Storey: Electronics: A Systems Approach 4th Ed.

Last time

- Mesh analysis
- AC circuits

This time

- Review R, L and C
- Review AC circuits
- Examples of impedance (Chapter 6)
- Power in AC circuits (Chapter 7)

Review of Resistors, Inductors and Capacitors

- Resistors, where R is the resistance (in Ohm) Chapter 3.5

$$V = RI \qquad R_{series} = R_1 + R_2 \qquad \frac{1}{R_{parllel}} = \frac{1}{R_1} + \frac{1}{R_2}$$

- Inductors, where L is the inductance (in Henry) Chapter 5.5

$$V = L\frac{\mathrm{d}I}{\mathrm{d}t} \qquad L_{series} = L_1 + L_2 \qquad \frac{1}{L_{parllel}} = \frac{1}{L_1} + \frac{1}{L_2}$$

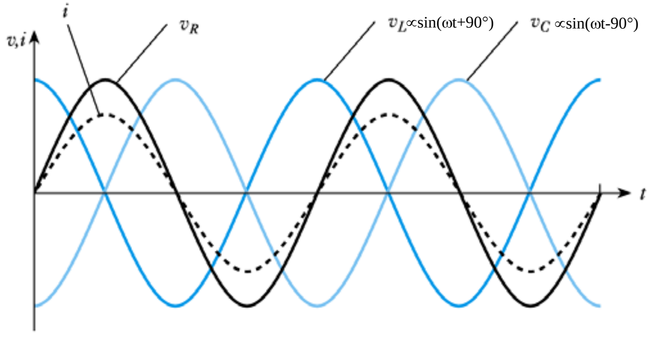
- Capacitors, where C is the capacitance (in Farad) Chapter 4.7

$$V = \frac{Q}{C} = \frac{1}{C}\int I \cdot dt \;\;\therefore\;\; I = C \cdot \frac{dV}{dt} \qquad \frac{1}{C_{Series}} = \frac{1}{C_1} + \frac{1}{C_2} \qquad C_{parallel} = C_1 + C_2$$

Key Points of AC circuits

- A sinusoidal voltage waveform can be described by the equation $v = V_p \sin(\omega t + \phi)$
- The voltage across a resistor is *in phase with* the current, the voltage across an inductor *leads* the current by 90°, and the voltage across a capacitor *lags* the current by 90°
- The reactance of an inductor $X_L = \omega L$
- The reactance of a capacitor $X_C = 1/\omega C$
- The relationship between current and voltage in circuits containing reactance can be described by its impedance
- The use of impedance is simplified by the use of complex notation

CIVIL (C⇒I leads V; L ⇐V lead I) or ELI the ICE man (E leads I in L, I leads E in C)

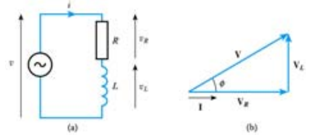


7.5

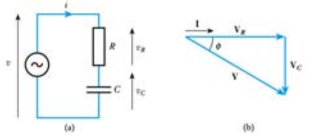
Phasor analysis of RL and RC series circuits

Current the same in both ∴ what is the voltage?

- Current R = Current L ⇒ E leads I in L ⇒ positive φ



- Current R = Current C ⇒ I leads E in C (∴ E lags I in C) ⇒ negative φ



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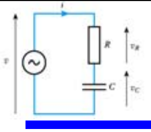
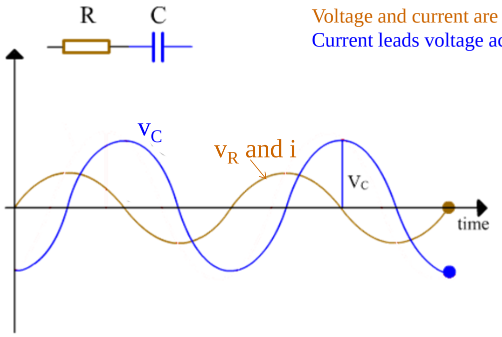
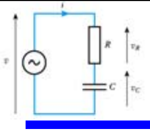


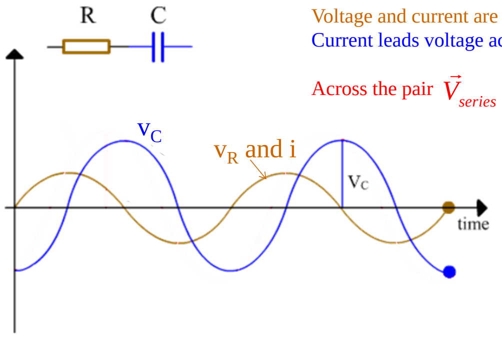


Voltage and current are in phase across resistor

Current leads voltage across the capacitor

7.7

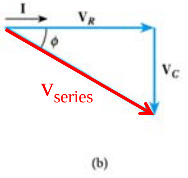




Voltage and current are in phase across resistor

Current leads voltage across the capacitor

Across the pair $\vec{V}_{series} = \vec{V}_R + \vec{V}_C$



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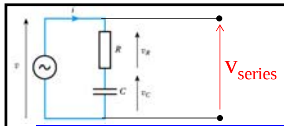
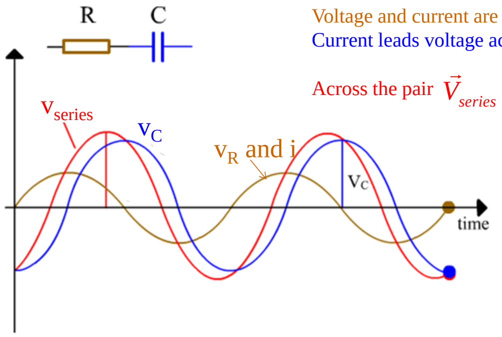
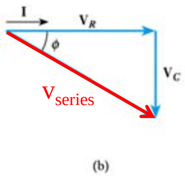


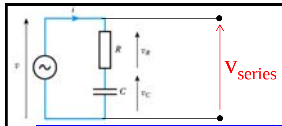


Voltage and current are in phase across resistor

Current leads voltage across the capacitor

Across the pair $\vec{V}_{series} = \vec{V}_R + \vec{V}_C$





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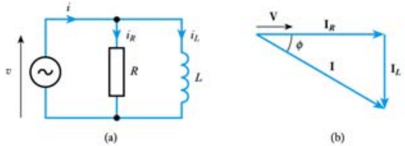
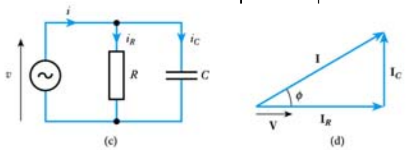
Phasor analysis of RL and RC parallel circuits

Voltage the same over both ∴ what is the current?

- Voltage constant ⇒ E leads I in L (∴ I lags E in L) ⇒ negative φ



- Voltage constant ⇒ I leads E in C ⇒ positive φ



What if we have R, L and C in both series and parallel?

- Phasors get messy
- But, phasors look like complex numbers
  - Vertical axis imaginary with j = *(-1)½*
- We will combine them using the *Impedance*, which we derive from the *Reactance*

Reactance is ratio of V_p to I_p

BUT WE MUST ACCOUNT FOR PHASE TO GET v(t)!

- The reactance of a component can be used in much the same way as resistance:
  - For a resistor $V_p = I_p X_R = I_p (R)$
  - for an inductor $V_p = I_p X_L = I_p (\omega L)$
  - for a capacitor $V_p = I_p X_C = I_p \left(\frac{1}{\omega C}\right)$

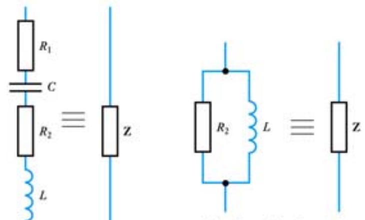
Note, for DC (ω=0) we only get V and I across a resistor
But V= 0 across an inductor
And I = 0 across a capacitor

Account for phase with Complex Notation

6.6

- Take the impedance of a resistor as purely real, an inductor to be +imaginary, and a capacitor to be - imaginary
- We represent impedance using complex notation as:
- Resistors: $\mathbf{Z_R} = R$
- Inductors: $\mathbf{Z_L} = jX_L = j\omega L$
- Capacitors: $\mathbf{Z_C} = -jX_C = -j\frac{1}{\omega C} = \frac{1}{j\omega C}$

- **Series and parallel combinations of impedances**
  - impedances combine in the same way as resistors



$$\mathbf{Z} = \mathbf{Z}_{R_1} + \mathbf{Z}_C + \mathbf{Z}_{R_2} + \mathbf{Z}_L = R_1 - jX_C + R_2 + jX_L = (R_1 + R_2) + j(\omega L - \frac{1}{\omega C})$$

(a)

$$\frac{1}{\mathbf{Z}} = \frac{1}{\mathbf{Z}_R} + \frac{1}{\mathbf{Z}_L} = \frac{1}{R} + \frac{1}{jX_L}$$

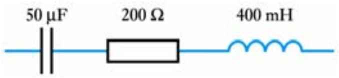
$$\mathbf{Z} = \frac{R \times jX_L}{R + jX_L} = \frac{R \times j\omega L}{R + j\omega L}$$

(b)

- **Manipulating complex impedances**
  - complex impedances can be *added*, *subtracted*, *multiplied* and *divided* in the same way as other complex quantities
  - they can also be expressed in a range of forms such as the **rectangular**, **polar** and **exponential** forms
  - if you are unfamiliar with the manipulation of complex quantities (or would like a little revision on this topic) see **Appendix D** of the course text which gives a tutorial on this subject

- **Example** – see **Example 6.7** in the course text

Determine the complex impedance of this circuit at a frequency of 50 Hz



At 50Hz, the angular frequency $\omega = 2\pi f = 2 \times \pi \times 50 = 314$ rad/s

Therefore

$$\mathbf{Z} = \mathbf{Z_C} + \mathbf{Z_R} + \mathbf{Z_L} = R + j(X_L - X_C) = R + j(\omega L - \frac{1}{\omega C})$$

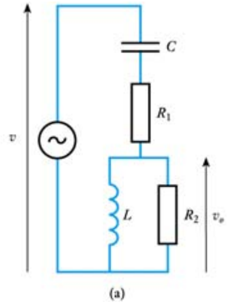
$$= 200 + j(314 \times 400 \times 10^{-3} - \frac{1}{314 \times 50 \times 10^{-6}})$$

$$= 200 + j62 \text{ ohms}$$

- **A further example**

A more complex task is to find the output voltage of this circuit

The analysis of this circuit, and a numerical example based on it, are given in **Section 6.6.4** and **Example 6.8** of the course text



Further Study

Video 6B Further Study

- The Further Study section at the end of Chapter 6 looks at the characteristics of a simple reactive circuit.
- See if you can model the behaviour of the circuit at a single frequency and then watch the video to check your analysis.

Power in AC Circuits

Chapter 7

- Introduction
- Power in Resistive Components
- Power in Capacitors
- Power in Inductors
- Circuits with Resistance and Reactance
- Active and Reactive Power
- Power Factor Correction
- Three-Phase Systems
- Power Measurement

7.1

Introduction

- The instantaneous power dissipated in a component is a product of the instantaneous voltage and the instantaneous current

$$p = vi$$

- In a resistive circuit the voltage and current are in phase – calculation of p is straightforward
- In reactive circuits, there will normally be some phase shift between v and i, and calculating the power becomes more complicated

7.2

Power in Resistive Components

- Suppose a voltage $v = V_p \sin \omega t$ is applied across a resistance R. The resultant current i will be

$$i = \frac{v}{R} = \frac{V_P \sin \omega t}{R} = I_P \sin \omega t$$

- The result power p will be

$$\begin{aligned} p &= vi \\ &= V_P \sin \omega t \times I_P \sin \omega t \\ &= V_P I_P (\sin^2 \omega t) \\ &= V_P I_P (\frac{1 - \cos 2\omega t}{2}) \end{aligned}$$

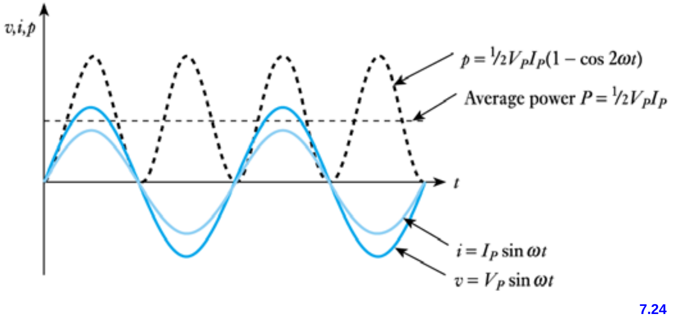
From Chapter 2
$V_{avg} = (2/\pi) \cdot V_p$
$V_{rms} = (1/\sqrt{2}) \cdot V_p$
Current the same

- The average value of (1 - cos $2\omega t$) is 1, so

$$\begin{aligned} \text{Average Power } P &= \frac{1}{2} V_P I_P \\ &= \frac{V_P}{\sqrt{2}} \times \frac{I_P}{\sqrt{2}} \\ &= VI \end{aligned}$$

where V and I are the **r.m.s. voltage and current**

- **Relationship between *v*, *i* and *p* in a resistor**



Power in Capacitors

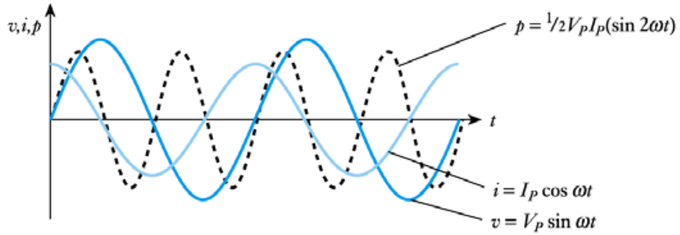
7.3

- From our discussion of capacitors we know that the current leads the voltage by 90°. Therefore, if a voltage $v = V_p \sin \omega t$ is applied across a capacitance C, the current will be given by $i = I_p \cos \omega t$
- Then

$$\begin{aligned} p &= vi \\ &= V_P \sin\omega t \times I_P \cos\omega t \\ &= V_P I_P (\sin\omega t \times \cos\omega t) \\ &= V_P I_P (\frac{\sin 2\omega t}{2}) \end{aligned}$$

- The average power is zero

- **Relationship between *v*, *i* and *p* in a capacitor**



Power in Inductors

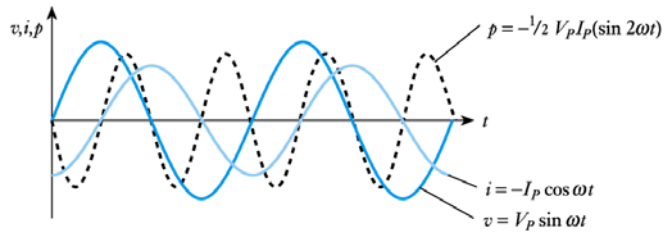
7.4

- From our discussion of inductors we know that the current lags the voltage by 90°. Therefore, if a voltage $v = V_p \sin \omega t$ is applied across an inductance L, the current will be given by $i = -I_p \cos \omega t$
- Therefore

$$\begin{aligned} p &= vi \\ &= V_P \sin\omega t \times -I_P \cos\omega t \\ &= -V_P I_P (\sin\omega t \times \cos\omega t) \\ &= -V_P I_P (\frac{\sin 2\omega t}{2}) \end{aligned}$$

- Again the average power is zero

- **Relationship between *v*, *i* and *p* in an inductor**



Trig relation :

$$\{\cos x - \cos(y)\} = -2\cdot\sin\left(\frac{x+y}{2}\right)\cdot\sin\left(\frac{x-y}{2}\right)$$

Circuit with Resistance and Reactance

7.5

- When a sinusoidal voltage $v = V_p \sin \omega t$ is applied across a circuit with resistance *and* reactance, the current will be of the general form $i = I_p \sin(\omega t - \phi)$
- Therefore, the instantaneous power, p is given by

$$\begin{aligned} p &= vi \\ &= V_P \sin \omega t \times I_P \sin(\omega t - \phi) \\ &= \frac{1}{2} V_P I_P \{\cos \phi - \cos(2\omega t - \phi)\} \end{aligned}$$

$$p = \frac{1}{2} V_P I_P \cos \phi - \frac{1}{2} V_P I_P \cos(2\omega t - \phi)$$

7.29

$$p = \frac{1}{2} V_P I_P \cos \phi - \frac{1}{2} V_P I_P \cos(2\omega t - \phi)$$

- The expression for p has two components
- The second part oscillates at 2ω and has an average value of zero over a complete cycle
  – this is the power that is stored in the reactive elements and then returned to the circuit within each cycle
- The first part represents the power dissipated in resistive components. **Average power dissipation** is

$$P = \frac{1}{2} V_P I_P (\cos \phi) = \frac{V_P}{\sqrt{2}} \times \frac{I_P}{\sqrt{2}} \times (\cos \phi) = VI \cos \phi$$

7.30

- The average power dissipation given by

$$P = \frac{1}{2} V_P I_P (\cos \phi) = VI \cos \phi$$

  is termed the **active power** in the circuit and is measured in watts (W)
- The product of the r.m.s. voltage and current VI is termed the **apparent power**, ***S***. To avoid confusion this is given the units of volt amperes (VA)

7.31

- From the above discussion it is clear that

$$\begin{aligned} P &= VI \cos\phi \\ &= S \cos\phi \end{aligned}$$

- In other words, the active power is the apparent power times the cosine of the phase angle.
- This cosine is referred to as the **power factor**

$$\frac{\text{Active power (in watts)}}{\text{Apparent power (in volt amperes)}} = \text{Power factor}$$

$$\text{Power factor} = \frac{P}{S} = \cos \phi$$

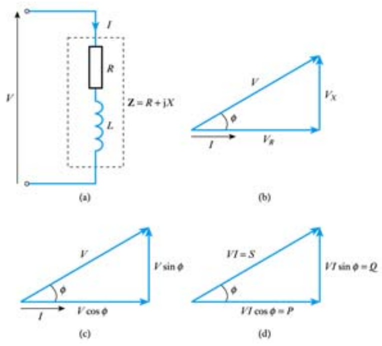
7.32

Active and Reactive Power

7.6

- When a circuit has resistive and reactive parts, the resultant power has 2 parts:
  - The first is *dissipated* in the resistive element. This is the **active power, *P***
  - The second is *stored* and *returned* by the reactive element. This is the **reactive power**, ***Q*** , which has units of **volt amperes reactive** or **var**
- While reactive power is not dissipated it does have an effect on the system
  - for example, it increases the current that must be supplied and increases losses with cables

- Consider an RL circuit
  - the relationship between the various forms of power can be illustrated using a power triangle



Applied voltage V
If V_R=Vcos(φ) then V_L=Vsin(φ) since there is a 90° phase shift between V_R and V_L

- Therefore

Active Power	$P = VI \cos \phi$	watts
Reactive Power	$Q = VI \sin \phi$	var
Apparent Power	$S = VI$	VA
	$S^2 = P^2 + Q^2$	

Power Factor Correction

7.7

- **Power factor** (Active/Apparent power) is particularly important in high-power applications
- Inductive loads have a *lagging* power factor
- Capacitive loads have a *leading* power factor
- Many high-power devices are inductive
  - a typical AC motor has a power factor of 0.9 lagging
  - the total load on the national grid is 0.8-0.9 lagging
  - this leads to major inefficiencies
  - power companies therefore penalize industrial users who introduce a poor power factor

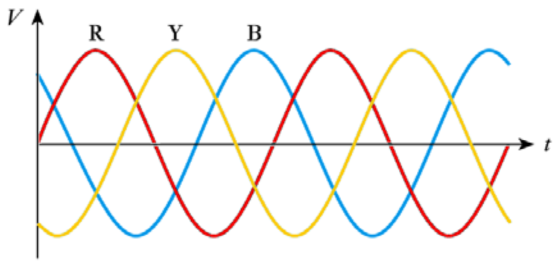
- The problem of poor power factor is tackled by adding additional components to bring the power factor back closer to unity
  - a capacitor of an appropriate size in parallel with a lagging load can ‘cancel out’ the inductive element
  - this is **power factor correction**
  - a capacitor can also be used in series but this is less common (since this alters the load voltage)
  - for examples of power factor correction see **Examples 7.2 and 7.3** in the course text

Three-Phase Systems

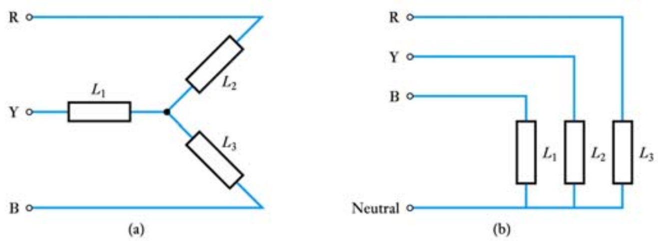
7.8

- So far, our discussion of AC systems has been restricted to **single-phase** arrangement
  - as in conventional domestic supplies
- In high-power industrial applications we often use **three-phase** arrangements
  - these have three supplies, differing in phase by 120°
  - phases are labeled **red**, **yellow** and **blue** (**R**, **Y** & **B**)

- Relationship between the phases in a three-phase arrangement



- Three-phase arrangements may use either 3 or 4 conductors



Power Measurement

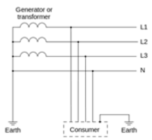
7.9

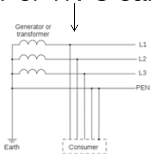
- When using AC, *power* is determined not only by the r.m.s. values of the **voltage** and **current**, but also by the **phase angle** (which determines the **power factor**)
  - consequently, you cannot determine the power from independent measurements of current and voltage
- In **single-phase systems** power is normally measured using an **electrodynamic wattmeter**
  - measures power directly using a single meter which effectively multiplies instantaneous current and voltage

- In **three-phase systems** we need to sum the power taken from the various phases
  - in three-wire arrangements we can deduce the total power from measurements using 2 wattmeters
  - in a four-wire system it may be necessary to use 3 wattmeters
  - in balanced systems (systems that take equal power from each phase) a single wattmeter can be used, its reading being multiplied by 3 to get the total power

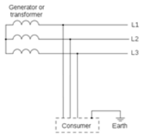
Who cares?

- In Europe, the standard is TT or TN-C earthing





- In Norway (and perhaps Albania) it is IT earthing



Buy equipment with TT or TN-C

You will need an additional transformer to run it in Norway

Further Study

Video 7B Further Study

- The Further Study section at the end of Chapter 7 is concerned with power factor correction for a high-power motor.
- Your task is to calculate the size of capacitor needed to achieve a specific power factor.
- Do the sums and then watch the video.

Key Points

- In resistive circuits the average power is equal to VI, where V and I are r.m.s. values
- In a capacitor the current *leads* the voltage by 90° and the average power is zero
- In an inductor the current *lags* the voltage by 90° and the average power is zero
- In circuits with both resistive and reactive elements, the average power is $VI \cos \phi$
- The term $\cos \phi$ is called the power factor
- Power factor correction is important in high-power systems
- High-power systems often use three-phase arrangements