Solution 1

$$P(x|\theta)=\prod_{i=1}^{d}\theta_i^{x_i}(1-\theta)^{1-x_i}$$

To find the maximum likelyhood estimate for θ we need to take partial derivative of the distribution for θ and set it to zero. But first we will take log of the distribution because it is easier to work with and max likellyhood would not be effected by it.

$$\log(P(x|\theta))=\ell(\theta)=\log\theta\sum_{i=1}^{d}x_i+\log(1-\theta)\sum_{i=1}^{d}(1-\theta)$$

$$\frac{\partial\ell(\theta)}{\partial\theta}=\frac{\sum_{i=1}^{d}x_i}{\theta}-\frac{\sum_{i=1}^{d}(1-x_i)}{1-\theta}\qquad\frac{\sum_{i=1}^{d}x_i}{\theta}-\frac{\sum_{i=1}^{d}(1-x_i)}{1-\theta}=0$$

=> $\frac{\sum_{i=1}^{d}x_i}{\theta}=\frac{d-\sum_{i=1}^{d}x_i}{1-\theta}$ => $\sum_{i=1}^{d}x_i-\sum_{i=1}^{d}\theta x_i=\theta d-\sum_{i=1}^{d}\theta x_i$ => $\theta=\frac{1}{d}\sum_{i=1}^{d}x_i$

Solution 2

Computer Exercise

a) Univariate Normal Distribution is

$$P(X|\mu,\sigma^2)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp(-\frac{(X-\mu)^2}{2\sigma^2})$$

Log Likelyhood of univariate normal distribution

$$\log(P(X|\mu,\sigma^2))=\frac{-N}{2}\log(2\pi\sigma^2)-\frac{1}{2\sigma^2}\sum_{n=1}^{N}(x_n-\mu)^2$$

Partial derivation with respect to μ

$$\frac{\partial}{\partial\mu}(\frac{-N}{2}\log(2\pi\sigma^2)-\frac{1}{2\sigma^2}\sum_{n=1}^{N}(x_n-\mu)^2)=\frac{\partial}{\partial\mu}(\frac{-N}{2}\log(2\pi\sigma^2))+\frac{\partial}{\partial\mu}(\frac{-1}{2\sigma^2}\sum_{n=1}^{N}(x_n-\mu)^2)$$

$$=0+\frac{\partial}{\partial\mu}(\frac{-1}{2\sigma^2}\sum_{n=1}^{N}(x_n-\mu)^2)$$

$$=\frac{\partial}{\partial\mu}\left(\frac{-1}{2\sigma^2}\sum_{n=1}^{N}(x_n-\mu)^2\right)=\sum_{n=1}^{N}\frac{\partial}{\partial\mu}\left(\frac{-1}{2\sigma^2}(x_n-\mu)^2\right)=\sum_{n=1}^{N}\left(0+\left(-\frac{1}{2\sigma^2}\right)\frac{\partial}{\partial\mu}(x_n-\mu)^2\right)$$

$$=-\frac{1}{2\sigma^2}\sum_{n=1}^{N}\left(\frac{\partial}{\partial\mu}(x_n-\mu)^2\right)=-\frac{1}{2\sigma^2}\sum_{n=1}^{N}2(x_n-\mu).-1=\frac{1}{\sigma^2}\sum_{n=1}^{N}(x_n-\mu)$$

Now we can find the MLE of μ by setting this derivation to zero

$$0=\frac{1}{\sigma^2}\sum_{n=1}^{N}(x_n-\mu)=\sum_{n=1}^{N}(x_n-\mu)=\sum_{n=1}^{N}x_n-\sum_{n=1}^{N}\mu=\sum_{n=1}^{N}x_n-N\mu$$

$\mu=\frac{1}{N}\sum_{n=1}^{N}x_n$ Maximum likelyhood estimator for μ

b)