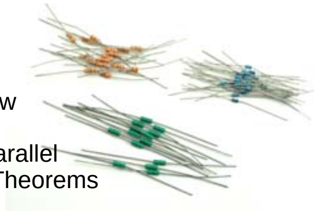
Resistance and DC Circuits

Chapter 3

- Introduction
- Current and Charge
- Voltage Sources
- Current Sources
- Resistance and Ohm's Law
- Kirchhoff's Laws
- Resistors in Series and Parallel
- Thévenin's and Norton's Theorems
- Superposition
- Nodal Analysis
- Mesh Analysis
- Solving Simultaneous Circuit Equations
- Choice of Techniques



 3.1

Introduction

3.1

- Many circuits can be analysed, and in some cases designed, using little more than Ohm's law
- However, in some cases we need some additional techniques and these are discussed in this lecture.
- We begin by reviewing some of the basic elements that we use to describe our circuits

 3.2

Current and Charge

3.2

- An electric **current** is a flow of electric **charge**

$$I = \frac{dQ}{dt}$$

- At an atomic level a current is a flow of **electrons**
  - each electron has a charge of 1.6×10^{-19} coulombs
  - conventional current flows in the opposite direction
- Rearranging above expression gives

$$Q = \int I dt$$

- For constant current

$$Q = I \times t$$

 3.3

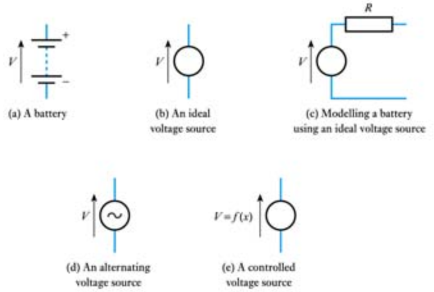
Voltage Sources

3.3

- A voltage source produces an **electromotive force** (**e.m.f.**) which causes a current to flow within a circuit
  - unit of e.m.f. is the **volt**
  - a volt is the potential difference between two points when a joule of energy is used to move one coulomb of charge from one point to the other
- Real voltage sources, such as batteries have resistance associated with them
  - in analysing circuits we use **ideal voltage sources**
  - we also use **controlled** or **dependent voltage sources**

 3.4

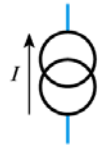
- Voltage sources



Current Sources

3.4

- We also sometimes use the concept of an **ideal current source**
  - unrealisable, but useful in circuit analysis
  - can be a fixed current source, or a **controlled** or **dependent current source**
  - while an ideal voltage source has *zero* output resistance, an ideal current source has *infinite* output resistance



Resistance and Ohm's Law

3.5

- **Ohm's law**

$$V \propto I$$

  - constant of proportionality is the resistance R
  - hence

$$V = IR \qquad I = \frac{V}{R} \qquad R = \frac{V}{I}$$

  - current through a resistor causes power dissipation

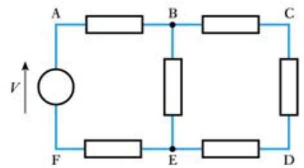
$$P = IV \qquad P = \frac{V^2}{R} \qquad P = I^2R$$

Kirchhoff's Laws

Video 3A

3.7

- **Node**
  - a point in a circuit where two or more circuit components are joined
- **Loop**
  - any closed path that passes through no node more than once
- **Mesh**
  - a loop that contains no other loop
- **Examples:**
  - A, B, C, D, E and F are *nodes*
  - the paths ABEFA, BCDEB and ABCDEFA are *loops*
  - ABEFA and BCDEB are *meshes*



- **Current Law**

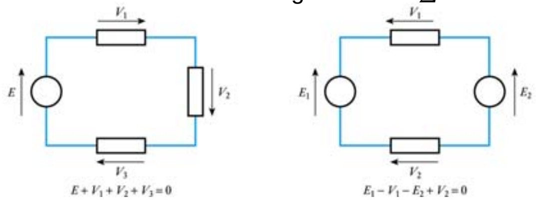
  At any instant, the algebraic sum of all the currents flowing into any node in a circuit is zero

  – if currents flowing *into* the node are positive, currents flowing *out of* the node are negative, then $\sum I = 0$

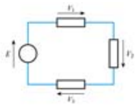
- **Voltage Law**

  At any instant the algebraic sum of all the voltages around any loop in a circuit is zero

  – if clockwise voltage arrows are positive and anticlockwise arrows are negative then $\sum V = 0$



In General: Single Loop Voltage Law



- The current *i(t)* is:

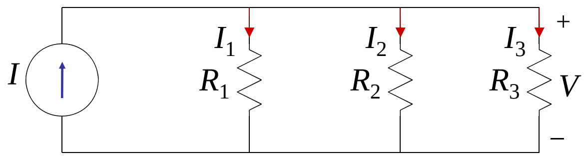
$$i(t) = \frac{\Sigma V_{Si}}{\Sigma R_j} = \frac{sum\ of\ \ voltage\ sources}{sum\ of\ \ resis\tan ces} = \frac{E}{\Sigma R_j}$$

- This approach works for any single loop circuit with voltage sources and resistors.
- Resistors in series can be replaced by:

$$R_{series} = R_1 + R_2 + \cdots + R_N = \sum R_j$$

Resistors in Parallel-Current Law

Same voltage across all the elements (have the same node)- the elements are in parallel

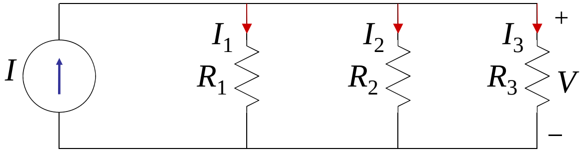


How do we find I_1, I_2, and I_3?

Apply KCL at the Top Node

$$I = I_1 + I_2 + I_3$$

But Ohm's Law: $I_1 = \dfrac{V}{R_1}$ $\quad I_2 = \dfrac{V}{R_2}$ $\quad I_3 = \dfrac{V}{R_3}$



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Solve for *V*

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$V = \frac{I}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}} \qquad \text{Ohm's Law:} \quad R_{eq} = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}$$

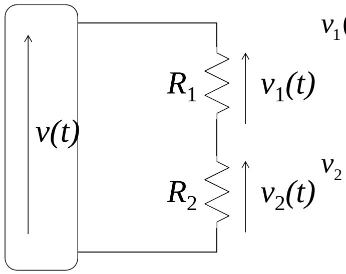
Definition: Parallel - *the elements share the same two end nodes (Voltage is the same across all the elements)*

3.14

Voltage Division

Two resistors **in series** with a voltage $v(t)$ across them

KVL: $v(t) - v_1(t) - v_2(t) = 0$ *Ohms Law:* $i(t) = v(t)/(R_1 + R_2)$



$$v_1(t) = i(t) \cdot R_1$$

$$\boxed{v_1(t) = v(t)\frac{R_1}{R_1 + R_2}}$$

$$v_2(t) = i(t) \cdot R_2 = v(t) - v_1(t)$$

$$\boxed{v_2(t) = v(t)\frac{R_2}{R_1 + R_2}}$$

3.15

In General: Voltage Division

Consider *N* resistors **in series**:

$$V_{R_i}(t) = \frac{R_i}{\sum R_j} \sum V_{S_k}(t)$$

Source voltage(s) are divided between the resistors in direct proportion to their resistances

3.16

Application of Voltage Divider

- Photomultiplier Tube
  - ~3000 V Potential Difference in 10-14 steps
  - Gain of 10^6 with low noise

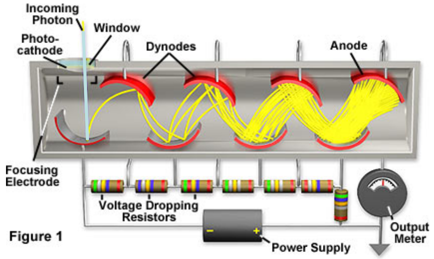


Figure 1

3.8

Thévenin's and Norton's Theorems

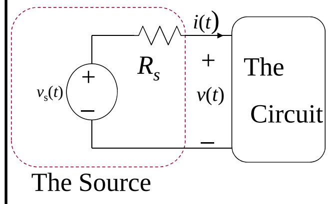
- **Thévenin's Theorem**

  As far as its appearance from outside is concerned, any two terminal network of resistors and energy sources can be replaced by a series combination of an ideal voltage source V and a resistor R, where V is the open-circuit voltage of the network and R is the voltage that would be measured between the output terminals if the energy sources were removed and replaced by their internal resistance

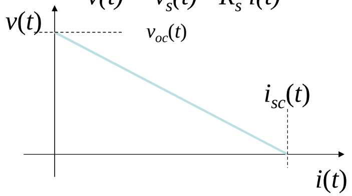
Thevenin's Theorem 2

- Any linear circuit with sources (dependent and/or independent) and resistors ***can be replaced by an equivalent circuit containing a single voltage source and a single resistor.***
- Thevenin's theorem implies that we can replace arbitrarily complicated networks with simple networks for purposes of analysis.
- If we have a single "load" resistor which is subject to change, we simplify circuit to a simple Thevenin equivalent with a single resistor and can recalculate easily for many different loads

A Realistic Source Model



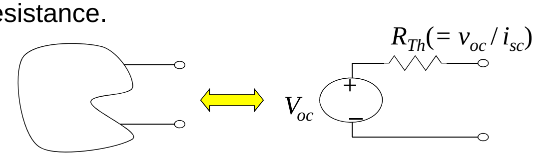
$v(t) = v_s(t) - R_s\, i(t)$



- Since the **open circuit voltage** and the **short circuit current** determine where the I-V line crosses both axes, they completely define the line.
- Any circuit that has the same I-V characteristics is an equivalent circuit.

Implications

- We use Thevenin's theorem to justify the concept of input and output resistance for amplifier circuits.
- We model transducers as equivalent sources and resistances.
- We model stereo speakers as an equivalent resistance.



Circuit with independent sources

Thevenin equivalent circuit

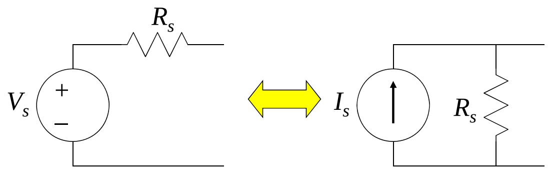
- **Norton's Theorem**

  As far as its appearance from outside is concerned, any two terminal network of resistors and energy sources can be replaced by a parallel combination of an ideal current source I and a resistor R, where I is the short-circuit current of the network and R is the voltage that would be measured between the output terminals if the energy sources were removed and replaced by their internal resistance

Norton's Theory 2

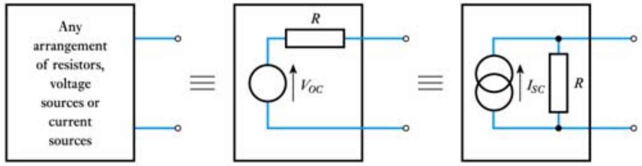
- Any Thevenin equivalent circuit is in turn equivalent to a current source in parallel with a resistor [source transformation].
- A current source in parallel with a resistor is called a Norton equivalent circuit.
- Finding a Norton equivalent circuit requires essentially the same process as finding a Thevenin equivalent circuit.

Source Transformation



$$V_s = R_s I_s$$

$$I_s = \frac{V_s}{R_s}$$



Thévenin equivalent circuit

Norton equivalent circuit

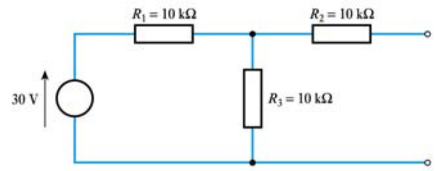
– from the Thévenin equivalent circuit $I_{SC} = \dfrac{V_{OC}}{R}$

– hence for *either* circuit $R = \dfrac{V_{OC}}{I_{SC}}$

3.25

- **Example** – see **Example 3.3** from course text

  Determine Thévenin and Norton equivalent circuits of the following circuit



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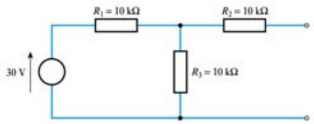
Example (continued)



– if nothing is connected across the output no current will flow in R_2 so there will be no voltage drop across it. Hence V_o is determined by the voltage source and the potential divider formed by R_1 and R_3. Hence

$$V_{OC} = \frac{30}{2} = 15\ \text{V}$$

– if the output is shorted to ground, R_2 is in parallel with R_3 and the current taken from the source is 30V/15 kΩ = 2 mA. This will divide equally between R_2 and R_3 so the output current, and so

$$I_{sc} = 1\text{mA}$$

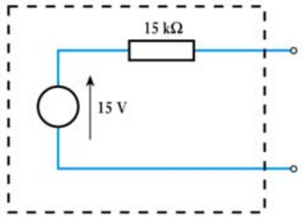
– the resistance in the equivalent circuit is therefore given by

$$R = \frac{V_{oc}}{I_{sc}} = \frac{15\ \text{V}}{1\text{mA}} = 15\ \text{k}\Omega$$

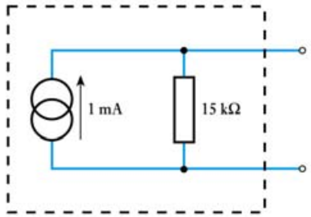
3.27

Example (continued)

– hence equivalent circuits are:



Thévenin equivalent circuit



Norton equivalent circuit

3.28

3.9

Superposition

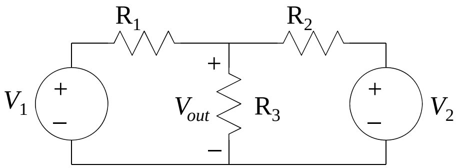
- **Principle of superposition**

  In any linear network of resistors, voltage sources and current sources, each voltage and current in the circuit is equal to the algebraic sum of the voltages or currents that would be present if each source were to be considered separately. When determining the effects of a single source the remaining sources are replaced by their internal resistance

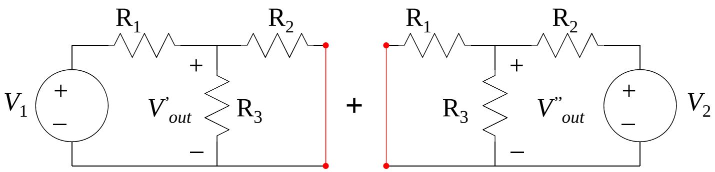
How to Apply Superposition

- To find the contribution due to an individual independent source, zero out the other independent sources in the circuit.
  - Voltage source ⇒ short circuit.
  - Current source ⇒ open circuit.
- Solve the resulting circuit using your favorite technique(s).

The Summing Circuit



Superposition



$I' = V_1/(R_1+R_{2//3})$

$V'_{out} = I' R_{2//3}$

Where

$R_{2//3} = R_2R_3/(R_2+R_3)$

$I'' = V_2/(R_2+R_{1//3})$

$V''_{out} = I\, R_{1//3}$

Where

$R_{1//3} = R_1R_3/(R_1+R_3)$

Use of Superposition

$$V_{out} = V'_{out} + V''_{out}$$

If $R_1=R_2=R_3=1\text{k}\Omega$

then

$$V'_{out} = V_1 /3$$

$$V''_{out} = V_2/3$$

$$V_{out} = V'_{out} + V''_{out} = V_1/3 + V_2/3$$

Summary and next time

- Nomenclature and symbols
- Circuit analysis using:
  - Ohms Law
  - Kirchoff's voltage law
  - Kirchoff's current law
- Further analysis using Thevenin and Norton theorems
- Superposition
- Next time
  - Further example of Superposition, Thevenin and Norton theorems
  - Mesh and Nodal analysis
  - AC circuits