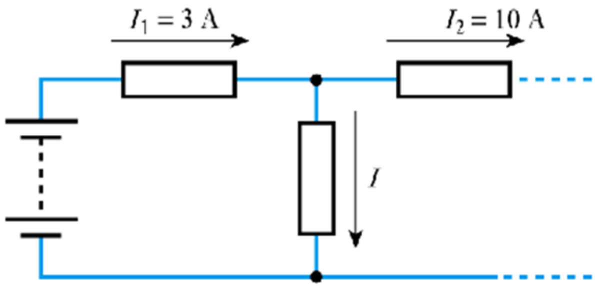
Problem set 1 TFY4185 Måleteknikk Issued 2 September 2015

1. Which of the following is a correct statement of Ohm's law?

a) I = R / V **b) I = V / R** c) V = I / R d) R = VI

Just manipulate V=I·R!

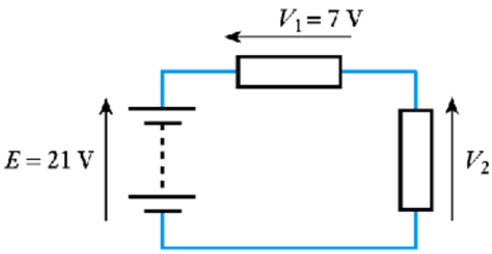
2. Calculate the current I in the following circuit:



a) -7 A b) 13 A c) 7 A d) 13 A

KCL $\Rightarrow$ *3A-10A-I=0* $\therefore$ *I = -7A*

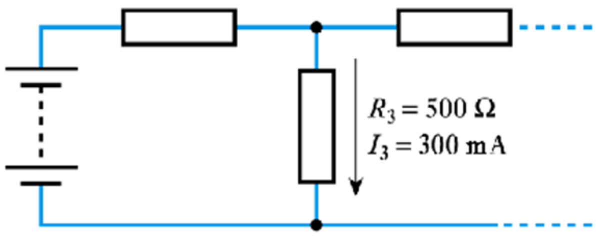
3. Calculate the voltage V_2 in the following circuit:



a) 28 V b) -14 V **c) 14 V** d) -7 V

KVL $\Rightarrow$ *-12V + 7V +V_2 = 0* $\therefore$ *V_2= 14 V*

4. **Calculate the power dissipated in R_3 in the following circuit:**



a) 150 mW   b) 45 mW   **c) 45 W**   d) 150 W

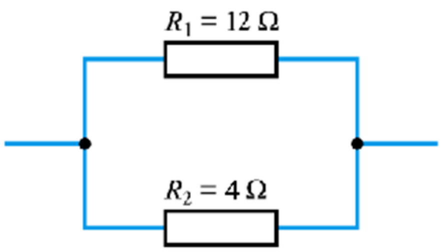
$P = I^2 \cdot R \Rightarrow (300x10^{-3}\ A)^2 \cdot 500\ \Omega = 45\ W$

5. **Calculate the effective resistance of the following combination:**

a) 49 Ω   b) 39 Ω   c) 12 Ω   d) 52 Ω

Series resistance (same current flows through each, but different voltage at each resistor)
$R_{tot} = \Sigma R_i = 10\ \Omega + 12\ \Omega + 5\ \Omega + 22\ \Omega = 49\ \Omega$

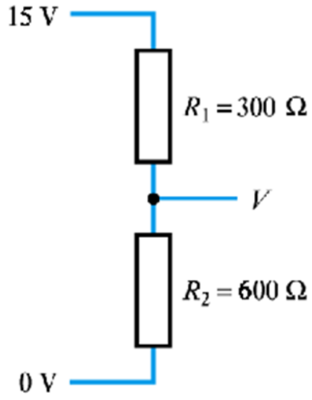
6. **Calculate the effective resistance of the following combination:**



a) 6 Ω   b) 16 Ω   **c) 3 Ω**   d) 8 Ω

Parallel resistance (same voltage across each resistor, but different current can flow through each resistor)
$1/R_{tot} = \Sigma(1/R_i) = 1/(12\ \Omega) + 1/(4\ \Omega) \Rightarrow R_{tot} = 3\ \Omega$

7. **Calculate the output voltage *V* of the following circuit:**



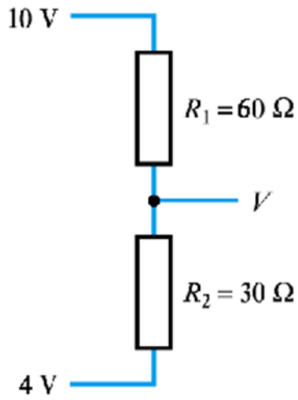
<u>a) 10 V</u> b) 3 V c) 5 V d) 6 V

Voltage Divider: The same current, I, flows through the 2 resistors. $\therefore I = 15V/R_{tot}$. *Since the same current is flowing through all the resistors, they are in series and* $R_{tot} = \Sigma R_i = 300\Omega + 600\Omega = 900\Omega$. *Although it is not necessary to calculate this current, it will be:* $I = 15\ V/\ 900\Omega = 0.0167\ A$.

The voltage drop across resistor 1 is $15V - V = I \cdot R_1$, *and the voltage drop across resistor 2 is* $V - 0V = I \cdot R_2$. *Since we know that* $I = 15V/R_{tot}$, *we see that the second expression yields:* $V = 15V/R_{tot} \cdot R_2$ *immediately as* $V = 15V/900\Omega \cdot 600\Omega = 10V$

Note, we could have used the voltage drop across resistor 1 to solve for V as: $V = 15V - 15V/R_{tot} \cdot R_1 = 10V$. *We just took advantage of the fact that the lower potential end of* R_2 *was at 0V to save us some calculation.*

8. Calculate the output voltage *V* of the following circuit:

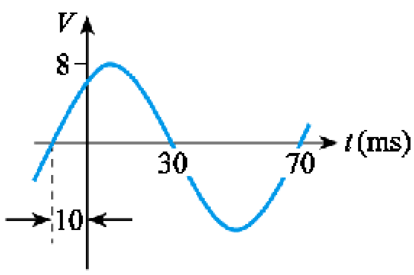


a) 2 V b) 4 V **<u>c) 6 V</u>** d) 8 V

Just the same as above, a Voltage Divider: The same current, I, flows through the 2 resistors. $\therefore$ $I = (10V-4V)/R_{tot}$. Since the same current is flowing through all the resistors, they are in series and $R_{tot} = \Sigma R_i = 30\Omega + 60\Omega = 90\Omega$. Although it is not necessary to calculate this current, it will be: $I = 6\text{ V}/ 90\Omega = 0.067$ A.

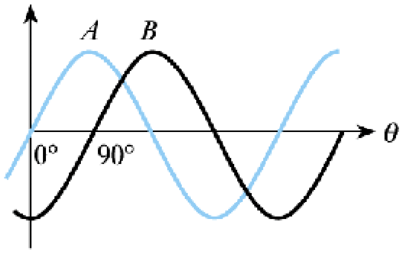
The voltage drop across resistor 1 is $10\text{V} - V = I \cdot R_1$, and the voltage drop across resistor 2 is $V - 4V = I \cdot R_2$. It does not matter which one we choose to calculate V. That is, both give:
(1) $V = 10V - (10V-4V)/R_{tot} \cdot R_1 = 6\text{V}$ (2) $V = (10V-4V)/R_{tot} \cdot R_2 + 4V = 6V$

9. **Which of the following equations describes the waveform shown here?**



a) v = 8 sin (90t - π / 4) b) v = 8 sin (79t - π / 4)
<u>c) v = 8 sin (79t + π / 4)</u> d) v = 8 sin (90t + π / 4)

10. In the following graph, waveform A lags waveform B by 90 degrees. True or false?



*This is **false** since A is starting up from our reference before B.*

11. What is the average value of a sinusoidal voltage that has a peak value of 15 V?

a) 0 V **b) 9.56 V** c) 10.6 V d) 19.1 V

In electrical engineering, the average value of a sinusoidal voltage is defined as: $V_{avg} = \frac{1}{T}\int_0^T \left|V_p \cdot \sin\omega \cdot t\right| dt$. *If we integrate the sine over a half of a cycle, this is the same as integrating the |sine| over many cycles. This gives:* $\frac{1}{\pi}\int_0^\pi \left|V_p \cdot \sin\omega \cdot t\right| dt = \frac{2 \cdot V_p}{\pi} = 0.637 V_p$. *In this case,* $V_{avg} = 0.637 \cdot 15V = 9.56V$.

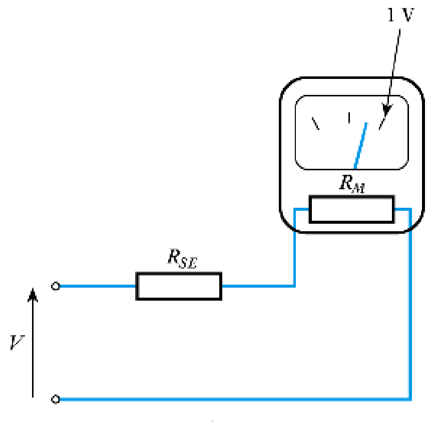
The wisdom of this definition comes from the fact that saying that the average value of a 220V AC signal is 0V does not make grabbing a live 220V line any less lethal!

12. A current of 5 A r.m.s is passed through a resistor of 5 Ω. What power will be dissipated in the resistor?

a) 20 W b) 50 W **c) 100 W** d) 200 W

For AC signals, the power is defined as: $P = (I_{rms})^2 \cdot R = I_{rms} \cdot V_{rms} = (V_{rms})^2/R$. *The first relation gives* $P = (25A_{rms})^2 \cdot 5\Omega = 125W$. *The closest, and therefore the best answer is c).*

13. A moving-coil meter produces a full-scale deflection for a current of 100 µA and has a resistance of 500 Ω. Select a series resistor to turn this device into a voltmeter with an full scale deflection of 1 V.

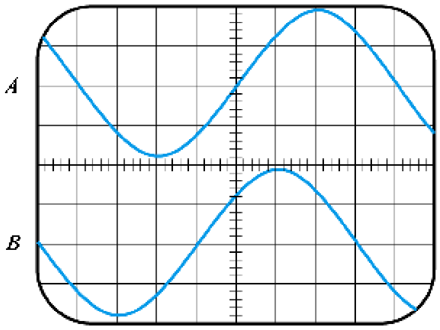


a) 8.5 kΩ b) 9 kΩ **c) 9.5 kΩ** d) 10 kΩ

The full scale deflection current, $I_{fsd} = 100\mu A = 1x10^{-4}A$ with a meter resistance $R_m = 500\Omega$. This FSD occurs when the voltage across the meter is: $V_{fsd} = I_{fsd} \cdot R_m = 0.05$ V. Now we will measure a signal of $V = 1V$ with series resistance R_{SE} so that we still get the same V_{fsd} across the meter. This is yet another voltage divider where we specify the voltage drop across R_m as $V_{fsd} = 0.05V$ and are asked to specify R_{SE} for a given input voltage, $V = 1V$.

The current through the two resistors is the same and $I = V/(R_{SE} + R_m)$. The voltage drop across the series resistor should be: $(V-V_{fsd}) = I \cdot R_{SE}$, and the voltage drop across the meter will be $(V_{fsd}-0V) = I \cdot R_m$. The latter saves a subtraction and we have that $V_{fsd} = [V/(R_{SE} + R_m)] \cdot R_m$. You can either substitute in $V = 1V$, $V_{fsd} = 0.05V$ and $R_m = 500\Omega$, and do simple arithmetic to get $R_{SE} = 9.5$ kΩ, or do simple algebra on the equation to recover the book formula $R_{SE} = V \cdot R_m/V_{fsd} - R_m = V/I_{fsd} - R_m$, and substitute in there.

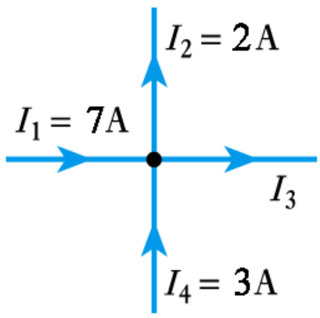
14. What is the relationship between the two waveforms shown here?



a) A leads B by π / 4 b) A lags B by π / 2 **<u>c) A lags B by π / 4</u>** d) A leads B by π / 2

Again, relative to the reference you have. Since all phases are uncertain by 2π, you could say B leads by 3π/4, but the convention is to use the smallest phase shift and adjust who is leading!

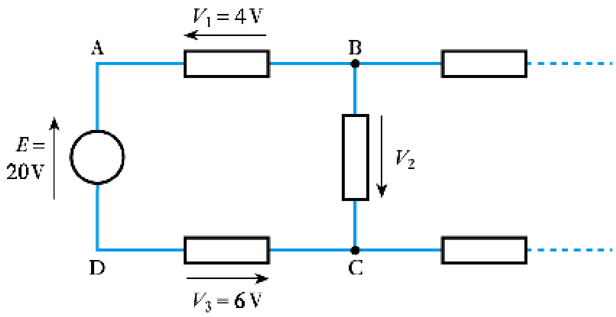
15. Calculate the current I_3 in the following circuit.



a) -8 A b) -2 A c) 2 A **<u>d) 8 A</u>**

KCL $\Rightarrow$ 7A + 3A – 2A – I_3 = 0 $\therefore I_3$ = 8A

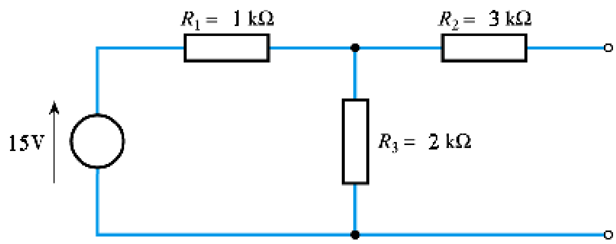
16. **Calculate the voltage V_2 in the following circuit.**



a) -10 V b) 10 V c) 12 V d) 18 V

KVL $\Rightarrow -20V + 4V - V_2 + 6V = 0 \therefore V_2 = -10$ V.
This negative sign means that the arrow should be the other way around in the drawing!

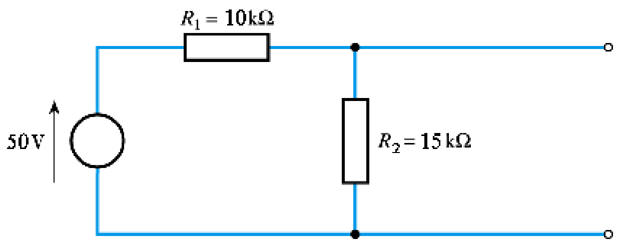
17. Determine the open-circuit output voltage of the following circuit.



a) 5 V b) 6.82 V c) 8.18 V **d) 10 V**

Since no current flows in R_2, there is no voltage drop and the voltage drop across R_3 is the open circuit voltage (i.e. the front side of R_2 is at the same potential as the back side). This is, wait for it, yet another voltage divider! By now you should be able to write down the voltage drop across R_3 as: $V_{oc} = V/(R_1+R_3) \cdot R_3 = 15V/(3k\Omega) \cdot 2k\Omega = 10V$

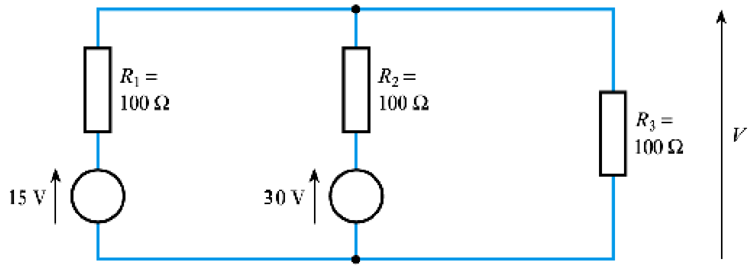
18. **Determine the short-circuit current of the following circuit.**



a) 2 mA b) 3.33 mA **c) 5 mA** d) 6.67 mA

If I short out the terminals, R_2 is in parallel with a wire (which has 0 Ω resistance). The equivalent resistance of this would be $R_{tot} = (1/R_2 + 1/0\Omega)^{-1} = 1/\infty = 0\Omega$. So this effectively replaces R_2 by a wire, and the current through R_1 is $I = V/R_1 = 50V/(1x10^4\Omega) = 5x10^{-3}$ A.

19. Use the principle of superposition to determine the output voltage V of the following circuit (you should be able to do this using mental arithmetic).



a) 5 V b) 7.5 V c) 12 V **d) 15 V**

Mental arithmetic aside, to do superposition, we replace all voltage sources (except the one for which you are calculating) by wires, and replace all current sources (except the one for which you are calculating) by open circuits. Here, let us first short out the 30V power supply with a wire. Then going from the 15V power supply we have R_1 in series with the parallel pair of R_2 and R_3. Or R_1 in series with R_{23}, where $R_{23} = (1/R_2 + 1/R_3)^{-1}$. The total resistance is therefore $R_{tot} = R_1 + R_{23} = R_1 + R_2 \cdot R_3/(R_2+R_3)$, and the mental arithmetic is that for two resistors of the same value in parallel, the effective parallel resistance is half the resistance. Thus, $R_{tot} = 100\Omega + ½ \cdot 100\Omega = 150\Omega$. The total current is therefore $I = 15V/150\Omega = 0.1A$.

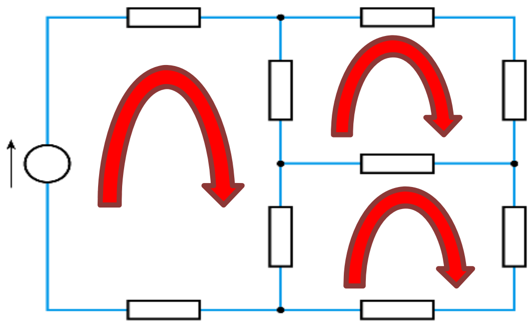
This current is split between R_2 and R_3, but in this special case, where they have the same value of resistance, the current is split equally between the R_2 and R_3 branches. Hence, the current through R_3, $I_3 = ½ \cdot 0.1A = 0.05A$, and the voltage across R_3 is:
$V_3 = I_3 \cdot R_3 = 0.05A \cdot 100\Omega = 5V$.

Now we put the 30V power supply back in the circuit, and calculate the voltage across R_3

when we short out the 15V power supply. Now we see we go from the 30V power supply across R_2 in series with the parallel pair R_1 and R_3. Again, since $R_1 = R_2 = R_3 = 100\Omega$, this is going to be the same total resistance as we did before ($R_{tot} = 150\Omega$), the current will be twice as much (since the power supply has twice the voltage), so $I_{tot} = 0.2A$, and this will be split equally across the R_1 and R_3 branches, giving $I_3 = 0.1A$. Therefore, this power supply puts a voltage across R_3 of: $V_3 = I_3 \cdot R_3 = 0.1A \cdot 100\Omega = 10V$. The total voltage across R_3 is therefore the 5V from the 15V power supply, plus the 10V from the 30V power supply, or 15V.

The mental arithmetic part is that 2 resistors of equal value in parallel gives you a net resistance of half the resistor value, and the current will split equally between the two resistor paths. The next realization is that shorting out either the 15V supply or the 30V supply gives the same resistor network with the same values, but 2x the voltage will give 2x the current, and thus 2x the voltage drop across R_3.

20. How many meshes are present within the following circuit?



a) 1 b) 2 **<u>c) 3</u>** d) 4

Any other loop we draw will contain at least one of these loops, and hence would not be a mesh.