Strengthening by Grain Size Reduction

7.20 Small-angle grain boundaries are not as effective in interfering with the slip process as are high-angle grain boundaries because there is not as much crystallographic misalignment in the grain boundary region for small-angle, and therefore not as much change in slip direction.

7.21 Hexagonal close packed metals are typically more brittle than FCC and BCC metals because there are fewer slip systems in HCP.

7.22 These three strengthening mechanisms are described in Sections 7.8, 7.9, and 7.10.

7.35 During cold-working, the grain structure of the metal has been distorted to accommodate the deformation. Recrystallization produces grains that are equiaxed and smaller than the parent grains.

7.36 (a) The driving force for recrystallization is the difference in internal energy between the strained and unstrained material.

(b) The driving force for grain growth is the reduction in grain boundary energy as the total grain boundary area decreases.

7.41 This problem calls for us to calculate the yield strength of a brass specimen after it has been heated to an elevated temperature at which grain growth was allowed to occur; the yield strength (150 MPa) was given at a grain size of 0.01 mm. It is first necessary to calculate the constant k_y in Equation 7.7 as

$$k_y = \frac{\sigma_y - \sigma_0}{d^{-1/2}}$$

$$= \frac{150\ \text{MPa} - 25\ \text{MPa}}{(0.01\ \text{mm})^{-1/2}} = 12.5\ \text{MPa} - \text{mm}^{1/2}$$

Next, we must determine the average grain size after the heat treatment. From Figure 7.25 at 500°C after 1000 s (16.7 min) the average grain size of a brass material is about 0.016 mm. Therefore, calculating σ_y at this new grain size using Equation 7.7 we get

$$\sigma_y = \sigma_0 + k_y d^{-1/2}$$

$$= 25\ \text{MPa} + \left(12.5\ \text{MPa - mm}^{1/2}\right)(0.016\ \text{mm})^{-1/2} = 124\ \text{MPa}\ \ (18{,}000\ \text{psi})$$

7.D4 This problem asks us to determine which of copper, brass, and a 1040 steel may be cold-worked so as to achieve a minimum yield strength of 310 MPa (45,000 psi) while maintaining a minimum ductility of 27%EL. For each of these alloys, the minimum cold work necessary to achieve the yield strength may be determined from Figure 7.19(a), while the maximum possible cold work for the ductility is found in Figure 7.19(c). These data are tabulated below.

	Yield Strength (> 310 MPa)	Ductility (> 27%EL)
Steel	Any %CW	Not possible
Brass	> 15%CW	< 18%CW
Copper	> 38%CW	< 10%CW

Thus, only brass is a possible candidate since for this alloy only there is an overlap of %CW's to give the required minimum yield strength and ductility values.

8.7 This problem asks us to determine the stress level at which an a wing component on an aircraft will fracture for a given fracture toughness $\left(26\ \text{MPa}\sqrt{\text{m}}\right)$ and maximum internal crack length (6.0 mm), given that fracture occurs for the same component using the same alloy at one stress level (112 MPa) and another internal crack length (8.6 mm). It first becomes necessary to solve for the parameter Y for the conditions under which fracture occurred using Equation 8.5. Therefore,

$$Y = \frac{K_{Ic}}{\sigma\sqrt{\pi a}} = \frac{26\ \text{MPa}\sqrt{\text{m}}}{(112\ \text{MPa})\sqrt{(\pi)\left(\dfrac{8.6\ \text{x}\ 10^{-3}\ \text{m}}{2}\right)}} = 2.0$$

Now we will solve for σ_c using Equation 8.6 as

$$\sigma_c = \frac{K_{Ic}}{Y\sqrt{\pi a}} = \frac{26\ \text{MPa}\sqrt{\text{m}}}{(2.0)\sqrt{(\pi)\left(\dfrac{6\ \text{x}\ 10^{-3}\ \text{m}}{2}\right)}} = 134\ \text{MPa}\quad (19{,}300\ \text{psi})$$

8.8 For this problem, we are given values of K_{Ic} $\left(82.4\ \text{MPa}\sqrt{\text{m}}\right)$, σ (345 MPa), and Y (1.0) for a large plate and are asked to determine the minimum length of a surface crack that will lead to fracture. All we need do is to solve for a_c using Equation 8.7; therefore

$$a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{Y\sigma}\right)^2 = \frac{1}{\pi}\left[\frac{82.4\ \text{MPa}\sqrt{\text{m}}}{(1.0)(345\ \text{MPa})}\right]^2 = 0.0182\ \text{m} = 18.2\ \text{mm} \quad (0.72\ \text{in.})$$

Cyclic Stresses (Fatigue)

The *S*-*N* Curve

8.14 (a) Given the values of σ_m (70 MPa) and σ_a (210 MPa) we are asked to compute σ_{max} and σ_{min}. From Equation 8.14

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = 70 \text{ MPa}$$

Or,

$$\sigma_{max} + \sigma_{min} = 140 \text{ MPa}$$

Furthermore, utilization of Equation 8.16 yields

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = 210 \text{ MPa}$$

Or,

$$\sigma_{max} - \sigma_{min} = 420 \text{ MPa}$$

Simultaneously solving these two expressions leads to

$$\sigma_{max} = 280 \text{ MPa} \quad (40,000 \text{ psi})$$
$$\sigma_{min} = -140 \text{ MPa} \quad (-20,000 \text{ psi})$$

(b) Using Equation 8.17 the stress ratio *R* is determined as follows:

$$R = \frac{\sigma_{min}}{\sigma_{max}} = \frac{-140 \text{ MPa}}{280 \text{ MPa}} = -0.50$$

(c) The magnitude of the stress range σ_r is determined using Equation 8.15 as

$$\sigma_r = \sigma_{max} - \sigma_{min} = 280 \text{ MPa} - (-140 \text{ MPa}) = 420 \text{ MPa} \quad (60,000 \text{ psi})$$

8.15 This problem asks that we determine the minimum allowable bar diameter to ensure that fatigue failure will not occur for a 1045 steel that is subjected to cyclic loading for a load amplitude of 66,700 N (15,000 $\mathrm{lb_f}$). From Figure 8.34, the fatigue limit stress amplitude for this alloy is 310 MPa (45,000 psi). Stress is defined in Equation 6.1 as $\sigma = \frac{F}{A_0}$. For a cylindrical bar

$$A_0 = \pi\left(\frac{d_0}{2}\right)^2$$

Substitution for A_0 into the Equation 6.1 leads to

$$\sigma = \frac{F}{A_0} = \frac{F}{\pi\left(\frac{d_0}{2}\right)^2} = \frac{4F}{\pi d_0^2}$$

We now solve for d_0, taking stress as the fatigue limit divided by the factor of safety. Thus

$$d_0 = \sqrt{\frac{4F}{\pi\left(\frac{\sigma}{N}\right)}}$$

$$= \sqrt{\frac{(4)(66{,}700 \text{ N})}{(\pi)\left(\frac{310 \times 10^6 \text{ N/m}^2}{2}\right)}} = 23.4 \times 10^{-3} \text{ m} = 23.4 \text{ mm} \quad (0.92 \text{ in.})$$

8.16 We are asked to determine the fatigue life for a cylindrical 2014-T6 aluminum rod given its diameter (6.4 mm) and the maximum tensile and compressive loads (+5340 N and –5340 N, respectively). The first thing that is necessary is to calculate values of σ_{max} and σ_{min} using Equation 6.1. Thus

$$\sigma_{max} = \frac{F_{max}}{A_0} = \frac{F_{max}}{\pi\left(\frac{d_0}{2}\right)^2}$$

$$= \frac{5340 \text{ N}}{(\pi)\left(\frac{6.4 \text{ x } 10^{-3} \text{ m}}{2}\right)^2} = 166 \text{ x } 10^6 \text{ N/m}^2 = 166 \text{ MPa} \quad (24,400 \text{ psi})$$

$$\sigma_{min} = \frac{F_{min}}{\pi\left(\frac{d_0}{2}\right)^2}$$

$$= \frac{-5340 \text{ N}}{(\pi)\left(\frac{6.4 \text{ x } 10^{-3} \text{ m}}{2}\right)^2} = -166 \text{ x } 10^6 \text{ N/m}^2 = -166 \text{ MPa} \quad (-24,400 \text{ psi})$$

Now it becomes necessary to compute the stress amplitude using Equation 8.16 as

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{166 \text{ MPa} - (-166 \text{ MPa})}{2} = 166 \text{ MPa} \quad (24,400 \text{ psi})$$

From Figure 8.34, for the 2014-T6 aluminum, the number of cycles to failure at this stress amplitude is about 1 x 10^7 cycles.

Crack Initiation and Propagation

Factors That Affect Fatigue Life

8.24 (a) With regard to size, beachmarks are normally of macroscopic dimensions and may be observed with the naked eye; fatigue striations are of microscopic size and it is necessary to observe them using electron microscopy.

(b) With regard to origin, beachmarks result from interruptions in the stress cycles; each fatigue striation is corresponds to the advance of a fatigue crack during a single load cycle.