HW 4

Portfolio

2/26/2021

pacman::p\_load(pacman, tseries, tidyverse, knitr)  
knitr::opts\_chunk$set(message = FALSE)

#### Question 4.2

mu1 = 0.05  
mu2 = 0.02  
sigma1 = 0.1  
sigma2 = 0.05  
  
   
  
rho\_12 = 0.4  
  
wt = seq(from=0, to=1, by=0.01)  
  
N = length(wt) # number of weight values   
var\_p = rep(NA,N) # holding vector  
mu\_p = rep(NA,N)  
  
# portfolio mean  
for (i in 1:N){  
 mu\_p[i] = wt[i]\*mu1 + (1-wt[i])\*mu2 }  
  
# portfolio varience  
for (i in 1:N){  
 var\_p[i] = wt[i]^2\*sigma1^2 + (1-wt[i])^2\*sigma2^2 + 2\*wt[i]\*(1-wt[i])\*rho\_12\*sigma1\*sigma2 }  
  
mu\_p

## [1] 0.0200 0.0203 0.0206 0.0209 0.0212 0.0215 0.0218 0.0221 0.0224 0.0227  
## [11] 0.0230 0.0233 0.0236 0.0239 0.0242 0.0245 0.0248 0.0251 0.0254 0.0257  
## [21] 0.0260 0.0263 0.0266 0.0269 0.0272 0.0275 0.0278 0.0281 0.0284 0.0287  
## [31] 0.0290 0.0293 0.0296 0.0299 0.0302 0.0305 0.0308 0.0311 0.0314 0.0317  
## [41] 0.0320 0.0323 0.0326 0.0329 0.0332 0.0335 0.0338 0.0341 0.0344 0.0347  
## [51] 0.0350 0.0353 0.0356 0.0359 0.0362 0.0365 0.0368 0.0371 0.0374 0.0377  
## [61] 0.0380 0.0383 0.0386 0.0389 0.0392 0.0395 0.0398 0.0401 0.0404 0.0407  
## [71] 0.0410 0.0413 0.0416 0.0419 0.0422 0.0425 0.0428 0.0431 0.0434 0.0437  
## [81] 0.0440 0.0443 0.0446 0.0449 0.0452 0.0455 0.0458 0.0461 0.0464 0.0467  
## [91] 0.0470 0.0473 0.0476 0.0479 0.0482 0.0485 0.0488 0.0491 0.0494 0.0497  
## [101] 0.0500

var\_p

## [1] 0.00250000 0.00249085 0.00248340 0.00247765 0.00247360 0.00247125  
## [7] 0.00247060 0.00247165 0.00247440 0.00247885 0.00248500 0.00249285  
## [13] 0.00250240 0.00251365 0.00252660 0.00254125 0.00255760 0.00257565  
## [19] 0.00259540 0.00261685 0.00264000 0.00266485 0.00269140 0.00271965  
## [25] 0.00274960 0.00278125 0.00281460 0.00284965 0.00288640 0.00292485  
## [31] 0.00296500 0.00300685 0.00305040 0.00309565 0.00314260 0.00319125  
## [37] 0.00324160 0.00329365 0.00334740 0.00340285 0.00346000 0.00351885  
## [43] 0.00357940 0.00364165 0.00370560 0.00377125 0.00383860 0.00390765  
## [49] 0.00397840 0.00405085 0.00412500 0.00420085 0.00427840 0.00435765  
## [55] 0.00443860 0.00452125 0.00460560 0.00469165 0.00477940 0.00486885  
## [61] 0.00496000 0.00505285 0.00514740 0.00524365 0.00534160 0.00544125  
## [67] 0.00554260 0.00564565 0.00575040 0.00585685 0.00596500 0.00607485  
## [73] 0.00618640 0.00629965 0.00641460 0.00653125 0.00664960 0.00676965  
## [79] 0.00689140 0.00701485 0.00714000 0.00726685 0.00739540 0.00752565  
## [85] 0.00765760 0.00779125 0.00792660 0.00806365 0.00820240 0.00834285  
## [91] 0.00848500 0.00862885 0.00877440 0.00892165 0.00907060 0.00922125  
## [97] 0.00937360 0.00952765 0.00968340 0.00984085 0.01000000

tibble(mean=mu\_p, variance=var\_p ) %>%   
 round(digits=5)

## # A tibble: 101 x 2  
## mean variance  
## <dbl> <dbl>  
## 1 0.02 0.0025   
## 2 0.0203 0.00249  
## 3 0.0206 0.00248  
## 4 0.0209 0.00248  
## 5 0.0212 0.00247  
## 6 0.0215 0.00247  
## 7 0.0218 0.00247  
## 8 0.0221 0.00247  
## 9 0.0224 0.00247  
## 10 0.0227 0.00248  
## # ... with 91 more rows

#### Question 4.3

sigma1 = 0.1  
sigma2 = 0.05  
  
   
  
  
rho\_12 = 0.4  
  
# weights  
wt1 = seq(-1,1,length.out = 21)  
N = length(wt1)  
  
# holding vectors  
mu\_p = rep(NA,N)   
var\_p = rep(NA,N)  
  
# Compute mean and variance  
for (i in 1:N){  
 # mean  
 mu\_p[i] = wt1[i]\*mu1 + (1-wt1[i])\*mu2  
 # variance  
 var\_p[i] = wt1[i]^2\*sigma1^2 + (1-wt1[i])^2\*sigma2^2 + 2\*wt1[i]\*(1-wt1[i])\*rho\_12\*sigma1\*sigma2}   
  
var\_p

## [1] 0.012000 0.010285 0.008740 0.007365 0.006160 0.005125 0.004260 0.003565  
## [9] 0.003040 0.002685 0.002500 0.002485 0.002640 0.002965 0.003460 0.004125  
## [17] 0.004960 0.005965 0.007140 0.008485 0.010000

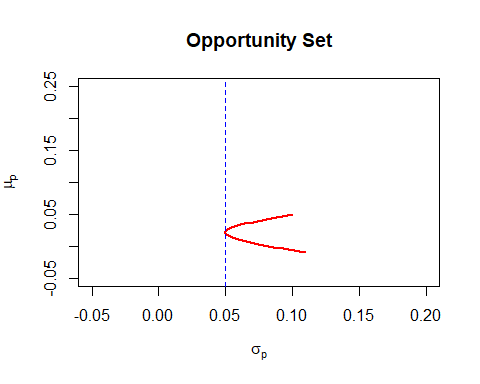
sigma = sqrt(var\_p)  
sigma

## [1] 0.10954451 0.10141499 0.09348797 0.08581958 0.07848567 0.07158911  
## [7] 0.06526868 0.05970762 0.05513620 0.05181699 0.05000000 0.04984977  
## [13] 0.05138093 0.05445181 0.05882176 0.06422616 0.07042727 0.07723341  
## [19] 0.08449852 0.09211406 0.10000000

tibble(mu\_p,var\_p,sigma,w1=wt1, w2=(1-wt1))

## # A tibble: 21 x 5  
## mu\_p var\_p sigma w1 w2  
## <dbl> <dbl> <dbl> <dbl> <dbl>  
## 1 -0.01 0.012 0.110 -1 2   
## 2 -0.007 0.0103 0.101 -0.9 1.9   
## 3 -0.004 0.00874 0.0935 -0.8 1.8   
## 4 -0.00100 0.00736 0.0858 -0.7 1.7   
## 5 0.002 0.00616 0.0785 -0.6 1.6   
## 6 0.00500 0.00513 0.0716 -0.5 1.5   
## 7 0.008 0.00426 0.0653 -0.400 1.4   
## 8 0.0110 0.00356 0.0597 -0.300 1.30  
## 9 0.014 0.00304 0.0551 -0.200 1.2   
## 10 0.017 0.00269 0.0518 -0.100 1.1   
## # ... with 11 more rows

plot(sigma,mu\_p, type="l",lwd=2,  
 col=2, ylim = c(-0.05,0.25),  
 xlim = c(-0.05,0.2),  
 xlab = expression(sigma[p]),   
 ylab = expression(mu[p]),  
 main = "Opportunity Set")  
abline(v=min(sigma),lty=2,col=4)



#### Question 4.4

# Synthetic data  
mu1 = 0.02  
mu2 = 0.04  
sigma1 = 0.05  
sigma2 = 0.06  
  
  
  
wt1 = wt2 = 0.5  
rho = seq(from=-1, to=1, length.out = 100)  
  
mu\_p = (wt1\*mu1 + (1-wt1)\*mu2)  
mu\_p

## [1] 0.03

# Derived values  
N = length(rho) # number of coorelation values   
var = rep(NA,N) # holding vector  
  
# portfolio variance   
for (i in 1:N){  
 var[i] = wt1^2\*sigma1^2 + (1-wt1)^2\*sigma2^2 + 2\*wt1\*(1-wt1)\*rho[i]\*sigma1\*sigma2 }  
var

## [1] 2.500000e-05 5.530303e-05 8.560606e-05 1.159091e-04 1.462121e-04  
## [6] 1.765152e-04 2.068182e-04 2.371212e-04 2.674242e-04 2.977273e-04  
## [11] 3.280303e-04 3.583333e-04 3.886364e-04 4.189394e-04 4.492424e-04  
## [16] 4.795455e-04 5.098485e-04 5.401515e-04 5.704545e-04 6.007576e-04  
## [21] 6.310606e-04 6.613636e-04 6.916667e-04 7.219697e-04 7.522727e-04  
## [26] 7.825758e-04 8.128788e-04 8.431818e-04 8.734848e-04 9.037879e-04  
## [31] 9.340909e-04 9.643939e-04 9.946970e-04 1.025000e-03 1.055303e-03  
## [36] 1.085606e-03 1.115909e-03 1.146212e-03 1.176515e-03 1.206818e-03  
## [41] 1.237121e-03 1.267424e-03 1.297727e-03 1.328030e-03 1.358333e-03  
## [46] 1.388636e-03 1.418939e-03 1.449242e-03 1.479545e-03 1.509848e-03  
## [51] 1.540152e-03 1.570455e-03 1.600758e-03 1.631061e-03 1.661364e-03  
## [56] 1.691667e-03 1.721970e-03 1.752273e-03 1.782576e-03 1.812879e-03  
## [61] 1.843182e-03 1.873485e-03 1.903788e-03 1.934091e-03 1.964394e-03  
## [66] 1.994697e-03 2.025000e-03 2.055303e-03 2.085606e-03 2.115909e-03  
## [71] 2.146212e-03 2.176515e-03 2.206818e-03 2.237121e-03 2.267424e-03  
## [76] 2.297727e-03 2.328030e-03 2.358333e-03 2.388636e-03 2.418939e-03  
## [81] 2.449242e-03 2.479545e-03 2.509848e-03 2.540152e-03 2.570455e-03  
## [86] 2.600758e-03 2.631061e-03 2.661364e-03 2.691667e-03 2.721970e-03  
## [91] 2.752273e-03 2.782576e-03 2.812879e-03 2.843182e-03 2.873485e-03  
## [96] 2.903788e-03 2.934091e-03 2.964394e-03 2.994697e-03 3.025000e-03

tibble(coorelation = rho, mean=mu\_p, variance=var )

## # A tibble: 100 x 3  
## coorelation mean variance  
## <dbl> <dbl> <dbl>  
## 1 -1 0.03 0.000025   
## 2 -0.980 0.03 0.0000553  
## 3 -0.960 0.03 0.0000856  
## 4 -0.939 0.03 0.000116   
## 5 -0.919 0.03 0.000146   
## 6 -0.899 0.03 0.000177   
## 7 -0.879 0.03 0.000207   
## 8 -0.859 0.03 0.000237   
## 9 -0.838 0.03 0.000267   
## 10 -0.818 0.03 0.000298   
## # ... with 90 more rows

#### Question 4.9

# Weight of minimum-variance portfolio  
w = function(s1, s2, rho){  
 w1 = (s2^2-rho\*s1\*s2)/(s1^2+s2^2 - 2\*rho\*s1\*s2)  
}  
   
  
# Portfolio variance   
vp = function(w, s1, s2, rho){  
 v = w^2\*s1^2 + (1-w)^2\* s2^2 + 2\*w\*(1-w)\*rho\*s1\*s2  
}  
  
sigma1 = 0.1  
sigma2 = 0.05  
  
   
  
rho\_12 = 0.4  
  
# Weights  
wt1 = w(sigma1,sigma2,rho\_12)  
   
  
   
  
# Portfolio's minimum variance  
v = vp(wt1,sigma1,sigma2,rho\_12)  
  
   
  
tibble(wt1, Var = v) %>%  
 round(7)

## # A tibble: 1 x 2  
## wt1 Var  
## <dbl> <dbl>  
## 1 0.0588 0.00247

#### Question 4.9 another method

sigma1 = 0.1  
sigma2 = 0.05  
  
  
  
rho\_12 = 0.4  
q = rho\_12\*sigma2\*sigma1  
q

## [1] 0.002

w = seq(0,1,0.01)  
  
  
   
x = w^2\*sigma1^2 + (1-w)^2\*sigma2^2 + 2\*q\*(1-w)\*w  
  
w

## [1] 0.00 0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09 0.10 0.11 0.12 0.13 0.14  
## [16] 0.15 0.16 0.17 0.18 0.19 0.20 0.21 0.22 0.23 0.24 0.25 0.26 0.27 0.28 0.29  
## [31] 0.30 0.31 0.32 0.33 0.34 0.35 0.36 0.37 0.38 0.39 0.40 0.41 0.42 0.43 0.44  
## [46] 0.45 0.46 0.47 0.48 0.49 0.50 0.51 0.52 0.53 0.54 0.55 0.56 0.57 0.58 0.59  
## [61] 0.60 0.61 0.62 0.63 0.64 0.65 0.66 0.67 0.68 0.69 0.70 0.71 0.72 0.73 0.74  
## [76] 0.75 0.76 0.77 0.78 0.79 0.80 0.81 0.82 0.83 0.84 0.85 0.86 0.87 0.88 0.89  
## [91] 0.90 0.91 0.92 0.93 0.94 0.95 0.96 0.97 0.98 0.99 1.00

x

## [1] 0.00250000 0.00249085 0.00248340 0.00247765 0.00247360 0.00247125  
## [7] 0.00247060 0.00247165 0.00247440 0.00247885 0.00248500 0.00249285  
## [13] 0.00250240 0.00251365 0.00252660 0.00254125 0.00255760 0.00257565  
## [19] 0.00259540 0.00261685 0.00264000 0.00266485 0.00269140 0.00271965  
## [25] 0.00274960 0.00278125 0.00281460 0.00284965 0.00288640 0.00292485  
## [31] 0.00296500 0.00300685 0.00305040 0.00309565 0.00314260 0.00319125  
## [37] 0.00324160 0.00329365 0.00334740 0.00340285 0.00346000 0.00351885  
## [43] 0.00357940 0.00364165 0.00370560 0.00377125 0.00383860 0.00390765  
## [49] 0.00397840 0.00405085 0.00412500 0.00420085 0.00427840 0.00435765  
## [55] 0.00443860 0.00452125 0.00460560 0.00469165 0.00477940 0.00486885  
## [61] 0.00496000 0.00505285 0.00514740 0.00524365 0.00534160 0.00544125  
## [67] 0.00554260 0.00564565 0.00575040 0.00585685 0.00596500 0.00607485  
## [73] 0.00618640 0.00629965 0.00641460 0.00653125 0.00664960 0.00676965  
## [79] 0.00689140 0.00701485 0.00714000 0.00726685 0.00739540 0.00752565  
## [85] 0.00765760 0.00779125 0.00792660 0.00806365 0.00820240 0.00834285  
## [91] 0.00848500 0.00862885 0.00877440 0.00892165 0.00907060 0.00922125  
## [97] 0.00937360 0.00952765 0.00968340 0.00984085 0.01000000

which(x==min(x))

## [1] 7

min\_var = x[7]  
wt = w[7]  
tibble(wt1 = wt , Var = min\_var) %>%  
 round(7)

## # A tibble: 1 x 2  
## wt1 Var  
## <dbl> <dbl>  
## 1 0.06 0.00247

tangWt = function(rf, m1, m2, s1, s2, rho){  
 num = (m1-rf)\*s2^2 - (m2-rf)\*rho\*s1\*s2  
 denom = (m2-rf)\*s1^2 + (m1-rf)\*s2^2 - ((m1-rf) + (m2-rf))\*rho\*s1\*s2  
 w = num/denom  
}

#### Question 4.14

mu1 = 0.05  
mu2 = 0.02  
sigma1 = 0.1  
sigma2 = 0.05  
rho\_12 = 0.4  
rf = 0.005  
  
wt\_tan = tangWt(rf,mu1,mu2,sigma1,sigma2,rho\_12)  
tibble(asset1\_wt = wt\_tan, asset2\_wt = (1-wt\_tan)) %>% round(3)

## # A tibble: 1 x 2  
## asset1\_wt asset2\_wt  
## <dbl> <dbl>  
## 1 0.579 0.421