HW7 Question 3

Efficient Portfolio Theory

3/19/2021

pacman::p\_load(pacman, tidyverse, tseries, knitr)  
knitr::opts\_chunk$set(message = FALSE)

##Question 3

# mean vector  
mu = c(0.04, 0.08, 0.02, 0.10, 0.03, 0.06)   
  
# covariance matrix  
Sigma = matrix(   
 c(0.2, 0.02, 0.03, 0.04, 0.05, 0.06,  
 0.02, 0.5, 0.06, 0.08, 0.1, 0.12 ,   
 0.03, 0.06, 0.2, 0.12, 0.15, 0.18,  
 0.04, 0.08, 0.12, 0.8, 0.2, 0.24,   
 0.05, 0.1, 0.15, 0.2 , 1.2, 0.3,  
 0.06, 0.12, 0.18, 0.24, 0.3, 0.8),   
 byrow = T, nrow = 6)  
  
#a  
# minimum-risk frontier  
A = cbind(c(1,1,1,1,1,1),mu)  
t(A)

## [,1] [,2] [,3] [,4] [,5] [,6]  
## 1.00 1.00 1.00 1.0 1.00 1.00  
## mu 0.04 0.08 0.02 0.1 0.03 0.06

library(quadprog)   
  
mrf1 = solve.QP(Dmat = 2\*Sigma, # 2 x covariance matrix  
 dvec = mu, # avg return  
 Amat = A, # constraint coefficients  
 bvec = c(1, 0.04), # values of constraints  
 meq = 2) # treat 2 constraints as qualities  
  
wts = mrf1$solution  
wts

## [1] 0.439528705 0.144725437 0.393804096 0.045170464 0.005286312  
## [6] -0.028515014

mean = sum(mrf1$solution \* mu)  
std\_dev = (mrf1$solution %\*% Sigma %\*% mrf1$solution) ^ 0.5  
  
tibble(mean, std\_dev) %>%  
 round(4)

## # A tibble: 1 x 2  
## mean std\_dev[,1]  
## <dbl> <dbl>  
## 1 0.04 0.321

#b  
# minimum-variance portfolio.  
# find unnormalized weights  
w01 = solve(Sigma, c(1,1,1,1,1,1))  
w01

## [1] 4.2705163 1.3539048 3.9472209 0.3837381 0.0599900 -0.2991168

# normalize the weights  
w\_mv1 = w01/sum(w01)  
w\_mv1

## [1] 0.43952295 0.13934433 0.40624928 0.03949445 0.00617419 -0.03078520

#c  
#tangency portfolio,  
w\_T = solve(Sigma, mu-0.01)/sum(solve(Sigma, mu-0.01))  
w\_T #weight vector of tangency portfolio

## [1] 0.43984460 0.44012824 -0.28938995 0.35676215 -0.04345489 0.09610984

tibble(weight\_vector\_of\_tangency\_portfolio = w\_T)

## # A tibble: 6 x 1  
## weight\_vector\_of\_tangency\_portfolio  
## <dbl>  
## 1 0.440   
## 2 0.440   
## 3 -0.289   
## 4 0.357   
## 5 -0.0435  
## 6 0.0961

#This tangency portfolio has the Sharpe ratio  
sum(w\_T\*mu)/(w\_T%\*%Sigma%\*%w\_T)^.5

## [,1]  
## [1,] 0.1669602