Department of Physics, NTNU

Homework 1
TFY4210/FY8916 Quantum field theory of many-particle systems
Spring 2020.

Problem 1

In class, we have seen how to formulate a second-quantized version of the Hamiltonian of a non-relativistic many-particle system in a general basis,

$$H = \sum_{\lambda_1,\lambda_2} \varepsilon_{\lambda 1,\lambda_2}\ c^\dagger_{\lambda_1} c_{\lambda_2} + \sum_{\lambda_1,\lambda_2\lambda_3,\lambda_4} v_{\lambda_1,\lambda_2\lambda_3,\lambda_4}\ c^\dagger_{\lambda_1}\ c^\dagger_{\lambda_2} c_{\lambda_3}\ c_{\lambda_4}$$

where $\varepsilon_{\lambda_1,\lambda_2}$ and $v_{\lambda_1,\lambda_2\lambda_3,\lambda_4}$ are matrix elements of the one-particle and two-particle contributions to the Hamiltonian, computed using a complete set of basis functions $\{\phi_\lambda(x)\}$.

a) Write down an expression for the Hamitonian in a general basis for the case where the system is materially open and coupled to particle reservoir. (In the above version, it is materially closed).

b) Give an explicit form of this Hamiltonian using a Bloch-basis for $\{\phi_\lambda(x)\}$

$$\phi_\lambda(x) = \phi_{\mathbf{k},\sigma}(\mathbf{r}, s) = \frac{1}{\sqrt{V}}\ e^{i\mathbf{k}\cdot\mathbf{r}}\ u_{\mathbf{k}}(\mathbf{r})\ \chi_\sigma(s)$$

in the notation used in class. (Hint: The Bloch functions may be taken to be eigenfunctions of the one-particle problem).

Problem 2

Show by an explicit calculation that the following relations hold for the spin-part of the wavefunction $\chi_\sigma(s)$ of $S = 1/2$ particles:

$$\begin{aligned} \sum_\sigma \chi^*_\sigma(s)\chi_\sigma(s') &= \delta_{s,s'} \\ \sum_s \chi^*_\sigma(s)\chi_{\sigma'}(s) &= \delta_{\sigma,\sigma'} \end{aligned}$$

Problem 3

Consider the following two-body potential that could enter into the two-particle contribution to a many-body Hamiltonian

$$V(\mathbf{r}) = V(r) = \frac{A}{r}\ \exp(-r/\lambda_{TF})$$

Compute the Fourier-transform of this potential. Give an interpretation of the quantity λ_{TF}. Here A is a dimensionful constant that we need not specify further.