the capacitor is 0 (one could also look at it from the point of view that the instantaneous change in current brought about by closing the switch is a current changing infinitely fast, and therefore represents an infinite frequency. The impedance of a capacitor to an infinite frequency,$-j/(\omega \cdot C)$, is zero. No impedance=no voltage drop!). As the current flows the charge on the capacitor grows, and hence the voltage slowly grows and eventually reaches 15V. The equation for the change in voltage is:
$v = V_f+(V_i-V_f)\cdot e^{-t/\tau}$.
Here $V_i = 0$, $V_f = 15V$, and $\tau = R \cdot C = 50x10^3\ \Omega \cdot 33x10^{-6}\ F = 1.65$ sec(/rad). After 3 seconds, the voltage would be: $v = 15(1- e^{-3/1.65})\ V = 15 \cdot 0.84\ V = 12.6\ V$.

24) The switch in the following circuit closes at $t = 0$. If the inductor is initially de-energised, calculate the time at which the current in the coil reaches 150 mA.

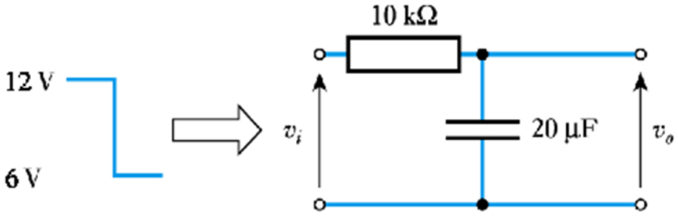


a) 1.44 ms b) 4.82 ms **c) 6.93 ms** d) 12.7 ms

Here when the switch is first closed, the current, initially zero, tries to start up. Since the inductor tries to stabilize the current, it will try to hold the current at zero with an induced voltage across its coils. Since the current is zero, there is no voltage drop across the resistor and all the voltage appears across the inductor (We can also visualize this, as above, as an instantaneous change in the current is a current changing infinitely fast, and therefore represents an infinite frequency. The impedance of an inductor, $j\omega \cdot L$ would therefore be infinite, and all the voltage drops occurs across the inductor.). The current in the system gradually builds up, and the voltage induced in the inductor drops to zero as the current slows its change. After some time, all the voltage is across the resistor, and the final current is $I_f = 10V/50\Omega$ Thus, the current at any time is given by:
$i = I_f+(I_i-I_f)\cdot e^{-t/\tau}$. *Solving this for t gives:* $t = -\tau \cdot ln[(i - i_f)/(i_i - i_f)]$.

Here $I_i = 0$, $I_f = 10V/50\Omega = 0.2A$, and $\tau = L/R = 250x10^{-3}$ H/50Ω = 0.005 sec(/rad). After t seconds, the current would be: $i = 0.15A = 0.2(1- e^{-t/0.005s})$ A. solving for t gives6.93ms

25) Derive an expression for the output voltage of the following circuit, for the period after $t = 0$.



a) $v = 12 - 6e^{-t/0.2}$ b) $v = 6 - 6e^{-t/0.2}$ **c) $v = 6 + 6e^{-t/0.2}$** d) $v = 12 - 12e^{-t/0.2}$

$v = V_f+(V_i-V_f)\cdot e^{-t/\tau}$. $V_i = 12$ V; $V_f = 6$ V; $T = R\cdot C = 10x10^3\Omega \cdot 20x10^{-6}F = 0.2$ sec(/rad)
$\Rightarrow v = 6 + (12-6)e^{-t/0.2} = 6 + 6e^{-t/0.2}$