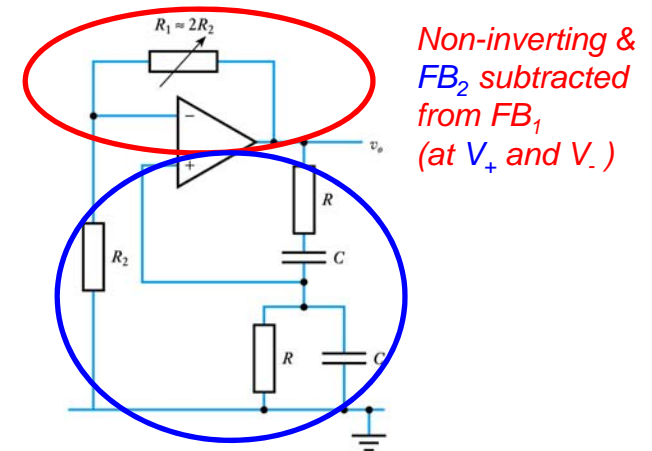
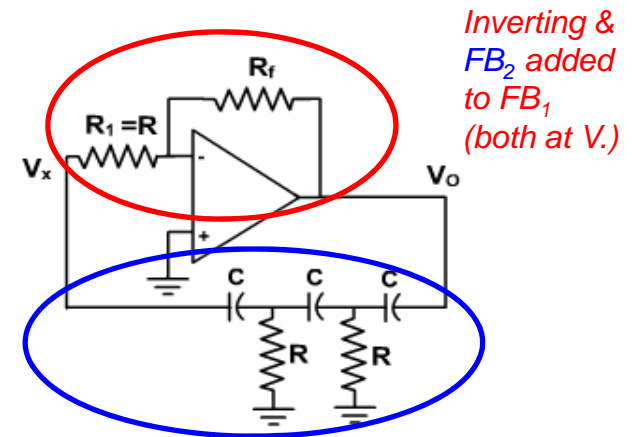


Last time: Key points oscillators

- Positive feedback is used in analogue and digital systems
- A primary use is in the production of oscillators
- The requirement for oscillation is that the loop gain AB must have a magnitude of 1, and a phase shift of 180° (or 180° plus some integer multiple of 360°)
- This can be achieved using a circuit that produces a phase shift of 180° subtracted from a non-inverting amplifier's feedback (or added to an inverting amplifier's feedback)
- Alternatively, it can be achieved using a circuit that produces a phase shift of 0° subtracted from an inverting amplifier's feedback (or added to a non-inverting amplifier's feedback)
- For good frequency stability we often use crystals
- Care must be taken to ensure the stability of all feedback systems

Sine-wave oscillator systems

- Two feedback loops
- FB_1 purely resistive gain
 - $V_{fb1} = G_1 \cdot V_o$, where $G_1 = \text{constant}$
- FB_2 has $G_2(f)$ with phase(f)
 - V_{fb2} Cancels V_{fb1} , but only when V_o changes at $f=f_o$
- Near saturation, V_o slows or stops ($A \rightarrow 0 \Rightarrow f \rightarrow 0 \Rightarrow G_2 \rightarrow 0$)
- FB_2 no longer cancels FB_1 and V_o reverses



How do we create a sine wave?

(from TI- Application Report SLOA060 - March 2001)

As the phase shift approaches 180° and $|A\beta| \rightarrow 1$, the output voltage of the now-unstable system tends to infinity but, of course, is limited to finite values by an energy-limited power supply.

When the output voltage approaches either power rail, the active devices in the amplifiers change gain. This causes the value of A to change and forces $A\beta$ away from the singularity; thus the trajectory towards an infinite voltage slows and eventually halts.

At this stage, one of three things can occur:

- (i) Nonlinearity in saturation or cutoff causes the system to become stable and lock up at the current power rail.
- (ii) The initial change causes the system to saturate (or cutoff) and stay that way for a long time before it becomes linear and heads for the opposite power rail. This produces highly distorted oscillations (usually quasi-square waves), the resulting oscillators being called relaxation oscillators
- (iii) The system stays linear and reverses direction, heading for the opposite power rail. **This produces a sine-wave oscillator.**



23.3

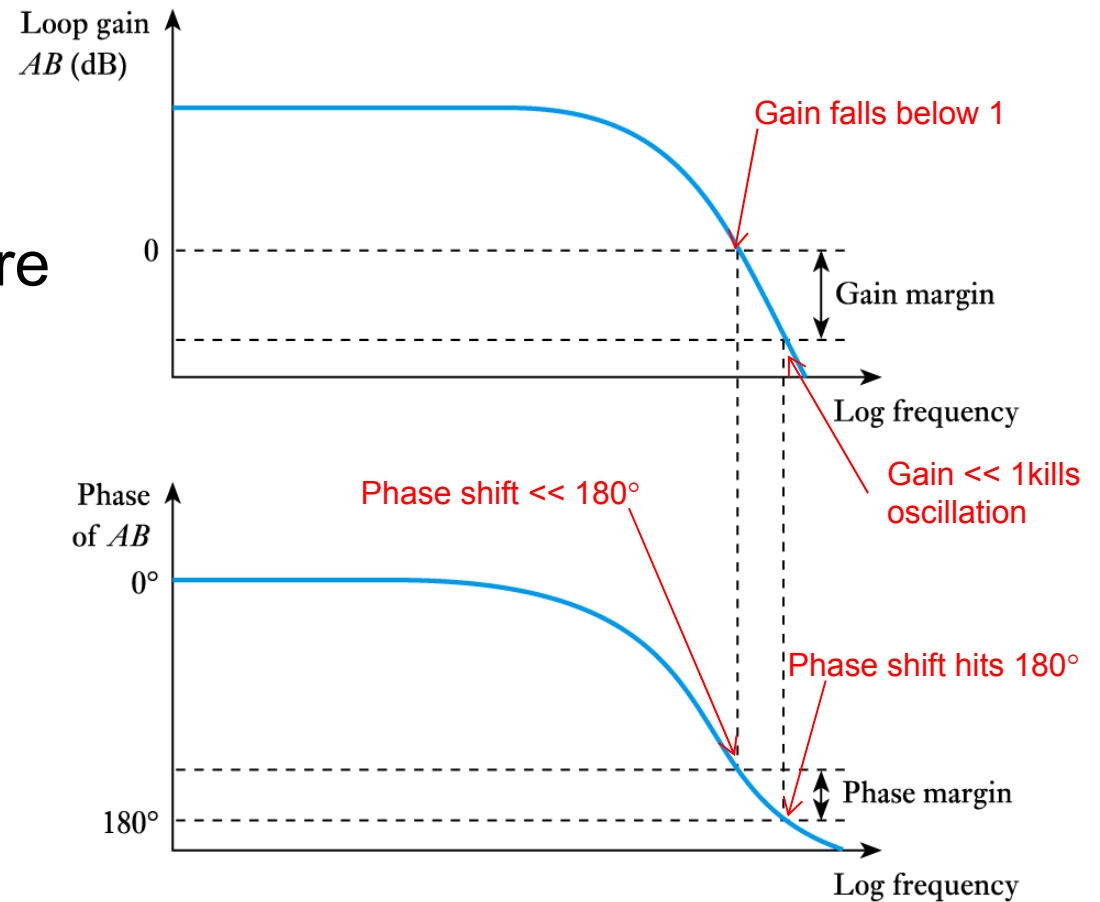
Stability

- The gain of all real amplifiers falls at high frequencies and this produces a phase shift of the feedback
- Unintended feedback within a circuit or **stray capacitance** or **stray inductance** can produce feedback phase shifts
- All multi-stage amplifiers will produce 180° of phase shift at some frequency
- To ensure stability we must ensure that the Barkhausen conditions for oscillation are *not* met
 - That is, $|AB|=1$ and 180° phase shift of B
 - to guarantee this we must ensure that the gain falls below unity before the phase shift reaches 180°

24.4

■ Gain and phase margins

- these are a measure of the stability of a circuit



- QUESTIONS?

Digital systems

- Introduction
- Binary quantities and variables
- Logic gates
- Boolean algebra
- Combinational logic
- Boolean algebraic manipulation
- Algebraic simplification
- Karnaugh maps
- Automated methods of minimisation
- Propagation delay and hazards
- Number systems and binary arithmetic
- Numeric and alphabetic codes
- Examples of combinational logic design



Introduction



Video 24A



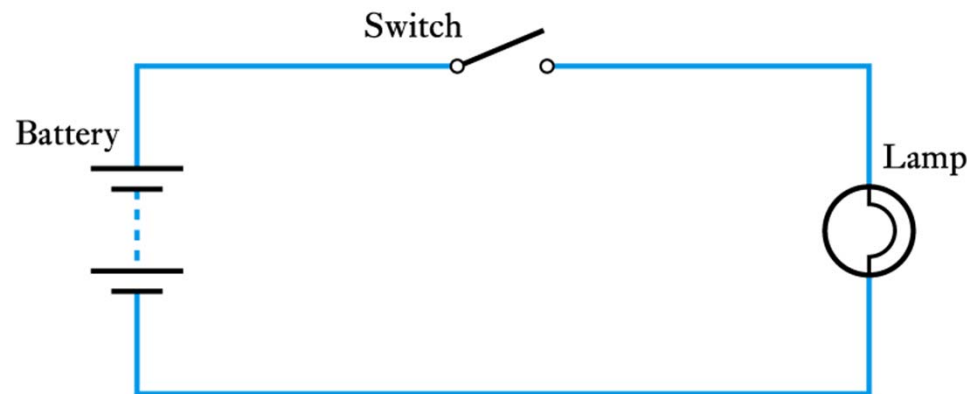
24.1

- Digital systems are concerned with digital signals
- Digital signals can take many forms
- Here we will concentrate on **binary signals** since these are the most common form of digital signals
 - can be used individually
 - perhaps to represent a single binary quantity or the state of a single switch
 - can be used in combination
 - to represent more complex quantities

24.8

Binary quantities and variables

- A **binary quantity** is one that can take only 2 states



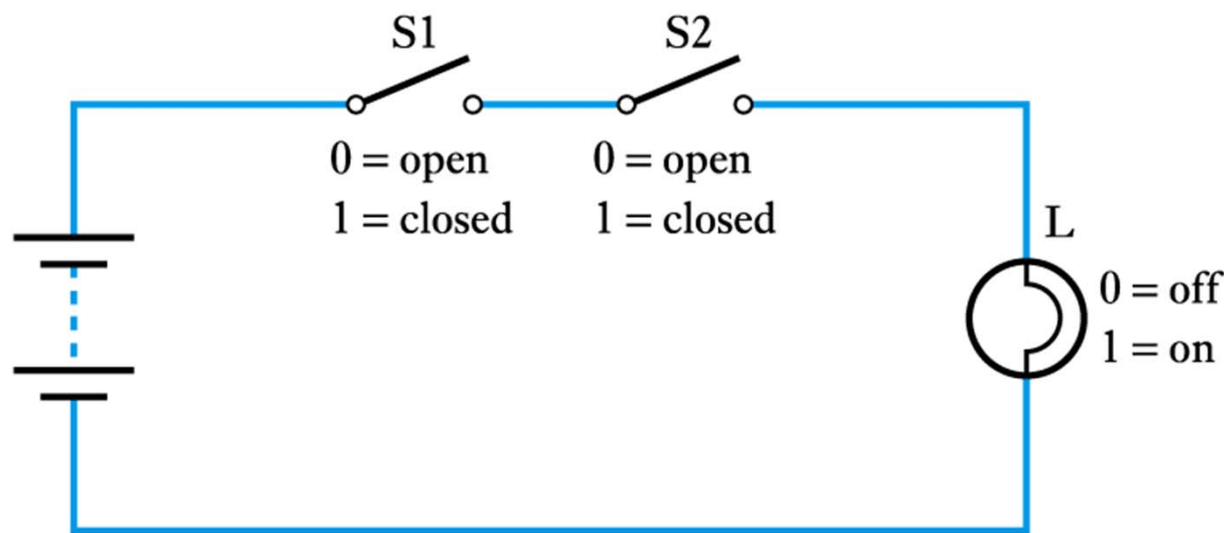
A simple binary arrangement

| S | L |
|--------|-----|
| OPEN | OFF |
| CLOSED | ON |

| S | L |
|---|---|
| 0 | 0 |
| 1 | 1 |

A truth table

- A binary arrangement with two switches in series



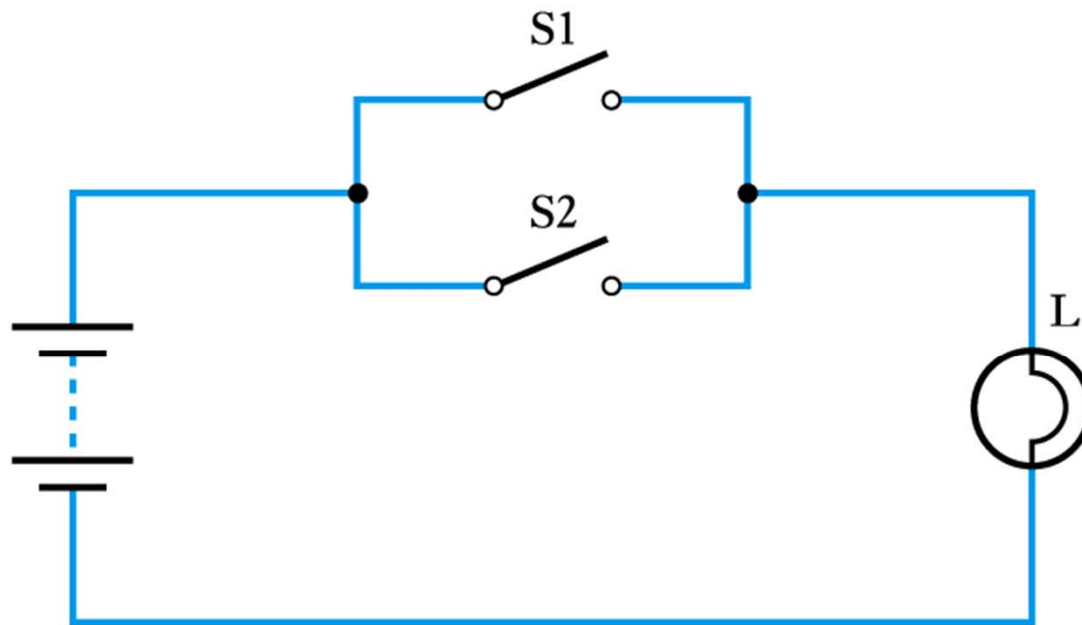
(a) Circuit

| S1 | S2 | L |
|----|----|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

(b) Truth table

$$L = S1 \text{ AND } S2$$

- A binary arrangement with two switches in parallel



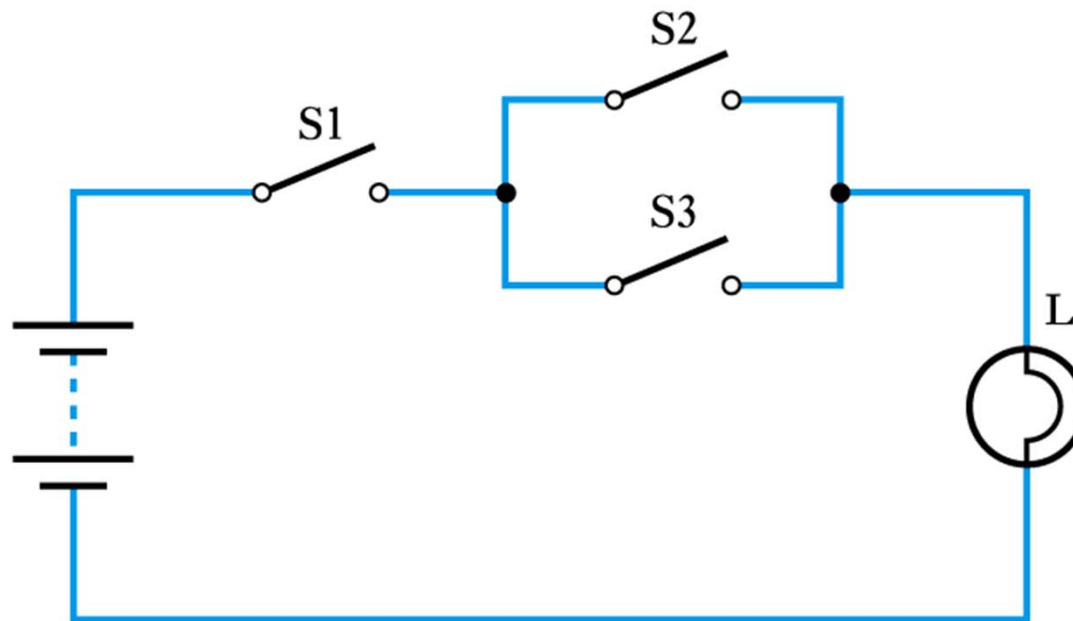
(a) Circuit

| S1 | S2 | L |
|----|----|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

(b) Truth table

$$L = S1 \text{ OR } S2$$

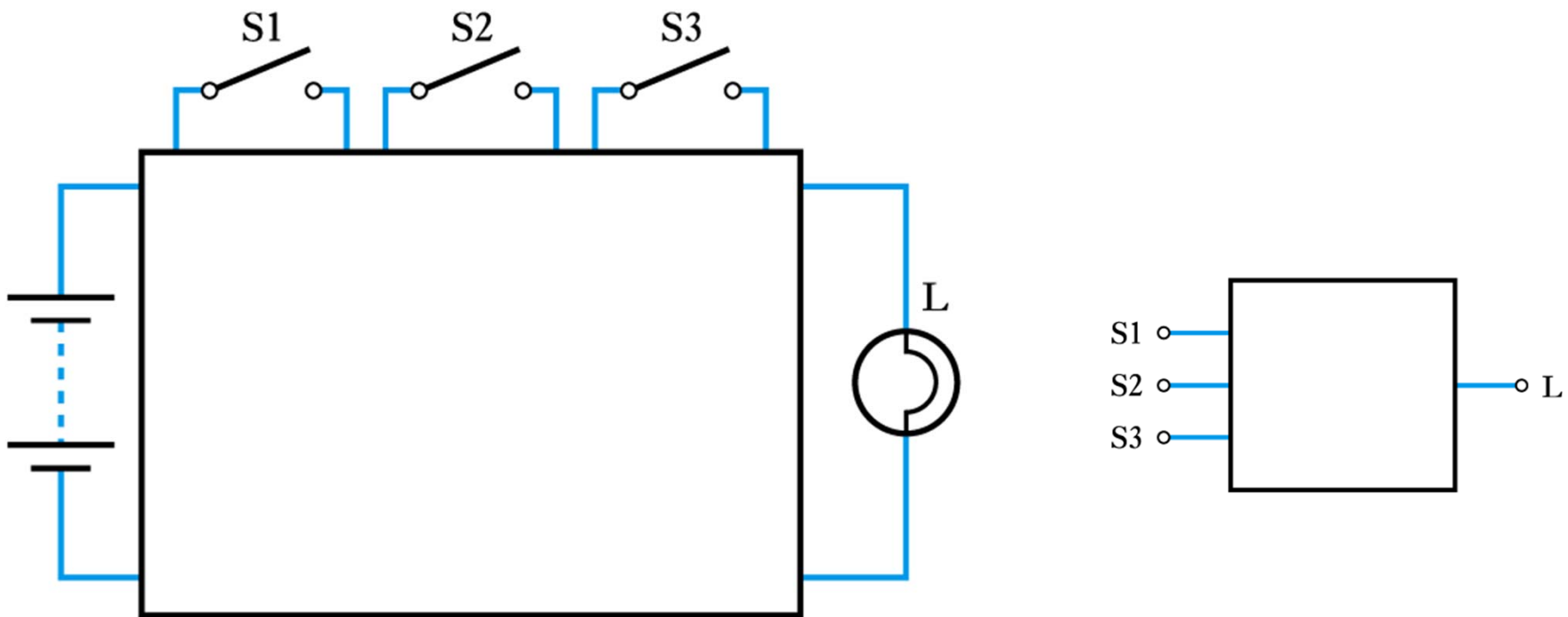
- A series/parallel arrangement



| S1 | S2 | S3 | L |
|----|----|----|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

$$L = S1 \text{ AND } (S2 \text{ OR } S3)$$

-
- Representing an unknown network with a truth table



Use this to look at Logic gates



Video 24B



24.3

- The building blocks used to create digital circuits are **logic gates**
- There are three elementary logic gates and a range of other simple gates
- Each gate has its own **logic symbol** which allows complex functions to be represented by a logic diagram
- The function of each gate can be represented by a **truth table** or using **Boolean notation**

24.14

■ The AND gate



(a) Circuit symbol

| A | B | C |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

(b) Truth table

Count up the inputs in binary to ensure all possibilities covered

$$C = A \cdot B$$

(c) Boolean expression

■ The OR gate



(a) Circuit symbol

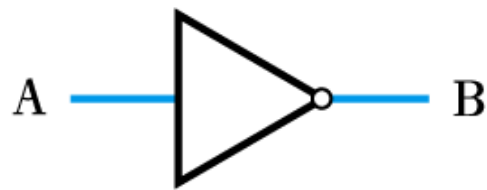
| A | B | C |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

(b) Truth table

$$C = A + B$$

(c) Boolean expression

- The NOT gate (or inverter)



(a) Circuit symbol

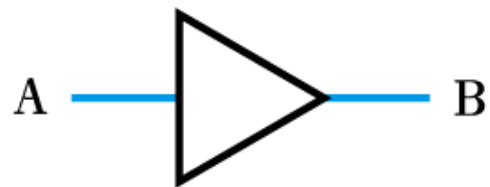
| A | B |
|---|---|
| 0 | 1 |
| 1 | 0 |

(b) Truth table

$$B = \bar{A}$$

(c) Boolean expression

- A logic buffer gate



(a) Circuit symbol

| A | B |
|---|---|
| 0 | 0 |
| 1 | 1 |

(b) Truth table

$$B = A$$

(c) Boolean expression

■ The NAND gate



(a) Circuit symbol

| A | B | C |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

(b) Truth table

$$C = \overline{A \cdot B}$$

(c) Boolean expression

■ The NOR gate



(a) Circuit symbol

| A | B | C |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

(b) Truth table

$$C = \overline{A + B}$$

(c) Boolean expression

■ The Exclusive OR gate



(a) Circuit symbol

| A | B | C |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

(b) Truth table

$$C = A \oplus B$$

(c) Boolean expression

■ The Exclusive NOR gate



(a) Circuit symbol









| A | B | C |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |


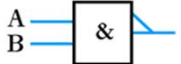

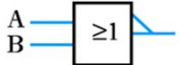

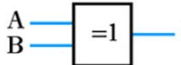

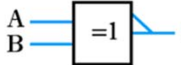
(b) Truth table

$$C = \overline{A \oplus B}$$

(c) Boolean expression

SUMMARY

| Function | Symbol | Alternative symbol | Boolean expression | Truth table | | | | | | | | | | | | | | | |
|----------|---|---|--------------------|--|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Buffer |  |  | $B = A$ | <table><tr><th>A</th><th>B</th></tr><tr><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td></tr></table> | A | B | 0 | 0 | 1 | 1 | | | | | | | | | |
| A | B | | | | | | | | | | | | | | | | | | |
| 0 | 0 | | | | | | | | | | | | | | | | | | |
| 1 | 1 | | | | | | | | | | | | | | | | | | |
| NOT |  |  | $B = \bar{A}$ | <table><tr><th>A</th><th>B</th></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td></tr></table> | A | B | 0 | 1 | 1 | 0 | | | | | | | | | |
| A | B | | | | | | | | | | | | | | | | | | |
| 0 | 1 | | | | | | | | | | | | | | | | | | |
| 1 | 0 | | | | | | | | | | | | | | | | | | |
| AND |  |  | $C = A \cdot B$ | <table><tr><th>A</th><th>B</th><th>C</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table> | A | B | C | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 |
| A | B | C | | | | | | | | | | | | | | | | | |
| 0 | 0 | 0 | | | | | | | | | | | | | | | | | |
| 0 | 1 | 0 | | | | | | | | | | | | | | | | | |
| 1 | 0 | 0 | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | | | | | | | | | | | | | | | | | |
| OR |  |  | $C = A + B$ | <table><tr><th>A</th><th>B</th><th>C</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table> | A | B | C | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| A | B | C | | | | | | | | | | | | | | | | | |
| 0 | 0 | 0 | | | | | | | | | | | | | | | | | |
| 0 | 1 | 1 | | | | | | | | | | | | | | | | | |
| 1 | 0 | 1 | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | | | | | | | | | | | | | | | | | |

| Function | Symbol | Alternative symbol | Boolean expression | Truth table | | | | | | | | | | | | | | | |
|---------------|---|---|-----------------------------|--|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| NAND |  |  | $C = \overline{A \cdot B}$ | <table><tr><th>A</th><th>B</th><th>C</th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table> | A | B | C | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 |
| A | B | C | | | | | | | | | | | | | | | | | |
| 0 | 0 | 1 | | | | | | | | | | | | | | | | | |
| 0 | 1 | 1 | | | | | | | | | | | | | | | | | |
| 1 | 0 | 1 | | | | | | | | | | | | | | | | | |
| 1 | 1 | 0 | | | | | | | | | | | | | | | | | |
| NOR |  |  | $C = \overline{A + B}$ | <table><tr><th>A</th><th>B</th><th>C</th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table> | A | B | C | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |
| A | B | C | | | | | | | | | | | | | | | | | |
| 0 | 0 | 1 | | | | | | | | | | | | | | | | | |
| 0 | 1 | 0 | | | | | | | | | | | | | | | | | |
| 1 | 0 | 0 | | | | | | | | | | | | | | | | | |
| 1 | 1 | 0 | | | | | | | | | | | | | | | | | |
| Exclusive OR |  |  | $C = A \oplus B$ | <table><tr><th>A</th><th>B</th><th>C</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table> | A | B | C | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 |
| A | B | C | | | | | | | | | | | | | | | | | |
| 0 | 0 | 0 | | | | | | | | | | | | | | | | | |
| 0 | 1 | 1 | | | | | | | | | | | | | | | | | |
| 1 | 0 | 1 | | | | | | | | | | | | | | | | | |
| 1 | 1 | 0 | | | | | | | | | | | | | | | | | |
| Exclusive NOR |  |  | $C = \overline{A \oplus B}$ | <table><tr><th>A</th><th>B</th><th>C</th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table> | A | B | C | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 |
| A | B | C | | | | | | | | | | | | | | | | | |
| 0 | 0 | 1 | | | | | | | | | | | | | | | | | |
| 0 | 1 | 0 | | | | | | | | | | | | | | | | | |
| 1 | 0 | 0 | | | | | | | | | | | | | | | | | |
| 1 | 1 | 1 | | | | | | | | | | | | | | | | | |



24.4

Boolean algebra

- **Boolean constants**
 - these are '0' (false) and '1' (true)
- **Boolean variables**
 - variables that can only take the values '0' or '1'
- **Boolean functions**
 - each of the logic functions (such as AND, OR and NOT) are represented by symbols as described above
- **Boolean theorems**
 - a set of **identities** and **laws** – see text **Table 24.2** for details

24.24

Boolean theorems

Boolean identities

AND function

$$0 \cdot 0 = 0$$

$$0 \cdot 1 = 0$$

$$1 \cdot 0 = 0$$

$$1 \cdot 1 = 1$$

$$A \cdot 0 = 0$$

$$0 \cdot A = 0$$

$$A \cdot 1 = A$$

$$1 \cdot A = A$$

$$A \cdot A = A$$

$$A \cdot \bar{A} = 0$$

OR function

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = 1$$

$$A + 0 = A$$

$$0 + A = A$$

$$A + 1 = 1$$

$$1 + A = 1$$

$$A + A = A$$

$$A + \bar{A} = 1$$

NOT function

$$\bar{0} = 1$$

$$\bar{1} = 0$$

$$\bar{\bar{A}} = A$$

Boolean laws

Commutative law

$$AB = BA$$

$$A + B = B + A$$

Absorption law

$$A + AB = A$$

$$A(A + B) = A$$

Distributive law

$$A(B + C) = AB + AC$$

$$A + BC = (A + B)(A + C)$$

De Morgan's law

$$\overline{A + B} = \bar{A} \cdot \bar{B}$$

$$\overline{A \cdot B} = \bar{A} + \bar{B}$$

Associative law

$$A(BC) = (AB)C$$

$$A + (B + C) = (A + B) + C$$

Note also

$$A + \bar{A}B = A + B$$

$$A(\bar{A} + B) = AB$$



Video 24C



24.5

Combinational logic

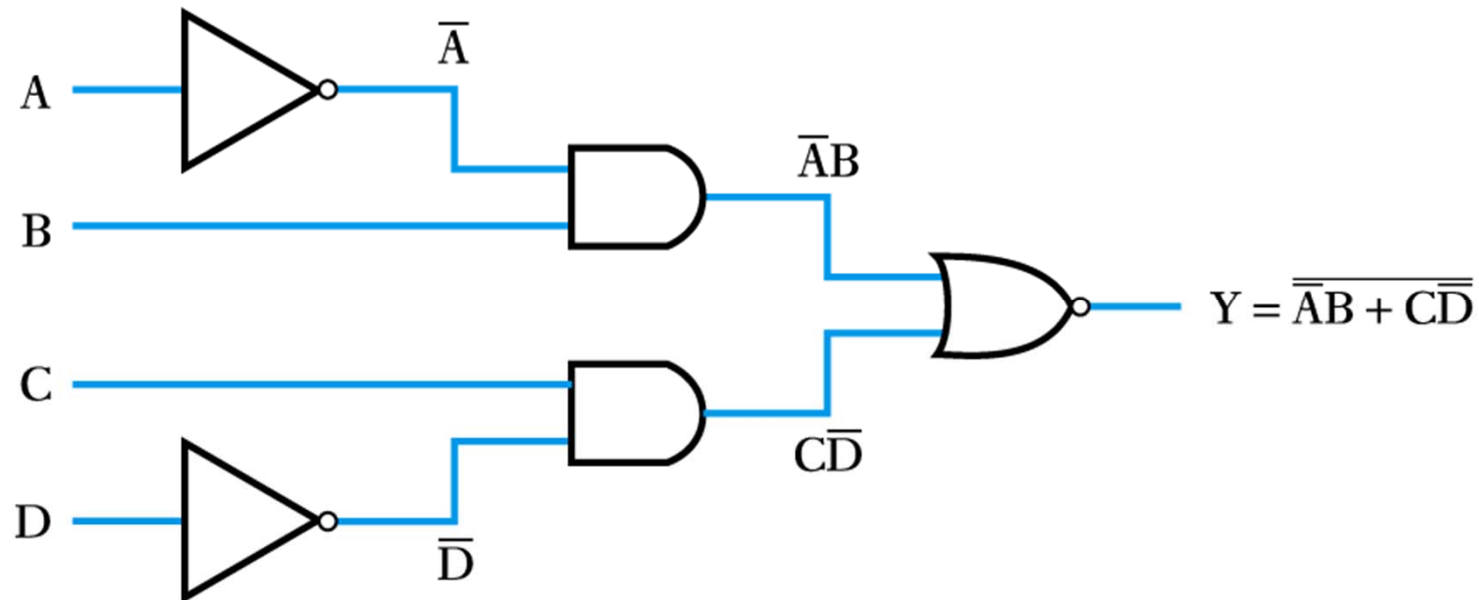
- Digital systems may be divided into two broad categories:
 - **combinational logic**
 - where the outputs are determined solely by the current states of the inputs
 - **sequential logic**
 - where the outputs are determined not only by the current inputs but also by the sequence of inputs that led to the current state
- In this lecture we will look at **combination logic**

24.26

- Implementing a function from a Boolean expression

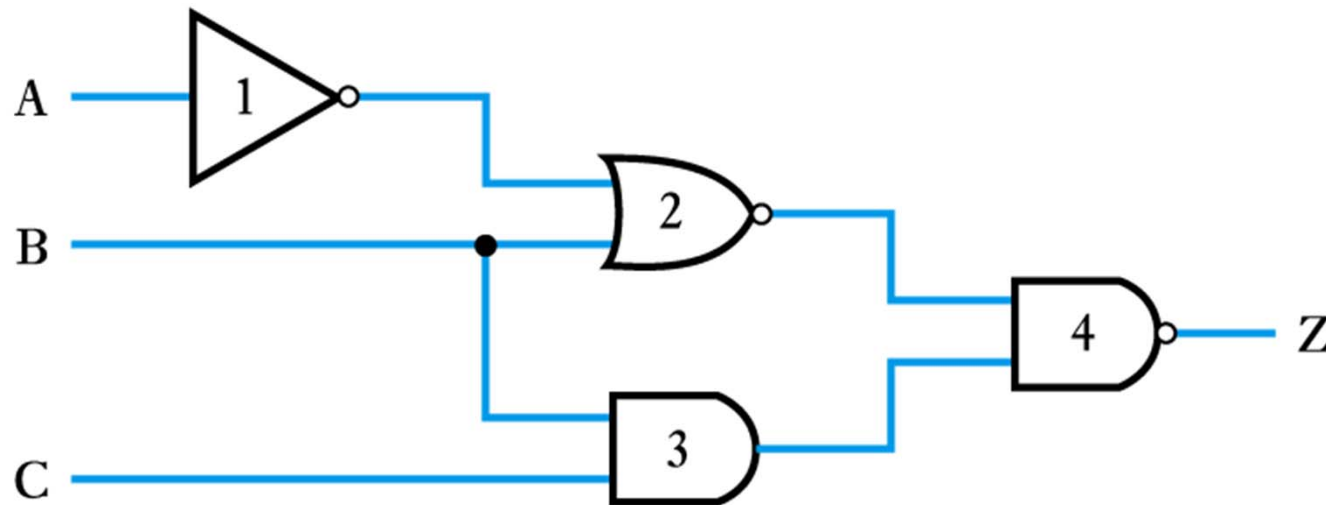
Example – see **Example 24.2** in the course text

Implement the function $Y = \overline{\overline{A}B + C\overline{D}}$



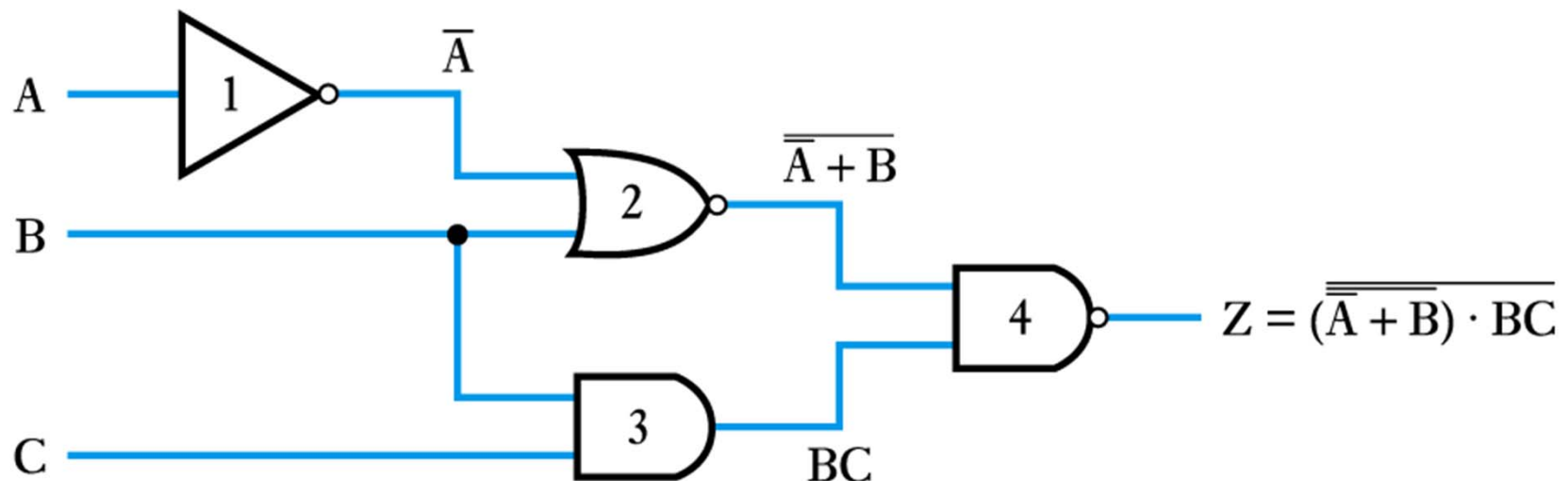
-
- **Generating a Boolean expression from a logic diagram**

Example – see **Example 24.3** in the course text



Example (continued)

- work progressively from the inputs to the output adding logic expressions to the output of each gate in turn



- **Implementing a logic function from a description**

Example – see **Example 24.4** in the course text

The operation of the Exclusive OR gate can be stated as:

“The output should be true if either of its inputs are true, but not if both inputs are true”

This can be rephrased as:

“The output is true if A OR B is true, AND if A AND B are NOT true.”

We can write this in Boolean notation as

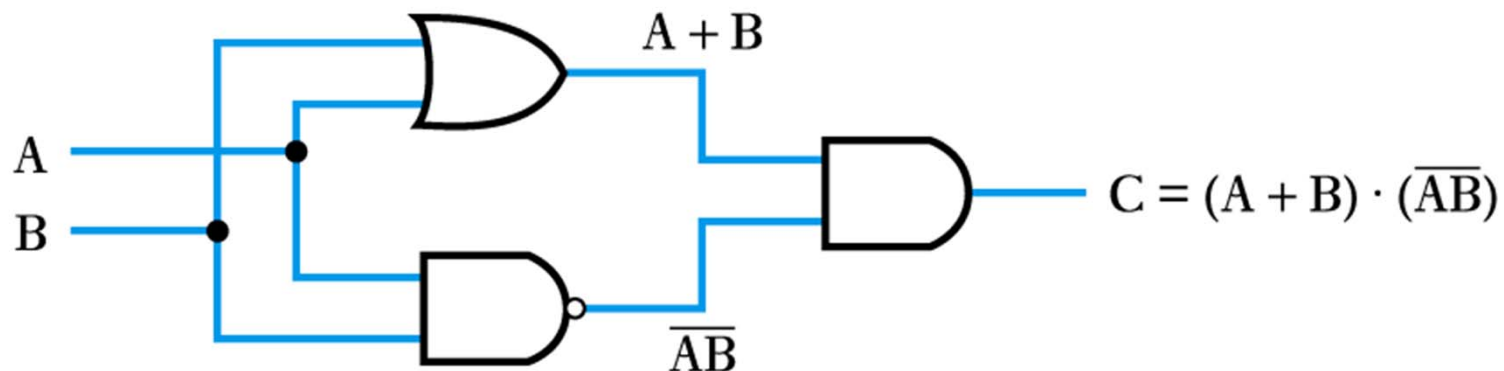
$$X = (A + B) \bullet \overline{(AB)}$$

Example (continued)

The logic function

$$X = (A + B) \cdot \overline{AB}$$

can then be implemented as before



■ Implementing a logic function from a truth table

Example – see **Example 24.6** in the course text

Implement the function of the following truth table

| A | B | C | X |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

– first write down a Boolean expression for the output as the sum of the **minterms**

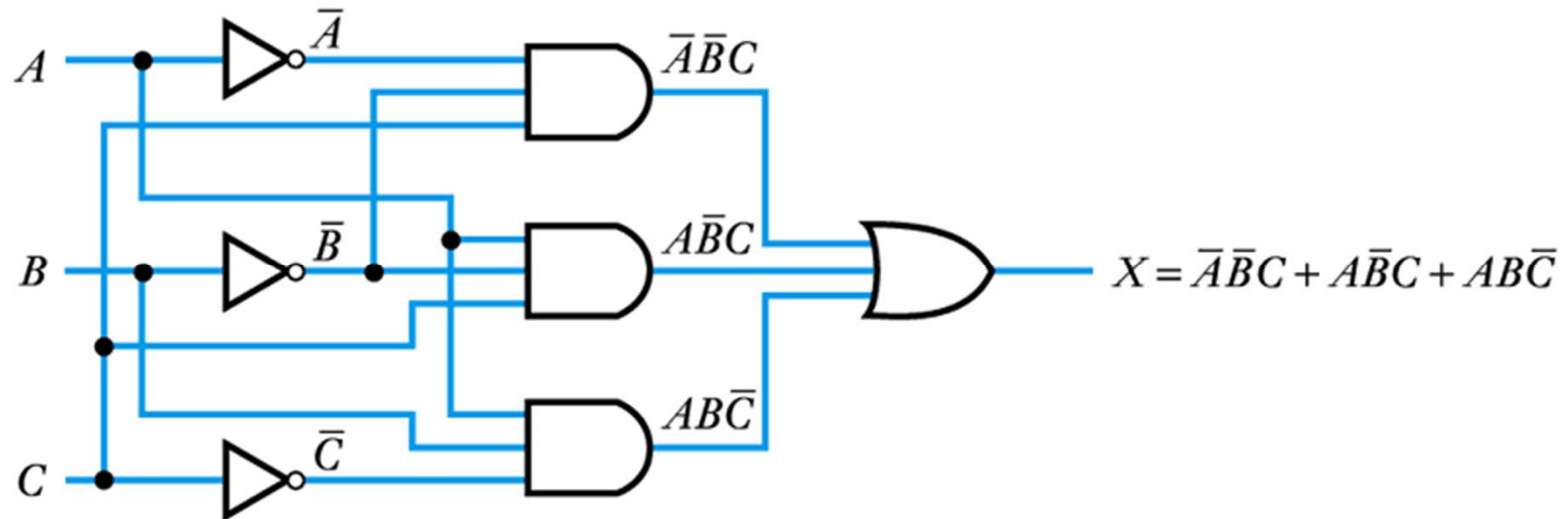
– then implement as before

– in this case

$$X = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C$$

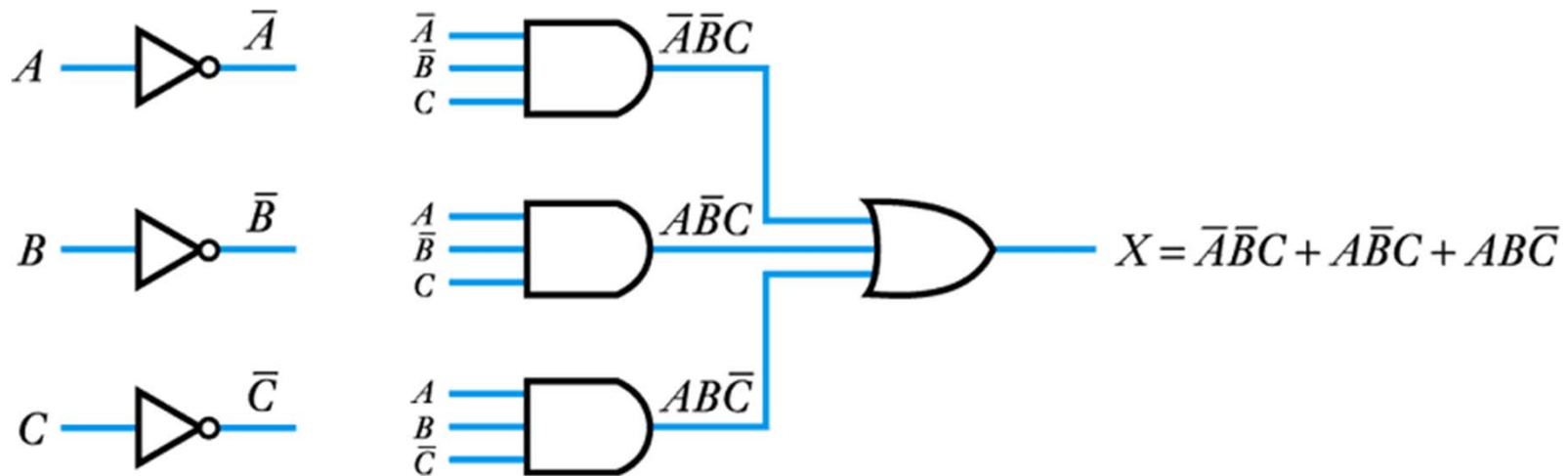

Example (continued)

The logic function $X = \bar{A}\bar{B}C + A\bar{B}C + AB\bar{C}$ can then be implemented as before



Example (continued)

- Complex logic diagrams are often made easier to understand by the use of labels, rather than showing complex interconnections – the earlier circuit becomes





Video 24D



24.7

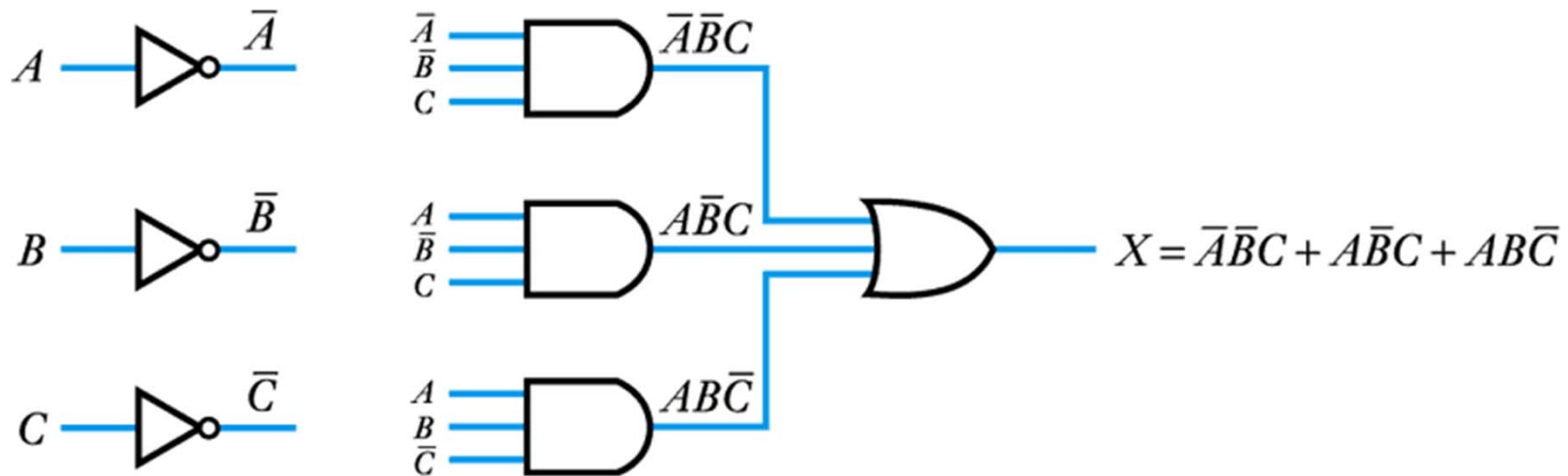
Algebraic simplification

- All combination circuits can be described in a **sum-of-products** form
 - This consists of a number of terms (**minterms** or **products**) that are OR'ed together
 - Examples include
$$\overline{A}B + \overline{A}B$$
$$XYZ + \overline{X}Y\overline{Z} + X\overline{Y}Z$$
$$AB\overline{C}D + \overline{A}\overline{B}C + \overline{A}BCD + ABC\overline{D}$$
 - The sum-of-products must *not* include inversions of a series of terms, as in
$$\overline{ABCD} + \overline{ABCD}$$

24.35

Example (continued)

- Complex logic diagrams are often made easier to understand by the use of labels, rather than showing complex interconnections – the earlier circuit becomes

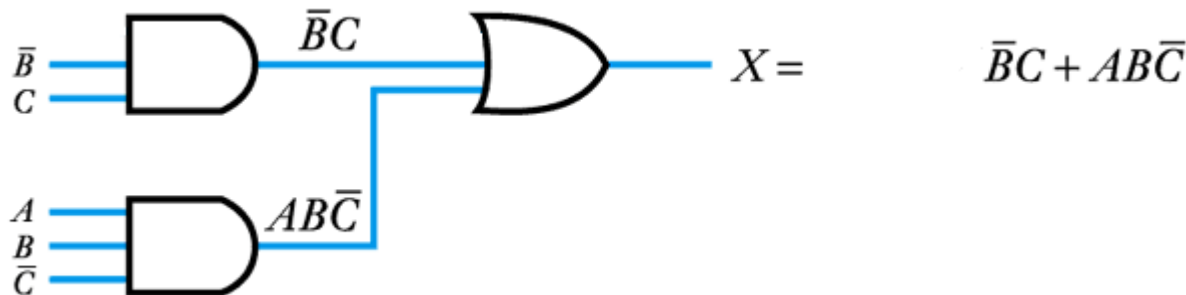


Allows us to simplify the circuit

- We can use Boolean laws to simplify expression and the circuit

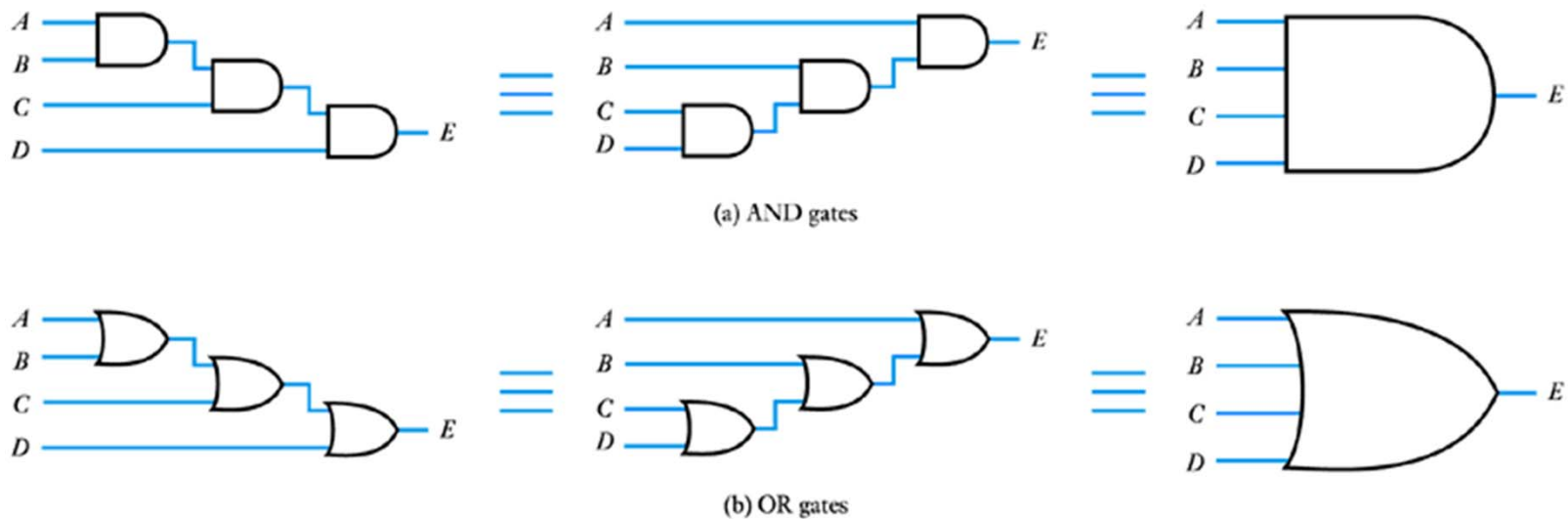
$$\begin{aligned} X &= \bar{A}\bar{B}C + A\bar{B}C + AB\bar{C} \\ &= \bar{B}C(A + \bar{A}) + AB\bar{C} \\ &= \bar{B}C + AB\bar{C} \end{aligned}$$

- So a circuit with identical logical properties is:



Boolean algebraic manipulation

- We can use the various laws of Boolean algebra to change the form of circuits to meet space available
 - the following diagram shows the implications of the associative laws



Next time: Digital systems

- Introduction
- Binary quantities and variables
- Logic gates
- Boolean algebra
- Combinational logic
- Boolean algebraic manipulation
- Algebraic simplification
- Karnaugh maps
- Automated methods of minimisation
- Propagation delay and hazards
- Number systems and binary arithmetic
- Numeric and alphabetic codes
- Examples of combinational logic design



Key points

- Logic circuits are usually implemented using logic gates
- Circuits in which the output is determined solely by the current inputs are termed combinational logic circuits
- Logic functions can be described by truth tables or using Boolean algebraic notation
- Boolean expressions can often be simplified by algebraic manipulation, or using techniques such as Karnaugh maps
- Binary digits may be combined to form digital words that can be processed using binary arithmetic
- Several codes can be used to represent different forms of information